Introduction to data analysis (CMM020)

- Statement for Audio and Video Learning Resources
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- If you require accurate captions as part of your reasonable adjustments, please contact the Inclusion Centre to discuss your requirements.

Introduction to Data Analysis

Ref: D. Diez, M. Cetinkaya-Rundel, C. Barr, *OpenIntro Statistics* (4th Edition), OpenIntro, 2019 [available for download at www.openintro.org/book/os/, accessed 27/01/2020]

Content

- Data
- Probability basics
- Statistics
  - Descriptive statistics for univariate distributions

Data

- Nominal
  - values are symbolic, e.g. desk, table, bed, wardrobe
  - no relation between nominal values
  - Boolean attributes are a special case
    - 0 and 1 or True and False
  - also called categorical, enumerated or discrete
- Ordinal
  - values are ordered, e.g. small, medium, large, x-large
  - small < medium < large < x-large
  - but difference between 2 values is not meaningful

… data

- Interval – Discrete quantitative data
  - Quantities are ordered
  - measured in fixed equal units
    - E.g. years 2001, 2002, 2003, 2004
  - difference between values meaningful: 2005 - 2004
    - but sum or product is not always meaningful: 2005 + 2004
- Ratio – Continuous quantitative data
  - Depth of the North sea in different areas …
  - 1.233m, 100.785m, …

Nominal vs. Ordinal

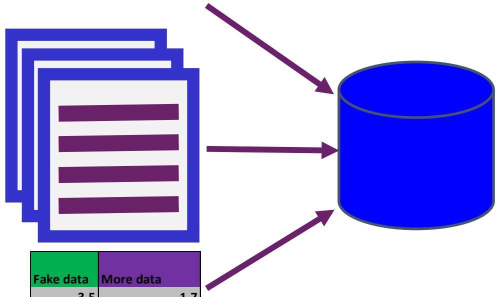
- Questionnaire responses as nominal
  - Strongly Disagree
  - Disagree
  - Neither agree nor disagree
  - Agree
  - Strongly agree

- Questionnaire responses as ordinal
  - Strongly Disagree < Disagree < Neither agree nor disagree
  - Neither agree nor disagree < Agree < Strongly agree

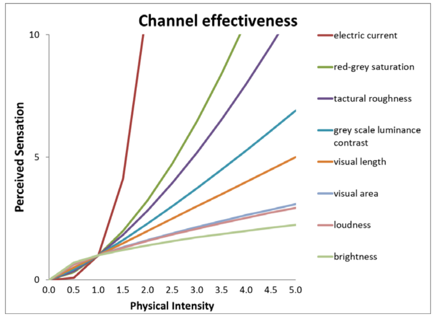
Data Processes

- Collection
  - From one or more sources
  - In one or more formats
  - Probably over a period of time
- Preparation
  - Get the right data into the right format for the desired analysis
- Analysis and visualisation
  - Make sense of the data

	Electric	Red-grey saturation	Tactual roughness	Grey scale luminance contrast
n=	3.5	1.7	1.5	1.2
0	0	0	0	0
0.5	0.088388	0.307786103	0.353553391	0.435275282
1	1	1	1	1
1.5	4.133514	1.99230186	1.837117307	1.626707657
2	11.31371	3.249009585	2.828427125	2.29739671
2.5	24.70529	4.747861206	3.952847075	3.002811085
3	46.76537	6.47300784	5.196152423	3.737192819
3.5	80.21178	8.412317884	6.547900427	4.49657305
4	128	10.55606329	8	5.278031643
4.5	193.3053	12.89618556	9.545941546	6.079320173
5	279.5085	15.42584657	11.18033989	6.898648307



Fake data	More data
3.5	1.7
0	0
0.088388	0.307786103
1	1
4.133514	1.99230186
11.31371	3.249009585
24.70529	4.747861206
46.76537	6.47300784
80.21178	8.412317884
128	10.55606329



Data collection

- From one or more sources
- In one or more formats
- Probably over a period of time
- May use both internal and external sources
  - E.g. sales data (internal to company)
    weather data (external, e.g. from met office)
- Collection itself may need to be planned.

Data preparation

- Preparing data for analysis and visualisation is difficult and demanding
  - Data may need assembling, integrating, aggregating and cleaning
    - if data set is huge, a sample may be used
    - Sample must be representative
- Various types of data may be used
  - Nominal and numeric are most common

… data preparation

- **Wrangling** - transforming data into another format to make it more suitable and valuable for a task
- **Cleansing** (cleaning) - detecting and correcting errors in the data.
- **Scraping** - automatic extraction of data from a data source.
- **Integration** - combining data from several disparate sources into a (useful) dataset.
    - Often challenging due to differences in practices

      E.g. measures by week vs. measures by month

Missing data

- Missing data may be unknown, unrecorded, irrelevant
- Causes
  - Equipment faults
  - Difficult to acquire (e.g. age, income)
  - Measurement is not possible
- The fact that a value is missing may be informative
  - e.g. missing test in medical examination
  - BUT this is NOT usually the case
- Represented in R as *NA*

… missing data

- What action is taken when data is missing?
  - Depends on the data
    - Ignore record
    - Data imputation
      - Replace by average, median, mode (see below)
      - Use machine learning techniques to derive likely value.
        - E.g. same value as most “similar” record

Inaccurate / incorrect values

- **Noise / outliers**
- Errors and omissions which do not affect original purpose of data collection
  - E.g. age of bank customers not important
  - E.g. Customers IDs not important
- Typographical errors in nominal attributes
  - e.g. Pepsi vs Pepsi-cola
- Deliberate errors
  - People may lie about their mental health history
  - Companies may downplay their impact on the environment
- Duplicates
  - Analysis may be very sensitive to this
- Outlier – sudden vibration causes sensor misreading.

Data analysis

- Make sense of data
  - Data exploration
    - Initial examination of data
    - Usually with target questions in mind
  - Data analysis and visualisation

… data analysis

- **Inference**
  - Using sample data to draw conclusions about a population.
- **Prediction**
  - Forecast - future values from past data.
  - Estimate - unseen values from seen values.
- **Causal Analysis (cause – effect)**
  - Does x cause y?
  - Hard!
  - E.g. in London people using umbrellas lived longer
    - Umbrella → longer life? …
    - It not because people use umbrellas, it is because they are wealthier than others
- Present the results – often visualisations
  - To a variety of audiences.
- Visualisation may be used for all tasks

Analysis and Visualisation

- Although taught separately, they are part of the same process
- Visualisation guides analysis and helps ascertain whether results of analysis make sense.
- Visualisation and summary statistics very useful for
  - Exploratory Data Analysis
  - Presentation of Results

Data Analysis and Probability

- What is the most likely explanation of the data?
- If something appears in the data – how likely is it to be real, rather than coincidence?
- Questions like these lead to discussion of **probability**
  - A quantitative expression of how likely some event is.
- Consequently, any statistical analysis involves probability

Probability

- P(A) is the probability of an event A
  - If A is certain to occur, $\boldsymbol{P(A) = 1}$
  - If A cannot occur, $\boldsymbol{P(A) = 0}$
  - Otherwise, $\boldsymbol{0 \leq P(A) \leq 1}$
  - Often converted to percentages, from 0% to 100%
- Variations of the Probability Concept
  - **Classical Probability** involves counting the ways in which an event can occur (often cards, dice, selecting *m* from *n*,…)
  - **Empirical Probability** is estimated from experimental evidence (e.g. 3 successes in 5 trials [3/5] equals 0.6)
  - **Subjective Probability** is an individual's estimate (guessing!)

Probability rules

- Complementary Events
  - Assume A is an event and $\bar{A}$ (not A) is event A not happening

$$\boldsymbol{P(A) + P(\bar{A}) = 1}$$

- General Rule for Combination of Events
  - For events A and B

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B) \leq 1}$$

- Mutually Exclusive Events
  - If events A and B cannot occur together then

$$\boldsymbol{P(A\ and\ B) = 0}$$
$$\boldsymbol{P(A\ or\ B) = P(A) + P(B) \leq 1}$$

Conditional Probability

- Conditional Probability
  - **The probability of A given** that B has happened is $P(A|B)$
  - E.g. Probability of a hypothesis being true, given the evidence
    - $P(A|B) = P(A \text{ and } B)/P(B)$
    - $P(A \text{ and } B) = P(A|B) * P(B) = P(B|A) * P(A)$

- Independent Events
  - The probability of A is unaffected by the success or failure of B
    - Observing B tells us nothing about A
  - Then A and B are **independent** events
    - $P(A|B) = P(A|\bar{B}) = P(A)$

$\bar{B}$ means "not B", i.e. B is false.

Conditional Probability Example

- A Geologist estimates that the probabilities of striking oil at locations A and B are
  - $P(A) = 0.6$
  - $P(B) = 0.5$
- If the events were independent, then
  - $P(A \text{ and } B) = 0.6 * 0.5 = 0.3$
- But the geologist estimates
  - $P(A \text{ and } B) = 0.4$
- A strike at one location increases the probability of a strike at the other
  - $P(A|B) = P(A \text{ and } B)/P(B) = 0.4/0.5 = 0.8$
  - $P(B|A) = P(A \text{ and } B)/P(A) = 0.4/0.6 = 0.667$

Probability Distributions

- Take an experiment or trial with exactly n possible (mutually exclusive) outcomes
- Then the probabilities of these outcomes sum to one
- The set of probabilities is a discrete probability distribution
- Important standard examples:
  - Binomial (total successes in repeated trials, n=2)
  - [Poisson]
  - Uniform – all probabilities equal to 1/n

Continuous Probability Distributions

- A variable x can take any value in an interval
- $p(x)$ is a probability density function describing the relative probability of values of x
- $P(a < x < b)$ is given by the area under the graph of $p(x)$ between a and b
  - The integral, if you've done calculus
- The total area under the graph is one
- Important Standard Examples
  - Normal (Gaussian or Bell Curve)
  - Uniform over a finite interval e.g. [0,1]

Descriptive Statistics

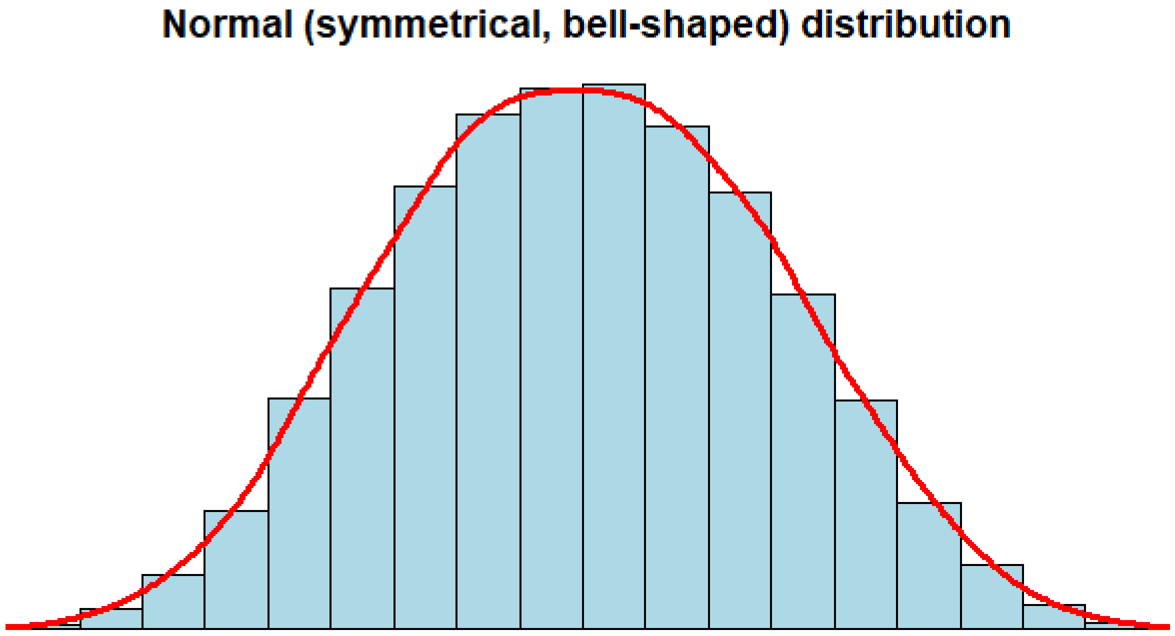
- Visualise Data
  - Tables
  - Charts
- Summarise Data
  - Measures of Location (Averages)
  - Measures of Dispersion (Spread)
  - And many more…
- Main Uses
  - Exploratory Data Analysis
  - Presenting Results

Univariate Distributions

- To examine a univariate (1-D or single variable) distribution:
  - Tabulate the **frequency or relative frequency** of each distinct value
  - Construct a (relative) frequency bar chart or a histogram
- Calculate summary statistics to indicate typical values and spread of values

Normal distribution

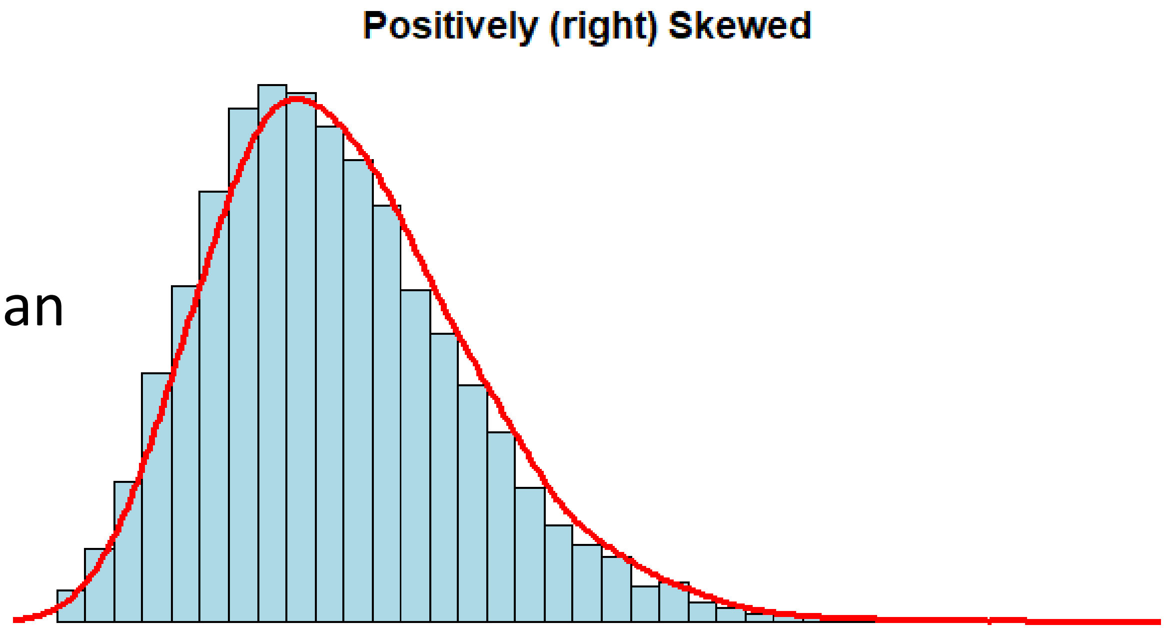
- Normal distribution
  - Gaussian
  - Bell shaped
  - mean=median
- The height indicates the number of values falling in that range.
- Most frequent value(s) – top of bell



Skewed distribution - right

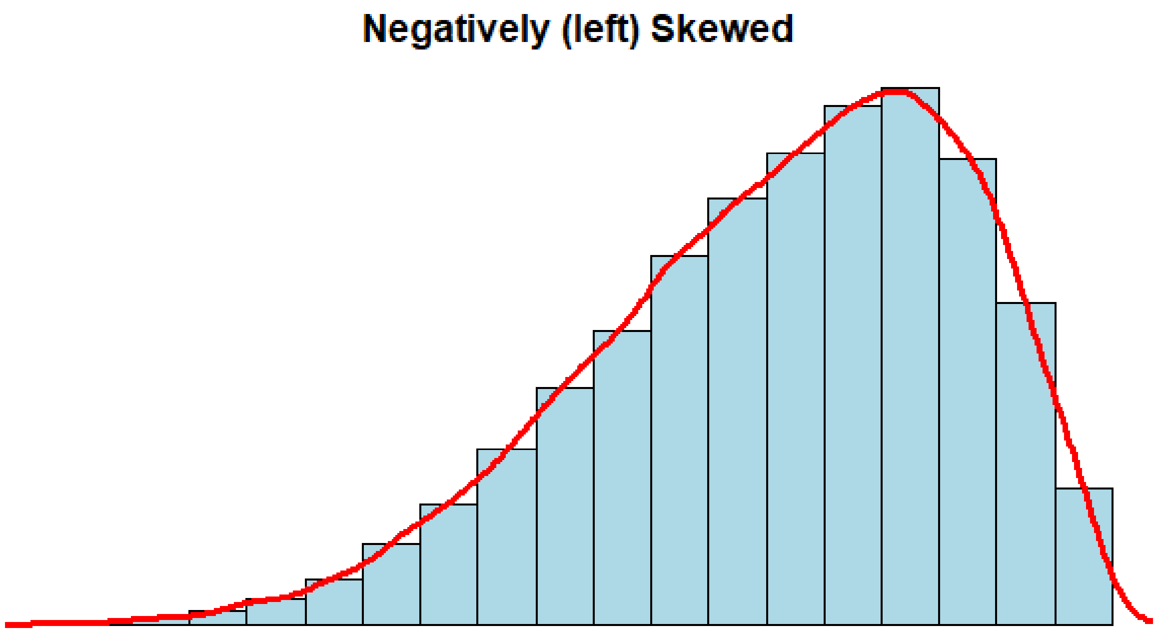
Right (Positive) Skew

- RH tail is longer or fatter
- Top half of data is further from median than bottom half
- Mean to the right of median



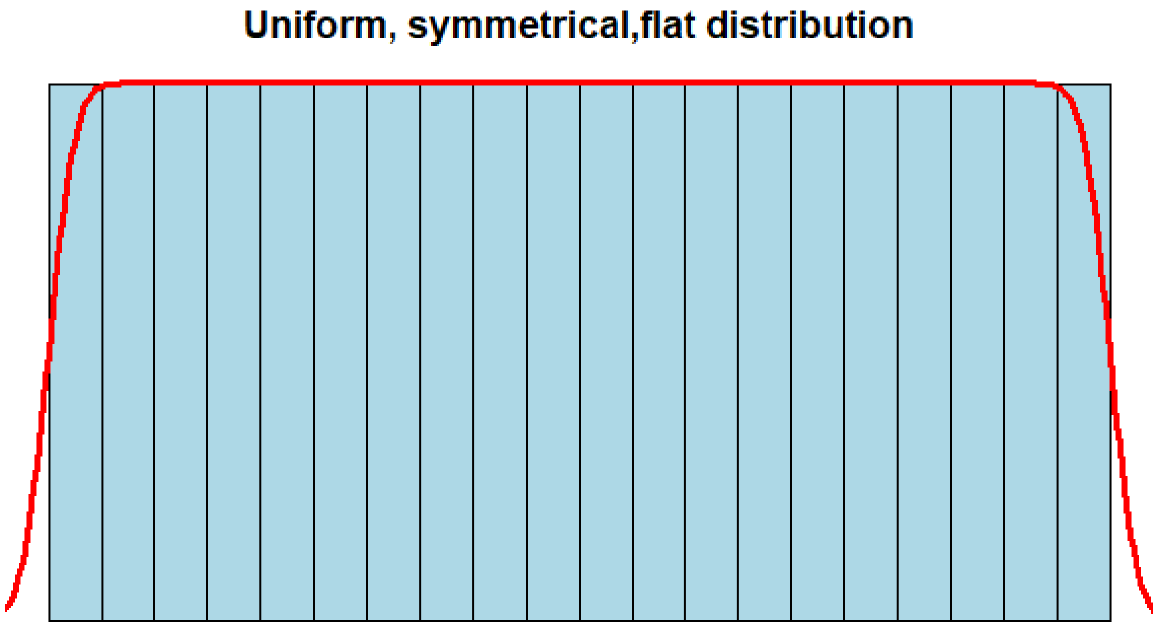
Skewed distribution - left

- Left (negative) skew
  - LH tail is longer or fatter
  - Bottom half of data is further from median than top half
  - Mean to the left of median



Uniform, symmetrical distribution

All values have similar frequency



Statistical Models

- Statistical procedures are usually based on a statistical model of the data and the underlying population.
- These models have probability distributions as inputs and outputs.
- Typically, random variables take values drawn from given probability distributions
- The normal distribution is particularly important.

Commonly Encountered Datasets

- **One-dimensional distribution** (univariate – one column)
  - E.g. Average daily time online – one piece of data per user
- **Two-dimensional distribution** (bivariate – two columns)
  - Number of accesses to website vs. total time spent on it
- **Time Series Data** (one of the dimensions is time)
  - Accesses to a website throughout the year
- **Rectangular Multivariate Data**
  - Several measurements on each class member
  - Can be viewed as a table or treated as a matrix

Univariate (one variable) Distributions

- To examine a univariate (1-D or single variable) distribution:
  - Tabulate the **frequency or relative frequency** of each distinct value
  - Construct a (relative) frequency bar chart or a histogram
- Calculate summary statistics to indicate typical values and spread of values
- Measures divided into
  - Measures of location
  - Measures of dispersion

Measures of Location

- Measures of central tendency
  - Typical value
    - But what is typical?
  - Mean (average) $\bar{x}$
    - Add up data and divide by number of items
  - Median
    - Middle value of ordered data
  - Mode
    - Most common data value
- Quartiles Q1, Q2, Q3
  - Divide data into 4 equally sized groups

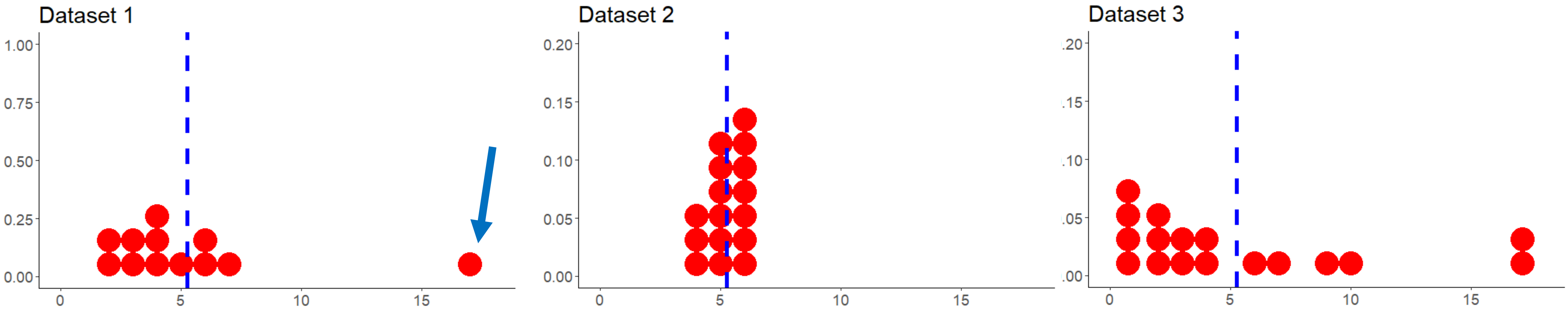
Mean (μ or $\bar{x}$)

- Sum of values divided by number of values

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Same mean, different picture

- Dataset 1: 2, 2, 3, 4, 5, 4, 6, 7, 3, 17, 4, 6 → mean 5.25
- Dataset 2: 4, 4, 5, 5, 6, 6, 5, 5, 5, 6, 4, 5, 6, 6, 6, 6 → mean 5.25
- Dataset 3: 0.5,0.5,1,1,2,2,3,4,10,2 ,7,3,17,4,6,9, 17.25 → mean 5.25

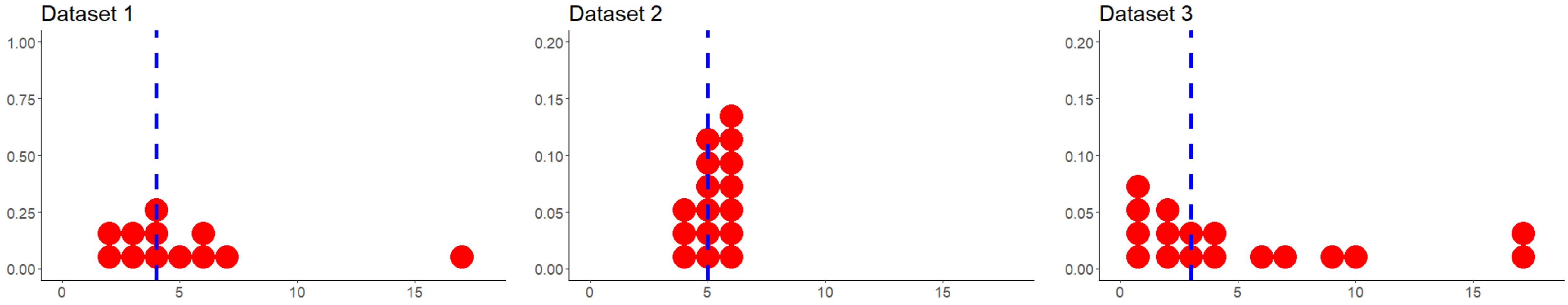


Median (η)

- Median = “(N+1)/2”th value
  - N=7, median is value at position(7+1)/2, i.e. 4th value
  - N=10, median is 5.5th value,
    - so average 5th and 6th values
  - Data must be in order
  - For a frequency distribution, we need to count cumulative frequency up to middle number
- Can also be defined for ordinal data, but not nominal data

… Median example

- Dataset 1: 2, 2, 3, 4, 5, 4, 6, 7, 3, 17, 4, 6 → median 4
- Dataset 2: 4, 4, 5, 5, 6, 6, 5, 5, 5, 6, 4, 5, 6, 6, 6, 6 → median 5
- Dataset 3: 0.5,0.5,1,1,2,2,3,4,10,2 ,7,3,17,4,6,9, 17.25 → median 3



Mode

- Most frequent value
- Disadvantages
  - May be several values of equal frequency
  - May not be near middle of distribution
  - In many applications, mode is not useful
- However, does apply to all data types, including nominal
- E.g. Browser users – frequency counts

chrome	edge	firefox	other	safari	samsung Int.
173	9	16	19	54	9

Mode is chrome, as it is the most frequent value.

Raw data example

- 3, 7, 5, 2, 4, 5, 3, 8, 3, 7
  - N = 10
- Mean

$$\bar{x} = \frac{3+7+2+5+4+5+3+8+3+7}{10} = \frac{47}{10} = 4.7$$

- Median
  - Order data: 2, 3, 3, 3, **4, 5**, 5, 7, 7, 8
  - Take value at position (10+1)/2 = 5.5
  - So average 5^{th} and 6^{th} values
    - 5.5^{th} value = (4+5)/2 = 4.5
- Mode is 3

Frequency Distribution – one-way table

Values	x_i	5	10	15	20
Frequency counts	f_i	13	8	6	9

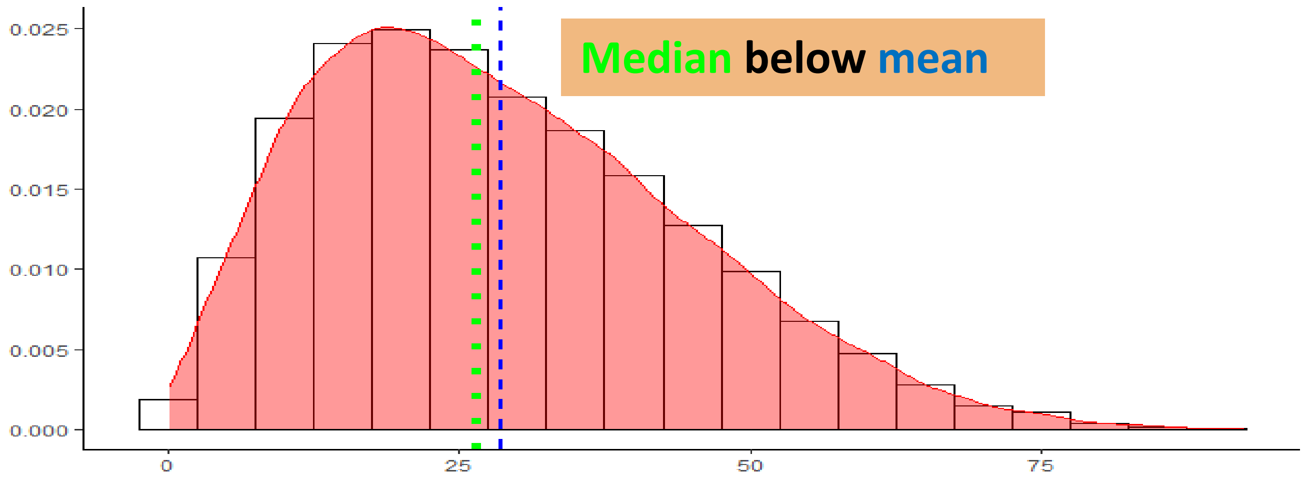
Value 20 appears 9 times

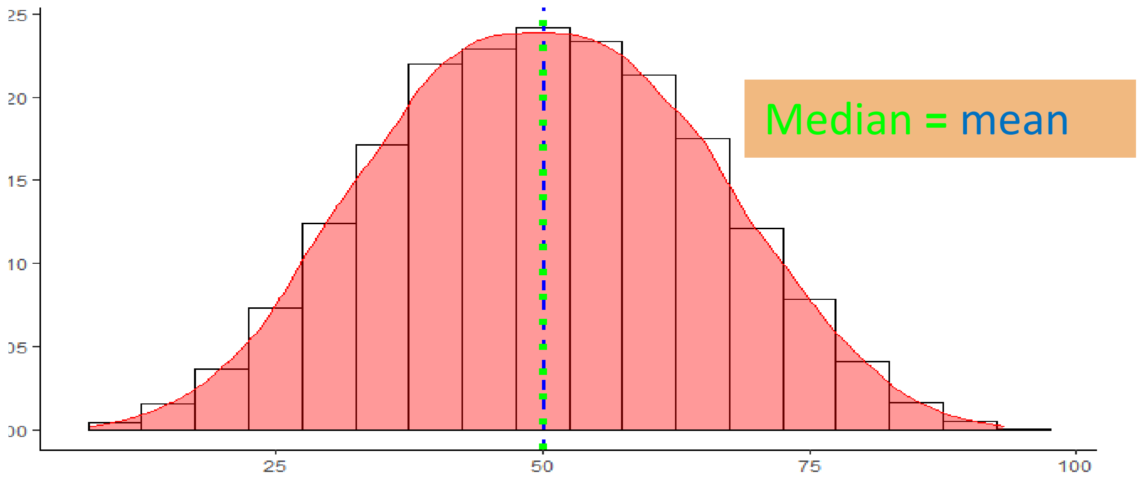
13+8+6+9 = 36 data values

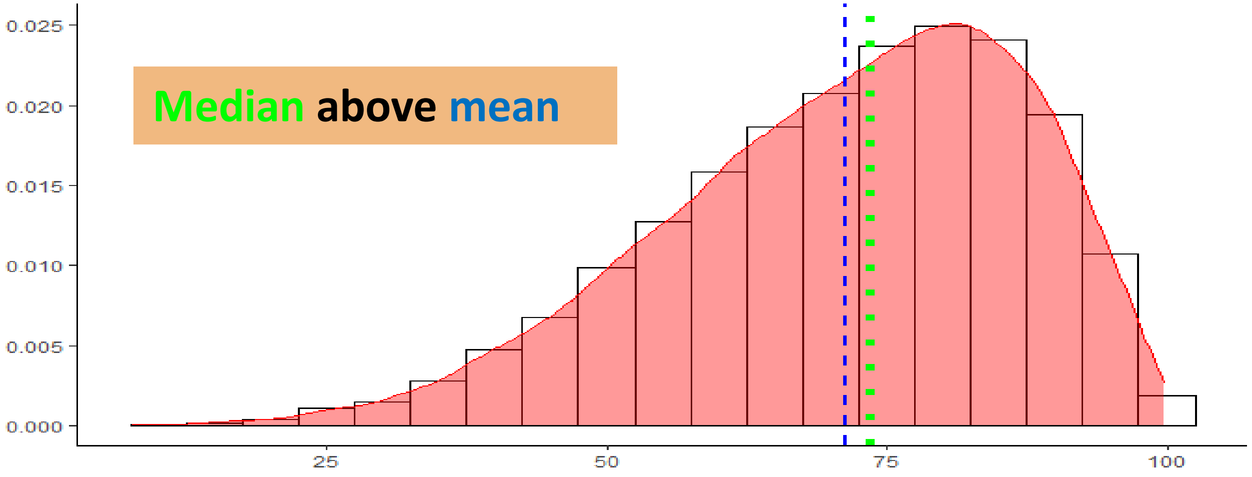
$$\bar{x} = \frac{13\times 5 + 8\times 10 + 6\times 15 + 9\times 20}{13+8+6+9} = \frac{415}{36} = 11.53$$

- Median = (37/2)th value = 18.5th value
  - 18th and 19th are both 10, so median is 10
- Mode is 5

Distributions vs mean and median values







Measures of Dispersion

- Range
- Inter-quartile range (IQR)
- Standard deviation and variance
- 5 point summary - boxplots

Quartiles

- The quartile provide information about the location of the data
  - Q1 is the median of bottom half of data (25% data is equal or below Q1)
  - Q2 is the median for the data (50% of data is equal or below Q2)
  - Q3 is the median of top half (75% data is equal or below Q3)
  - [There are some differences in the way these are calculated depending on method, leading to different results]
- Q1, Q2, Q3 Divide data in four
  - Confusingly, the quarters are sometimes called quartiles
- Used with minimum and maximum value to provide 5-point description of data
  - Min, Q1, Q2, Q3, Max
  - See below boxplots

Quantiles

- **Quartiles** can be generalised to any number of divisions
- Dividing values are **Quantiles**
- **Percentiles** divide the data into 100 equal parts
- **Deciles** divide the data into 10 equal parts

- Balance of detail vs effective summary

Measures of Dispersion (spread)

- **Range:** maximum value – minimum value
- **Inter-quartile range:** range of middle half, Q3 – Q1
  - Often paired with median
- **5- Number Summary {**Min, Q1, Q2, Q3, Max**}**
- **Standard deviation**
  - Pairs with mean
  - **σ** = population SD
  - **S** = sample SD

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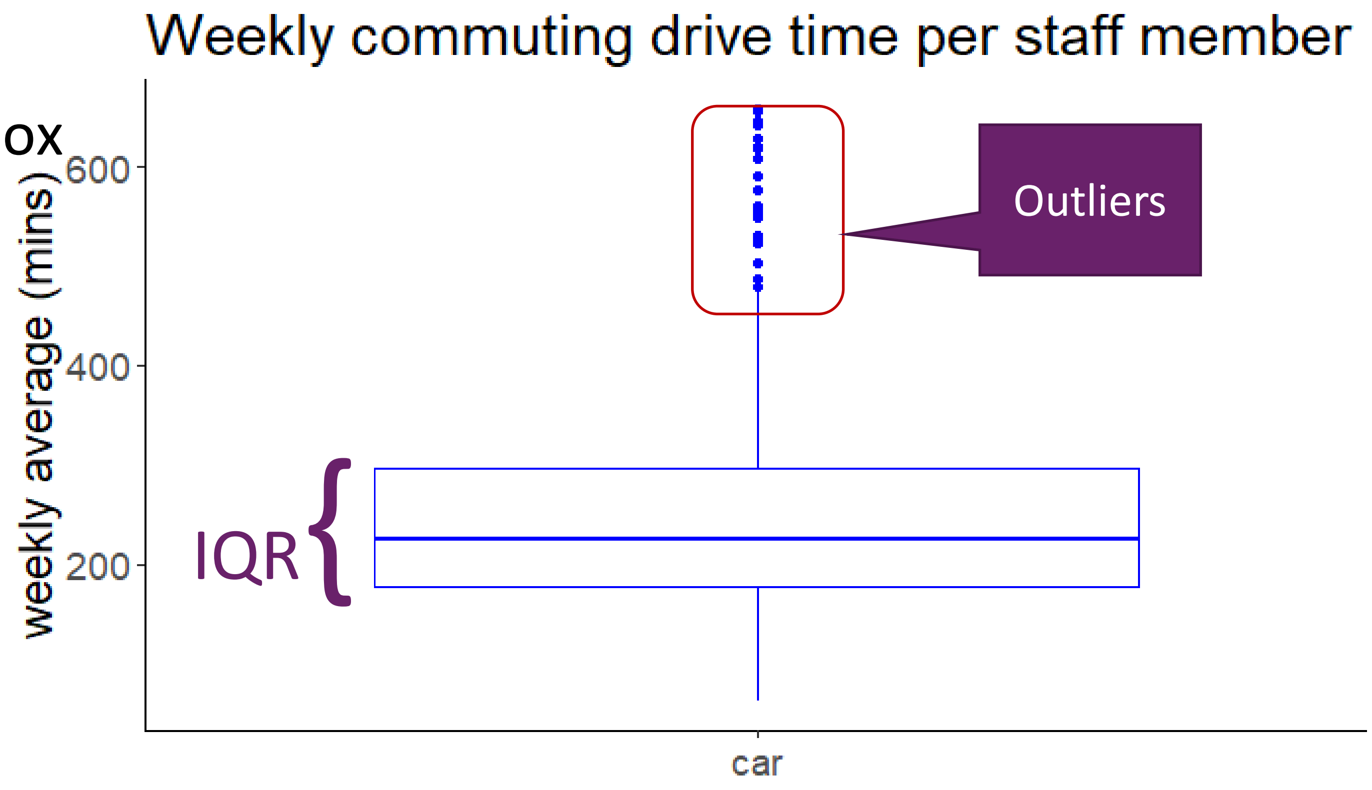
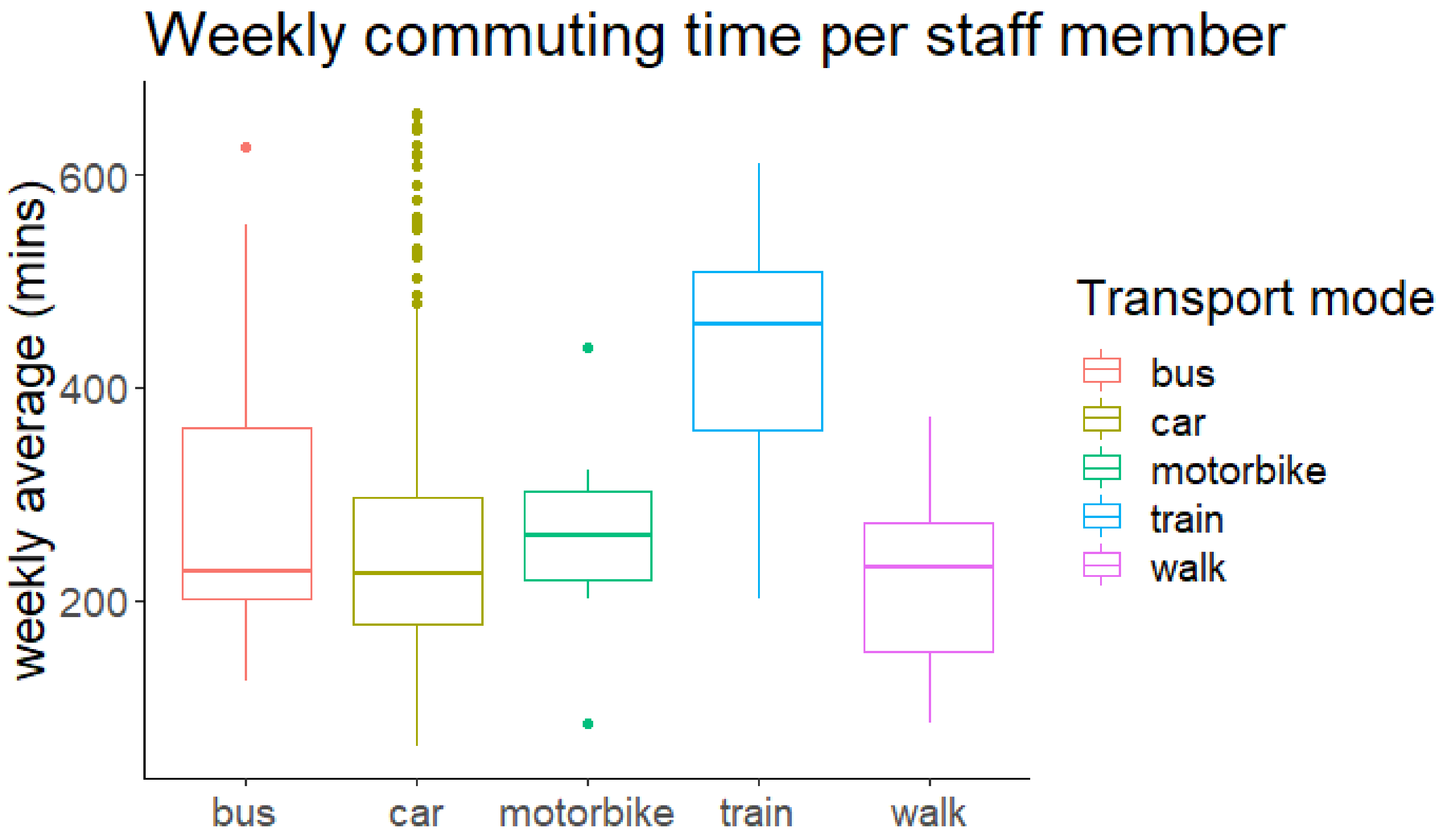
Boxplot

- **IQR** is difference between bottom and top of each box (blue area).
- **Q2 = Median** is horizontal bar in each box
- **Q1** is bottom of box
- **Q3** is top of box
- **Min** is bottom of bottom line (if no outliers)
- **Max** is top of top line (if no outliers)
- Outliers indicated by dots
- **5 point summary**

  {min, Q1,median, Q3, max}



Boxplot example



Outliers

- Unusually high/low value compared to the rest of the data in the dataset
  - Data incorrectly recorded (needs correction or removal)
  - Data should not have been included in dataset (does not belong - remove)
  - Data is correct and belongs in the dataset, but is unusual – skewness
    - Retain?
- Outliers
  - Below **lower inner fence**: Q1 – 1.5 * IQR
  - Above **upper inner fence**: Q3 + 1.5 * IQR

IQR and 5 number summary example

- Data
  - 3, 7, 5, 2, 4, 5, 3, 8, 3, 7
- Sorted data
  - 2, 3 ,3, 3, 4, 5, 5, 7, 7, 8
- Range = max – min = 8 - 2 = 6
- Median (Q2) = (4 + 5)/2 = 4.5
- Q1 = 3
- Q3 = 7
- IQR = 7 – 3 = 4
- 5-number summary {2,3,4.5,7,8}

Standard deviation

- Indicates spread of data
- Low value
  - Data is close to the mean
- Large value
  - Data is spread more
- Is a low SD value better than a high SD value?
  - Depends on the problem domain
  - Just because the data is more disperse it does not mean it is worse.
    - E.g. Time of the day at which each student starts using computer resources
    - Ideally start times are spread (up to a point) throughout the day so there is more resource availability per student -> higher SD

Standard Deviation: population and sample

Population standard deviation σ_N

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{N}}$$

Sample standard deviation $s = \sigma_{N-1}$

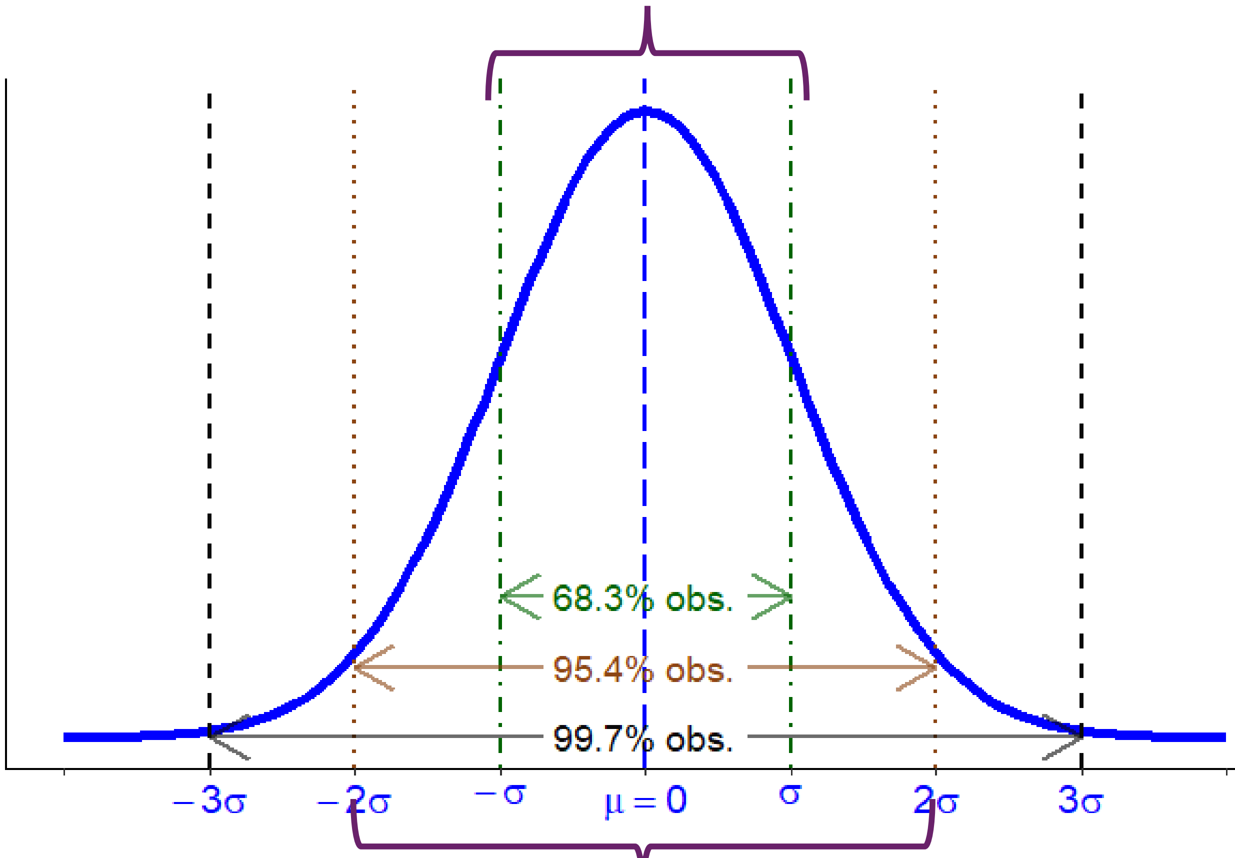
$$\sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{N-1}}$$

- σ is for entire **population**
- s is for **sample** of larger population

Interpreting σ for a normal distribution

- For a normal distribution
  - **~68%** of observations will fall within 1 standard deviation of the mean
  - **~95%** of observations will fall within 2 standard deviations of the mean
  - almost **all** observations will fall within 3 standard deviations of the mean

One SD from mean 68% data



Two SDs from mean 95% data

Interpretation of sample standard deviation

- It is possible that no observations fall within one standard deviation of the mean
  - In $[\bar{x} - \sigma, \ \bar{x} + \sigma]$

- At least **3/4** of observations (75%) fall within 2 standard deviations of the mean
  - In $[\bar{x} - 2\sigma, \ \bar{x} + 2\sigma]$

- At least **8/9** of observations (89%) fall within 3 standard deviations of the mean.
  - In $[\bar{x} - 3\sigma, \ \bar{x} + 3\sigma]$

Standard deviation example

- Data
  - 3, 7, 5, 2, 4, 5, 3, 8, 3, 7 with $\bar{x} = \mathbf{4.7}$

$$\sigma_N = \sqrt{\frac{(3-4.7)^2 + (7-4.7)^2 + \ldots + (7-4.7)^2}{10}}$$

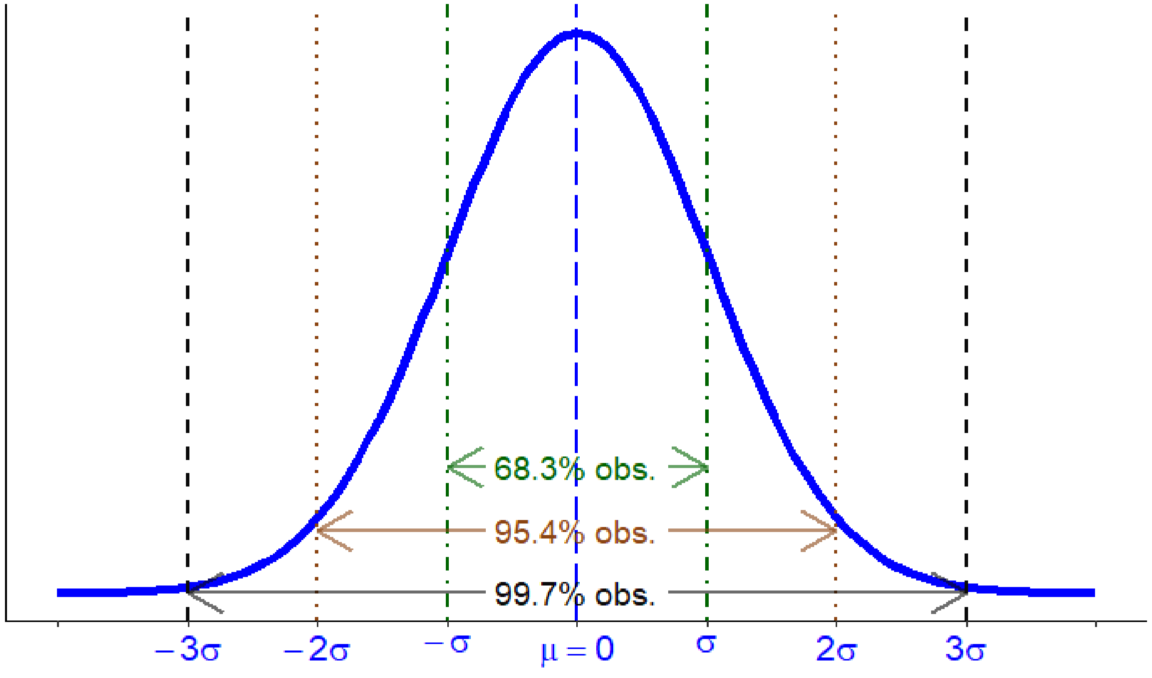
$$= \sqrt{\frac{38.1}{10}} = \sqrt{3.81} = 1.95$$

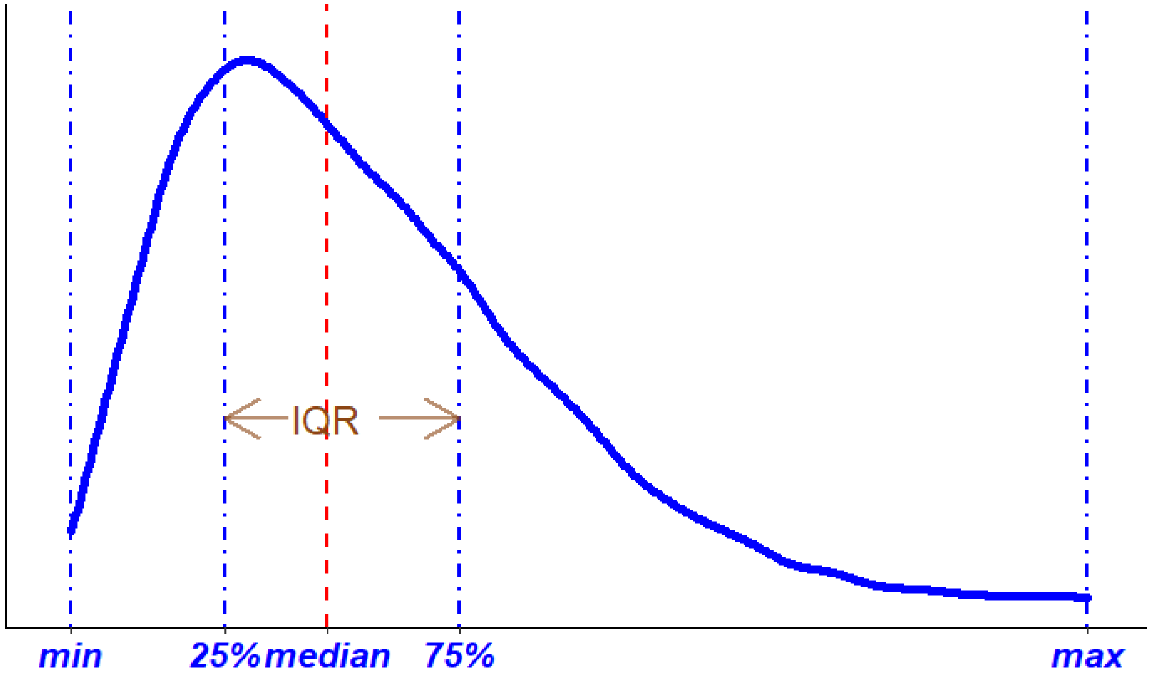
Variance vs Standard Deviation

- Variance = σ^2
- Sample Variance = s^2

What measures?

- For **normal distributions**
  - Summarise data using **mean** and **standard deviation**.
- For **skewed distributions**
  - Summarise data using **median** and **interquartile range** as they are more representative of data centre and spread.
  - But also use mean and standard deviation in addition.
  - Or five number.





Summary

- Data analysis depends on types of data
- Data cannot be analysed without 2 previous processes
  - Collection
  - Preparation
- Basics of probability and stats can tell important information about data
- Important measures are
  - Mean
  - Median
  - Mode
  - Range
  - Standard deviation
  - Interquartile range
  - 5-point summary {min, Q1, Median, Q3, max}