Introducing Neural Networks

Neural networks, also called **Artificial Neural Networks** (though it seems, in recent years, we've dropped the "artificial" part), are a type of machine learning often conflated with deep learning. The defining characteristic of a *deep* neural network is having two or more **hidden layers** — a concept that will be explained shortly, but these hidden layers are ones that the neural network controls. It's reasonably safe to say that most neural networks in use are a form of deep learning.

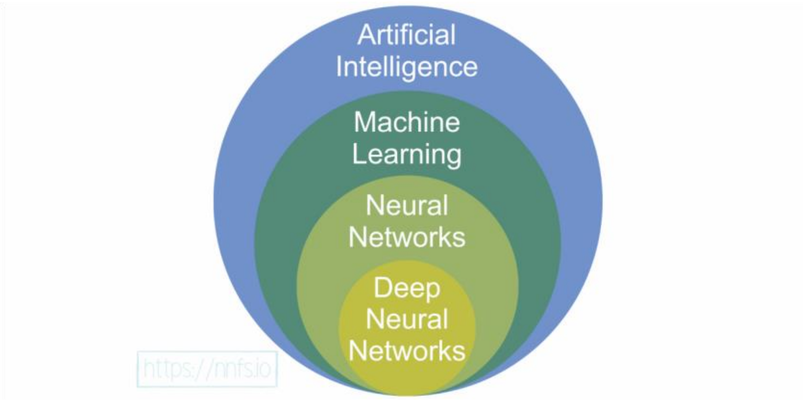


Fig. Depicting the various fields of artificial intelligence and where they fit in overall.

A Brief History

Since the advent of computers, scientists have been formulating ways to enable machines to take input and produce desired output for tasks like **classification** and **regression**. Additionally, in general, there's **supervised** and **unsupervised** machine learning. Supervised machine learning is used when you have pre-established and labeled data that can be used for training. Let's say you have sensor data for a server with metrics such as upload/download rates, temperature, and humidity, all organized by time for every 10 minutes. Normally, this server operates as intended and has no outages, but sometimes parts fail and cause an outage. We might collect data and then divide it into two classes: one class for times/observations when the server is operating normally, and another class for times/observations when the server is experiencing an outage. When the server is failing, we want to label that sensor data leading up to failure as data that preceded a failure. When the server is operating normally, we simply label that data as "normal."

What each sensor measures in this example is called a feature. A group of features makes up a feature set (represented as vectors/arrays), and the values of a feature set can be referred to as a sample. Samples are fed into neural network models to train them to fit desired outputs from these inputs or to predict based on them during the inference phase.

The "normal" and "failure" labels are **classifications** or **labels** You may also see these referred to as **targets** or **ground-truths** while we fit a machine learning algorithm. These targets are the classifications that are the *goal* or *target,* known to be *true and correct*, for the algorithm to learn. For this example, the aim is to eventually train an algorithm to read sensor data and accurately predict when a failure is imminent. This is just one example of supervised learning in the form of classification. In addition to classification, there's also regression, which is used to predict numerical values, like stock prices. There's also unsupervised machine learning, where the machine finds structure in data without knowing the labels/classes ahead of time. There are additional concepts (e.g., reinforcement learning and semi-supervised machine learning) that fall under the umbrella of neural networks.

Neural networks were conceived in the 1940s, but figuring out how to train them remained a mystery for 20 years. The concept of **backpropagation** (explained later) came in the 1960s, but neural networks still did not receive much attention until they started winning competitions in 2010. Since then, neural networks have been on a meteoric rise due to their sometimes seemingly magical ability to solve problems previously deemed unsolvable, such as image captioning, language translation, audio and video synthesis, and more.

Currently, neural networks are the primary solution to most competitions and challenging technological problems like self-driving cars, calculating risk, detecting fraud, and early cancer detection, to name a few.

What is a Neural Network?

"Artificial" neural networks are inspired by the organic brain, translated to the computer. It's not a perfect comparison, but there are neurons, activations, and lots of interconnectivity, even if the underlying processes are quite different.

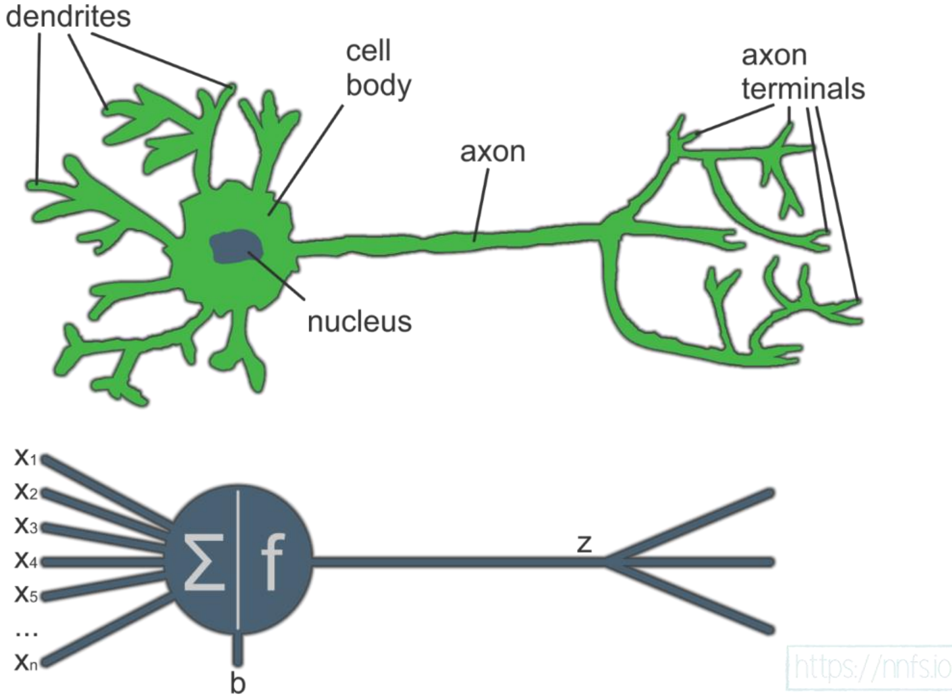
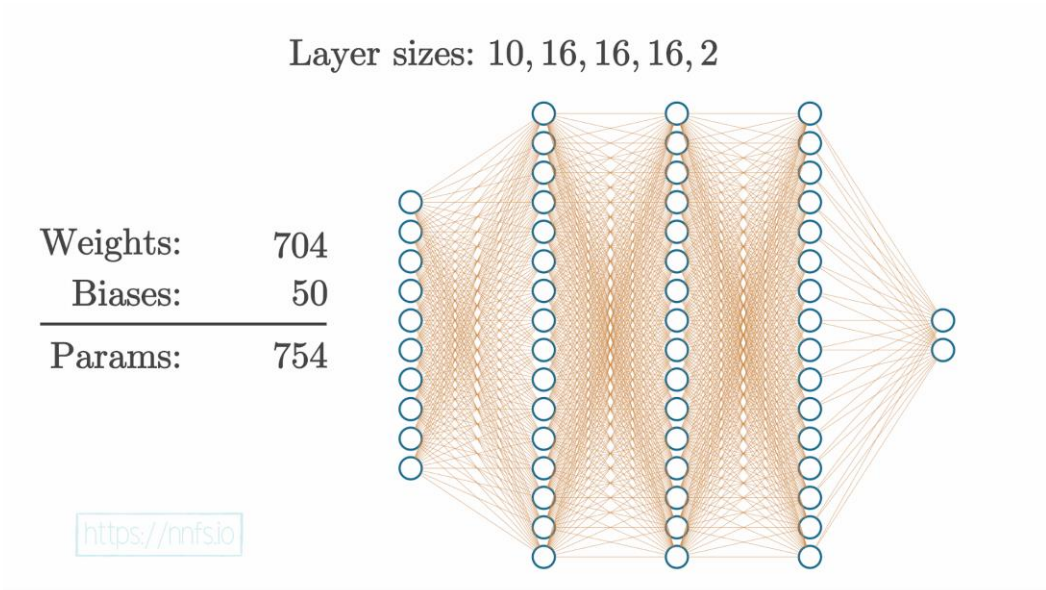


Fig. Comparing a biological neuron to an artificial neuron.

A single neuron by itself is relatively useless, but, when combined with hundreds or thousands (or many more) of other neurons, the interconnectivity produces relationships and results that frequently outperform any other machine learning methods.



It might seem rather complicated when you look at it this way. Neural networks are considered to be "black boxes" in that we often have no idea *why* they reach the conclusions they do. We do understand *how* they do this, though.

Dense layers, the most common layers, consist of interconnected neurons. In a dense layer, each neuron of a given layer is connected to every neuron of the next layer, which means that its output value becomes an input for the next neurons. Each connection between neurons has a weight associated with it, which is a trainable factor of how much of this input to use, and this weight gets multiplied by the input value. Once all of the *inputs·weights* flow into our neuron, they are summed, and a bias, another trainable parameter, is added. The purpose of the bias is to offset the output positively or negatively, which can further help us map more real-world types of dynamic data.

The concept of weights and biases can be thought of as "knobs" that we can tune to fit our model to data. In a neural network, we often have thousands or even millions of these parameters tuned by the optimizer during training. Some may ask, "why not just have biases or just weights?" Biases and weights are both tunable parameters, and both will impact the neurons' outputs, but they do so in different ways. Since weights are multiplied, they will only change the magnitude or even completely flip the sign from positive to negative, or vice versa. *Output = weight·input+bias* is not unlike the equation for a line $y = mx+b$. We can visualize this with:

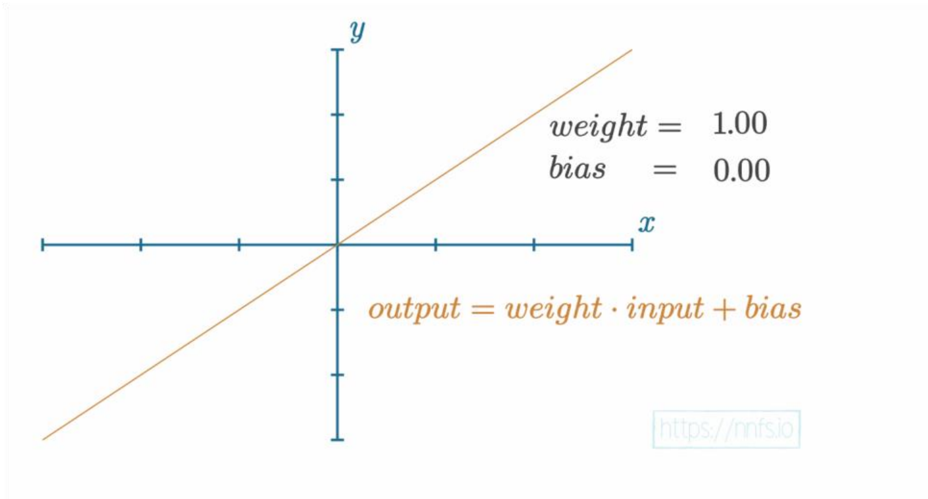


Fig. Graph of a single-input neuron's output with a weight of 1, bias of 0 and input *x*.

Adjusting the weight will impact the slope of the function:

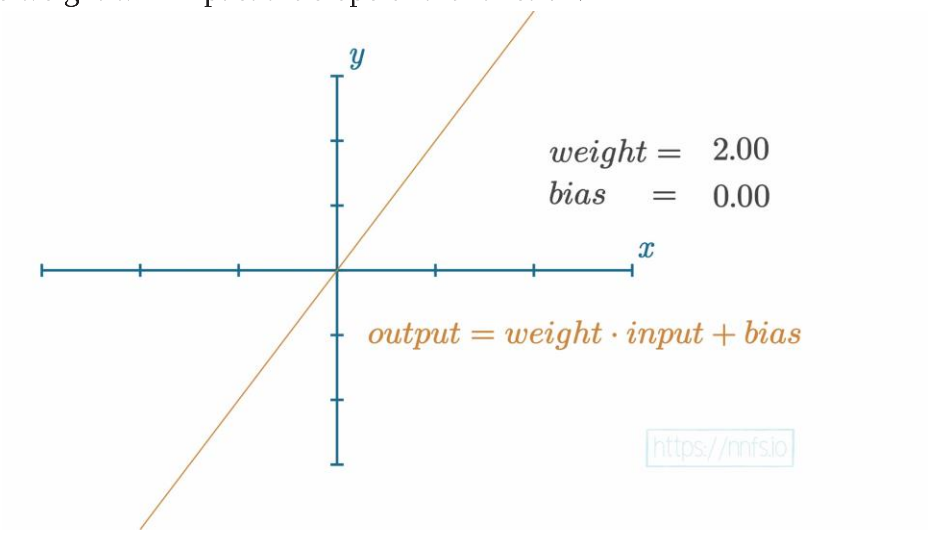


Fig. Graph of a single-input neuron's output with a weight of 2, bias of 0 and input *x*.

As we increase the value of the weight, the slope will get steeper. If we decrease the weight, the slope will decrease. If we negate the weight, the slope turns to a negative:

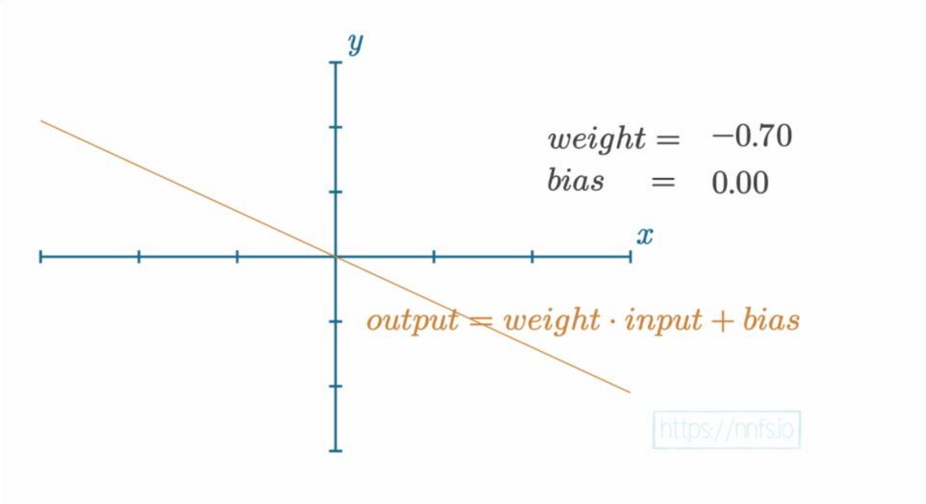


Fig. Graph of a single-input neuron's output with a weight of -0.70, bias of 0 and input *x*.

This should give you an idea of how the weight impacts the neuron's output value that we get from *inputs·weights+bias* Now, how about the bias parameter? The bias offsets the overall function. For example, with a weight of 1.0 and a bias of 2.0:

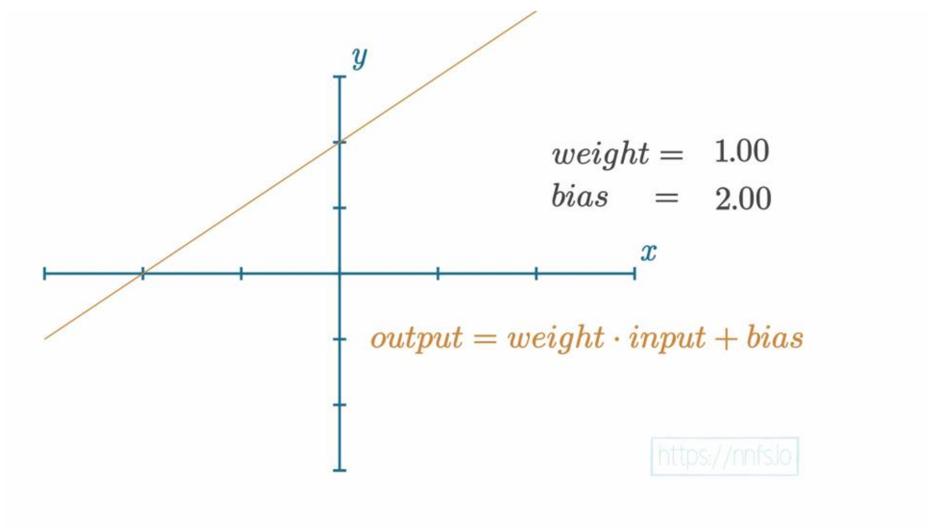


Fig. Graph of a single-input neuron's output with a weight of 1, bias of 2 and input *x*.

As we increase the bias, the function output overall shifts upward. If we decrease the bias, then the overall function output will move downward. For example, with a negative bias:

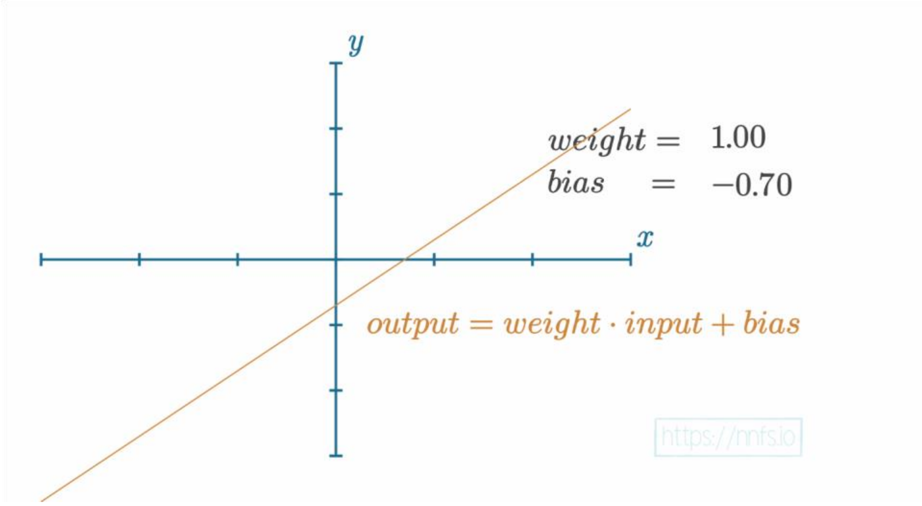


Fig. Graph of a single-input neuron’s output with a weight of 1.0, bias of -0.70 and input *x*.

As you can see, weights and biases help to impact the outputs of neurons, but they do so in slightly different ways. This will make even more sense when we cover **activation functions** in later lessons. Still, you can hopefully already see the differences between weights and biases and how they might individually help to influence output. Why this matters will be conveyed shortly.

As a very general overview, the step function meant to mimic a neuron in the brain, either “firing” or not — like an on-off switch. In programming, an on-off switch as a function would be called a **step function** because it looks like a step if we graph it.

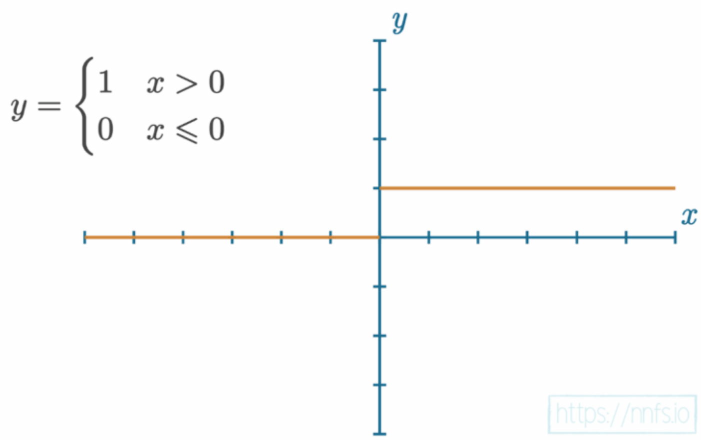


Fig 1.09: Graph of a step function.

For a step function, if the neuron’s output value, which is calculated by *sum(inputs · weights) + bias*, is greater than 0, the neuron fires (so it would output a 1). Otherwise, it does not fire and would pass along a 0. The formula for a single neuron might look something like: `output = sum(inputs * weights) + bias`

We then usually apply an activation function to this output, noted by *activation()*:

```
output = activation(output)
```

While you can use a step function for your activation function, we tend to use something slightly more advanced. Neural networks of today tend to use more informative activation functions (rather than a step function), such as the **Rectified Linear** (ReLU) activation function, which we shall cover later. Each neuron's output could be a part of the ending output layer, as well as the input to another layer of neurons. While the full function of a neural network can get very large, let's start with a simple example with 2 hidden layers of 4 neurons each.

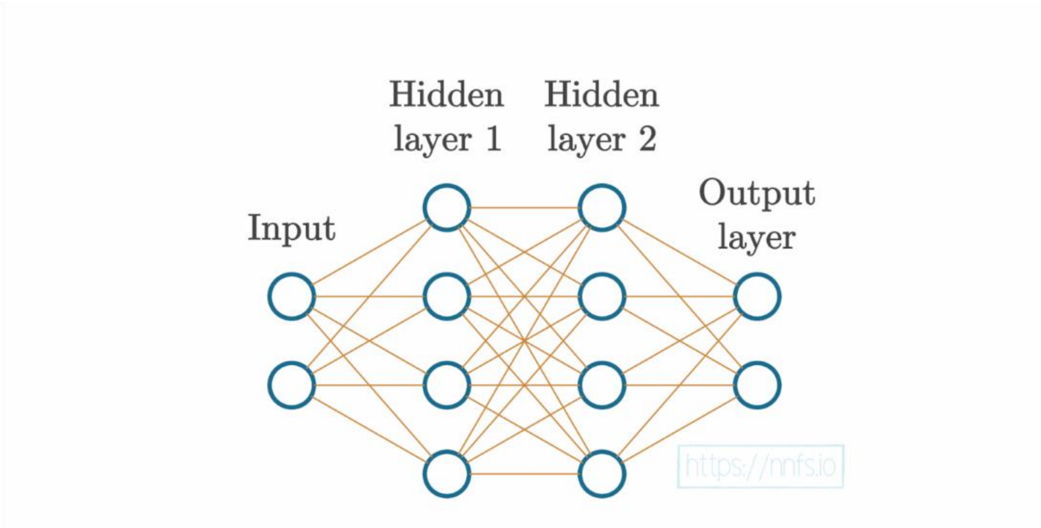


Fig 1.10: Example basic neural network.

Along with these 2 hidden layers, there are also two more layers here — the input and output layers. The input layer represents your actual input data, for example, pixel values from an image or data from a temperature sensor. While this data can be "raw" in the exact form it was collected, you will typically **preprocess** your data through functions like **normalization** and **scaling**, and your input needs to be in numeric form. Concepts like scaling and normalization will be covered later. However, it is common to preprocess data while retaining its features and having the values in similar ranges between 0 and 1 or -1 and 1. To achieve this, you will use either or both scaling and normalization functions. The output layer is whatever the neural network returns. With classification, where we aim to predict the class of the input, the output layer often has as many neurons as the training dataset has classes, but can also have a single output neuron for binary (two classes) classification. We'll discuss this type of model later and, for now, focus on a classifier that uses a separate output neuron per each class. For example, if our goal is to classify a collection of pictures as a “dog” or “cat,” then there are two classes in total. This means our output layer will consist of two neurons; one neuron associated with “dog” and the other with “cat.” You could also have just a single output neuron that is “dog” or “not dog.”

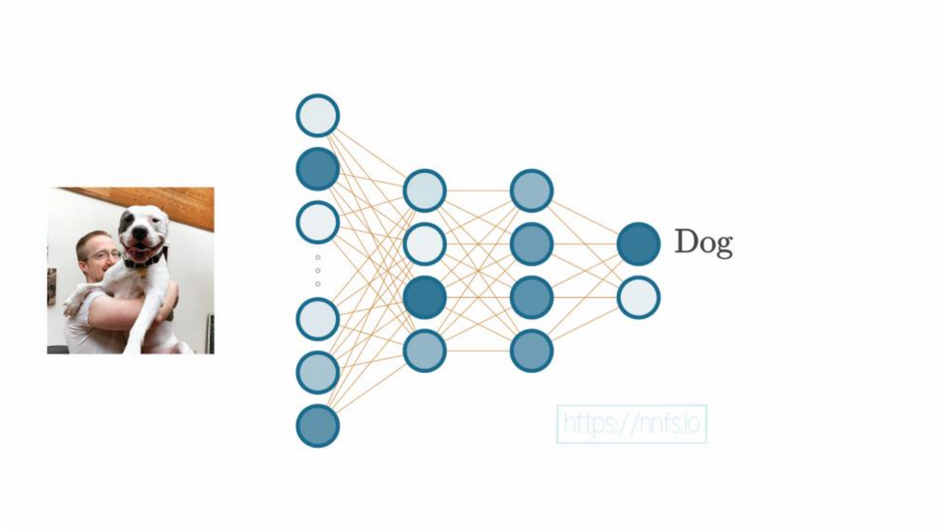


Fig. Visual depiction of passing image data through a neural network, getting a classification

For each image passed through this neural network, the final output will have a calculated value in the “cat” output neuron, and a calculated value in the “dog” output neuron. The output neuron that received the highest score becomes the class prediction for the image used as input.

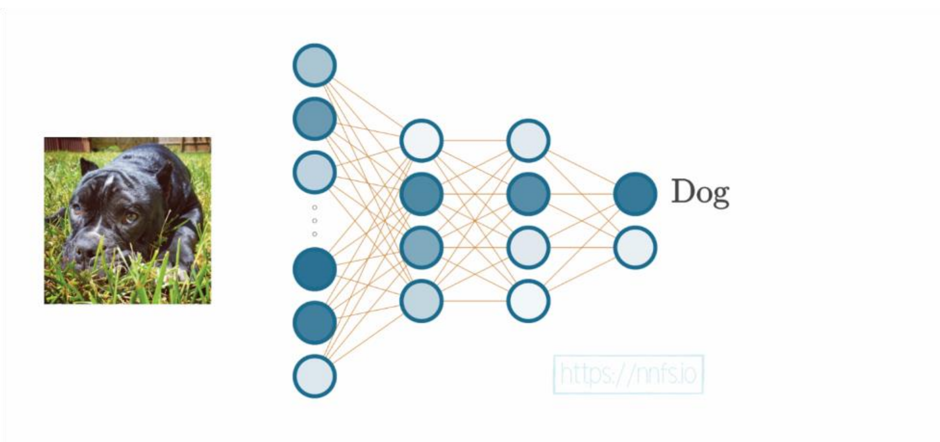


Fig. Visual depiction of passing image data through a neural network, getting a classification

The thing that makes neural networks appear challenging is the math involved and how scary it can sometimes look. For example, let’s imagine a neural network, and take a journey through what’s going on during a simple forward pass of data, and the math behind it. Neural networks are really only a bunch of math equations that we, programmers, can turn into code. For this, do not worry about understanding everything. The idea here is to give you a high-level impression of what's going on overall.

There may be some functions there that you don't understand yet. For example, maybe you do not know what a log function is, but this is something simple that we'll cover. Then we have a sum operation, an exponentiating operation (again, you may not exactly know what this does, but it's nothing hard). Then we have a dot product, which is still just about understanding how it works, there's nothing there that is over your head if you know how multiplication works! Finally, we have some transposes, noted as .T, which, again, once you learn what that operation does, is not a challenging concept. Once we've separated each of these elements, learning what they do and how they work, suddenly, things will not appear to be as daunting or foreign. Nothing in this forward pass requires education beyond basic high school algebra!

A typical neural network has thousands or even up to millions of adjustable **parameters** (weights and biases). In this way, neural networks act as enormous functions with vast numbers of **parameters**. The concept of a long function with millions of variables that could be used to solve a problem isn't all too difficult. With that many variables related to neurons, arranged as interconnected layers, we can imagine there exist some combinations of values for these variables that will yield desired outputs. Finding that combination of parameter (weight and bias) values is the challenging part.

The end goal for neural networks is to adjust their weights and biases (the parameters), so when applied to a yet-unseen example in the input, they produce the desired output. When supervised machine learning algorithms are trained, we show the algorithm examples of inputs and their associated desired outputs. One major issue with this concept is **overfitting** — when the algorithm only learns to fit the training data but doesn't actually "understand" anything about underlying input-output dependencies. The network basically just "memorizes" the training data.

Thus, we tend to use "in-sample" data to train a model and then use "out-of-sample" data to validate an algorithm (or a neural network model in our case). Certain percentages are set aside for both datasets to partition the data. For example, if there is a dataset of 100,000 samples of data and labels, you will immediately take 10,000 and set them aside to be your "out-of-sample" or "validation" data. You will then train your model with the other 90,000 in-sample or "training" data and finally validate your model with the 10,000 out-of-sample data that the model hasn't yet seen. The goal is for the model to not only accurately predict on the training data, but also to be similarly accurate while predicting on the withheld out-of-sample validation data.

This is called **generalization**, which means learning to fit the data instead of memorizing it. The idea is that we "train" (slowly adjusting weights and biases) a neural network on many examples of data. We then take out-of-sample data that the neural network has never been presented with and hope it can accurately predict on these data too.

You should now have a general understanding of what neural networks are, or at least what the objective is, and how we plan to meet this objective. To train these neural networks, we calculate how “wrong” they are using algorithms to calculate the error (called **loss**), and attempt to slowly adjust their parameters (weights and biases) so that, over many iterations, the network gradually becomes less wrong. The goal of all neural networks is to generalize, meaning the network can see many examples of never-before-seen data, and accurately output the values we hope to achieve. Neural networks can be used for more than just classification. They can perform regression (predict a scalar, singular, value), clustering (assign unstructured data into groups), and many other tasks. Classification is just a common task for neural networks.

Coding Neurons

A Single Neuron

Let's say we have a single neuron, and there are three inputs to this neuron. As in most cases, when you initialize parameters in neural networks, our network will have weights initialized randomly, and biases set as zero to start. Why we do this will become apparent later on. The input will be either actual training data or the outputs of neurons from the previous layer in the neural network. We're just going to make up values to start with as input for now:

```
inputs = [1, 2, 3]
```

Each input also needs a weight associated with it. Inputs are the data that we pass into the model to get desired outputs, while the weights are the parameters that we'll tune later on to get these results. Weights are one of the types of values that change inside the model during the training phase, along with biases that also change during training. The values for weights and biases are what get "trained," and they are what make a model actually work (or not work). We'll start by making up weights for now. Let's say the first input, at index 0, which is a 1, has a weight of 0.2, the second input has a weight of 0.8, and the third input has a weight of -0.5. Our input and weights lists should now be:

```
inputs = [1, 2, 3]

weights = [0.2, 0.8, -0.5]
```

Next, we need the bias. At the moment, we're modeling a single neuron with three inputs. Since we're modeling a single neuron, we only have one bias, as there's just one bias value per neuron. The bias is an additional tunable value but is not associated with any input in contrast to the weights. We'll randomly select a value of 2 as the bias for this example:

```
inputs = [1, 2, 3]

weights = [0.2, 0.8, -0.5]

bias = 2
```

This neuron sums each input multiplied by that input's weight, then adds the bias. All the neuron does is take the fractions of inputs, where these fractions (weights) are the adjustable parameters, and adds another adjustable parameter — the bias — then outputs the result. Our output would be calculated up to this point like:

```
output = (inputs[0]*weights[0] + inputs[1]*weights[1] +
inputs[2]*weights[2] + bias)
```

```
print(output)

>>>
2.3
```

The output here should be **2.3**. We will use `>>>` to denote output.

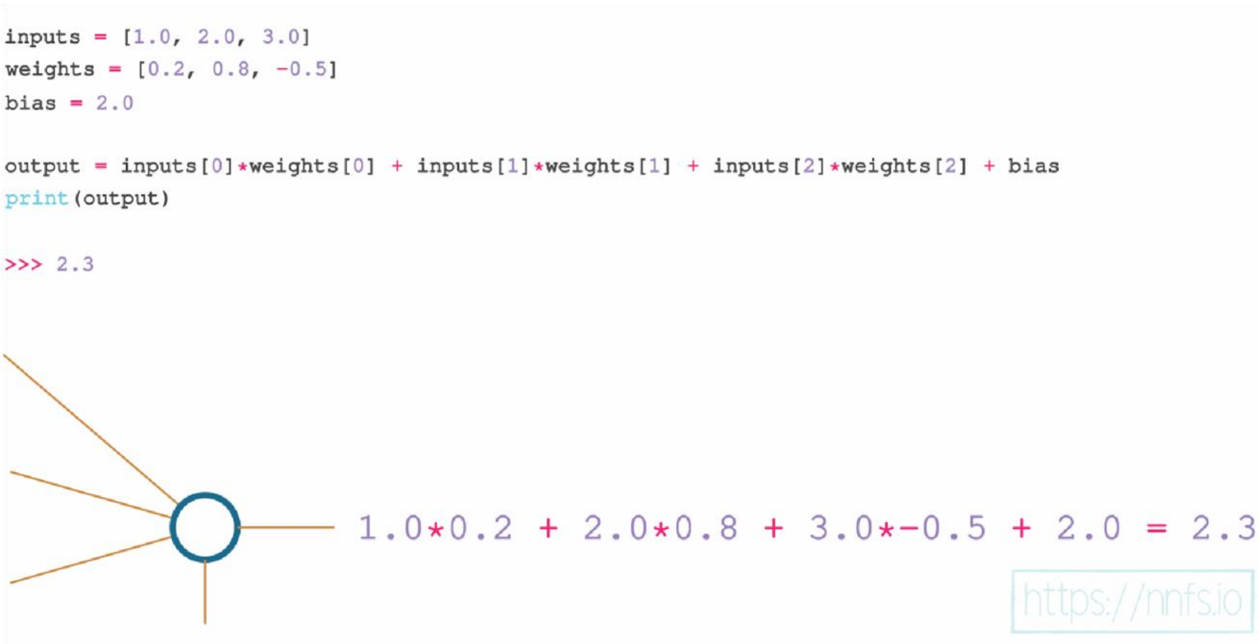


Fig. Visualizing the code that makes up the math of a basic neuron.

What might we need to change if we have 4 inputs, rather than the 3 we've just shown? Next to the additional input, we need to add an associated weight, which this new input will be multiplied with. We'll make up a value for this new weight as well. Code for this data could be:

```
inputs = [1.0, 2.0, 3.0, 2.5]

weights = [0.2, 0.8, -0.5, 1.0] bias = 2.0
```

Which could be depicted visually as:

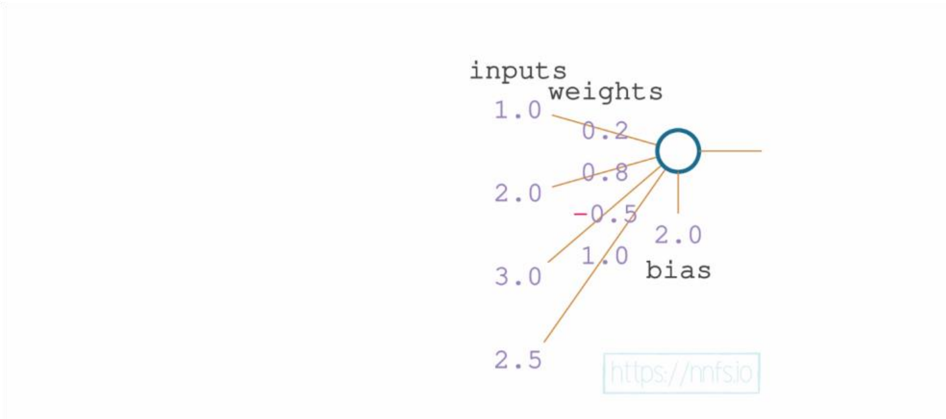


Fig. Visualizing how the inputs, weights, and biases from the code interact with the neuron.

All together in code, including the new input and weight, to produce output:

```
inputs = [1.0, 2.0, 3.0, 2.5]

weights = [0.2, 0.8, -0.5, 1.0]

bias = 2.0

output = (inputs[0]*weights[0]+inputs[1]*weights[1]+
inputs[2]*weights[2] +  inputs[3]*weights[3] + bias)
print(output)

>>>
4.8
```

Visually:

```python
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [0.2, 0.8, -0.5, 1.0]
bias = 2.0

output = inputs[0]*weights[0] + inputs[1]*weights[1] + inputs[2]*weights[2] + inputs[3]*weights[3] + bias
print(output)

>>> 4.8
```

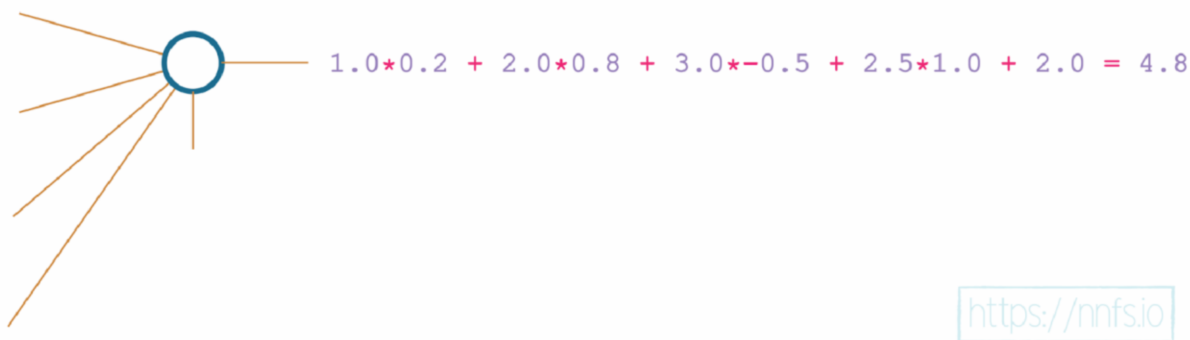


Fig. Visualizing the code that makes up a basic neuron, with 4 inputs this time.

A Layer of Neurons

Neural networks typically have layers that consist of more than one neuron. Layers are nothing more than groups of neurons. Each neuron in a layer takes exactly the same input — the input given to the layer (which can be either the training data or the output from the previous layer), but contains its own set of weights and its own bias, producing its own unique output. The layer's output is a set of each of these outputs — one per each neuron. Let's say we have a scenario with 3 neurons in a layer and 4 inputs:

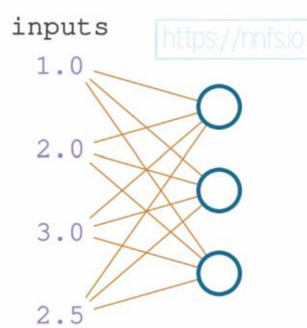


Fig. Visualizing a layer of neurons with common input.

We'll keep the initial 4 inputs and set of weights for the first neuron the same as we've been using so far. We'll add 2 additional, made up, sets of weights and 2 additional biases to form 2 new neurons for a total of 3 in the layer. The layer's output is going to be a list of 3 values, not just a single value like for a single neuron.

```python
inputs = [1, 2, 3, 2.5]
```

```
weights1 = [0.2, 0.8, -0.5, 1]
weights2 = [0.5, -0.91, 0.26, -0.5]
weights3 = [-0.26, -0.27, 0.17, 0.87]
bias1 = 2
bias2 = 3 bias3 = 0.5
outputs = [          # Neuron 1
inputs[0]*weights1[0] +  inputs[1]*weights1[1] + inputs[2]*weights1[2] +
inputs[3]*weights1[3] + bias1,
# Neuron 2:
inputs[0]*weights2[0] + inputs[1]*weights2[1] + inputs[2]*weights2[2] +
inputs[3]*weights2[3] + bias2,
        # Neuron 3:
inputs[0]*weights3[0] + inputs[1]*weights3[1] + inputs[2]*weights3[2] +
inputs[3]*weights3[3] + bias3
]
print(outputs)
>>>
[4.8, 1.21, 2.385]
```

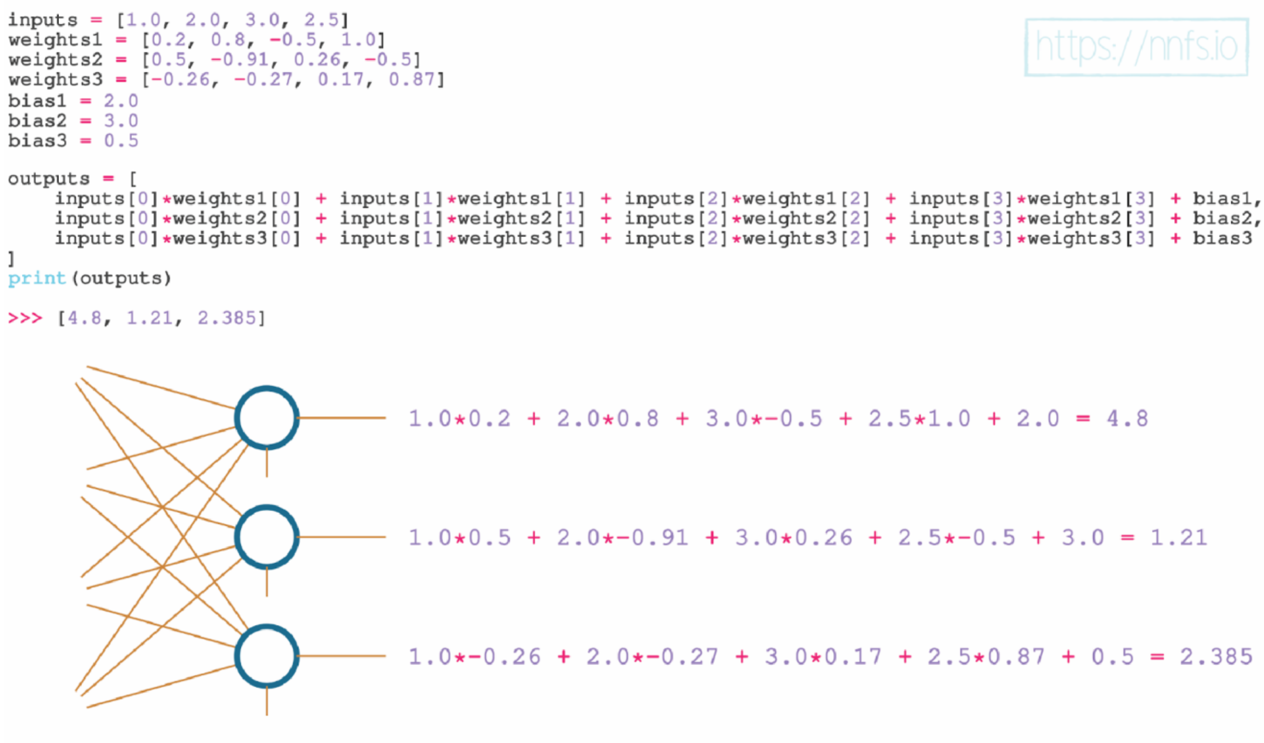


Fig. Code, math and visuals behind a layer of neurons.

In this code, we have three sets of weights and three biases, which define three neurons. Each neuron is "connected" to the same inputs. The difference is in the separate weights and bias that each neuron applies to the input. This is called a **fully connected** neural network — every neuron in the current layer has connections to every neuron from the previous layer. This is a very common type of neural network, but it should be noted that there is no requirement to fully connect everything like this. At this point, we have only shown code for a single layer with very few neurons. Imagine coding many more layers and more neurons. This would get very challenging to code using our current methods. Instead, we could use a loop to scale and handle dynamically-sized inputs and layers. We've turned the separate weight variables into a list of weights so we can iterate over them, and we changed the code to use loops instead of the hardcoded operations.

```
inputs = [1, 2, 3, 2.5]
weights = [[0.2, 0.8, -0.5, 1],
           [0.5, -0.91, 0.26, -0.5],
           [-0.26, -0.27, 0.17, 0.87]]
biases = [2, 3, 0.5]

# Output of current layer
layer_outputs = []
# For each neuron
for neuron_weights, neuron_bias in zip(weights, biases):
    # Zeroed output of given neuron
    neuron_output = 0
    # For each input and weight to the neuron
    for n_input, weight in zip(inputs, neuron_weights):
        # Multiply this input by associated weight
        # and add to the neuron's output variable
        neuron_output += n_input*weight
    # Add bias
    neuron_output += neuron_bias
    # Put neuron's result to the layer's output list
    layer_outputs.append(neuron_output)

print(layer_outputs)
```

```
>>>
[4.8, 1.21, 2.385]
```

This does the same thing as before, just in a more dynamic and scalable way. If you find yourself confused at one of the steps, `print()` out the objects to see what they are and what's happening. The `zip()` function lets us iterate over multiple iterables (lists in this case) simultaneously. Again, all we're doing is, for each neuron (the outer loop in the code above, over neuron weights and biases), taking each input value multiplied by the associated weight for that input (the inner loop in the code above, over inputs and weights), adding all of these together, then adding a bias at the end. Finally, sending the neuron's output to the layer's output list.

That's it! How do we know we have three neurons? Why do we have three? We can tell we have three neurons because there are 3 sets of weights and 3 biases. When you make a neural network of your own, you also get to decide how many neurons you want for each of the layers. You can combine however many inputs you are given with however many neurons that you desire.

With our above code that uses loops, we could modify our number of inputs or neurons in our layer to be whatever we wanted, and our loop would handle it. As we said earlier, it would be a disservice not to show NumPy here since Python alone doesn't do matrix/tensor/array math very efficiently. But first, the reason the most popular deep learning library in Python is called "TensorFlow" is that it's all about doing operations on **tensors**.

Tensors, Arrays and Vectors

What are "tensors?"

Tensors are *closely-related to* arrays. If you interchange tensor/array/matrix when it comes to machine learning, people probably won't give you too hard of a time. But there are subtle differences, and they are primarily either the context or attributes of the tensor object. To understand a tensor, let's compare and describe some of the other data containers in Python (things that hold data). Let's start with a list. A Python list is defined by comma-separated objects contained in brackets. So far, we've been using lists.

This is an example of a simple list:

```
l = [1,5,6,2]
```

A list of lists:

```
lol = [[1,5,6,2],
       [3,2,1,3]]
```

A list of lists of lists!

```
lolol = [[[1,5,6,2],
          [3,2,1,3]],
         [[5,2,1,2],
          [6,4,8,4]],
         [[2,8,5,3],
          [1,1,9,4]]]
```

Everything shown so far could also be an array or an array representation of a tensor. A list is just a list, and it can do pretty much whatever it wants, including:

```
another_list_of_lists = [[4,2,3],
                         [5,1]]
```

The above list of lists cannot be an array because it is not **homologous**. A list of lists is homologous if each list along a dimension is identically long, and this must be true for each dimension. In the case of the list shown above, it's a 2-dimensional list. The first dimension's length is the number of sublists in the total list (2). The second dimension is the length of each of those sublists (3, then 2). In the above example, when reading across the "row" dimension (also called the second dimension), the first list is 3 elements long, and the second list is 2 elements long — this is not homologous and, therefore, cannot be an array. While failing to be consistent in one dimension is enough to show that this example is not homologous, we could also read down the "column" dimension (the first dimension); the first two columns are 2 elements long while the third column only contains 1 element. Note that every dimension does not necessarily need to be the same length; it is perfectly acceptable to have an array with 4 rows and 3 columns (i.e., 4x3).

A matrix is pretty simple. It's a rectangular array. It has columns and rows. It is two dimensional. So a matrix can be an array (a 2D array). Can all arrays be matrices? No. An array can be far more than just columns and rows, as it could have four dimensions, twenty dimensions, and so on.

```
list_matrix_array = [[4,2],
                     [5,1],
                     [8,2]]
```

The above list could also be a valid matrix (because of its columns and rows), which automatically means it could also be an array. The "shape" of this array would be 3x2, or more formally described as a shape of *(3, 2)* as it has 3 rows and 2 columns.

To denote a shape, we need to check every dimension. As we've already learned, a matrix is a 2-dimensional array. The first dimension is what's inside the most outer brackets, and if we look at the above matrix, we can see 3 lists there: `[4,2]`, `[5,1]`, and `[8,2]`; thus, the size in this dimension is 3 and each of those lists has to be the same shape to form an array (and matrix in this case). The next dimension's size is the number of elements inside this more inner pair of brackets, and we see that it's 2 as all of them contain 2 elements.

With 3-dimensional arrays, like in *lolol* below, we'll have a 3rd level of brackets:

```
lolol = [[[1,5,6,2],
          [3,2,1,3]],
         [[5,2,1,2],
          [6,4,8,4]],
         [[2,8,5,3],
          [1,1,9,4]]]
```

The first level of this array contains 3 matrices:

```
[[1,5,6,2],
  [3,2,1,3]]

[[5,2,1,2],
 [6,4,8,4]]
```

And

```
[[2,8,5,3],
 [1,1,9,4]]
```

That's what's inside the most outer brackets and the size of this dimension is then 3. If we look at the first matrix, we can see that it contains 2 lists — `[1,5,6,2]` and `[3,2,1,3]` so the size of this dimension is 2 — while each list of this inner matrix includes 4 elements. These 4 elements make up the 3rd and last dimension of this matrix since there are no more inner brackets. Therefore, the shape of this array is *(3, 2, 4)* and it's a 3-dimensional array, since the shape contains 3 dimensions

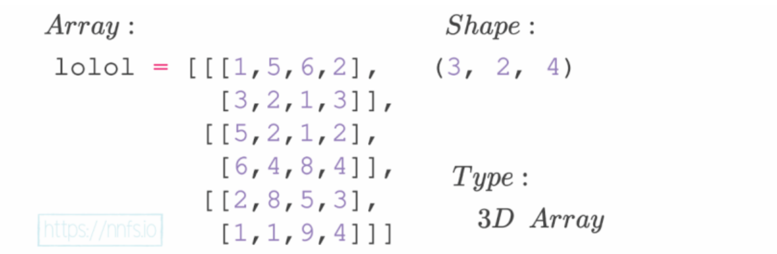


Fig. Example of a 3-dimensional array.

Finally, what's a tensor? When it comes to the discussion of tensors versus arrays in the context of computer science, pages and pages of debate have ensued. This intense debate appears to be caused by the fact that people are arguing from entirely different places. There's no question that a tensor is not just an array, but the real question is: "What is a tensor, to a computer scientist, in the context of deep learning?" We believe that we can solve the debate in one line:

A tensor object is an object that can be represented as an array

What this means is, as programmers, we can (and will) treat tensors as arrays in the context of deep learning, and that's really all the thought we have to put into it. Are all tensors *just* arrays? No, but they are represented as arrays in our code, so, to us, they're only arrays, and this is why there's so much argument and confusion.

Now, what is an array? An array can be defined as an ordered homologous container for numbers, and mostly use this term when working with the NumPy package since that's what the main data structure is called within it. A linear array, also called a 1-dimensional array, is the simplest example of an array, and in plain Python, this would be a list. Arrays can also consist of multi-dimensional data, and one of the best-known examples is what we call a matrix in mathematics, which we'll represent as a 2-dimensional array. Each element of the array can be accessed using a tuple of indices as a key, which means that we can retrieve any array element.

We need to learn one more notion — a vector. Put simply, a vector in math is what we call a list in Python or a 1-dimensional array in NumPy. Of course, lists and NumPy arrays do not have the same properties as a vector, but, just as we can write a matrix as a list of lists in Python, we can also write a vector as a list or an array! Additionally, we'll look at the vector algebraically (mathematically) as a set of numbers in brackets. This is in contrast to the physics perspective, where the vector's representation is usually seen as an arrow, characterized by a magnitude and a direction.

Dot Product and Vector Addition

Let's now address vector multiplication, as that's one of the most important operations we'll perform on vectors. We can achieve the same result as in our pure Python implementation of multiplying each element in our inputs and weights vectors element-wise by using a **dot product**, which we'll explain shortly Traditionally, we use dot products for **vectors** (yet another name for a container), and we can certainly refer to what we're doing here as working with vectors just as we can call them "tensors." Nevertheless, this seems to add to the mysticism of neural networks — like they're these objects out in a complex multi-dimensional vector space that we'll never understand. Keep thinking of vectors as arrays — a 1-dimensional array is just a vector (or a list in Python).

Because of the sheer number of variables and interconnections made, we can model very complex and non-linear relationships with non-linear activation functions, and truly feel like wizards, but this might do more harm than good. Yes, we will be using the "dot product," but we're doing this because it results in a clean way to perform the necessary calculations. It's nothing more in-depth than that — as you've already seen, we can do this math with far more rudimentary-sounding words. When multiplying vectors, you either perform a dot product or a cross product. A cross product results in a vector while a dot product results in a scalar (a single value/number).

First, let's explain what a dot product of two vectors is. Mathematicians would say:

$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n$$

A dot product of two vectors is a sum of products of consecutive vector elements. Both vectors must be of the same size (have an equal number of elements).

Let's write out how a dot product is calculated in Python. For it, you have two vectors, which we can represent as lists in Python. We then multiply their elements from the same index values and then add all of the resulting products. Say we have two lists acting as our vectors:

```
a = [1, 2, 3]
b = [2, 3, 4]
```

To obtain the dot product:

```python
dot_product = a[0]*b[0] + a[1]*b[1] + a[2]*b[2]

print(dot_product)

>>>
20
```

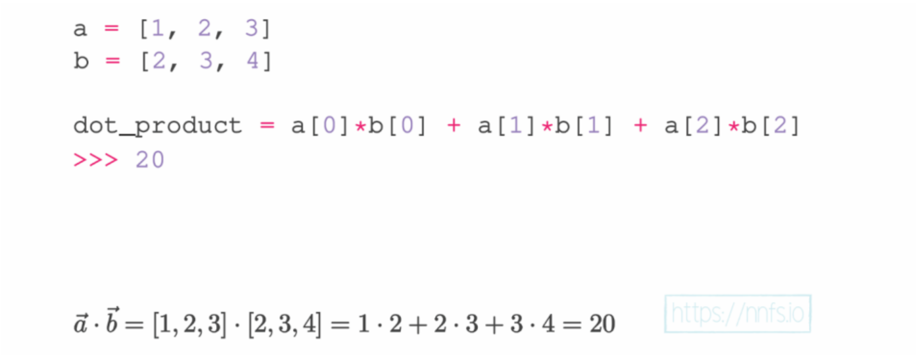


Fig . Math behind the dot product example.

Now, what if we called *a* “inputs” and *b* “weights?” Suddenly, this dot product looks like a succinct way to perform the operations we need and have already performed in plain Python. We need to multiply our weights and inputs of the same index values and add the resulting values together. The dot product performs this exact type of operation; thus, it makes lots of sense to use here. Returning to the neural network code, let’s make use of this dot product. Plain Python does not contain methods or functions to perform such an operation, so we’ll use the NumPy package, which is capable of this, and many more operations that we’ll use in the future.

We’ll also need to perform a vector addition operation in the not-too-distant future. Fortunately, NumPy lets us perform this in a natural way — using the plus sign with the variables containing vectors of the data. The addition of the two vectors is an operation performed element-wise, which means that both vectors have to be of the same size, and the result will become a vector of this

size as well. The result is a vector calculated as a sum of the consecutive vector elements:
$\vec{a} + \vec{b} = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n]$

A Single Neuron with NumPy

Let’s code the solution, for a single neuron to start, using the dot product and the addition of the vectors with NumPy. This makes the code much simpler to read and write (and faster to run):

```python
import numpy as np
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [0.2, 0.8, -0.5, 1.0]
bias = 2.0
outputs = np.dot(weights, inputs) + bias
```

```
print(outputs)

>>>
4.8
```

```
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [0.2, 0.8, -0.5, 1.0]
bias = 2.0

outputs = np.dot(weights, inputs) + bias



np.dot([0.2, 0.8, -0.5, 1.0], [1.0, 2.0, 3.0, 2.5]) =
= 0.2*1.0 + 0.8*2.0 + -0.5*3.0 + 1.0*2.5 = 2.8
```


Fig. Visualizing the math of the dot product of inputs and weights for a single neuron.

```
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [0.2, 0.8, -0.5, 1.0]
bias = 2.0

outputs = np.dot(weights, inputs) + bias
>>> 4.8



np.dot(weights, inputs) + bias = 2.8 + 2.0 = 4.8
```


Fig. Visualizing the math summing the dot product and bias.

A Layer of Neurons with NumPy

Now we're back to the point where we'd like to calculate the output of a layer of 3 neurons, which means the weights will be a matrix or list of weight vectors. In plain Python, we wrote this as a list of lists. With NumPy, this will be a 2-dimensional array, which we'll call a matrix. Previously with the 3-neuron example, we performed a multiplication of those weights with a list containing inputs, which resulted in a list of output values — one per each neuron.

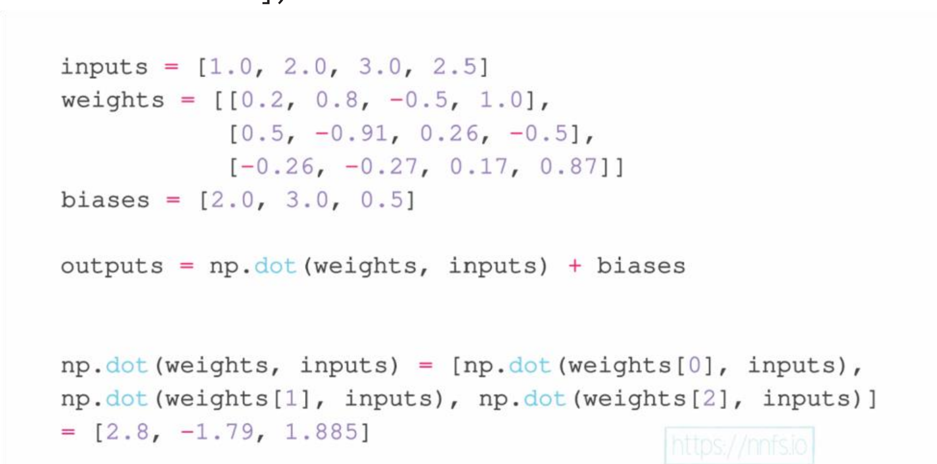
We also described the dot product of two vectors, but the weights are now a matrix, and we need to perform a dot product of them and the input vector. NumPy makes this very easy for us — treating this matrix as a list of vectors and performing the dot product one by one with the vector of inputs, returning a list of dot products.

The dot product's result, in our case, is a vector (or a list) of sums of the weight and bias products for each of the neurons. From here, we still need to add corresponding biases to them. The biases can be easily added to the result of the dot product operation as they are a vector of the same size. We can also use the plain Python list directly here, as NumPy will convert it to an array internally.

Previously, we had calculated outputs of each neuron by performing a dot product and adding a bias, one by one. Now we have changed the order of those operations — we're performing dot product first as one operation on all neurons and inputs, and then we are adding a bias in the next operation. When we add two vectors using NumPy, each i-th element is added together, resulting in a new vector of the same size. This is both a simplification and an optimization, giving us simpler and faster code.

```
import numpy as np
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [[0.2, 0.8, -0.5, 1],
           [0.5, -0.91, 0.26, -0.5],
           [-0.26, -0.27, 0.17, 0.87]]
biases = [2.0, 3.0, 0.5]
layer_outputs = np.dot(weights, inputs) + biases print(layer_outputs)

>>>
array([4.8   1.21   2.385])
```

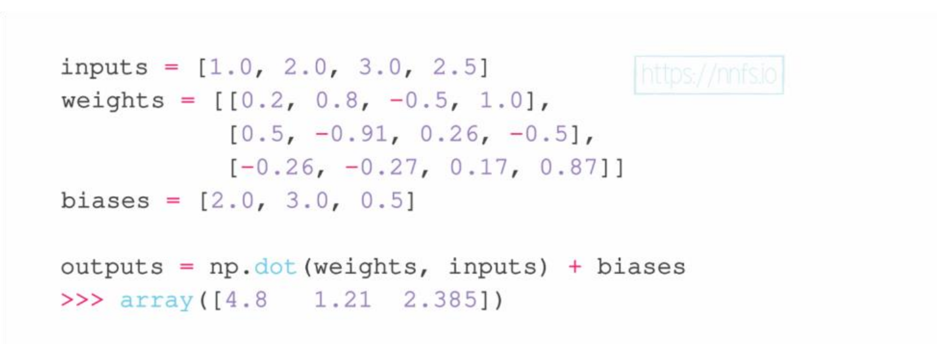


```
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [[0.2, 0.8, -0.5, 1.0],
           [0.5, -0.91, 0.26, -0.5],
           [-0.26, -0.27, 0.17, 0.87]]
biases = [2.0, 3.0, 0.5]

outputs = np.dot(weights, inputs) + biases


np.dot(weights, inputs) = [np.dot(weights[0], inputs),
np.dot(weights[1], inputs), np.dot(weights[2], inputs)]
= [2.8, -1.79, 1.885]
```


Fig. Code and visuals for the dot product applied to the layer of neurons.



```
inputs = [1.0, 2.0, 3.0, 2.5]
weights = [[0.2, 0.8, -0.5, 1.0],
           [0.5, -0.91, 0.26, -0.5],
           [-0.26, -0.27, 0.17, 0.87]]
biases = [2.0, 3.0, 0.5]

outputs = np.dot(weights, inputs) + biases
>>> array([4.8   1.21   2.385])
```


Fig. Code and visuals for the sum of the dot product and bias with a layer of neurons.

This syntax involving the dot product of weights and inputs followed by the vector addition of bias is the most commonly used way to represent this calculation of *inputs·weights+bias*. To explain the order of parameters we are passing into *npdot()*, we should think of it as whatever comes first will decide the output shape. In our case, we are passing a list of neuron weights first and then the inputs, as our goal is to get a list of neuron outputs. As we mentioned, a dot product of a matrix and a vector results in a list of dot products. The *npdot()* method treats the matrix as a list of vectors and performs a dot product of each of those vectors with the other vector. In this example, we used that property to pass a matrix, which was a list of neuron weight vectors and a vector of inputs and get a list of dot products — neuron outputs.

A Batch of Data

To train, neural networks tend to receive data in **batches** So far, the example input data have been only one sample (or **observation**) of various features called a feature set:

```
inputs = [1, 2, 3, 2.5]
```

Here, the `[1, 2, 3, 2.5]` data are somehow meaningful and descriptive to the output we desire. Imagine each number as a value from a different sensor all simultaneously. Each of these values is a feature observation datum, and together they form a **feature set instance**, also called an **observation**, or most commonly, a **sample**.

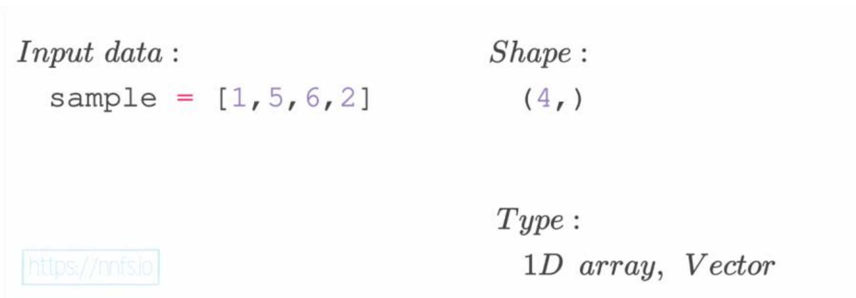


Fig. Visualizing a 1D array.

Often, neural networks expect to take in many **samples** at a time for two reasons. One reason is that it's faster to train in batches in parallel processing, and the other reason is that batches help with generalization during training. If you fit (perform a step of a training process) on one sample at a time, you're highly likely to keep fitting to that individual sample, rather than slowly producing general tweaks to weights and biases that fit the entire dataset. Fitting or training in batches gives you a higher chance of making more meaningful changes to weights and biases

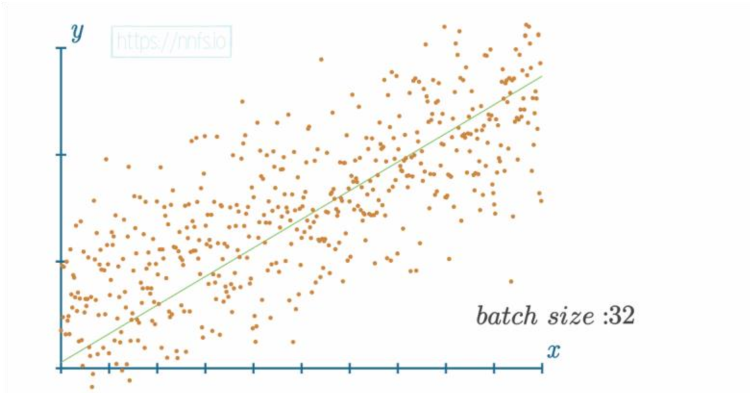


Fig. Example of a linear equation fitting batches of 32 chosen samples.

An example of a batch of data could look like:

Input data :

```
batch = [[1,5,6,2],
         [3,2,1,3],
         [5,2,1,2],
         [6,4,8,4],
         [2,8,5,3],
         [1,1,9,4],
         [6,6,0,4],
         [8,7,6,4]]
```

Shape :

(8, 4)

Type :

2D Array, Matrix

Recall that in Python, and in our case, lists are useful containers for holding a sample as well as multiple samples that make up a batch of observations. Such an example of a batch of observations, each with its own sample, looks like:

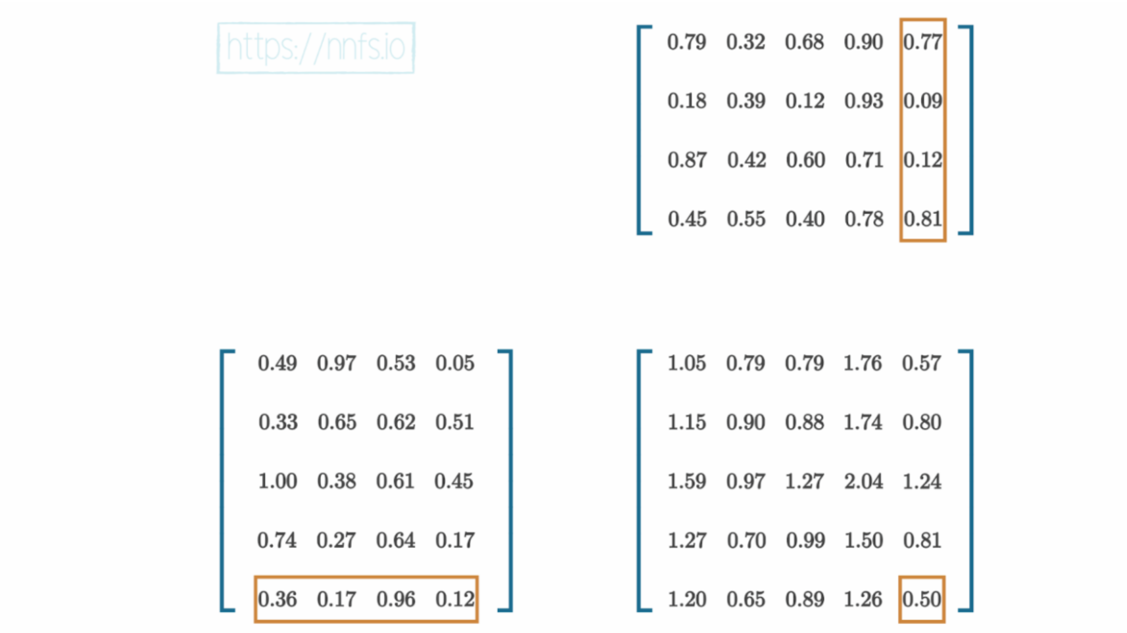
```
inputs = [[1, 2, 3, 2.5], [2, 5, -1, 2], [-1.5, 2.7, 3.3, -0.8]]
```

This list of lists could be made into an array since it is homologous. Note that each "list" in this larger list is a sample representing a feature set. `[1, 2, 3, 2.5]`, `[2, 5, -1, 2]`, and `[-1.5, 2.7, 3.3, -0.8]` are all **samples**, and are also referred to as **feature set instances** or **observations**.

We have a matrix of inputs and a matrix of weights now, and we need to perform the dot product on them somehow, but how and what will the result be? Similarly, as we performed a dot product on a matrix and a vector, we treated the matrix as a list of vectors, resulting in a list of dot products. In this example, we need to manage both matrices as lists of vectors and perform dot products on all of them in all combinations, resulting in a list of lists of outputs, or a matrix; this operation is called the **matrix product**.

Matrix Product

The **matrix product** is an operation in which we have 2 matrices, and we are performing dot products of all combinations of rows from the first matrix and the columns of the 2nd matrix, resulting in a matrix of those atomic **dot products**:



To perform a matrix product, the size of the second dimension of the left matrix must match the size of the first dimension of the right matrix. For example, if the left matrix has a shape of *(5, 4)* then the right matrix must match this 4 within the first shape value *(4, 7)*. The shape of the resulting array is always the first dimension of the left array and the second dimension of the right array, *(5, 7)*. In the above example, the left matrix has a shape of *(5, 4)*, and the upper-right matrix has a shape of *(4, 5)*. The second dimension of the left array and the first dimension of the second array are both *4*, they match, and the resulting array has a shape of *(5, 5)*.

To elaborate, we can also show that we can perform the matrix product on vectors. In mathematics, we can have something called a column vector and row vector, which we'll explain better shortly. They're vectors, but represented as matrices with one of the dimensions having a size of 1:

$$a = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

a is a row vector. It looks very similar to a vector *a* (with an arrow above it) described earlier along with the vector product. The difference in notation between a row vector and vector are commas between values and the arrow above symbol *a* is missing on a row vector. It's called a row vector as it's a vector of a row of a matrix. *b*, on the other hand, is called a column vector because it's a column of a matrix. As row and column vectors are technically matrices, we do not denote them with vector arrows anymore.

When we perform the matrix product on them, the result becomes a matrix as well, like in the previous example, but containing just a single value, the same value as in the dot product example we have discussed previously:

$$ab = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \end{bmatrix}$$

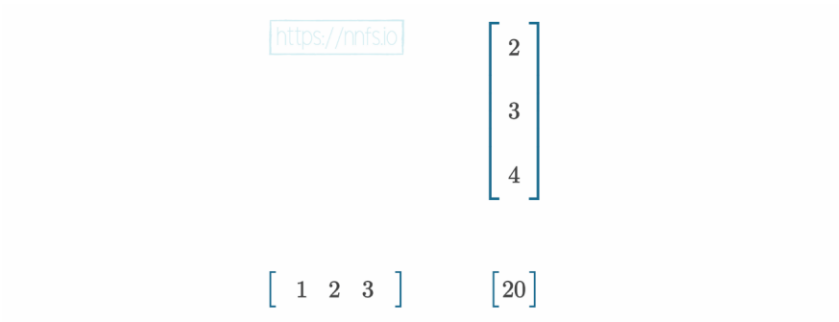


Fig. Product of row and column vectors.

In other words, row and column vectors are matrices with one of their dimensions being of a size of 1; and, we perform the **matrix product** on them instead of the **dot product**, which results in a matrix containing a single value. In this case, we performed a matrix multiplication of matrices with shapes *(1, 3)* and *(3, 1)*, then the resulting array has the shape *(1, 1)* or a size of *1x1*.

Transposition for the Matrix Product

How did we suddenly go from 2 vectors to row and column vectors? We used the relation of the dot product and matrix product saying that a dot product of two vectors equals a matrix product of a row and column vector (the arrows above the letters signify that they are vectors):

$$\vec{a} \cdot \vec{b} = ab^T$$

We also have temporarily used some simplification, not showing that column vector *b* is actually a **transposed** vector *b*. The proper equation, matching the dot product of vectors *a* and *b* written as matrix product should look like:

$$ab^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \end{bmatrix}$$

Here we introduced one more new operation — **transposition**. Transposition simply modifies a matrix in a way that its rows become columns and columns become rows:

$$\begin{bmatrix} 00 & 01 & 02 & 03 & 04 \\ 05 & 06 & 07 & 08 & 09 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}^{T} = \begin{bmatrix} 00 & 05 & 10 & 15 \\ 01 & 06 & 11 & 16 \\ 02 & 07 & 12 & 17 \\ 03 & 08 & 13 & 18 \\ 04 & 09 & 14 & 19 \end{bmatrix}$$

Fig. Example of an array transposition.

$$\begin{bmatrix} 0.49 & 0.97 & 0.53 & 0.05 & 0.33 \\ 0.65 & 0.62 & 0.51 & 1.00 & 0.38 \\ 0.61 & 0.45 & 0.74 & 0.27 & 0.64 \\ 0.17 & 0.36 & 0.17 & 0.96 & 0.12 \\ 0.79 & 0.32 & 0.68 & 0.90 & 0.77 \end{bmatrix}^{T} = \begin{bmatrix} 0.49 & 0.65 & 0.61 & 0.17 & 0.79 \\ 0.97 & 0.62 & 0.45 & 0.36 & 0.32 \\ 0.53 & 0.51 & 0.74 & 0.17 & 0.68 \\ 0.05 & 1.00 & 0.27 & 0.96 & 0.90 \\ 0.33 & 0.38 & 0.64 & 0.12 & 0.77 \end{bmatrix}$$

Fig. Another example of an array transposition.

Now we need to get back to row and column vector definitions and update them with what we have just learned.

A row vector is a matrix whose first dimension's size (the number of rows) equals 1 and the second dimension's size (the number of columns) equals *n* — the vector size. In other words, it's a 1×n array or array of shape (1, n):

$$a = \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_n \end{bmatrix}$$

With NumPy and with 3 values, we would define it as:

```
np.array([[1, 2, 3]])
```

Note the use of double brackets here. To transform a list into a matrix containing a single row (perform an equivalent operation of turning a vector into row vector), we can put it into a list and create numpy array:

```
a = [1, 2, 3]
print(np.array([a]))
>>>
array([[1, 2, 3]])
```

Again, note that we encase `a` in brackets before converting to an array in this case.
Or we can turn it into a 1D array and expand dimensions using one of the NumPy abilities:

```
a = [1, 2, 3]
print(np.expand_dims(np.array(a), axis=0))
>>> array([[1, 2, 3]])
```

Where *npexpand_dims()* adds a new dimension at the index of the *axis*.

A column vector is a matrix where the second dimension's size equals 1, in other words, it's an array of shape (n, 1):

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_n \end{bmatrix}$$

With NumPy it can be created the same way as a row vector, but needs to be additionally transposed — transposition turns rows into columns and columns into rows:

$$\begin{bmatrix} b_1 & b_2 & b_3 & \dots & b_n \end{bmatrix}^T = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_n \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_n \end{bmatrix}^T = \begin{bmatrix} b_1 & b_2 & b_3 & \dots & b_n \end{bmatrix}$$

To turn vector *b* into row vector *b*, we'll use the same method that we used to turn vector *a* into row vector *a*, then we can perform a transposition on it to make it a column vector *b*:

$$b = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$b^T = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

With NumPy code:

```
import numpy as np
a = [1, 2, 3] b = [2, 3, 4]
a = np.array([a]) b =
np.array([b]).T
print(np.dot(a, b))
```

```
>>> array([[20]])
```

We have achieved the same result as the dot product of two vectors, but performed on matrices and returning a matrix — exactly what we expected and wanted. It's worth mentioning that NumPy does not have a dedicated method for performing matrix product — the dot product and matrix product are both implemented in a single method: *npdot()*.

As we can see, to perform a matrix product on two vectors, we took one as is, transforming it into a row vector, and the second one using transposition on it to turn it into a column vector. That allowed us to perform a matrix product that returned a matrix containing a single value. We also performed the matrix product on two example arrays to learn how a matrix product works — it creates a matrix of dot products of all combinations of row and column vectors.

A Layer of Neurons & Batch of Data w/ NumPy

Let's get back to our inputs and weights — when covering them, we mentioned that we need to perform dot products on all of the vectors that consist of both input and weight matrices. As we have just learned, that's the operation that the matrix product performs. We just need to perform transposition on its second argument, which is the weights matrix in our case, to turn the row vectors it currently consists of into column vectors.

Initially, we were able to perform the dot product on the inputs and the weights without a transposition because the weights were a matrix, but the inputs were just a vector. In this case, the dot product results in a vector of atomic dot products performed on each row from the matrix and this single vector. When inputs become a batch of inputs (a matrix), we need to perform the matrix product. It takes all of the combinations of rows from the left matrix and columns from the right matrix, performing the dot product on them and placing the results in an output array. Both arrays have the same shape, but, to perform the matrix product, the shape's value from the index 1 of the first matrix and the index 0 of the second matrix must match — they don't right now.

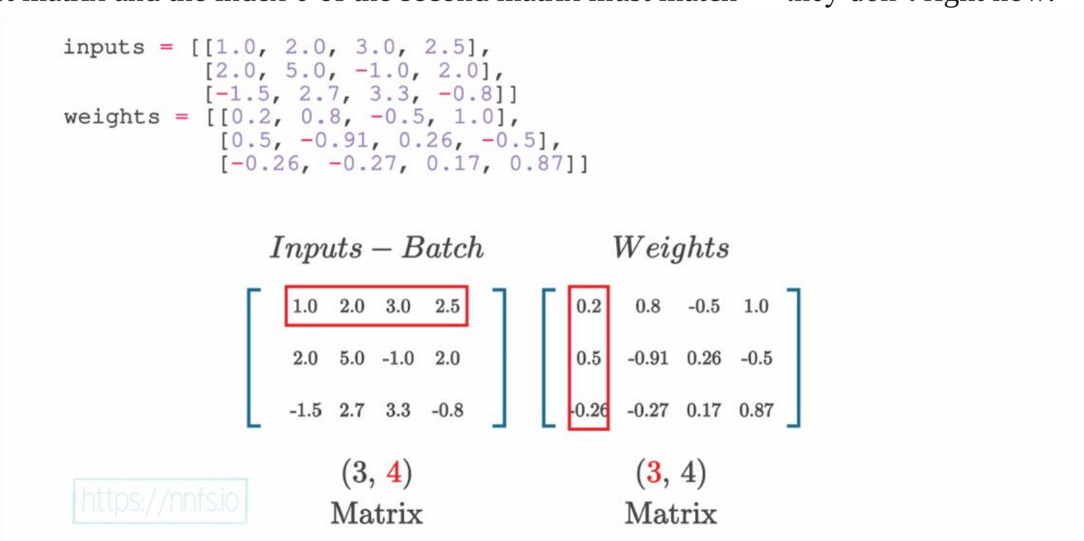


Fig 2.18: Depiction of why we need to transpose to perform the matrix product.

If we transpose the second array, values of its shape swap their positions.

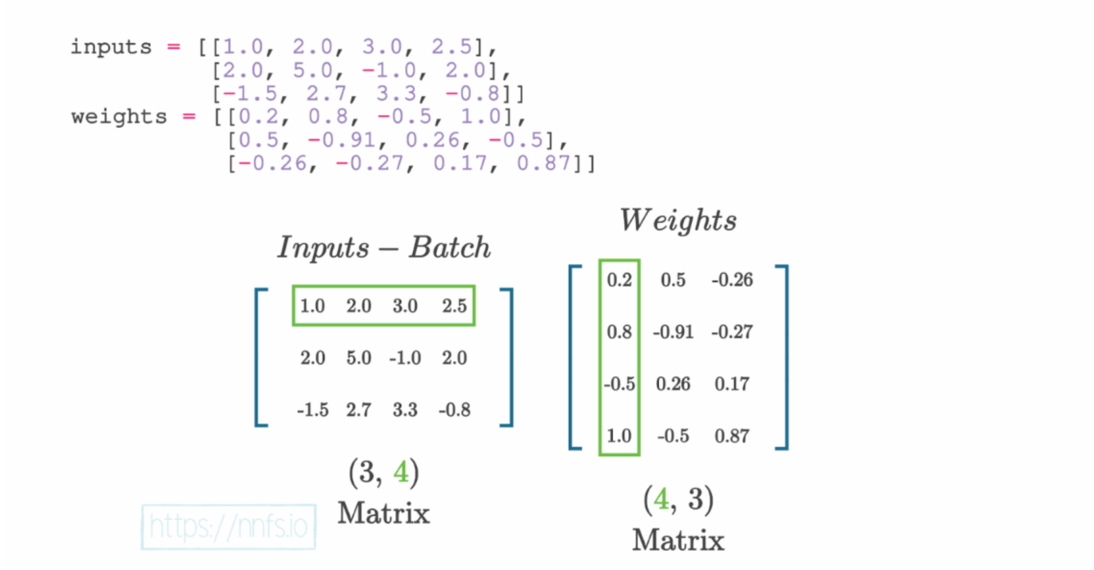


Fig. After transposition, we can perform the matrix product.

If we look at this from the perspective of the input and weights, we need to perform the dot product of each input and each weight set in all of their combinations. The dot product takes the row from the first array and the column from the second one, but currently the data in both arrays are row-aligned. Transposing the second array shapes the data to be column-aligned. The matrix product of inputs and transposed weights will result in a matrix containing all atomic dot products that we need to calculate. The resulting matrix consists of outputs of all neurons after operations performed on each input sample:

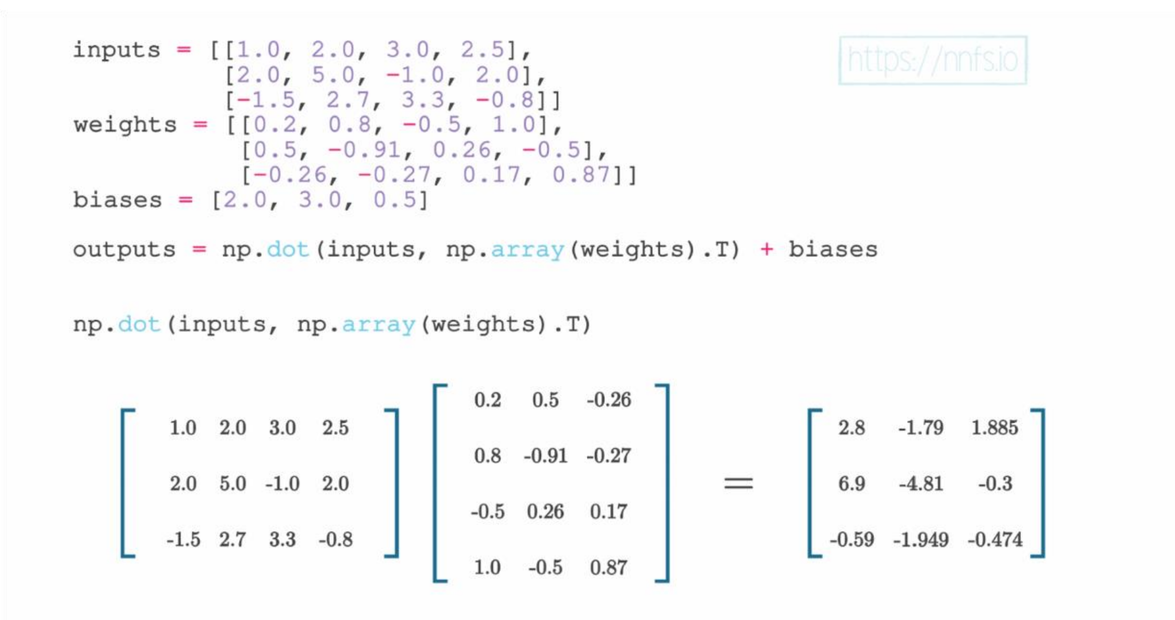


Fig. Code and visuals depicting the dot product of inputs and transposed weights.

We mentioned that the second argument for *npdot()* is going to be our transposed weights, so first will be inputs, but previously weights were the first parameter. We changed that here. Before, we were modeling neuron output using a single sample of data, a vector, but now we are a step forward when we model layer behavior on a batch of data. We could retain the current parameter order, but, as we'll soon learn, it's more useful to have a result consisting of a list of layer outputs per each sample than a list of neurons and their outputs sample-wise. We want the resulting array to be sample-related and not neuron-related as we'll pass those samples further through the network, and the next layer will expect a batch of inputs.

We can code this solution using NumPy now. We can perform *npdot()* on a plain Python list of lists as NumPy will convert them to matrices internally. We are converting weights ourselves though to perform transposition operation first, *T* in the code, as plain Python list of lists does not support it. Speaking of biases, we do not need to make it a NumPy array for the same reason — NumPy is going to do that internally.

Biases are a list, though, so they are a 1D array as a NumPy array. The addition of this bias vector to a matrix (of the dot products in this case) works similarly to the dot product of a matrix and vector that we described earlier; The bias vector will be added to each row vector of the matrix. Since each column of the matrix product result is an output of one neuron, and the vector is going to be added to each row vector, the first bias is going to be added to each first element of those vectors, second to second, etc. That's what we need — the bias of each neuron needs to be added to all of the results of this neuron performed on all input vectors (samples).

```
inputs = [[1.0, 2.0, 3.0, 2.5],
          [2.0, 5.0, -1.0, 2.0],
          [-1.5, 2.7, 3.3, -0.8]]
weights = [[0.2, 0.8, -0.5, 1.0],
           [0.5, -0.91, 0.26, -0.5],
           [-0.26, -0.27, 0.17, 0.87]]
biases = [2.0, 3.0, 0.5]

outputs = np.dot(inputs, np.array(weights).T) + biases
>>> array([[ 4.8    1.21   2.385],
           [ 8.9   -1.81   0.2  ],
           [ 1.41   1.051  0.026]])
```


$$\begin{bmatrix} 2.8 & -1.79 & 1.885 \\ 6.9 & -4.81 & -0.3 \\ -0.59 & -1.949 & -0.474 \end{bmatrix} + \begin{bmatrix} 2.0 & 3.0 & 0.5 \end{bmatrix} = \begin{bmatrix} 4.8 & 1.21 & 2.385 \\ 8.9 & -1.81 & 0.2 \\ 1.41 & 1.051 & 0.026 \end{bmatrix}$$

Fig. Code and visuals for inputs mulitplied by the weights, plus the bias.

Now we can implement what we have learned into code:

```
import numpy as np

inputs = [[1.0, 2.0, 3.0, 2.5],
```

```
            [2.0, 5.0, -1.0, 2.0],
            [-1.5, 2.7, 3.3, -0.8]]
  weights = [[0.2, 0.8, -0.5, 1.0],
            [0.5, -0.91, 0.26, -0.5],
          [-0.26, -0.27, 0.17, 0.87]]
          biases = [2.0, 3.0, 0.5]
layer_outputs = np.dot(inputs, np.array(weights).T) + biases
print(layer_outputs)

  >>>
  array([[ 4.8    1.21   2.385],        [
  8.9   -1.81   0.2  ],
        [ 1.41   1.051  0.026]])
```

As you can see, our neural network takes in a group of samples (inputs) and outputs a group of predictions. If you've used any of the deep learning libraries, this is why you pass in a list of inputs (even if it's just one feature set) and are returned a list of predictions, even if there's only one prediction.