**Development of a predictive model for warpage prediction in thermoplastic injection molding**

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**Supplementary Information (SI):** The warpage model developed through Pi-Buckingham technique

The dimensions of the variables are as follows:

* Warpage (W): L
* Melt temperature :
* Mold temperature :
* Packing pressure :
* Packing time :
* Injection time :
* Volume shrinkage :
* Sink mark :
* Average velocity :
* Flow front speed :

**Table S1**: The repeated variables dimension matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Variable** | **Symbol** | **Mass (M)** | **Length (L)** | **Time (T)** | **Temperature (** |
| Warpage | W | 0 | 1 | 0 | 0 |
| Melt Temperature |  | 0 | 0 | 0 | 1 |
| Mold Temperature |  | 0 | 0 | 0 | 1 |
| Packing Pressure |  | 1 | -1 | -2 | 0 |
| Packing Time |  | 0 | 0 | 1 | 0 |
| Injection Time |  | 0 | 0 | 1 | 0 |
| Volumetric Shrinkage |  | 0 | 0 | 0 | 0 |
| Sink Mark Depth |  | 0 | 1 | 0 | 0 |
| Average Velocity |  | 0 | 1 | -1 | 0 |
| Flow Front Speed |  | 0 | 1 | -1 | 0 |

Applying the application of Pi-Buckingham theorem, the number of dimensionless groups is:

Where:

n = 10 (The number of considered variables),

r = 4 (The number of fundamental dimensions).

The dimensionless group is a product of the considered variables to the unknown exponents:

The following equation is expressed based on:

1. Mass (M):
2. Length (L):
3. Time (T):
4. Temperature :

6 independent variables left; the subset was selected to express the rest in terms of these.

Temperature ratio:

Pressure ratio: (Density, , was introduced as a common property in flow-related conditions).

Time ratio:

Sink mark ratio: ( is the allowable depth of the mold cavity, that is, allowance depth).

Velocity ratio:

Warpage normalization:

The warpage, , of the dimensionless groups is expressed as:

**Table S2**: The experimental design runs of the process parameters

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Runs | Melt temperature (℃) | Mold temperature (℃) | Packing pressure (N/mm2) | Packing time (s) | Injection time (s) | Warpage (mm) | Volumetric shrinkage (%) | Sink mark (mm) | Average velocity (mm/s) | Flow front speed (mm/s) |
| 1 | 280 | 40 | 40 | 10 | 1.5 | 0.1902 | 17.93 | 0.0355 | 16938 | 9335 |
| 2 | 300 | 50 | 35 | 10 | 1 | 0.2005 | 21.65 | 0.0377 | 36091 | 2726 |
| 3 | 300 | 50 | 35 | 10 | 1.5 | 0.1873 | 19.61 | 0.0377 | 23726 | 10310 |
| 4 | 280 | 50 | 35 | 10 | 1 | 0.2227 | 19.95 | 0.041 | 23419 | 2789 |
| 5 | 280 | 40 | 35 | 5 | 1 | 0.2113 | 19.88 | 0.0411 | 22103 | 2784 |
| 6 | 280 | 40 | 35 | 10 | 1.5 | 0.1974 | 17.93 | 0.0416 | 13312 | 7471 |
| 7 | 280 | 50 | 40 | 5 | 1 | 0.2142 | 19.95 | 0.0386 | 29977 | 2789 |
| 8 | 280 | 40 | 35 | 5 | 1.5 | 0.1974 | 17.93 | 0.0416 | 13312 | 7471 |
| 9 | 300 | 40 | 35 | 10 | 1.5 | 0.175 | 19.46 | 0.0332 | 21825 | 12878 |
| 10 | 300 | 50 | 40 | 10 | 1 | 0.1941 | 21.65 | 0.0361 | 41841 | 2726 |
| 11 | 280 | 50 | 40 | 10 | 1 | 0.2142 | 19.95 | 0.0386 | 29977 | 2789 |
| 12 | 280 | 50 | 40 | 5 | 1.5 | 0.1976 | 18.06 | 0.0371 | 17766 | 6723 |
| 13 | 300 | 40 | 40 | 10 | 1.5 | 0.1682 | 19.45 | 0.0318 | 27679 | 12673 |
| 14 | 300 | 40 | 40 | 5 | 1 | 0.1805 | 21.57 | 0.0318 | 40916 | 2722 |
| 15 | 300 | 40 | 40 | 5 | 1.5 | 0.1682 | 19.45 | 0.0318 | 27679 | 12673 |
| 16 | 280 | 50 | 35 | 5 | 1.5 | 0.2062 | 18.05 | 0.0408 | 13937 | 5635 |
| 17 | 280 | 50 | 35 | 10 | 1.5 | 0.2062 | 18.05 | 0.0408 | 13937 | 5635 |
| 18 | 300 | 50 | 40 | 5 | 1.5 | 0.1791 | 19.59 | 0.036 | 30053 | 10404 |
| 19 | 300 | 50 | 35 | 5 | 1 | 0.2005 | 21.65 | 0.0377 | 36091 | 2726 |
| 20 | 300 | 50 | 35 | 5 | 1.5 | 0.1873 | 19.61 | 0.0377 | 23726 | 10310 |
| 21 | 280 | 40 | 35 | 10 | 1 | 0.2113 | 19.88 | 0.0411 | 22103 | 2784 |
| 22 | 300 | 40 | 35 | 10 | 1 | 0.1883 | 21.57 | 0.0346 | 35248 | 2722 |
| 23 | 280 | 50 | 40 | 10 | 1.5 | 0.1976 | 18.06 | 0.0371 | 17766 | 6723 |
| 24 | 300 | 50 | 40 | 5 | 1 | 0.1941 | 21.65 | 0.0361 | 41841 | 2726 |
| 25 | 280 | 40 | 40 | 5 | 1 | 0.203 | 19.88 | 0.0351 | 28191 | 2784 |
| 26 | 280 | 50 | 35 | 5 | 1 | 0.2227 | 19.95 | 0.041 | 23419 | 2789 |
| 27 | 280 | 40 | 40 | 10 | 1 | 0.203 | 19.88 | 0.0351 | 28191 | 2784 |
| 28 | 300 | 40 | 40 | 10 | 1 | 0.1805 | 21.57 | 0.0318 | 40916 | 2722 |
| 29 | 300 | 50 | 40 | 10 | 1.5 | 0.1791 | 19.59 | 0.036 | 30053 | 10404 |
| 30 | 300 | 40 | 35 | 5 | 1.5 | 0.175 | 19.46 | 0.0332 | 21825 | 12878 |
| 31 | 280 | 40 | 40 | 5 | 1.5 | 0.1902 | 17.93 | 0.0355 | 16998 | 9307 |
| 32 | 300 | 40 | 35 | 5 | 1 | 0.1883 | 21.57 | 0.0346 | 35248 | 2722 |

**Table S3**: Regression results of the ratio parameters

OLS Regression Results

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Dep. Variable: Warpage R-squared: 0.858

Model: OLS Adj. R-squared: 0.830

Method: Least Squares F-statistic: 31.32

Date: Thu, 13 Mar 2025 Prob (F-statistic): 3.24e-10

Time: 01:19:47 Log-Likelihood: 121.36

No. Observations: 32 AIC: -230.7

Df Residuals: 26 BIC: -221.9

Df Model: 5

Covariance Type: nonrobust

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coef std err t P>|t| [0.025 0.975]

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const 0.1170 0.030 3.948 0.001 0.056 0.178

temp\_ratio -0.0022 0.002 -1.212 0.236 -0.006 0.002

pressure\_ratio 0.0010 0.001 2.050 0.051 -2.72e-06 0.002

time\_ratio 0.0003 0.000 0.508 0.615 -0.001 0.001

sink\_mark\_ratio 117.9697 27.052 4.361 0.000 62.363 173.576

flow\_front\_speed\_ratio -0.0442 0.010 -4.583 0.000 -0.064 -0.024

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Omnibus: 7.515 Durbin-Watson: 2.067

Prob(Omnibus): 0.023 Jarque-Bera (JB): 2.650

Skew: 0.342 Prob(JB): 0.266

Kurtosis: 1.767 Cond. No. 2.81e+05

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**Table S3**: Regression results of the process parameters

OLS Regression Results

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Dep. Variable: Warpage R-squared: 0.839

Model: OLS Adj. R-squared: 0.783

Method: Least Squares F-statistic: 14.99

Date: Sat, 22 Mar 2025 Prob (F-statistic): 1.89e-07

Time: 16:31:26 Log-Likelihood: 89.614

No. Observations: 32 AIC: -161.2

Df Residuals: 23 BIC: -148.0

Df Model: 8

Covariance Type: nonrobust

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coef std err t P>|t| [0.025 0.975]

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const 3.4633 1.389 2.494 0.020 0.590 6.336

melt\_temperature -0.0125 0.005 -2.721 0.012 -0.022 -0.003

mold\_temperature -0.0043 0.001 -4.587 0.000 -0.006 -0.002

packing\_pressure -0.0232 0.009 -2.617 0.015 -0.041 -0.005

packing\_time 0.0038 0.001 3.125 0.005 0.001 0.006

injection\_time 0.4887 0.119 4.106 0.000 0.242 0.735

sink\_mark 1.8874 2.968 0.636 0.531 -4.253 8.028

average\_velocity 1.921e-05 6.83e-06 2.814 0.010 5.09e-06 3.33e-05

flow\_front\_speed 3.508e-06 4.04e-06 0.868 0.394 -4.85e-06 1.19e-05

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Omnibus: 5.612 Durbin-Watson: 1.993

Prob(Omnibus): 0.060 Jarque-Bera (JB): 3.966

Skew: 0.754 Prob(JB): 0.138

Kurtosis: 3.838 Cond. No. 2.88e+07

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