Topology Optimization of Truss Structures to Redesign Solid Components

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Abstract. One of the most common ways to redesign solid components into lightweight designs is to substitute the solid material with a truss structure. Trusses can be found in a wide range of dimensions, from stents in blood arteries to ensure the blood flow to the patients heart all the way up to architectural structures like bridges, roofs or the Eiffel tower as a popular example. This paper treats the implementation of an algorithm which is capable of redesigning solid components into lightweight truss structures by optimizing the topology of a Ground Structure Method based truss. The Finite Element Model is further able to allow concave boundaries and voids in the design domain by defining restriction zones for the truss elements. The algorithm is based on an Optimality Criterion aiming for a fully stressed truss to obtain minimum weight. For a sufficient start design, the optimized results are near optimal in shape and volume with errors $< 1\%$ in relation to the Michell structures.

Keywords: Topology optimization · Truss optimization · Lightweight truss · Ground Structure Method · Michell structures

1 Introduction

Since the ongoing innovation of additive manufacturing is extending the limits of freedom for the design of mechanical structures, engineers intend to use this possibility to produce optimized components with high material efficiency. Hence today optimization algorithms with different approaches and objectives are available in a rich variety. When inspecting topology optimized structures by the homogenization method, for instance, the material is mostly arranged on lattices which are following the load paths of the boundary conditions [12,1,13]. Therefore the approach used in this thesis discretizes a continuous domain of solid material with truss elements. To obtain results of higher optimality, a more powerful and controllable discretization method is used to generate this ground structure stated in section 2 which is then optimized by a topology optimization algorithm outlined in section 3. This approach of identifying the optimal set of elements in a richly filled truss is available in various publications and literature [15,16,8,4]. Michell has developed a design theory for truss structures, based on Maxwell's load path theorem, which can be used for benchmark modeling to verify the solutions provided by the algorithm as shown in section 4 [10]. His theory is capable of calculating the minimum material volume for distinct load cases – so called Michell structures – wherein every truss element is stressed at the maximum permissible amount to not waste any material.

The methods used to implement this algorithm into an existing FEM framework are combined from different sources with the focus on a minimum of computational complexity. The first part of the implementation treats the finite element modeling of the start design. The second part unveils the topology optimization algorithm.

2 Ground Structure Method

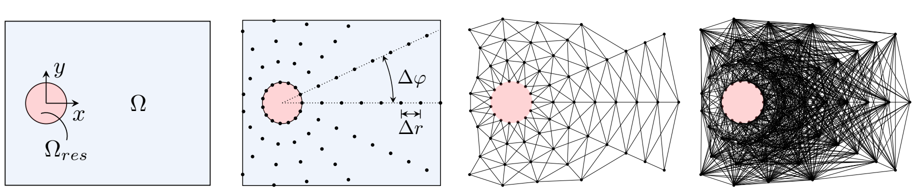
The Ground Structure Method used from Zegard & Paulino for plane and spatial truss structures with the option of restricted zones is based on the connectivity of a set of nodal points [15,16]. Restriction zones do not allow the existence of truss elements. Figure 1 depicts the generation of a densely filled truss in a solution domain Ω. In the shown geometry for the Michell cantilever with circular support, the domain is filled with 66 equidistant nodes along the polar coordinates (r, φ). To obtain the base mesh from figure 1c, the directly neighbored nodes are connected by an element according to the connectivity matrix $\mathbf{C}^0$ which is filled by

$$c^0_{pq} = \begin{cases} 1 & \text{if nodes } p \text{ and } q \text{ are direct neighbors} \\ 0 & \text{if } p = q \text{ or otherwise} \end{cases} \tag{1}$$

and higher connectivity levels n_{lvl}, like in figure 1d for instance, can be reached by the matrix multiplication of the connectivity matrix from the previous level

$$\mathbf{C}^{n_{lvl}} = \mathbf{C}^{n_{lvl}-1}\mathbf{C}^{n_{lvl}-1}. \tag{2}$$

With every level of connectivity, the neighborhood of each node is expanded by one neighbor farther away which will lead to a fast enrichment in the number of elements. To avoid superimposition, only new potential members are added to



(a) Geometry (b) Nodes $|\mathcal{N}^{(0)}| = 66$ (c) Base mesh $n_{lvl} = 0$ (d) Ground structure $n_{lvl} = 2$

Fig. 1. Generating the ground structure from a polar distributed set of nodes in a solution domain Ω with a circular restriction zone Ω_{res}

the structure if they are not collinear to an existing element. This collinearity check has to be performed between each level step up for all existing elements $\overline{pq}$ at node p to the new candidate $\overline{pi}$ with the new node i. If the angles between their directional cosine vectors is

$$\cos(\angle pqi) = \left|\hat{\mathbf{d}}_{pq}\hat{\mathbf{d}}_{pi}\right| \leq \epsilon_\theta \qquad \hat{\mathbf{d}}_{pq} = \frac{\mathbf{p}-\mathbf{q}}{||\mathbf{p}-\mathbf{q}||} \tag{3}$$

with the collinearity tolerance $\epsilon_\theta \lesssim 1$, the element will be added to the structure. All principles and formulas so far are further valid and easily projected to an equidistant spherical (r, φ, θ) and Cartesian node distribution along (x, y, z).

Restriction zones are considered via collision detection methods from the field of algorithmic geometry [15,5]. The restriction zones are defined as elementary geometric figures (circle, rectangle, sphere, cuboid and cylinder) and the truss elements are interpreted as a line. A pleasant collection of the source code for plane geometries along with a visualization is provided by Thompson [14]. Elements colliding with Ω_{res} are eliminated from the structure. This is helpful when modeling concave domains, like the Michell cantilever with circular support from figure 1, to avoid elements crossing the circular area, for instance, or simply just to define void regions in the domain.

3 Optimization algorithm

Since the material density is assumed constant for all elements, the weight optimization problem is formulated as the minimization of the truss volume

$$\min\left(V = \sum l_i a_i\right) \tag{4}$$

so that

$$\begin{aligned} a^l < a_i &\leq a^u \\ |\sigma_i| &\leq \sigma^u \qquad i \in \mathcal{B}_{phys} \end{aligned} \tag{5}$$

where a_i are the cross-sectional areas, l_i the lengths and σ_i the stresses of the elements. The principle of the implemented topology optimization algorithm is to aim for a homogeneous stress distribution. In the case of truss structures it is often called the Fully Stressed Design (FSD) since the minimum of required material is found in a statically determinate system with all elements stressed at the maximum permissible amount. Even hyperstatic systems, like the richly filled ground structure used here, will ideally drop redundant elements in the optimization process and result in a statically determinate truss with their element stresses

$$\sigma_i = \sigma^u \qquad i \in \mathcal{B}_{phys} \tag{6}$$

where $\mathcal{B}_{phys}$ is the index set of all physically active elements (bars) [7]. All dispensable elements which are identified during the optimization are penalized into a fictitious domain Ω_{fict} by dropping their cross-sectional areas to near zero. Therefore Ω_{fict} is a soft environment wherein the stiff physical domain

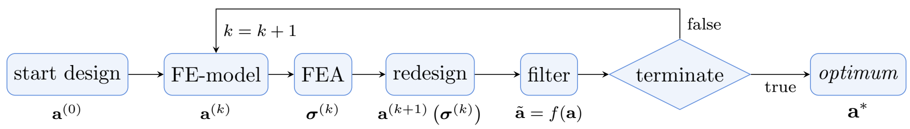


Fig. 2. The implemented topology optimization algorithm

Ω_{phys} is embedded. In this application the fictitious domain approach is in use to simulate the elimination of elements to avoid rebuilding the linear system of equations and is based on the approach used in the Finite Cell Method [11].

Figure 2 shows the iterative algorithm flow where a Finite Element (FE) model is initialized with the start design from section 2. The elements normal stresses $\boldsymbol{\sigma}$ are provided per Finite Element Analysis (FEA) to redesign the cross-sectional areas $\mathbf{a}$ based on a modified formula of Baier [3]

$$a_i^{(k+1)} = \begin{cases} \min\left(a_i^{(k)} \frac{\sigma_i^{(k)}}{\sigma^u},\ a^u\right) & \text{if } a_i^{(k+1)} > a^l \\ \mu = 10^{-12} & \text{if } a_i^{(k+1)} \leq a^l \end{cases} \qquad i \in \mathcal{B} \tag{7}$$

where elements are transferred into the fictitious domain Ω_{fict} by penalization with μ if their stress capacity $\frac{\sigma_i}{\sigma^u}$ is low enough to shrink the elements cross-sectional area below a^l. To improve the contouring between the physical and fictitious domain a filtering is performed by the Heavyside function [6].

To declare a design as optimal, a termination criterion is formulated by three subcriteria. Most importantly all elements have to be fully stressed, as stated in equation (6), secondly the model volume has to be converged and lastly the change in the cross-sectional areas between the iterations has to be zero to obtain a thinned out structure with a statically determinate truss. The third subcriterion is calculated by the Euclidean distance between the normalized design variable vectors $\mathbf{a}^{(k+1)}$ and $\mathbf{a}^{(k)}$ as a scalar value for the redesign change in the model.

4 Verification

As mentioned in section 1, when optimizing truss structures the ability of calculating the minimum model volume to compare the optimized results against some Michell benchmarks exists [10]. One of the most famous one is shown in figure 3 along with the original shape solution from Michell so that the visual solution can be verified as well as the quantitative. The minimum volume for this benchmark is calculated by

$$V^{opt} = |\mathbf{p}| r_o \ln \frac{r_o}{r_i} \left(\frac{1}{\sigma_t^u} + \frac{1}{\sigma_c^u} \right) \qquad \sigma_t^u = \sigma_c^u = \sigma^u \tag{8}$$

where the upper stress limits for tension and compression are defined as equal for this experiment. Since the optimality of the resulting design is obviously depending on how rich and diverse the ground structure is filled with elements, the

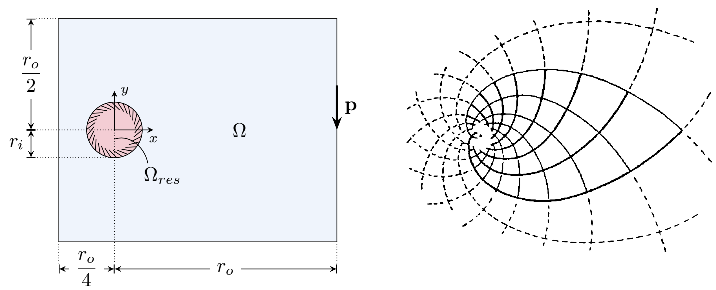


Fig. 3. Model definition of the Michell cantilever with circular support and the original Michell structure on the right [10]

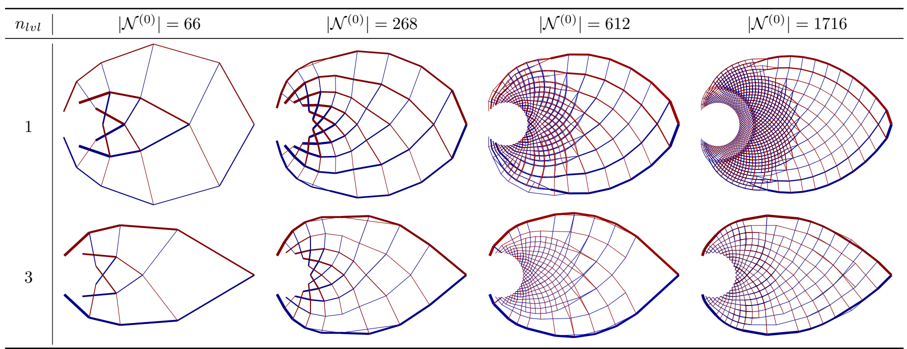


Fig. 4. Optimized results of the Michell cantilever with circular support

experiment parameters are the connectivity level n_{lvl} and the number of nodes distributed in the start design $|\mathcal{N}^{(0)}|$. Comparing the visual results in figure 4 to the original Michell structure in figure 3, the influence of both parameters is clearly visible. While refining the nodal distribution, the outer shape is more and more matching the original structure. At this point, it is important to know that Michell structures are theoretically filled with an infinite number of elements, arranged like the scheme implies in the original image. The visual comparison is therefore more focused on the outer shape and the pure orthogonal crossing of elements inside. Regarding the influence of the ground structure connectivity, especially the finest configuration exposes its effect. Due to the insufficient connectivity level, the ground structure can not provide suitable elements to build the appropriate load paths. An increase of n_{lvl} directly leads to a structure of higher visual optimality. In terms of the quantitative verification, figure 5 reveals the convergence of the model volume to the minimum volume provided by equation (8) if both, connectivity level and number of nodes, are increasing. However, an increase of the connectivity does not necessarily have to result in a higher optimality. Coarser start designs are reaching their maximum connectivity quite fast so that there are no additional elements gained by increasing the level.

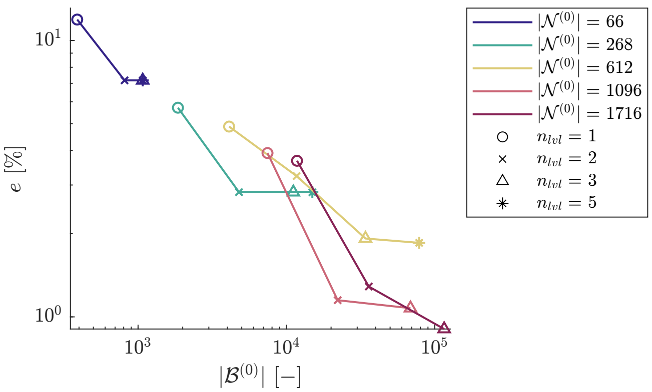


Fig. 5. Relative error e of the optimized solutions V^* to the Michell optimum V^{opt} over the number of elements in the ground structure at $k = 0$

5 Conclusion and outlook

This implementation has shown quite satisfying results with errors lower 1% for such a relatively simplistic algorithm. Nevertheless solutions of high optimality are expensive in terms of computational effort in the generation of the ground structure as well as the optimization itself. Besides the quantitative verification via the model volume, the visual appearance can be a kind of misleading if not observed with sufficient detail. As figure 6 depicts, the finest and richest configuration of the experiment lacks in contouring between the physical and fictitious domain. Thin elements which are attending the more thicker load carrying elements cannot be eliminated due to their similarity. An improved ground structure or post-processing methods would be useful to further clear out the structure. An improved ground structure could be a pre-optimized model. The stress field of the design domain as a continuous Finite Element Model could provide the orientation of the truss elements in the ground structure. Some approaches in this direction are already available with different methods in use, revealing much more clearness in the optimized design [2,9].

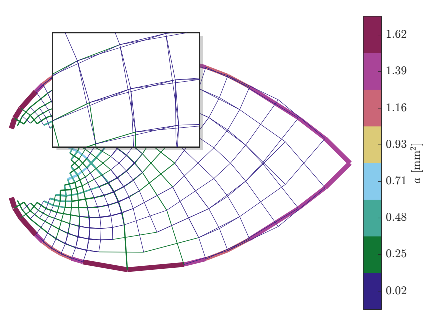


Fig. 6. Magnified visualization of the insufficient contouring in densely filled models

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