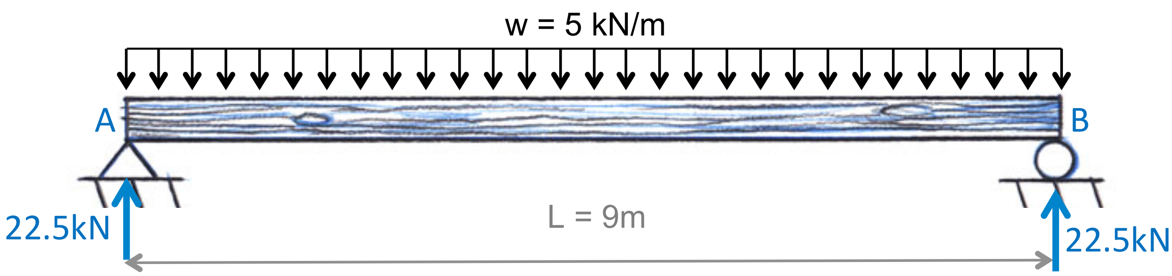
Beam Bridge Example:

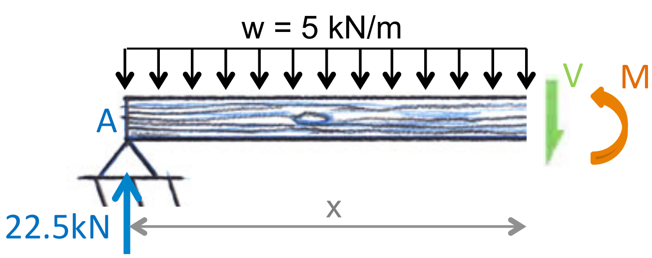
Modeling and Analysis of Beams

We've looked at a cable-stayed bridge, a suspension bridge and a truss bridge. Let's add a beam bridge and see how it compares. I'm going to assume that the span is 9m and the applied load is 5kN/m as we did with the truss bridge.



As with the truss bridge the total downward load will be 5.0kN/m * 9m = 45kN, which will be distributed evenly to each of the supports, so 22.5kN acting upward at each support.

Let's look at a free-body diagram of the beam so we can determine internal shear forces and bending moments. I'm going to cut the beam at a variable distance x. Where do you expect the shear force to be a maximum? How about the bending moment?



Let's look at equilibrium when x=1m, starting with vertical equilibrium.
$\Sigma F_y = 22.5\text{kN} - V - (5\text{kN/m})(x=1\text{m}) = 0;$
V = 22.5kN-5kN = **17.5kN** = **V**

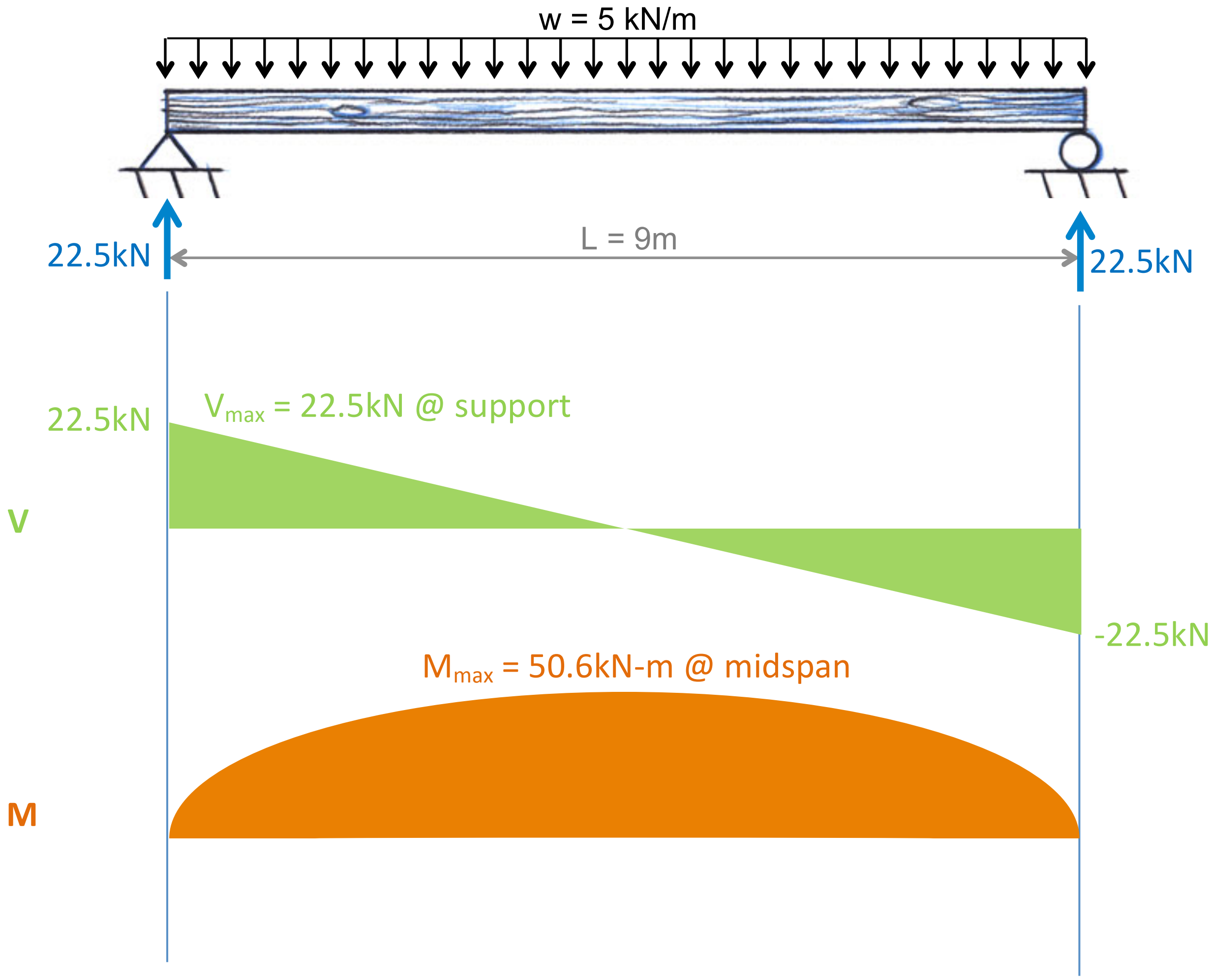
Next let's use rotational equilibrium about the support:

$\Sigma M_A = (V=17.5\text{kN})\ (x=1\text{m}) - M + (5\text{kN/m})(x=1\text{m})((x=1\text{m})/2) = 0$
M = internal bending moment in the beam at x=1m = **20kN-m = M**

What will happen to the shear and bending moment when we increase the distance from the support to x=2m? Let's see.

$\Sigma F_y = 22.5\text{kN} - V - (5\text{kN/m})(x=2\text{m}) = 0;$
V = 22.5kN-10kN = **12.5kN** = **V**
$\Sigma M_A = (V=12.5\text{kN})\ (x=2\text{m}) - M + (5\text{kN/m})(x=2\text{m})((x=2)/2) = 0$
M = internal bending moment in the beam at x=2m = **35kN-m = M**

The shear force decreases and the bending moment increases. Does this help you figure out the location of the maximum shear force and bending moment? You can keep checking different points if you wish using the same approach. A graphical representation of the variation in the shear and moment along the length of the beam is given below.

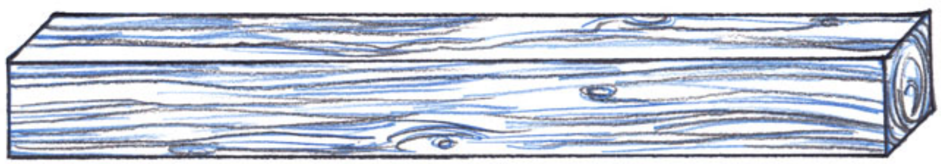


Again, we don't expect you to be able to draw these diagrams but we're hoping you can use them to help you determine the maximum shear and moment values and locations. You can use the beam simulator to produce the diagrams. The nice thing about the diagrams is that you can quickly see the location and value of the maximum shear forces and moments: in this case the maximum shear force is 22.5kN at the supports and the maximum bending moment is 50.6kN-m at the midspan (or middle) of the beam.

The engineer would then need to select the material, size and shape for the beam such that it will not fail in shear, bending, or deflection. Bending often is the most critical piece so I'm going to focus on bending here. And I'm going to use wood as the material. This helps since I can look up (or test) the allowable stress for wood – I'm going to use an allowable stress for wood of 30,000 kN/m^2. The equation we use for wood to determine the stress due to bending is:

$$\sigma_{bending} = \frac{Mc}{I} < \sigma_{allowable} = 30{,}000 kN/m^2$$

This should look familiar from when we looked at tensile and compressive stresses. The main difference is that we're using a bending stress equation rather than an axial stress equation. Note that both I and c are dependent on the shape of the cross-section, making it difficult to solve directly for one of them. One way around this is to use a value called the section modulus or S, which equals I/c. Let's assume for now that we're going to use a solid rectangular cross-section for this bridge. The moment of inertia of a rectangular cross section is $I=bh^3/12$ and $c=h/2$ so the section modulus for a rectangular cross-section is $bh^2/6 = S$.



This still leaves us with two unknowns but we can make some assumptions. Let's try a beam with a width (b) of 200mm. Using this assumption we can calculate the Section Modulus (S) and then using the allowable stress equation, determine the required height of the cross-section.

$$S = \frac{(0.2m)(h^2)}{6} = 0.033h^2$$

$$\sigma = \frac{Mc}{I} = \frac{M}{S} = \frac{50.6kN-m}{0.033h^2} < \sigma_{allowable} = 30{,}000kN/m^2$$

h = 0.225m = 225mm

One option for a beam bridge is to use a wood beam with a solid rectangular cross-section that is 200mm wide x 225mm high.