Modeling and Analysis of Columns
Washington Monument

The Washington Monument is an obelisk designed and built in honor of President George Washington. The Washington Monument, completed in 1888, is the tallest obelisk in the world that is completely constructed of stone, a combination of granite, marble and bluestone gneiss. Robert Mills, an architect, was selected to design the Washington Monument through a design competition.

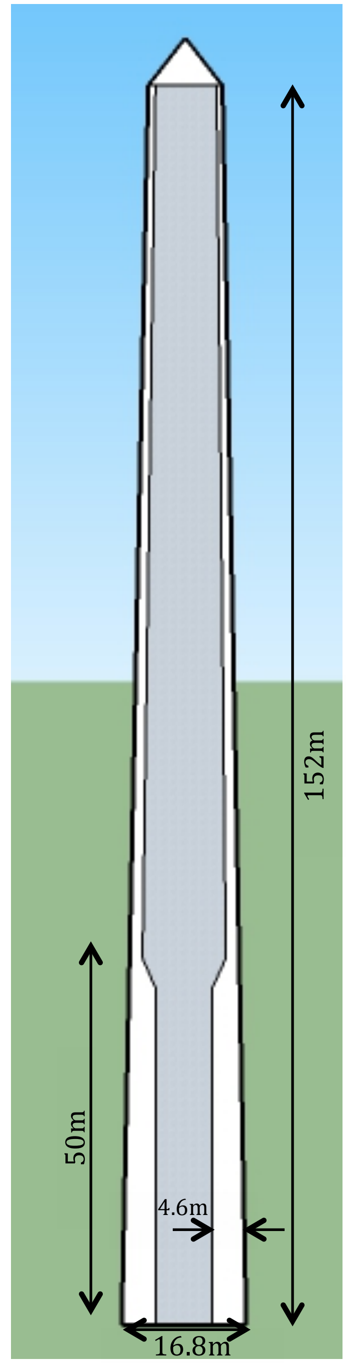
The Washington Monument has a square, tubular, tapering cross-section, with thicker walls at the base to help resist the weight of the stone above. It is considered a pure column because it really only resists compressive forces. While it is subjected to wind loads as well, these loads are very small relative to the vertical compressive forces resulting from the weight of the stone. An earthquake in 2011 did cause some damage to the Washington Monument but it has been repaired and re-opened in May of 2014.

David P. Billington, an emeritus professor of civil engineering at Princeton University classifies the Washington as 'Structural Art' and has written and spoken extensively about it. To learn more about the Washington Monument and other works of Structural Art check out the following book and lecture:

- *The Tower and the Bridge* by David P. Billington.
- Lecture by David P. Billington about the Washington Monument and Eiffel Tower: http://alumni.princeton.edu/learntravel/lectures/videodetail/index.xml?videoid=267

Dimensions (as measured by the National Park Service, and listed on wikipedia):

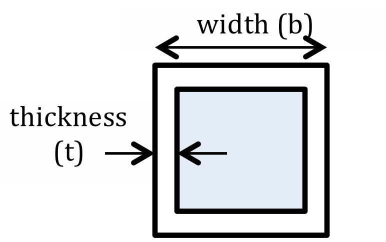
- Total height of monument: 555 ft 5 1/8 in (169 m)
- Height from lobby to observation level: 500 ft (152 m)
- Width at base of monument: 55 ft 1 1/2 in (16.8 m)
- Width at top of shaft: 34 ft 5 in (10.5 m)
- Thickness of monument walls at base: 15 ft (4.6 m)
- Thickness of monument walls at observation level: 18 in (457 mm)
- Weight of foundation: 41,341 tons
- Total weight of monument (including foundation): 90,854 tons (82,421 t)
- Total weight of monument at base (without foundation): 49,513 tons = 440,490kN
- Weight of the capstone at top: 3300lb = 14.7kN



Dimensions at 50m:
b=14.8m
t=2.6m

Dimensions at base:
b=16.8m
t=4.6m

Dimensions at midheight:
b=13.4m



So how do we expect the Washington Monument to fail? In compression or buckling? And since it is tapered, where should we check the compressive stress and buckling load?

Let's start with compression. Since the compressive force applied to the Washington Monument is due primarily to the self-weight of the stones used to construct it the force will be highest at the base so one critical section to check is at the base. To calculate the compressive stress at the base we simply need to take the total weight of the stone at the base, which is 440,490kN, and divide by the cross-sectional area at the base. The cross-sectional area at the base is that of a square tube with an inner dimension of 7.6m and an outer dimension of 16.8m.

<u>Compressive stress at base</u>:

Load, P, at base = 440,490kN
Cross-sectional Area = 16.8m*16.8m – 7.6m*7.6m = 224.5m^2
Compressive stress at base = 440,490kN/224.5m^2 = 1962kN/m^2 = 1962kPa

kPa = kiloPascals, a unit of stress = kiloNewtons/meter^2
The allowable compressive stress of stone is approximately 100,000kPa or 100x10^6 Pa, which is much higher than the applied compressive stress so I do not expect the base of the Washington Monument to crush.

Another location to check, however, is at the location ~50m above the base where the cross-sectional area of the monument is smaller. At this location the load due to the stone above is approximated as 270,000kN. Thus at this location the compressive stress maybe calculated as follows.

Compressive stress at location 50m above base:

P=270,000kN

Cross-sectional area = 14.8m*14.8m-9.6m*9.6m = 126.9m^2

Compressive stress at 50m from base = 270,000kN/126.9m^2 = 2,128kN/m^2=2,128kPa

The compressive stress at a location 50m up from the base is higher than at the base due to the reduced size of the cross-section but the stress is still well below the allowable compressive stress of the stone. So I do not expect the monument to fail there either.

What about buckling? Do you think the Washington Monument will fail in buckling? Let's check. But where should we check? It is a tapered column so the cross-section varies. While there are more precise equations we could use I'm going to make an approximation here and use the cross-sectional dimensions at the midheight of the monument to calculate the moment of inertia. Note that I'm using 2L for the length of the column/monument rather than L; this is because the Washington Monument is fixed at the base and free at the top rather than pinned at both ends (this type of fixity causes a different deformed shape when it buckles).

Critical buckling load for the Washington Monument:

$E_{granite}$ = 80x10^9Pa=80x10^6kPa

L = 169m

$I = \frac{\mathbf{13.4^4-9.6^4}}{\mathbf{12}} = \mathbf{1900}\text{m}^4$

Allowable load before buckling = $\frac{\pi^2 EI}{(2L)^2} = \frac{\pi^2(80x10^9Pa)(1900m^4)}{(2*169m)^2}$=13.13x$10^6$kN

This load is much higher than the applied load due to the weight of the Washington Monument so I don't expect any buckling issues either.

The weight of the stone of the Washington Monument is so great that compression is a bigger issue than buckling. Note that the Washington Monument was damaged during the 2011 Virginia Earthquake due to lateral or horizontal motions rather than due to vertical compressive loads (see Concept 6 for a discussion of the earthquake response of structures).