

So I'm going to look at a simple truss. And my goal is to find the forces in all the members. So the force in member AC, AB, and BC. To do that, I'm first going to find the reactions. You actually don't always have to find the reactions in a truss, but externally we can find the reactions. So let's see, I'll look at these three reactions. So I've put it on a pin supported A and a roller supported B. So I can use equilibrium.

So if I sum my moments about this point A, that just means again that I don't want it to rotate about point A. Get this 100 kilonewton force. 100 kilonewtons acting at 4 meters, trying to rotate it one direction and this reaction force-- let's call it B_y , A_y , and A_x . The B_y is trying to counteract that if we're rotating about this point A.

So that is minus B_y times 3 meters all equals 0. Hey I can solve for B_y . B_y is going to be 100 kilonewtons times $4/3$, 4 meters over 3 meters, or 133.3 kilonewtons. So that is this reaction force. So I'm going to design that reaction to be able to support that much load. I can then use horizontal equilibrium and vertical equilibrium, summing forces horizontally and vertically, to solve for the reactions at A_x and A_y .

I'm looking at this. The only things acting in the Y direction are A_y and B_y , so we can see that those have to be equal and opposite. My negative here just means that I assumed upward and it's really a downward force to balance out the reaction at the other side. And then A_x and 100 are the only things acting. So A_x , or the horizontal reaction at this point, has to be able to support 100 kilonewtons.

And again, I have a negative on it, just meaning it's going in the opposite direction. So that's my starting point. But what I really want is to look internally at the truss. So I want to draw a free body diagram so that I cut through some of the members. And that's going to expose the forces on the bars. So really what I'm doing here is drawing a free body diagram for joint C. If I look at that joint, I've got this is member AC and here is member BC. This joint has the load on it, so we need to put that on our free body diagram. 100 kilonewtons. And I have these two forces, F_{AC} and F_{BC} .

Now I always draw these pulling away from the joint, which means I'm assuming that it's tension, if I get a negative, it means it's pushing on the joint and it's in compression. And that's going to be another check. Looking at this truss with this force pushing this way, I expect BC to be in compression. So I

should get a negative quantity when I solve for this one. And that one will have to be intention. But we can see once we look at the numbers.

So looking at this and trying to do equilibrium, I have two options here, summing forces horizontally and vertically. I'm going to sum forces horizontally first because I only have two things acting, and one of them is unknown. So I only have FAC in my equation. If I sum things horizontally, I get 100 kilonewtons from the load, that's the positive direction. And I'm going to subtract FAC, and I could go in and find this angle. But I can also use the geometry here. This happens to be a 3:4:5 triangle, one of my favorite to use, because I can look at it and see that the ratio acting horizontally is going to be 3/5.

You could also find this angle and take the cosine of it, but I'm going to use that this is 3/5. And that whole thing equals 0. Make sure I did that right. This one going to the right, that one going to the left, times 3/5, I can solve for FAC. And FAC is going to be 100 kilonewtons. Change that. 100 kilonewtons times 5/3. So FAC is a positive quantity. You expected it to be positive. And it's 167 kilonewtons. So that tells us this internal force in here, FAC, is 167 kilonewtons.

I can then progress and do is sum forces in the y direction. Sum force in the y direction. I have just these two forces. They're both acting downwards, so I'll assume downward is positive. FBC plus FAC. And now the ratio of this based on this geometry is 4/5. And that all equals 0. I know FAC, so I can put that in for that value and solve for FBC is going to equal-- I'm going to move it to the other side. Minus 167 times 4/5, and that is FBC is going to be a negative 133 kilonewtons.

So that tells me the quantity there. So now I know this force. That's a positive. This one's going to be pushing on the joint, is compression 133. I know two of the bar forces. I could now go down to another joint, this joint, and solve for my final member force. So I have a reaction of 133.3 kilonewtons. This one's actually going to be really easy. FBC, we know it's pushing on the joint. This is FBC is 133.3 kilonewtons. Those two balance. That's good. Makes me happy.

My only unknown here is this one, FAB. And looking at this joint, I would sum force this horizontally to solve for this. And there's nothing else acting horizontally. So this one actually has to be 0. So the force in member AB is 0. We still needed our truss to hold everything together. And for different load cases, it would probably have a force in it. For this load case, it happens to be what we call a zero force member.

But our goal is to solve for the forces, and we've gotten the three forces. FAC in tension, FBC in compression and a zero force member.

So we looked at this truss earlier when we calculated the force in each of the three bars, or members of the truss. And we found these three forces. So FAC has a tension force of 167 kilonewtons. FBC 133 kilonewtons. And FAB for this loading we found to be a zero force member. So our next goal is to design this truss, so actually figure out what type of material we want to use and the size we want to use and the shape. So different scenarios.

You could try lots of different things. I'm going to start out with some assumptions. So I'm going to assume that we're going to use steel. That'll help us pick allowable values and modulus of elasticity when we need it. I'm also going to make the assumption that I'm going to use a square cross section for the truss. So I'm going to assume that my cross section is square. That I'll make it a little easier to solve for dimensions.

So if nothing else, give us a starting point on the approximate size, and then we can start playing with hollow tubes that are different cross sections. So those are my assumptions. I'm also going to assume that all three members are going to have the same size. This is a small enough truss that it's probably, construction wise, easiest to just use the same member. We could use anything really for AB since it doesn't carry any force.

So I'll design the truss based on the forces in AC and BC. AC has the higher force, so let's start there. I'm going to look at the force in member AC, we know, is 167 kilonewtons. This member is in tension, so if we remember back to tension, what we're going to have to do is look at the stress. So the stress in this member AC, since its in tension is just the force over the area, and the force in this case is 167 kilonewtons divided by the area, which is what we're looking for, so I'll just leave that in there for now.

And we're going to keep that less than the allowable stress, which in this case is the allowable stress for steel. I can look up the yield stress for steel, which is 250,000 kilonewtons per meter squared. I'm going to divide that by 2. I'm going to use a factor safety of 2. So I'll want to keep my stress is less than 125,000 kilonewtons per meter squared.

This equation is going allow me now to solve for this cross sectional area. So I can find that the cross sectional area is going to have to be greater-- moving this over and stress to the other side-- 167

kilonewtons over 125,000 kilonewtons per meter squared. We can solve for that area. That area ends up being 0.0013 meters squared.

Knowing that I have a square cross section, I can now solve for the dimensions since the area of a square is just b squared. So this is going to equal to b squared, and I can solve for b . And b ends up being 36.5 millimeters.

So steel bar that has a square cross section with dimensions 36 by 36.5 by 36.5 would be sufficient for AC. Now I want to look at member BC. One of the key pieces I missed here is that this is a negative 133 because this member is in compression. So why does that make a difference? In tension, I'm just going to pull on it and look at the stress.

As I push on this one, since it's in compression, it could yield by a compression failure. Or it could yield by buckling. And that's the key with this one. So I'm going to look, I need also to look at buckling with member BC and make sure that's OK. The stress should be fine. Steel has the same allowable stress in tension and in compression, and this has a smaller force. So this cross section would be sufficient for a compression failure. But you would want to make sure you checked buckling.

So P critical in this case will be $\pi^2 EI$ over L squared. The length of this member, it's really just acting as a column in this truss, the length was 4 meters. So I'd use that. I would look up the modulus of elasticity of steel. Moment of inertia is a function of the cross section. So a function of this shape, which is going to be b to the fourth over 12.

And then I can solve for this. It turns out to be 219 kilonewtons, which is less than the 133. But my factor of safety is going to be smaller than this, too. Your factor of safety would be 219 over 133, so it would be slightly less than 2. So I would expect this to be the critical member in this truss. So this to fail before I have a tension failure.

I feel pretty comfortable with this size, though. And that's how I would choose the size to use for my truss.