

Owl's Tree Example: Modeling and Analysis of Columns



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Can Owl's tree support the additional load from the planned library? How many books can be put in the library?

Owl's "library" tree is:

- 35m tall
- 0.5m in diameter
- weighs 85kN

For this analysis, I will assume that the cross-section of the tree is constant, circular, and solid. Thus the cross-sectional properties of Owl's tree are as follows:

$$A = \text{cross-sectional area} = \pi r^2 = \pi(0.25\text{m})^2 = 0.20\text{m}^2$$

$$I = \text{moment of inertia} = \frac{\pi r^4}{4} = \frac{\pi(0.25)^4}{4} = 0.0031\text{m}^4$$



The compressive stress at the base of Owl's tree with no additional load (σ_{tree}) is the total force divided by the cross-sectional area as follows:

$$\sigma_{\text{tree}} = 85\text{kN}/0.20\text{m}^2 = 425 \text{ kN/m}^2$$

We know through testing that the allowable stress ($\sigma_{\text{tree allowable}}$) of Owl's tree is 30,000kN/m² so I don't think Owl's tree will crush:

$$\sigma_{\text{tree allowable}} = 30,000 \text{ kN/m}^2$$

$$E = \text{Modulus of Elasticity} = 7 \times 10^9 \text{ Pa}$$

Now let's check buckling.

The buckling load for Owl's tree is:

$$P_{\text{cr,tree}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (7 \times 10^6 \text{ kN/m}^2) (0.0031\text{m}^4)}{(35\text{m})^2} = 175 \text{ kN}$$

The buckling load, or load at which the tree will buckle, is greater than the self-weight of the tree, which is 85kN, so Owl shouldn't have a buckling problem either. Note that I used the full height of the tree, 35m, in the buckling equation, which means I'm assuming Owl will put the library at the top of the tree, the worst-case scenario. The library doesn't actually get put there, which is a good choice. Note that I'm also assuming that the base of the tree and the top of the tree are both pinned connections for simplicity.



The good news is that the tree that Owl has selected can safely support its own weight. But how much additional weight will it be able to support? It is much closer to buckling than to failing in compression so let's focus on buckling.

The tree weighs 85kN and it will buckle when the load reaches 175kN. That means that Owl can add 90kN. But how many books is that? Plus the structure to support the books?

Let's assume that the structure required to support Owl's library plus all of the furniture will weigh 30kN. That leaves 60kN for books. Assuming an average book weighs about 0.025kN means Owl can include quite a few books in the library, about 2400 books: $60\text{kN} / 0.025\text{kN/book}$. That's a lot of books but Owl does love to read...

