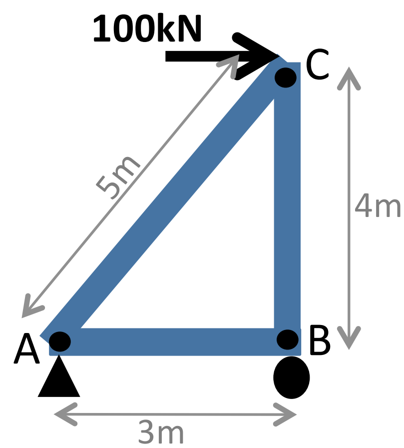
Simple Truss Example:

Modeling and Analysis of Trusses

THAYER SCHOOL OF ENGINEERING AT DARTMOUTH

Simple Truss – Calculate Forces



Let's start with a simple, three-member truss and calculate the force in each member of the truss when a load of 100kN is applied.

I'm going to start by calculating the support reactions. Note that you don't always have to calculate the support reactions to determine the member forces. I could have started by looking at joint C and not calculated the support reactions but I'm going to calculate them so we have a complete picture of the forces and what is happening in this truss.

I'm using a triangle to represent a support that provides both horizontal and vertical resistance and a circle to represent a support that provides only vertical resistance. Remember that we need 3 external support forces in order for a structure to be stable. I've named the three reaction forces A_x, A_y, and B_y.

I'll start with rotational equilibrium, using joint A as my point of rotation. Thus:

$$\Sigma M_A = 100kN\ (4m) - By\ (3m) = 0$$

$$B_y = 100kN\ (4m)\ /\ (3m) = 133.3kN$$

Now I'm going to start looking at each of the joints and enforcing equilibrium there. I'll start by calculating the angle, θ. Note: atan means arctan.

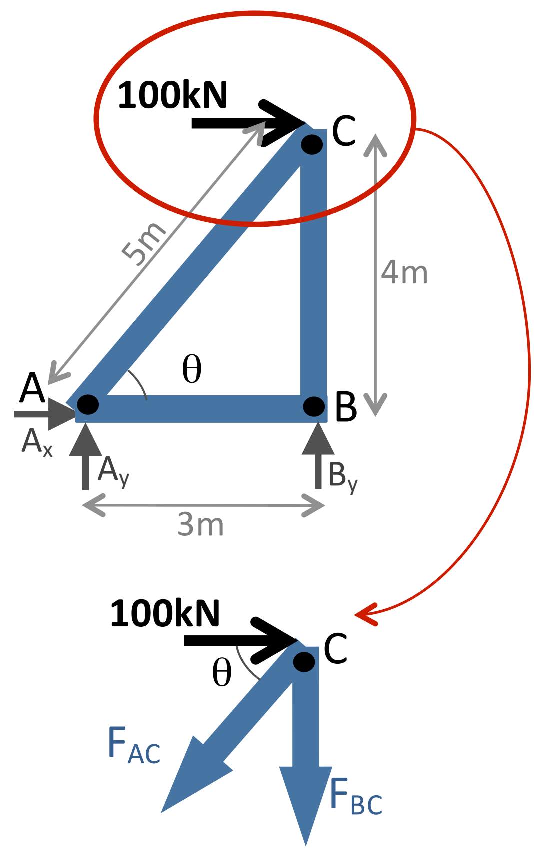
$$\theta = \text{atan}(4m/3m) = 53.1°$$

To calculate the cosine and sine I can use the angle but alternatively, I can use geometry and the fact that this is a 3-4-5 triangle:

$$\cos(\theta) = \cos(53.1°) = 3/5$$

$$\sin(\theta) = \sin(53.1°) = 4/5$$

Now let's look at each joint individually and draw a free-body diagram for each joint, starting with joint C. When I draw the arrows representing the member forces, I always draw the arrows pulling away from the joint, thus assuming the members are in tension. If I get a negative value then I know that that member is in compression.



Free-body diagram of Joint C

Now we can apply equilibrium to this joint, joint C. I'll start with equilibrium in the horizontal direction:

$$\Sigma F_x = 100\text{kN} - F_{AC}\cos(\theta) = 100\text{kN} - F_{AC}(3/5) = 0$$

F_{AC} = 100kN (5/3) = 166.7kN (Note that I get a positive value so member AC is in tension, which I expected).

Next we can use vertical equilibrium to determine the force in member BC. I expect this member to be in compression so I'm expecting a negative value.

$$\Sigma F_y = -F_{BC} - F_{AC}\sin(\theta) = -F_{BC} - (166.7\text{kN})(4/5) = 0$$

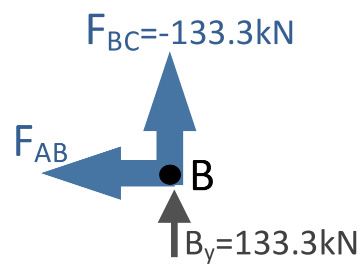
$$F_{BC} = -166.7\text{kN}(4/5) = -133.3\text{kN}$$

Now we just need to find F_{AB} and we'll know all the forces in the truss. I could look at either joint A or joint B. I'm going to look at joint B because there are fewer forces acting there but I should get the same answer if I look at joint A and I could use joint A to check my results.

Using horizontal equilibrium we find:

$$\Sigma F_x = -F_{AB} = 0$$

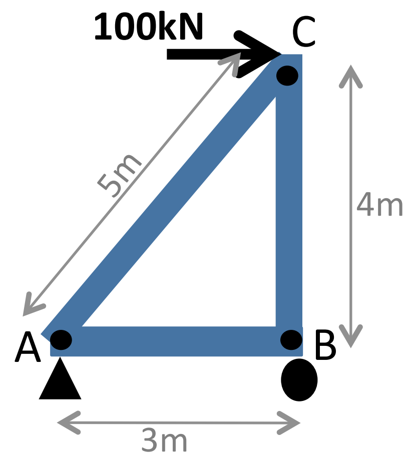
F_{AB} carries no force for this loading condition. This type of member is sometimes referred to as a zero-force member. It is needed in the truss in order for the truss to be stable but carries no force.



Free-body diagram of Joint B

Simple Truss – Design

We found the forces in our simple truss but now I want to design the size and material for the members such that they are adequate to resist the forces we calculated.



The forces in the truss are as follows:

$F_{AB} = 0$
$F_{AC} = 166.7\text{kN}$
$F_{BC} = -133.3\text{kN}$

Let's design a square cross-section for the members of the truss. I'll use the same size for all of the members; I could use a much smaller size for member AB since it doesn't carry any force but for a truss this size it is probably better to use the same size and material for all of the members.

It looks like the critical truss member is member AC since the force in member AC is the highest so let's start there. To determine the size needed for this member we'll compare the stress to the allowable stress. Since we've decided to try to use a steel member we know the yield stress, 250,000kN/m^2. Let's use a factor of safety of 2 so the allowable stress is 125,000kN/m^2.

Stress in the member = force/area = 166.7kN/A < allowable stress = 125,000kN/m^2

$A > 166.7\text{kN}/125{,}000\text{kN/m}^2 = 0.00133\text{m}^2$

$A_{square} = b^2$, where b is the length of one side of the square

$b = \sqrt{0.00133m^2} = 0.0365\text{m} = 36.5\text{mm}$

A square cross-section made of steel that is 36.5mmx36.5mm is adequate for member AC and AB but will it work for member BC? The force in member BC is lower at -133.3kN but member BC is in compression thus member BC has the potential to buckle so we should check to make sure this is adequate to resist buckling. We can look up the modulus of elasticity (E) for steel and the formula for the moment of inertia (I). We know that the length of truss member BC is 4m.

$E_{steel} = 200 \times 10^6 \text{ kN/m}^2$

$I_{square} = b^4/12 = (36.5^4)/12 = 177.5\times10^4\text{mm}^4 = 1.775\times10^{-6}\text{ m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\text{x}10^6\text{kN/m}^2)(1.775\text{x}10^{-6}\text{m}^4)}{(4\text{m})^2} = 219.0\text{ kN}$$

We find that the critical buckling load is 219kN, which is greater than the compressive force in member BC so our steel member with a square cross-section that is 36.5mm x 36.5mm is adequate and we have a factor of safety against buckling of 219kN/133.3 = 1.64.