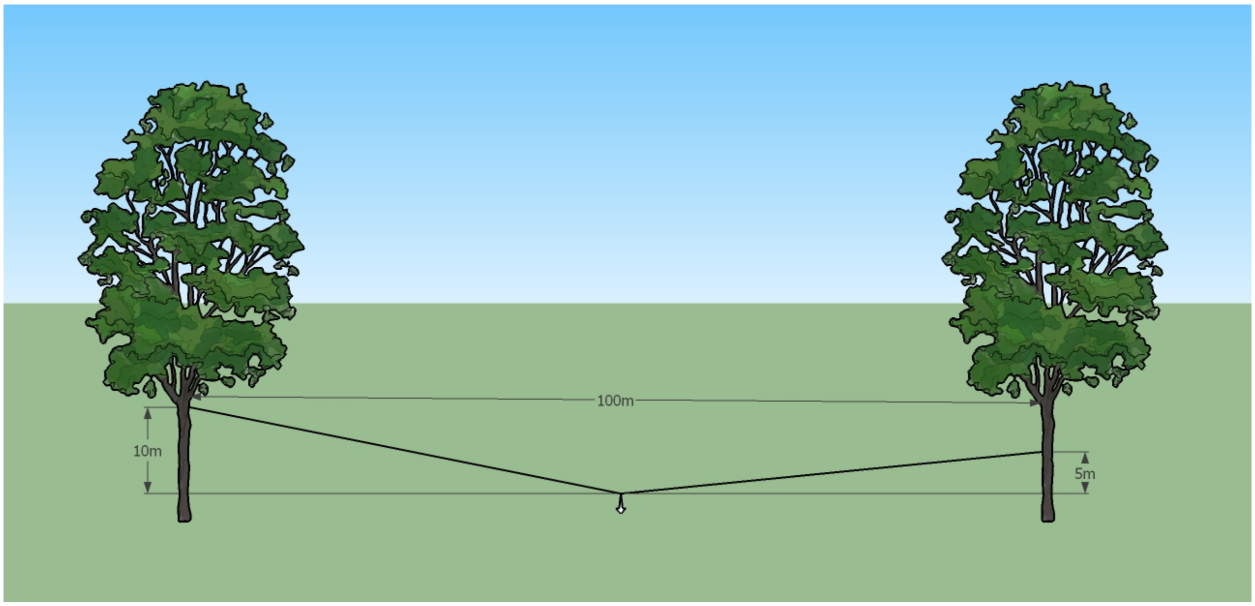
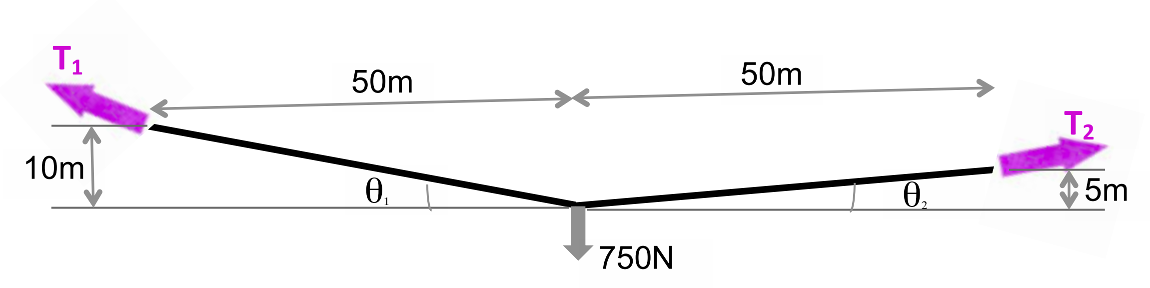
Modeling and Analysis of Funicular Forms
Zipline

My children want a zipline in the backyard. We've found a site and have a design for the supports but need to determine the cable type and diameter. We'll attach the cable to two trees that are 100m apart with a 5m drop in elevation. I'm going to analyze the zipline when a person is at the middle, assuming a sag of 5m and assuming the person weighs 750N. My model for the zipline is shown below:





Our goal here is to determine the type and diameter of the cable for the zipline.

Let's first determine the angles, θ_1 and θ_2:

θ_1 = atan(10m/50m) = 11.3°

θ_2 = atan(5m/50m) = 5.7°

Knowing the angles we can use equilibrium to find the tension forces in the cable. I'll start with equilibrium in the horizontal direction:

$\Sigma F_x = T_2 \cos 5.7° - T_1 \cos 11.3° = 0$

$\Rightarrow T_1 = T_2 \cos 5.7° / \cos 11.3°$

$T_1 = 1.0147\ T_2$ *T_1 needs to be slightly higher due to the difference in the angles.

Now let's apply equilibrium in the vertical direction:

$\Sigma F_y = T_1 \sin 11.3° + T_2 \sin 5.7° - 750N = 0$

Substituting $T_1 = 1.0147\ T_2$:

$T_1 \sin 11.3° + 1.0147 T_1 \sin 5.7° - 750N = 0$

We can solve for T_1:

$0.297 T_1 = 750\ N$

T_1 = 2524.5 N (note that the force in the cable is much higher than the applied load due to the shallow angle of the cable).

Since T_1 is the higher of the two forces, we'll use it, 2524.5 N, as the critical force in the cable.

Now we need to design the actual cable. Let's assume we're going to use steel with an allowable stress of 150,000kN/m^2. Using this value we can figure out how large the cable needs to be since:

Force/Area < Allowable Stress

Area > Force/Allowable Stress = 2.53 kN / 150,000 kN/m^2

Area = 1.683×10^{-5}m^2 = 16.83mm^2

Assuming a circular cross-section we can calculate the diameter of the cable required:

Area = 16.83mm^2 = π * radius^2

Radius = 2.3 mm

Diameter = 4.6mm

I'll feel better if we include a safety factor; I'm going to use a factor of 3, which means I'd specify the cable diameter to be 4.6mm*3 or 13.8mm.

The cable should be made of steel with an allowable stress of 150,000kN/m^2 and a diameter of 13.8mm. This is a fairly large cable; I could use a higher strength steel to reduce this diameter.