

## Bone/Femur Example: Modeling and Analysis of Columns

The femur or thigh bone is typically the strongest and longest bone in the human body. One common way to model a femur bone is as a column.

How much of a compression load can a typical femur support before it fractures?

Femur measurements for a typical human:

Length =  $L = 47\text{cm}$   
Outer Diameter =  $3.5\text{cm}$   
Outer Radius =  $r_o = \text{outer diameter}/2 = 1.75\text{cm}$   
Inner Diameter =  $2.0\text{cm}$   
Inner Radius =  $r_i = \text{inner diameter}/2 = 1.0\text{cm}$

Calculated properties:

To calculate the cross-sectional area and moment of inertia of the femur I'm going to assume that it has a hollow, circular cross-section.

$$A = \text{cross - sectional area} = \pi(r_o^2 - r_i^2) = \pi(1.75\text{cm}^2 - 1.0\text{cm}^2) = 6.48\text{cm}^2$$

$$I = \text{moment of inertia} = \frac{\pi(r_o^4 - r_i^4)}{4} = 6.58\text{cm}^4$$

Published material properties for bone (<http://www.engineeringtoolbox.com/>):

Modulus of Elasticity =  $E = 10 \times 10^6 \text{ kN/m}^2$

Allowable Compressive Stress of bone =  $\sigma_c = 170 \times 10^3 \text{ kN/m}^2$

**How much load can the femur support before it fails in compression?:**

$$\begin{aligned} \text{Allowable load in compression} &= \text{Allowable compressive stress of bone} * \text{cross-sectional area} \\ &= \sigma_c * A = 170 \times 10^3 \text{ kN/m}^2 * 6.48\text{cm}^2 * (1\text{m}/100\text{cm})^2 = 110,000\text{N} = 110\text{kN} \end{aligned}$$

**How much load can the femur support before it fails in buckling?:**

$$\text{Allowable load before buckling} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (10 \times 10^6 \text{ kN/m}^2) (6.58\text{cm}^4) (\frac{\text{m}}{100\text{cm}})^4}{(47\text{cm})^2 * (\frac{1\text{m}}{100\text{cm}})^2} = 29\text{kN}$$

Based on these numbers a typical femur will fail or break in buckling before compression when the load reaches 29kN. 29kN is a fairly high load, much greater than the weight of a human. So how do femurs break? They typically break due to a horizontal load or a combination of vertical and horizontal loads such as from a fall or some type of impact. A column in compression is more likely to fail when a lateral load is applied.