Bone/Femur Example:
Modeling and Analysis of Columns

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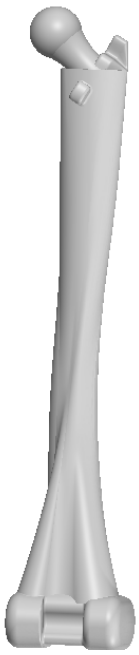
The femur or thigh bone is typically the strongest and longest bone in the human body. One common way to model a femur bone is as a column.

How much of a compression load can a typical femur support before it fractures?



Femur measurements for a typical human:

Length = L = 47cm
Outer Diameter = 3.5cm
Outer Radius = r_o = outer diameter/2 = 1.75cm
Inner Diameter = 2.0cm
Inner Radius = r_i = inner diameter/2 = 1.0cm

Calculated properties:

To calculate the cross-sectional area and moment of inertia of the femur I'm going to assume that it has a hollow, circular cross-section.

$\boldsymbol{A = \text{cross} - \text{sectional area} = \pi(r_o^2 - r_i^2) = \pi(1.75cm^2 - 1.0cm^2)}$=6.48cm^2

$\boldsymbol{I = \text{moment of inertia} = \frac{\pi(r_o^4 - r_i^4)}{4}}$= 6.58cm^4

Published material properties for bone (http://www.engineeringtoolbox.com/):

Modulus of Elasticity = E = 10x10^6 kN/m^2
Allowable Compressive Stress of bone = σ_c = 170x10^3 kN/m^2

How much load can the femur support before it fails in compression?:

Allowable load in compression = Allowable compressive stress of bone * cross-sectional area
= σ_c *A = 170x10^3 kN/m^2 * 6.48cm^2 *(1m/100cm)^2 = 110,000N = 110kN

How much load can the femur support before it fails in buckling?:

Allowable load before buckling = $\frac{\pi^2 EI}{L^2} = \frac{\pi^2(10x10^6 kN/m^2)(6.58cm^4)(\frac{m}{100cm})^4}{(47cm)^2 * (\frac{1m}{100cm})^2}$=29kN

Based on these numbers a typical femur will fail or break in buckling before compression when the load reaches 29kN. 29kN is a fairly high load, much greater than the weight of a human. So how do femurs break? They typically break due to a horizontal load or a combination of vertical and horizontal loads such as from to a fall or some type of impact. A column in compression is more likely to fail when a lateral load is applied.