

So I hope through experimentation you were able to get a better feel for columns and maybe discover what factors effect how a column will behave. So how do columns fail? It turns out there's two different ways a column can fail. They can buckle. Buckling just means it displaces laterally or horizontally, in this case. The middle is displacing.

Long, slender columns like this one will tend to buckle. You could also have a compression failure. And a compression failure is actually the material itself crushing or yielding. These shorter, wider columns will fail in compression. So how do we predict failure? I'm going to start with a compression failure. And I'm going to use a small blue foam beam.

When I push on it, if I push hard enough I can shorten the column. That downward force when I push on it, I call that a compressive force. And I often model that with a downward arrow. And, again, if I push hard enough, I can cause failure. But I can't push hard enough on this one to cause failure. I did bring a piece of chalk, also a fairly short, wide column with a circular cross section.

And I'm not strong enough to push it to cause failure, but I think if I use my hammer I can apply a large enough load to cause it to fail. And that will mean that my applied force and stress is higher than the allowable.

When I apply compressive force to a column, it causes a stress on the column. What is stress? The definition is a force per unit area. But what does that mean? If we use this blue foam column as an example, when I push on the top that's applying a compressive force, I usually denote that with a capital P and a downward arrow. That's my force at the top. And that force ends up getting distributed throughout the column and over the column area as it gets internal stresses.

Since this is a column with a circular cross section, that cross sectional area, which I denote as a capital A, will be π times this radius squared. So that's easily calculatable. And then I can use those values to calculate my compressive stress. So my compressive stress would be the applied load, P, divided by the cross sectional area.

The units will be force over area, or newtons per meter squared, as an example. Failure in compression, which is a yielding or a crushing of the material is something we want to avoid. So

engineers will look up allowable stresses for different materials. So different materials will have different values for allowable stress. Wood has a much lower allowable stress than say steel.

So when I look up those values, I can calculate an applied stress that I expect in my column and compare it to the allowable stress for the material I expect to use, and that will help me design and analyze my column.

Now let's consider buckling. So what is buckling again? Buckling is when I push on a column and it displaces laterally or horizontally. But when will that happen? And what are the factors that will effect when a column will buckle? Will a longer column buckle at a higher or lower load than a shorter column? And how much of a factor is the length in the behavior?

Other things that we can consider are material type, cross sectional shape, and cross sectional area. Which of these will have an effect on the load at which a column will buckle? The load at which a column will buckle is referred to as the buckling load. It's also referred to as the Euler critical buckling load. It's named after mathematician Leonard Euler who discovered or derived the formula in 1757.

And if we look at this equation, the equation has $P_{critical} = \frac{\pi^2 EI}{L^2}$. E is a material property, so different materials will have different modulus of elasticity values, which is what E stands for. So the modulus of elasticity, or E, for steel will be much higher than the modulus of elasticity for wood, as an example.

Moment of inertia, or I, in that equation is a property of the section type. So it will be the section shape really is what the moment of inertia is a function of. We'll discuss moment of inertia more when we discuss bending. For now it's fine to just understand that there's a relationship between shape and the moment of inertia. As an example, a hollow tube will have a much higher moment of inertia than a solid tube with the same cross sectional area. Thus the hollow tube will be able to support much load before it buckles.

And then L is the length of the height of the column. So when we look at a column, there's two main things we look at. We look at compressive stress and buckling load. We're going to go over the equations that go along with those. I'll just draw a simple column here with a rectangular cross section. So that's a cross section. I'll give it some height.

So this is my column. This would be my cross sectional area. And it will have a certain height or length. I

usually use length for the column. I call that L . So the two different things we look at, again, are compressive stress and buckling load. And we have two equations that govern that. So when I'm looking at compressive stress, I usually use the symbol σ .

And to calculate the compressive stress in a column it would be force over area. So that would be applied force. So there'd be a downward applied force, P say. And that's my applied force. And then the area is a cross sectional area, or A is a cross sectional area. That would be in meters squared, inches squared, some type of length squared.

We calculate this value. So for a given column we'll typically know the cross sectional area and we'll know the applied force. And we'll know the material or we'll be designing the material. So I want to keep this stress less than the allowable stress of the material. So if we're dealing with steel, it would have a higher allowable stress than say wood, and we would play around with these numbers to make sure that it didn't fail in compression.

The other thing we have to look at is buckling. We usually look at the buckling load, which I'll refer to as $P_{critical}$. And that value is $\pi^2 EI / L^2$. I'm not going to derive that equation here, but it was derived in the 1700s. π is a constant. We have E , which is our modulus elasticity.

Just as a reminder, that's a function of the material. So different materials will have different modulus of elasticity values. Steel, again, will have a higher modulus than say wood. We have moment of inertia. Moment of inertia is a function not of the material, but of the cross section. So the shape. So this rectangular cross section will have a certain moment of inertia and that'll be different than if it were circular or hollow tube.

And then L is just the length. So using these equations or using this critical buckling load, if I know the material I can look up E . If I know the shape of the cross section, I can look up the moment of inertia. And I'll typically know the height. I can calculate this critical buckling load. And I'll want to make sure that the applied load is less than this critical buckling load.

So you'll want $P_{applied}$ to be less than that critical buckling load so that you can make sure it doesn't buckle. So those would be the two things we'd look at if we're designing a column.

Go ahead and experiment with the column simulator in the next segment of the course to see if you can build a tall but strong column. And check out the column case studies. In addition to more standard columns and buildings, trees, bones, lamps, and many other things can be modeled as columns.