Leverone Field House at Dartmouth

Case Study of a Thin Shell Barrel Vault Structure

THAYER SCHOOL OF ENGINEERING AT DARTMOUTH

Background

The Leverone Field House facility was completed in 1962 in response to the cold New England winters. It provided athletes space to train indoors for the first time at Dartmouth. Leverone, which is shown in Figure 1, was commissioned by Nathaniel Leverone (class of 1906), founder of the Automatic Canteen Company of America. Richard W. Olmstead (class of 1932 and Thayer class of 1933), a construction manager for Dartmouth at the time, recruited Italian engineer Pier Luigi Nervi for the project after seeing his work in Rome. Olmstead admired Nervi's "repetition of cool, crystalline geometric units to create refined and poetic concrete buildings" (Meacham and Mehling, 2008, p. 103). Nervi writes in his book *Aesthetics and Technology in Building* that "The kind request of Mr. Olmstead for the design of a Field House for Dartmouth College in Hanover, New Hampshire, gave me the opportunity to design a large vaulted structure using precast elements." (Nervi, 1966, p. 107).

Figure 1. Leverone Field House

Nervi had already tackled what was most challenging about Leverone, the construction of a large unobstructed area, in his previous work. His plan for Dartmouth modified the circular design for the Palazzetto dello Sport that he designed for the 1960 Olympic Games in Rome to the rectangular dimensions of the field house. Dartmouth alumni magazine classified the work as "gothic style brought up to date." (*Dartmouth Alumni Magazine*, 1961).

Pier Luigi Nervi *(1891-1979)*

Originally from Sondrio in Italy, Nervi, shown in Figure 2, graduated from the Civil Engineering School in Bologna in 1913. Nervi's perspective on building goes beyond simple engineering to encompass issues of aesthetics and constructability. He was truly an integrated designer and has been called a "poet in concrete." In addition to his careful attention to aesthetics, Nervi developed detailed construction plans for his buildings.

Nervi balanced intuition with mathematics while designing and was inspired by Roman and Renaissance architecture to apply ribbing and vaulting to his designs. He used prefabrication and simple geometry to create innovative designs.

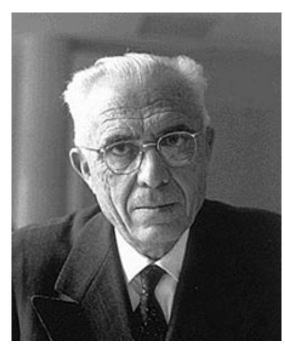

Figure 2. Pier Luigi Nervi

Barrel Vaults

Barrel vaults are curved structural elements used to span between walls or columns, often to form a roof structure as shown in Figure 3. Barrel vaults use single curvature, often semi-circular or parabolic, rather than double curvature like domes or hyperbolic paraboloids and may be thought of as a series of arches linked together to form a three-dimensional element. Thin-shell barrel vaults resist applied loads predominately either through ***bending behavior*** or ***arch action***, meaning they either act like a beam or a series of arches.

Figure 3. The Cloisters, NY

The Leverone Field House

The Leverone Field House covers an area that is 108.8 meters (357 feet) by 78.9 meters (259 feet), providing 8584m^2 (over 2 acres) of space; enough space for indoor track facilities, a weight room, and an indoor practice area for football, lacrosse, soccer, softball, golf, and rugby. The interior of Leverone is shown in Figure 4.

Figure 4. Interior of the Leverone Field House

A barrel vault with a parabolic shape is used for the roof of the Leverone Field House (and walls) – the roof itself spans 66.8 meters (219 feet), with a height of 13.4 meters (44 feet). Nervi designed it as a series of connected, short barrel vaults that resist loads through arch action.

To successfully design a series of short barrel vaults there needs to be a way to resist the lateral thrust forces that are generated – such as a thick edge beam or wall, or individual thrust elements. Nervi uses a series of concrete 'thrust elements' in the Leverone Field House to resist the lateral forces from each arch. Figure 5 shows a side elevation view of Nervi's design of the Leverone Field House (copied from original drawings done by Nervi's studio and stored with the Rauner Special Collections at Dartmouth College) – you can see the thrust elements repeating every 5.7 meters (18 feet 8 inches). Figure 6 shows the details of these thrust elements.

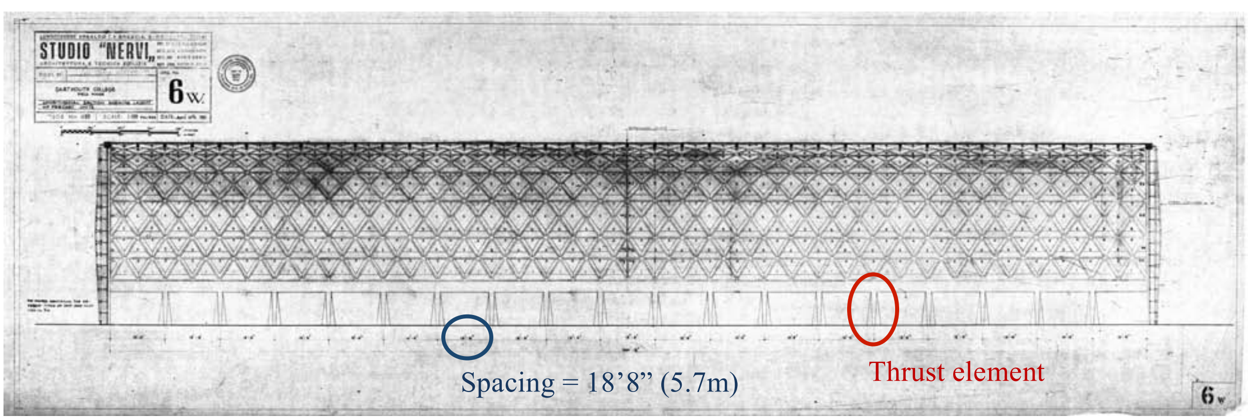

Figure 5. Side Elevation of the Leverone Field House

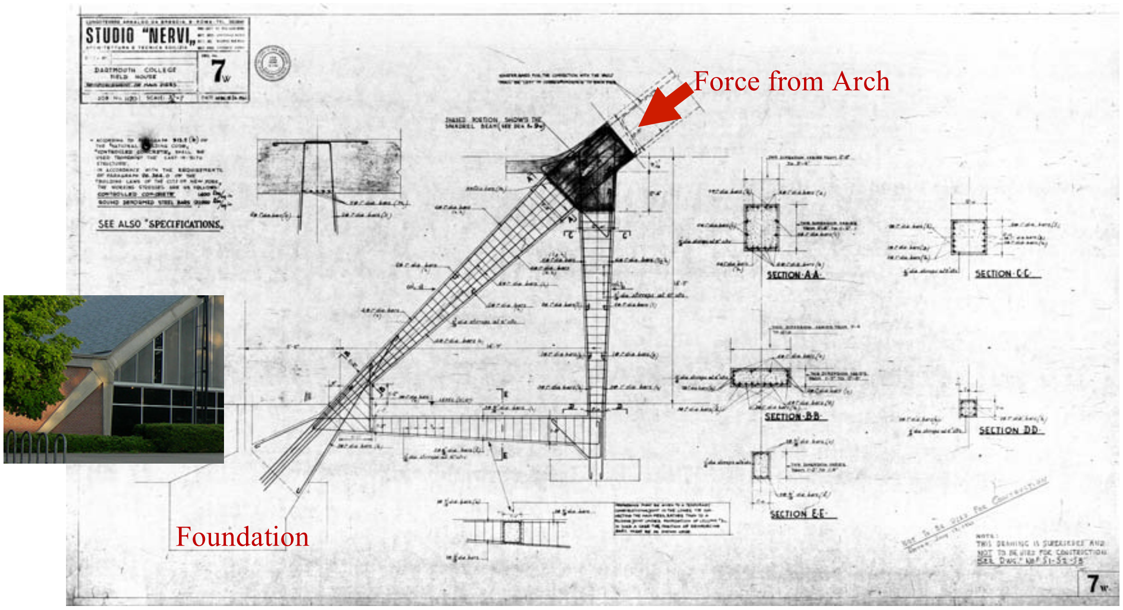

Figure 6. Thrust Elements of the Leverone Field House

Loads and Modeling

Based on Nervi's calculations, many of which are available through the Rauner Special Collections at Dartmouth College, Nervi considered three main load cases for the Leverone Field House:

1. Dead load (self-weight) plus full snow load over the entire span,
2. Dead load (self-weight) plus snow load on half of the span, and
3. Snow load on half of the span.

We will focus on the first load case (self-weight plus full snow over the entire span) in this case study. Based on Nervi's calculation of the self-weight and amount of snow expected in the area, the dead and snow loads considered for case 1 are shown below – the snow load is constant at 1.9 kN/m^2 (40 lb/ft^2), while the dead load varies from 4.6 kN/m^2 (95 lb/ft^2) at the crown of the arch to 7.9kN/m^2 (165lb/ft^2) at the ends or springing points as shown in Figure 7. Nervi converted the self-weight and dead load to horizontal projections to simplify the calculations. Note that the model shown in Figure 7 is a two-dimensional model of an *elementary arch*, or one segment of Leverone. Each elementary arch is 5.7 meters (18 feet 8 inches) wide.

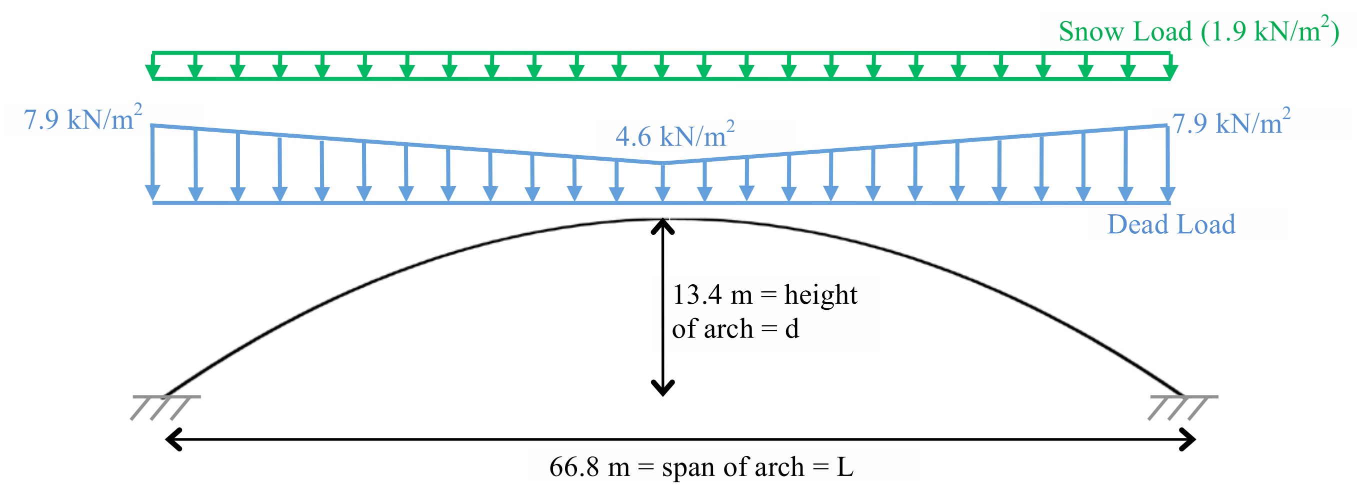

Figure 7. Two-dimensional Model of the Leverone Field House under Load Case 1

Based on Nervi's notes (again available through Rauner Special Collections), he modeled the base of each elementary arch as fixed as shown in Figure 7; this seems appropriate based on the details shown in Figure 6. But with both ends of the arch assumed fixed it is a ***statically indeterminate*** system, meaning that using statics alone (or $\Sigma F_x=0$, $\Sigma F_y=0$, and $\Sigma M=0$) we will not be able to find the internal forces and moments. We'll need something else: either 1) to make some additional assumptions about the structure, or 2) use deformation and compatibility equations. Based on Nervi's calculations, he initially made some assumptions about the structural form to simplify the analysis and then proceeded to confirm these general results with more detailed calculations.

Force in the Arch

In order to simplify the analysis, Nervi assumed that under the full dead plus snow load (as shown in Figure 7) the form of the arch is ***funicular***, meaning that it is the ideal shape for carrying the applied loads. More importantly, it means that the loading then causes pure compression in the arch and no bending moments, which greatly simplifies the analysis of the arch. While this assumption is not entirely correct it will give a fairly close approximation and is a good first analysis.

Making the assumption that the shape of the arch is funicular for the dead plus snow load leads to the assumption that there is no bending, only compressive forces, in the arch. Let's calculate this compressive force, or thrust force, by drawing a free-body diagram for half of the arch, as shown in Figure 8, and summing the forces and moments about the support point at the base. Note that when calculating the loads for Figure 8, I calculated the loads acting on a single segment of the arch, which is 5.7 meters (18 feet 8 inches) wide – giving a snow load of 10.9 kN/m (746.8 lb/ft) and dead loads from 25.9 kN/m (1773.7 lb/ft) at the crown to 45.0 kN/m (3080.6 lb/ft) at the support. As shown in Figures 8 and 9, the reactions are limited to a compressive force, C, acting in the arch and no moments because we are assuming the arch is funicular for the given loading.

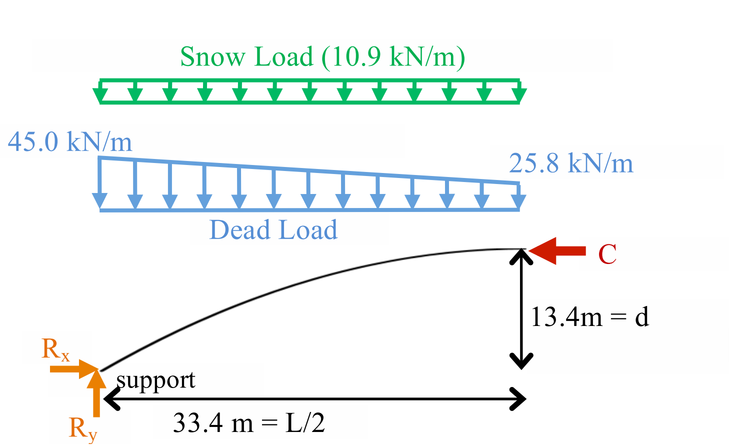

Figure 8. Free-body Diagram of Half of the Arch

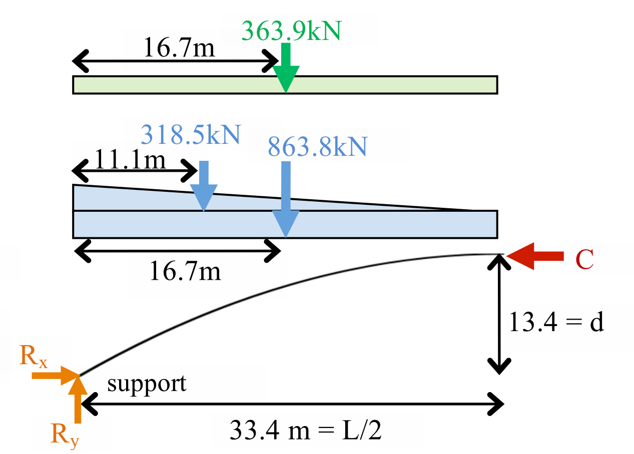

Figure 9. Free-body Diagram of Half of the Arch with Equivalent Point Loads

In Figure 9, I've replaced the distributed snow and dead loads with equivalent point loads to simplify the calculations. For the dead load, which is trapezoidal, I separated it into a rectangular and triangular load and calculated two equivalent loads as shown in Figure 9.

The next step is to calculate the compressive force, C. By summing moments clockwise about the support point we get the following equation (see Figure 9):

$$\sum \mathbf{M}_{\text{support}} = (\mathbf{363.9kN})(\mathbf{16.7m}) + (\mathbf{318.5kN})(\mathbf{11.1m}) + (\mathbf{863.8kN})(\mathbf{16.7m}) - \mathbf{C}(\mathbf{13.4m}) = \mathbf{0} \quad \text{(Eq. 1)}$$

Solving Equation 1 for the compressive force (**C**):

$$\frac{\mathbf{25866kN-m}}{\mathbf{13.4m}} = \mathbf{C} = \mathbf{1,800kN} \quad \text{(Eq. 2)}$$

Thus the compressive force in the arch is 1,800kN.

Shell Thickness

Once we have the maximum compressive force in the arch 1,800 kN (404,000 lb), we should be able to approximate the thickness of the shell. Since the arch is assumed to be in compression only, we can simply take the force over the area to calculate the stress. For the area of the arch, we will use the width of the elementary arch (5.7 m = 18.67ft) multiplied by the thickness of the shell (t). The next question is what we should use for the allowable compressive stress. Nervi used concrete and specified a compressive stress of 24,100 kN/m^2 (3500psi). Experimental testing showed that the average compressive stress was closer to 27,600 kN/m^2 (4000psi) but we'll use 24,100 kN/m^2 (3500psi) since that is what Nervi used. But we'll also want to use a factor of safety. If we use a factor of safety of 2, that gives us an allowable compressive stress value of 12,050 kN/m^2 (1750psi). Now we can calculate the thickness of the shell:

$$\boldsymbol{\sigma} = \frac{\mathbf{1,800kN}}{\mathbf{t(5.7m)}} < \mathbf{12,050\ kN/m^2} = \boldsymbol{\sigma}_{\mathbf{allow}} \quad \text{(Eq. 3)}$$

Solving Equation 3 for *t* gives a required shell thickness of 0.026m (1.03inches) – very thin! And that includes a factor of safety of 2!?

Do you think the thickness of the shell could really be 0.026m (~1 inch)? Did we calculate something incorrectly? Are we missing something? *The something that we're missing or neglecting is buckling – a 0.026 m (~1 inch) shell is adequate to resist the compressive forces without crushing but not necessarily large enough to resist buckling. Think of a column under compressive loads – how will it 'fail?' It will likely buckle. The same is true for a long, thin arch – buckling may be an issue.*

Buckling

Swiss engineer Heinz Isler, who studied thin concrete shells extensively, developed an equation to determine the critical buckling load for shells. The equation, while based in science, was heuristically derived from his 35 years of experience designing over 1500 thin shell concrete domes. For a shell to be stable the critical buckling load ($P_{cr,\ shell}$) must be greater than the expected load times a factor of safety. The equation developed by Isler for the critical buckling load of a shell is given in Equation 4.

$$P_{cr,\ shell} = C{*}E{*}(t/r)^x \quad \text{(Eq. 4)}$$

where: $P_{cr,\ shell}$= critical buckling load for a shell
C= a constant developed from experience
E= the modulus of elasticity of the concrete
t= the thickness of the shell
r= the radius of curvature of the shell
x=a factor related to the shape of the shell.

Equation 4 should be somewhat familiar as it is comparable to the equation used for column buckling. While we won't calculate the critical buckling load for Leverone in this case study, what are some ways that we might be able to reduce the chance of buckling? Here are some options:

- Reduce the load (not really possible in this case),
- Reduce the length (but then we don't meet the specifications),
- Increase the thickness (okay, but then we increase the weight),
- Add supports (but they might get in the way of the open space)…

Nervi solved the problem by adding ribs. Thus, he was able to keep the shell itself relatively thin but added ribs or beams along the shell. The ribs increase the moment of inertia of the shell and reduce the risks of buckling.

Rib Design

Nervi chose to address buckling by increasing the thickness and adding ribs to the shell. Ribs increase the moment of inertia of the cross section, and thus increase the critical buckling load (as demonstrated in Equation 5) without increasing the overall thickness by too much. Rather than use ribs that are linear, which is common and is shown in Figure 10, Nervi chose to use diagonal ribs (see Figure 5 for an interior view of Leverone and the diagonal ribs). The diagonal ribs create a more interesting pattern and provide additional strength to resist buckling (Nervi is famous for his diagonal ribs…part of the reason he is known as a 'poet in concrete'). Using a diagonal pattern, Nervi was able to include two ribs in each direction for every 5.7 m (18 feet 8 inches) elementary arch as shown in Figure 11 – arches in one direction are shown in blue and in the other direction orange (try cutting a vertical section through any of the bays – you always cut through two orange and two blue ribs).

Figure 10. Hershey Park Arena

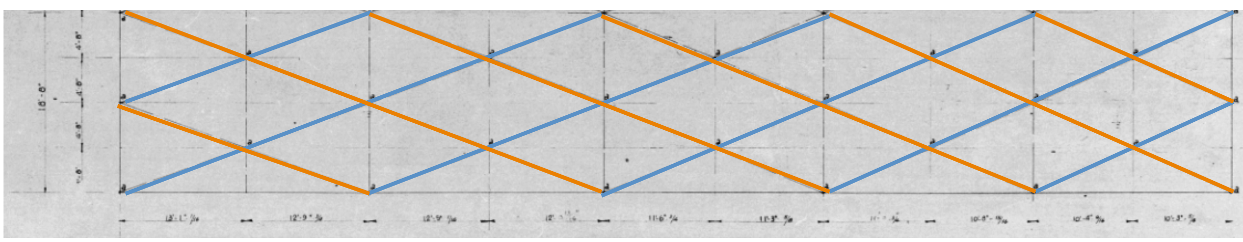

Figure 11. Plan View of the Diagonal Ribs of Leverone for a Single Arch

In his analyzes, Nervi equated the diagonal ribs to two equivalent linear ribs for every 5.7 m (18 feet 8 inches) or 1 equivalent rib for every 2.85 m (9 feet 4 inches) section. He used the equivalent T-section shown in Figure 12 for his more detailed analyses, which included one rib for every 2.85 m.

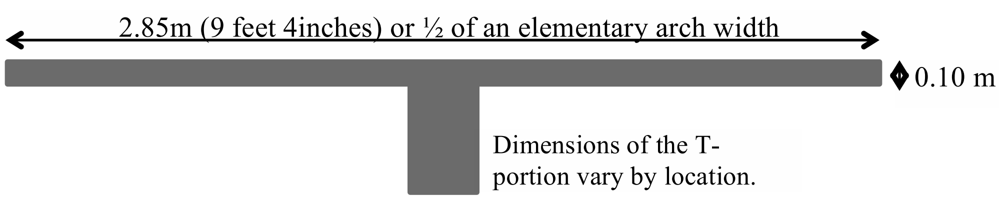

Figure 12. T-section of an Elementary Arch

References

Billington, David P. (1982). *Thin Shell Concrete Structures*, second edition. McGraw-Hill Book Company, NY.

Dartmouth Alumni Magazine. “Nervi Designs a Field House.” May 1961: 20-21.

Meacham, Scott and Mehling, Joseph (2008). Dartmouth College: an Architectural Tour. New York: Princeton Architectural Review.

Nervi, Pier Luigi (1960). *Aesthetics and Technology in Building*. Harvard University Press, Cambridge Massachusetts.

Salvadori, Mario and Heller, Robert (1963). *Structure in Architecture*. Prentice-Hall Inc., Englewood Cliffs, NJ.

Robbins, Tony (1976). *Engineering A New Architecture*. New Haven: Yale:58-9