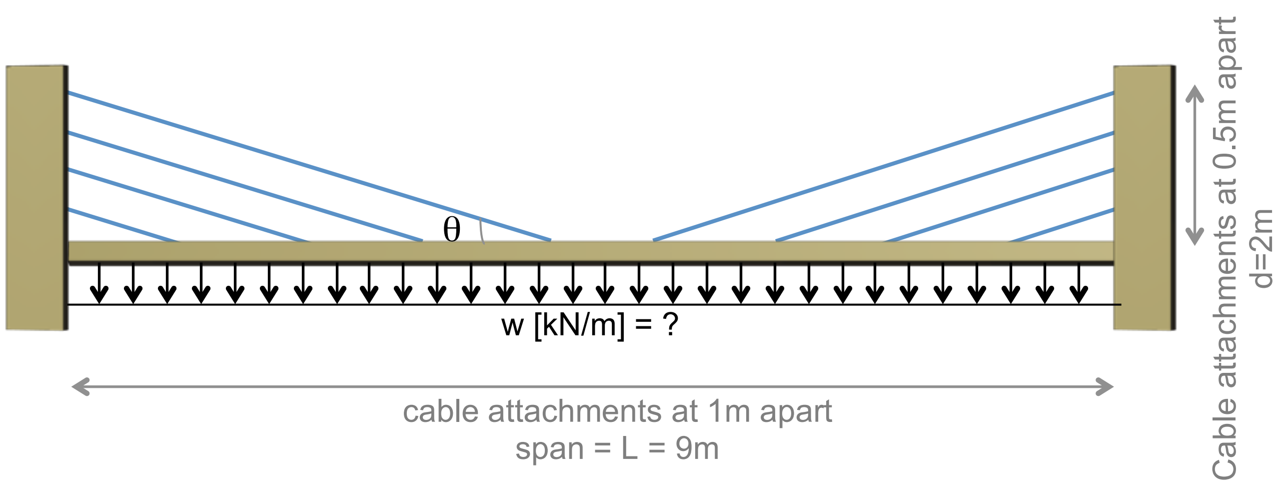
Bridge Example:
Allowable Load For Owl's Cable-Stayed Bridge

Owl did a nice job building a cable-stayed bridge. Let's help out by calculating how much load the bridge can support. I'm going to focus on the cables in this analysis but when we talk about beams later in the course we will analyze the deck of the bridge.

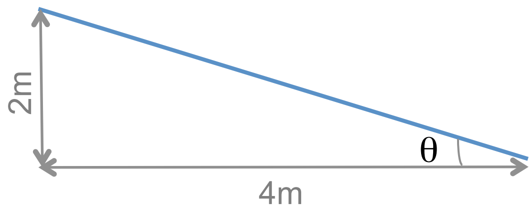
Owl used steel cables that are 6.4mm in diameter and have a breaking strength of 20kN.

Owl used a harp design for the cable-stayed bridge so that will simplify our analysis a bit. But we'll also look at how the design would change if Owl used a fan design.

Model of Owl's cable-stayed bridge:

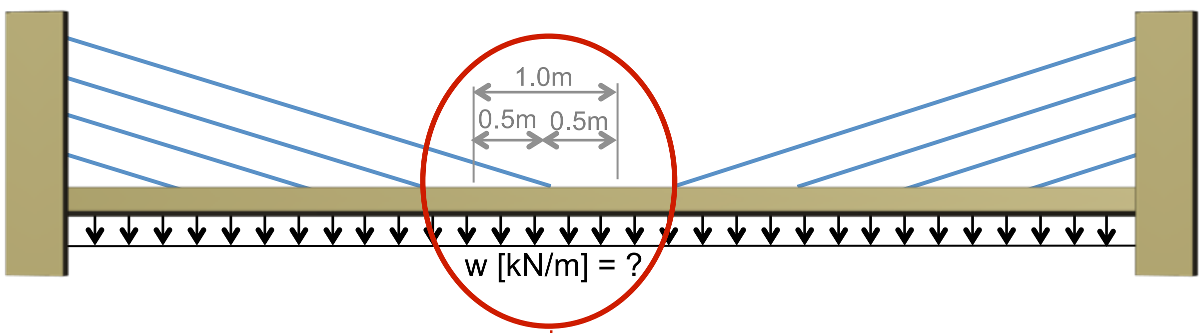


Let's take a look at one of the cables. I'm going to start by looking at one of the longest cables in the middle. Based on the geometry of Owl's bridge, we can determine the angle of the cable.

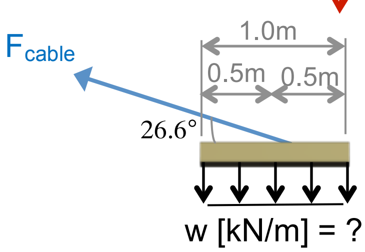


$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{\boldsymbol{2m}}{\boldsymbol{4m}}\right) = 26.6°$$

Using this geometry and the spacing of the cables we should be able to use equilibrium to determine the force in the longest cable. We'll assume that each cable carries a proportion of the weight from the bridge deck based on the distance to the adjacent cables. Thus, I'm going to assume that this longer cable supports the weight of the deck 0.5m to the left of its attachment point and 0.5m to the right of its attachment point for a total length of 1.0m. This distance is often referred to as the tributary length.



Free-body diagram of the cable attachment:



Vertical equilibrium:

$\Sigma F_y = F_{cable}$*sin(26.6°) – w*1m = 0

I have 2 unknowns here, F_{cable} and w, but we know that the breaking strength of the cables that Owl used is 20kN. Let's use a factor of safety of 2.0, meaning that we'll use an allowable strength of 20kN/2.0.

Thus, F_{cable} = 20kN/2.0 = 10kN. Substituting F_{cable} = 10kN in the above equation we can solve for w, the distributed load:

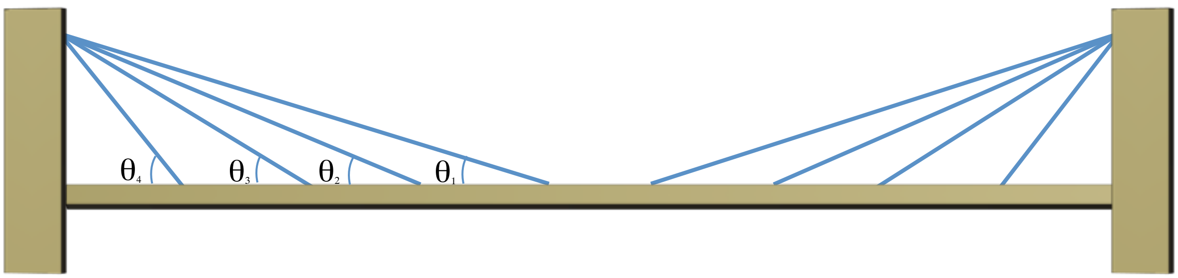
w = F_{cable}*sin(26.6°)/1m = 10kN * sin(26.6°)÷1m= **4.48kN/m**

Based on this analysis, the longest cables of Owl's cable-stayed bridge can support a load of 4.48kN/m. Are these the critical cables? How will the load vary in the other cables? It turns out that since Owl used a harp design for the bridge with even spacing, each of the cables is attached at the same angle so the allowable load will be the same for each cable.

Note that the other end of each cable is attached to the trees (which act as the towers of the bridge). The cables will apply both vertical and horizontal forces on the trees. The vertical forces will cause compression, which we'll discuss in Concept 3, and the horizontal forces will cause bending, which we'll discuss in Concept 5.

How would the results vary if Owl had used a fan design for the cable-stayed bridge?

Alternate Design (fan):



The main difference between this fan design and the harp design that Owl used for the cable-stayed bridge is the angle of the cables. With a fan design all of the angles will be different. Which cable will be most critical? The cable with the smallest or largest angle? It turns out that the cable with the smallest or shallowest angle will have the highest force. The attachment between the longest cable and the deck will be the smallest so this cable will be the critical cable to check.