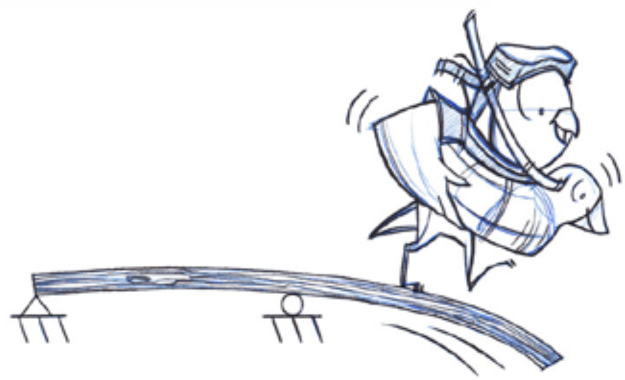
Owl's Diving Board Example:

Modeling and Analysis of Beams

Let's look at Owl's diving board.

Owl's diving board may be modeled as a propped cantilever beam by replacing the connection at the tree with a pinned support that resists both vertical and horizontal forces. The angled member may be represented by a roller support that resists vertical loads.



Let's assume that Owl weighs 30N and the dimensions are as shown, 2m between the supports and 1m past the support to the load.

We can use equilibrium to find the support reactions:

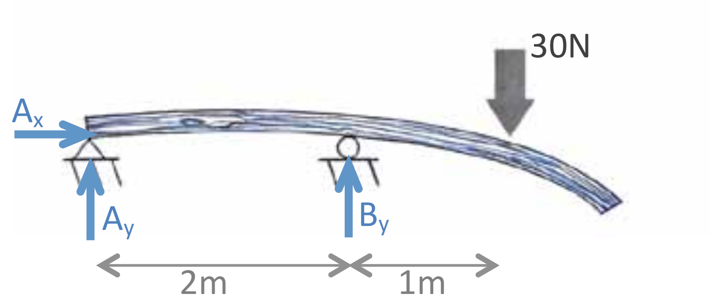
ΣM_A = 30N (3m) – By (2m) = 0

B_y = 90N-m / 2m = **45N = B_y**

ΣF_x = **A_x = 0**

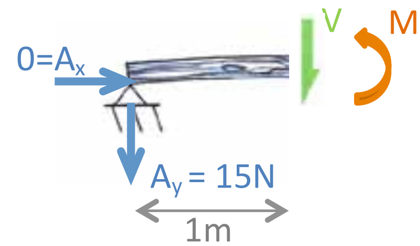
ΣF_y = A_y + (B_y=45N) -30N = 0;

A_y = –15N



Now we know how much load the connections need to be able to resist, but we don't know what is happening internally to the beam. We need to understand the forces and moments acting in the beam in order to determine the right materials and sizing. To do this, engineers draw free-body diagrams, not unlike those we drew for trusses but now cutting through the beam.

Let's start with a free-body diagram through the beam at the midspan between A and B, or at a distance of 1m from A. Note that I've reversed the direction of A_y, replacing the negative on the upward force with a downward force (the negative sign just means that the direction is reversed). Internally, we will have a shear force and a bending moment at the cut, internal surface.



Now we can again use equilibrium to solve for the internal shear force, V, and the internal moment, M.

Let's start with vertical equilibrium:
$\Sigma F_y = V + 15N = 0;$
$V = -15N$

Thus the shear force across the surface is 15N, upward. What will happen to this force as the distance from A changes? Meaning we get further from the support? Will the internal shear force increase? Decrease? Or remain the same? It turns out that it will remain the same since in this case the shear force was not a function of the distance. The shear force, V, between supports A and B will be an upward 15N.

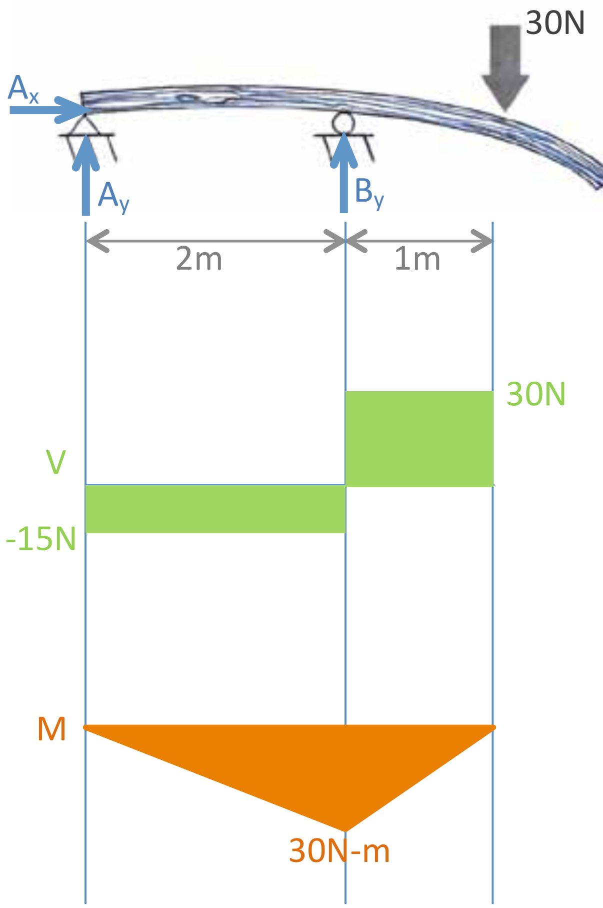
Next let's use rotational equilibrium about point A:

$\Sigma M_A = V\,(1m) - M = 0$
M = internal bending moment in the beam = V*1m = (-15N)(1m) = **-15N-m = M**

Thus the bending moment at 1m from the support is 15kN-m, clockwise. What will happen to this moment as the distance from A changes? Meaning we get further from the support? Will the internal moment increase? Decrease? Or remain the same? It turns out that it will increase since the moment is a function of the distance. Typically the further away from the support or applied load the greater the moment.

Engineers often draw diagrams to represent the variation of the shear and moment along the length of a beam; these diagrams are called shear and moment diagrams. Here is a shear and moment diagram for Owl's diving board. You could draw this diagram by creating additional free-body diagrams at different points and then recording the shear and moment at each location. From the diagram, we see that the maximum shear force in Owl's diving board is 30N at the free end (beyond support B) and the maximum bending moment, 30N-m, occurs in the beam at support B. Bending moment tends to govern the design of most beams, so I'd be most worried about the moment at support B and would design the material type and cross-sectional shape such that it was able to support a moment of 30N-m.

We don't expect you to be able to draw shear and bending moment diagrams. Use the beam simulator to determine the shear force and bending moment along the length of different beams with different loadings.



V_{max} = 30N

M_{max} = 30N-m @ support B