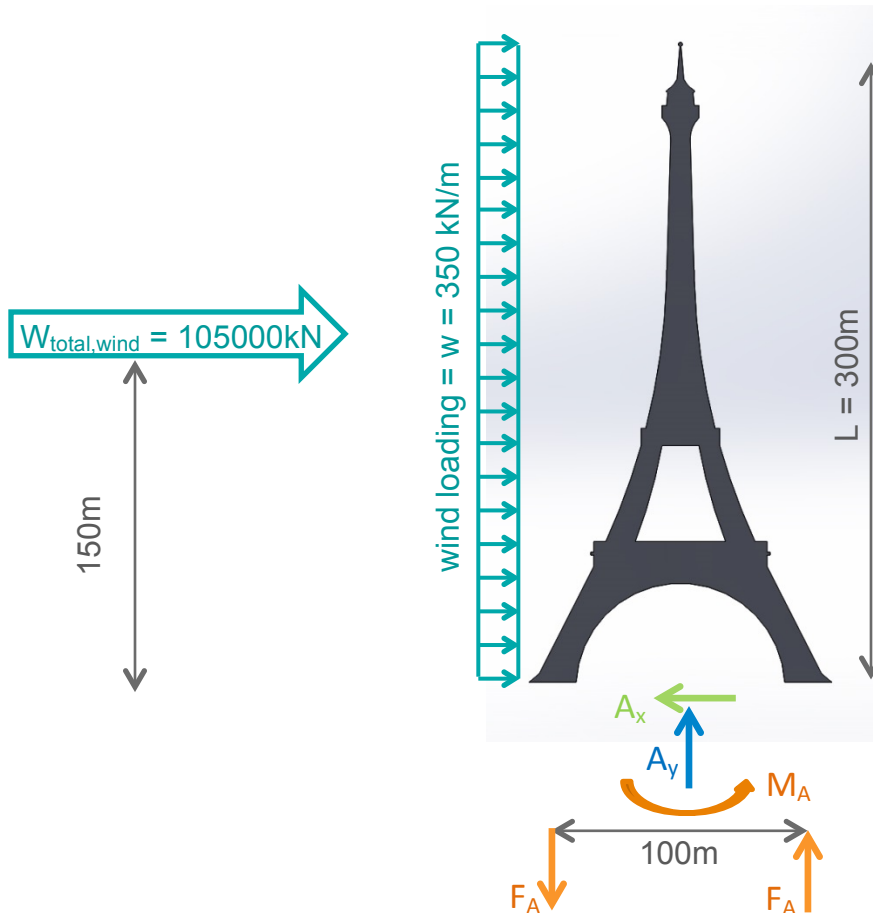


Eiffel Tower Example:



Modeling and Analysis of Beams

Let's look at one more 'beam' – the Eiffel Tower. The Eiffel Tower acts like a cantilever beam when subjected to wind loads and has an interesting form. Let's assume that the wind loading on the Eiffel Tower may be represented by an evenly distributed load of 350kN/m. The total resultant load is thus 350kN/m * 300m or 105,000kN, acting at a height of 150m from the base of the Eiffel Tower.



First let's calculate the reactions at the base of the Eiffel Tower. Since we're modeling it as a cantilever beam all three of our reaction forces are at the base: A_x , A_y , and M_A .

$\Sigma F_y = A_y = 0$; for this loading case

$\Sigma F_x = 105,000\text{kN} - A_x = 0$

$A_x = 105,000\text{kN}$

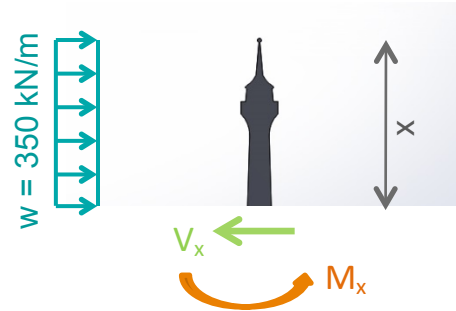
$\Sigma M_A = (105,000\text{kN})(150\text{m}) - M_A = 0$

$M_A = 15,750,000\text{kN-m}$

The moment at the base, M_A , can actually be replaced with two equal forces, F_A , acting at each leg of Tower.

$F_A = 15750000\text{kN-m}/100\text{m} = \mathbf{157,500\text{kN} = F_A}$

How does the shear force and moment vary along the height of the Eiffel Tower? Let's consider a portion of the top of the Tower.



If I cut the Eiffel Tower at a variable height, x , I can determine the shear and moment as a function of this variable. Let's use equilibrium.

$$\Sigma F_x = (350 \text{ kN/m})(x) - V_x = 0$$

$$V_x = 350x \text{ kN}$$

$$\Sigma M_x = (350 \text{ kN})(x)(x/2) - M_x = 0$$

$$M_x = 350x^2/2 \text{ kN-m}$$

Thus we see that the shear force in the tower varies linearly from zero at the top to a maximum at the base and the moment varies parabolically from zero at the top to a maximum at the base. It turns out that the shape of the Eiffel Tower is also parabolic, making it an ideal shape to resist wind loads. Below are shear and moment diagrams for the Eiffel Tower, which are simply graphical representations of the equations for shear and moment along the height.

