Bridges Example:
Suspension Bridge vs. a Cable-Stayed Bridge

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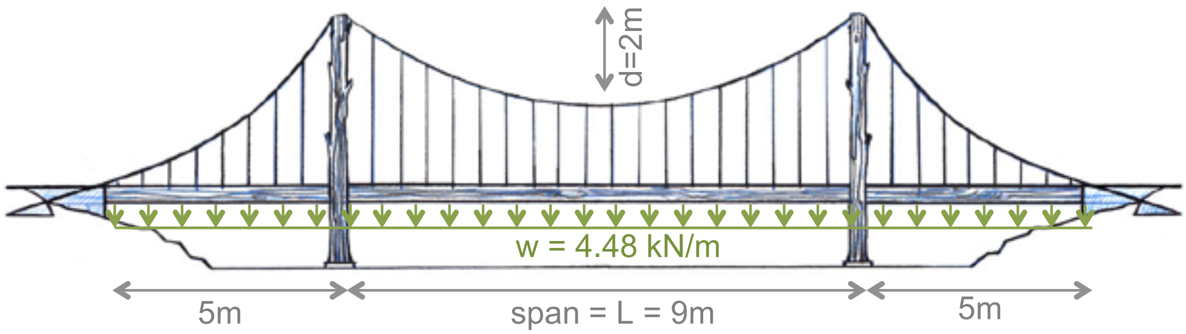
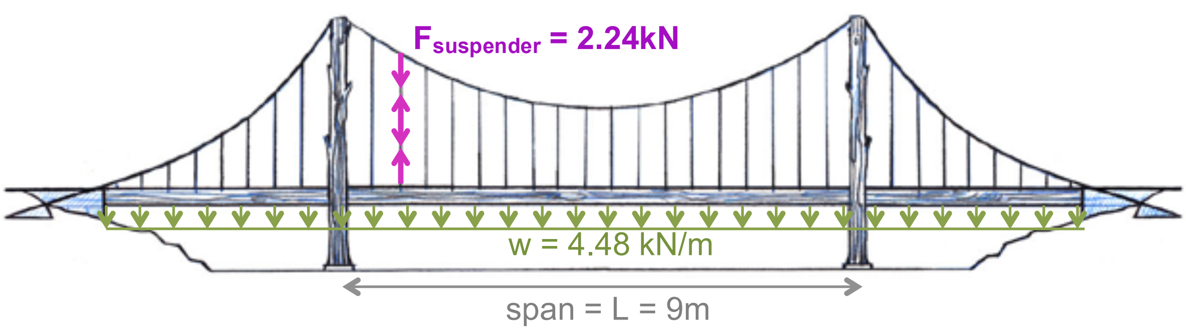
A suspension bridge doesn't really make sense for Owl's site but let's look at a suspension bridge with similar dimensions so we can make a few comparisons between a suspension bridge and Owl's cable-stayed bridge. Rather than computing the load that this suspension bridge can support let's apply the maximum load that the cable-stayed bridge was able to support, 4.48kN/m, and determine the size of the cables required. I'm going to take this approach since the size of the suspender cables and main cables of a suspension bridge tend to be different sizes.



In a suspension bridge the distributed load from the deck is transferred through the suspenders or vertically oriented cables to the main cables. This suspension bridge has 18 suspenders and I'm going to assume that each supports an equal share of the load:
Total load the over 9m main span = 4.48kN/m * 9m = 40.3kN

Load to each suspender = Total load/number of suspenders =
40.3kN/18suspenders = **2.24kN/suspender**



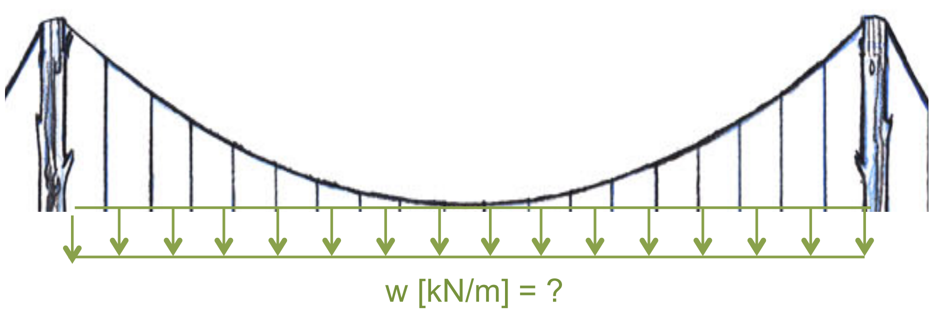
Knowing the force in each suspender we should be able to determine the diameter of the cables required. Let's use steel with the same strength as we did for the cable-stayed bridge and a factor of safety of 2 again. The breaking strength of the cables with a diameter of 6.4mm was 20kN so the breaking stress = $20kN/(\pi(0.0064m/2)^2) = 621{,}700kN/m^2$. If we divide this by 2.0 (factor of safety) we have an allowable stress for the steel of $310{,}850kN/m^2$. Thus, we will need to use cables with a diameter of:

Stress = $\sigma = P/A = 2.24kN/A <$ stress allowable = $310{,}850kN/m^2$

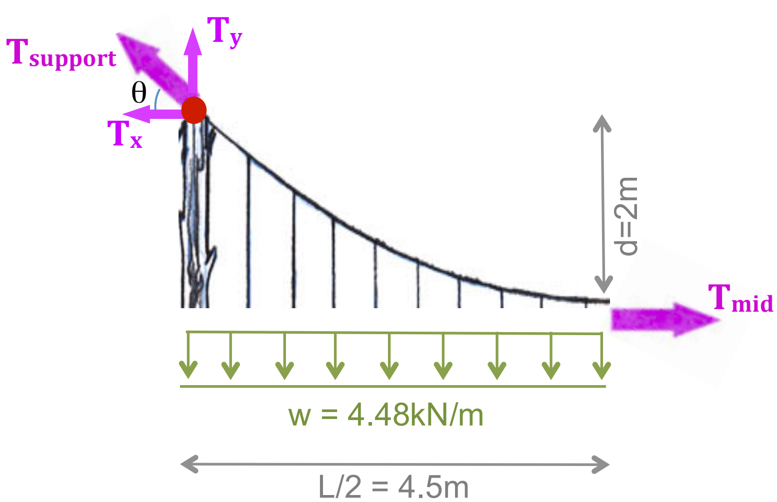
$A > 2.24kN/310{,}850kN/m^2 = 7.21 \times 10^{-6} m^2 = \pi\ (\text{radius})^2 \quad \Rightarrow$ **diameter = 3.03mm**

The diameter of the suspenders is less than half of the diameter required for the cable-stayed bridge; this makes sense since we're using more than twice as many cables.

We can draw free-body diagrams for each of the individual suspenders but an engineer will typically assume that the suspenders are frequent enough that they can be modeled with a distributed load. Thus, the distributed load may really be modeled as acting directly on the main cables of the bridge. Next I'm going to focus on the main cables of the bridge.



In order to determine the tension force in the main cable, I am going to focus half of the structure as shown below:



For the suspension bridge to be in equilibrium there must be no motion in the horizontal direction (x), vertical direction (y), and no rotation. I'm going to start with rotational equilibrium this time. I'm using the support at the left as my point of rotation – this point is indicated by a red dot in the model. When I'm looking at rotational equilibrium, I use something called a moment, which we'll discuss more in Concept 5. But a moment is a tendency to cause rotation and is calculated as a force multiplied by the perpendicular distance between the force and the point of rotation. In this case the distributed load will cause a clockwise rotation about the support (red dot) that must be counteracted by a counterclockwise moment caused the tension in the rope at the mid-span (T_{mid}).

$$\Sigma M_{support} = w^*(L/2)^*(L/4) - T_{mid} {}^*d = wL^2/8 - T_{mid}{}^*d = 0$$
$$T_{mid} = wL^2/(8^*d) = 4.48\text{kN/m}^*(9\text{m})^2/(8\text{m}^*2) = \mathbf{22.7kN}$$

$$\Sigma F_x = T_{mid} - T_x = 0$$
$$T_x = T_{mid} = \mathbf{22.7kN}$$

$$\Sigma F_y = T_y - w(L/2) = 0$$
$$T_y = w(L/2) = 4.48\text{kN/m}^*4.5\text{m} = \mathbf{20.16kN}$$

The tension in the main cable will be greatest at the support and may be calculated using the fact that $T_y^2 + T_x^2$ must equal $T_{support}^2$; use the Pythagorean Theorem.

$$T_{support}^2 = T_y^2 + T_x^2 = (20.16\text{kN})^2 + (22.68\text{kN})^2$$

$$\mathbf{T_{support} = 30.35kN}$$

So the maximum force in the main cables is 30.35kN, which is much higher than the force in the cables of the cable-stayed bridge but this makes sense since there are 8 cables sharing the load in the cable-stayed bridge and only one main cable supporting the load in the suspension bridge. What size cable do we need?

Let's again use an allowable stress for the steel of 310,850kN/m^2. Thus we will need to use cables with a diameter of:

$$\text{Stress} = \sigma = P/A = 30.35\text{kN}/A < \text{stress allowable} = 310{,}850\text{kN/m}^2$$
$$A > 30.35\text{kN}/310{,}850\text{kN/m}^2 = 3.11\times10^{-5}\text{m}^2 = \pi\,(\text{radius})^2 \quad \Rightarrow \quad \textbf{diameter = 11.15mm}$$

The diameter of the main cables needs to be 11.15mm, which is larger than the cables of the cable-stayed bridge; the cables of the cable-stayed bridge were 6.4mm in diameter.

Suspension bridges in general tend to use more steel as we see here but the main advantage of suspension bridges comes in the towers. The main cables of the suspension bridges typically are not connected directly to the towers but rather to abutments at the ends, thus the towers carry only compressive loads rather than compressive ***and*** bending loads. Towers in suspension bridges tend to be lighter than those for cable-stayed bridges and suspension bridges can typically span further. Both types of bridges have advantages and disadvantages and make sense in different situations.