AUS-S-1 T-2 C
Arthur Dent
Earth

Theory
Two Problems in Mechanics
Cover sheet

Please return this cover sheet together with all the related question sheets.

Nonlinear Dynamics in Electric Circuits (10 points)

Please read the general instructions in the separate envelope before you start this problem.

Introduction

Bistable non-linear semiconducting elements (e.g. thyristors) are widely used in electronics as switches and generators of electromagnetic oscillations. The primary field of applications of thyristors is controlling alternating currents in power electronics, for instance rectification of AC current to DC at the megawatt scale. Bistable elements may also serve as model systems for self-organization phenomena in physics (this topic is covered in part B of the problem), biology (see part C) and other fields of modern nonlinear science.

Goals

To study instabilities and nontrivial dynamics of circuits including elements with non-linear $I-V$ characteristics. To discover possible applications of such circuits in engineering and in modeling of biological systems.

Part A. Stationary states and instabilities (3 points)

Fig. 1 shows the so-called **S-shaped** $I-V$ characteristics of a non-linear element X. In the voltage range between $U_{\mathrm{h}} = 4.00\ \mathrm{V}$ (the holding voltage) and $U_{\mathrm{th}} = 10.0\ \mathrm{V}$ (the threshold voltage) this $I-V$ characteristics is multivalued. For simplicity, the graph on Fig. 1 is chosen to be piece-wise linear (each branch is a segment of a straight line). In particular, the line in the upper branch touches the origin if it is extended. This approximation gives a good description of real thyristors.

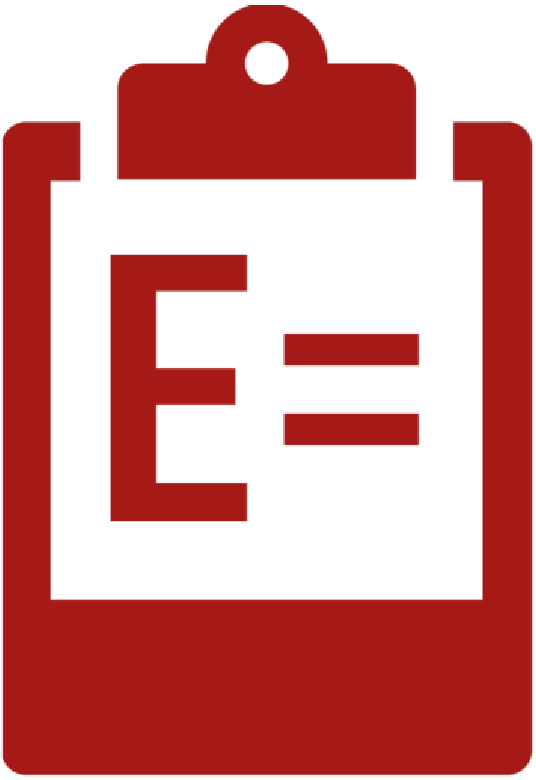

Figure 1: $I-V$characteristics of the non-linear element X.

A.1	Using the graph, determine the resistance R_{on} of the element X on the upper branch of the $I-V$ characteristics, and R_{off} on the lower branch, respectively. The middle branch is described by the equation $$I = I_0 - \frac{U}{R_{\text{int}}}. \quad (1)$$ Find the values of the parameters I_0 and R_{int}.	0.4pt

The element X is connected in series (see Fig.2) with a resistor R, an inductor L and an ideal voltage source $\mathcal{E}$. One says that the circuit is in a stationary state if the current is constant in time, $I(t) = \text{const}$.

Figure 2: Circuit with element X, resistor R, inductor L and voltage source $\mathcal{E}$.

A.2	What are the possible numbers of stationary states that the circuit of Fig. 2 may have for a fixed value of $\mathcal{E}$ and for $R = 3.00\ \Omega$? How does the answer change for $R = 1.00\ \Omega$?	1pt

A.3	Let $R = 3.00\ \Omega, L = 1.00\ \mu\text{H}$ and $\mathcal{E} = 15.0\ \text{V}$ in the circuit shown in Fig. 2. Determine the values of the current $I_{\text{stationary}}$ and the voltage $V_{\text{stationary}}$ on the non-linear element X in the stationary state.	0.6pt

The circuit in Fig. 2 is in the stationary state with $I(t) = I_{\text{stationary}}$. This stationary state is said to be stable if after a small displacement (increase or decrease in the current), the current returns towards the stationary state. And if the system keeps moving away from the stationary state, it is said to be unstable.

A.4	Use numerical values of the question **A.3** and study the stability of the stationary state with $I(t) = I_{\text{stationary}}$. Is it stable or unstable?	1pt

Part B. Bistable non-linear elements in physics: radio transmitter (5 points)

We now investigate a new circuit configuration (see Fig. 3). This time, the non-linear element X is connected in parallel to a capacitor of capacitance $C = 1.00\ \mu\mathrm{F}$. This block is then connected in series to a resistor of resistance $R = 3.00\ \Omega$ and an ideal constant voltage source of voltage $\mathcal{E} = 15.0\ \mathrm{V}$. It turns out that this circuit undergoes oscillations with the non-linear element X jumping from one branch of the $I - V$ characteristics to another over the course of one cycle.

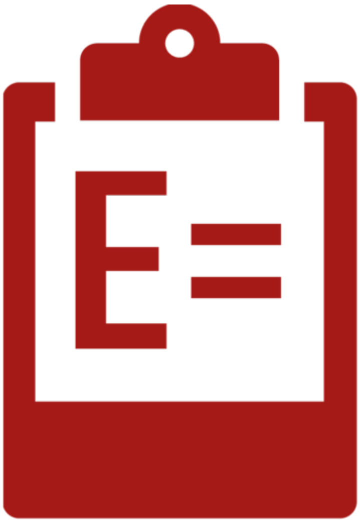

Figure 3: Circuit with element X, capacitor C, resistor R and voltage source $\mathcal{E}$.

B.1	Draw the oscillation cycle on the $I - V$ graph, including its direction (clockwise or anticlockwise). Justify your answer with equations and sketches.	1.8pt

B.2	Find expressions for the times t_1 and t_2 that the system spends on each branch of the $I - V$ graph during the oscillation cycle. Determine their numerical values. Find the numerical value of the oscillation period T assuming that the time needed for jumps between the branches of the $I - V$ graph is negligible.	1.9pt

B.3	Estimate the average power P dissipated by the non-linear element over the course of one oscillation. An order of magnitude is sufficient.	0.7pt

The circuit in Fig. 3 is used to build a radio transmitter. For this purpose, the element X is attached to one end of a linear antenna (a long straight wire) of length s. The other end of the wire is free. In the antenna, an electromagnetic standing wave is formed. The speed of electromagnetic waves along the antenna is the same as in vacuum. The transmitter is using the main harmonic of the system, which has period T of question **B.2**.

B.4	What is the optimal value of s assuming that it cannot exceed $1\ \mathrm{km}$?	0.6pt

Part C. Bistable non-linear elements in biology: neuristor (2 points)

In this part of the problem, we consider an application of bistable non-linear elements to modeling of biological processes. A neuron in a human brain has the following property: when excited by an external signal, it makes one single oscillation and then returns to its initial state. This feature is called excitability. Due to this property, pulses can propagate in the network of coupled neurons constituting the nerve systems. A semiconductor chip designed to mimic excitability and pulse propagation is called a *neuristor* (from neuron and transistor).

We attempt to model a simple neuristor using a circuit that includes the non-linear element X that we investigated previously. To this end, the voltage $\mathcal{E}$ in the circuit of Fig. 3 is decreased to the value $\mathcal{E}' = 12.0\ \mathrm{V}$. The oscillations stop, and the system reaches its stationary state. Then, the voltage is rapidly increased back to the value $\mathcal{E} = 15.0\ \mathrm{V}$, and after a period of time τ (with $\tau < T$) is set again to the value $\mathcal{E}'$ (see Fig. 4). It turns out that there is a certain critical value $\tau_{\mathrm{crit.}}$, and the system shows qualitatively different behavior for $\tau < \tau_{\mathrm{crit}}$ and for $\tau > \tau_{\mathrm{crit}}$.

Figure 4: Voltage of the voltage source as a function of time.

C.1	Sketch the graphs of the time dependence of the current $I_X(t)$ on the non-linear element X for $\tau < \tau_{\mathrm{crit}}$ and for $\tau > \tau_{\mathrm{crit}}$.	1.2pt

C.2	Find the expression and the numerical value of the critical time τ_{crit} for which the scenario switches.	0.6pt

C.3	Is the circuit with $\tau = 1.00 \times 10^{-6}$ s a neuristor?	0.2pt

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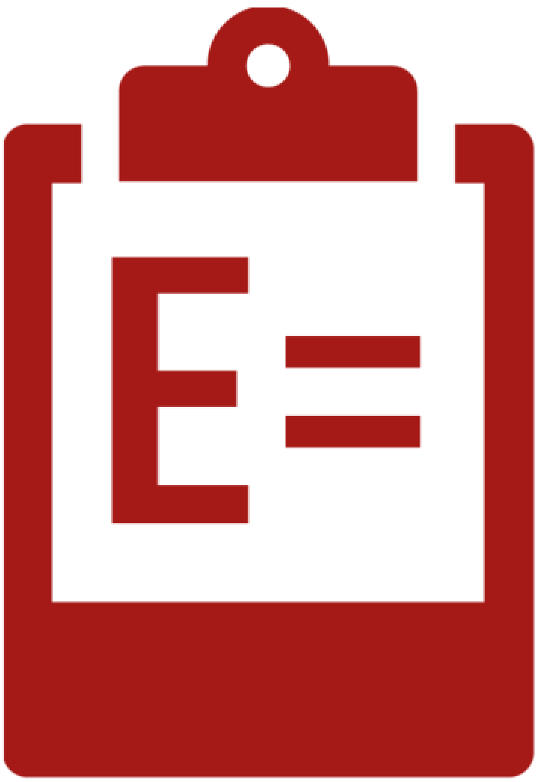

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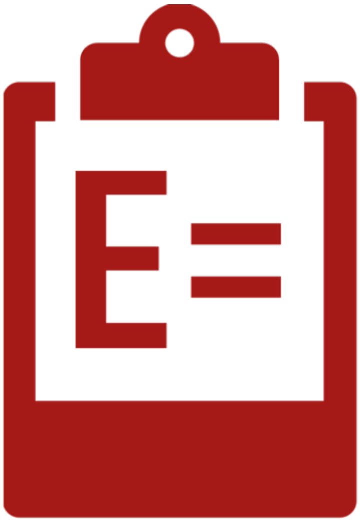

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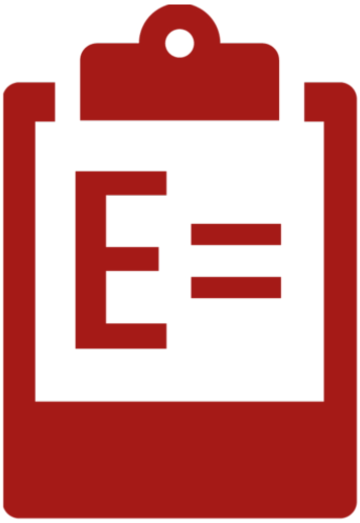

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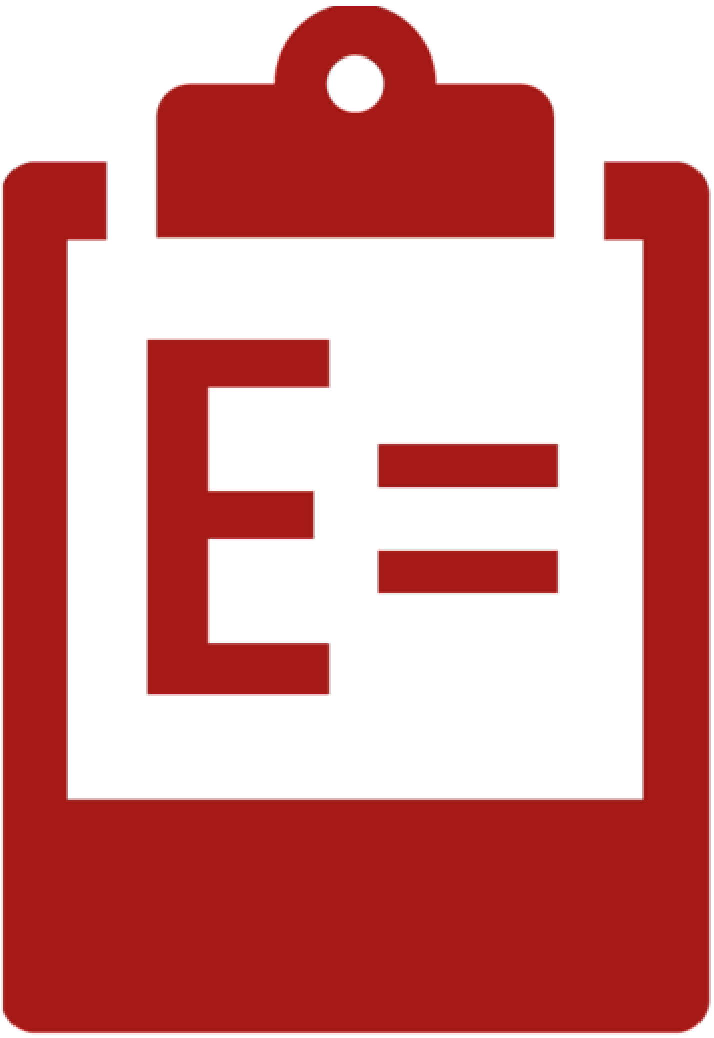

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Nonlinear Dynamics in Electric Circuits (10 points)

Part A. Stationary states and instabilities (3 points)

A.1 (0.4 pt)

$R_{\text{on}} =$

$R_{\text{off}} =$

$I_0 =$

$R_{\text{int}} =$

A.2 (1 pt)

Possible numbers of stationary states for $R = 3.00\ \Omega$:

Possible numbers of stationary states for $R = 1.00\ \Omega$:

A.3 (0.6 pt)

$I_{\text{stationary}} =$

$V_{\text{stationary}} =$

A.4 (1 pt)

Behaviour for $I(t=0) > I_{\text{stationary}}$:

Behaviour for $I(t=0) < I_{\text{stationary}}$:

Is the stationary state: ☐ stable? ☐ unstable?

Part B. Bistable non-linear elements in physics: radio transmitter (5 points)

B.1 (1.8 pt)

Justification:

B.2 (1.9 pt)

Formula of $t_1 =$

Numerical value of $t_1 =$

Formula of $t_2 =$

Numerical value of $t_2 =$

Numerical value of $T =$

B.3 (0.7 pt)

$P \approx$

B.4 (0.6 pt)

$s =$

Part C. Bistable non-linear elements in biology: neuristor (2 points)

C.1 (1.2 pt)

Sketch for $\tau < \tau_{\text{crit}}$:

Sketch for $\tau > \tau_{\text{crit}}$:

C.2 (0.6 pt)

Formula of $\tau_{\text{crit}} =$

Numerical value of $\tau_{\text{crit}} =$

C.3 (0.2 pt)

Is the circuit a neuristor? ☐ Yes ☐ No