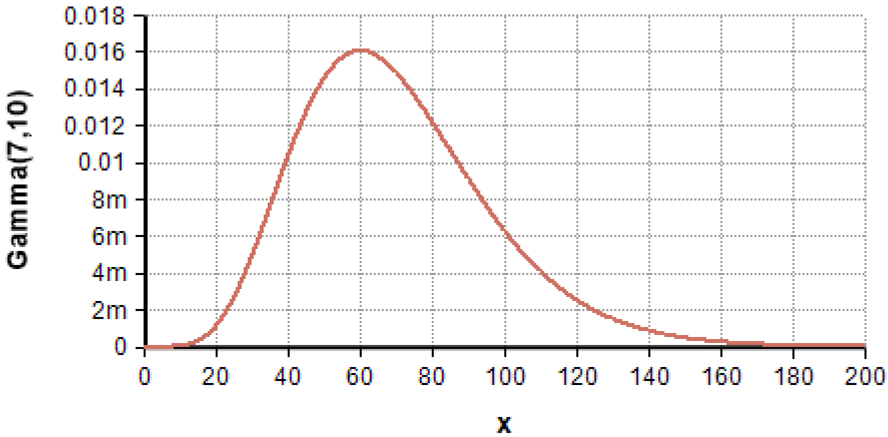
# Gamma distribution

The **Gamma distribution** is a continuous (/index.php?title=Category%3AContinuous_distributions), positive-only (/index.php?title=Category%3ASemi-bounded_distributions), unimodal (/index.php?title=Category%3AUnimodal_distributions) distribution that encodes the time required for «alpha» events to occur in a Poisson (/index.php?title=Poisson) process with mean arrival time of «beta»




(/index.php?title=File%3AGamma(7,10).png)

Use the Gamma (/index.php?title=Gamma) distribution with «alpha» > 1 if you have a sharp lower bound of zero but no sharp upper bound, a single mode, and a positive skew. The LogNormal (/index.php?title=LogNormal) distribution is also an option in this case. Gamma (/index.php?title=Gamma)() is especially appropriate when encoding arrival times for sets of events. A gamma distribution with a large value for «alpha» is also useful when you wish to use a bell-shaped curve for a positive-only quantity.

The gamma distribution is bounded below by zero (all sample points are positive) and is unbounded from above. It has a theoretical mean of `alpha*beta` and a theoretical variance of `alpha*beta^2`. When «alpha» > 1, the distribution is unimodal with the mode at `(alpha - 1)*beta`. An exponential distribution results when `alpha = 1`. As $\alpha \to \infty$, the gamma distribution approaches a normal distribution in shape.

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## Functions

**Note**

Some textbooks use `Rate = 1/beta`, instead of «beta», as the scale parameter.

### Gamma(alpha*, beta, over*)

The distribution function. Use this to describe a quantity that is gamma-distributed with shape parameter «alpha» and scale parameter «beta». The scale parameter, «beta», is optional and defaults to `beta = 1`.

### Dens_Gamma(x, alpha*, beta*)

To use this, you need to add the Distribution Densities Library (/index.php?title=Distribution_Densities_Library) to your model.

The analytic probability density of the Gamma (/index.php?title=Gamma) distribution at «x». Returns

$$p(x) = \frac{\beta^{-\alpha} x^{\alpha-1} \exp(-x/\beta)}{\Gamma(\alpha)}$$

### CumGamma(x, alpha*, beta*)

To use this, you need to add the Distribution Densities Library (/index.php?title=Distribution_Densities_Library) to your model, or use GammaI (/index.php?title=GammaI) instead.

The cumulative density up to «x», given for $x > 0$ by

$$F(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \beta^{-\alpha} t^{\alpha-1} \exp(-t/\beta) dt$$

This is also the same as the regularized incomplete gamma function, computed by the function GammaI (/index.php?title=GammaI).

### CumGammaInv(p, alpha*, beta*)

To use this, you need to add the Distribution Densities Library (/index.php?title=Distribution_Densities_Library) to your model, or use GammaIInv (/index.php?title=GammaIInv) instead.

The analytic inverse cumulative probability function (quantile function). Returns the $p^{th}$ fractile/quantile/percentile for the gamma distribution. Same as the inverse incomplete gamma function, GammaIInv (/index.php?title=GammaIInv).

## When to use

Use the Gamma (/index.php?title=Gamma) distribution with «alpha» > 1 if you have a sharp lower bound of zero but no sharp upper bound, a single mode, and a positive skew. The LogNormal (/index.php?title=LogNormal) distribution is also an option in this case. Gamma (/index.php?title=Gamma)() is especially appropriate when encoding arrival times for sets of events. A gamma distribution with a large value for «alpha» is also useful when you wish to use a bell-shaped curve for a positive-only quantity.

## Statistics

The theoretical statistics (i.e., in the absence of sampling error) for the gamma distribution are as follows.

- $0 < x < \infty$
- $\alpha, \beta > 0$
- Mean (/index.php?title=Mean) = $\alpha\beta$
- Mode (/index.php?title=Mode) = $Max([0, \alpha - 1)\beta])$

- Variance (/index.php?title=Variance) = $\alpha\beta^2$
- SDeviation (/index.php?title=SDeviation) = $\beta\sqrt{\alpha}$
- Skewness (/index.php?title=Skewness) = $2/\sqrt{\alpha}$
- Kurtosis (/index.php?title=Kurtosis) = $6/\alpha$

# Parameter Estimation

Suppose `X` contains sampled historical data indexed by `I`. To estimate the parameters of the gamma distribution that best fits this sampled data, the following parameter estimation formulae can be used:

```
alpha := Mean (/index.php?title=Mean)(X, I)^2/Variance (/index.php?title=Variance)(X, I)
beta  := Variance (/index.php?title=Variance)(X, I)/Mean (/index.php?title=Mean)(X, I)
```

The above is not the maximum likelihood parameter estimation, which turns out to be rather complex (see Wikipedia (http://en.wikipedia.org/wiki/Gamma_Distribution#Maximum_likelihood_estimation)). However, in practice the above estimation formula perform excellently and are so convenient that more complicated methods are hardly justified.

The Gamma distribution with an «offset» has the form:

Gamma (/index.php?title=Gamma)(alpha, beta) - offset

To estimate all three parameters, the following heuristic estimation can be used:

```
alpha  := 4/Skewness (/index.php?title=Skewness)(X, I)^2
offset := Mean (/index.php?title=Mean)(X, I) - SDeviation (/index.php?title=SDeviation)(X, I)*Sqrt
(/index.php?title=Sqrt)(alpha)
beta  := Variance (/index.php?title=Variance)(X, I)/(Mean (/index.php?title=Mean)(X, I) - offset)
```

# History

- The analytic functions, DensGamma (/index.php?title=DensGamma), CumGamma (/index.php?title=CumGamma), and CumGammaInv (/index.php?title=CumGammaInv) were added as built-in functions to Analytica 5.2 (/index.php?title=Analytica_5.2).
- The underscore in `Dens_Gamma` function in the Distribution Densities Library (/index.php?title=Distribution_Densities_Library) was dropped for the built-in function.
- In Analytica 5.0 (/index.php?title=Analytica_5.0), the analytic functions CumGamma (/index.php?title=CumGamma) and CumGammaInv (/index.php?title=CumGammaInv) were added to the Distribution Densities Library (/index.php?title=Distribution_Densities_Library). Although they are identical to the incomplete gamma function and its inverse, GammaI (/index.php?title=GammaI) and GammaIInv (/index.php?title=GammaIInv), and hence entirely redundant, the addition was done to match the naming convention used for all other distributions.
- GammaI (/index.php?title=GammaI) and GammaIInv (/index.php?title=GammaIInv) were added as built-in functions in Analytica 2.0.

# See Also

- Erlang (/index.php?title=Erlang)
- Gamma_m_sd (/index.php?title=Gamma_m_sd)
- GammaI (/index.php?title=GammaI) -- cumulative density at «x», incomplete gamma function
- GammaIInv (/index.php?title=GammaIInv) -- inverse cumulative density
- GammaFn (/index.php?title=GammaFn) -- the gamma function
- Beta (/index.php?title=Beta)

- Exponential (/index.php?title=Exponential)
- LogNormal (/index.php?title=LogNormal) -- and above, related distributions
- SDeviation (/index.php?title=SDeviation)
- Parametric continuous distributions (/index.php?title=Parametric_continuous_distributions)
- Distribution Densities Library (/index.php?title=Distribution_Densities_Library)



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