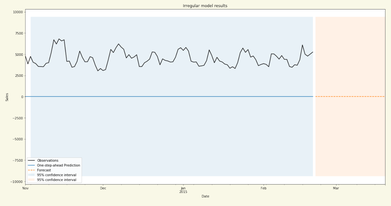
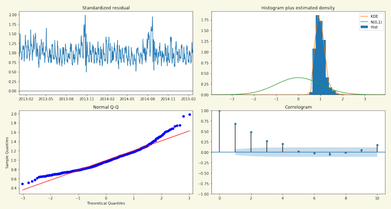
Time Series Modelling with Unobserved Components

Irregular

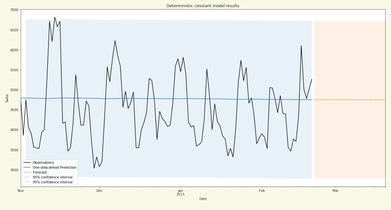
$$y_t = \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

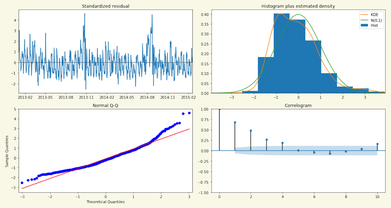




Deterministic Constant

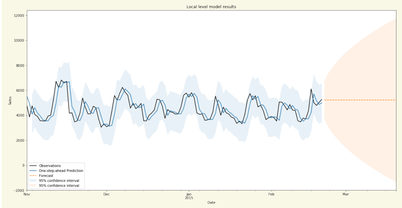
$$y_t = \mu + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

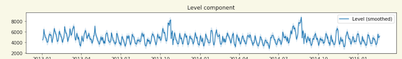




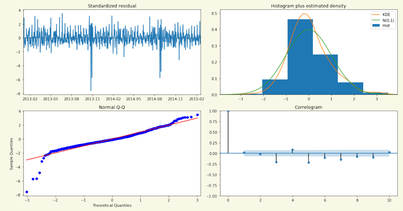
Local Level

$$y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_{t+1} = \mu_t + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$
$$\mu_1 \sim \mathcal{N}(a_1, P_1)$$



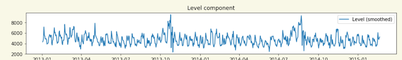


Note: The first 1 observations are not shown, due to approximate diffuse initialization.

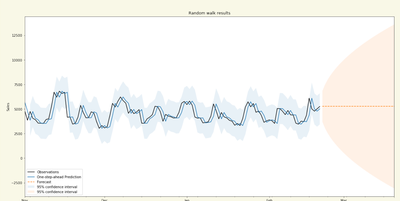


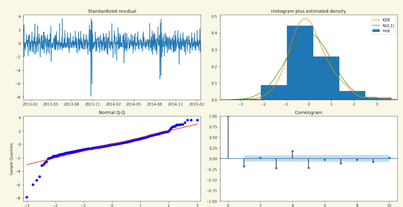
Random Walk

$$y_t = \mu_t$$
$$\mu_t = \mu_{t-1} + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$



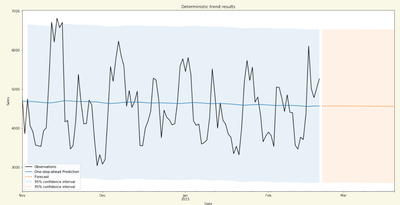
Note: The first 1 observations are not shown, due to approximate diffuse initialization.

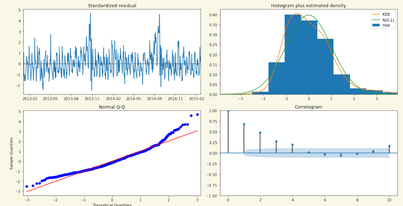


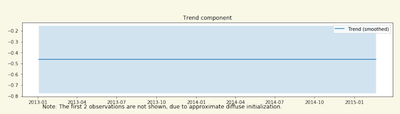


Deterministic Trend

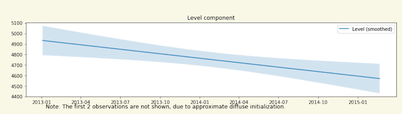
$$y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta$$







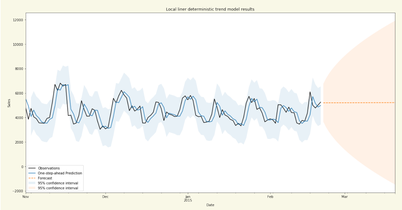
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

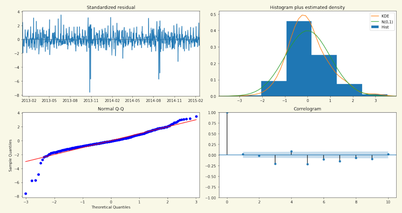


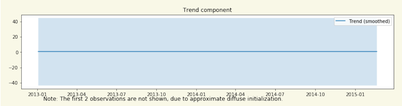
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

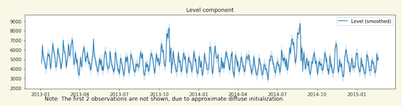
Local Linear Deterministic Trend

$$y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$





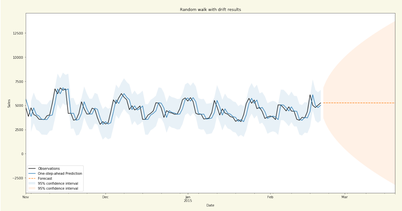


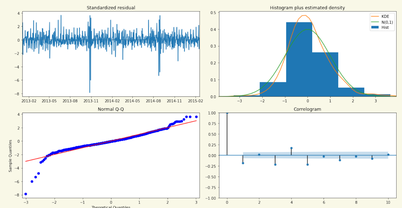


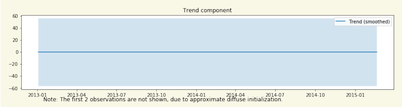
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

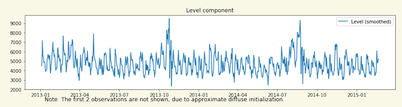
Random Walk With Drift

$$y_t = \mu_t$$
$$\mu_t = \mu_{t-1} + \beta + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$





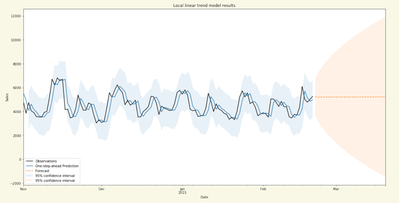


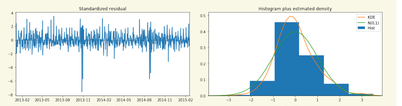


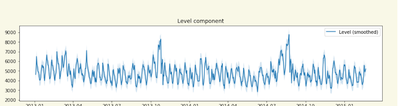
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

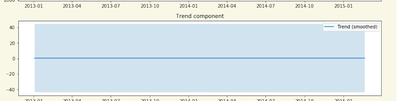
Local Linear Trend

$$y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$
$$\beta_t = \beta_{t-1} + \zeta_t, \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$





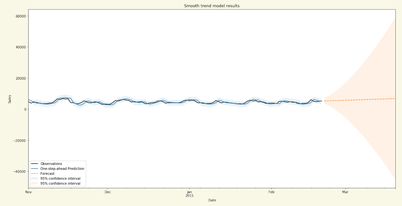


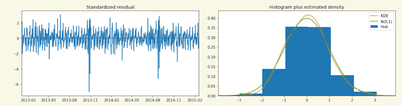


Note: The first 2 observations are not shown, due to approximate diffuse initialization.

Smooth Trend

$$y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta_{t-1}$$
$$\beta_t = \beta_{t-1} + \zeta_t, \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$

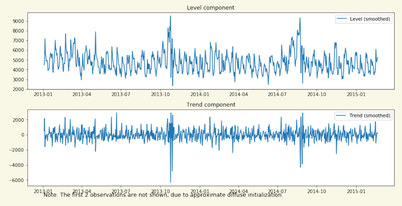




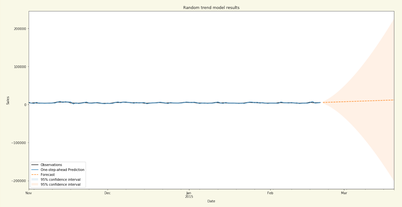
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

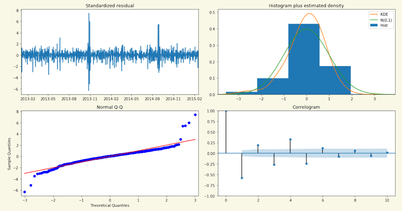
Random Trend

$$y_t = \mu_t$$
$$\mu_t = \mu_{t-1} + \beta_{t-1}$$
$$\beta_t = \beta_{t-1} + \zeta_t, \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$



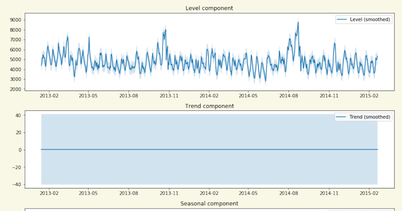
Note: The first 2 observations are not shown, due to approximate diffuse initialization.

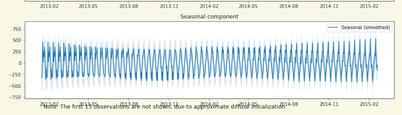




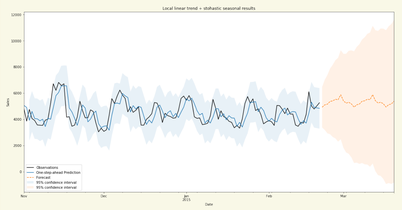
Local Linear Trend with Seasonal

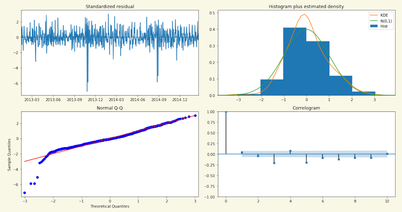
$$y_t = \mu_t + \gamma_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$
$$\beta_t = \beta_{t-1} + \zeta_t, \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$
$$\mu_1 \sim \mathcal{N}(a_1, P_1)$$
$$\gamma_t = -\sum_{j=1}^{s-1} \gamma_{t+1-j} + \omega_t, \omega_t \sim \mathcal{N}(0, \sigma_\omega^2)$$





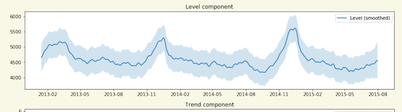
Note: The first 11 observations are not shown, due to approximate diffuse initialization.





Local Linear Trend with Seasonal and Regressors

$$y_t = \mu_t + \gamma_t + \delta_1^T x_t + \delta_2^T z_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$
$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$$
$$\beta_t = \beta_{t-1} + \zeta_t, \zeta_t \sim \mathcal{N}(0, \sigma_\zeta^2)$$
$$\mu_1 \sim \mathcal{N}(a_1, P_1)$$
$$\gamma_t = -\sum_{j=1}^{s-1} \gamma_{t+1-j} + \omega_t, \omega_t \sim \mathcal{N}(0, \sigma_\omega^2)$$



Note: The first 11 observations are not shown, due to approximate diffuse initialization.

Olya Gnilova

Higher School of Economics, 2017