### 0.1 Syntax

$$\langle foo \rangle$$
 ::=  $\langle bar \rangle [\langle baz \rangle] | \langle wat \rangle$ 

### 0.2 Static rules for expressions

$$\overline{A \vdash_{\mathtt{sfrm}} x} \quad \text{WF-Var}$$

$$\overline{A \vdash_{\mathtt{sfrm}} v} \quad \text{WF-Value}$$

$$\underline{(x, f) \in A}_{A \vdash_{\mathtt{sfrm}} x, f} \quad \text{WF-Field}$$

#### 0.3 Static rules for formulas

# 0.4 Static footprint

```
\begin{array}{lll} \texttt{static-footprint}(\texttt{true}) &= \emptyset \\ \\ \texttt{static-footprint}(e_1 = e_2) &= \emptyset \\ \\ \texttt{static-footprint}(e_1 \neq e_2) &= \emptyset \\ \\ \texttt{static-footprint}(\texttt{acc}(x.f)) &= \{(x,f)\} \\ \\ \texttt{static-footprint}(\phi_1 * \phi_2) &= \texttt{static-footprint}(\phi_1) \cup \texttt{static-footprint}(\phi_2) \end{array}
```

#### 0.5 Hoare

$$\frac{\Gamma \vdash \{\phi_p\}s_1\{\phi_{q1}\} \quad \phi_{q1} \implies \phi_{q2} \quad \Gamma \vdash \{\phi_{q2}\}s_2\{\phi_r\}}{\Gamma \vdash \{\phi_p\}s_1; s_2\{\phi_r\}} \quad \text{H-Sec}$$

$$\frac{\Gamma x = C) \qquad \mathtt{fields}(C) = \{\overline{f_i}\}}{\Gamma \vdash \{\phi\}x := \mathsf{new}\ C\{\overline{\mathtt{acc}(x.f_i)} * x \neq \mathtt{null} * \phi\}} \quad \text{H-NewObj}$$

$$\frac{\phi \implies \mathtt{acc}(x.f) * x \neq \mathtt{null}}{\Gamma \vdash \{\phi\} x.f := y \{\phi * x.f = y\}} \quad \text{H-FieldAssign}$$

$$\frac{\phi' = \phi[e/x] \qquad \emptyset \vdash_{\mathtt{sfrm}} \phi' \qquad \mathtt{static-footprint}(\phi') \vdash_{\mathtt{sfrm}} e}{\Gamma \vdash \{\phi'\}x := e\{\phi\}} \qquad \text{H-VarAssign}$$

$$\frac{1}{\Gamma \vdash \{\phi\} \text{return } x \{\phi * \text{result} = x\}} \quad \text{H-Return}$$

$$\frac{\Gamma y = C) \qquad \phi \implies y \neq null * \phi_p * \phi_r) \qquad \phi_p = \texttt{mpre}(C, m)[y, \overline{z}/\texttt{this}, \overline{X}]) \qquad \phi_q = \texttt{mpost}(C, m)[y, \overline{z}, x/\texttt{this}, \overline{X}]}{\Gamma \vdash \{\phi\}x := y.m(\overline{z})\{\phi_q * \phi_r\}}$$

$$\frac{\phi \implies \phi'}{\Gamma \vdash \{\phi\} \text{assert } \phi' \{\phi\}} \quad \text{H-Assert}$$

$$\frac{\phi \implies \phi' * \phi_r \qquad \emptyset \vdash_{\mathtt{sfrm}} \phi_r}{\Gamma \vdash \{\phi\} \mathtt{release} \ \phi' \{\phi_r\}} \quad \text{H-Release}$$

# 0.6 Dynamic rules for expressions

$$\overline{H, \rho \vdash x \Downarrow \rho(x)}$$
 EE-Var

$$\overline{H, \rho \vdash v \Downarrow v}$$
 EE-Value

$$\frac{H, \rho \vdash x \Downarrow o}{H, \rho \vdash x.f \Downarrow G(o)(f)} \quad \text{EE-Acc}$$

### 0.7 Dynamic rules for formulas

$$H, \rho, A \models \mathsf{true}$$
 EA-True

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 = v_2}{H, \rho, A \vDash e_1 = e_2} \quad \text{EA-Equal}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 = v_2}{H, \rho, A \vDash e_1 = e_2} \quad \text{EA-NEqual}$$

$$\frac{H, \rho \vdash x \Downarrow o \quad (o, f) \in A}{H, \rho, A \vDash \mathsf{acc}(x.f)} \quad \text{EA-Acc}$$

$$\frac{A_1 = A \backslash A_2 \qquad H, \rho, A_1 \vDash \phi_1 \qquad H, \rho, A_2 \vDash \phi_2}{H, \rho, A \vDash \phi_1 * \phi_2} \quad \text{EA-SepOp}$$

### 0.8 Dynamic footprint

$$\begin{array}{ll} \operatorname{footprint}_{H,\rho}(\operatorname{true}) & = \emptyset \\ \operatorname{footprint}_{H,\rho}(e_1 = e_2) & = \emptyset \\ \operatorname{footprint}_{H,\rho}(e_1 \neq e_2) & = \emptyset \\ \operatorname{footprint}_{H,\rho}(\operatorname{acc}(e.f)) & = \{(o,f)\} \text{ where } H, \rho \vdash e \Downarrow o \\ \operatorname{footprint}_{H,\rho}(\phi_1 * \phi_2) & = \operatorname{footprint}_{H,\rho}(\phi_1) \cup \operatorname{footprint}_{H,\rho}(\phi_2) \end{array}$$

## 0.9 Small-step semantics

TODO

#### 0.10 Theorems

Hoare preserves self-framing

$$\begin{split} \forall \; \Gamma, \phi_1, \phi_2, s : \Gamma \vdash \{\phi_1\} s \{\phi_2\} \\ &\implies \mathsf{static\text{-}footprint}(\phi_1) \vdash_{\mathsf{sfrm}} \phi_1 \\ &\implies \mathsf{static\text{-}footprint}(\phi_2) \vdash_{\mathsf{sfrm}} \phi_2 \end{split}$$

#### Hoare progress

$$\begin{split} \forall \; \Gamma, \phi_1, \phi_2, s, H_1, \rho_1, A_1 : \Gamma \vdash \{\phi_1\} s \{\phi_2\} \\ &\Longrightarrow \; H_1, \rho_1, A_1 \vDash \phi_1 \\ &\Longrightarrow \; \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S) \end{split}$$

Hoare preservation

$$\forall \ \Gamma, \phi_1, \phi_2, s, H_1, H_2, \rho_1, \rho_2, A_1, A_2 : \Gamma \vdash \{\phi_1\} s \{\phi_2\}$$

$$\Longrightarrow H_1, \rho_1, A_1 \vDash \phi_1$$

$$\Longrightarrow (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S)$$

$$\Longrightarrow H_2, \rho_2, A_2 \vDash \phi_2$$