

1 Syntax

<i>program</i>	$::= \overline{cls} \ \bar{s}$
<i>cls</i>	$::= \text{class } C \ \{\overline{field} \ \overline{method}\}$
<i>field</i>	$::= T \ f;$
<i>method</i>	$::= T \ m(T \ x) \ \text{contract} \ \{\bar{s}\}$
<i>contract</i>	$::= \text{requires } \phi; \ \text{ensures } \phi;$
<i>T</i>	$::= \text{int} \mid C$
<i>s</i>	$::= x.f := y; \mid x := e; \mid x := \text{new } C; \mid x := y.m(z);$ $\mid \text{return } x; \mid \text{assert } \phi; \mid \text{release } \phi; \mid T \ x;$
ϕ	$::= \text{true} \mid e = e \mid e \neq e \mid \text{acc}(e.f) \mid x : T \mid \phi * \phi$
<i>e</i>	$::= v \mid x \mid e.f$
<i>x</i>	$::= \text{this} \mid \text{result} \mid \langle \text{other} \rangle$
<i>v</i>	$::= o \mid n \mid \text{null}$
<i>n</i>	$\in \mathbb{Z}$
<i>H</i>	$\in (o \multimap (C, \overline{(f \multimap v)}))$
ρ	$\in (x \multimap v)$
<i>A_s</i>	$::= \overline{(e, f)}$
<i>A_d</i>	$::= \overline{(o, f)}$
<i>S</i>	$::= (\rho, A_d, \bar{s}) \cdot S \mid \text{nil}$

2 Assumptions

All the rules in the following sections are implicitly parameterized over a *program* that is well-formed.

2.0.1 Well-formed program (*program* OK)

$$\frac{\overline{cls_i \text{ OK}}}{(\overline{cls_i} \ \bar{s}) \text{ OK}} \text{ OKPROGRAM}$$

2.0.2 Well-formed class (*cls* OK)

$$\frac{\text{unique } field\text{-names} \quad \text{unique } method\text{-names} \quad \overline{method_i \text{ OK in } C}}{(\text{class } C \ \{\overline{field_i} \ \overline{method_i}\}) \text{ OK}} \text{ OKCLASS}$$

2.0.3 Well-formed method (*method* OK in *C*)

$$\frac{\begin{array}{c} FV(\phi_1) \subseteq \{x, \text{this}\} \\ FV(\phi_2) \subseteq \{x, \text{this}, \text{result}\} \quad \vdash \{x : T_x * \text{this} : C * \phi_1\} \bar{s} \{x : T_x * \text{this} : C * \text{result} : T_m * \phi_2\} \\ \emptyset \vdash_{\text{sfrm}} \phi_1 \quad \emptyset \vdash_{\text{sfrm}} \phi_2 \quad \neg \text{writesTo}(s_i, x) \end{array}}{(T_m \ m(T_x \ x) \ \text{requires } \phi_1; \ \text{ensures } \phi_2; \ \{\bar{s}\}) \text{ OK in } C} \text{ OKMETHOD}$$

3 Static semantics

3.1 Expressions (*A_s* \vdash_{sfrm} *e*)

$$\frac{}{A \vdash_{\text{sfrm}} x} \text{ WFVAR}$$

$$\frac{}{A \vdash_{\text{sfrm}} v} \text{WFVALUE}$$

$$\frac{(e, f) \in A \quad A \vdash_{\text{sfrm}} e}{A \vdash_{\text{sfrm}} e.f} \text{WFFIELD}$$

3.2 Formulas ($A_s \vdash_{\text{sfrm}} \phi$)

$$\frac{}{A \vdash_{\text{sfrm}} \text{true}} \text{WFTTRUE}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 = e_2)} \text{WFEQUAL}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 \neq e_2)} \text{WFNEQUAL}$$

$$\frac{A \vdash_{\text{sfrm}} e}{A \vdash_{\text{sfrm}} \text{acc}(e.f)} \text{WFAcc}$$

$$\frac{}{A \vdash_{\text{sfrm}} (x : T)} \text{WFTYPE}$$

$$\frac{A_s \vdash_{\text{sfrm}} \phi_1 \quad A_s \cup [\phi_1] \vdash_{\text{sfrm}} \phi_2}{A_s \vdash_{\text{sfrm}} \phi_1 * \phi_2} \text{WFSEPOP}$$

3.2.1 Implication ($\phi_1 \Rightarrow \phi_2$)

Conservative approx. of $\phi_1 \Rightarrow \phi_2$.

3.3 Footprint ($[\phi] = A_s$)

$$\begin{array}{ll} [\text{true}] & = \emptyset \\ [e_1 = e_2] & = \emptyset \\ [e_1 \neq e_2] & = \emptyset \\ [\text{acc}(e.f)] & = \{(e, f)\} \\ [\phi_1 * \phi_2] & = [\phi_1] \cup [\phi_2] \end{array}$$

3.4 Type ($\phi \vdash e : T$)

$$\frac{}{\phi \vdash n : \text{int}} \text{STVALNUM}$$

$$\frac{}{\phi \vdash \text{null} : T} \text{STVALNULL}$$

$$\frac{\phi \Longrightarrow (x : T)}{\phi \vdash x : T} \text{STVAR}$$

$$\frac{\phi \vdash e : C \quad \vdash C.f : T}{\phi \vdash e.f : T} \text{STFIELD}$$

3.5 Hoare ($\vdash \{\phi\} \bar{s} \{\phi\}$)

$$\frac{\vdash \{\phi_p\} s_1 \{\phi_{q1}\} \quad \phi_{q1} \Longrightarrow \phi_{q2} \quad \emptyset \vdash_{\text{sfrm}} \phi_{q2} \quad \vdash \{\phi_{q2}\} s_2 \{\phi_r\}}{\vdash \{\phi_p\} s_1; s_2 \{\phi_r\}} \text{HSEC}$$

$$\frac{\phi \Longrightarrow \phi' \quad \emptyset \vdash_{\text{sfrm}} \phi' \quad x \notin FV(\phi') \quad \phi \vdash x : C \quad \text{fields}(C) = \bar{f}}{\vdash \{\phi\} x := \text{new } C \{ \text{acc}(x, f_i) * x : C * (x \neq \text{null}) * \phi' \}} \text{HNEWOBJ}$$

$$\frac{\phi \Longrightarrow \text{acc}(x.f) * (x \neq \text{null}) * \phi' \quad \emptyset \vdash_{\text{sfrm}} \phi' \quad \phi \vdash x : C \quad \phi \vdash y : T \quad \vdash C.f : T}{\vdash \{\phi\} x.f := y \{ x : C * \text{acc}(x.f) * (x \neq \text{null}) * (x.f = y) * \phi' \}} \text{HFIELDASSIGN}$$

$$\frac{\phi \Longrightarrow \phi' \quad \emptyset \vdash_{\text{sfrm}} \phi' \quad x \notin FV(\phi') \quad x \notin FVe(e) \quad \phi \vdash x : T \quad \phi \vdash e : T \quad [\phi'] \vdash_{\text{sfrm}} e}{\vdash \{\phi\} x := e \{ \phi' * (x = e) \}} \text{HVARASSIGN}$$

$$\frac{\phi \Longrightarrow \phi' \quad \emptyset \vdash_{\text{sfrm}} \phi' \quad \text{result} \notin FV(\phi') \quad \phi \vdash x : T \quad \phi \vdash \text{result} : T}{\vdash \{\phi\} \text{return } x \{ \text{result} : T * (\text{result} = x) * \phi' \}} \text{HRETURN}$$

$$\frac{\begin{array}{l} \phi \vdash y : C \quad \text{mmethod}(C, m) = T_r \ m(T_p \ z) \text{ requires } \phi_{pre}; \text{ ensures } \phi_{post}; \{ _ \} \\ \phi \vdash x : T_r \quad \phi \vdash z' : T_p \quad \phi \Longrightarrow (y \neq \text{null}) * \phi_p * \phi_r \quad \emptyset \vdash_{\text{sfrm}} \phi_r \quad x \notin FV(\phi_r) \\ @listDistinct(x, x \cdot y \cdot z' \cdot \emptyset) \quad \phi_p = \phi_{pre}[y, z' / \text{this}, z] \quad \phi_q = \phi_{post}[y, z', x / \text{this}, z, \text{result}] \end{array}}{\vdash \{\phi\} x := y.m(z') \{ \phi_q * \phi_r \}} \text{HAPP}$$

$$\frac{\phi_1 \Longrightarrow \phi_2}{\vdash \{\phi_1\} \text{assert } \phi_2 \{ \phi_1 \}} \text{HASSERT}$$

$$\frac{\phi_1 \Longrightarrow \phi_2 * \phi_r \quad \emptyset \vdash_{\text{sfrm}} \phi_r}{\vdash \{\phi_1\} \text{release } \phi_2 \{ \phi_r \}} \text{HRELEASE}$$

$$\frac{\phi \Longrightarrow \phi' \quad \emptyset \vdash_{\text{sfrm}} \phi' \quad x \notin FV(\phi')}{\vdash \{\phi\} T \ x \{ x : T * (x = \text{defaultValue}(T)) * \phi' \}} \text{HDECLARE}$$

3.5.1 Hoare revisited - pre-grad minification

$$\frac{\vdash \{\phi_p\}s_1\{\phi_{q1}\} \quad \phi_{q1} \implies \phi_{q2} \quad \emptyset \vdash_{\text{sfrm}} \phi_{q2} \quad \vdash \{\phi_{q2}\}s_2\{\phi_r\}}{\vdash \{\phi_p\}s_1; s_2\{\phi_r\}} \text{HSEC}$$

$$\frac{\text{NotInFV}(x, \phi) \quad \text{fields}(C) = \bar{f}}{\vdash \{(x : C) * \phi\}x := \text{new } C\{\text{acc}(x, f_i)(x : C) * (x \neq \text{null}) * \phi\}} \text{HNEWOBJ}$$

$$\frac{\vdash C.f : T}{\vdash \{(x : C) * (y : T) * \phi * \text{acc}(x.f)\}x.f := y\{(x : C) * \text{acc}(x.f) * (x.f = y) * \phi\}} \text{HFIELDASSIGN}$$

$$\frac{\text{NotInFV}(x, \phi) \quad x \notin \text{FV}(e) \quad [e : T]_{T'}}{\vdash \{(x : T) * \llbracket e : T \rrbracket_{T'} * \phi\}x := e\{\llbracket e : T \rrbracket_{T'} * \phi * (x = e)\}} \text{HVARASSIGN}$$

$$\frac{\text{NotInFV}(\text{result}, \phi)}{\vdash \{(x : T) * (\text{result} : T) * \phi\}\text{return } x\{(\text{result} : T) * (\text{result} = x) * \phi\}} \text{HRETURN}$$

$$\frac{\text{mmethod}(C, m) = T_r \ m(T_p \ z) \ \text{requires } \phi_{pre}; \ \text{ensures } \phi_{post}; \ \{_\} \quad \text{NotInFV}(x, \phi_r) \quad x \neq y \wedge x \neq z'}{\vdash \{(x : T_r) * (y : C) * (z' : T_p) * \phi_r * (y \neq \text{null}) * \phi_{pre}[y, z'/\text{this}, z]\}x := y.m(z')\{\phi_r * \phi_{post}[y, z', x/\text{this}, z, \text{result}]\}} \text{H}$$

$$\frac{\phi \implies \phi'}{\vdash \{\phi\}\text{assert } \phi'\{\phi\}} \text{HASSERT}$$

$$\frac{}{\vdash \{\phi * \phi'\}\text{release } \phi'\{\phi\}} \text{HRELEASE}$$

$$\frac{\text{NotInFV}(x, \phi)}{\vdash \{\phi\}T \ x\{(x : T) * (x = \text{defaultValue}(T)) * \phi\}} \text{HDECLARE}$$

3.5.2 Hoare revisited - pre-grad minification with simple HSec

$$\frac{\vdash \{\phi_p\}s_1\{\phi_q\} \quad \vdash \{\phi_q\}s_2\{\phi_r\}}{\vdash \{\phi_p\}s_1; s_2\{\phi_r\}} \text{HSEC}$$

(other rules not printed here: think of previous subsection, but with self-framed implication on every pre-condition)

4 Dynamic semantics

4.1 Expressions ($H, \rho \vdash e \Downarrow v$)

$$\frac{}{H, \rho \vdash x \Downarrow \rho(x)} \text{EEVAR}$$

$$\frac{}{H, \rho \vdash v \Downarrow v} \text{EEVALUE}$$

$$\frac{H, \rho \vdash e \Downarrow o}{H, \rho \vdash e.f \Downarrow H(o)(f)} \text{EEACC}$$

4.2 Formulas ($H, \rho, A \models \phi$)

$$\frac{}{H, \rho, A \models \mathbf{true}} \text{EATrue}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 = v_2}{H, \rho, A \models (e_1 = e_2)} \text{EAEqual}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 \neq v_2}{H, \rho, A \models (e_1 \neq e_2)} \text{EANEqual}$$

$$\frac{H, \rho \vdash e \Downarrow o \quad (o, f) \in A}{H, \rho, A \models \mathbf{acc}(e.f)} \text{EAAcc}$$

$$\frac{\rho(x) = v \quad H \vdash v : T}{H, \rho, A \models (x : T)} \text{EAType}$$

$$\frac{A_1 = A \setminus A_2 \quad H, \rho, A_1 \models \phi_1 \quad H, \rho, A_2 \models \phi_2}{H, \rho, A \models \phi_1 * \phi_2} \text{EASEPop}$$

We give a denotational semantics of formulas as $\llbracket \phi \rrbracket = \{ (H, \rho, A) \mid H, \rho, A \models \phi \}$

Note: ϕ satisfiable $\iff \llbracket \phi \rrbracket \neq \emptyset$

4.2.1 Implication ($\phi_1 \implies \phi_2$)

$$\phi_1 \implies \phi_2 \iff \forall H, \rho, A : H, \rho, A \models \phi_1 \implies H, \rho, A \models \phi_2$$

Drawn from def. of entailment in “A Formal Semantics for Isorecursive and Equirecursive State Abstractions”.

4.3 Footprint ($\llbracket \phi \rrbracket_{H, \rho} = A_d$)

$$\begin{aligned} \llbracket \mathbf{true} \rrbracket_{H, \rho} &= \emptyset \\ \llbracket e_1 = e_2 \rrbracket_{H, \rho} &= \emptyset \\ \llbracket e_1 \neq e_2 \rrbracket_{H, \rho} &= \emptyset \\ \llbracket \mathbf{acc}(x.f) \rrbracket_{H, \rho} &= \{(o, f)\} \text{ where } H, \rho \vdash x \Downarrow o \\ \llbracket \phi_1 * \phi_2 \rrbracket_{H, \rho} &= \llbracket \phi_1 \rrbracket_{H, \rho} \cup \llbracket \phi_2 \rrbracket_{H, \rho} \end{aligned}$$

4.4 Small-step ($(H, S) \rightarrow (H', S)$)

$$\frac{H, \rho \vdash x \Downarrow o \quad H, \rho \vdash y \Downarrow v_y \quad (o, f) \in A \quad H' = H[o \mapsto [f \mapsto v_y]]}{(H, (\rho, A, x.f := y; \bar{s}) \cdot S) \rightarrow (H', (\rho, A, \bar{s}) \cdot S)} \text{ESFieldAssign}$$

$$\frac{H, \rho \vdash e \Downarrow v \quad \rho' = \rho[x \mapsto v]}{(H, (\rho, A, x := e; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESVarAssign}$$

$$\frac{\text{fields}(C) = \bar{T} \ \bar{f} \quad \rho' = \rho[x \mapsto o] \quad A' = A * (\bar{o}, \bar{f}_i) \quad H' = H[o \mapsto \overline{[f \mapsto \text{defaultValue}(\bar{T})]}]}{(H, (\rho, A, x := \mathbf{new} \ C; \bar{s}) \cdot S) \rightarrow (H', (\rho', A', \bar{s}) \cdot S)} \text{ESNewObj}$$

$$\frac{H, \rho \vdash x \Downarrow v_x \quad \rho' = \rho[\mathbf{result} \mapsto v_x]}{(H, (\rho, A, \mathbf{return} \ x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESRETURN}$$

$$\frac{\begin{array}{c} H, \rho \vdash y \Downarrow o \\ H, \rho \vdash z \Downarrow v \quad H(o) = (C, _) \quad \text{mmethod}(C, m) = T_r \ m(T \ w) \text{ requires } \phi; \text{ ensures } _; \{\bar{r}\} \\ \rho' = [\mathbf{result} \mapsto \mathbf{defaultValue}(T_r), \mathbf{this} \mapsto o, w \mapsto v] \quad H, \rho', A \models \phi \quad A' = \lfloor \phi \rfloor_{H, \rho'} \end{array}}{(H, (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho', A', \bar{r}) \cdot (\rho, A \setminus A', x := y.m(z); \bar{s}) \cdot S))} \text{ESAPP}$$

$$\frac{\begin{array}{c} H(o) = (C, _) \quad \text{mpost}(C, m) = \phi \quad H, \rho \vdash y \Downarrow o \\ H, \rho', A' \models \phi \quad A'' = \lfloor \phi \rfloor_{H, \rho'} \quad H, \rho' \vdash \mathbf{result} \Downarrow v_r \end{array}}{(H, (\rho', A', \emptyset) \cdot (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho[x \mapsto v_r], A * A'', \bar{s}) \cdot S))} \text{ESAPPFINISH}$$

$$\frac{H, \rho, A \models \phi}{(H, (\rho, A, \mathbf{assert} \ \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A, \bar{s}) \cdot S)} \text{ESASSERT}$$

$$\frac{H, \rho, A \models \phi \quad A' = A \setminus \lfloor \phi \rfloor_{H, \rho}}{(H, (\rho, A, \mathbf{release} \ \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A', \bar{s}) \cdot S)} \text{ESRELEASE}$$

$$\frac{\rho' = \rho[x \mapsto \mathbf{defaultValue}(T)]}{(H, (\rho, A, T \ x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESDECLARE}$$

5 Gradualization

5.1 Syntax

5.1.1 Gradual formula

$$\tilde{\phi} ::= \phi \mid ? * \phi$$

Note: consider $?$ in other positions as “self-framing delimiter”, but with semantically identical meaning. As long as $?$ is only legal in the front though: $\phi_1 * \tilde{\phi}_2$ propagates the $?$ to the very left in case $\tilde{\phi}_2$ contains one.

5.1.2 Type judgment expansion

Motivation: Materialize and combine the requirements on ϕ .

Designed so that

$$\begin{aligned} \phi \vdash e : T \quad \wedge \quad \lfloor \phi \rfloor \vdash_{\mathbf{sfrm}} e \\ \iff \\ [e : T]_C \quad \wedge \quad \phi \implies \llbracket e : T \rrbracket_C \end{aligned}$$

Expand into premise: $[e : T]_C$

$$\begin{aligned} [v : T]_C &= T = C \quad \wedge \quad \vdash v : T \\ [x : T]_C &= T = C \\ [e.f : T]_C &= [e : C']_C \quad \wedge \quad \vdash C'.f : T \end{aligned}$$

Expand into formula: $\llbracket e : T \rrbracket_C$

$$\begin{aligned} \llbracket v : T \rrbracket_C &= \mathbf{true} \\ \llbracket x : T \rrbracket_C &= (x : C) \\ \llbracket e.f : T \rrbracket_C &= \llbracket e : T \rrbracket_C * \mathbf{acc}(e.f) \end{aligned}$$

5.2 Concretization

Syntax $\hat{\phi} :=$ self-framed and satisfiable ϕ

$$\begin{aligned} \gamma(\hat{\phi}) &= \{ \hat{\phi} \} \\ \gamma(? * \phi') &= \{ \hat{\phi} \mid \hat{\phi} \implies \phi' \} \text{ if } \phi' \text{ satisfiable} \\ \gamma(\phi) &\text{ undefined otherwise} \end{aligned}$$

5.3 Abstraction

$$\begin{aligned} \alpha(\emptyset) &\text{ undefined} \\ \alpha(\{ \phi \}) &= \phi \\ \alpha(\overline{\phi} \text{ with maximum element } \phi) &= ? * \phi \\ \alpha(\overline{\phi}) &= ? \text{ otherwise} \end{aligned}$$

5.4 Gradual Lifting

5.4.1 Self framing

$$\frac{A \vdash_{\text{sfrm}} \phi}{A \widetilde{\vdash}_{\text{sfrm}} \phi} \text{ GSFRMNONGRAD}$$

$$\frac{}{A \widetilde{\vdash}_{\text{sfrm}} ? * \phi} \text{ GSFRMGRAD}$$

5.4.2 Implication

$$\frac{\phi_1 \implies \phi_2}{\phi_1 \widetilde{\implies} \phi_2} \text{ GIMPLNONGRAD}$$

$$\frac{\hat{\phi}_m \implies \phi_2 \quad \hat{\phi}_m \implies \phi_1}{? * \phi_1 \widetilde{\implies} \phi_2} \text{ GIMPLGRAD}$$

$\hat{\phi}_m$ is evidence!

Consistent transitivity

While \implies is transitive, $\widetilde{\implies}$ is generally not.

But maybe not even necessary with smarter hoare rules?

5.4.3 Equality

$$\frac{\phi_1 = \phi_2}{\phi_1 \approx \phi_2} \text{ GEQSTATIC}$$

$$\frac{\begin{array}{c} \text{at least one of } \widetilde{\phi}_1 \text{ or } \widetilde{\phi}_2 \text{ contains ?} \\ \widetilde{\phi}_1 \widetilde{\implies} \widetilde{\phi}_2 \quad \widetilde{\phi}_2 \widetilde{\implies} \widetilde{\phi}_1 \end{array}}{\widetilde{\phi}_1 \approx \widetilde{\phi}_2} \text{ GEQGRADUAL}$$

5.4.4 Hoare and evidence

Discussion/Considerations:

- The post-condition- ϕ seems to inherit its gradual-ness from implication, which itself does not care about whether its second argument is gradual or not...
- Gradual

Example:

$$\frac{\emptyset \vdash_{\text{sfrm}} \tilde{\phi}' \quad x \notin FV(\tilde{\phi}') \quad x \notin FV(e) \quad \epsilon \vdash \tilde{\phi} \Rightarrow \tilde{\phi}' \quad \epsilon \vdash \tilde{\phi} \vdash x : T \quad \epsilon \vdash \tilde{\phi} \vdash e : T \quad \epsilon \vdash [\tilde{\phi}'] \vdash_{\text{sfrm}} e}{\vdash \{\tilde{\phi}\}x := e\{\tilde{\phi}' * (x = e)\}} \text{GHVARASSIGN}$$

Collapsing (hidden) gradual implications into a single one:

$$\frac{\epsilon \vdash \tilde{\phi} \Rightarrow (x : T) * \llbracket e : T \rrbracket_C * \tilde{\phi}' \quad \emptyset \vdash_{\text{sfrm}} \llbracket e : T \rrbracket_C * \tilde{\phi}' \quad x \notin FV(\tilde{\phi}') \quad x \notin FV(e) \quad [e : T]_C}{\vdash \{\tilde{\phi}\}x := e\{\llbracket e : T \rrbracket_C * \tilde{\phi}' * (x = e)\}} \text{GHVARASSIGN}$$

When shifting implication responsibility to GHSec:

$$\frac{x \notin FV(\tilde{\phi}') \quad x \notin FV(e) \quad [e : T]_C}{\vdash \{(x : T) * \llbracket e : T \rrbracket_C * \tilde{\phi}'\}x := e\{\llbracket e : T \rrbracket_C * \tilde{\phi}' * (x = e)\}} \text{GHVARASSIGN}$$

Example derivation:

$$\begin{aligned} & \{(x : T) * (y : C) * \text{acc}(y.a) * \text{acc}(y.a.b) * \text{acc}(y.a.b.c) * \tilde{\phi}'\} \\ & \{(x : T) * \llbracket y.a.b.c : T \rrbracket_C * \tilde{\phi}'\} \\ & x := y.a.b.c; \quad \begin{array}{l} x \notin FV(\tilde{\phi}') \\ x \notin FV(y.a.b.c) \\ [y.a.b.c : T]_C = \vdash C_y = C \wedge \vdash C_y.a : C_a \wedge \vdash C_a.b : C_b \wedge \vdash C_b.c : T \end{array} \\ & \{\llbracket y.a.b.c : T \rrbracket_C * \tilde{\phi}' * (x = y.a.b.c)\} \\ & \{(y : C) * \text{acc}(y.a) * \text{acc}(y.a.b) * \text{acc}(y.a.b.c) * \tilde{\phi}' * (x = y.a.b.c)\} \end{aligned}$$

5.4.5 GHFieldAssign

$$\frac{\begin{array}{l} \vdash_{\text{sfrm}} \phi \quad \vdash C.f : T \\ \tilde{\phi}_1 \approx (x : C) * (y : T) * (x \neq \text{null}) * \phi * \text{acc}(x.f) \quad \tilde{\phi}_2 \approx (x : C) * \text{acc}(x.f) * (x \neq \text{null}) * (x.f = y) * \phi \end{array}}{\vdash \{\tilde{\phi}_1\}x.f := y\{\tilde{\phi}_2\}} \text{GHFIELDASSIGN}$$

5.4.6 GHSec - sound but obviously not complete!

$$\frac{\tilde{\vdash}\{\tilde{\phi}_p\}s_1\{\tilde{\phi}_q\} \quad \phi_{q1} \Rightarrow \phi_{q2} \quad \emptyset \vdash_{\text{sfrm}} \phi_{q2} \quad \tilde{\vdash}\{\phi_{q2}\}s_2\{\tilde{\phi}_r\}}{\tilde{\vdash}\{\tilde{\phi}_p\}s_1; s_2\{\tilde{\phi}_r\}} \text{GHSEC}$$

5.4.7 HFieldAssign (deterministic alternative)

$$\frac{\begin{array}{l} \vdash C.f : T \\ \phi_1 \Rightarrow (x : C) * (y : T) * \text{acc}(x.f) \quad \phi_2 = (x : C) * \text{acc}(x.f) * (x.f = y) * \phi_1[\mathbf{w/o} \text{acc}(x.f)] \end{array}}{\vdash \{\phi_1\}x.f := y\{\phi_2\}} \text{HFIELDASSIGN}$$

Note: $\phi[\mathbf{w/o} \text{acc}(x.f)]$ removes $\text{acc}(x.f)$ and all uses of $x.f$ from ϕ . The result is self-framed given that ϕ is.

Attention: This version is weaker than the other (pairwise equivalent) versions of HFieldAssign!

Explanation: Above operator may remove more information than necessary from ϕ .

Example:

- Given: $\phi_1 = \text{acc}(x.f) * (x.f = a) * (x.f = b)$
- Goal: $\phi_2 \implies (a = b)$
- **not provable** with this deterministic version of HFieldAssign
- **provable** with all other versions

5.4.8 GHFieldAssign (deterministic alternative)

(= gradual lifting of GHFieldAssign as function)

$$\frac{\widetilde{\phi}_2 = \alpha(\{\phi_2 \mid \phi_1 \in \gamma(\widetilde{\phi}_1) \wedge \vdash \{\phi_1\}x.f := y\{\phi_2\}\})}{\vdash \{\widetilde{\phi}_1\}x.f := y\{\widetilde{\phi}_2\}} \text{GHFIELDASSIGN}$$

Which should be equivalent to this:

$$\frac{\phi_1 \implies (x : C) * (y : T) * \text{acc}(x.f) \quad \vdash C.f : T \quad \phi_2 = (x : C) * \text{acc}(x.f) * (x.f = y) * \phi_1[\mathbf{w/o} \text{acc}(x.f)]}{\vdash \{\phi_1\}x.f := y\{\phi_2\}} \text{GHFA1}$$

$$\frac{\widetilde{\phi}_1 \widetilde{\implies}_{\phi_m} (x : C) * (y : T) * \text{acc}(x.f) \quad \vdash C.f : T \quad \phi_2 = (x : C) * \text{acc}(x.f) * (x.f = y) * \phi_m[\mathbf{w/o} \text{acc}(x.f)]}{\vdash \{? * \phi_1\}x.f := y\{? * \phi_2\}} \text{GHFA2}$$

Which should be summarizable as this:

$$\frac{\widetilde{\phi}_1 \widetilde{\implies}_{\phi_m} (x : C) * (y : T) * \text{acc}(x.f) \quad \vdash C.f : T \quad \widetilde{\phi}_2 = (x : C) * \text{acc}(x.f) * (x.f = y) * \widetilde{\phi_m}[\mathbf{w/o} \text{acc}(x.f)]}{\vdash \{\widetilde{\phi}_1\}x.f := y\{\widetilde{\phi}_2\}} \text{GHFA2}$$

5.4.9 GHFieldAssign (implication alternative)

$$\frac{\vdash_{\text{sfrm}} \phi \quad \vdash C.f : T \quad \phi_1 \implies (x : C) * (y : T) * \phi * \text{acc}(x.f) \quad \phi_2 = (x : C) * \text{acc}(x.f) * (x.f = y) * \phi}{\vdash \{\phi_1\}x.f := y\{\phi_2\}} \text{HFIELDASSIGN}$$

$$\frac{\vdash_{\text{sfrm}} \phi \quad \vdash C.f : T \quad \widetilde{\phi}_1 \widetilde{\implies} (x : C) * (y : T) * \phi * \text{acc}(x.f) \quad \widetilde{\phi}_2 \approx (x : C) * \text{acc}(x.f) * (x.f = y) * \phi}{\vdash \{\widetilde{\phi}_1\}x.f := y\{\widetilde{\phi}_2\}} \text{GHFIELDASSIGN}$$

Note: With this alternative rule design $\widetilde{\implies}$ is consistently used with static formulas as second argument. This plays nicely with the fact that $\widetilde{\implies}$ does not care about the gradualness of that argument. Might make sense to define lifting of \implies as lifting on only the first parameter in the first place.

Minimum runtime checks: For $\widetilde{\phi}_1 \widetilde{\implies} \widetilde{\phi}_2$ to hold at runtime, practically just ϕ_2 needs to hold. So that would be a valid assertion to check. Yet, we know statically that ϕ_1 holds, so we can remove everything from the runtime check that is implied by ϕ_1 . So in a sense, we only need to check $\phi_2 \setminus \phi_1$ at runtime (the operator can be an approximation).

5.5 Theorems

5.5.1 Soundness of α

$$\forall \bar{\phi} : \bar{\phi} \subseteq \gamma(\alpha(\bar{\phi}))$$

5.5.2 Optimality of α

$$\forall \bar{\phi}, \tilde{\phi} : \bar{\phi} \subseteq \gamma(\tilde{\phi}) \implies \gamma(\alpha(\bar{\phi})) \subseteq \gamma(\tilde{\phi})$$

6 Theorems

6.1 Invariant $invariant(H, \rho, A_d, \phi)$

6.1.1 Phi valid

$$\vdash_{\text{sfrm}} \phi$$

6.1.2 Phi holds

$$H, \rho, A_d \models \phi$$

6.1.3 Types preserved

$$\begin{aligned} & \forall e, T : \phi \vdash e : T \\ \implies & H, \rho \vdash e : T \end{aligned}$$

6.1.4 Heap consistent

$$\begin{aligned} & \forall o, C, \mu, f, T : H(o) = (C, \mu) \\ \implies & \text{fieldType}(C, f) = T \\ \implies & H, \rho \vdash \mu(f) : T \end{aligned}$$

6.1.5 Heap not total

$$\begin{aligned} & \exists o_{\min} : \\ & \forall o \geq o_{\min} : o \notin \text{dom}(H) \\ & \quad \wedge \forall f, (o, f) \notin A \end{aligned}$$

6.2 Soundness

6.2.1 Progress

$$\begin{aligned} \forall \dots : & \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies & invariant(H_1, \rho_1, A_1, \phi_1) \\ \implies & \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \end{aligned}$$

6.2.2 Preservation

$$\begin{aligned} \forall \dots : & \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies & invariant(H_1, \rho_1, A_1, \phi_1) \\ \implies & (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \\ \implies & invariant(H_2, \rho_2, A_2, \phi_2) \end{aligned}$$