

# 1 Syntax

<i>program</i>	$::= \overline{cls} \ \overline{s}$
<i>cls</i>	$::= \text{class } C \ \{\overline{field} \ \overline{method}\}$
<i>field</i>	$::= T \ f;$
<i>method</i>	$::= T \ m(T \ x) \ \text{contract } \{\overline{s}\}$
<i>contract</i>	$::= \text{requires } \phi; \ \text{ensures } \phi;$
<i>T</i>	$::= \text{int} \mid C$
<i>s</i>	$::= x.f := y; \mid x := e; \mid x := \text{new } C; \mid x := y.m(z);$ $\mid \text{return } x; \mid \text{assert } \phi; \mid \text{release } \phi; \mid T \ x;$
$\phi$	$::= \text{true} \mid e = e \mid e \neq e \mid \text{acc}(x.f) \mid x : T \mid \phi * \phi$
<i>e</i>	$::= v \mid x \mid e.f$
<i>x</i>	$::= \text{this} \mid \text{result} \mid \langle \text{other} \rangle$
<i>v</i>	$::= o \mid n \mid \text{null}$
<i>n</i>	$\in \mathbb{Z}$
<i>H</i>	$\in (o \multimap (C, \overline{(f \multimap v)}))$
$\rho$	$\in (x \multimap v)$
<i>A<sub>s</sub></i>	$::= \overline{(x, f)}$
<i>A<sub>d</sub></i>	$::= \overline{(o, f)}$
<i>S</i>	$::= (\rho, A_d, \overline{s}) \cdot S \mid \text{nil}$

## 2 Static semantics

### 2.1 Expressions ( $A_s \vdash_{\text{sfrm}} e$ )

$$\frac{}{A \vdash_{\text{sfrm}} x} \text{WFVAR}$$

$$\frac{}{A \vdash_{\text{sfrm}} v} \text{WFVALUE}$$

$$\frac{(x, f) \in A}{A \vdash_{\text{sfrm}} x.f} \text{WFFIELD}$$

### 2.2 Formulas ( $A_s \vdash_{\text{sfrm}} \phi$ )

$$\frac{}{A \vdash_{\text{sfrm}} \text{true}} \text{WFTTRUE}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 = e_2)} \text{WFEQUAL}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 \neq e_2)} \text{WFNEQUAL}$$

$$\frac{}{A \vdash_{\text{sfrm}} \text{acc}(x, f)} \text{WFAcc}$$

$$\frac{}{A \vdash_{\text{sfrm}} x : T} \text{WFTYPE}$$

$$\frac{A_s \vdash_{\text{sfrm}} \phi_1 \quad A_s \cup [\phi_1] \vdash_{\text{sfrm}} \phi_2}{A_s \vdash_{\text{sfrm}} \phi_1 * \phi_2} \text{WFSEPOP}$$

### 2.2.1 Implication ( $\phi_1 \dot{\Rightarrow} \phi_2$ )

Conservative approx. of  $\phi_1 \Rightarrow \phi_2$ .

### 2.3 Footprint ( $[\phi] = A_s$ )

$$\begin{array}{ll} [\text{true}] & = \emptyset \\ [e_1 = e_2] & = \emptyset \\ [e_1 \neq e_2] & = \emptyset \\ [\text{acc}(x.f)] & = \{(x, f)\} \\ [\phi_1 * \phi_2] & = [\phi_1] \cup [\phi_2] \end{array}$$

### 2.4 Type ( $\phi \vdash e : T$ )

$$\frac{}{\phi \vdash v_T : T} \text{STVALUE}$$

$$\frac{\phi \Rightarrow (x : T)}{\phi \vdash x : T} \text{STVAR}$$

$$\frac{\phi \vdash e : C \quad \vdash C.f : T}{\phi \vdash e.f : T} \text{STFIELD}$$

## 2.5 Hoare ( $\vdash \{\phi\} \bar{s} \{\phi\}$ )

$$\begin{array}{c}
\frac{\vdash \{\phi_p\} s_1 \{\phi_{q1}\} \quad \phi_{q1} \implies \phi_{q2} \quad \vdash \{\phi_{q2}\} s_2 \{\phi_r\}}{\vdash \{\phi_p\} s_1; s_2 \{\phi_r\}} \text{HSEC} \\
\\
\frac{\phi \vdash x : C \quad \text{fields}(C) = \bar{f}}{\vdash \{\phi\} x := \text{new } C \{(\text{acc}(x, \bar{f}_i) * ((x \neq \text{null}) * \phi))\}} \text{HNEWOBJ} \\
\\
\frac{\phi \vdash x : C \quad \text{fields}(C) = \bar{f}}{\vdash \{\phi\} x := \text{new } C \{\overline{\text{acc}(x, f_i)} * (x \neq \text{null}) * \phi\}} \text{HNEWOBJ} \\
\\
\frac{\phi \vdash x : C \quad \vdash C.f : T \quad \phi \vdash y : T \quad \phi \implies \text{acc}(x.f) \quad \phi \implies (x \neq \text{null})}{\vdash \{\phi\} x.f := y \{\phi * (x.f = y)\}} \text{HFIELDASSIGN} \\
\\
\frac{\phi_1 \vdash x : T \quad \phi_1 \vdash e : T \quad \phi_1 = \phi_2[e/x] \quad \emptyset \vdash_{\text{sfrm}} \phi_1 \quad [\phi_1] \vdash_{\text{sfrm}} e}{\vdash \{\phi_1\} x := e \{\phi_2\}} \text{HVARASSIGN} \\
\\
\frac{\phi \vdash x : T \quad \phi \vdash \text{result} : T}{\vdash \{\phi\} \text{return } x \{\phi * (\text{result} = x)\}} \text{HRETURN} \\
\\
\frac{\begin{array}{l} \phi \vdash y : C \quad \text{mmethod}(C, m) = T_r \ m(T_p \ z) \text{ requires } \phi_{pre}; \text{ ensures } \phi_{post}; \{\_\} \\ \phi \vdash x : T_r \quad \phi \vdash z' : T_p \quad \phi \implies (y \neq \text{null}) \\ \phi \implies \phi_p * \phi_r \quad \phi_p = \phi_{pre}[y, z'/\text{this}, z] \quad \phi_q = \phi_{post}[y, z', x/\text{this}, z, \text{result}] \end{array}}{\vdash \{\phi\} x := y.m(z') \{\phi_q * \phi_r\}} \text{HAPP} \\
\\
\frac{\phi_1 \implies \phi_2}{\vdash \{\phi_1\} \text{assert } \phi_2 \{\phi_1\}} \text{HASSERT} \\
\\
\frac{\phi_1 \implies \phi_2 * \phi_r \quad \emptyset \vdash_{\text{sfrm}} \phi_r}{\vdash \{\phi_1\} \text{release } \phi_2 \{\phi_r\}} \text{HRELEASE} \\
\\
\frac{x \notin \text{dom}(\phi_1) \quad \phi_2 = \phi_1 * x : T * (x = \text{defaultValue}(T))}{\vdash \{\phi_1\} T \ x \{\phi_2\}} \text{HDECLARE} \\
\\
\frac{}{\vdash \{\emptyset\} \text{int this} \{\emptyset\}} \text{HDUMM}
\end{array}$$

### 3 Dynamic semantics

#### 3.1 Expressions ( $H, \rho \vdash e \Downarrow v$ )

$$\frac{}{H, \rho \vdash x \Downarrow \rho(x)} \text{EEVAR}$$

$$\frac{}{H, \rho \vdash v \Downarrow v} \text{EEVALUE}$$

$$\frac{H, \rho \vdash x \Downarrow o}{H, \rho \vdash x.f \Downarrow H(o)(f)} \text{EEACC}$$

#### 3.2 Formulas ( $H, \rho, A \models \phi$ )

$$\frac{}{H, \rho, A \models \text{true}} \text{EATRUE}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 = v_2}{H, \rho, A \models (e_1 = e_2)} \text{EAEQUAL}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 \neq v_2}{H, \rho, A \models (e_1 \neq e_2)} \text{EANEQUAL}$$

$$\frac{\rho(x) = o \quad (o, f) \in A}{H, \rho, A \models \text{acc}(x, f)} \text{EAAcc}$$

$$\frac{\rho(x) = (\text{exist}Tv)}{H, \rho, A \models x : T} \text{EATYPE}$$

$$\frac{A_1 = A \setminus A_2 \quad H, \rho, A_1 \models \phi_1 \quad H, \rho, A_2 \models \phi_2}{H, \rho, A \models \phi_1 * \phi_2} \text{EASEPOP}$$

##### 3.2.1 Implication ( $\phi_1 \implies \phi_2$ )

$$\phi_1 \implies \phi_2 \quad \iff \quad \forall H, \rho, A : H, \rho, A \models \phi_1 \implies H, \rho, A \models \phi_2$$

Drawn from def. of entailment in “A Formal Semantics for Isorecursive and Equirecursive State Abstractions”.

### 3.3 Footprint ( $\lfloor \phi \rfloor_{H,\rho} = A_d$ )

$$\begin{array}{ll}
\lfloor \mathbf{true} \rfloor_{H,\rho} & = \emptyset \\
\lfloor e_1 = e_2 \rfloor_{H,\rho} & = \emptyset \\
\lfloor e_1 \neq e_2 \rfloor_{H,\rho} & = \emptyset \\
\lfloor \mathbf{acc}(e.f) \rfloor_{H,\rho} & = \{(o, f)\} \text{ where } H, \rho \vdash e \Downarrow o \\
\lfloor \phi_1 * \phi_2 \rfloor_{H,\rho} & = \lfloor \phi_1 \rfloor_{H,\rho} \cup \lfloor \phi_2 \rfloor_{H,\rho}
\end{array}$$

### 3.4 Type ( $H, \rho \vdash e : T$ )

$$\frac{H, \rho \vdash e \Downarrow v_T}{H, \rho \vdash e : T} \text{DTEVAL}$$

### 3.5 Small-step ( $(H, S) \rightarrow (H', S')$ )

$$\frac{H, \rho \vdash x \Downarrow o \quad H, \rho \vdash y \Downarrow v_y \quad (o, f) \in A \quad H' = H[o \mapsto [f \mapsto v_y]]}{(H, (\rho, A, x.f := y; \bar{s}) \cdot S) \rightarrow (H', (\rho, A, \bar{s}) \cdot S)} \text{ESFIELDASSIGN}$$

$$\frac{H, \rho \vdash e \Downarrow v \quad \rho' = \rho[x \mapsto v]}{(H, (\rho, A, x := e; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESVARASSIGN}$$

$$\frac{\rho' = \rho[x \mapsto o] \quad o \notin \text{dom}(H) \quad \text{fields}(C) = \bar{T} \bar{f} \quad A' = A * (\bar{o}, \bar{f}_i) \quad H' = H[o \mapsto [f \mapsto \text{defaultValue}(T)]]}{(H, (\rho, A, x := \mathbf{new} \ C; \bar{s}) \cdot S) \rightarrow (H', (\rho', A', \bar{s}) \cdot S)} \text{ESNEWOBJ}$$

$$\frac{H, \rho \vdash x \Downarrow v_x \quad \rho' = \rho[\mathbf{result} \mapsto v_x]}{(H, (\rho, A, \mathbf{return} \ x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESRETURN}$$

$$\frac{H, \rho \vdash y \Downarrow o \quad H, \rho \vdash z \Downarrow v \quad H(o) = (C, \_) \quad \text{mmethod}(C, m) = T_r \ m(T \ w) \text{ requires } \phi; \text{ ensures } \_; \{\bar{r}\} \quad \rho' = [\mathbf{result} \mapsto \text{defaultValue}(T_r), \mathbf{this} \mapsto o, w \mapsto v] \quad H, \rho', A \models \phi \quad A' = \lfloor \phi \rfloor_{H,\rho'}}{(H, (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho', A', \bar{r}) \cdot (\rho, A \setminus A', x := y.m(z); \bar{s}) \cdot S)} \text{ESAPP}$$

$$\frac{\text{mpost}(C, m) = \phi \quad H, \rho' \vdash \mathbf{result} \Downarrow v_r \quad H(o) = (C, \_) \quad A'' = \lfloor \phi \rfloor_{H,\rho'} \quad H, \rho' \vdash \mathbf{result} \Downarrow v_r}{(H, (\rho', A', \emptyset) \cdot (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho[x \mapsto v_r], A * A'', \bar{s}) \cdot S)} \text{ESAPPFINISH}$$

$$\frac{H, \rho, A \models \phi}{(H, (\rho, A, \mathbf{assert} \ \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A, \bar{s}) \cdot S)} \text{ESASSERT}$$

$$\frac{H, \rho, A \models \phi \quad A' = A \setminus \lfloor \phi \rfloor_{H, \rho}}{(H, (\rho, A, \text{release } \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A', \bar{s}) \cdot S)} \text{ESRELEASE}$$

$$\frac{\rho' = \rho[x \mapsto \text{defaultValue}(T)]}{(H, (\rho, A, T \ x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESDECLARE}$$

## 4 Theorems

### 4.1 Invariant $\text{invariant}(H, \rho, A_d, \phi)$

#### 4.1.1 Heap consistent

$$\begin{aligned} \forall x, o, C : \rho(x) = o_C &\implies \\ \exists f_C, m : \text{fields}(C) = f_C & \\ \wedge H(o_C) = (C, m) & \\ \wedge (\forall (T, f) \in f_C : H, \rho \vdash m(f) : T) & \end{aligned}$$

#### 4.1.2 Phi holds

$$H, \rho, A_d \models \phi$$

#### 4.1.3 Types preserved

$$\begin{aligned} \forall e, T : \phi \vdash e : T \\ \implies H, \rho \vdash e : T \end{aligned}$$

## 4.2 Soundness

### 4.2.1 Progress

$$\begin{aligned} \forall \dots : \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies \text{invariant}(H_1, \rho_1, A_1, \phi_1) \\ \implies \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \end{aligned}$$

### 4.2.2 Preservation

$$\begin{aligned} \forall \dots : \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies \text{invariant}(H_1, \rho_1, A_1, \phi_1) \\ \implies (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \\ \implies \text{invariant}(H_2, \rho_2, A_2, \phi_2) \end{aligned}$$