1 Syntax

$$
\begin{array}{ll}
\mathit{program} & ::= \overline{\mathit{cls}}\ \overline{s} \\
\mathit{cls} & ::= \texttt{class}\ C\ \{\overline{\mathit{field}}\ \overline{\mathit{method}}\} \\
\mathit{field} & ::= T\ f; \\
\mathit{method} & ::= T\ m(T\ x)\ \mathit{contract}\ \{\overline{s}\} \\
\mathit{contract} & ::= \texttt{requires}\ \phi;\ \texttt{ensures}\ \phi; \\
T & ::= \texttt{int} \mid C \\
s & ::= x.f := y; \mid x := e; \mid x := \texttt{new}\ C; \mid x := y.m(z); \mid \texttt{return}\ x; \mid \texttt{assert}\ \phi; \mid \texttt{release}\ \phi; \\
\phi & ::= \texttt{true} \mid e = e \mid e \neq e \mid \texttt{acc}(x.f) \mid \phi * \phi \\
e & ::= v \mid x \mid e.f \\
x & ::= \texttt{this} \mid \texttt{result} \mid \langle \mathit{other} \rangle
\end{array}
$$

$$
\begin{array}{ll}
\Gamma & ::= (x \mapsto T) \\
H & ::= (o \mapsto (C, \overline{(f \mapsto v)})) \\
\rho & ::= (x \mapsto v) \\
A_s & ::= \overline{(x, f)} \\
A_d & ::= \overline{(o, f)} \\
S & ::= (\rho, A_d, \overline{s}) \cdot S \mid \mathit{nil}
\end{array}
$$

2 Static semantics

2.1 Expressions ($A_s \vdash_{\texttt{sfrm}} e$)

$$\frac{}{A \vdash_{\texttt{sfrm}} x}\ \textsc{WFVar}$$

$$\frac{}{A \vdash_{\texttt{sfrm}} v}\ \textsc{WFValue}$$

$$\frac{(x, f) \in A}{A \vdash_{\texttt{sfrm}} x.f}\ \textsc{WFField}$$

2.2 Formulas ($A_s \vdash_{\texttt{sfrm}} \phi$)

$$\frac{}{A \vdash_{\texttt{sfrm}} \texttt{true}}\ \textsc{WFTrue}$$

$$\frac{A \vdash_{\texttt{sfrm}} e_1 \qquad A \vdash_{\texttt{sfrm}} e_2}{A \vdash_{\texttt{sfrm}} e_1 = e_2} \textsc{WFEqual}$$

$$\frac{A \vdash_{\texttt{sfrm}} e_1 \qquad A \vdash_{\texttt{sfrm}} e_2}{A \vdash_{\texttt{sfrm}} e_1 \neq e_2} \textsc{WFNEqual}$$

$$\frac{}{A \vdash_{\texttt{sfrm}} \texttt{acc}(x, f)} \textsc{WFAcc}$$

$$\frac{A_s \vdash_{\texttt{sfrm}} \phi_1 \qquad A_s \cup \texttt{static-footprint}(\phi_1) \vdash_{\texttt{sfrm}} \phi_2}{A_s \vdash_{\texttt{sfrm}} \phi_1 * \phi_2} \quad \text{WF-SepOp}$$

2.2.1 Implication ($\phi_1 \stackrel{\cdot}{\Longrightarrow} \phi_2$)

Conservative approx. of $\phi_1 \implies \phi_2$.

2.3 Footprint ($\texttt{static-footprint}(\phi) = A_s$)

$$\begin{aligned}
&\texttt{static-footprint}(\texttt{true}) &&= \emptyset \\
&\texttt{static-footprint}(e_1 = e_2) &&= \emptyset \\
&\texttt{static-footprint}(e_1 \neq e_2) &&= \emptyset \\
&\texttt{static-footprint}(\texttt{acc}(x.f)) &&= \{(x, f)\} \\
&\texttt{static-footprint}(\phi_1 * \phi_2) &&= \texttt{static-footprint}(\phi_1) \cup \texttt{static-footprint}(\phi_2)
\end{aligned}$$

2.4 Hoare ($\Gamma \vdash \{\phi\}\overline{s}\{\phi\}$)

$$\frac{\Gamma \vdash \{\phi_p\}s_1\{\phi_{q1}\} \qquad \phi_{q1} \implies \phi_{q2} \qquad \Gamma \vdash \{\phi_{q2}\}s_2\{\phi_r\}}{\Gamma \vdash \{\phi_p\}s_1; s_2\{\phi_r\}} \quad \text{H-Sec}$$

$$\frac{\Gamma(x) = C \qquad \texttt{fields}(C) = \{\overline{f_i}\}}{\Gamma \vdash \{\phi\}x := \texttt{new}\ C\{\overline{\texttt{acc}(x.f_i)} * x \neq \texttt{null} * \phi\}} \quad \text{H-NewObj}$$

$$\frac{\phi \implies \texttt{acc}(x.f) * x \neq \texttt{null}}{\Gamma \vdash \{\phi\}x.f := y\{\phi * x.f = y\}} \quad \text{H-FieldAssign}$$

$$\frac{\phi' = \phi[e/x] \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad \texttt{static-footprint}(\phi') \vdash_{\texttt{sfrm}} e}{\Gamma \vdash \{\phi'\} x := e \{\phi\}} \quad \text{H-VarAssign}$$

$$\frac{}{\Gamma \vdash \{\phi\}\texttt{return } x\{\phi * \texttt{result} = x\}} \quad \text{H-Return}$$

$$\frac{\Gamma(y) = C \qquad \phi \implies y \neq null * \phi_p * \phi_r \qquad \phi_p = \texttt{mpre}(C, m)[y, \overline{z}/\texttt{this}, \overline{X}] \qquad \phi_q = \texttt{mpost}(C, m)[y, \overline{z}, x/\texttt{th}}{\Gamma \vdash \{\phi\} x := y.m(\overline{z}) \{\phi_q * \phi_r\}}$$

$$\frac{\phi \implies \phi'}{\Gamma \vdash \{\phi\}\texttt{assert } \phi'\{\phi\}} \quad \text{H-Assert}$$

$$\frac{\phi \implies \phi' * \phi_r \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_r}{\Gamma \vdash \{\phi\}\texttt{release } \phi'\{\phi_r\}} \quad \text{H-Release}$$

3 Dynamic semantics

3.1 Expressions ($H, \rho \vdash e \Downarrow v$)

$$\frac{}{H, \rho \vdash x \Downarrow \rho(x)} \quad \text{EE-Var}$$

$$\frac{}{H, \rho \vdash v \Downarrow v} \quad \text{EE-Value}$$

$$\frac{H, \rho \vdash x \Downarrow o}{H, \rho \vdash x.f \Downarrow H(o)(f)} \quad \text{EE-Acc}$$

3.2 Formulas $(H, \rho, A \vDash \phi)$

$$\frac{}{H, \rho, A \vDash \texttt{true}} \quad \text{EA-True}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 = v_2}{H, \rho, A \vDash e_1 = e_2} \quad \text{EA-Equal}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 = v_2}{H, \rho, A \vDash e_1 = e_2} \quad \text{EA-NEqual}$$

$$\frac{H, \rho \vdash x \Downarrow o \qquad (o, f) \in A}{H, \rho, A \vDash \texttt{acc}(x.f)} \quad \text{EA-Acc}$$

$$\frac{A_1 = A \backslash A_2 \qquad H, \rho, A_1 \vDash \phi_1 \qquad H, \rho, A_2 \vDash \phi_2}{H, \rho, A \vDash \phi_1 * \phi_2} \quad \text{EA-SepOp}$$

3.2.1 Implication $(\phi_1 \implies \phi_2)$

$$\phi_1 \implies \phi_2 \qquad \iff \qquad \forall H, \rho, A : H, \rho, A \vDash \phi_1 \implies H, \rho, A \vDash \phi_2$$

Drawn from def. of entailment in "A Formal Semantics for Isorecursive and Equirecursive State Abstractions".

3.3 Footprint $(\texttt{footprint}_{H,\rho}(\phi) = A_d)$

$$\begin{aligned}
&\texttt{footprint}_{H,\rho}(\texttt{true}) && = \emptyset \\
&\texttt{footprint}_{H,\rho}(e_1 = e_2) && = \emptyset \\
&\texttt{footprint}_{H,\rho}(e_1 \neq e_2) && = \emptyset \\
&\texttt{footprint}_{H,\rho}(\texttt{acc}(e.f)) && = \{(o, f)\} \text{ where } H, \rho \vdash e \Downarrow o \\
&\texttt{footprint}_{H,\rho}(\phi_1 * \phi_2) && = \texttt{footprint}_{H,\rho}(\phi_1) \cup \texttt{footprint}_{H,\rho}(\phi_2)
\end{aligned}$$

3.4 Small-step $((H, S) \rightarrow (H, S))$

$$\frac{H, \rho \vdash x \Downarrow o \qquad H, \rho \vdash y \Downarrow v_y \qquad (o, f) \in A \qquad H' = H[o \mapsto [f \mapsto v_y]]}{(H, (\rho, A, x.f := y; \overline{s}) \cdot S) \rightarrow (H', (\rho, A, \overline{s}) \cdot S)} \ \textsc{ESFieldAssign}$$

$$\frac{H, \rho \vdash e \Downarrow v \qquad \rho' = \rho[x \mapsto v]}{(H, (\rho, A, x := e; \overline{s}) \cdot S) \rightarrow (H, (\rho', A, \overline{s}) \cdot S)} \ \textsc{ESVarAssign}$$

$$\frac{\begin{array}{c} H(o) = \bot \qquad \texttt{fields}(C) = \overline{f} \\ \rho' = \rho[x \mapsto o] \qquad A' = A * \overline{(o, f_i)} \qquad H' = H[o \mapsto [\overline{((f_i, \texttt{null}))}]] \end{array}}{(H, (\rho, A, x := \texttt{ new } C; \overline{s}) \cdot S) \rightarrow (H', (\rho', A', \overline{s}) \cdot S)} \textsc{ESNewObj}$$

$$\frac{H, \rho \vdash x \Downarrow v_x \qquad \rho' = \rho[\texttt{result} \mapsto v_x]}{(H, (\rho, a, \texttt{return } x; \overline{s}) \cdot S) \rightarrow (H, (\rho', a, \overline{s}) \cdot S)} \textsc{ESReturn}$$

$$\frac{\begin{array}{c} H, \rho \vdash y \Downarrow o \qquad H, \rho \vdash z \Downarrow v \qquad H(o) = (C, fvf) \\ \texttt{mbody}(C, m) = \overline{r} \qquad \texttt{mparam}(C, m) = (T, w) \qquad \texttt{mpre}(C, m) = \phi \\ \rho' = [\texttt{this} \mapsto o, w \mapsto v] \qquad H, \rho', A \vDash \phi \qquad A' = \texttt{footprint}_{H,\rho'}(\phi) \end{array}}{(H, (\rho, A, x := y.m(z); \overline{s}) \cdot S) \rightarrow (H, (\rho', A', \overline{r}) * (\rho, A \setminus A', x := y.m(z); \overline{s}) \cdot S)} \textsc{ESApp}$$

$$\frac{\begin{array}{c} \texttt{mpost}(C, m) = \phi \\ H, \rho', A' \vDash \phi \qquad A'' = \texttt{footprint}_{H,\rho'}(\phi) \qquad H, \rho' \vdash \texttt{result} \Downarrow v_r \end{array}}{(H, (\rho', A', \emptyset) * (\rho, A, x := y.m(z); \overline{s}) \cdot S) \rightarrow (H, (\rho[x \mapsto v_r], A * A'', \overline{s}) \cdot S)} \textsc{ESAppFinish}$$

$$\frac{H, \rho, A \vDash \phi}{(H, (\rho, A, \texttt{assert } \phi; \overline{s}) \cdot S) \rightarrow (H, (\rho, A, \overline{s}) \cdot S)} \textsc{ESAssert}$$

$$\frac{H, \rho, A \vDash \phi \qquad A' = A \setminus \texttt{footprint}_{H,\rho}(\phi)}{(H, (\rho, A, \texttt{release } \phi; \overline{s}) \cdot S) \rightarrow (H, (\rho, A', \overline{s}) \cdot S)} \textsc{ESRelease}$$

4 Theorems

Hoare preserves self-framing

$$\begin{aligned} &\forall\ \Gamma, \phi_1, \phi_2, s : \Gamma \vdash \{\phi_1\} s \{\phi_2\} \\ &\qquad \implies \texttt{static-footprint}(\phi_1) \vdash_{\texttt{sfrm}} \phi_1 \\ &\qquad \implies \texttt{static-footprint}(\phi_2) \vdash_{\texttt{sfrm}} \phi_2 \end{aligned}$$

Hoare progress

$$
\begin{aligned}
\forall\ \Gamma, \phi_1, \phi_2, s, H_1, \rho_1, A_1 : \Gamma \vdash \{\phi_1\} s \{\phi_2\} \\
&\implies H_1, \rho_1, A_1 \vDash \phi_1 \\
&\implies \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \to^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S)
\end{aligned}
$$

Hoare preservation

$$
\begin{aligned}
\forall\ \Gamma, \phi_1, \phi_2, s, H_1, H_2, \rho_1, \rho_2, A_1, A_2 : \Gamma \vdash \{\phi_1\} s \{\phi_2\} \\
&\implies H_1, \rho_1, A_1 \vDash \phi_1 \\
&\implies (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \to^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S) \\
&\implies H_2, \rho_2, A_2 \vDash \phi_2
\end{aligned}
$$