

1 Syntax

<i>program</i>	$::= \overline{cls} \ \overline{s}$
<i>cls</i>	$::= \text{class } C \ \{\overline{field} \ \overline{method}\}$
<i>field</i>	$::= T \ f;$
<i>method</i>	$::= T \ m(T \ x) \ \text{contract} \ \{\overline{s}\}$
<i>contract</i>	$::= \text{requires } \phi; \ \text{ensures } \phi;$
<i>T</i>	$::= \text{int} \mid C$
<i>s</i>	$::= x.f := y; \mid x := e; \mid x := \text{new } C; \mid x := y.m(z);$ $\mid \text{return } x; \mid \text{assert } \phi; \mid \text{release } \phi; \mid T \ x;$
ϕ	$::= \text{true} \mid e = e \mid e \neq e \mid \text{acc}(x.f) \mid x : T \mid \phi * \phi$
<i>e</i>	$::= v \mid x \mid e.f$
<i>x</i>	$::= \text{this} \mid \text{result} \mid \langle \text{other} \rangle$
<i>v</i>	$::= o \mid n \mid \text{null}$
<i>n</i>	$\in \mathbb{Z}$
<i>H</i>	$\in (o \multimap (C, \overline{(f \multimap v)}))$
ρ	$\in (x \multimap v)$
<i>A_s</i>	$::= \overline{(x, f)}$
<i>A_d</i>	$::= \overline{(o, f)}$
<i>S</i>	$::= (\rho, A_d, \overline{s}) \cdot S \mid \text{nil}$

2 Static semantics

2.1 Expressions ($A_s \vdash_{\text{sfrm}} e$)

$$\frac{}{A \vdash_{\text{sfrm}} x} \text{WFVAR}$$

$$\frac{}{A \vdash_{\text{sfrm}} v} \text{WFVALUE}$$

$$\frac{(x, f) \in A}{A \vdash_{\text{sfrm}} x.f} \text{WFFIELD}$$

2.2 Formulas ($A_s \vdash_{\text{sfrm}} \phi$)

$$\frac{}{A \vdash_{\text{sfrm}} \text{true}} \text{WFTTRUE}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 = e_2)} \text{WFEQUAL}$$

$$\frac{A \vdash_{\text{sfrm}} e_1 \quad A \vdash_{\text{sfrm}} e_2}{A \vdash_{\text{sfrm}} (e_1 \neq e_2)} \text{WFNEQUAL}$$

$$\frac{}{A \vdash_{\text{sfrm}} \text{acc}(x.f)} \text{WFAcc}$$

$$\frac{}{A \vdash_{\text{sfrm}} x : T} \text{WFTYPE}$$

$$\frac{A_s \vdash_{\text{sfrm}} \phi_1 \quad A_s \cup [\phi_1] \vdash_{\text{sfrm}} \phi_2}{A_s \vdash_{\text{sfrm}} \phi_1 * \phi_2} \text{WFSEPOP}$$

2.2.1 Implication ($\phi_1 \dot{\Rightarrow} \phi_2$)

Conservative approx. of $\phi_1 \Rightarrow \phi_2$.

2.3 Footprint ($[\phi] = A_s$)

$$\begin{array}{ll} [\text{true}] & = \emptyset \\ [e_1 = e_2] & = \emptyset \\ [e_1 \neq e_2] & = \emptyset \\ [\text{acc}(x.f)] & = \{(x, f)\} \\ [\phi_1 * \phi_2] & = [\phi_1] \cup [\phi_2] \end{array}$$

2.4 Type ($\phi \vdash e : T$)

$$\frac{}{\phi \vdash v_T : T} \text{STVALUE}$$

$$\frac{\phi \Rightarrow (x : T)}{\phi \vdash x : T} \text{STVAR}$$

$$\frac{\phi \vdash e : C \quad \vdash C.f : T}{\phi \vdash e.f : T} \text{STFIELD}$$

2.5 Hoare ($\vdash \{\phi\} \bar{s} \{\phi\}$)

$$\frac{\vdash \{\phi_p\} s_1 \{\phi_{q1}\} \quad \phi_{q1} \implies \phi_{q2} \quad \vdash \{\phi_{q2}\} s_2 \{\phi_r\}}{\vdash \{\phi_p\} s_1; s_2 \{\phi_r\}} \text{HSEC}$$

$$\frac{\phi \vdash x : C \quad \mathbf{fields}(C) = \bar{f}}{\vdash \{\phi\} x := \mathbf{new } C \{\mathbf{acc}(x, f_i) * (x \neq \mathbf{null}) * \phi\}} \text{HNEWOBJ}$$

$$\frac{\phi \vdash x : C \quad \vdash C.f : T \quad \phi \vdash y : T \quad \phi \implies \mathbf{acc}(x.f) \quad \phi \implies (x \neq \mathbf{null})}{\vdash \{\phi\} x.f := y \{\phi * (x.f = y)\}} \text{HFIELDASSIGN}$$

$$\frac{\phi_1 \vdash x : T \quad \phi_1 \vdash e : T \quad \phi_1 = \phi_2[e/x] \quad \emptyset \vdash_{\mathbf{sfrm}} \phi_1 \quad \lfloor \phi_1 \rfloor \vdash_{\mathbf{sfrm}} e}{\vdash \{\phi_1\} x := e \{\phi_2\}} \text{HVARASSIGN}$$

$$\frac{\phi \vdash x : T \quad \phi \vdash \mathbf{result} : T}{\vdash \{\phi\} \mathbf{return } x \{\phi * (\mathbf{result} = x)\}} \text{HRETURN}$$

$$\frac{\begin{array}{l} \phi \vdash y : C \quad \mathbf{mmethod}(C, m) = T_r \ m(T_p \ z) \ \mathbf{requires } \phi_{pre}; \ \mathbf{ensures } \phi_{post}; \ \{_\} \\ \phi \vdash x : T_r \quad \phi \vdash z' : T_p \quad \phi \implies (y \neq \mathbf{null}) \\ \phi \implies \phi_p * \phi_r \quad \phi_p = \phi_{pre}[y, z'/\mathbf{this}, z] \quad \phi_q = \phi_{post}[y, z', x/\mathbf{this}, z, \mathbf{result}] \end{array}}{\vdash \{\phi\} x := y.m(z') \{\phi_q * \phi_r\}} \text{HAPP}$$

$$\frac{\phi_1 \implies \phi_2}{\vdash \{\phi_1\} \mathbf{assert } \phi_2 \{\phi_1\}} \text{HASSERT}$$

$$\frac{\phi_1 \implies \phi_2 * \phi_r \quad \emptyset \vdash_{\mathbf{sfrm}} \phi_r}{\vdash \{\phi_1\} \mathbf{release } \phi_2 \{\phi_r\}} \text{HRELEASE}$$

$$\frac{x \notin \mathbf{dom}(\phi_1) \quad \phi_2 = \phi_1 * x : T * (x = \mathbf{defaultValue}(T))}{\vdash \{\phi_1\} T \ x \{\phi_2\}} \text{HDECLARE}$$

3 Dynamic semantics

3.1 Expressions ($H, \rho \vdash e \Downarrow v$)

$$\frac{}{H, \rho \vdash x \Downarrow \rho(x)} \text{EEVAR}$$

$$\frac{}{H, \rho \vdash v \Downarrow v} \text{EEVALUE}$$

$$\frac{H, \rho \vdash x \Downarrow o}{H, \rho \vdash x.f \Downarrow H(o)(f)} \text{EEACC}$$

3.2 Formulas ($H, \rho, A \models \phi$)

$$\frac{}{H, \rho, A \models \mathbf{true}} \text{EATRUE}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 = v_2}{H, \rho, A \models (e_1 = e_2)} \text{EAEQUAL}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \quad H, \rho \vdash e_2 \Downarrow v_2 \quad v_1 \neq v_2}{H, \rho, A \models (e_1 \neq e_2)} \text{EANEQUAL}$$

$$\frac{\rho(x) = o \quad (o, f) \in A}{H, \rho, A \models \mathbf{acc}(x.f)} \text{EAACC}$$

$$\frac{\rho(x) = v_T}{H, \rho, A \models x : T} \text{EATYPE}$$

$$\frac{A_1 = A \setminus A_2 \quad H, \rho, A_1 \models \phi_1 \quad H, \rho, A_2 \models \phi_2}{H, \rho, A \models \phi_1 * \phi_2} \text{EASEPOP}$$

3.2.1 Implication ($\phi_1 \implies \phi_2$)

$$\phi_1 \implies \phi_2 \quad \iff \quad \forall H, \rho, A : H, \rho, A \models \phi_1 \implies H, \rho, A \models \phi_2$$

Drawn from def. of entailment in “A Formal Semantics for Isorecursive and Equirecursive State Abstractions”.

3.3 Footprint ($\lfloor \phi \rfloor_{H, \rho} = A_d$)

$$\begin{aligned} \lfloor \mathbf{true} \rfloor_{H, \rho} &= \emptyset \\ \lfloor e_1 = e_2 \rfloor_{H, \rho} &= \emptyset \\ \lfloor e_1 \neq e_2 \rfloor_{H, \rho} &= \emptyset \\ \lfloor \mathbf{acc}(e.f) \rfloor_{H, \rho} &= \{(o, f)\} \text{ where } H, \rho \vdash e \Downarrow o \\ \lfloor \phi_1 * \phi_2 \rfloor_{H, \rho} &= \lfloor \phi_1 \rfloor_{H, \rho} \cup \lfloor \phi_2 \rfloor_{H, \rho} \end{aligned}$$

3.4 Type $(H, \rho \vdash e : T)$

$$\frac{H, \rho \vdash e \Downarrow v_T}{H, \rho \vdash e : T} \text{DTEVAL}$$

3.5 Small-step $((H, S) \rightarrow (H', S))$

$$\frac{H, \rho \vdash x \Downarrow o \quad H, \rho \vdash y \Downarrow v_y \quad (o, f) \in A \quad H' = H[o \mapsto [f \mapsto v_y]]}{(H, (\rho, A, x.f := y; \bar{s}) \cdot S) \rightarrow (H', (\rho, A, \bar{s}) \cdot S)} \text{ESFIELDASSIGN}$$

$$\frac{H, \rho \vdash e \Downarrow v \quad \rho' = \rho[x \mapsto v]}{(H, (\rho, A, x := e; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESVARASSIGN}$$

$$\frac{\rho' = \rho[x \mapsto o] \quad o \notin \text{dom}(H) \quad \text{fields}(C) = \bar{T} \bar{f} \quad A' = A * (\bar{o}, \bar{f}_i) \quad H' = H[o \mapsto [f \mapsto \text{defaultValue}(T)]]}{(H, (\rho, A, x := \text{new } C; \bar{s}) \cdot S) \rightarrow (H', (\rho', A', \bar{s}) \cdot S)} \text{ESNEWOBJ}$$

$$\frac{H, \rho \vdash x \Downarrow v_x \quad \rho' = \rho[\text{result} \mapsto v_x]}{(H, (\rho, A, \text{return } x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESRETURN}$$

$$\frac{H, \rho \vdash y \Downarrow o \quad H, \rho \vdash z \Downarrow v \quad H(o) = (C, _) \quad \text{mmethod}(C, m) = T_r \ m(T \ w) \text{ requires } \phi; \text{ ensures } _; \{\bar{r}\} \quad \rho' = [\text{result} \mapsto \text{defaultValue}(T_r), \text{this} \mapsto o, w \mapsto v] \quad H, \rho', A \models \phi \quad A' = [\phi]_{H, \rho'}}{(H, (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho', A', \bar{r}) \cdot (\rho, A \setminus A', x := y.m(z); \bar{s}) \cdot S)} \text{ESAPP}$$

$$\frac{\text{mpost}(C, m) = \phi \quad H, \rho \vdash y \Downarrow o \quad H(o) = (C, _) \quad H, \rho', A' \models \phi \quad A'' = [\phi]_{H, \rho'} \quad H, \rho' \vdash \text{result} \Downarrow v_r}{(H, (\rho', A', \emptyset) \cdot (\rho, A, x := y.m(z); \bar{s}) \cdot S) \rightarrow (H, (\rho[x \mapsto v_r], A * A'', \bar{s}) \cdot S)} \text{ESAPPFINISH}$$

$$\frac{H, \rho, A \models \phi}{(H, (\rho, A, \text{assert } \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A, \bar{s}) \cdot S)} \text{ESASSERT}$$

$$\frac{H, \rho, A \models \phi \quad A' = A \setminus [\phi]_{H, \rho}}{(H, (\rho, A, \text{release } \phi; \bar{s}) \cdot S) \rightarrow (H, (\rho, A', \bar{s}) \cdot S)} \text{ESRELEASE}$$

$$\frac{\rho' = \rho[x \mapsto \text{defaultValue}(T)]}{(H, (\rho, A, T \ x; \bar{s}) \cdot S) \rightarrow (H, (\rho', A, \bar{s}) \cdot S)} \text{ESDECLARE}$$

4 Gradualization

4.1 Syntax

$$\tilde{\phi} ::= \phi \mid \phi * ?$$

4.2 Concretization A?

$$\begin{aligned} \gamma(\phi) &= \{ \phi' \mid \phi' \iff \phi \} \\ \gamma(\phi * ?) &= \{ \phi' \mid \exists \phi_x : \phi * \phi_x \iff \phi' \} \\ &= \{ \phi' \mid \phi' \implies \phi \} \end{aligned}$$

4.3 Abstraction (to show: set of ϕ s is poset)

$$\alpha(\bar{\phi}) = (\sqcap \bar{\phi}) * ?$$

4.4 Theorems

4.4.1 Soundness

$$\forall \bar{\phi} : \bar{\phi} \subseteq \gamma(\alpha(\bar{\phi}))$$

4.4.2 Optimality

$$\forall \bar{\phi}, \tilde{\phi} : \bar{\phi} \subseteq \gamma(\tilde{\phi}) \implies \gamma(\tilde{\phi}) \subseteq \gamma(\alpha(\bar{\phi}))$$

4.5 Concretization B?

$$\begin{aligned} \gamma(\phi) &= \{ (H, \rho, A) \mid H, \rho, A \models \phi \} \\ \gamma(\phi * ?) &= \{ (H, \rho, A) \mid H, \rho, A \models \phi \} \end{aligned}$$

5 Theorems

5.1 Invariant $\text{invariant}(H, \rho, A_d, \phi)$

5.1.1 Heap consistent

$$\begin{aligned} \forall x, o, C : \rho(x) = o_C &\implies \\ \exists f_C, m : \text{fields}(C) &= f_C \\ \wedge H(o_C) &= (C, m) \\ \wedge (\forall (T, f) \in f_C : H, \rho &\vdash m(f) : T) \end{aligned}$$

5.1.2 Phi holds

$$H, \rho, A_d \models \phi$$

5.1.3 Types preserved

$$\begin{aligned} & \forall e, T : \phi \vdash e : T \\ \implies & H, \rho \vdash e : T \end{aligned}$$

5.2 Soundness

5.2.1 Progress

$$\begin{aligned} \forall \dots : & \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies & \text{invariant}(H_1, \rho_1, A_1, \phi_1) \\ \implies & \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \end{aligned}$$

5.2.2 Preservation

$$\begin{aligned} \forall \dots : & \vdash \{\phi_1\} s' \{\phi_2\} \\ \implies & \text{invariant}(H_1, \rho_1, A_1, \phi_1) \\ \implies & (H_1, (\rho_1, A_1, s'; \bar{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \bar{s}) \cdot S) \\ \implies & \text{invariant}(H_2, \rho_2, A_2, \phi_2) \end{aligned}$$