1 Syntax

$$
\begin{array}{lll}
\mathit{program} & ::= \overline{\mathit{cls}}\ \overline{s} \\
\mathit{cls} & ::= \texttt{class}\ C\ \{\overline{\mathit{field}}\ \overline{\mathit{method}}\} \\
\mathit{field} & ::= T\ f; \\
\mathit{method} & ::= T\ m(T\ x)\ \mathit{contract}\ \{\overline{s}\} \\
\mathit{contract} & ::= \texttt{requires}\ \phi;\ \texttt{ensures}\ \phi; \\
T & ::= \texttt{int} \mid C \\
s & ::= x.f := y; \mid x := e; \mid x := \texttt{new}\ C; \mid x := y.m(z); \\
 & \mid \texttt{return}\ x; \mid \texttt{assert}\ \phi; \mid \texttt{release}\ \phi; \mid T\ x; \\
\phi & ::= \texttt{true} \mid e = e \mid e \neq e \mid \texttt{acc}(e.f) \mid x : T \mid \phi * \phi \\
e & ::= v \mid x \mid e.f \\
x & ::= \texttt{this} \mid \texttt{result} \mid \langle \mathit{other} \rangle \\
v & ::= o \mid n \mid \texttt{null} \\
n & \in\ \ \mathbb{Z} \\
 & \\
H & \in\ \ (o \rightharpoonup (C, \overline{(f \rightharpoonup v)})) \\
\rho & \in\ \ (x \rightharpoonup v) \\
A_s & ::= \overline{(e, f)} \\
A_d & ::= \overline{(o, f)} \\
S & ::= (\rho, A_d, \overline{s}) \cdot S \mid \mathit{nil}
\end{array}
$$

2 Assumptions

All the rules in the following sections are implicitly parameterized over a *programp* that is well-formed.

2.0.1 Well-formed program ($\mathit{program}$ $\texttt{OK}$)

$$
\frac{\overline{\mathit{cls}_i\ \texttt{OK}}}{(\overline{\mathit{cls}_i}\ \overline{s})\ \texttt{OK}}\ \textsc{OKProgram}
$$

2.0.2 Well-formed class (cls $\texttt{OK}$)

$$
\frac{\text{unique } \mathit{field}\text{-names} \qquad \text{unique } \mathit{method}\text{-names} \qquad \overline{\mathit{method}_i\ \texttt{OK in}\ C}}{(\texttt{class}\ C\ \{\overline{\mathit{field}_i}\ \overline{\mathit{method}_i}\})\ \texttt{OK}}\ \textsc{OKClass}
$$

2.0.3 Well-formed method (method $\texttt{OK in}$ C)

$$
\frac{\begin{array}{c} FV(\phi_1) \subseteq \{x, \texttt{this}\} \\ FV(\phi_2) \subseteq \{x, \texttt{this}, \texttt{result}\} \qquad \vdash \{x : T_x * \texttt{this} : C * \phi_1\}\overline{s}\{x : T_x * \texttt{this} : C * \texttt{result} : T_m * \phi_2\} \\ \emptyset \vdash_{\texttt{sfrm}} \phi_1 \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_2 \qquad \overline{\neg \mathit{writesTo}(s_i, x)} \end{array}}{(T_m\ m(T_x\ x)\ \texttt{requires}\ \phi_1;\ \texttt{ensures}\ \phi_2;\ \{\overline{s}\})\ \texttt{OK in}\ C}\ \textsc{OKMethod}
$$

3 Static semantics

3.1 Expressions ($A_s \vdash_{\texttt{sfrm}} e$)

$$
\frac{}{A \vdash_{\texttt{sfrm}} x}\ \textsc{WFVar}
$$

$$\frac{}{A \vdash_{\texttt{sfrm}} v}\ \textsc{WFValue}$$

$$\frac{(e, f) \in A \qquad A \vdash_{\texttt{sfrm}} e}{A \vdash_{\texttt{sfrm}} e.f}\ \textsc{WFField}$$

3.2 Formulas ($A_s \vdash_{\texttt{sfrm}} \phi$)

$$\frac{}{A \vdash_{\texttt{sfrm}} \texttt{true}}\ \textsc{WFTrue}$$

$$\frac{A \vdash_{\texttt{sfrm}} e_1 \qquad A \vdash_{\texttt{sfrm}} e_2}{A \vdash_{\texttt{sfrm}} (e_1 = e_2)}\ \textsc{WFEqual}$$

$$\frac{A \vdash_{\texttt{sfrm}} e_1 \qquad A \vdash_{\texttt{sfrm}} e_2}{A \vdash_{\texttt{sfrm}} (e_1 \neq e_2)}\ \textsc{WFNEqual}$$

$$\frac{A \vdash_{\texttt{sfrm}} e}{A \vdash_{\texttt{sfrm}} \texttt{acc}(e.f)}\ \textsc{WFAcc}$$

$$\frac{}{A \vdash_{\texttt{sfrm}} (x : T)}\ \textsc{WFType}$$

$$\frac{A_s \vdash_{\texttt{sfrm}} \phi_1 \qquad A_s \cup \lfloor \phi_1 \rfloor \vdash_{\texttt{sfrm}} \phi_2}{A_s \vdash_{\texttt{sfrm}} \phi_1 * \phi_2}\ \textsc{WFSepOp}$$

3.2.1 Implication ($\phi_1 \stackrel{.}{\Longrightarrow} \phi_2$)

Conservative approx. of $\phi_1 \implies \phi_2$.

3.3 Footprint ($\lfloor \phi \rfloor = A_s$)

$$\begin{aligned}
\lfloor \texttt{true} \rfloor &= \emptyset \\
\lfloor e_1 = e_2 \rfloor &= \emptyset \\
\lfloor e_1 \neq e_2 \rfloor &= \emptyset \\
\lfloor \texttt{acc}(e.f) \rfloor &= \{(e, f)\} \\
\lfloor \phi_1 * \phi_2 \rfloor &= \lfloor \phi_1 \rfloor \cup \lfloor \phi_2 \rfloor
\end{aligned}$$

3.4 Type ($\phi \vdash e : T$)

$$\frac{}{\phi \vdash n : \texttt{int}}\ \textsc{STValNum}$$

$$\frac{}{\phi \vdash \texttt{null} : T}\ \textsc{STValNull}$$

$$\frac{\phi \implies (x:T)}{\phi \vdash x:T}\ \textsc{STVar}$$

$$\frac{\phi \vdash e:C \qquad \vdash C.f:T}{\phi \vdash e.f:T}\ \textsc{STField}$$

3.5 Hoare ($\vdash \{\phi\}\overline{s}\{\phi\}$)

$$\frac{\vdash \{\phi_p\}s_1\{\phi_{q1}\} \qquad \phi_{q1} \implies \phi_{q2} \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_{q2} \qquad \vdash \{\phi_{q2}\}s_2\{\phi_r\}}{\vdash \{\phi_p\}s_1; s_2\{\phi_r\}}\ \textsc{HSec}$$

$$\frac{\phi \implies \phi' \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad x \notin FV(\phi') \qquad \phi \vdash x:C \qquad \texttt{fields}(C) = \overline{f}}{\vdash \{\phi\}x := \texttt{new}\ C\{\overline{\texttt{acc}(x,f_i)*}x:C*(x \neq \texttt{null})*\phi'\}}\ \textsc{HNewObj}$$

$$\frac{\phi \implies \texttt{acc}(x.f)*(x \neq \texttt{null})*\phi' \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad \phi \vdash x:C \qquad \phi \vdash y:T \qquad \vdash C.f:T}{\vdash \{\phi\}x.f := y\{x:C*\texttt{acc}(x.f)*(x \neq \texttt{null})*(x.f = y)*\phi'\}}\ \textsc{HFieldAssign}$$

$$\frac{\phi \implies \phi' \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad x \notin FV(\phi') \qquad x \notin FVe(e) \qquad \phi \vdash x:T \qquad \phi \vdash e:T \qquad \lfloor \phi' \rfloor \vdash_{\texttt{sfrm}} e}{\vdash \{\phi\}x := e\{\phi' * (x = e)\}}\ \textsc{HVarAssign}$$

$$\frac{\phi \implies \phi' \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad \texttt{result} \notin FV(\phi') \qquad \phi \vdash x:T \qquad \phi \vdash \texttt{result}:T}{\vdash \{\phi\}\texttt{return}\ x\{\texttt{result}:T*(\texttt{result} = x)*\phi'\}}\ \textsc{HReturn}$$

$$\frac{\begin{array}{c}\phi \vdash y:C \qquad \texttt{mmethod}(C,m) = T_r\ m(T_p\ z)\ \texttt{requires}\ \phi_{pre};\ \texttt{ensures}\ \phi_{post};\ \{\_\} \\ \phi \vdash x:T_r \qquad \phi \vdash z':T_p \qquad \phi \implies (y \neq \texttt{null})*\phi_p*\phi_r \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_r \qquad x \notin FV(\phi_r) \\ @listDistinct(x, x\cdot y\cdot z'\cdot\emptyset) \qquad \phi_p = \phi_{pre}[y,z'/\texttt{this},z] \qquad \phi_q = \phi_{post}[y,z',x/\texttt{this},z,\texttt{result}]\end{array}}{\vdash \{\phi\}x := y.m(z')\{\phi_q * \phi_r\}}\ \textsc{HApp}$$

$$\frac{\phi_1 \implies \phi_2}{\vdash \{\phi_1\}\texttt{assert}\ \phi_2\{\phi_1\}}\ \textsc{HAssert}$$

$$\frac{\phi_1 \implies \phi_2 * \phi_r \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_r}{\vdash \{\phi_1\}\texttt{release}\ \phi_2\{\phi_r\}}\ \textsc{HRelease}$$

$$\frac{\phi \implies \phi' \qquad \emptyset \vdash_{\texttt{sfrm}} \phi' \qquad x \notin FV(\phi')}{\vdash \{\phi\}T\ x\{x:T*(x = \texttt{defaultValue}(T))*\phi'\}}\ \textsc{HDeclare}$$

3.5.1 Hoare revisited - pre-grad minification

$$\frac{\vdash \{\phi_p\}s_1\{\phi_{q1}\} \qquad \phi_{q1} \implies \phi_{q2} \qquad \emptyset \vdash_{\texttt{sfrm}} \phi_{q2} \qquad \vdash \{\phi_{q2}\}s_2\{\phi_r\}}{\vdash \{\phi_p\}s_1; s_2\{\phi_r\}}\ \textsc{HSec}$$

$$\frac{x \notin FV(\phi) \qquad \texttt{fields}(C) = \overline{f}}{\vdash \{(x : C) * \phi\}x := \texttt{new}\ C\{\overline{\texttt{acc}(x, f_i) *}(x : C) * (x \neq \texttt{null}) * \phi\}}\ \textsc{HNewObj}$$

$$\frac{\vdash C.f : T}{\vdash \{(x : C) * (y : T) * (x \neq \texttt{null}) * \phi * \texttt{acc}(x.f)\}x.f := y\{(x : C) * \texttt{acc}(x.f) * (x \neq \texttt{null}) * (x.f = y) * \phi\}}\ \textsc{HFieldAs}$$

$$\frac{x \notin FV(\phi) \qquad x \notin FV(e) \qquad [e : T]_{T'}}{\vdash \{(x : T) * [\![e : T]\!]_{T'} * \phi\}x := e\{[\![e : T]\!]_{T'} * \phi * (x = e)\}}\ \textsc{HVarAssign}$$

$$\frac{\texttt{result} \notin FV(\phi)}{\vdash \{(x : T) * (\texttt{result} : T) * \phi\}\texttt{return}\ x\{(\texttt{result} : T) * (\texttt{result} = x) * \phi\}}\ \textsc{HReturn}$$

$$\frac{\texttt{mmethod}(C, m) = T_r\ m(T_p\ z)\ \texttt{requires}\ \phi_{pre};\ \texttt{ensures}\ \phi_{post};\ \{\_\} \qquad x \notin FV(\phi_r) \qquad x \neq y \land x \neq z'}{\vdash \{(x : T_r) * (y : C) * (z' : T_p) * \phi_r * (y \neq \texttt{null}) * \phi_{pre}[y, z'/\texttt{this}, z]\}x := y.m(z')\{\phi_r * \phi_{post}[y, z', x/\texttt{this}, z, \texttt{result}]\}}\ \textsc{H}$$

$$\frac{\phi \implies \phi'}{\vdash \{\phi\}\texttt{assert}\ \phi'\{\phi\}}\ \textsc{HAssert}$$

$$\frac{}{\vdash \{\phi * \phi'\}\texttt{release}\ \phi'\{\phi\}}\ \textsc{HRelease}$$

$$\frac{x \notin FV(\phi)}{\vdash \{\phi\}T\ x\{(x : T) * (x = \texttt{defaultValue}(T)) * \phi\}}\ \textsc{HDeclare}$$

4 Dynamic semantics

4.1 Expressions ($H, \rho \vdash e \Downarrow v$)

$$\frac{}{H, \rho \vdash x \Downarrow \rho(x)}\ \textsc{EEVar}$$

$$\frac{}{H, \rho \vdash v \Downarrow v}\ \textsc{EEValue}$$

$$\frac{H, \rho \vdash e \Downarrow o}{H, \rho \vdash e.f \Downarrow H(o)(f)}\ \textsc{EEAcc}$$

4.2 Formulas ($H, \rho, A \vDash \phi$)

$$\frac{}{H, \rho, A \vDash \texttt{true}} \text{ EAT}\textsc{rue}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 = v_2}{H, \rho, A \vDash (e_1 = e_2)} \text{ EAE}\textsc{qual}$$

$$\frac{H, \rho \vdash e_1 \Downarrow v_1 \qquad H, \rho \vdash e_2 \Downarrow v_2 \qquad v_1 \neq v_2}{H, \rho, A \vDash (e_1 \neq e_2)} \text{ EANE}\textsc{qual}$$

$$\frac{H, \rho \vdash e \Downarrow o \qquad (o, f) \in A}{H, \rho, A \vDash \texttt{acc}(e.f)} \text{ EAA}\textsc{cc}$$

$$\frac{\rho(x) = v \qquad H \vdash v : T}{H, \rho, A \vDash (x : T)} \text{ EAT}\textsc{ype}$$

$$\frac{A_1 = A \backslash A_2 \qquad H, \rho, A_1 \vDash \phi_1 \qquad H, \rho, A_2 \vDash \phi_2}{H, \rho, A \vDash \phi_1 * \phi_2} \text{ EAS}\textsc{ep}\text{O}\textsc{p}$$

We give a denotational semantics of formulas as $[\![\phi]\!] = \{\ (H, \rho, A) \mid H, \rho, A \vDash \phi\ \}$
Note: ϕ satisfiable $\iff [\![\phi]\!] \neq \emptyset$

4.2.1 Implication ($\phi_1 \implies \phi_2$)

$$\phi_1 \implies \phi_2 \qquad \iff \qquad \forall H, \rho, A : H, \rho, A \vDash \phi_1 \implies H, \rho, A \vDash \phi_2$$

Drawn from def. of entailment in "A Formal Semantics for Isorecursive and Equirecursive State Abstractions".

4.3 Footprint ($\lfloor \phi \rfloor_{H,\rho} = A_d$)

$$\begin{aligned}
\lfloor \texttt{true} \rfloor_{H,\rho} &= \emptyset \\
\lfloor e_1 = e_2 \rfloor_{H,\rho} &= \emptyset \\
\lfloor e_1 \neq e_2 \rfloor_{H,\rho} &= \emptyset \\
\lfloor \texttt{acc}(x.f) \rfloor_{H,\rho} &= \{(o, f)\} \text{ where } H, \rho \vdash x \Downarrow o \\
\lfloor \phi_1 * \phi_2 \rfloor_{H,\rho} &= \lfloor \phi_1 \rfloor_{H,\rho} \cup \lfloor \phi_2 \rfloor_{H,\rho}
\end{aligned}$$

4.4 Small-step ($(H, S) \to (H, S)$)

$$\frac{H, \rho \vdash x \Downarrow o \qquad H, \rho \vdash y \Downarrow v_y \qquad (o, f) \in A \qquad H' = H[o \mapsto [f \mapsto v_y]]}{(H, (\rho, A, x.f := y; \overline{s}) \cdot S) \to (H', (\rho, A, \overline{s}) \cdot S)} \text{ ESF}\textsc{ield}\text{A}\textsc{ssign}$$

$$\frac{H, \rho \vdash e \Downarrow v \qquad \rho' = \rho[x \mapsto v]}{(H, (\rho, A, x := e; \overline{s}) \cdot S) \to (H, (\rho', A, \overline{s}) \cdot S)} \text{ ESV}\textsc{ar}\text{A}\textsc{ssign}$$

$$\frac{\texttt{fields}(C) = \overline{T}\ \overline{f} \qquad \rho' = \rho[x \mapsto o] \qquad \begin{array}{c} o \notin \texttt{dom}(H) \\ A' = A * \overline{(o, f_i)} \end{array} \qquad H' = H[o \mapsto [\overline{f \mapsto \texttt{defaultValue}(T)}]]}{(H, (\rho, A, x := \texttt{new}\ \ C; \overline{s}) \cdot S) \to (H', (\rho', A', \overline{s}) \cdot S)} \text{ ESN}\textsc{ew}\text{O}\textsc{bj}$$

$$\frac{H, \rho \vdash x \Downarrow v_x \qquad \rho' = \rho[\texttt{result} \mapsto v_x]}{(H, (\rho, A, \texttt{return}\ x; \overline{s}) \cdot S) \rightarrow (H, (\rho', A, \overline{s}) \cdot S)} \text{ ESReturn}$$

$$\frac{\begin{array}{c} H, \rho \vdash y \Downarrow o \\ H, \rho \vdash z \Downarrow v \qquad H(o) = (C, \_) \qquad \texttt{mmethod}(C, m) = T_r\ m(T\ w)\ \texttt{requires}\ \phi;\ \texttt{ensures}\ \_;\ \{\overline{r}\} \\ \rho' = [\texttt{result} \mapsto \texttt{defaultValue}(T_r), \texttt{this} \mapsto o, w \mapsto v] \qquad H, \rho', A \vDash \phi \qquad A' = \lfloor \phi \rfloor_{H, \rho'} \end{array}}{(H, (\rho, A, x := y.m(z); \overline{s}) \cdot S) \rightarrow (H, (\rho', A', \overline{r}) \cdot (\rho, A \setminus A', x := y.m(z); \overline{s}) \cdot S)} \text{ ESApp}$$

$$\frac{\begin{array}{c} H, \rho \vdash y \Downarrow o \\ H(o) = (C, \_) \qquad \texttt{mpost}(C, m) = \phi \qquad H, \rho', A' \vDash \phi \qquad A'' = \lfloor \phi \rfloor_{H, \rho'} \qquad H, \rho' \vdash \texttt{result} \Downarrow v_r \end{array}}{(H, (\rho', A', \emptyset) \cdot (\rho, A, x := y.m(z); \overline{s}) \cdot S) \rightarrow (H, (\rho[x \mapsto v_r], A * A'', \overline{s}) \cdot S)} \text{ ESAppFinish}$$

$$\frac{H, \rho, A \vDash \phi}{(H, (\rho, A, \texttt{assert}\ \phi; \overline{s}) \cdot S) \rightarrow (H, (\rho, A, \overline{s}) \cdot S)} \text{ ESAssert}$$

$$\frac{H, \rho, A \vDash \phi \qquad A' = A \setminus \lfloor \phi \rfloor_{H, \rho}}{(H, (\rho, A, \texttt{release}\ \phi; \overline{s}) \cdot S) \rightarrow (H, (\rho, A', \overline{s}) \cdot S)} \text{ ESRelease}$$

$$\frac{\rho' = \rho[x \mapsto \texttt{defaultValue}(T)]}{(H, (\rho, A, T\ x; \overline{s}) \cdot S) \rightarrow (H, (\rho', A, \overline{s}) \cdot S)} \text{ ESDeclare}$$

5 Gradualization

5.1 Syntax

5.1.1 Gradual formula

$$\widetilde{\phi} \quad ::= \quad \phi \mid ? * \phi$$

Note: consider ? in other positions as "self-framing delimiter", but with semantically identical meaning.
As long as ? is only legal in the front though: $\phi_1 * \widetilde{\phi_2}$ propagates the ? to the very left in case $\widetilde{\phi_2}$ contains one.

5.1.2 Type judgment expansion

Motivation: Materialize and combine the requirements on ϕ.
Designed so that

$$\phi \vdash e : T \quad \wedge \quad \lfloor \phi \rfloor \vdash_{\texttt{sfrm}} e$$
$$\Longleftrightarrow$$
$$[e : T]_C \quad \wedge \quad \phi \implies [\![e : T]\!]_C$$

Expand into premise: $[e : T]_C$

$$\begin{aligned} [v : T]_C &= T = C \quad \wedge \quad \vdash v : T \\ [x : T]_C &= T = C \\ [e.f : T]_C &= [e : C']_C \quad \wedge \quad \vdash C'.f : T \end{aligned}$$

Expand into formula: $[\![e : T]\!]_C$

$$\begin{aligned} [\![v : T]\!]_C &= \texttt{true} \\ [\![x : T]\!]_C &= (x : C) \\ [\![e.f : T]\!]_C &= [\![e : T]\!]_C * \texttt{acc}(e.f) \end{aligned}$$

5.2 Concretization

Syntax $\hat{\phi}$:= self-framed and satisfiable ϕ

$$\gamma(\hat{\phi}) = \{ \hat{\phi} \}$$
$$\gamma(? * \phi') = \{ \hat{\phi} \mid \hat{\phi} \implies \phi' \} \text{ if } \phi' \text{ satisfiable}$$
$$\gamma(\phi) \text{ undefined otherwise}$$

5.3 Abstraction

$$\alpha(\emptyset) \text{ undefined}$$
$$\alpha(\{ \phi \}) = \phi$$
$$\alpha(\overline{\phi} \text{ with maximum element } \phi) = ? * \phi$$
$$\alpha(\overline{\phi}) = ? \quad \text{otherwise}$$

5.4 Gradual Lifting

5.4.1 Self framing

$$\frac{A \vdash_{\mathtt{sfrm}} \phi}{A \mathrel{\widetilde{\vdash_{\mathtt{sfrm}}}} \phi} \text{GSfrmNonGrad}$$

$$\frac{}{A \mathrel{\widetilde{\vdash_{\mathtt{sfrm}}}} ? * \phi} \text{GSfrmGrad}$$

5.4.2 Implication

$$\frac{\phi_1 \implies \phi_2}{\phi_1 \mathrel{\widetilde{\implies}} \widetilde{\phi_2}} \text{GImplNonGrad}$$

$$\frac{\hat{\phi_m} \implies \phi_2 \qquad \hat{\phi_m} \implies \phi_1}{? * \phi_1 \mathrel{\widetilde{\implies}} \widetilde{\phi_2}} \text{GImplGrad}$$

$\hat{\phi_m}$ is evidence!

Consistent transitivity
While $\implies$ is transitive, $\widetilde{\implies}$ is generally not.
But maybe not even necessary with smarter hoare rules?

5.4.3 Hoare and evidence

Discussion/Considerations:

- The post-condition-ϕ seems to inherit its gradual-ness from implication, which itself does not care about whether its second argument is gradual or not...
- Gradual

Example:

$$\frac{\emptyset \vdash_{\mathtt{sfrm}} \widetilde{\phi'} \qquad x \notin FV(\widetilde{\phi'}) \qquad x \notin FV(e) \qquad \begin{array}{c}\epsilon \vdash \widetilde{\phi} \mathrel{\widetilde{\implies}} \widetilde{\phi'} \\ \epsilon \vdash \widetilde{\phi} \vdash x : T\end{array} \qquad \epsilon \vdash \widetilde{\phi} \vdash e : T \qquad \epsilon \vdash \lfloor \widetilde{\phi'} \rfloor \vdash_{\mathtt{sfrm}} e}{\vdash \{\widetilde{\phi}\} x := e \{\widetilde{\phi'} * (x = e)\}} \text{GHVarAssign}$$

Collapsing (hidden) gradual implications into a single one:

$$\frac{\epsilon \vdash \widetilde{\phi} \stackrel{\sim}{\Longrightarrow} (x:T) * [\![e:T]\!]_C * \widetilde{\phi}' \qquad \emptyset \vdash_{\texttt{sfrm}} [\![e:T]\!]_C * \widetilde{\phi}' \qquad x \notin FV(\widetilde{\phi}') \qquad x \notin FV(e) \qquad [e:T]_C}{\vdash \{\widetilde{\phi}\} x := e \{[\![e:T]\!]_C * \widetilde{\phi}' * (x = e)\}} \text{ GHVarAssig}$$

When shifting implication responsibility to GHSec:

$$\frac{x \notin FV(\widetilde{\phi}') \qquad x \notin FV(e) \qquad [e:T]_C}{\vdash \{(x:T) * [\![e:T]\!]_C * \widetilde{\phi}'\} x := e \{[\![e:T]\!]_C * \widetilde{\phi}' * (x = e)\}} \text{ GHVarAssign}$$

Example derivation:

$\{(x:T) * (y:C) * \texttt{acc}(y.a) * \texttt{acc}(y.a.b) * \texttt{acc}(y.a.b.c) * \widetilde{\phi}'\}$

$\{(x:T) * [\![y.a.b.c:T]\!]_C * \widetilde{\phi}'\}$

$x := y.a.b.c;$ $\quad x \notin FV(\widetilde{\phi}')$
$\quad x \notin FV(y.a.b.c)$
$\quad [y.a.b.c:T]_C = \ \vdash C_y = C \ \wedge \ \vdash C_y.a : C_a \ \wedge \ \vdash C_a.b : C_b \ \wedge \ \vdash C_b.c : T$

$\{[\![y.a.b.c:T]\!]_C * \widetilde{\phi}' * (x = y.a.b.c)\}$

$\{(y:C) * \texttt{acc}(y.a) * \texttt{acc}(y.a.b) * \texttt{acc}(y.a.b.c) * \widetilde{\phi}' * (x = y.a.b.c)\}$

5.5 Theorems

5.5.1 Soundness of α

$$\forall \overline{\phi} : \overline{\phi} \subseteq \gamma(\alpha(\overline{\phi}))$$

5.5.2 Optimality of α

$$\forall \overline{\phi}, \widetilde{\phi} : \overline{\phi} \subseteq \gamma(\widetilde{\phi}) \implies \gamma(\alpha(\overline{\phi})) \subseteq \gamma(\widetilde{\phi})$$

6 Theorems

6.1 Invariant $invariant(H, \rho, A_d, \phi)$

6.1.1 Phi valid

$$\vdash_{\texttt{sfrm}} \phi$$

6.1.2 Phi holds

$$H, \rho, A_d \vDash \phi$$

6.1.3 Types preserved

$$\begin{aligned} &\forall e, T : \phi \vdash e : T \\ &\implies H, \rho \vdash e : T \end{aligned}$$

6.1.4 Heap consistent

$$\begin{aligned} &\forall o, C, \mu, f, T : H(o) = (C, \mu) \\ &\implies \texttt{fieldType}(C, f) = T \\ &\implies H, \rho \vdash \mu(f) : T \end{aligned}$$

6.1.5 Heap not total

$$\begin{aligned}
&\exists o_{min}: \\
&\forall o \geq o_{min} : o \notin \mathtt{dom}(H) \\
&\qquad \wedge \forall f, (o, f) \notin A
\end{aligned}$$

6.2 Soundness

6.2.1 Progress

$$\begin{aligned}
\forall \ldots : \quad & \vdash \{\phi_1\} s' \{\phi_2\} \\
\implies & \mathit{invariant}(H_1, \rho_1, A_1, \phi_1) \\
\implies & \exists H_2, \rho_2, A_2 : (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S)
\end{aligned}$$

6.2.2 Preservation

$$\begin{aligned}
\forall \ldots : \quad & \vdash \{\phi_1\} s' \{\phi_2\} \\
\implies & \mathit{invariant}(H_1, \rho_1, A_1, \phi_1) \\
\implies & (H_1, (\rho_1, A_1, s'; \overline{s}) \cdot S) \rightarrow^* (H_2, (\rho_2, A_2, \overline{s}) \cdot S) \\
\implies & \mathit{invariant}(H_2, \rho_2, A_2, \phi_2)
\end{aligned}$$