Gradual Verification

with Implicit Dynamic Frames

Master Thesis of

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Gradual Verification

(with Implicit Dynamic Frames)

```
int getFour(int i)
    requires ?; // haven't figured that one out, yet
    ensures  result = 4;
{
    i := i + 1;
    return i;
}
```

Motivation

- Program verification (against some specification)
- Two flavors: static & dynamic

```
// spec: callable only if (this.balance >= amount)
void withdrawCoins(int amount)
{
   // business logic
   this.balance -= amount;
}
```

Dynamic Verification

- runtime checks
- testing techniques
- guarantee compliance **at runtime**

```
void withdrawCoins(int amount)
{
   assert this.balance >= amount;
   // business logic
   this.balance -= amount;
}
```

Dynamic Verification - Drawbacks

- runtime checks — runtime overhead
- testing techniques — additional efforts
- guarantee compliance **at runtime** — pot. late detection

```
void withdrawCoins(int amount)
    requires this.balance >= amount;
{
    // business logic
    this.balance -= amount;
}
```

Static Verification

- declarative
- formal logic
- guarantee compliance **in advance**

```
void withdrawCoins(int amount)
    requires this.balance >= amount;
{
    // business logic
    this.balance -= amount;
}
```

Static Verification - Drawbacks

- declarative
- formal logic
- guarantee compliance **in advance**

limited syntax
decidability

annotation pressure

```
void withdrawCoins(int amount)
    requires this.balance >= amount;
    ensures  this.balance = old(this.balance) – amount;
{
    // business logic
    this.balance -= amount;
}
```

Solution? Static + Dynamic

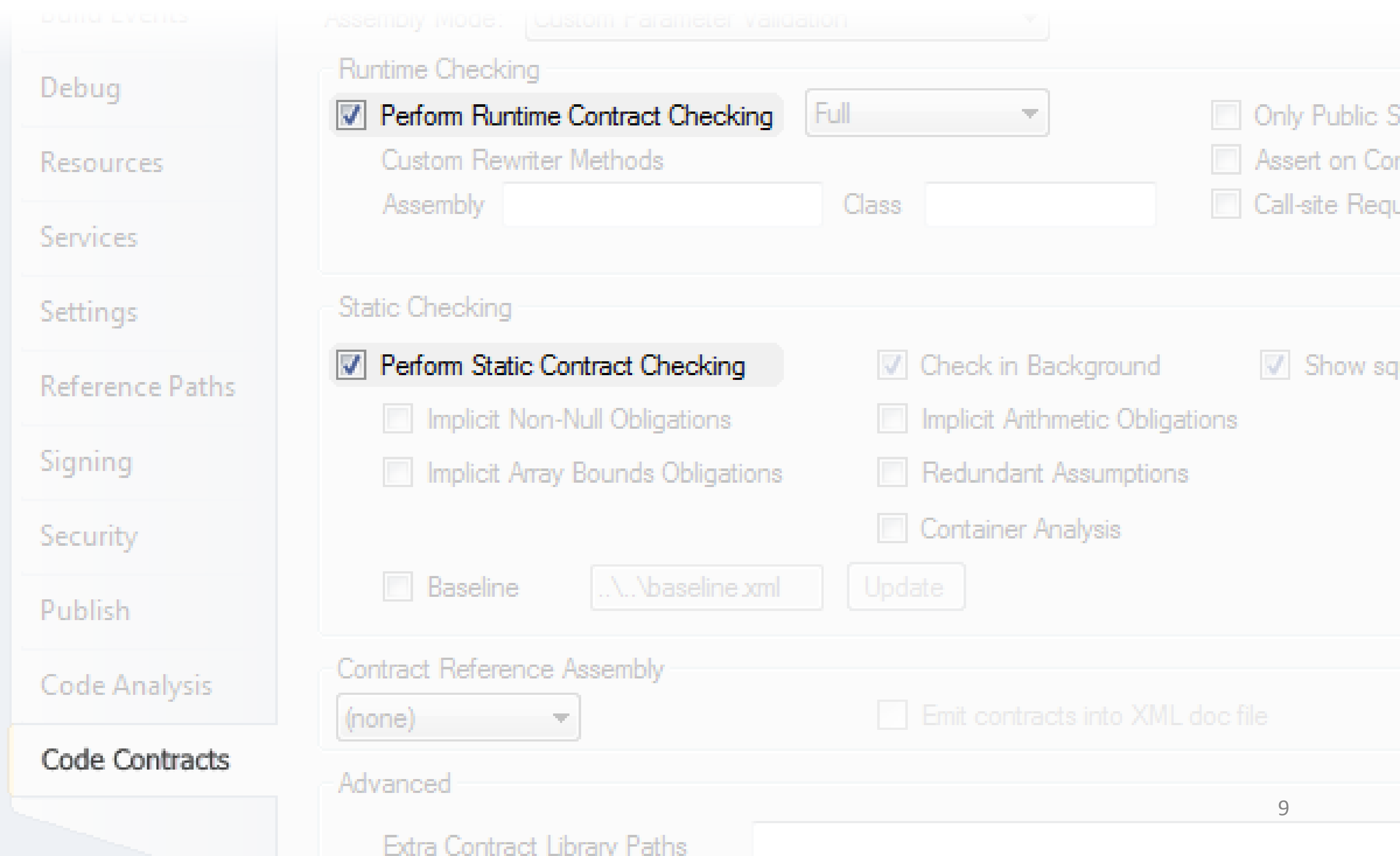
Means:

- “relaxed” static verification (warnings on failure)
- turn contracts into runtime assertions

Notable implementations:

- Java with JML annotations
  - “ESC/Java” for static verification
  - “JML4c” for dynamic verification
- Code Contracts for .NET (by RiSE, MSR)

Solution? Static + Dynamic



Solution! Static ⊕ Dynamic

"Static Typing Where Possible,
Dynamic Typing When Needed" (Erik Meijer)

```
void withdrawCoins(int amount)
    requires this.balance >= amount;
{
    …
}
…
acc.balance = 100;
acc.withdrawCoins(50); // can prove acc.balance >= 50
acc.withdrawCoins(30); // can't prove acc.balance >= 30
acc.withdrawCoins(30); // can't prove acc.balance >= 30
```

Approach

Syntax

$s \in \textsc{Stmt}$

$\phi \in \textsc{Formula}$

Program State

$\pi \in \textsc{ProgramState}$

Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \models \phi$

Soundness

Gradualization

Syntax

$\widetilde{s} \in \widetilde{\textsc{Stmt}}$

$\widetilde{\phi} \in \widetilde{\textsc{Formula}}$

Program State

$\widetilde{\pi} \in \widetilde{\textsc{ProgramState}}$

Semantics

Static $\widetilde{\vdash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}\}$

Dynamic $\widetilde{\pi} \mathrel{\widetilde{\longrightarrow}} \widetilde{\pi}$

Formula $\widetilde{\pi} \mathrel{\widetilde{\models}} \widetilde{\phi}$

Soundness

Statically Verified Language

Syntax

$s \in \textsc{Stmt}$

$\phi \in \textsc{Formula}$

Program State

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Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \vDash \phi$

Soundness

$$e \quad ::= \quad x \ \mid\ n \ \mid\ e_1 + e_2$$

$$s \quad ::= \quad x \ \texttt{:=}\ e \ \mid\ \texttt{assert}\ \phi \ \mid\ s_1\texttt{;}\ s_2$$

$$\phi \quad ::= \quad \texttt{true} \ \mid\ (e_1\ \texttt{=}\ e_2) \ \mid\ \phi_1 \wedge \phi_2$$

$$= (\textsc{Var} \rightharpoonup \mathbb{N}_0) \times \textsc{Stmt}$$

Statically Verified Language

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Formula $\pi \vDash \phi$

Soundness

$$\frac{x\ \texttt{:=}\ e}{\vdash \{\phi[e/x]\}\ x\ \texttt{:=}\ e\ \{\phi\}}\ \textsc{HAssign}$$

$$\frac{\phi \Rightarrow \phi_a}{\vdash \{\phi\}\ \texttt{assert}\ \phi_a\ \{\phi\}}\ \textsc{HAssert}$$

$$\frac{\begin{array}{c}\phi_{q1} \Rightarrow \phi_{q2} \\ \vdash \{\phi_p\}\ s_1\ \{\phi_{q1}\} \qquad \vdash \{\phi_{q2}\}\ s_2\ \{\phi_r\}\end{array}}{\vdash \{\phi_p\}\ s_1\texttt{;}\ \ s_2\ \{\phi_r\}}\ \textsc{HSeq}$$

Statically Verified Language

Syntax

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Program State

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Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \models \phi$

Soundness

$$\frac{\mathcal{N}_\sigma(e) = n}{\langle \sigma, x\ \texttt{:=}\ e\texttt{;}\ \ s\rangle \longrightarrow \langle \sigma[x \mapsto n], s\rangle}\ \textsc{SsAssign}$$

$$\frac{\langle \sigma, \texttt{assert}\ \ \phi_a\texttt{;}\ \ s\rangle \models \phi_a}{\langle \sigma, \texttt{assert}\ \ \phi_a\texttt{;}\ \ s\rangle \longrightarrow \langle \sigma, s\rangle}\ \textsc{SsAssert}$$

Statically Verified Language

Syntax

$s \in \textsc{Stmt}$

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Program State

$\pi \in \textsc{ProgramState}$

Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \vDash \phi$

Soundness

$$\frac{\vdash \{\phi\}\ s\ \{\phi'\}}{\vDash \{\phi\}\ s\ \{\phi'\}}\ \textsc{Soundness}$$

$$\vDash \{\phi\}\ s\ \{\phi'\}$$

$$\overset{\text{def}}{\iff}$$

$$\forall \pi, \pi'.\ \pi \xrightarrow{s} \pi' \land \pi \vDash \phi \implies \pi' \vDash \phi'$$

Approach

Syntax

$s \in \textsc{Stmt}$

$\phi \in \textsc{Formula}$

Program State

$\pi \in \textsc{ProgramState}$

Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \vDash \phi$

Soundness

Gradualization

Syntax

$\widetilde{s} \in \widetilde{\textsc{Stmt}}$

$\widetilde{\phi} \in \widetilde{\textsc{Formula}}$

Program State

$\widetilde{\pi} \in \widetilde{\textsc{ProgramState}}$

Semantics

Static $\widetilde{\vdash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}\}$

Dynamic $\widetilde{\pi} \ \widetilde{\longrightarrow}\ \widetilde{\pi}$

Formula $\widetilde{\pi}\ \widetilde{\vDash}\ \widetilde{\phi}$

Soundness

Gradualization – Goal 1/3

Introduction of wildcard formula ?

- placeholder for arbitrary (satisfiable) formula
- enables Hoare deduction despite incomplete information
- enables gradual annotation of programs (? as default)

Formula Precision

$$\widetilde{\phi_1} \sqsubseteq \widetilde{\phi_2} \overset{\text{def}}{\iff} \gamma(\widetilde{\phi_1}) \subseteq \gamma(\widetilde{\phi_2})$$

Gradualization – Goal 2/3

Compatibility with static language

- don't reject source code that was accepted before — static
- observable behavior is not changed — dynamic

$$\textsc{Formula} \subseteq \widetilde{\textsc{Formula}} \qquad \textsc{Stmt} \subseteq \widetilde{\textsc{Stmt}}$$

$$\vdash \{\phi\}\ s\ \{\phi'\} \implies \widetilde{\vdash} \{\phi\}\ s\ \{\phi'\}$$

$$\pi \xrightarrow{s} \pi' \implies \exists \pi''.\ \pi \overset{s}{\widetilde{\longrightarrow}} \pi'' \wedge (\forall \phi.\ \pi' \vDash \phi \implies \pi'' \vDash \phi)$$

Gradualization – Goal 3/3

Gradual guarantee (Siek et al.), adapted

Reducing precision will not

- introduce verification failure — static
- change observable behavior — dynamic

$$\text{Given} \quad \widetilde{\phi_1} \sqsubseteq \widetilde{\phi_2} \wedge \widetilde{\phi_1'} \sqsubseteq \widetilde{\phi_2'} \wedge \widetilde{s_1} \sqsubseteq \widetilde{s_2} \wedge \widetilde{\pi_1} \sqsubseteq \widetilde{\pi_2}$$

$$\widetilde{\vdash} \{\widetilde{\phi_1}\} \; \widetilde{s_1} \; \{\widetilde{\phi_1'}\} \implies \widetilde{\vdash} \{\widetilde{\phi_2}\} \; \widetilde{s_2} \; \{\widetilde{\phi_2'}\}$$

$$\widetilde{\pi_1} \xrightarrow{\widetilde{s_1}} \widetilde{\pi_1'} \implies \exists \widetilde{\pi_2'}.\; \widetilde{\pi_2} \xrightarrow{\widetilde{s_2}} \widetilde{\pi_2'} \wedge \widetilde{\pi_1'} \sqsubseteq \widetilde{\pi_2'}$$

Gradual Predicate Lifting

Introduction

$$\forall \phi_1, \phi_2 \in \textsc{Formula}.\ P(\phi_1, \phi_2) \implies \widetilde{P}(\phi_1, \phi_2)$$

Monotonicity

$$\forall \widetilde{\phi_1}, \widetilde{\phi_2}, \widetilde{\phi_1'}, \widetilde{\phi_2'} \in \widetilde{\textsc{Formula}}.\ \widetilde{\phi_1} \sqsubseteq \widetilde{\phi_1'} \wedge \widetilde{\phi_2} \sqsubseteq \widetilde{\phi_2'} \wedge \widetilde{P}(\widetilde{\phi_1}, \widetilde{\phi_2}) \implies \widetilde{P}(\widetilde{\phi_1'}, \widetilde{\phi_2'})$$

$$P(\phi_1, \phi_a, \phi_2) \stackrel{\text{def}}{=} \phi_1 = \phi_2 \wedge \phi_1 \Rightarrow \phi_a$$

$$\widetilde{P}(\widetilde{\phi_1}, \widetilde{\phi_a}, \widetilde{\phi_2}) \stackrel{\text{def}}{=} \widetilde{\phi_1} \approx \widetilde{\phi_2} \wedge \widetilde{\phi_1} \mathrel{\widetilde{\Rightarrow}} \widetilde{\phi_a}$$

$$\widetilde{\phi_1} \approx \widetilde{\phi_2} \stackrel{\text{def}}{=} \widetilde{\phi_1} = \widetilde{\phi_2} \vee \widetilde{\phi_1} = \textsf{?} \vee \widetilde{\phi_2} = \textsf{?}$$

$$\widetilde{\phi_1} \mathrel{\widetilde{\Rightarrow}} \widetilde{\phi_2} \stackrel{\text{def}}{=} \widetilde{\phi_1} \Rightarrow \widetilde{\phi_2} \vee \widetilde{\phi_1} = \textsf{?} \vee \widetilde{\phi_2} = \textsf{?}$$

$$\frac{\phi \Rightarrow \phi_a}{\vdash \{\phi\}\ \texttt{assert}\ \phi_a\ \{\phi\}}\ \textsc{HAssert}$$

$$\frac{}{\widetilde{\vdash} \{\widetilde{\phi_1}\}\ \texttt{assert}\ \widetilde{\phi_a}\ \{\widetilde{\phi_2}\}}\ \widetilde{\textsc{H}}\textsc{Assert}$$

Gradual Predicate Lifting

Introduction

$$\forall \phi_1, \phi_2 \in \textsc{Formula}.\ P(\phi_1, \phi_2) \implies \widetilde{P}(\phi_1, \phi_2)$$

Monotonicity

$$\forall \widetilde{\phi_1}, \widetilde{\phi_2}, \widetilde{\phi_1'}, \widetilde{\phi_2'} \in \widetilde{\textsc{Formula}}.\ \widetilde{\phi_1} \sqsubseteq \widetilde{\phi_1'} \land \widetilde{\phi_2} \sqsubseteq \widetilde{\phi_2'} \land \widetilde{P}(\widetilde{\phi_1}, \widetilde{\phi_2}) \implies \widetilde{P}(\widetilde{\phi_1'}, \widetilde{\phi_2'})$$

(Optimality)

$\widetilde{P}$ is smallest predicate closed under above rules

⇔

$$\widetilde{P}(\widetilde{\phi_1}, \widetilde{\phi_2}) \iff \exists \phi_1 \in \gamma(\widetilde{\phi_1}), \phi_2 \in \gamma(\widetilde{\phi_2}).\ P(\phi_1, \phi_2)$$

Gradual Predicate Lifting

$$
\begin{aligned}
& \widetilde{\pi} \mathrel{\widetilde{\vDash}} \widetilde{\phi} \\
\iff & \exists \pi \in \gamma(\widetilde{\pi}), \phi \in \gamma(\widetilde{\phi}).\ \pi \vDash \phi \\
\iff & \exists \phi \in \gamma(\widetilde{\phi}).\ \widetilde{\pi} \vDash \phi \\
\iff & \widetilde{\pi} \vDash \widetilde{\phi} \lor \widetilde{\phi} = \texttt{?}
\end{aligned}
$$

$$
\frac{\widetilde{\pi} \vDash \phi}{\widetilde{\pi} \mathrel{\widetilde{\vDash}} \phi}\ \textsc{EvalPhiStatic} \qquad\qquad \frac{}{\widetilde{\pi} \mathrel{\widetilde{\vDash}} \texttt{?}}\ \textsc{EvalPhiStatic}
$$

$$
\widetilde{P}(\widetilde{\phi_1}, \widetilde{\phi_2}) \iff \exists \phi_1 \in \gamma(\widetilde{\phi_1}), \phi_2 \in \gamma(\widetilde{\phi_2}).\ P(\phi_1, \phi_2)
$$

Gradual Function Lifting

Introduction

$$\forall \phi \in \textsc{Formula}.\ f(\phi) \sqsubseteq \widetilde{f}(\phi)$$

Monotonicity

$$\forall \widetilde{\phi_1}, \widetilde{\phi_2} \in \widetilde{\textsc{Formula}}.\ \widetilde{\phi_1} \sqsubseteq \widetilde{\phi_2} \implies \widetilde{f}(\widetilde{\phi_1}) \sqsubseteq \widetilde{f}(\widetilde{\phi_2})$$

(Optimality)

$\widetilde{f}$ has most precise return values among all liftings

$\Leftrightarrow$ $\widetilde{f}(\widetilde{\phi}) = \alpha(\overline{f}(\gamma(\widetilde{\phi})))$ where $\langle \alpha, \gamma \rangle$ is $\{\ \overline{f}\ \}$-partial Galois connection

Gradual Partial Function Lifting

Introduction

$$\forall \phi \in \textsc{Formula} \cap \mathsf{dom}(f).\ f(\phi) \sqsubseteq \widetilde{f}(\phi)$$

Monotonicity

$$\forall \widetilde{\phi_1}, \widetilde{\phi_2} \in \widetilde{\textsc{Formula}}.\ \widetilde{\phi_1} \sqsubseteq \widetilde{\phi_2} \land \widetilde{\phi_1} \in \mathsf{dom}(\widetilde{f}) \implies \widetilde{f}(\widetilde{\phi_1}) \sqsubseteq \widetilde{f}(\widetilde{\phi_2})$$

(Optimality)

$\widetilde{f}$ has smallest domain and most precise return values among all liftings

$\Leftrightarrow$ $\quad \widetilde{f}(\widetilde{\phi}) = \alpha(\overline{f}(\gamma(\widetilde{\phi})))$ $\quad$ where $\langle \alpha, \gamma \rangle$ is $\{\ \overline{f}\ \}$-partial Galois connection

Gradual Partial Function Lifting

$$\frac{\langle \sigma, \texttt{assert}\ \ \phi_a\,;\ \ s\rangle \vDash \phi_a}{\langle \sigma, \texttt{assert}\ \ \phi_a\,;\ \ s\rangle \longrightarrow \langle \sigma, s\rangle}\ \textsc{SsAssert}$$

$$\frac{\langle \sigma, \texttt{assert}\ \ \widetilde{\phi_a}\,;\ \ s\rangle \mathrel{\widetilde{\vDash}} \widetilde{\phi_a}}{\langle \sigma, \texttt{assert}\ \ \widetilde{\phi_a}\,;\ \ s\rangle \mathrel{\widetilde{\longrightarrow}} \langle \sigma, s\rangle}\ \textsc{SsAssert}$$

⇔ $\widetilde{f}(\widetilde{\phi}) = \alpha(\overline{f}(\gamma(\widetilde{\phi})))$ where $\langle \alpha, \gamma \rangle$ is $\{\ \overline{f}\ \}$-partial Galois connection

Bonus: { F }-partial Galois connection

Definition 28 (Partial Galois connection). *Let* $(C, \sqsubseteq_C)$ *and* $(A, \sqsubseteq_A)$ *be two posets,* $\mathcal{F}$ *a set of operators on* C, $\alpha : C \rightharpoonup A$ *a partial function and* $\gamma : A \to C$ *a total function. The pair* $\langle \alpha, \gamma \rangle$ *is an* $\mathcal{F}$*-partial Galois connection if and only if:*

1. *If* $\alpha(c)$ *is defined, then* $c \sqsubseteq_C \gamma(\alpha(c))$, *and*
2. *If* $\alpha(c)$ *is defined, then* $c \sqsubseteq_C \gamma(a)$ *implies* $\alpha(c) \sqsubseteq_A a$, *and*
3. *For all* $F \in \mathcal{F}$ *and* $c \in C$, $\alpha(F(\gamma(c)))$ *is defined.*

This definition can be generalized for a set $\mathcal{F}$ *of arbitrary* n*-ary operators.*

Gradual Verification - Approach

Syntax

$s \in \textsc{Stmt}$

$\phi \in \textsc{Formula}$

Program State

$\pi \in \textsc{ProgramState}$

Semantics

Static $\vdash \{\phi\}\ s\ \{\phi\}$

Dynamic $\pi \longrightarrow \pi$

Formula $\pi \vDash \phi$

Soundness

Gradualization

Syntax

$\widetilde{s} \in \widetilde{\textsc{Stmt}}$

$\widetilde{\phi} \in \widetilde{\textsc{Formula}}$

Program State

$\widetilde{\pi} \in \widetilde{\textsc{ProgramState}}$

Semantics

Static $\widetilde{\vdash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}\}$

Dynamic $\widetilde{\pi} \mathrel{\widetilde{\longrightarrow}} \widetilde{\pi}$

Formula $\widetilde{\pi} \mathrel{\widetilde{\vDash}} \widetilde{\phi}$

Soundness

Gradual Verification - Approach

Syntax		**Syntax**
$s \in \textsc{Stmt}$	$\longrightarrow$	$\widetilde{s} \in \widetilde{\textsc{Stmt}}$
$\phi \in \textsc{Formula}$	syntax extension $\longrightarrow$ abstract interpretation	$\widetilde{\phi} \in \widetilde{\textsc{Formula}}$
Program State		**Program State**
$\pi \in \textsc{ProgramState}$	$\longrightarrow$	$\widetilde{\pi} \in \widetilde{\textsc{ProgramState}}$
Semantics		**Semantics**
Static $\vdash \{\phi\}\ s\ \{\phi\}$	predicate lifting $\longrightarrow$	Static $\widetilde{\vdash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}\}$
Dynamic $\pi \longrightarrow \pi$	partial function lifting $\longrightarrow$	Dynamic $\widetilde{\pi} \mathrel{\widetilde{\longrightarrow}} \widetilde{\pi}$
Formula $\pi \models \phi$	predicate lifting $\longrightarrow$	Formula $\widetilde{\pi} \mathrel{\widetilde{\models}} \widetilde{\phi}$
Soundness	$\longrightarrow$	**Soundness**

Gradual Soundness

$$\frac{\vdash \{\phi\}\ s\ \{\phi'\}}{\vDash \{\phi\}\ s\ \{\phi'\}}\ \textsc{Soundness}$$

$$\vDash \{\phi\}\ s\ \{\phi'\}$$

$$\overset{\text{def}}{\iff}$$

$$\forall \pi, \pi'.\ \pi \xrightarrow{s} \pi' \land \pi \vDash \phi \implies \pi' \vDash \phi'$$

$$\frac{\widetilde{\vdash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}'\}}{\widetilde{\vDash} \{\widetilde{\phi}\}\ \widetilde{s};\ \texttt{assert}\ \widetilde{\phi}'\ \{\widetilde{\phi}'\}}\ \widetilde{\textsc{S}}\textsc{oundness}$$

$$\widetilde{\vDash} \{\widetilde{\phi}\}\ \widetilde{s}\ \{\widetilde{\phi}'\}$$

$$\overset{\text{def}}{\iff}$$

$$\forall \widetilde{\pi}, \widetilde{\pi}'.\ \widetilde{\pi} \overset{\widetilde{s}}{\widetilde{\longrightarrow}} \widetilde{\pi}' \land \widetilde{\pi}\ \widetilde{\vDash}\ \widetilde{\phi} \implies \widetilde{\pi}'\ \widetilde{\vDash}\ \widetilde{\phi}'$$

$$\widetilde{\vdash} \{?\}\ \texttt{y := 4}\ \{\texttt{(x = 2)} \land \texttt{(y = 4)}\}$$

$$\widetilde{\vDash} \{?\}\ \texttt{y := 4; assert (x = 2)}\ \{\texttt{(x = 2)} \land \texttt{(y = 4)}\}$$

Gradual Verification – Put to the Test

$$\dfrac{\widetilde{\phi_1} \mathrel{\widetilde{\Rightarrow}} \widetilde{\phi_2} \qquad \mathrel{\widetilde{\vdash}} \{?\}\ \texttt{y := 2}\ \{\widetilde{\phi_1}\} \qquad \mathrel{\widetilde{\vdash}} \{\widetilde{\phi_2}\}\ \texttt{x := 3}\ \{(\texttt{x = 3}) \land (\texttt{y = 2})\}}{\mathrel{\widetilde{\vdash}} \{?\}\ \texttt{y := 2; x := 3}\ \{(\texttt{x = 3}) \land (\texttt{y = 2})\}}\ \widetilde{\mathrm{HS}}\textsc{eq}$$

Implicit Dynamic Frames

- Gradual Frame rule