Gradually Verified Language with Recursive Predicates

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Contents

1 Grammar

$$
\begin{array}{rcl}
x, y, z & \in & \mathit{VAR} \\
v & \in & \mathit{VAL} \\
e & \in & \mathit{EXPR} \\
s & \in & \mathit{STMT} \\
o & \in & \mathit{LOC} \\
f & \in & \mathit{FIELDNAME} \\
m & \in & \mathit{METHODNAME} \\
C, D & \in & \mathit{CLASSNAME} \\
\alpha & \in & \mathit{PREDNAME} \\
P & ::= & \overline{\mathit{cls}}\ s \\
\mathit{cls} & ::= & \texttt{class}\ C\ \texttt{extends}\ D\ \{\overline{\mathit{field}}\ \overline{\mathit{pred}}\ \overline{\mathit{method}}\} \\
\mathit{field} & ::= & T\ f; \\
\mathit{pred} & ::= & \texttt{predicate}\ \alpha_C(\overline{T\ x}) = \widetilde{\phi} \\
T & ::= & \texttt{int} \mid \texttt{bool} \mid C \mid \top \\
\mathit{method} & ::= & T\ m(\overline{T\ x})\ \texttt{dynamically}\ \mathit{contract}\ \texttt{statically}\ \mathit{contract}\ \{s\} \\
\mathit{contract} & ::= & \texttt{requires}\ \widetilde{\phi}\ \texttt{ensures}\ \widetilde{\phi} \\
\oplus & ::= & + \mid - \mid * \mid \backslash \mid \&\& \mid \ || \\
\odot & ::= & \neq \mid = \mid < \mid > \mid \leq \mid \geq \\
s & ::= & \texttt{skip} \mid s_1\ ;\ s_2 \mid T\ x \mid x := e \mid \texttt{if}\ (e)\ \{s_1\}\ \texttt{else}\ \{s_2\} \\
 & & \mid \texttt{while}\ (e)\ \texttt{invariant}\ \widetilde{\phi}\ \{s\} \mid x.f := y \mid x := \texttt{new}\ C \mid y := z.m(\overline{x}) \\
 & & \mid y := z.m_C(\overline{x}) \mid \texttt{assert}\ \phi \mid \texttt{release}\ \phi \mid \texttt{hold}\ \phi\ \{s\} \mid \texttt{fold}\ A \mid \texttt{unfold}\ A \\
e & ::= & v \mid x \mid e \oplus e \mid e \odot e \mid e.f \\
x & ::= & \texttt{result} \mid \mathit{id} \mid \texttt{old}(\mathit{id}) \mid \texttt{this} \\
v & ::= & n \mid o \mid \texttt{null} \mid \texttt{true} \mid \texttt{false} \\
A & ::= & \alpha(\overline{e}) \mid \alpha_C(\overline{e}) \\
\circledast & ::= & \wedge \mid * \\
\phi & ::= & e \mid A \mid \texttt{acc}(e.f) \mid \phi \circledast \phi \mid (\texttt{if}\ e\ \texttt{then}\ \phi\ \texttt{else}\ \phi) \mid (\texttt{unfolding}\ A\ \texttt{in}\ \phi) \\
\widetilde{\phi} & ::= & \phi \mid ? * \phi
\end{array}
$$

2 Well-formedness

3 Aliasing

3.1 Definitions

An **object variable** is one of the following:

- a class instance variable i.e. a variable v such that $v : C$ for some class C,
- a class instance field reference i.e. a field reference $e.f$ where $e.f : C$ for some class C,
- `null` as a value such that `null` $: C$ for some class C.

Let $\mathcal{O}$ be a set of object variables. An $O \subset \mathcal{O}$ **aliases** if and only if each $o \in O$ refers to the same memory in the heap as each other, written propositionally as

$$\forall o, o' \in O : o = o' \iff \mathsf{aliases}(O)$$

While it is possible to keep track of negated aliasings (of the form $\sim \mathsf{aliases}\,\{o_\alpha\}$), this will not be needed for either aliasing tree construction or self-framing desicions. So, it will not be tracked i.e. $x \neq y$ does not contribute anything to an aliasing context.

3.2 Aliasing Context

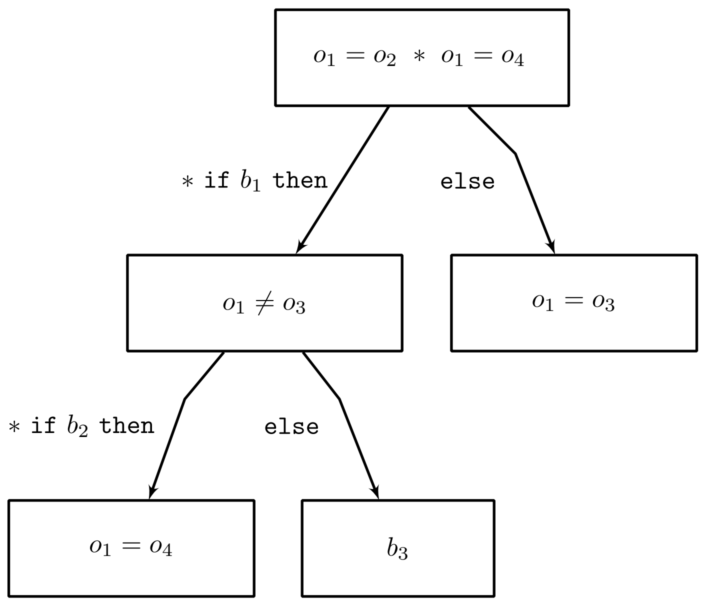
Let ϕ be a formula. The **aliasing context** $\mathcal{A}$ of ϕ is a tree of set of aliasing proposition about aliasing of object variables that appear in ϕ. $\mathcal{A}$ needs to be a tree because the conditional and unfolding sub-formulas that may appear in ϕ allow for branching aliasing contexts not expressible flatly at the top level. In the case of conditionals i.e. sub-formulas of the form `if` e `then` ϕ_1 `else` ϕ_2, two branches sprout from the original context. In the case of unfoldings i.e. sub-formulas of the form `unfolding` $\alpha_C(\overline{e})$ `in` ϕ, one branch sprouts from the original context. Each node in the tree corresponds to a set of aliasing propositions, and each branch refers to a branch of a unique conditional in ϕ. The parts of the tree are labeled in such a way that modularly allows a specified sub-formula of ϕ to be matched to the unique aliasing sub-context that corresponds to it. For example, consider the following formula:

$$\begin{aligned}
\phi := {} & (o_1 = o_2) \; * \\
& (\texttt{if}\ (b_1) \\
& \quad \texttt{then}\ (\\
& \qquad (o_1 \neq o_3) \; * \\
& \qquad (\texttt{if}\ (b_2) \\
& \qquad\quad \texttt{then}\ (o_1 = o_4) \\
& \qquad\quad \texttt{else}\ (b_3))) \\
& \quad \texttt{else}\ (o_1 = o_3)) \; * \\
& (o_1 = o_4)
\end{aligned}$$

where b_1, b_2 are arbitrary boolean expressions that do not assert aliasing propositions. ϕ has a formula-structure represented by the tree in figure **??**. The formula-structure tree for ϕ corresponds node-for-node and edge-for-edge to the aliasing context tree in figure **??**.

More generally, for ϕ a formula and ϕ' a sub-formula of ϕ, write $\mathcal{A}_\phi(\phi')$ as the **total**

Figure 1: Formula structure tree for ϕ.



aliasing context of ϕ' which includes aliasing propositions inherited from its ancestors in the aliasing context tree of ϕ. These aliasing contexts are combined via $\sqcup$ which will be defined in the next section. For example, the total aliasing context at the sub-formula $(o_1 = o_4)$ of ϕ is:

$$\mathcal{A}_\phi(o_1 = o_4) := \{\mathsf{aliased}\,\{o_1, o_2, o_4\}\}$$

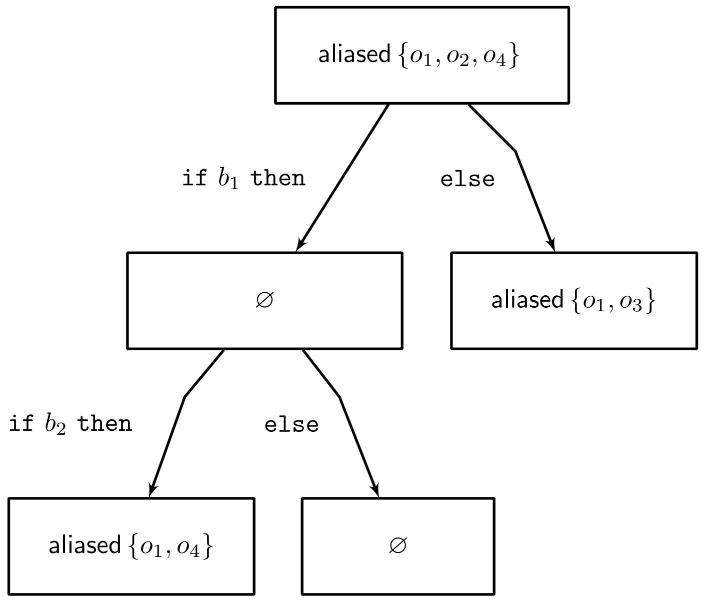
along with the fact that it has no child branches. Usually $\mathcal{A}_{\phi_{\mathsf{root}}}(\phi')$ is abbreviated to $\mathcal{A}(\phi')$ when the top level formula ϕ is implicit and ϕ' is a sub-formula of ϕ_{root}.

An aliasing context $\mathcal{A}$ may entail $\mathsf{aliased}(O)$ for some $O \subset \mathcal{O}$. Since $\mathcal{A}$ is efficiently represented as a set of propositions about sets, it may be the case that $\mathsf{aliased}(O) \notin \mathcal{A}$ yet still the previous judgement holds. For example, this is true when $\exists O' \subset \mathcal{O}$ such that $O \subset O'$ and $\mathsf{aliased}(O') \in \mathcal{A}$. So, the explicit definition for making this judgement is as follows:

$$\mathcal{A} \vdash \mathsf{aliased}(O) \iff \exists O' \subset \mathcal{O} : (O \subset O') \ \wedge \ (\mathsf{aliased}(O') \in \mathcal{A})$$

The notations $\mathsf{aliased}(O) \in \mathcal{A}$ is a little misleading because $\mathcal{A}$ is in fact a tree and not just a set. To be explicit, $\mathsf{aliased}(O) \in \mathcal{A}$ is defined to be set membership of the set of aliasing propositions in the total aliasing context at $\mathcal{A}$.

Figure 2: $\mathcal{A}(\phi)$, the aliasing context tree for ϕ.



3.3 Constructing an Aliasing Context

An aliasing context of a formula ϕ is a tree, where nodes represent local aliasing contexts and branches represent the branches of conditional sub-formulas nested in ϕ. So, an aliasing context is defined structurally as

$$\mathcal{A} \ ::= \langle A,\ \{l_\alpha : \mathcal{A}_\alpha\}\rangle$$

where A is a set of propositions about aliasing and the $l_\alpha : \mathcal{A}_\alpha$ are the nesting aliasing contexts that correspond to the branches of conditionals and unfoldings directly nested in ϕ, the l_α being labels for each child context.

Given a root formula ϕ_{root}, the aliasing context of ϕ_{root} is written $\mathcal{A}(\phi_{\mathsf{root}})$. With the root invariant, the following recursive algorithm constructs $\mathcal{A}(\phi)$ for any sub-formula of ϕ_{root}

(including $\mathcal{A}(\phi_{\mathsf{root}})$).

$$
\begin{array}{lll}
\mathcal{A}(\phi) & := & \mathsf{match}\ \phi\ \mathsf{with} \\
& & v \qquad\qquad\qquad\quad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & x \qquad\qquad\qquad\quad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & e_1\ \&\&\ e_2 \qquad\quad \mapsto\ \mathcal{A}(e_1) \sqcup \mathcal{A}(e_2) \\
& & e_1\ ||\ e_2 \qquad\qquad \mapsto\ \mathcal{A}(\texttt{if}\ e_1\ \texttt{then}\ \texttt{true}\ \texttt{else}\ e_2) \\
& & e_1 \oplus e_2 \qquad\qquad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & o_1 = o_2 \qquad\qquad \mapsto\ \langle \{\mathsf{aliases}\,\{o_1, o_2\}\}\,,\ \varnothing \rangle \\
& & e_1 \odot e_2 \qquad\qquad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & e.f \qquad\qquad\qquad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & \texttt{acc}(e.f) \qquad\quad\ \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & \phi_1 * \phi_2 \qquad\qquad \mapsto\ \mathcal{A}(\phi_1) \sqcup \mathcal{A}(\phi_2) \\
& & \phi_1 \wedge \phi_2 \qquad\qquad \mapsto\ \mathcal{A}(\phi_1) \sqcup \mathcal{A}(\phi_2) \\
& & \alpha_C(\overline{e}) \qquad\qquad \mapsto\ \langle \varnothing,\ \varnothing \rangle \\
& & \texttt{if}\ e\ \texttt{then}\ \phi_1\ \texttt{else}\ \phi_2 \ \mapsto\ \langle \varnothing,\ \{e : \mathcal{A}(e) \sqcup \mathcal{A}(\phi_1),\ \sim e : (\mathcal{A}(\sim e)) \sqcup \mathcal{A}(\phi_2)\} \rangle \\
& & \texttt{unfolding}\ \alpha_C(\overline{e})\ \texttt{in}\ \phi' \ \mapsto\ \langle \varnothing,\ \{\texttt{unfolding}(\alpha_C(\overline{e})) : \mathcal{A}(\mathsf{unfold}\ \alpha_C(\overline{e})) \sqcup \mathcal{A}(\phi')\} \rangle
\end{array}
$$

Note the following:

- $\mathcal{A}(\phi_{\mathsf{root}})$ is implicitly unioned with the discrete aliasing context $\{\{o\} : o \in \mathcal{O}\}$. This convention yields that each $o \in \mathcal{O}$ is always considered an alias of itself.
- The $\sim e$ expression in the result of the rule for $\mathcal{A}(\texttt{if}\ e\ \texttt{then}\ \phi_1\ \texttt{else}\ \phi_2)$ means to negate the boolean expression of e
- The $e_1\ ||\ e_2$ expression is translated into $\texttt{if}\ e_1\ \texttt{then}\ \texttt{true}\ \texttt{else}\ e_2$ for the purpose of aliasing. So, boolean or operations in forumals yield branching just like conditional expressions.
- The $\mathsf{unfold}\ \alpha_C(\overline{e})$ expression in the result of the rulle for $\mathcal{A}(\texttt{unfolding}\ \alpha_C(\overline{e})\ \texttt{in}\ \phi')$ is translated to a single unfolding of the body of $\alpha_C(\overline{e})$ with the arguments substituted appropriately.

As examples,

$$
\begin{aligned}
\mathcal{A}(\sim (x = y)) = \mathcal{A}(x \neq y) &= \langle \varnothing,\ \varnothing \rangle \\
\mathcal{A}(\sim (x \neq y)) = \mathcal{A}(x = y) &= \langle \{\mathsf{aliased}\,\{x, y\}\}\,,\ \varnothing \rangle
\end{aligned}
$$

Context union, $\sqcup$, and context intersection, $\sqcap$, are operations that combine aliasing contexts and are defined below.

$$
\begin{aligned}
&\langle A_1,\ \{l_\alpha : \mathcal{A}_\alpha\} \rangle \sqcup \langle A_2,\ \{l_\beta : \mathcal{A}_\beta\} \rangle := \\
&\qquad \langle \{\mathsf{aliased}\,\{o' \mid \forall o' : (A_1 \vdash \mathsf{aliased}\,\{o, o'\})\ \vee\ (A_2 \vdash \mathsf{aliased}\,\{o, o'\})\} \mid \forall o\}\,, \\
&\qquad \{l_\alpha : \mathcal{A}_\alpha\} \cup \{l_\beta : \mathcal{A}_\beta\} \rangle
\end{aligned}
$$

$$
\begin{aligned}
&\langle A_1,\ \{l_\alpha : \mathcal{A}_\alpha\} \rangle \sqcap \langle A_2,\ \{l_\beta : \mathcal{A}_\beta\} \rangle := \\
&\qquad \langle \{\mathsf{aliased}\,\{o' \mid \forall o' : (A_1 \vdash \mathsf{aliased}\,\{o, o'\})\ \wedge\ (A_2 \vdash \mathsf{aliased}\,\{o, o'\})\} \mid \forall o\}\,, \\
&\qquad \{l_\alpha : \mathcal{A}_\alpha\} \cap \{l_\beta : \mathcal{A}_\beta\} \rangle
\end{aligned}
$$

4 Framing

4.1 Definitions

For framing, a formula is considered inside a **permission context**, a set of permissions, where a **permission** π is to do one of the following:

- to reference $e.f$, written $\mathsf{accessed}(e.f)$.
- to assume $\alpha_C(\overline{e})$, written $\mathsf{assumed}(\alpha_C(\overline{e}))$. This allows the a single unrolling of $\alpha_C(\overline{e})$. Explicitly, an instance of $\mathsf{assumed}(\alpha_C(\overline{e}))$ in a set of permissions Π may be expanded into $\Pi \cup \mathsf{granted}(\dots)$ where $\dots$ is replaced with a single unrolling of the body of $\alpha_C(\overline{e})$ with the arguments substituted appropriately[1].

Let ϕ be a formula. ϕ may **require** a permission π. For example, the formula $e.f = 1$ requires $\mathsf{accessed}(e.f)$, because it references $e.f$. The set of all permissions that ϕ requires is called the **requirements** of ϕ. ϕ may also **grant** a permission π. For example, the formula $\mathsf{acc}(e.f)$ grants the permission $\mathsf{accessed}(e.f)$.

Altogether, ϕ is **framed** by a set of permissions Π if all permissions required by ϕ are either in Π or granted by ϕ. The proposition that Π frames ϕ is written

$$\Pi \vDash_I \phi$$

Of course, ϕ may grant some of the permissions it requires but not all. The set of permissions that ϕ requires but does not grant is called the **footprint** of ϕ. The footprint of ϕ is written

$$\lfloor \phi \rfloor$$

Finally, a ϕ is called **self-framing** if and only if for any set of permissions Π, $\Pi \vDash_I \phi$. The proposition that ϕ is self-framing is written

$$\vdash_{\mathsf{frm}I} \phi$$

Note that $\vdash_{\mathsf{frm}I} \phi \iff \varnothing \vDash_I \phi$, in other words ϕ is self-framing if and only if it grants all of the permissions it requires. Or in other words still, $\lfloor \phi \rfloor = \varnothing$.

[1] As demonstrated by this description, $\mathsf{assumed}$ predicates are really just a useful shorthand and not a fundamentally new type of permission. The only kind fundamental kind of permission is $\mathsf{accessed}$.

4.2 Deciding Framing

Deciding $\Pi \vDash_I \phi$ must take into account the requirements, granteds, and aliases contained in Π and the sub-formulas of ϕ. The following recursive algorithm decides $\Pi \vDash_I \phi_{root}$, where $\mathcal{A}$ is implicitly assumed to be the top-level aliasing context (where the top-level in this context is the level that ϕ_{root} exists at in the program).

$$
\begin{array}{lcll}
\Pi \vDash_I \phi & \iff & \textsf{match } \phi \textsf{ with} & \\
 & & v & \mapsto \top \\
 & & x & \mapsto \top \\
 & & e_1 \oplus e_2 & \mapsto \Pi \vDash_I e_1, e_2 \\
 & & e_1 \odot e_2 & \mapsto \Pi \vDash_I e_1, e_2 \\
 & & e.f & \mapsto (\Pi \vDash_I e) \ \wedge \ (\Pi \vdash \textsf{accessed}_\phi(e.f)) \\
 & & \texttt{acc}(e.f) & \mapsto (\Pi \vDash_I e) \\
 & & \phi_1 \circledast \phi_2 & \mapsto (\Pi \cup \textsf{granted}(\phi_2) \vDash_I \phi_1) \ \wedge \\
 & & & \quad\ (\Pi \cup \textsf{granted}(\phi_1) \vDash_I \phi_2) \\
 & & \alpha_C(e_1, \dots, e_k) & \mapsto \Pi \vDash_I e_1, \dots, e_2 \\
 & & \texttt{if } e \texttt{ then } \phi_1 \texttt{ else } \phi_2 & \mapsto \Pi \vDash_I e, \phi_1, \phi_2 \\
 & & \texttt{unfolding } \alpha_C(\overline{e}) \texttt{ in } \phi' & \mapsto (\Pi \vDash_I \alpha_C(\overline{e})) \ \wedge \ (\Pi \vdash \textsf{assumed}_\phi(\alpha_C(\overline{e}))) \ \wedge \ (\Pi \vDash_I \phi') \\
 & & & \\
\textsf{granted}(\phi) & := & \textsf{match } \phi \textsf{ with} & \\
 & & e & \mapsto \varnothing \\
 & & \texttt{acc}(e.f) & \mapsto \{\textsf{accessed}(e.f)\} \\
 & & \phi_1 \circledast \phi_2 & \mapsto \textsf{granted}(\phi_1) \cup \textsf{granted}(\phi_2) \\
 & & \alpha_C(\overline{e}) & \mapsto \{\textsf{assumed}(\alpha_C(\overline{e}))\} \\
 & & \texttt{if } e \texttt{ then } \phi_1 \texttt{ else } \phi_2 & \mapsto \textsf{granted}(\phi_1) \cap \textsf{granted}(\phi_2) \\
 & & \texttt{unfolding } \alpha_C(\overline{e}) \texttt{ in } \phi' & \mapsto \textsf{granted}(\phi')
\end{array}
$$

Where $\textsf{accessed}_\phi$ and $\textsf{assumed}_\phi$ indicate the respective propositions considered within the total alias context (including inherited aliasing contexts). More explicitly,

$$
\begin{array}{rl}
\Pi \vdash \textsf{accessed}_\phi(o.f) & \iff \exists \textsf{accessed}(o'.f) \in \Pi : \mathcal{A}(\phi) \vdash \textsf{aliased}\ \{o, o'\} \\
\Pi \vdash \textsf{assumed}_\phi(\alpha_C(e_1, \dots, e_k)) & \iff \exists \textsf{assumed}(\alpha_C(e'_1, \dots, e'_k)) \in \Pi : \forall i : \mathcal{A}(\phi) \vdash \textsf{aliased}\ \{e_i, e'_i\}
\end{array}
$$

4.3 Examples

In the following examples, assume that the considered formulas are well-formed.

Example 1

Define

$$\phi_{\mathsf{root}} := x = y * \mathsf{acc}(x.f) * \mathsf{acc}(y.f).$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \{\mathsf{aliased}\,\{x, y\}\}\,,\ \varnothing \rangle.$$

And so,

$$\begin{aligned}
\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\
&\iff \varnothing \vDash_I x = y * \mathsf{acc}(x.f) * \mathsf{acc}(y.f) \\
&\iff (\mathsf{granted}(\mathsf{acc}(x.f) * \mathsf{acc}(y.f)) \vDash_I x = y) \wedge \\
&\qquad (\mathsf{granted}(x = y * \mathsf{acc}(y.f)) \vDash_I \mathsf{acc}(x.f)) \wedge \\
&\qquad (\mathsf{granted}(x = y * \mathsf{acc}(x.f)) \vDash_I \mathsf{acc}(y.f)) \\
&\iff \top \wedge \\
&\qquad (\mathsf{granted}(x = y * \mathsf{acc}(y.f)) \vDash_I x) \wedge \\
&\qquad (\mathsf{granted}(x = y * \mathsf{acc}(x.f)) \vDash_I y) \\
&\iff \top \wedge \top \wedge \top \\
&\iff \top
\end{aligned}$$

Example 2

Define

$$\phi_{\mathsf{root}} := \mathtt{acc}(x.f) \ * \ (\mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathit{true}\ \mathtt{else}\ \mathtt{acc}(x.f))$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{x.f = 1 : \langle \varnothing,\ \varnothing \rangle,\ x.f \neq 1 : \langle \varnothing,\ \varnothing \rangle\} \rangle$$

And so,

$$\begin{aligned}
\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\
&\iff \varnothing \vDash_I \mathtt{acc}(x.f) \ * \ (\mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathit{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \\
&\iff (\mathsf{granted}(\mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \vDash_I \mathtt{acc}(x.f)) \wedge \\
&\qquad (\mathsf{granted}(\mathtt{acc}(x.f)) \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \\
&\iff (\mathsf{granted}(\mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \vDash_I x) \wedge \\
&\qquad (\mathsf{granted}(\mathtt{acc}(x.f)) \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \\
&\iff \top \ \wedge \ (\mathsf{granted}(\mathtt{acc}(x.f)) \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \\
&\iff \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ \mathtt{true}\ \mathtt{else}\ \mathtt{acc}(x.f)) \\
&\iff \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I x.f = 1) \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I \mathtt{true}) \wedge \\
&\qquad (\{\mathsf{accessed}(x.f)\} \vDash_I \mathtt{acc}(x.f) \\
&\iff \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{\phi_{\mathsf{root}}}(x.f)) \wedge \\
&\qquad \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I x) \\
&\iff \top \ \wedge \ \top \ \wedge \ \top \ \wedge \ \top \\
&\iff \top
\end{aligned}$$

Example 3

Define

$$\phi_{\mathsf{root}} := \mathsf{acc}(x.f) \;*\; x = y \;*\; y.f = 1$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \{\mathsf{aliased}\,\{x, y\}\}\,,\; \varnothing \rangle$$

And so,

$$\begin{aligned}
\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\
&\iff \varnothing \vDash_I \mathsf{acc}(x.f) \;*\; x = y \;*\; y.f = 1 \\
&\iff (\mathsf{granted}(x = y * y.f = 1) \vDash_I \mathsf{acc}(x.f)) \wedge \\
&\qquad (\mathsf{granted}(\mathsf{acc}(x.f) * y.f = 1) \vDash_I x = y) \wedge \\
&\qquad (\mathsf{granted}(\mathsf{acc}(x.f) * x = y) \vDash_I y.f = 1) \\
&\iff (\mathsf{granted}(x = y * y.f = 1) \vDash_I x) \wedge \\
&\qquad (\mathsf{granted}(\mathsf{acc}(x.f) * y.f = 1) \vDash_I x, y) \wedge \\
&\qquad (\mathsf{granted}(\mathsf{acc}(x.f) * x = y) \vDash_I y.f) \\
&\iff \top \;\wedge\; \top \wedge\; (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{\phi_{\mathsf{root}}}(y.f)) \\
&\iff \top \;\wedge\; \top \wedge\; \top \\
&\iff \top
\end{aligned}$$

Example 4

Define

$$
\begin{array}{l}
\texttt{class}\ \mathsf{List}\ \{\\
\quad \texttt{int}\ \mathit{head};\\
\quad \mathsf{List}\ \mathit{tail};\\
\quad \texttt{predicate}\ \mathsf{List}(l) =\\
\qquad l \neq \texttt{null}\ *\ \texttt{acc}(l.\mathit{head})\ *\ \texttt{acc}(l.\mathit{tail})\ *\\
\qquad \texttt{if}\ l.\mathit{tail} = \texttt{null}\ \texttt{then}\ \texttt{true}\ \texttt{else}\ \mathsf{List}(l.\mathit{tail});\\
\}
\end{array}
$$

$$\phi_{\mathsf{root}} := \mathsf{List}(l)\ *\ \texttt{unfolding}\ \mathsf{List}(l)\ \texttt{in}\ l.\mathit{head} = 1$$

Then

$$
\begin{aligned}
\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{&\texttt{unfolding}(\mathsf{List}(l)) :\\
&\{\langle \varnothing,\ \{t.\mathit{tail} = \texttt{null} : \langle \{\mathsf{aliased}\,\{t.\mathit{tail}, \texttt{null}\}\},\ \varnothing\rangle,\ t.\mathit{tail} \neq \texttt{null} : \langle \varnothing,\ \varnothing\rangle\}\rangle\}\}\rangle
\end{aligned}
$$

And so,

$$
\begin{aligned}
\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}}\\
&\iff \varnothing \vDash_I \mathsf{List}(l)\ *\ \texttt{unfolding}\ \mathsf{List}(l)\ \texttt{in}\ l.\mathit{head} = 1\\
&\iff (\mathsf{granted}(\texttt{unfolding}\ \mathsf{List}(l)\ \texttt{in}\ l.\mathit{head} = 1) \vDash_I \mathsf{List}(l))\ \wedge\\
&\qquad (\mathsf{granted}(\mathsf{List}(l)) \vDash_I \texttt{unfolding}\ \mathsf{List}(l)\ \texttt{in}\ l.\mathit{head} = 1)\\
&\iff \top\ \wedge\ (\{\mathsf{assumed}(\mathsf{List}(l))\} \vDash_I \texttt{unfolding}\ \mathsf{List}(l)\ \texttt{in}\ l.\mathit{head} = 1)\\
&\iff \top\ \wedge\ (\mathsf{granted}(l \neq \texttt{null}\ *\ \texttt{acc}(l.\mathit{head})\ *\ \texttt{acc}(l.\mathit{tail})\ *\\
&\qquad\quad \texttt{if}\ l.\mathit{tail} = \texttt{null}\ \texttt{then}\ \texttt{true}\ \texttt{else}\ \mathsf{List}(l.\mathit{tail})) \vDash_I\\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{(expansion of } \mathsf{assumed}(\mathsf{List}(l)))\\
&\qquad l.\mathit{head} = 1)\\
&\iff \top\ \wedge\ (\{\mathsf{accessed}(l.\mathit{head}), \mathsf{accessed}(l, \mathit{tail})\}) \vDash_I l.\mathit{head} = 1)\\
&\iff \top\ \wedge\ (\{\mathsf{accessed}(l.\mathit{head}), \mathsf{accessed}(l, \mathit{tail})\}) \vdash \mathsf{accessed}_{\phi_{\mathsf{root}}}(l.\mathit{head}))\\
&\iff \top\ \wedge\ \top\\
&\iff \top
\end{aligned}
$$

Example 5

Define

$$\phi_{\mathsf{root}} := \texttt{if } x = \texttt{null} \texttt{ then } \texttt{true} \texttt{ else } (\texttt{acc}(x.f) * x.f = 1)$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{x = \texttt{null} : \langle \{\mathsf{aliased}\,\{x, \texttt{null}\}\}\,,\ \varnothing \rangle, x \neq \texttt{null} : \langle \varnothing,\ \varnothing \rangle \} \rangle$$

And so,

$$\begin{aligned}
\vdash_{\mathsf{frm}\,I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\
&\iff \varnothing \vDash_I \texttt{if } x = \texttt{null} \texttt{ then } \texttt{true} \texttt{ else } (\texttt{acc}(x.f) * x.f = 1) \\
&\iff (\varnothing \vDash_I x = \texttt{null}) \ \wedge\ (\varnothing \vDash_I \texttt{true}) \ \wedge\ (\varnothing \vDash_I \texttt{acc}(x.f) * x.f = 1) \\
&\iff \top \ \wedge\ \top \ \wedge (\mathsf{granted}(x.f = 1) \vDash_I \texttt{acc}(x.f)) \ \wedge (\mathsf{granted}(\texttt{acc}(x.f)) \vDash_I x.f = 1) \\
&\iff \top \ \wedge\ \top \ \wedge (\varnothing \vDash_I \texttt{acc}(x.f)) \ \wedge (\{\mathsf{accessed}(x.f)\} \vDash_I x.f = 1) \\
&\iff \top \ \wedge\ \top \ \wedge (\varnothing \vDash_I x) \ \wedge \\
&\qquad (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{x.f}(x.f)) \ \wedge (\{\mathsf{accessed}(x.f)\} \vDash_I 1) \\
&\iff \top \wedge \top \wedge \top \wedge \top \wedge \top \\
&\iff \top
\end{aligned}$$

Example 6

Use the definition of List from example 4. Define

$$\begin{aligned}\phi_{\mathsf{root}} &:= \mathtt{acc}(x.f)\ *\ \phi_1\ * \phi_2\\ \phi_1 &:= \mathtt{if}\ x.f = 1\ \mathtt{then}\ x = y\ \mathtt{else}\ \mathtt{true}\\ \phi_2 &:= \mathtt{if}\ x.f = 1\ \mathtt{then}\ y.f = 1\ \mathtt{else}\ \mathtt{true}\end{aligned}$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{x.f = 1 : \langle \{\mathsf{aliased}\,\{x, y\}\}\,,\ \varnothing\rangle,\ x.f \neq 1 : \langle \varnothing,\ \varnothing\rangle\}\rangle$$

And so,

$$\begin{aligned}\vdash_{\mathsf{frm}I}\ \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \mathtt{acc}(x.f)\ *\ \phi_1\ * \phi_2\\ &\iff (\mathsf{granted}(\phi_1 * \phi_2) \vDash_I \mathtt{acc}(x.f))\ \land\ (\mathsf{granted}(\mathtt{acc}(x.f) * \phi_2) \vDash_I \phi_1) \land\\ &\qquad (\mathsf{granted}(\mathtt{acc}(x.f) * \phi_1) \vDash_I \phi_2)\\ &\iff (\mathsf{granted}(\phi_1 * \phi_2) \vDash_I x) \land\\ &\qquad (\{\mathsf{accessed}(x.f)\} \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ x = y\ \mathtt{else}\ \mathtt{true}) \land\\ &\qquad (\{\mathsf{accessed}(x.f)\} \vDash_I \mathtt{if}\ x.f = 1\ \mathtt{then}\ y.f = 1\ \mathtt{else}\ \mathtt{true})\\ &\iff \top\ \land\ (\{\mathsf{accessed}(x.f)\} \vDash_I (x.f = 1), (x = y), (\mathtt{true})) \land\\ &\qquad (\{\mathsf{accessed}(x.f)\} \vDash_I (x.f = 1), (y.f = 1), (\mathtt{true}))\\ &\iff \top\ \land\ (\{\mathsf{accessed}(x.f)\} \vDash_I x.f)\ \land\ (\{\mathsf{accessed}(x.f)\} \vDash_I x.f) \land\\ &\qquad (\{\mathsf{accessed}(x.f)\} \vDash_I y.f)\\ &\iff \top\ \land\ (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{\phi_{\mathsf{root}}}(x.f)) \land\\ &\qquad (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{y.f=1}(y.f))\\ &\iff \top\ \land\ \top\ \land\ \top \qquad (\star)\\ &\iff \top\end{aligned}$$

($\star$): $\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{y.f=1}(y.f) \iff \top$ since $\mathcal{A}(y.f = 1) \vdash \mathsf{aliased}\,\{x, y\}$ because $\mathcal{A}(y.f = 1)$ and $\mathcal{A}(x = y)$ are combined into a single branch of $\mathcal{A}(\phi_{\mathsf{root}})$, as they have the same conditions.

Example 7

Define

$$\phi_{\mathsf{root}} := \texttt{if } x = y \texttt{ then } \texttt{acc}(x.f) \texttt{ else } x.f = 2$$

Then

$$\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{x = y : \langle \{\mathsf{aliased}\,\{x, y\}\}\,,\ \varnothing\rangle,\ x \neq y : \langle \varnothing,\ \varnothing\rangle\}\rangle$$

And so,

$$\begin{aligned}
\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\
&\iff \varnothing \vDash_I \texttt{if } x = y \texttt{ then } \texttt{acc}(x.f) \texttt{ else } x.f = 2 \\
&\iff \varnothing \vDash_I (x = y), (\texttt{acc}(x.f)), (x.f = 2) \\
&\iff \top \ \wedge\ (\varnothing \vDash_I x) \ \wedge\ (\varnothing \vDash_I x.f) \\
&\iff \top \ \wedge\ \top \ \wedge\ (\varnothing \vdash \mathsf{accessed}_{x.f=2}(x.f)) \\
&\iff \top \ \wedge\ \top \ \wedge\ \bot \\
&\iff \bot
\end{aligned}$$

Example 8

Define

$$\texttt{predicate}\ \mathsf{aliasChoice}(x,y,z) := \ x = y \mid\mid x = z$$

$$\begin{aligned}\phi_{\mathsf{root}} &:= \texttt{acc}(x.f) \ * \ \mathsf{aliasChoice}(x,y,z) \ * \ \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1 \\ \phi_1 &:= y.f = 1 \mid\mid z.f = 1\end{aligned}$$

Then

$$\begin{aligned}\mathcal{A}(\phi_{\mathsf{root}}) = \langle \varnothing,\ \{&\texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) : \\ &\{x = y : \langle \{\mathsf{aliased}\,\{x,y\}\}\,,\ \varnothing\rangle, \\ &\ x \neq y : \langle \{\mathsf{aliased}\,\{x,z\}\}\,,\ \varnothing\rangle, \\ &\ y.f = 1 : \langle \varnothing,\ \varnothing\rangle, \\ &\ y.f \neq 1 : \langle \varnothing,\ \varnothing\rangle\}\}\rangle\}\end{aligned}$$

Note that the $x = y \mid\mid x = z$ in the body of aliasChoice is translated to $\texttt{if}\ x = y\ \texttt{then}\ \texttt{true}\ \texttt{else}\ x = z$ when construction $\mathcal{A}(\phi_1)$. And so,

$$\begin{aligned}\vdash_{\mathsf{frm}I} \phi_{\mathsf{root}} &\iff \varnothing \vDash_I \phi_{\mathsf{root}} \\ &\iff \varnothing \vDash_I \texttt{acc}(x.f) \ * \ \mathsf{aliasChoice}(x,y,z) \ * \ \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1 \\ &\iff (\mathsf{granted}(\mathsf{aliasChoice}(x,y,z) \ * \ \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1) \vDash_I \texttt{acc}(x.f)) \ \wedge \\ &\qquad (\mathsf{granted}(\texttt{acc}(x.f) \ * \ \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1) \vDash_I \mathsf{aliasChoice}(x,y,z)) \ \wedge \\ &\qquad (\mathsf{granted}(\texttt{acc}(x.f) \ * \ \mathsf{aliasChoice}(x,y,z)) \vDash_I \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1) \\ &\iff \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I \mathsf{aliasChoice}(x,y,z)) \ \wedge \\ &\qquad (\{\mathsf{accessed}(x.f), \mathsf{assumed}(\mathsf{aliasChoice}(x,y,z))\} \vDash_I \texttt{unfolding}(\mathsf{aliasChoice}(x,y,z)) \ \texttt{in} \ \phi_1) \\ &\iff \top \ \wedge \ \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I y.f = 1 \mid\mid z.f = 1) \\ &\iff \top \ \wedge \ \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I y.f) \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vDash_I z.f) \\ &\iff \top \ \wedge \ \top \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{y.f=1}(y.f)) \ \wedge \ (\{\mathsf{accessed}(x.f)\} \vdash \mathsf{accessed}_{z.f=1}(z.f)) \quad (\star) \\ &\iff \top \wedge \top \wedge \bot \wedge \bot \\ &\iff \bot\end{aligned}$$

($\star$): The acccessed to $y.f, z.f$ are not framed because it is statically undetermined which branch of $x = y \mid\mid x = z$ will be taken. The case could arise that $x = z$ and then when checking the condition $y.f = 1$ there is not access to $y.f$. The idea of the original formula can be correctly captured in one of the following revisions:

$$\begin{aligned}\phi'_{\mathsf{root}} &:= \texttt{acc}(x.f) \ * \ (x = y \mid\mid x = z) \ * \ \texttt{if}\ x = y\ \texttt{then}\ y.f = 1\ \texttt{else}\ z.f = 1 \\ \phi'_{\mathsf{root}} &:= \texttt{acc}(x.f) \ * \ \texttt{if}\ x = y\ \texttt{then}\ y.f = 1\ \texttt{else}\ (\texttt{if}\ x = z\ \texttt{then}\ z.f = 1\ \texttt{else}\ \texttt{false})\end{aligned}$$

For example. the $z.f = 1$ will be framed because the aliasing context of the $x \neq y$ branch of $(x = y \mid\mid x = z)$ will be combined with the aliasing context of the $x \neq y$ branch of $(\texttt{if}\ x = y\ \texttt{then}\ y.f = 1\ \texttt{else}\ z.f = 1)$, yielding $\mathsf{aliased}\,\{x,z\}$ in $z.f = 1$. The similar case holds for the $x = y$ branches combining to allow the aliasing to frame $y.f = 1$.

5 Satisfiability

6 Implication

7 Weakest Predonditions

7.1 Concrete Weakest Liberal Precondition (WLP) Rules

$$\mathsf{WLP} : \textsc{Statement} \times \textsc{FrmSatFormula} \to \textsc{FrmSatFormula}$$

$$\mathsf{WLP}(s, \phi) := \text{ math } s \text{ with}$$

$$\left| \begin{array}{lll}
\texttt{skip} & \mapsto & \phi \\
s_1\texttt{;}\ s_2 & \mapsto & \mathsf{WLP}(s_1,\ \mathsf{WLP}(s_2, \phi)) \\
T\ x & \mapsto & \phi\ \textit{(assert that } x \textit{ does not appear in } \phi\textit{)} \\
x\ \texttt{:=}\ e & \mapsto & \lfloor e \rfloor\ \wedge\ [e/x]\phi \\
x\ \texttt{:=}\ \texttt{new}\ C & \mapsto & [\mathsf{new}(C)/x]\phi \\
x.f\ \texttt{:=}\ y & \mapsto & \lfloor x.f \rfloor\ \wedge\ [y/x.f]\phi \\
y\ \texttt{:=}\ z.m_C(\overline{e}) & \mapsto & \lfloor \overline{e} \rfloor\ \wedge\ z\ \texttt{!=}\ \texttt{null}\ \wedge \\
 & & [z/\texttt{this},\ \overline{e/x}]\mathsf{pre}(m_C) * \\
 & & \mathsf{handleMethodPermissions}(m_C, \phi) \\
\texttt{if}\ (e)\ \{s_{\text{the}}\}\ \texttt{else}\ \{s_{\text{els}}\} & \mapsto & \texttt{if}\ (e)\ \texttt{then}\ \mathsf{WLP}(s_{\text{the}}, \phi)\ \texttt{else}\ \mathsf{WLP}(s_{\text{els}}, \phi) \\
\texttt{while}\ (e)\ \texttt{invariant}\ \phi_{\text{inv}}\ \{s_{\text{bod}}\} & \mapsto & \lfloor e \rfloor\ \wedge\ \phi_{\text{inv}}\ \wedge \\
 & & \texttt{if}\ (e)\ \texttt{then}\ \mathsf{WLP}(s_{\text{bod}}, \phi_{\text{inv}})\ \texttt{else}\ \phi \\
\texttt{assert}\ \phi_{\text{ass}} & \mapsto & \lfloor \phi_{\text{ass}} \rfloor\ \wedge\ \phi_{\text{ass}}\ \wedge\ \phi \\
\texttt{hold}\ \phi_{\text{hol}}\ \{s_{\text{bod}}\} & \mapsto & \text{(unimplemented)} \\
\texttt{release}\ \phi_{\text{rel}} & \mapsto & \text{(unimplemented)} \\
\texttt{unfold}\ \alpha_C(\overline{e}) & \mapsto & \lfloor \overline{e} \rfloor\ \wedge\ [\mathsf{unfolded}(\alpha_C(\overline{e}))/\alpha_C(\overline{e}), \\
 & & \ \phi'/\texttt{unfolding}\ \alpha_C(\overline{e})\ \texttt{in}\ \phi']\phi \\
\texttt{fold}\ \alpha_C(\overline{e}) & \mapsto & \lfloor \overline{e} \rfloor\ \wedge\ [\alpha_C(\overline{e})/\mathsf{unfolded}(\alpha_C(\overline{e}))]\phi
\end{array} \right.$$

Since WLP takes a framed, satisfiable formula and yields a framed, satisfiable formula, there is an implicit check that asserts these properties before and after WLP is computed.

7.1.1 Utility Functions

The implementations of the functions in this section can be made much more efficient than the naive definition here in mathematical notation. For example, calculating the footprint of expressions and formulas can avoid redundancy by not generating permission-subformulas that are already satisfied. This can be implemented as implicit in $\wedge$ by a wrapper $\wedge_{\text{wrap}}$ operation in some way similar to this:

$$\phi \ \wedge_{\text{wrap}} \ \phi' := \begin{cases} \phi & \text{if } \phi \implies \phi' \\ \phi \wedge \phi' & \text{otherwise} \end{cases}$$

The following functions are useful abbreviations for common constructs.

$$\begin{array}{lcl}
\mathsf{new}(C) & := & \text{an object that is a new instance of class } C, \\
 & & \text{where all fields are assigned to their default values} \\
\mathsf{unfolded}(\alpha_C(\overline{e})) & := & [\overline{e/x}]\mathsf{body}(\alpha_C) \\
\mathsf{pre}(z.m_C(\overline{e})) & := & [z/\texttt{this},\ \overline{e/x}]\mathsf{pre}(m_C) \\
\mathsf{pre}(m_C) & := & \text{the static-contract pre-condition of } m_C \\
\mathsf{post}(z.m_C(\overline{e})) & := & [z/\texttt{this},\ \overline{e/\texttt{old}(x)}]\mathsf{post}(m_C) \\
\mathsf{post}(m_C) & := & \text{the static-contract post-condition of } m_C \\
\mathsf{body}(\alpha_C) & := & \text{the body formula of } \alpha_C
\end{array}$$

The footprint function, $\lfloor\cdot\rfloor$, generates a formula containing all the permissions necessary to frame its argument. With efficient implementations of a wrapped $\wedge$, this can result in the smallest such formula.

$$\begin{array}{lcl}
\lfloor e \rfloor & := & \text{match } e \text{ with} \\
 & & \left| \begin{array}{lcl} e.f & \mapsto & \lfloor e' \rfloor \ \wedge \ e' \ \texttt{!= null} \ \wedge \ \texttt{acc}(e'.f) \\ e_1 \oplus e_2 & \mapsto & \lfloor e_1 \rfloor \ \wedge \ \lfloor e_2 \rfloor \\ e_1 \odot e_2 & \mapsto & \lfloor e_1 \rfloor \ \wedge \ \lfloor e_2 \rfloor \\ e & \mapsto & \texttt{true} \end{array} \right. \\
\lfloor \overline{e} \rfloor & := & \bigwedge \lfloor e \rfloor \\
\lfloor \phi \rfloor & := & \bigwedge \{ \lfloor e \rfloor : e \text{ appears in } \phi \} \ \wedge \\
 & & \bigwedge \{ \alpha_C(\overline{e}) : \texttt{unfolding } \alpha_C(\overline{e}) \texttt{ in } \phi' \text{ appears in } \phi \}
\end{array}$$

The $\mathsf{handleMethodPermissions}$ helper function assists in generating the WLP for a method call. It yields a formula that asserts the following:

(a) For all $\texttt{acc}(e.f)$ that appear in $\lfloor \phi \rfloor$, $\texttt{acc}(e.f)$ does not appear in $\mathsf{pre}(z.m_C(\overline{e})) \setminus \mathsf{post}(z.m_C(\overline{e}))$. This asserts that ϕ does not use access to a part of the heap whose access was required to call $z.m_C(\overline{e})$ and then not ensured after the call.

(b) For all $e.f$ that appear in ϕ (not including than those appearing in $\texttt{acc}(e.f)$), $\texttt{acc}(e.f)$ does not appear in $\mathsf{pre}(z.m_C(\overline{e})) \wedge \mathsf{post}(z.m_C(\overline{e}))$. This asserts that ϕ does not rely on the value of a part of the heap that was accessed by $z.m_C(\overline{e})$.

(c) For all $\texttt{unfolding}\ \alpha_C(\overline{e})\ \texttt{in}\ \phi'$ that appear in ϕ, $\alpha_C(\overline{e})$ does not appear in $\mathsf{pre}(z.m_C(\overline{e})) \wedge \mathsf{post}(z.m_C(\overline{e}))$. This asserst that ϕ does not rely on the truth of any predicates that were used in $z.m_C(\overline{e})$, as predicates may contain heap accesses.

$$\begin{aligned}
&\mathsf{handleMethodPermissions}(z.m_C(\overline{e}), \phi) := \\
&\qquad \bigwedge \{(\mathsf{pre}(z.m_C(\overline{e})) \setminus \mathsf{post}(z.m_C(\overline{e}))) * \mathtt{acc}(e.f) : \lfloor \phi \rfloor \implies \mathtt{acc}(e.f)\} \ \wedge \\
&\qquad \bigwedge \{(\mathsf{pre}(z.m_C(\overline{e})) \wedge \mathsf{post}(z.m_C(\overline{e}))) * \mathtt{acc}(e.f) : e.f \text{ appears in } \phi\} \ \wedge \\
&\qquad \bigwedge \{(\mathsf{pre}(z.m_C(\overline{e})) \wedge \mathsf{post}(z.m_C(\overline{e}))) * \alpha_C(\overline{e}) : \mathtt{unfolding}\ \alpha_C(\overline{e})\ \mathtt{in} \text{ appears in } \phi\}
\end{aligned}$$

TODO: define without function, $\phi \setminus e$.