C Equivalent Networks

The application of equivalent networks is a useful method for the solution of many tasks in acoustics. The method is applicable if the sound field at any value of the x co-ordinate in the “direction of propagation” has the same lateral distribution. Plane waves are just a special case.

The conception of *end corrections*, or, equivalently, *oscillating mass*, extends the range of application even to a space with contractions. The method of equivalent networks is based on the analogies (*electro-acoustic analogies*) with electrical circuits.

C.1 Fundamentals of Equivalent Networks

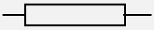
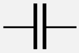
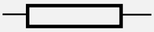
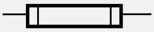
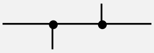
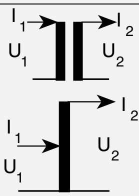
► *See also:* Mechel, Vol. II, Ch. 2 (1995)

Electromagnetic quantities and their relations (A is a cross-sectional area, or Ampere):

Table 1 Electromagnetic quantities and their relations

Quantities					
electric			magnetic		
Quantity	Relation	Dimension	Quantity	Relation	Dimension
Voltage	U	Volt, V	Current	I	Ampere, A
El. field strength	E	V/m	Magn. field strength	H	A/m
El. induction	$D = q/A$	$A \cdot s/m$	Magn. induction	$B = \Phi_m/A$	$V \cdot s/m$
Charge, flow	$q = \int I\,dt = D \cdot A = \Phi_e$	$A \cdot s$	Flow	$\Phi_m = \int U\,dt = B \cdot A$	$V \cdot s$
Voltage	$U = \int E\,ds$	V	Current	$I = \oint H \cdot ds$	A
Current	$I = dq/dt$	A	Voltage	$U = d\Phi_m/dt$	V
Capacity	$C = q/U = \Phi_e/U$	$A \cdot s/V$	Inductivity	$L = U/(dI/dt) = \Phi_m/(dI/dt)$	$V \cdot s/A$

Table 2 & 3 Passive electrical and mechanical circuit components

Element	Quantity	Symbol	Letter	Definition
Resistor	Resistance		R	$R = \frac{\Delta U}{I}$
Capacitor	Capacity		C	$C = \frac{\int I \cdot dt}{\Delta U}$
Coil	Inductivity		L	$L = \frac{\Delta U}{dI/dt}$
Complex Impedance			Z	$Z = \frac{\Delta U}{I}$
Complex Admittance			G	$G = \frac{I}{\Delta U} = \frac{1}{Z}$
Connection				$\Delta U = 0; \Delta I = 0$
Transformer			$u = \frac{w_2}{w_1}$	$u = \frac{U_2}{U_1} = \frac{I_1}{I_2}$

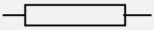
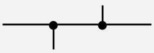
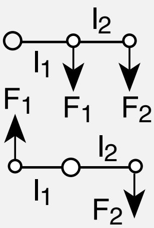
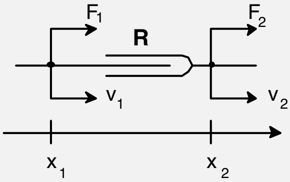
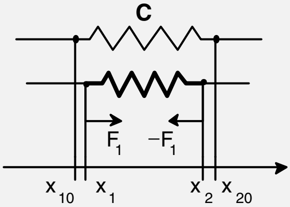
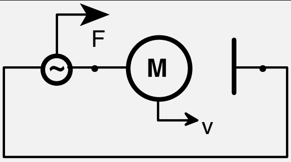
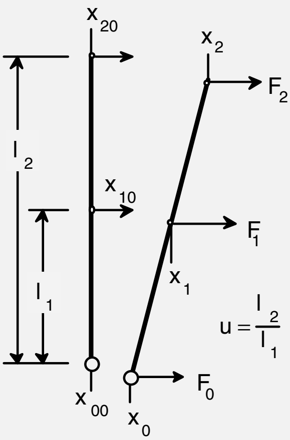
Element	Quantity	Symbol	Letter	Definition
Resistor	Friction		R	$R = \frac{\Delta F}{v}$
Spring	Compliance		C	$C = \frac{\int \Delta v dt}{\Delta F}$
Mass	Inertance		M	$M = \frac{\Delta F}{dv/dt}$
Complex Impedance			Z	$Z = \frac{\Delta F}{v}$
Complex Admittance			G	$G = \frac{v}{\Delta F} = \frac{1}{Z}$
Rigid Connection				$\Delta F = 0; \Delta v = 0$
Lever			$u = \frac{l_2}{l_1}$	$u = \frac{F_2}{F_1} = \frac{v_1}{v_2}$

Table 4 Defining relations for passive mechanical circuit elements

Element	Co-ordinates	Relations
Friction		$F = F_1 - F_2$ $v = v_1 - v_2$ $F = R \cdot v$
Spring		$\xi_1 = (x_1 - x_{10})$ $\xi_2 = (x_2 - x_{20})$ $\xi = \xi_1 - \xi_2$ $F = F_1 = \frac{1}{j\omega C} \cdot v$
Mass		$F = j\omega M \cdot v$
Lever		$\xi_{00} = x_0 - x_{00}$ $\xi_{10} = x_1 - x_{10}$ $\xi_{20} = x_2 - x_{20}$ $\xi_1 = \xi_{10} - \xi_{00}$ $\xi_2 = \xi_{20} - \xi_{00}$ $F_0 = -(F_1 + F_2)$ $F_2 = (l_1/l_2) \cdot F_1$ $\xi_2 = (l_2/l_1) \cdot \xi_1$

F = force;
v = velocity;
x = position;
ξ = deformation;
R = friction factor;
M = mass;
C = compliance;
l = length

The velocity of a resistance is the relative velocity of both ends of the resistance.

The velocity of a spring is the relative velocity of both ends of the spring.

The velocity of a mass is its velocity relative to the point on which the force source is supported.

The force acting on a resistance or a spring is the force difference at both ends of the element.

Boundary conditions:
Node theorem: The sum of all forces acting on an immaterial node point is zero.
Mesh theorem: The sum of the velocities in a closed mesh is zero.

A spring is supposed to have no mass; a mass is supposed to be incompressible.

A hard (or rigid) termination with v = 0 corresponds in electrical circuits
- to an open termination in the UK analogy (see below),
- to a short-circuited termination in the Uv analogy;

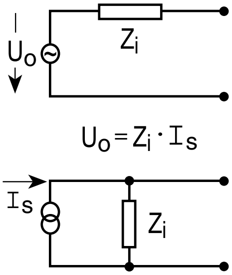
a soft (or pressure release) termination with F = 0 (or p = 0) corresponds in electrical circuits
- to a short-circuited termination in the UK analogy,
- to an open termination in the Uv analogy.

Rules:
- There is no force difference across a spring.
- There is no velocity difference across a mass.
- The second pole of a mass is at the point on which the driving force is supported.

Relations for levers with a leverage u: $\begin{pmatrix} v_2 \\ F_2 \end{pmatrix} = \begin{pmatrix} u & 0 \\ 0 & \frac{1}{u} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ F_1 \end{pmatrix}$.

Sources:

Helmholtz theorem:



Z_i	is the internal source impedance;
U_o	is the open-circuit source voltage;
I_s	is the short-circuit source current

Reciprocal networks:
A reciprocal network is composed of elements Z_r which follow from the elements Z of the original network by the rule $Z \rightarrow Z_r = r^2/Z^{*)}$ with the *reciprocal invariant* r. With suitably normalised impedances, one can take $r = 1$. Voltage sources change to current sources, and vice versa.

In both networks voltage transfer ratios ↔ current transfer ratios correspond to each other and have the same frequency response curves.

An advantage of the reciprocal network possibly is its easier conception and realisation.

The shape of the reciprocal network changes: a mesh changes to a node; a node changes to a mesh (see below for a more precise rule).

Table 5 Reciprocal electrical elements

← Reciprocal →			
Resistance	R	r^2/R	Reciprocal resistance
Inductivity	L	L/r^2	Capacity
Capacity	C	r^2C	Inductivity
Impedance	Z	r^2/Z	Admittance
Admittance	G	r^2G	Impedance
Voltage source	U, Z_i	U/r, r^2/Z_i	Current source
Current source	I, Z_i	rI, r^2/Z_i	Voltage source

Rule for the construction of a reciprocal network of networks, which can be drawn in one plane:

- Draw a point into every mesh of the original network and one point outside the network.
- Connect all pairs of points with each other by lines which cross circuit elements.
- Replace the crossed elements with their reciprocal elements.
- If necessary, redraw the reciprocal network in a better form.

*) See Preface to the 2nd edition.

Below is an example illustrating the application of this rule.

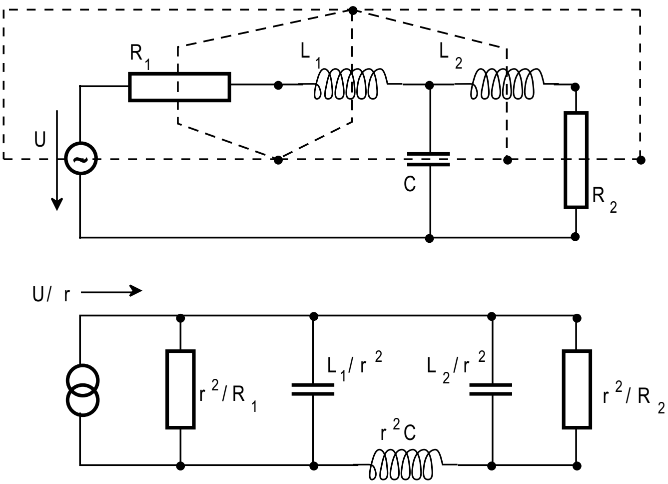


Figure 1 Example of reciprocal networks

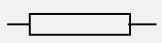
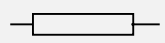
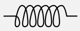
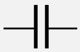
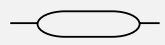
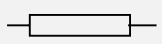
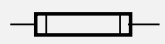
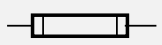
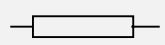
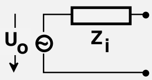
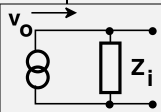
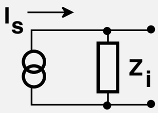
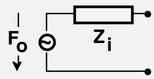
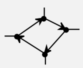
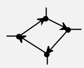
Electro-acoustic UK analogy:

Table 6 Corresponding elements in the UK-analogy

electrical				mechanical			
Voltage	**U**			Force	**F**		
Current	**I**			Velocity	**v**		
Resistance	$\mathbf{R_e}$		$U = R_e \cdot I$	Resistance	$\mathbf{R_m}$		$F = R_m \cdot v$
Coil	**L**		$U = j\omega L \cdot I$	Mass	**M**		$F = j\omega M \cdot v$
Condensator	$\mathbf{C_e}$		$U = \frac{1}{j\omega C_e} \cdot I$	Spring	$\mathbf{c_m}$		$F = \frac{1}{j\omega C_m \cdot v}$
Impedance	$\mathbf{Z_e}$		$U = Z_e \cdot I$	Impedance	$\mathbf{Z_m}$		$F = Z_m \cdot v$
Admittance	$\mathbf{G_e}$		$U = \frac{1}{G_e} \cdot I$	Admittance	$\mathbf{G_m}$		$F = \frac{1}{G_m} \cdot v$
Voltage source	U_o Z_i			Fource source	F_o Z_i		
Current source	I_s Z_i			Velocity source	v_s Z_i		
Node			$\sum I = 0$	Mesh			$\sum v = 0$
Mesh			$\sum U = 0$	Node			$\sum F = 0$

Electro-acoustic Uv analogy:

Table 7 Corresponding elements in the Uv analogy

Electrical				Mechanical			
Voltage	**U**			Velocity	**v**		
Current	**I**			Force	**F**		
Resistance	R_e		$U = R_e \cdot I$	Resistance	$1/R_m$		$v = \frac{1}{R_m} \cdot F$
Coil	**L**		$U = j\omega L \cdot I$	Spring	C_m		$v = j\omega C_m \cdot F$
Condensator	C_e		$U = \frac{1}{j\omega C_e} \cdot I$	Mass	**M**		$v = \frac{1}{j\omega M \cdot F}$
Impedance	Z_e		$U = Z_e \cdot I$	Admittance	G_m		$v = G_m \cdot F$
Admittance	G_e		$U = \frac{1}{G_e} \cdot I$	Impedance	Z_m		$v = \frac{1}{Z_m} \cdot F$
Voltage source				Velocity source			
Current source				Force source			
Node			$\sum I = 0$	Node			$\sum F = 0$
Mesh			$\sum U = 0$	Mesh			$\sum v = 0$

Networks in the UK and Uv analogy, respectively, are reciprocal to each other.

C.2 Distributed Network Elements

► *See also:* Mechel, Vol. II, Ch. 2 (1995)

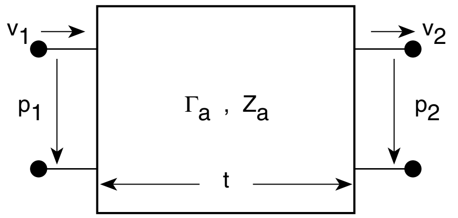
One distinguishes between "lumped" elements, as in *Sect. C.1*, with no sound propagation within one element, and "distributed" elements with internal sound propagation. Distributed elements are homogeneous, i.e. without change in the cross section and/or material. They are introduced into network analysis as four poles, whereas lumped elements are two poles.

Four poles themselves can be represented as equivalent networks, either as T networks or as Π networks. The four-pole representation is used for duct sections and/or layers with internal axial sound propagation. In the following formulas t is the duct section length or layer thickness t.

The axial propagation constant Γ_a is either the characteristic propagation constant of the medium in the duct or layer for a plane wave propagating in the axial direction, or the axial component for oblique propagation. Correspondingly, Z_a is the characteristic impedance of the medium, or the axial component. If the medium in the duct or layer is air, then $\Gamma_a = j \cdot k_0$; $Z_a = Z_0$.

Four-pole equations:

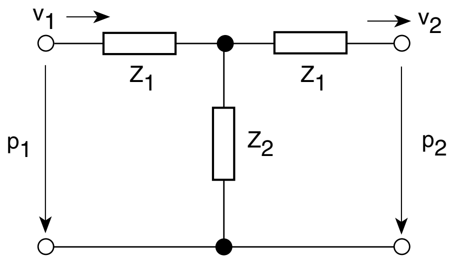
$$\begin{aligned} p_1 &= \cosh(\Gamma_a t) \cdot p_2 + Z_a \sinh(\Gamma_a t) \cdot v_2 \\ Z_a \cdot v_1 &= \sinh(\Gamma_a t) \cdot p_2 + Z_a \cosh(\Gamma_a t) \cdot v_2 \end{aligned} \tag{1}$$



Equivalent T-circuit impedances:

$$\begin{aligned} Z_1 &= Z_a \cdot \coth(\Gamma_a t) - Z_2 \\ &= Z_a \frac{\cosh(\Gamma_a t) - 1}{\sinh(\Gamma_a t)} \end{aligned} \tag{2}$$

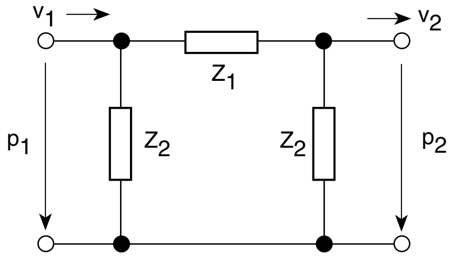
$$Z_2 = \frac{Z_a}{\sinh(\Gamma_a t)} \tag{3}$$



Equivalent Π-circuit impedances:

$$\begin{aligned} Z_1 &= Z_a \cdot \sinh(\Gamma_a t) \\ Z_2 &= \frac{Z_a \sinh(\Gamma_a t)}{\cosh(\Gamma_a t) - 1} \end{aligned} \tag{4}$$

$$\frac{1}{Z_2} = \frac{1}{Z_a \tanh(\Gamma_a t)} - \frac{1}{Z_1} \tag{5}$$



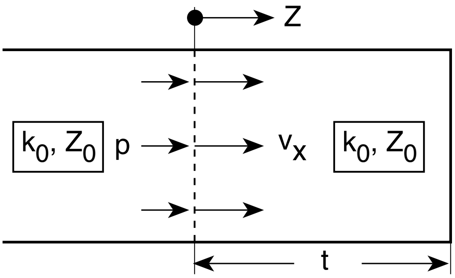
Some simple systems with distributed-network elements are displayed below.

Tube section with hard termination:

$$\frac{Z}{Z_0} = \frac{p}{v_x} = -j \ \cot(k_0 t),$$

$$Z \xrightarrow[t \ll \lambda_0]{} \frac{1}{j\omega} \frac{\rho_0 c_0^2}{t} = \frac{1}{j\omega C}, \tag{6}$$

$$C = \frac{t}{\rho_0 c_0^2}.$$



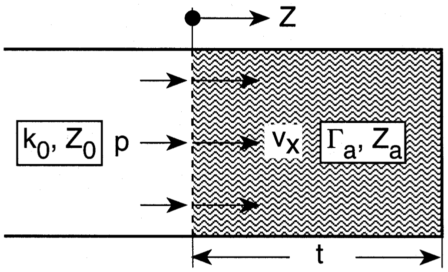
Remarks:

- For $t \ll \lambda_0$ a spring-type reactance;
- First resonance at $k_0 t = \pi/2$; $t = \lambda_0/4$;
- For $\pi/2 < k_0 t < \pi$ a mass-type reactance.

Tube section with hard termination, filled with porous material:

$$\frac{Z}{Z_0} = \frac{p}{v_x} = \frac{Z_a}{Z_0} \ \coth(\Gamma_a t),$$

$$Z \xrightarrow[t \ll \lambda_a]{} \frac{Z_a}{\Gamma_a t} \approx \frac{1}{j\omega} \frac{\rho_0 c_0^2}{\kappa \sigma t}. \tag{7}$$



Remarks:

- κ = adiabatic exponent of air;
- σ = porosity of porous material.

Tube section with open termination:

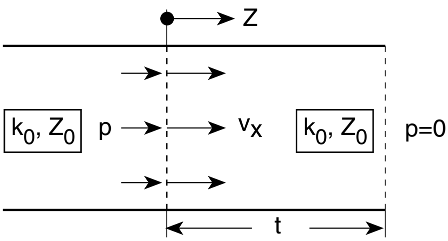
$$\frac{Z}{Z_0} = \frac{p}{v_x} = j\ \tan(k_0 t),$$

$$Z \xrightarrow[t \ll \lambda_0]{} j\,\omega\rho_0\,t = j\,\omega\,M, \tag{8}$$

$$M = \rho_0\,t.$$

Remarks:

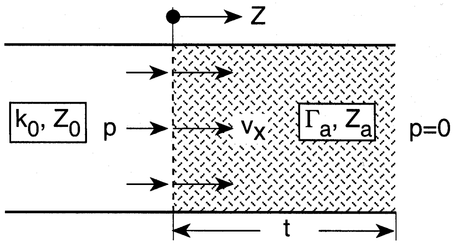
- The assumption p = 0 at the orifice is an approximation for narrow tubes;
- Without load of radiation impedance !;
- $t \ll \lambda_0$ mass-type reactance;
- $\pi/2 < k_0 t < \pi$ spring-type reactance.



Tube section with open termination, filled with porous material:

$$\frac{Z}{Z_0} = \frac{p}{v_x} = \frac{Z_a}{Z_0}\ \tanh(\Gamma_a t)$$

$$Z \xrightarrow[t \ll \lambda_a]{} \Gamma_a Z_a\,t \approx \frac{j}{\kappa\sigma} Z_0\,k_0 t \cdot (\Gamma_a/k_0)^2\,; \qquad \xrightarrow[E \ll 1]{} \frac{\Xi\,t}{\sigma} \quad ; \quad \xrightarrow[E \gg 1]{} \frac{j\,\omega\,\rho_0\,t}{\sigma} \tag{9}$$

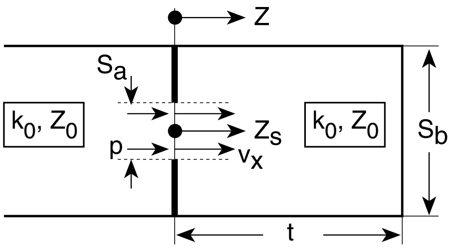


Remarks:

- The assumption p = 0 at the orifice is an approximation for narrow tubes;
- Without load of radiation impedance !;
- $E = \rho_0 f/\Xi$ absorber parameter;
- Ξ = flow resistivity of porous material;
- σ = porosity of porous material;
- λ_a = wavelength in absorber material;
- $t \ll \lambda_a$ and $E \ll 1$: $Z \approx$ resistance,
  and $E > 1$: $Z \approx$ mass reactance.

Tube terminated with Helmholtz resonator with thin resonator plate:

$$
\begin{aligned}
Z &= \frac{\langle p\rangle_{S_b}}{\langle v_x\rangle_{S_b}} \approx -j\,Z_0\ \cot(k_0 t) \xrightarrow[t\ll\lambda_0]{} \frac{1}{j\,\omega}\,\frac{\rho_0 c_0^2}{t}, \\
Z_s &= \frac{\langle p\rangle_{S_a}}{\langle v_x\rangle_{S_a}} \approx \sigma \cdot Z \xrightarrow[t\ll\lambda_0]{} \frac{1}{j\,\omega}\,\rho_0 c_0^2\,\frac{S_a}{V}.
\end{aligned}
\tag{10}
$$

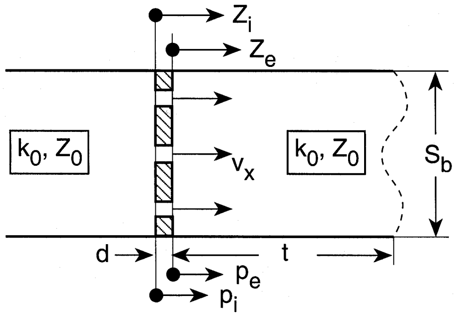


Remarks:

- End corrections of orifices neglected !;
- $S_a \cdot S_b \ll \lambda^2{}_0$;
- Z = "homogeneous" impedance;
- Z_s = interior orifice impedance;
- $s = S_a/S_b$ = resonator plate porosity;
- $V = t \cdot S_b$ = resonator volume.

Perforated plate in a tube:

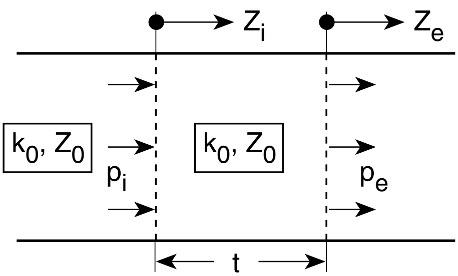
$$Z_i = Z_t + Z_e = \frac{1}{\sigma} Z_s + Z_e, \quad {}^{*)}$$
$$Z_t = \frac{1}{\sigma} Z_s. \tag{11}$$



Remarks:
- $d \ll \lambda_0$;
- Ze = "homogeneous" load impedance;
- Zi = "homogeneous" input impedance;
- Zt = "homogeneous" partition impedance of plate;
- Zs = partition impedance of perforations.

A layer of air (transformation of impedances by a layer):

$$\frac{Z_i}{Z_0} = \frac{j \tan(k_0 t) + Z_e/Z_0}{1 + j\, Z_e/Z_0 \cdot \tan(k_0 t)},$$
$$Z_i \xrightarrow[t \ll \lambda_0]{} \frac{j\,\omega\rho_0 + Z_e}{1 + Z_e \cdot j \dfrac{t}{\rho_0 c_0^2}}. \tag{12}$$



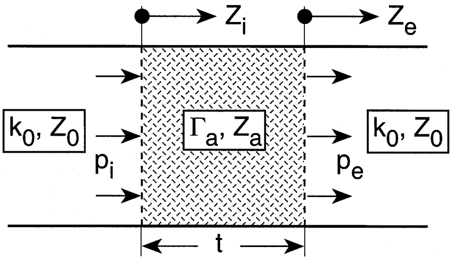
*) see Preface to the 2nd edition.

Remarks:

- Zi = “homogeneous” input impedance;
- Ze = “homogeneous” load impedance.

A layer of porous material (transformation of impedances by a layer):

$$\frac{Z_i}{Z_0} = \frac{\tanh(\Gamma_a t) + Z_e/Z_0}{1 + j\, Z_e/Z_0 \cdot \tanh(\Gamma_a t)}. \tag{13}$$



Remarks:

- Z_i = “homogeneous” input impedance;
- Z_e = “homogeneous” load impedance.

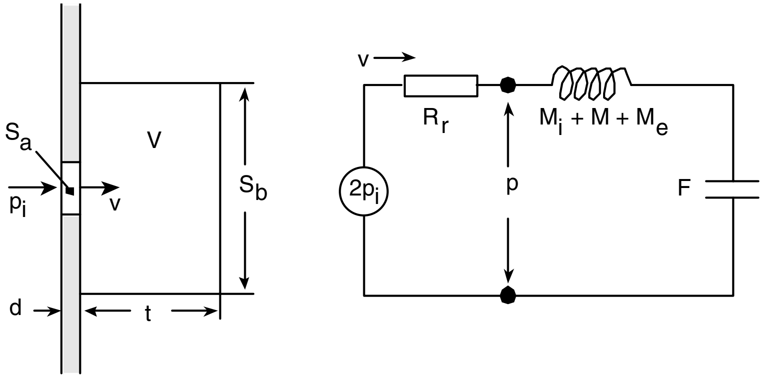
C.3 Elements with Constrictions

► *See also:* Mechel, Vol. II, Ch. 2 (1995)

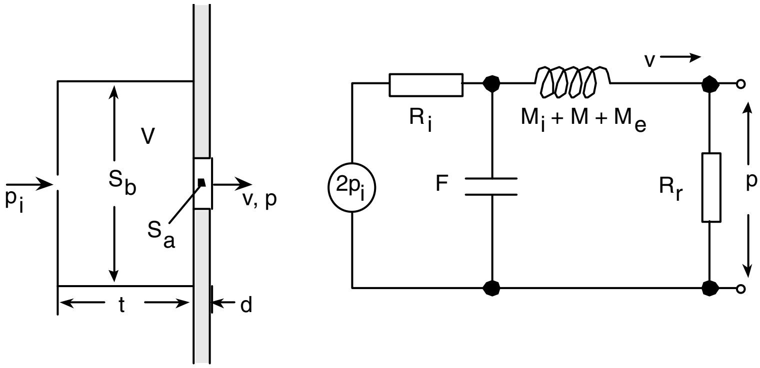
Equivalent networks can be used for sound in multilayer absorbers or in ducts without constrictions because the lateral sound distribution functions can be divided out. Nevertheless, constrictions can also be represented with equivalent networks if their lateral dimensions and the lateral dimensions in front of and behind the constriction are small compared to the wavelength or, more precisely, if no higher modes can propagate in the wide cross sections. Then the constrictions produce only near fields. These can be represented by *equivalent oscillating masses* M_i at the orifice on the side of sound incidence and M_e at the orifice of sound exit.

The equivalent oscillating mass is proportional to the *end correction*.

The following examples show two Helmholtz resonators, one excited by an incident wave p_i on the resonator, the other excited by a sound pressure p_i at the back side of the resonator volume.



R_r	represents the radiation resistance of the orifice near the incidence;
M_i	is the equivalent oscillating mass on the outer side;
	mass);
M_e	is the equivalent oscillating mass on the interior side
	mass)



R_i	represents the interior source resistance;
M_i	is the equivalent oscillating mass on the outer side;
M_e	is the equivalent oscillating mass on the interior side

C.4 Superposition of Multiple Sources in a Network

Helmholtz's theorem of superposition for multiple sources:

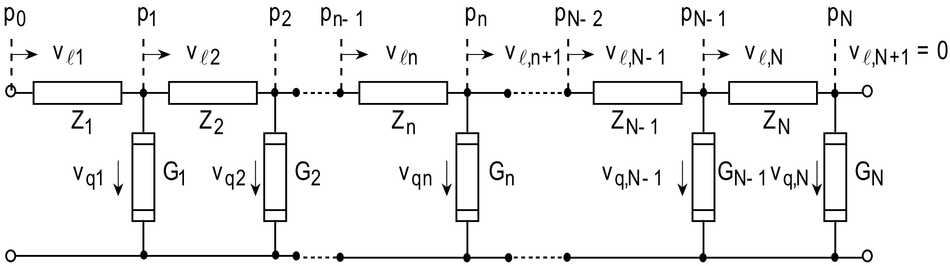
If a network is excited by more than one voltage source (current sources) with the same frequency, the state of the network with common excitation is a superposition of states in which only one source is active, and the network terminals at the other sources are short-circuited (open).

C.5 Chain Circuit

▸ *See also:* Mechel, Vol. II, Ch. 2 (1995)

A chain circuit is a useful representation of multilayer absorbers (see *Sect. D.4*).

A chain network consists of longitudinal impedances Z_n and lateral admittance G_n. Its sound pressures p_n in the nodes and velocities v_{ln} in the longitudinal elements, as well as v_{qn} in the transversal elements, can be evaluated by iteration.



If the network is open (as shown), i.e. $v_{l,N+1} = 0$, one begins with an assumed value $p_N = 1$.

The backward recursion is

$$\begin{aligned} v_{q,n} &= p_n \cdot G_n, \\ v_{\ell,n} &= v_{q,n} + v_{\ell,n+1}, \\ p_{n-1} &= p_n + v_{\ell,n} \cdot Z_n. \end{aligned} \tag{1}$$

One iterates over $n = N, N-1, \ldots, 1$. The last result is p_0. All field quantities are proportional to p_N. To replace this by p_0 as the reference pressure, divide all (saved) quantities by the value of p_0. If parameters Z_n, G_n are used and normalised with Z_0, the velocities are returned as $Z_0\, v_n$.

If the real network is terminated with a load impedance Z_{load}, add $1/Z_{load}$ to G_N, so the network to be evaluated is open again.

The input impedance of the network is $$Z = \frac{p_0}{v_{\ell,1}} = Z_1 + \frac{p_1}{v_{\ell,1}}. \tag{2}$$

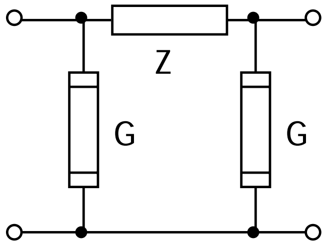
The impedance of the part of the network behind the node n is

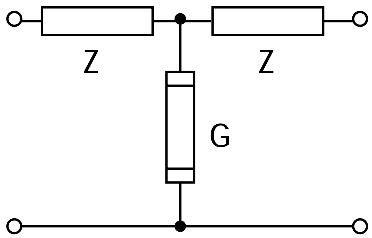
$$Y_n = \frac{p_n}{v_{\ell,n+1}};\ n = 1, 2, \ldots, N-1. \tag{3}$$

The load impedance at the node n is

$$X_n = \frac{p_n}{v_{\ell,n}} = \frac{p_n}{v_{q,n} + v_{\ell,n+1}};\ n = 1, 2, \ldots, N. \tag{4}$$

Suitable representations of a layer of material with thickness t, propagation constant Γ_a and wave impedance Z_a of the material are





$$Z = Z_a \cdot \sinh(\Gamma_a t);\ G = \frac{\cosh(\Gamma_a t) - 1}{Z_a \cdot \sinh(\Gamma_a t)}; \qquad G = \frac{\sinh(\Gamma_a t)}{Z_a};\ Z = \frac{Z_a}{\sinh(\Gamma_a t)}(\cosh(\Gamma_a t) - 1).$$

C.6 Partition Impedance of Orifices

The method of equivalent networks was originally designed for a sequence of layers without constrictions. The impedance at a layer boundary is defined by the ratio of sound pressure p and normal velocity v, both averaged over the boundary surface S, $Z = \langle p\rangle_S/\langle v\rangle_S$. A layer with constrictions (e.g. a plate with the neck of a resonator) can be included in the equivalent network scheme if an *orifice partition impedance* (or simply: *orifice impedance*) Z_M is added to the orifice of the constriction. This is a *partition impedance* of the type $Z_p = \langle \Delta p\rangle_S/\langle v\rangle_S$ with $\Delta p = (p_{front} - p_{back})$ the sound pressure drop across the plane of the orifice and v the particle velocity through the orifice (in the direction front→back).

Assume the area of the orifice is s (e.g. s = the cross-section area of a neck) with the porosity $\sigma = s/S$ (e.g. S = cross-section area of a single resonator). Let $Z_s = \langle p\rangle_s//\langle v\rangle_s$ be the impedance at the neck orifice (inside the neck) and $\underline{Z}_K = \langle p\rangle_S/\langle v\rangle_S$ the impedance in the "chamber" (e.g. resonator volume) in the orifice plane (inside the chamber). The underlining will serve to remind the reader that $\underline{Z}_K$ is a homogenised impedance. Then, from the conditions of continuity of volume flow and average pressure across the orifice plane follows for the orifice partition impedance

$$Z_M = Z_s - \sigma\underline{Z}_K = \langle p\rangle_s/\langle v\rangle_s - \sigma\underline{Z}_K. \tag{1}$$

For its evaluation a field analysis must be made of the sound field in front of and behind the orifice for a plane wave incident on the orifice. Due to reciprocity it is sufficient to consider the case of sound incidence from the side of the narrow section (neck). The orifice partition impedance will be the same for sound incidence in the opposite direction.

For orifices radiating into free space or ending in an empty chamber, the orifice partition impedance is purely reactive with the sign of a mass reactance. Mostly it is represented in the literature by the *end correction* $\Delta\ell/a$ (with $s = \pi a^2$) by the general interrelation

$$Z_M = j\,k_0 a \cdot \Delta\ell/a, \qquad (2)$$

i.e. $\Delta\ell/a$ represents the imaginary part of Z_M. A number of end corrections are given in *Sect. F.2, End corrections*, and in *Ch. H, "Compound Absorbers"*, of this book. However, the orifice partition impedance becomes complex in some important configurations, and its dependence on the parameters of the configuration is not simple enough for a formulation of Z_M as a regression polynomial of these parameters. Therefore, this section derives and presents the orifice partition impedance Z_M for a number of configurations as explicit formulas in short, tabular form. The derivation of an explicit formula (i.e. no solutions of systems of equations) requires in some configurations the approximate assumption that only plane waves propagate in the narrow section (neck). At higher frequencies, for which higher neck modes are popagating, the conception of the equivalent networks no longer works.

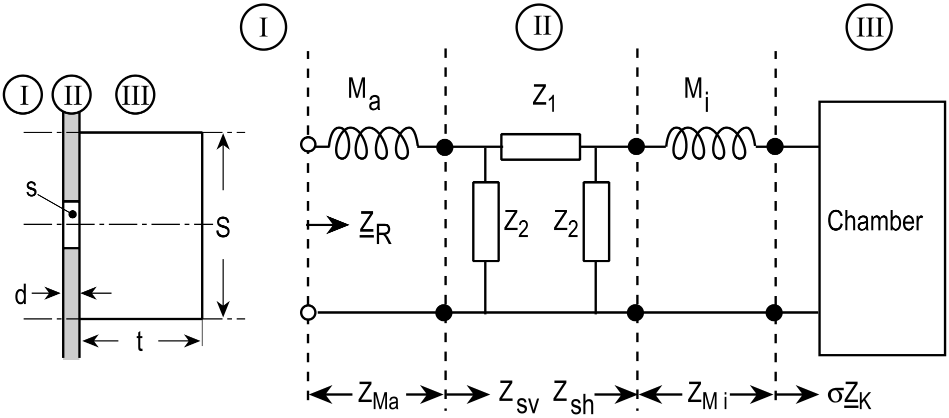
An important advantage of the orifice partition impedance Z_M lies in the fact that it can be combined with the partition impedance Z_F of (e.g.) a poro-elastic foil in the orifice by simple addition: $Z = Z_M + Z_F$.

Below:
- The use of Z_M is demonstrated in the chain of formulas of an equivalent network for a Helmholtz resonator (as an example).
- Then the typical procedure for deriving Z_M is illustrated for this configuration.
- Finally, the formulas for other configurations are collected in a table.

All impedances and admittances are supposed to be normalised with Z_0.

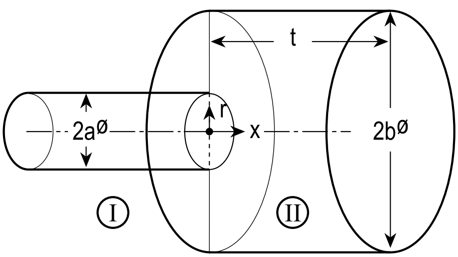
Use of Z_M in the equivalent network for a Helmholtz resonator

The sound field is subdivided into three zones:
(I) in front of the resonator
(II) in the neck
(III) in the chamber (resonator volume)



$\underline{Z}_R$	homogenised front-side impedance of the resonator array;
Z_{Ma}, Z_{Mi}	outer and interior orifice partition impedances;
Z_{sv}	entrance impedance of the neck (II);
Z_{sh}	exit impedance of the neck (II);
$\underline{Z}_K$	homogenised entrance impedance of the resonator chamber (III);
Z_1, Z_2	impedances of the Π-fourpole which represents the neck channel;
$\sigma = s/S$	porosity of the neck plate;
$\underline{G}_v = \sigma \cdot G_v$	homogenised front-side admittance of resonator; (3)
$G_v = 1/Z_v$	G_v = front-side neck entrance admittance; (4) Z_v = front-side neck entrance impedance;
$Z_v = Z_{Ma} + Z_{sv}$	Z_{Ma} = front-side orifice partition impedance; (5) Z_{sv} = front-side neck entrance impedance;
$Z_{sv} = Z_i \dfrac{Z_i \cdot \tanh(\Gamma_i d) + Z_{sh}}{Z_i + Z_{sh} \cdot \tanh(\Gamma_i d)}$	front-side neck entrance impedance for a narrow neck, Γ_i, Z_i characteristic capillary values; (6a)
$Z_{sv} = \dfrac{j \cdot \tan(k_0 d) + Z_{sh}}{1 + j \cdot Z_{sh} \cdot \tan(k_0 d)}$	front-side neck entrance impedance for a medium-wide neck (plane waves only); (6b)
$Z_{sh} = Z_{Mi} + Z_K$	exit impedance of neck; (7) Z_{Mi} = interior orifice partition impedance; $Z_K = \sigma \cdot \underline{Z}_K$ load impedance of neck; $\underline{Z}_K$ = homogenised chamber entrance impedance;
$Z_K = \sigma \cdot \underline{Z}_k = -j \dfrac{\sigma}{\tan(k_0 t)}$	load impedance for an empty chamber of depth t; (8)
$= Z_a \dfrac{\sigma}{\tanh(\Gamma_a t)}$	chamber filled with porous material, char. values Γ_a, Z_a; (9)
$= \sigma$	anechoic empty channel; (10)
$= \sigma \cdot Z_a$	channel filled with porous material; (11)
$Z_{Ma} \to Z_{Ma} + Z_{Fa}$ $Z_{Mi} \to Z_{Mi} + Z_{Fi}$	if orifice(s) is (are) covered with poro-elastic foil(s). (12)

Field analysis for a circular neck orifice in a circular, empty chamber (example)



Field formulation in neck (I) for an incident plane wave with arbitrary amplitude B_0 (e.g. $B_0 = 1$) and higher radial modes with amplitudes C_m reflected at the orifice at $x = 0$:

$$\begin{aligned} p_I(x,r) &= B_0 e^{-j k_0 x} + \sum_{m\geq 0} C_m e^{+\kappa_m x} J_0(\varepsilon_m r) \quad ; \quad \kappa_m^2 = \varepsilon_m^2 - k_0^2 \\ Z_0 v_{Ix}(x,r) &= \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} + j\sum_{m\geq 0} C_m \frac{\kappa_m}{k_0} e^{+\kappa_m x} J_0(\varepsilon_m r) \end{aligned} \tag{13}$$

The radial eigenvalues follow from the condition of zero radial particle velocity of each mode at the neck wall at $r = a$ and therefore are solutions of $J_1(\varepsilon_m a) = 0$; $m = 0, 1, 2, \ldots$; with $\varepsilon_0 a = 0$, thus $\kappa_0 = jk_0$.

Field formulation in chamber (II) with unknown mode amplitudes D_n:

$$\begin{aligned} p_{II}(x,r) &= \sum_{n\geq 0} D_n \cosh\left(\gamma_n(x-t)\right) J_0(\eta_n r) \quad ; \quad \gamma_n^2 = \eta_n^2 - k_0^2 \\ Z_0 v_{IIx}(x,r) &= \frac{j}{k_0}\frac{\partial p}{\partial x} = j\sum_{n\geq 0} D_n \frac{\gamma_n}{k_0} \sinh\left(\gamma_n(x-t)\right) J_0(\eta_n r) \end{aligned} \tag{14}$$

The radial-mode eigenvalues $\eta_n b$ are solutions of $J_1(\eta_n b) = 0$ (with $n = 0, 1, \ldots$); $\eta_0 b = 0$, i.e. $\eta_0 = 0$ und $\gamma_0 = jk_0$.

The modes in each zone are orthogonal to each other and have *mode norms* $N_m^{(I)}$, $N_n^{(II)}$:

$$\begin{aligned} \int_0^a J_0(\varepsilon_m r)\, J_0(\varepsilon_\mu r)\, r\,dr &= \begin{cases} 0 \ ; & m \neq \mu \\ N_m^{(I)} = \dfrac{a^2}{2} J_0^2(\varepsilon_m a) \xrightarrow[m=0]{} \dfrac{a^2}{2} \ ; & m = \mu \end{cases} \\ \int_0^b J_0(\eta_n r)\, J_0(\eta_\nu r)\, r\,dr &= \begin{cases} 0 \ ; & n \neq \nu \\ N_n^{(II)} = \dfrac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \dfrac{b^2}{2} \ ; & n = \nu \end{cases} \end{aligned} \tag{15}$$

Mode-coupling integrals over the orifice area $s = \pi a^2$ between modes from both zones are

$$\begin{aligned} S_{m,n} &= \int_0^a J_0(\varepsilon_m r)\, J_0(\eta_n r)\, r\,dr = a^2 \frac{(\varepsilon_m a)\, J_1(\varepsilon_m a)\, J_0(\eta_n a) - (\eta_n a)\, J_0(\varepsilon_m a)\, J_1(\eta_n a)}{(\varepsilon_m a)^2 - (\eta_n a)^2} \\ &= a^2 \frac{(\eta_n a)\, J_0(\varepsilon_m a)\, J_1(\eta_n a)}{(\eta_n a)^2 - (\varepsilon_m a)^2} \xrightarrow[n=0, m\neq 0]{} 0 \ ; \\ &\xrightarrow[m=0]{} a^2 \frac{J_1(\eta_n a)}{(\eta_n a)} \ ; \xrightarrow[m,n=0,0]{} \frac{a^2}{2} \end{aligned} \tag{16}$$

Integrals for average values over the cross sections vanish for higher modes and in the averge over s are special cases of the mode-coupling integrals $S_{m,0}$ for $n = 0$. Therefore, in the evaluation of the average values $< p >_s$ and $< Z_0 v_x >_s$ for the impedances, only the fundamental modes will contribute. This fact supports the approximation with only plane waves in the neck.

$$\begin{aligned}
&\int_0^a J_0(\varepsilon_m r)\cdot r\,dr = a^2\cdot\frac{J_1(\varepsilon_m a)}{\varepsilon_m a} = S_{m,0} = \begin{cases} 0 & ; \quad m>0 \\ a^2/2 & ; \quad m=0 \end{cases} \\
&\int_0^b J_0(\eta_n r)\cdot r\,dr = b^2\cdot\frac{J_1(\eta_n b)}{\eta_n b} = \begin{cases} 0 & ; \quad n>0 \\ b^2/2 & ; \quad n=0 \end{cases}
\end{aligned} \tag{17}$$

Matching the axial particle velocity at $x = 0$ requires

$$v_{IIx}(0,r) \overset{!}{=} \begin{cases} 0 & ; \quad r>a \\ v_{Ix}(0,r) & ; \quad r\le a \end{cases} \qquad \text{with the field formulations} \tag{18a}$$

$$-j\sum_{n\ge0} D_n\frac{\gamma_n}{k_0}\,\sinh(\gamma_n t)\;J_0(\eta_n r) \overset{!}{=} \begin{cases} 0 & ; \quad r>a \\ B_0 + j\sum\limits_{m\ge0} C_m\dfrac{\kappa_m}{k_0}J_0(\varepsilon_m r) & ; \quad r\le a. \end{cases} \tag{18b}$$

The range is $0 \le r \le b$. Therefore, multiply and integrate over this range, i.e.

left-hand side: $\int_0^b \ldots\cdot J_0(\eta_\nu r)\,r\,dr$; right-hand side: $\int_0^a \ldots\cdot J_0(\eta_\nu r)\,r\,dr$; $\nu = 0, 1, 2, \ldots$,

giving

$$-j\,D_\nu\frac{\gamma_\nu}{k_0}\,\sinh(\gamma_\nu t)\;N_\nu^{(II)} = B_0\,S_{0,\nu} + j\sum_{m\ge0} C_m\frac{\kappa_m}{k_0}\,S_{m,\nu} \quad ; \quad \nu = 0,\,1,\,2,\ldots. \tag{19}$$

Matching the sound pressure at $x = 0$ requires

$$p_{II}(0,r) \overset{!}{=} p_I(0,r) \quad ; \quad r\le a, \tag{20a}$$

$$\sum_{n\ge0} D_n\;\cosh(\gamma_n t)\;J_0(\eta_n r) = B_0 + \sum_{m\ge0} C_m\,J_0(\varepsilon_m r). \tag{20b}$$

The range is $0 \le r \le a$. Therefore, multiply and integrate over this range, i.e.

on both sides $\int_0^a \ldots\cdot J_0(\varepsilon_\mu r)\,r\,dr \quad ; \quad \mu = 0,\,1,\,2\,\ldots$; giving

$$\sum_{n\ge0} D_n\;\cosh(\gamma_n t)\;S_{\mu,n} = \left(\delta_{0,\mu}B_0 + C_\mu\right)\,N_\mu^{(I)} \quad ; \quad \mu = 0,\,1,\,2,\,\ldots \tag{21}$$

($\delta_{m,n}$ = Kronecker symbol). Equations (19) and (21) are two linear systems of equations for the unknown amplitudes C_m and D_n. One gets from (19) (changing $\nu \to n$)

$$D_n = \frac{j}{(\gamma_n/k_0)\ \sinh(\gamma_n t)\ N_n^{(II)}} \left[B_0\, S_{0,n} + j \sum_{m\geq 0} C_m \frac{\kappa_m}{k_0}\, S_{m,n} \right]. \tag{22}$$

Inserting this into (21) leads to the following linear system of equations for the C_m:

$$\begin{aligned} &\sum_{m>0} C_m \left[\frac{\kappa_m}{k_0} \sum_{n\geq 0} \frac{\coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} S_{m,n} S_{\mu,n} + \delta_{m,\mu} N_\mu^{(I)} \right] \\ &\quad = B_0 \left[j \sum_{n\geq 0} \frac{\coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} S_{0,n} S_{\mu,n} - \delta_{0,\mu} N_\mu^{(I)} \right] \quad ; \quad \mu = 0,\ 1,\ 2,\ \ldots \end{aligned} \tag{23}$$

With the solutions C_m inserted into (22) the D_n are are obtained; thus the sound field is known.

The homogenised chamber entrance impedance $\underline{Z}_K$ is

$$\underline{Z}_K = \frac{\langle p_{II0}(0,r)\rangle_b}{\langle Z_0 v_{II0x}(0,r)\rangle_b} = j\frac{D_0\ \cosh(\gamma_0 t)}{D_0\ \dfrac{\gamma_0}{k_0}\ \sinh(\gamma_0 t)} = \coth(\gamma_0 t) = -j\ \cot(k_0 t). \tag{24}$$

The neck exit impedance is

$$Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{B_0 + C_0}{B_0 - C_0}. \tag{25}$$

Thus, when knowing the amplitude C_0 of the reflected fundamental mode in the neck, the orifice partition impedance can be evaluated:

$$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K = Z_{sh} + j\,\sigma\ \cot(k_0 t). \tag{26}$$

Because only the amplitude C_0 of the fundamental mode (plane wave) in the mode sum (13) is used, an acceptable way to approximate medium-wide necks is to formulate the sound field in the neck only with plane waves. The computational advantage of this approximation lies in the fact that in such a case no system of equtions need be solved; the orifice partition impedance can be written in an explicit formula.

The following tables present solutions for different arrangements. They cover the following variations:

$$\text{Neck:}\left\{\begin{array}{l}\text{round}\\ \text{square}\end{array}\right.\left\{\begin{array}{l}\text{wide (mode sum)}\\ \text{medium-wide (only plane waves)}\end{array}\right.$$

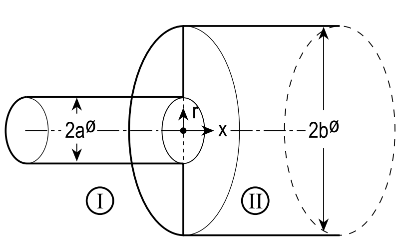
$$\text{Chamber:}\left\{\infty\text{ length}\left\{\begin{array}{l}\text{round}\quad\text{"tube"}\\ \left.\begin{array}{l}\text{square}\\ \text{rectangul.}\end{array}\right\}\text{"duct"}\end{array}\right.\left\{\begin{array}{l}\text{empty}\\ \text{filled with absorber}\end{array}\right.\right.$$

$$\text{Chamber:}\left\{\text{finite length}\left\{\begin{array}{l}\text{round}\\ \text{square}\\ \text{rectangul.}\end{array}\right.\text{"chamber"}\left\{\begin{array}{l}\text{empty}\\ \text{filled with absorber}\\ \text{front absorber layer}\\ \text{rear absorber layer}\end{array}\right.\right.$$

The tables show:

- A sketch of the arrangement and a short description;
- Formulations of the sound fields in the zones;
- Equations for the mode amplitudes;
- Mode norm integrals;
- Mode-coupling integrals;
- Orifice partition impedance Z_{Mi}.

The objects with the round chambers are suited as approximations for perforated panels with the perforations in a hexagonal arrangement; the objects with square or rectangular chambers are suited for similar arrangements of the perforations. Square necks are of interest in combination with square or rectangular chambers since their mode-coupling integrals are easier to evaluate.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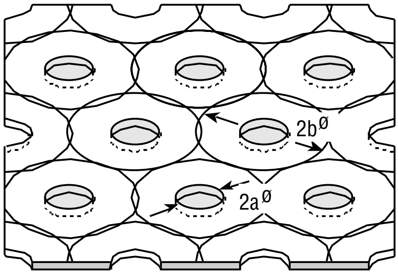
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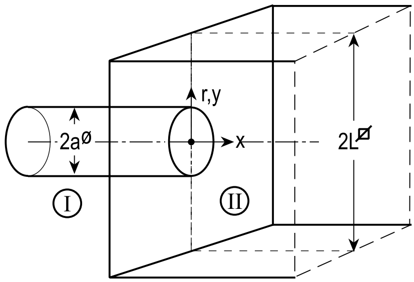
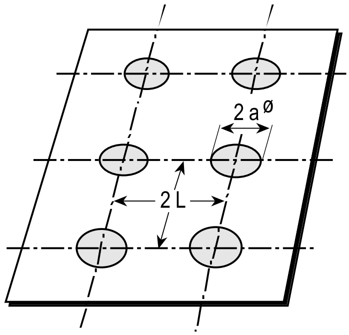
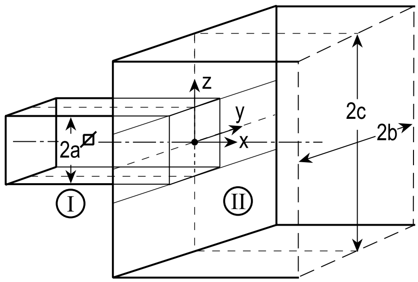

r,y
2aø
x
2L
I
II


2 aø
2 L


z
y
x
2a
2c
2b
I
II

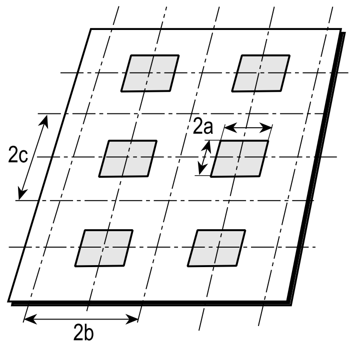

2a
2c
2b

Table 1 Wide round neck and round tube; full analysis

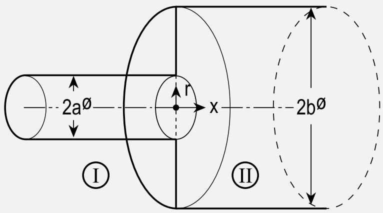
	A plane wave with amplitude B_0 in a round neck with diameter 2a is incident on an orifice to an empty round tube with 2b diameter. Both zones have ∞ lengths. A mode sum is reflected in the neck. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-jk_0x} + \sum_{m\geq 0} C_m e^{+\kappa_m x} J_0(\varepsilon_m r)\ ;\ \kappa_m^2 = \varepsilon_m^2 - k_0^2$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-jk_0x} + j\sum_{m\geq 0} C_m \frac{\kappa_m}{k_0} e^{+\kappa_m x} J_0(\varepsilon_m r)$
Eigenvalues $\varepsilon_m a$	$J_1(\varepsilon_m a) = 0;\ m = 0, 1, 2, \ldots;$ with $\varepsilon_0 a = 0$
Field formulation in (II)	$p_{II}(x,r) = \sum_{n\geq 0} D_n\ e^{-\gamma_n x} J_0(\eta_n r)\ ;\ \gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j\sum_{n\geq 0} D_n \frac{\gamma_n}{k_0}\ e^{-\gamma_n x} J_0(\eta_n r)$
Eigenvalues $\eta_n b$	$J_1(\eta_n b) = 0;\ n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
System of equations for C_m	$\sum_{m\geq 0} C_m \left[\delta_{\mu,m} N_\mu^{(I)} + \frac{\kappa_m}{\gamma_n}\sum_{n\geq 0}\frac{S_{m,n}\,S_{\mu,n}}{N_n^{(II)}}\right] = B_0\left(-\delta_{0,\mu}N_\mu^{(I)} + j\sum_{n\geq 0}\frac{S_{0,n}}{N_n^{(II)}\ \gamma_n/k_0}\right)\ ;\ \mu = 0, 1, 2, \ldots$
Equations for D_n	$D_n = j\,B_0 \frac{S_{0,n}}{N_n^{(II)}\ \gamma_n/k_0} - \sum_{m\geq 0} C_m \frac{\kappa_m}{\gamma_n}\frac{S_{m,n}}{N_n^{(II)}}$
Mode norms in (I)	$N_m^{(I)} = \int_0^a J_0^2(\varepsilon_m r)\, r\, dr = \frac{a^2}{2} J_0^2(\varepsilon_m a) \xrightarrow[m=0]{} \frac{a^2}{2}$
Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \frac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \frac{b^2}{2}$
Mode coupling (I)–(II)	$S_{m,n} = \int_0^a J_0(\varepsilon_m r)\, J_0(\eta_n r)\, r\, dr = a^2 \frac{(\eta_n a)\ J_0(\varepsilon_m a)\ J_1(\eta_n a)}{(\eta_n a)^2 - (\varepsilon_m a)^2}$ $\xrightarrow[n=0,\,m\neq 0]{} 0\ ;\ \xrightarrow[n=0,\,m=0]{} \frac{a^2}{2}\ ;\ \xrightarrow[m=0]{} a^2\frac{J_1(\eta_n a)}{(\eta_n a)}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K\ ;\ \underline{Z}_K = 1$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \sigma\left(1 + \frac{2}{D_0}\sum_{n>0} D_n J_1(\eta_n a)/(\eta_n a)\right)$

Table 2 Medium-wide round neck and round tube

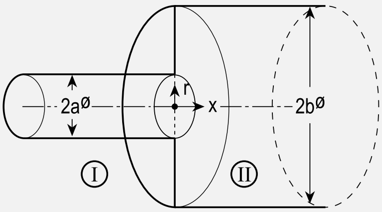
	A plane wave with amplitude B_0 in a round neck with diameter 2a is incident on an orifice to an empty round tube with 2b diameter. Both zones have ∞ lengths. A plane wave is reflected in the neck. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum_{n\geq 0} D_n\, e^{-\gamma_n x}\, J_0(\eta_n r) \;\; ; \;\; \gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j \sum_{n\geq 0} D_n \frac{\gamma_n}{k_0}\, e^{-\gamma_n x}\, J_0(\eta_n r)$ $J_1(\eta_n b) = 0; n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
Equation for C_0	$C_0 = -B_0 \left[1 - \frac{2j}{a^2} \sum_{n\geq 0} \frac{S_{0,n}^2}{N_n^{(II)}\, \gamma_n/k_0} \right] \times \left[1 + \frac{2j}{a^2} \sum_{n\geq 0} \frac{S_{0,n}^2}{N_n^{(II)}\, \gamma_n/k_0} \right]^{-1}$
Equations for D_n	$D_n = j \frac{S_{0,n}}{N_n^{(II)}\, \gamma_n/k_0} (B_0 - C_0)$
Mode norm in (I)	$N_0^{(I)} = \int_0^a r\, dr = \frac{a^2}{2}$
Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \frac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \frac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r)\, r\, dr = a^2 \frac{J_1(\eta_n a)}{(\eta_n a)} \;\; ; \;\; \xrightarrow[n=0]{} \frac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \;\; ; \;\; \underline{Z}_K = 1$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{B_0 + C_0}{B_0 - C_0}$

Table 3 Narrow round neck and round tube

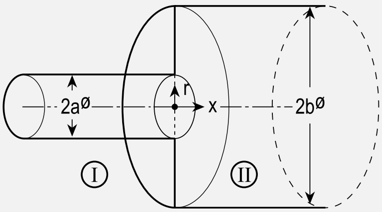
	A capillary wave with amplitude B_0 in a round neck with diameter 2a is incident on an orifice to an empty round tube with 2b diameter. Both zones have ∞ lengths. A fundamental capillary wave is reflected in the neck. Γ_i = capillary propagation constant; Z_i = (normalised) capillary wave impedance (see Sect. J.3). $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = [B_0 e^{-\Gamma_i x} + C_0 e^{\Gamma_i x}]\, J_0(\varepsilon_i r);\quad \varepsilon_i^2 = \Gamma_i^2 + k_\rho^2$ $Z_0 v_{Ix}(x,r) = \dfrac{-1}{\Gamma_i Z_i}\dfrac{\partial p}{\partial x} = \dfrac{1}{Z_i}[B_0 e^{-\Gamma_i x} - C_0 e^{\Gamma_i x}]\, J_0(\varepsilon_i r)$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum_{n\ge 0} D_n\, e^{-\gamma_n x}\, J_0(\eta_n r);\quad \gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = -j\sum_{n\ge 0} D_n \dfrac{\gamma_n}{k_0}\, e^{-\gamma_n x}\, J_0(\eta_n r)$ $J_1(\eta_n b) = 0;\ n = 0,1,2,\ldots;$ with $\eta_0\, b = 0$
Capillary characteristic values	$\left(\dfrac{\Gamma_i}{k_0}\right)^2 = -\dfrac{\rho_{eff}}{\rho_0}\cdot\dfrac{C_{eff}}{C_0};\quad \left(\dfrac{Z_i}{Z_0}\right)^2 = \dfrac{\rho_{eff}}{\rho_0}\Big/\dfrac{C_{eff}}{C_0}$ $\dfrac{\rho_{eff}}{\rho_0} = \dfrac{1}{1 - J_{1,0}(k_\nu a)};\quad \dfrac{C_{eff}}{C_0} = 1 + (\kappa - 1)\, J_{1,0}(k_{\alpha 0} a)$ $J_{1,0}(z) := 2\dfrac{J_1(z)}{z\cdot J_0(z)};\quad k_\nu^2 = -j\dfrac{\omega}{\nu};$ $k_{\alpha 0}^2 = -j\dfrac{\kappa\omega}{\alpha} = \kappa\ \Pr\cdot k_\nu^2$
Equation for C_0	$C_0 = -B_0\left[T_{0,0} - j\sum_{n\ge 0}\dfrac{T_{0,n} S_{0,n}}{Z_i N_n^{(II)}\,\gamma_n/k_0}\right] \times \left[T_{0,0} + j\sum_{n\ge 0}\dfrac{T_{0,n} S_{0,n}}{Z_i N_n^{(II)}\,\gamma_n/k_0}\right]^{-1}$
Equations for D_n	$D_n = j\dfrac{T_{0,n}}{Z_i\, N_n^{(II)}\,\gamma_n/k_0}\,(B_0 - C_0)$
Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \dfrac{b^2}{2}\, J_0^2(\eta_n b) \xrightarrow[n=0]{} \dfrac{b^2}{2}$
Mode coupling (I)–(II)	$T_{0,n} = \int_0^a J_0(\varepsilon_i r)\cdot J_0(\eta_n r)\, r\, dr \xrightarrow[n=0]{} a^2\dfrac{J_1(\varepsilon_i a)}{\varepsilon_i a}$ $= a^2\dfrac{\varepsilon_i a\, J_1(\varepsilon_i a)\, J_0(\eta_n a) - \eta_n a\, J_0(\varepsilon_i a)\, J_1(\eta_n a)}{(\varepsilon_i a)^2 - (\eta_n a)^2}$

Table 3 continued

Modes in (II) – average over $s=\pi a^2$	$S_{0,n} = \int_0^a J_0(\eta_n r)\, r\, dr = a^2 \dfrac{J_1(\eta_n a)}{\eta_n a} \xrightarrow[n=0]{} \dfrac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = 1$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = Z_i \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 4 Medium-wide round neck and round tube with absorber

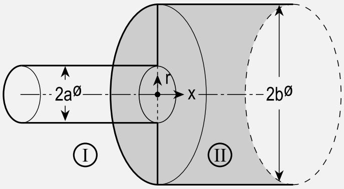
	A plane wave with amplitude B_0 in a round neck with diameter 2a is incident on an orifice to a round tube with 2b diameter, filled with porous absorber material. Both zones have ∞ lengths. A plane wave is reflected in the neck. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0} \dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum_{n\geq 0} D_n\, e^{-\gamma_n x} J_0(\eta_n r) \quad ; \quad \gamma_n^2 = \eta_n^2 + \Gamma_a^2$ $Z_0 v_{IIx}(x,r) = \dfrac{-1}{\Gamma_a Z_i} \dfrac{\partial p}{\partial x} = \dfrac{1}{\Gamma_i Z_i} \sum_{n\geq 0} D_n \dfrac{\gamma_n}{k_0} e^{-\gamma_n x} J_0(\eta_n r)$ $J_1(\eta_n b) = 0; n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
Equation for C_0	$C_0 = -B_0 \dfrac{S_{0,0} - \Gamma_i Z_i \sum_{n\geq 0} \dfrac{S_{0,n}^2}{N_n\, \gamma_n/k_0}}{S_{0,0} + \Gamma_i Z_i \sum_{n\geq 0} \dfrac{S_{0,n}^2}{N_n\, \gamma_n/k_0}}$
Equations for D_n	$D_n = \Gamma_i Z_i \dfrac{S_{0,n}}{N_n\, \gamma_n/k_0} (B_0 - C_0)$
Mode norms in (II)	$N_n = \int_0^b J_0^2(\eta_n r)\, r\, dr = \dfrac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \dfrac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r)\, r\, dr = a^2 \dfrac{J_1(\eta_n a)}{(\eta_n a)} \quad ; \quad \xrightarrow[n=0]{} \dfrac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K = Z_{sh} - \sigma Z_i$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 5 Wide round neck and round empty chamber; full analysis

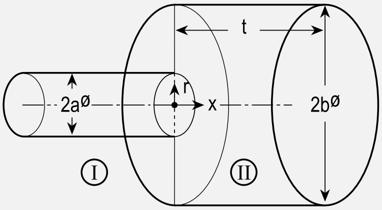
	A round neck with 2a diameter ends in a round empty chamber with 2b diameter and depth t. A plane wave with amplitude B_0 is incident in the neck; a mode sum is reflected. $\sigma = (a/b)^2$.
Field formulation in (I) Eigenvalues $\varepsilon_m a$	$p_I(x,r) = B_0 e^{-jk_0x} + \sum_{m\geq 0} C_m e^{+\kappa_m x} J_0(\varepsilon_m r)$; $\kappa_m^2 = \varepsilon_m^2 - k_0^2$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-jk_0x}$ $+j\sum_{m\geq 0} C_m \frac{\kappa_m}{k_0} e^{+\kappa_m x} J_0(\varepsilon_m r)$ $J_1(\varepsilon_m a) = 0;\ m = 0,1,2,\ldots;$ with $\varepsilon_0 a = 0$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum_{n\geq 0} D_n \cosh(\gamma_n(x-t))\ J_0(\eta_n r)$; $\gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x}$ $= j\sum_{n\geq 0} D_n \frac{\gamma_n}{k_0} \sinh(\gamma_n(x-t))\ J_0(\eta_n r)$ $J_1(\eta_n b) = 0;\ n = 0,1,2,\ldots;$ with $\eta_0 b = 0$
System of equations for C_m, μ= 0,1,2,…	$\sum_{m>0} C_m \left[\frac{\kappa_m}{k_0}\sum_{n\geq 0}\frac{\coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} S_{m,n} S_{\mu,n} + \delta_{m,\mu} N_\mu^{(I)}\right]$ $= B_0\left[j\sum_{n\geq 0}\frac{\coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} S_{0,n} S_{\mu,n} - \delta_{0,\mu} N_\mu^{(I)}\right]$
Equations for D_n	$D_n = \frac{j}{(\gamma_n/k_0)\ \sinh(\gamma_n t)\ N_n^{(II)}}$ $\times\left[B_0 S_{0,n} + j\sum_{m\geq 0} C_m \frac{\kappa_m}{k_0} S_{m,n}\right]$
Mode norms in (I)	$N_m^{(I)} = \int_0^a J_0^2(\varepsilon_m r)\, r\, dr = \frac{a^2}{2} J_0^2(\varepsilon_m a) \xrightarrow[m=0]{} \frac{a^2}{2}$
Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \frac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \frac{b^2}{2}$

Table 5 continued

Mode coupling (I)–(II)	$S_{m,n} = \int_0^a J_0(\varepsilon_m r)\, J_0(\eta_n r)\, r\, dr$ $= a^2 \frac{(\eta_n a)\ J_0(\varepsilon_m a)\ J_1(\eta_n a)}{(\eta_n a)^2 - (\varepsilon_m a)^2}$ $\xrightarrow[n=0,\, m\neq 0]{} 0\ ;\quad \xrightarrow[n=0,\, m=0]{} \frac{a^2}{2}\ ;\quad \xrightarrow[m=0]{} a^2 \frac{J_1(\eta_n a)}{(\eta_n a)}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = -j\cot(k_0 t)$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 6 Medium-wide round neck and round empty chamber

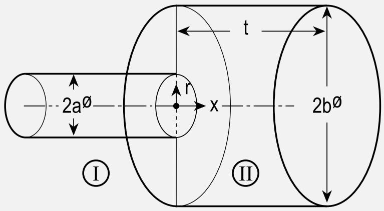
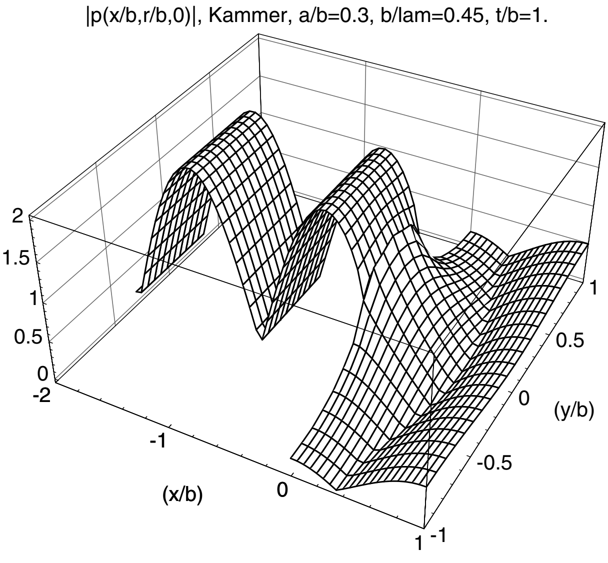
	A round neck with 2a diameter ends in a round empty chamber with 2b diameter and depth t. A plane wave with amplitude B_0 is incident in the neck; a plane wave is reflected. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-jk_0 x} + C_0 e^{+jk_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-jk_0 x} - C_0 e^{+jk_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum_{n\geq 0} D_n\ \cosh(\gamma_n(x-t))\ J_0(\eta_n r)\ ;$ $\gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x}$ $= j\sum_{n\geq 0} D_n \frac{\gamma_n}{k_0}\ \sinh(\gamma_n(x-t))\ J_0(\eta_n r)$ $J_1(\eta_n b) = 0; n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
Equation for C_0	$C_0 = B_0 \left[-N_0^{(I)} + j\sum_{n\geq 0} \frac{S_{0,n}^2 \coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} \right]$ $\times \left[N_0^{(I)} + j\sum_{n\geq 0} \frac{S_{0,n}^2 \coth(\gamma_n t)}{(\gamma_n/k_0)\ N_n^{(II)}} \right]^{-1}$
Equations for D_n	$D_n = j\ (B_0 - C_0)\ \frac{S_{0,n}}{(\gamma_n/k_0)\ \sinh(\gamma_n t)\ N_n^{(II)}}$

Table 6 continued

Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \frac{b^2}{2}\, J_0^2(\eta_n b) \xrightarrow[n=0]{} \frac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r) \cdot r\, dr = a^2 \frac{J_1(\eta_n a)}{\eta_n a} \xrightarrow[n=0]{} \frac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = -j\cot(k_0 t)$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$



Example of sound pressure matching at the orifice of a medium-wide neck and an empty chamber. $a/b = 0.3$; $b/\lambda_0 = 0.45$; $t/b = 1.0$

Table 7 Medium-wide round neck and round chamber with absorber

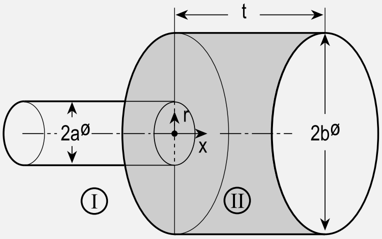
	A round neck with 2a diameter ends in a round chamber with 2b diameter and depth t, filled with porous absorber material. A plane wave with amplitude B_0 is incident in the neck; a plane wave is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-jk_0x} + C_0 e^{+jk_0x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-jk_0x} - C_0 e^{+jk_0x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,r) = \sum\limits_{n\geq 0} D_n \cosh(\gamma_n(x-t))\ J_0(\eta_n r)$; $\gamma_n^2 = \eta_n^2 + \Gamma_a^2$ $Z_0 v_{IIx}(x,r) = \dfrac{-1}{\Gamma_a Z_i}\dfrac{\partial p}{\partial x}$ $= \dfrac{-1}{\Gamma_i Z_i}\sum\limits_{n\geq 0} D_n \dfrac{\gamma_n}{k_0} \sinh(\gamma_n(x-t))\ J_0(\eta_n r)$ $J_1(\eta_n b) = 0; n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
Equation for C_0	$C_0 = B_0\left[-1 + 2\dfrac{\Gamma_i Z_i}{a^2}\sum\limits_{n\geq 0}\dfrac{S_{0,n}^2 \coth(\gamma_n t)}{(\gamma_n/k_0)\,N_n^{(II)}}\right]$ $\times\left[1 + 2\dfrac{\Gamma_i Z_i}{a^2}\sum\limits_{n\geq 0}\dfrac{S_{0,n}^2 \coth(\gamma_n t)}{(\gamma_n/k_0)\,N_n^{(II)}}\right]^{-1}$
Equations for D_n	$D_n = (B_0 - C_0)\,\Gamma_i Z_i \dfrac{S_{0,n}}{(\gamma_n/k_0)\sinh(\gamma_n t)\ N_n^{(II)}}$
Mode norms in (II)	$N_n^{(II)} = \int_0^b J_0^2(\eta_n r)\, r\, dr = \dfrac{b^2}{2} J_0^2(\eta_n b) \xrightarrow[n=0]{} \dfrac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r)\cdot r\ dr = a^2 \cdot \dfrac{J_1(\eta_n a)}{\eta_n a} \xrightarrow[n=0]{} \dfrac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K$; $\underline{Z}_K = Z_i \coth(\gamma_0 t)$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 8 Medium-wide neck and round chamber with front absorber layer

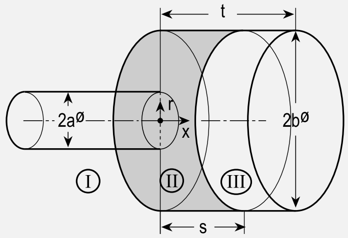
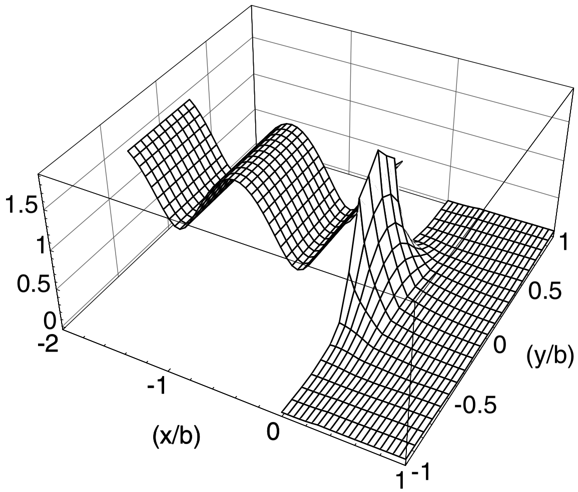
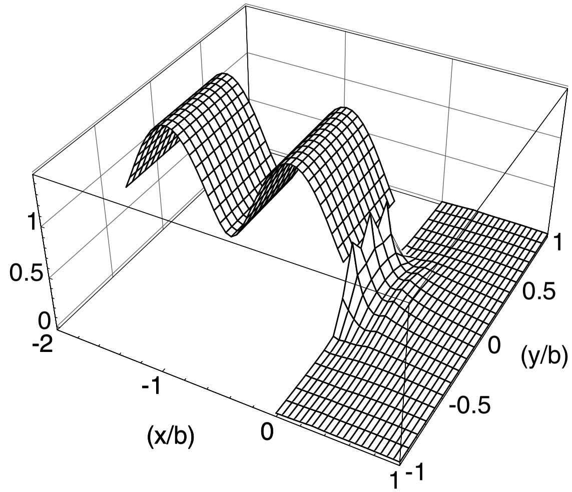
	A round neck with 2a diameter ends in a round chamber with 2b diameter and depth t, partially filled with porous absorber layer adjacent to the orifice. A plane wave with amplitude B_0 is incident in the neck; a plane wave is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II)	$p_{II}(x,r) = \sum_{n\geq 0} \left(D_n\, e^{-\gamma_n x} + E_n\, e^{+\gamma_n x}\right) J_0(\eta_n r)$; $\gamma_n^2 = \eta_n^2 + \Gamma_a^2$ $Z_0 v_{IIx}(x,r) = \frac{-1}{\Gamma_a Z_i}\frac{\partial p}{\partial x} = \frac{1}{\Gamma_i Z_i}\sum_{n\geq 0}\frac{\gamma_n}{k_0}\left(D_n\, e^{-\gamma_n x} - E_n\, e^{+\gamma_n x}\right) J_0(\eta_n r)$
Eigenvalues $\eta_n b$	$J_1(\eta_n b) = 0$; $n = 0, 1, 2, \ldots$; with $\eta_0 b = 0$
Field formulation in (III)	$p_{III}(x,r) = \sum_{n\geq 0} F_n \cosh(\kappa_n(x-t))\, J_0(\eta_n r)$; $\kappa_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = j\sum_{n\geq 0} F_n \frac{\kappa_n}{k_0} \sinh(\kappa_n(x-t))\, J_0(\eta_n r)$
Equation for C_0	$C_0 = B_0\left[-1 + \frac{2\Gamma_i Z_i}{a^2}\sum_{n\geq 0}\frac{k_0}{\gamma_n}\frac{S_{0,n}^2}{N_n}\frac{\gamma_n + j\,\Gamma_i Z_i\,\kappa_n \tanh(\gamma_n s)\,\tanh(\kappa_n(s-t))}{\gamma_n \tanh(\gamma_n s) + j\,\Gamma_i Z_i\,\kappa_n \tanh(\kappa_n(s-t))}\right]$ $\times\left[1 + \frac{2\Gamma_i Z_i}{a^2}\sum_{n\geq 0}\frac{k_0}{\gamma_n}\frac{S_{0,n}^2}{N_n}\frac{\gamma_n + j\,\Gamma_i Z_i\,\kappa_n \tanh(\gamma_n s)\,\tanh(\kappa_n(s-t))}{\gamma_n \tanh(\gamma_n s) + j\,\Gamma_i Z_i\,\kappa_n \tanh(\kappa_n(s-t))}\right]^{-1}$

Table 8 continued

Equations for D_n	$D_n = (B_0 - C_0)\,\Gamma_i Z_i \dfrac{k_0}{\gamma_n}\,\dfrac{S_{0,n}}{N_n}\,\dfrac{\gamma_n + j\,\Gamma_i Z_i\,\kappa_n\,\tanh(\kappa_n(s-t))}{\gamma_n\left(1-e^{-2\gamma_n s}\right) + j\,\Gamma_i Z_i\,\kappa_n\left(1+e^{-2\gamma_n s}\right)\,\tanh(\kappa_n(s-t))}$
Equations for E_n	$E_n = (B_0 - C_0)\,\Gamma_i Z_i \dfrac{k_0}{\gamma_n}\,\dfrac{S_{0,n}}{N_n}\,e^{-2\gamma_n s}\,\dfrac{\gamma_n - j\,\Gamma_i Z_i\,\kappa_n\,\tanh(\kappa_n(s-t))}{\gamma_n\left(1-e^{-2\gamma_n s}\right) + j\,\Gamma_i Z_i\,\kappa_n\left(1+e^{-2\gamma_n s}\right)\,\tanh(\kappa_n(s-t))}$
Equations for F_n	$F_n = \dfrac{D_n\,e^{-\gamma_n s} + E_n\,e^{+\gamma_n s}}{\cosh\,(\kappa_n(s-t))}$
Mode norms in (II)	$N_n = \int_0^b J_0^2(\eta_n r)\,r\,dr = \dfrac{b^2}{2}\,J_0^2(\eta_n b) \xrightarrow[n=0]{} \dfrac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r) r\,dr = a^2 \dfrac{J_1(\eta_n a)}{\eta_n a} \xrightarrow[n=0]{} \dfrac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{B_0 + C_0}{B_0 - C_0} = \dfrac{2\Gamma_i Z_i}{a^2} \sum_{n\geq 0} \dfrac{k_0}{\gamma_n}\,\dfrac{S_{0,n}^2}{N_n}\,\dfrac{\gamma_n + j\,\Gamma_i Z_i\,\kappa_n \tanh(\gamma_n s)\,\tanh(\kappa_n(s-t))}{\gamma_n \tanh(\gamma_n s) + j\,\Gamma_i Z_i\,\kappa_n\,\tanh(\kappa_n(s-t))}$ $\underline{Z}_K = \dfrac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = Z_i \dfrac{D_0 + E_0}{D_0 - E_0} = Z_i \dfrac{1 + j\,Z_i \tanh\left(\Gamma_i k_0 s\right)\,\tan\left(k_0(t-s)\right)}{\tanh\left(\Gamma_i k_0 s\right) + j\,Z_i\,\tan\left(k_0(t-s)\right)}$





Example of sound pressure and axial particle velocity matching at the orifice of a medium-wide neck in a round chamber with absorber layer at its entrance. Parameters: $a/b = 0.3$; $t/b = 1.0$; $s/t = 0.3$; $b/\lambda_0 = 0.45$; $\Xi b/Z_0 = 20.0$.

Table 9 Medium-wide neck and round chamber with rear absorber layer

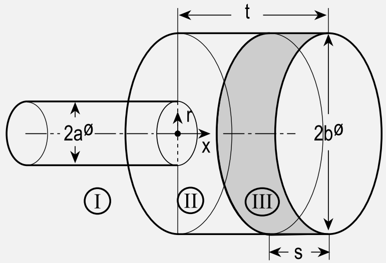
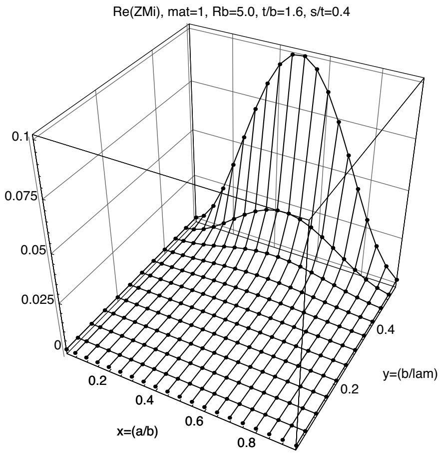
	A round neck with 2a diameter ends in a round chamber with 2b diameter and depth t, partially filled with porous absorber layer adjacent to the back side. A plane wave with amplitude B_0 is incident in the neck; a plane wave is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0\,;Z_i = Z_a/Z_0.\,\sigma = (a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II)	$p_{II}(x,r) = \sum_{n\geq 0}\left(D_n\, e^{-\gamma_n x} + E_n\, e^{+\gamma_n x}\right) J_0(\eta_n r)\ ;\ \gamma_n^2 = \eta_n^2 - k_0^2$ $Z_0 v_{IIx}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j\sum_{n\geq 0}\frac{\gamma_n}{k_0}\left(D_n\, e^{-\gamma_n x} - E_n\, e^{+\gamma_n x}\right) J_0(\eta_n r)$
Eigenvalues $\eta_n b$	$J_1(\eta_n b) = 0;\ n = 0, 1, 2, \ldots;$ with $\eta_0 b = 0$
Field formulation in (III)	$p_{III}(x,r) = \sum_{n\geq 0} F_n\ \cosh\,(\kappa_n(x-t))\ J_0(\eta_n r)\ ;\qquad \kappa_n^2 = \eta_n^2 + \Gamma_a^2$ $Z_0 v_{IIIx}(x,r) = \frac{-1}{\Gamma_a Z_i}\frac{\partial p}{\partial x} = \frac{-1}{\Gamma_i Z_i}\sum_{n\geq 0} F_n \frac{\kappa_n}{k_0}\ \sinh\,(\kappa_n(x-t))\ J_0(\eta_n r)$

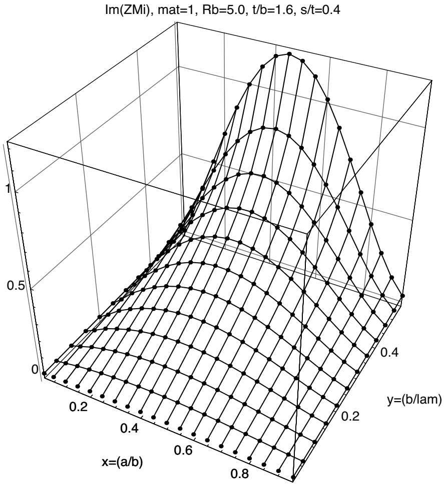
Table 9 continued

Equation for C_0	$C_0 = -B_0 \left[1 - \frac{2j}{a^2} \sum_{n \geq 0} \frac{k_0 S_{0,n}^2}{\gamma_n N_n} \frac{\gamma_n \Gamma_i Z_i + j\,\kappa_n \tanh(\kappa_n s) \tanh(\gamma_n (t-s))}{\gamma_n \Gamma_i Z_i \tanh(\gamma_n (t-s)) + j\,\kappa_n \tanh(\kappa_n s)} \right] \times \left[1 + \frac{2j}{a^2} \sum_{n \geq 0} \frac{k_0 S_{0,n}^2}{\gamma_n N_n} \frac{\gamma_n \Gamma_i Z_i + j\,\kappa_n \tanh(\kappa_n s) \tanh(\gamma_n (t-s))}{\gamma_n \Gamma_i Z_i \tanh(\gamma_n (t-s)) + j\,\kappa_n \tanh(\kappa_n s)} \right]^{-1}$
Equations for D_n	$D_n = (B_0 - C_0) \frac{j\,k_0 S_{0,n}}{\gamma_n N_n} \frac{\gamma_n \Gamma_i Z_i + j\,\kappa_n \tanh(\kappa_n s)}{\gamma_n \Gamma_i Z_i \left(1 - e^{-2\gamma_n (t-s)}\right) + j\,\kappa_n \tanh(\kappa_n s) \left(1 + e^{-2\gamma_n (t-s)}\right)}$
Equations for E_n	$E_n = (B_0 - C_0) \frac{j\,k_0 S_{0,n}}{\gamma_n N_n} e^{-2\gamma_n (t-s)} \frac{\gamma_n \Gamma_i Z_i - j\,\kappa_n \tanh(\kappa_n s)}{\gamma_n \Gamma_i Z_i \left(1 - e^{-2\gamma_n (t-s)}\right) + j\,\kappa_n \tanh(\kappa_n s) \left(1 + e^{-2\gamma_n (t-s)}\right)}$
Equations for F_n	$F_n = \frac{D_n\, e^{-\gamma_n (t-s)} + E_n\, e^{+\gamma_n (t-s)}}{\cosh(\kappa_n s)}$

Table 9 continued

Mode norms in (II)	$N_n = \int_0^b J_0^2(\eta_n r)\, r\, dr = \frac{b^2}{2}\, J_0^2(\eta_n b) \xrightarrow[n=0]{} \frac{b^2}{2}$
Mode coupling (I)–(II)	$S_{0,n} = \int_0^a J_0(\eta_n r) r\, dr = a^2 \frac{J_1(\eta_n a)}{\eta_n a} \xrightarrow[n=0]{} \frac{a^2}{2}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{B_0 + C_0}{B_0 - C_0}$ $= \frac{2j}{a^2} \sum_{n \geq 0} \frac{k_0 S_{0,n}^2}{\gamma_n N_n} \frac{\gamma_n \Gamma_i Z_i + j\, \kappa_n \tanh(\kappa_n s) \tanh(\gamma_n (t-s))}{\gamma_n \Gamma_i Z_i \tanh(\gamma_n(t-s)) + j\, \kappa_n \tanh(\kappa_n s)}$ $\underline{Z}_K = \frac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = \frac{D_0 + E_0}{D_0 - E_0} = \frac{Z_i + j\ \tanh(\Gamma_a s)\ \tan\left(k_0(t-s)\right)}{j\, Z_i \tan\left(k_0(t-s)\right) + \tanh(\Gamma_a s)}$





Example of variation of $\mathrm{Re}(Z_{Mi})$ and $\mathrm{Im}(Z_{Mi})$ over variables x= (a/b) and $y = (b/\lambda_0)$ for parameter values $R_b = \Xi b/Z_0 = 5.0$; $t/b = 1.6$; $s/t = 0.4$ (absorber = glass fibre, mat=1)

Table 10 Medium-wide round neck and square empty duct

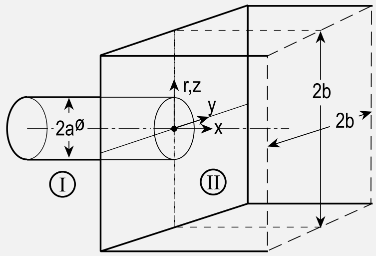
	A round neck with 2a diameter ends in an square empty duct with infinite length and 2b width. The incident wave in the neck with amplitude B_0 and the reflected wave with amplitude C_0 are plane waves. $\sigma = \pi/4(a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\, e^{-\gamma_{n,\nu}x}\cos(\eta_n y)\cos(\eta_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x}$ $= -j\sum_{n,\nu\geq 0} D_{n,\nu}\frac{\gamma_{n,\nu}}{k_0}e^{-\gamma_{n,\nu}x}\cos(\eta_n y)\cos(\eta_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\;;\; n = 0, 1, 2, \ldots\;;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \eta_\nu^2 - k_0^2$
Equation for C_0	$C_0 = B_0\left[-1 + \frac{j}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{j}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = j\frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}(B_0 - C_0)$
Mode norm in (I)	$N_0^{(I)} = \int_0^a r\,dr = \frac{a^2}{2}$

Table 10 continued

Mode norms in (II)	$N^{(II)}_{n,\nu} = \int\limits_{-b}^{+b}\int\limits_{-b}^{+b} \cos(\eta_n y)\ \cos(\eta_\nu z) \cdot \cos(\eta_{n'} y)\ \cos(\eta_{\nu'} z)\,dy\,dz$ $N^{(II)}_{n,\nu} = b^2 N^{(y)}_n \cdot N^{(z)}_\nu\ ;\ N^{(y)}_n = \begin{cases} 0\ ;\ n \neq n' \\ 1\ ;\ n = n' \neq 0 \\ 2\ ;\ n = n' = 0 \end{cases}\ ;$ $N^{(z)}_\nu = \begin{cases} 0\ ;\ \nu \neq \nu' \\ 1\ ;\ \nu = \nu' \neq 0 \\ 2\ ;\ \nu = \nu' = 0 \end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,\nu} = \iint\limits_s \cos(\eta_n y)\ \cos(\eta_\nu z)\,ds = 2\pi a^2 \dfrac{J_1\left(a\sqrt{\eta_n^2+\eta_\nu^2}\right)}{a\sqrt{\eta_n^2+\eta_\nu^2}} \xrightarrow[\eta_n=\eta_\nu=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K\quad ;\quad \underline{Z}_K = 1$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \dfrac{B_0 + C_0}{B_0 - C_0}$

Table 11 Medium-wide round neck and square duct filled with absorber

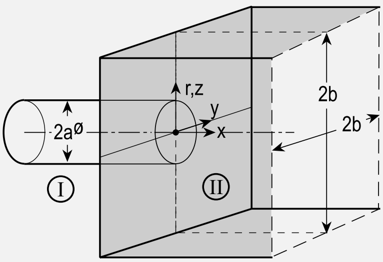
	A round neck with 2a diameter ends in a square duct with infinite length and 2b width, filled with porous absorber material. The incident wave in the neck with amplitude B_0 and the reflected wave with amplitude C_0 are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = \pi/4(a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$

Table 11 continued

Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\ e^{-\gamma_{n,\nu}x} \cos(\eta_n y)\ \cos(\eta_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{-1}{\Gamma_a Z_i}\frac{\partial p}{\partial x} = \frac{1}{\Gamma_i Z_i}\sum_{n,\nu\geq 0} D_{n,\nu}\frac{\gamma_{n,\nu}}{k_0}\ e^{-\gamma_{n,\nu}x}$ $\cdot \cos(\eta_n y)\ \cos(\eta_\nu z)$
Eigenvalues $\eta_n b$	$\sin(\eta_n b) = 0 \ \Rightarrow\ \eta_n b = n\pi\ ;\ \ n = 0,\ 1,\ 2, \ldots\ ;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \eta_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0 = B_0\left[-1 + \frac{\Gamma_i Z_i}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]$ $\times\left[1 + \frac{\Gamma_i Zi}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i \frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}(B_0 - C_0)$
Mode norm in (I)	$N_0^{(I)} = \int_0^a r\, dr = \frac{a^2}{2}$
Mode norms in (II)	$N_{n,\nu}^{(II)} = \int_{-b}^{+b}\int_{-b}^{+b} \cos(\eta_n y)\cos(\eta_\nu z)$ $\cdot\ \cos(\eta_{n'} y)\ \cos(\eta_{\nu'} z)\, dy\, dz$ $N_{n,\nu}^{(II)} = b^2 N_n^{(y)} \cdot N_\nu^{(z)}\ ;\ N_n^{(y)} = \begin{cases} 0\ ;\ n \neq n' \\ 1\ ;\ n = n' \neq 0 \\ 2\ ;\ n = n' = 0 \end{cases}\ ;$ $N_\nu^{(z)} = \begin{cases} 0\ ;\ \nu \neq \nu' \\ 1\ ;\ \nu = \nu' \neq 0 \\ 2\ ;\ \nu = \nu' = 0 \end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,\nu} = \iint_s \cos(\eta_n y)\ \cos(\eta_\nu z)\, ds$ $= 2\pi a^2 \frac{J_1\left(a\sqrt{\eta_n^2 + \eta_\nu^2}\right)}{a\sqrt{\eta_n^2 + \eta_\nu^2}} \xrightarrow[\eta_n = \eta_\nu = 0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K\ \ ;\ \ \underline{Z}_K = Z_i$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{B_0 + C_0}{B_0 - C_0}$

Table 12 Medium-wide round neck and square empty chamber

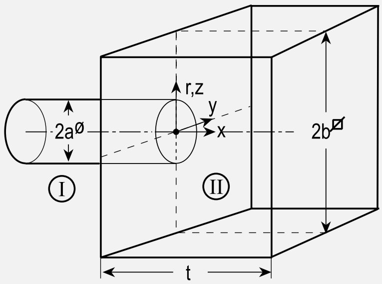
	A medium-wide round neck with 2a diameter ends in a square empty chamber with 2b side length and depth t. The incident and the reflected waves in the neck are plane waves. $\sigma = \pi/4(a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,y,z) = \sum_{n,\nu \geq 0} D_{n,\nu} \cosh\left(\gamma_{n,\nu}(x-t)\right)$ $\cdot \cos(\eta_n y)\cos(\eta_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x} = j \sum_{n,\nu \geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0} \sinh\left(\gamma_{n,\nu}(x-t)\right)$ $\cdot \cos(\eta_n y)\cos(\eta_\nu z)$ $\sin(\eta_n b) = 0 \quad \Rightarrow \quad \eta_n b = n\pi \quad ; \quad n = 0, 1, 2, \ldots \quad ;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \eta_\nu^2 - k_0^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{j}{\pi a^2} \sum_{n,\nu \geq 0} \frac{S_{n,\nu}^2 \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]$ $\times \left[1 + \frac{j}{\pi a^2} \sum_{n,\nu \geq 0} \frac{S_{n,\nu}^2 \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = j \frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\, \sinh\left(\gamma_{n,\nu} t\right) N_{n,\nu}^{(II)}} (B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = \int_{-b}^{+b}\int_{-b}^{+b} \cos(\eta_n y)\cos(\eta_\nu z) \cdot \cos(\eta_{n'} y)\cos(\eta_{\nu'} z)\, dy\, dz$ $N_{n,\nu}^{(II)} = b^2 N_n^{(y)} \cdot N_\nu^{(z)} \; ; \; N_n^{(y)} = \begin{cases} 0\,; & n \neq n' \\ 1\,; & n = n' \neq 0 \\ 2\,; & n = n' = 0 \end{cases} \;;$ $N_\nu^{(z)} = \begin{cases} 0\,; & \nu \neq \nu' \\ 1\,; & \nu = \nu' \neq 0 \\ 2\,; & \nu = \nu' = 0 \end{cases}$

Table 12 continued

Mode coupling (I)–(II) in $s=\pi a^2$	$S_{n,\nu} = \iint\limits_s \cos(\eta_n y)\,\cos(\eta_\nu z)\,ds$ $= 2\pi a^2 \dfrac{J_1\left(a\sqrt{\eta_n^2+\eta_\nu^2}\right)}{a\sqrt{\eta_n^2+\eta_\nu^2}} \xrightarrow[\eta_n=\eta_\nu=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = -j\cot(k_0 t)$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 13 Medium-wide round neck and square chamber with absorber

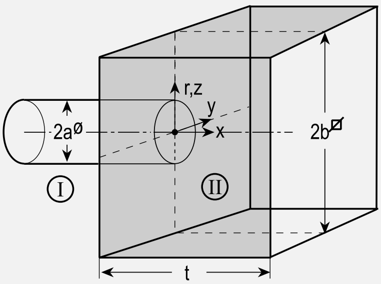
	A medium-wide round neck with 2a diameter ends in a square chamber with 2b side length and depth t, filled with porous absorber material. The incident and the reflected waves in the neck are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = \pi/4(a/b)^2$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-jk_0x} + C_0 e^{jk_0x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-jk_0x} - C_0 e^{jk_0x}$
Field formulation in (II) Eigenvalues $\eta_n b$	$p_{II}(x,y,z) = \sum\limits_{n,\nu\geq 0} D_{n,\nu}\,\cosh\left(\gamma_{n,\nu}(x-t)\right)\cos(\eta_n y)\,\cos(\eta_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \dfrac{-1}{\Gamma_i Z_i}\sum\limits_{n,\nu\geq 0} D_{n,\nu}\dfrac{\gamma_{n,\nu}}{k_0}\sinh\left(\gamma_{n,\nu}(x-t)\right)$ $\cdot \cos(\eta_n y)\,\cos(\eta_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\;;\; n = 0, 1, 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \eta_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \dfrac{\Gamma_i Z_i}{\pi a^2}\sum\limits_{n,\nu\geq 0}\dfrac{S_{n,\nu}^2\coth\left(\gamma_{n,\nu}t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]$ $\times\left[1 + \dfrac{\Gamma_i Z_i}{\pi a^2}\sum\limits_{n,\nu\geq 0}\dfrac{S_{n,\nu}^2\coth\left(\gamma_{n,\nu}t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i \dfrac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\,\sinh\left(\gamma_{n,\nu}t\right)\,N_{n,\nu}^{(II)}}(B_0 - C_0)$

Table 13 continued

Mode norms in (II)	$N^{(II)}_{n,v} = \int_{-b}^{+b}\int_{-b}^{+b} \cos(\eta_n y)\,\cos(\eta_v z) \cdot \cos(\eta_{n'} y)\,\cos(\eta_{v'} z)\,dy\,dz$ $N^{(II)}_{n,v} = b^2 N^{(y)}_n \cdot N^{(z)}_v\,;\; N^{(y)}_n = \begin{cases} 0\,;\; n \neq n' \\ 1\,;\; n = n' \neq 0 \\ 2\,;\; n = n' = 0 \end{cases}\;;$ $N^{(z)}_v = \begin{cases} 0\,;\; v \neq v' \\ 1\,;\; v = v' \neq 0 \\ 2\,;\; v = v' = 0 \end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,v} = \iint_s \cos(\eta_n y)\,\cos(\eta_v z)\,ds$ $= 2\pi a^2 \dfrac{J_1\left(a\sqrt{\eta_n^2+\eta_v^2}\right)}{a\sqrt{\eta_n^2+\eta_v^2}} \xrightarrow[\eta_n=\eta_v=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = Z_i \coth(\gamma_0 t)$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 14 Medium-wide round neck and rectangular empty duct

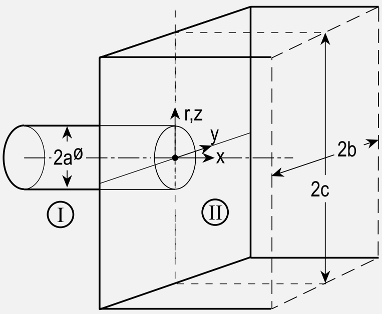
	A round neck with 2a diameter ends in a rectangular empty duct with ∞ length and sides 2b, 2c. The incident and the reflected waves in the neck are plane waves. $\sigma = \pi a^2/(4bc)$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$

Table 14 continued

Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\ e^{-\gamma_{n,\nu}x} \cos(\eta_n y)\ \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j \sum_{n,\nu\geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0} e^{-\gamma_{n,\nu}x}$ $\cdot \cos(\eta_n y)\ \cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \ \Rightarrow\ \eta_n b = n\pi\,;\ n = 0,\ 1,\ 2,\ldots;$ $\sin(\chi_\nu c) = 0 \ \Rightarrow\ \chi_\nu c = \nu\pi\,;\ \nu = 0,\ 1,\ 2,\ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{j}{\pi a^2} \sum_{n,\nu\geq 0} \frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{j}{\pi a^2} \sum_{n,\nu\geq 0} \frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0), N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = j\,\frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\,(B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = \int_{-c}^{+c}\int_{-b}^{+b} \cos(\eta_n y)\ \cos(\chi_\nu z)$ $\cdot \cos(\eta_{n'} y)\ \cos(\chi_{\nu'} z)\, dy\, dz$ $N_{n,\nu}^{(II)} = bc\, N_n^{(y)} \cdot N_\nu^{(z)};\ \ N_n^{(y)} = \begin{cases} 0\,;\ n \neq n' \\ 1\,;\ n = n' \neq 0 \\ 2\,;\ n = n' = 0 \end{cases}\ ;$ $N_\nu^{(z)} = \begin{cases} 0\,;\ \nu \neq \nu' \\ 1\,;\ \nu = \nu' \neq 0 \\ 2\,;\ \nu = \nu' = 0 \end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,\nu} = \iint_s \cos(\eta_n y)\ \cos(\chi_\nu z)\, ds$ $= 2\pi a^2 \frac{J_1\left(a\sqrt{\eta_n^2 + \chi_\nu^2}\right)}{a\sqrt{\eta_n^2 + \chi_\nu^2}} \xrightarrow[\eta_n = \chi_\nu = 0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad;\quad \underline{Z}_K = 1$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 15 Medium-wide round neck and rectangular duct with absorber

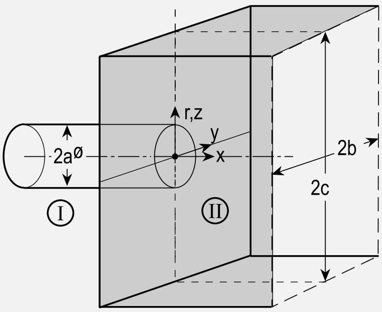
	A round neck with 2a diameter ends in a rectangular duct with ∞ length and sides 2b, 2c, filled with porous absorber material. The incident and the reflected waves in the neck are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = \pi a^2/(4bc)$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\; e^{-\gamma_{n,\nu} x} \cos(\eta_n y)\; \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{1}{\Gamma_i Z_i} \sum_{n,\nu\geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0}\; e^{-\gamma_{n,\nu} x}\; \cos(\eta_n y)\; \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\,;\; n = 0, 1, 2, \ldots;$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi\,;\; \nu = 0, 1, 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{\Gamma_i Z_i}{\pi a^2} \sum_{n,\nu\geq 0} \frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{\Gamma_i Z_i}{\pi a^2} \sum_{n,\nu\geq 0} \frac{S_{n,\nu}^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i \frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}} (B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = \int_{-c}^{+c}\int_{-b}^{+b} \cos(\eta_n y)\; \cos(\chi_\nu z) \cdot \cos(\eta_{n'} y)\; \cos(\chi_{\nu'} z)\, dy\, dz$ $N_{n,\nu}^{(II)} = bc\, N_n^{(y)} \cdot N_\nu^{(z)};\; N_n^{(y)} = \begin{cases} 0\,;\; n \neq n' \\ 1\,;\; n = n' \neq 0 \\ 2\,;\; n = n' = 0 \end{cases}\;;$ $N_\nu^{(z)} = \begin{cases} 0\,;\; \nu \neq \nu' \\ 1\,;\; \nu = \nu' \neq 0 \\ 2\,;\; \nu = \nu' = 0 \end{cases}$

Table 15 continued

Mode coupling (I)–(II) in $s=\pi a^2$	$S_{n,\nu} = \iint_s \cos(\eta_n y)\cos(\chi_\nu z)\,ds$ $= 2\pi a^2 \frac{J_1\left(a\sqrt{\eta_n^2+\chi_\nu^2}\right)}{a\sqrt{\eta_n^2+\chi_\nu^2}} \xrightarrow[\eta_n=\chi_\nu=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = Z_i$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{1+C_0/B_0}{1-C_0/B_0}$

Table 16 Medium-wide round neck and rectangular empty chamber

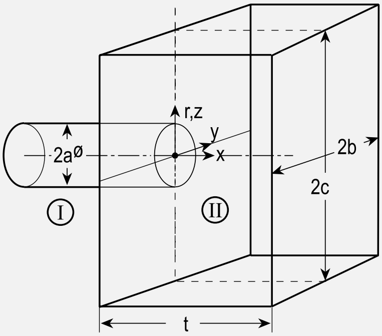
	A round neck with 2a diameter ends in a rectangular empty chamber with depth t and sides 2b, 2c. The incident and the reflected waves in the neck are plane waves. $\sigma = \pi a^2/(4bc)$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu} \cosh\left(\gamma_{n,\nu}(x-t)\right)\cos(\eta_n y)\cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = j \sum_{n,\nu\geq 0} D_{n,\nu}\frac{\gamma_{n,\nu}}{k_0}\sinh\left(\gamma_{n,\nu}(x-t)\right)\cos(\eta_n y)\cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\,;\; n = 0, 1, 2, \ldots;$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi\,;\; \nu = 0, 1, 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{j}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2 \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{j}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2 \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]^{-1}$

Table 16 continued

Equations for $D_{n,v}$	$D_{n,v} = j \dfrac{S_{n,v}}{(\gamma_{n,v}/k_0)\ \sinh(\gamma_{n,v} t)\ N^{(II)}_{n,v}} (B_0 - C_0)$
Mode norms in (II)	$N^{(II)}_{n,v} = \int_{-c}^{+c}\int_{-b}^{+b} \cos(\eta_n y)\ \cos(\chi_v z) \cdot \cos(\eta_{n'} y)\ \cos(\chi_{v'} z)\, dy\, dz$ $N^{(II)}_{n,v} = bc\, N^{(y)}_n \cdot N^{(z)}_v;\ \ N^{(y)}_n = \begin{cases} 0;\ n \neq n' \\ 1;\ n = n' \neq 0 \\ 2;\ n = n' = 0 \end{cases}$; $N^{(z)}_v = \begin{cases} 0;\ v \neq v' \\ 1;\ v = v' \neq 0 \\ 2;\ v = v' = 0 \end{cases}$
Mode coupling (I)–(II) in s= πa^2	$S_{n,v} = \iint_s \cos(\eta_n y)\ \cos(\chi_v z)\, ds$ $= 2\pi a^2 \dfrac{J_1\left(a\sqrt{\eta_n^2 + \chi_v^2}\right)}{a\sqrt{\eta_n^2 + \chi_v^2}} \xrightarrow[\eta_n = \chi_v = 0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$; $\underline{Z}_K = -j \cot(k_0 t)$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 17 Medium-wide round neck and rectangular chamber with absorber

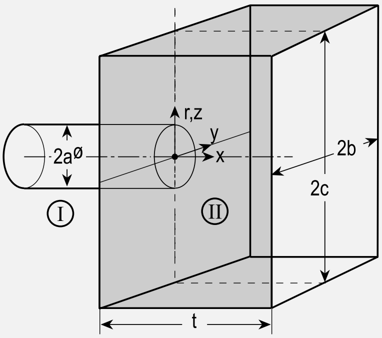
	A round neck with 2a diameter ends in a rectangular chamber with depth t and sides 2b, 2c, filled with porous absorber material. The incident and the reflected waves in the neck are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = \pi a^2/(4bc)$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x,r) = \dfrac{j}{k_0} \dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$

Table 17 continued

Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\ \cosh\left(\gamma_{n,\nu}(x-t)\right)\cos(\eta_n y)\ \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{-1}{\Gamma_i Z_i}\sum_{n,\nu\geq 0} D_{n,\nu}\frac{\gamma_{n,\nu}}{k_0}\sinh\left(\gamma_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\ \cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \ \Rightarrow\ \eta_n b = n\pi;\ n = 0,\ 1,\ 2,\ldots;$ $\sin(\chi_\nu c) = 0 \ \Rightarrow\ \chi_\nu c = \nu\pi;\ \nu = 0,\ 1,\ 2,\ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{\Gamma_i Z_i}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2\ \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{\Gamma_i Z_i}{\pi a^2}\sum_{n,\nu\geq 0}\frac{S_{n,\nu}^2\ \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i\,\frac{S_{n,\nu}}{(\gamma_{n,\nu}/k_0)\ \sinh\left(\gamma_{n,\nu} t\right)\ N_{n,\nu}^{(II)}}\,(B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = \int_{-c}^{+c}\int_{-b}^{+b}\cos(\eta_n y)\ \cos(\chi_\nu z)\ \cdot\ \cos(\eta_{n'} y)\ \cos(\chi_{\nu'} z)\,dy\,dz$ $N_{n,\nu}^{(II)} = bc\,N_n^{(y)}\cdot N_\nu^{(z)};\ \ N_n^{(y)} = \begin{cases} 0;\ n\neq n' \\ 1;\ n = n'\neq 0 \\ 2;\ n = n' = 0\end{cases}\ ;$ $N_\nu^{(z)} = \begin{cases} 0;\ \nu\neq \nu' \\ 1;\ \nu = \nu'\neq 0 \\ 2;\ \nu = \nu' = 0\end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,\nu} = \iint_s \cos(\eta_n y)\ \cos(\chi_\nu z)\,ds$ $= 2\pi a^2\frac{J_1\left(a\sqrt{\eta_n^2+\chi_\nu^2}\right)}{a\sqrt{\eta_n^2+\chi_\nu^2}} \xrightarrow[\eta_n=\chi_\nu=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K\ \ ;\ \ \underline{Z}_K = Z_i\coth(\Gamma_a t)$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 18 Medium-wide round neck and rectangular chamber with front absorber

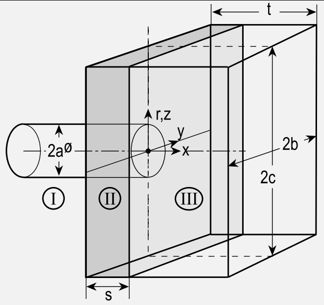
	A round neck with diameter 2a ends in a rectangular chamber with sides 2b, 2c and depth t, with an absorber layer of thickness s adjacent to the orifice. The incident and the reflected waves in the neck are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0\,; Z_i = Z_a/Z_0.\,\sigma = \pi a^2/(4bc)$.
Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu}x} + E_{n,\nu}\, e^{+\gamma_{n,\nu}x}\right) \cdot \cos(\eta_n y)\,\cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{-1}{\Gamma_a Z_{an}}\frac{\partial p}{\partial x} = \frac{1}{\Gamma_{an} Z_{an}} \sum_{n,\nu\geq 0} \frac{\gamma_{n,\nu}}{k_0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu}x} - E_{n,\nu}\, e^{+\gamma_{n,\nu}x}\right) \cdot \cos(\eta_n y)\,\cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\,;\; n = 0, 1, 2, \ldots;$ $\quad \gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi\,;\; \nu = 0, 1, 2, \ldots;$
Field formulation in (III) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{III}(x,y,z) = \sum_{n,\nu\geq 0} F_{n,\nu} \cosh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\,\cos(\chi_\nu z)$ $Z_0 v_{IIIx}(x,y,z) = j \sum_{n,\nu\geq 0} F_{n,\nu} \frac{\kappa_{n,\nu}}{k_0} \sinh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\,\cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi\,;\; n = 0, 1, 2, \ldots;$ $\quad \kappa_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi\,;\; \nu = 0, 1, 2, \ldots;$

Table 18 continued

Equation for C_0	$C_0/B_0 = \left[-1 + \frac{\Gamma_i Z_i}{\pi a^2} \times \sum_{n,v\geq 0} \frac{k_0}{\gamma_{n,v}} \frac{S_{n,v}^2}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\gamma_{n,v} s)\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v} \tanh(\gamma_{n,v} s) + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\kappa_{n,v}(s-t))}\right] \times \left[1 + \frac{\Gamma_i Z_i}{\pi a^2} \sum_{n,v\geq 0} \frac{k_0}{\gamma_{n,v}} \frac{S_{n,v}^2}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\gamma_{n,v} s)\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v} \tanh(\gamma_{n,v} s) + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\kappa_{n,v}(s-t))}\right]^{-1}$
Equations for $D_{n,v}$	$D_{n,v} = (B_0 - C_0)\,\Gamma_i Z_i \frac{k_0}{\gamma_{n,v}} \frac{S_{n,v}}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v}\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v}\left(1 - e^{-2\gamma_{n,v} s}\right) + j\,\Gamma_i Z_i\,\kappa_{n,v}\left(1 + e^{-2\gamma_{n,v} s}\right)\ \tanh(\kappa_{n,v}(s-t))}$
Equations for $E_{n,v}$	$E_{n,v} = (B_0 - C_0)\,\Gamma_i Z_i \frac{k_0}{\gamma_{n,v}} \frac{S_{n,v}}{N_{n,v}^{(II)}} e^{-2\gamma_{n,v} s} \frac{\gamma_{n,v} - j\,\Gamma_i Z_i\,\kappa_{n,v}\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v}\left(1 - e^{-2\gamma_{n,v} s}\right) + j\,\Gamma_i Z_i\,\kappa_{n,v}\left(1 + e^{-2\gamma_{n,v} s}\right)\ \tanh(\kappa_{n,v}(s-t))}$
Equations for $F_{n,v}$	$F_{n,v} = \frac{D_{n,v}\,e^{-\gamma_{n,v} s} + E_{n,v}\,e^{+\gamma_{n,v} s}}{\cosh\ (\kappa_{n,v}(s-t))}$
Mode norms in (II)	$N_{n,v}^{(II)} = \int_{-c}^{+c}\int_{-b}^{+b} \cos(\eta_n y)\ \cos(\chi_v z)\ \cdot\ \cos(\eta_{n'} y)\ \cos(\chi_{v'} z)\ dy\ dz$ $N_{n,v}^{(II)} = bc\,N_n^{(y)} \cdot N_v^{(z)};\ N_n^{(y)} = \begin{cases} 0;\ n \neq n' \\ 1;\ n = n' \neq 0 \\ 2;\ n = n' = 0 \end{cases}\ ;\ N_v^{(z)} = \begin{cases} 0;\ v \neq v' \\ 1;\ v = v' \neq 0 \\ 2;\ v = v' = 0 \end{cases}$

Table 18 continued

Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,v} = \iint_s \cos(\eta_n y)\,\cos(\chi_v z)\,ds = 2\pi a^2 \frac{J_1\left(a\sqrt{\eta_n^2+\chi_v^2}\right)}{a\sqrt{\eta_n^2+\chi_v^2}} \xrightarrow[\eta_n=\chi_v=0]{} \pi a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{B_0+C_0}{B_0-C_0} =$ $\frac{\Gamma_i Z_i}{\pi a^2}\sum_{n,v\geq 0}\frac{k_0}{\gamma_{n,v}}\frac{S_{n,v}^2}{N_{n,v}^{(II)}}\frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v}\tanh(\gamma_{n,v}s)\,\tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v}\tanh(\gamma_{n,v}s) + j\,\Gamma_i Z_i\,\kappa_{n,v}\tanh(\kappa_{n,v}(s-t))}$ $\underline{Z}_K = \frac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = Z_i\frac{D_{0,0}+E_{0,0}}{D_{0,0}-E_{0,0}} = Z_i\frac{1 + j\,Z_i\tanh(\Gamma_i k_0 s)\,\tan(k_0(t-s))}{\tanh(\Gamma_i k_0 s) + j\,Z_i\tan(k_0(t-s))}$

Table 19 Medium-wide round neck and rectangular chamber with rear absorber

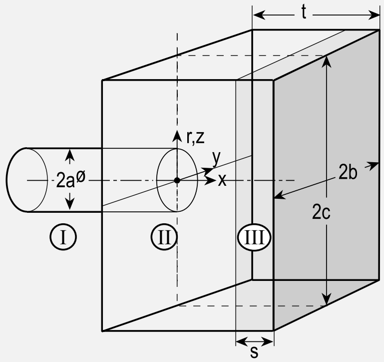
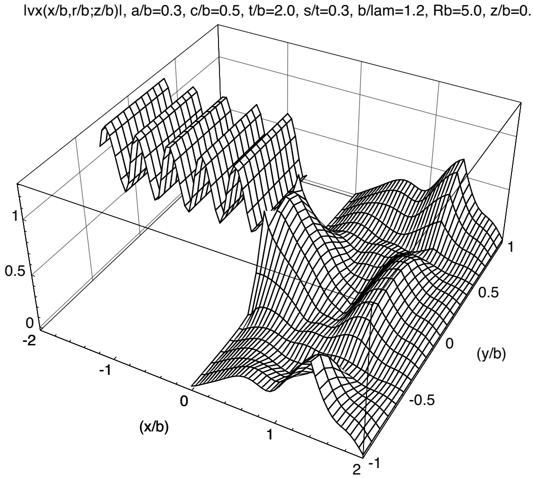
	A round neck with diameter 2a ends in a rectangular chamber with sides 2b, 2c and depth t, with an absorber layer of thickness s adjacent to the back side. The incident and the reflected waves in the neck are plane waves. Absorber characteristic values: $\Gamma_i = \Gamma_a/_0$; $Z_i = Z_a/Z_0$. $\sigma = \pi a^2/(4bc)$.

Table 19 continued

Field formulation in (I)	$p_I(x,r) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x,r) = \frac{j}{k_0}\frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu} x} + E_{n,\nu}\, e^{+\gamma_{n,\nu} x}\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j \sum_{n,\nu\geq 0} \frac{\gamma_{n,\nu}}{k_0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu} x} - E_{n,\nu}\, e^{+\gamma_{n,\nu} x}\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi;\ n = 0, 1, 2, \ldots;$ $\quad \gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi;\ \nu = 0, 1, 2, \ldots;$
Field formulation in (III)	$p_{III}(x,y,z) = \sum_{n,\nu\geq 0} F_{n,\nu} \cosh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $Z_0 v_{IIIx}(x,y,z) = \frac{-1}{\Gamma_i Z_i} \sum_{n,\nu\geq 0} F_{n,\nu} \frac{\kappa_{n,\nu}}{k_0} \sinh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi;\ n = 0, 1, 2, \ldots;$ $\quad \kappa_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi;\ \nu = 0, 1, 2, \ldots;$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{1}{\pi a^2} \sum_{n,\nu\geq 0} \frac{k_0}{\gamma_{n,\nu}} \frac{S_{n,\nu}^2}{N_{n,\nu}^{(II)}} \frac{j\,\Gamma_i Z_i\, \gamma_{n,\nu} - \kappa_{n,\nu} \tanh\left(\gamma_{n,\nu}(t-s)\right)\, \tanh\left(\kappa_{n,\nu} s\right)}{\Gamma_i Z_i\, \gamma_{n,\nu} \tanh\left(\gamma_{n,\nu}(t-s)\right) + j\, \kappa_{n,\nu} \tanh\left(\kappa_{n,\nu} s\right)}\right]$ $\times \left[1 + \frac{1}{\pi a^2} \sum_{n,\nu\geq 0} \frac{k_0}{\gamma_{n,\nu}} \frac{S_{n,\nu}^2}{N_{n,\nu}^{(II)}} \frac{j\,\Gamma_i Z_i\, \gamma_{n,\nu} - \kappa_{n,\nu} \tanh\left(\gamma_{n,\nu}(t-s)\right)\, \tanh\left(\kappa_{n,\nu} s\right)}{\Gamma_i Z_i\, \gamma_{n,\nu} \tanh\left(\gamma_{n,\nu}(t-s)\right) + j\, \kappa_{n,\nu} \tanh\left(\kappa_{n,\nu} s\right)}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = (B_0 - C_0) \frac{k_0}{\gamma_{n,\nu}} \frac{S_{n,\nu}}{N_{n,\nu}^{(II)}} \frac{j\,\Gamma_i Z_i\, \gamma_{n,\nu} - \kappa_{n,\nu} \tanh\left(\kappa_{n,\nu} s\right)}{\Gamma_i Z_i\, \gamma_{n,\nu} \left(1 - e^{-2\gamma_{n,\nu}(t-s)}\right) + j\, \kappa_{n,\nu} \left(1 + e^{-2\gamma_{n,\nu}(t-s)}\right) \tanh\left(\kappa_{n,\nu} s\right)}$

Table 19 continued

Equations for $E_{n,v}$	$E_{n,v} = (B_0 - C_0)\,\frac{k_0}{\gamma_{n,v}}\,\frac{S_{n,v}}{N^{(II)}_{n,v}}\,e^{-2\gamma_{n,v}(t-s)}\,\frac{j\,\Gamma_i Z_i\,\gamma_{n,v} + \kappa_{n,v}\,\tanh\left(\kappa_{n,v} s\right)}{\Gamma_i Z_i\,\gamma_{n,v}\left(1 - e^{-2\gamma_{n,v}(t-s)}\right) + j\,\kappa_{n,v}\left(1 + e^{-2\gamma_{n,v}(t-s)}\right)\,\tanh\left(\kappa_{n,v} s\right)}$
Equations for $F_{n,v}$	$F_{n,v} = \frac{D_{n,v}\,e^{-\gamma_{n,v}(t-s)} + E_{n,v}\,e^{+\gamma_{n,v}(t-s)}}{\cosh\left(\kappa_{n,v} s\right)}$
Mode norms in (II)	$N^{(II)}_{n,v} = \int_{-c}^{+c}\int_{-b}^{+b} \cos(\eta_n y)\,\cos(\chi_v z)\,\cdot\,\cos(\eta_{n'} y)\,\cos(\chi_{v'} z)\,dy\,dz$ $N^{(II)}_{n,v} = bc\,N^{(y)}_n \cdot N^{(z)}_v;\;\; N^{(y)}_n = \begin{cases} 0;\; n \neq n' \\ 1;\; n = n' \neq 0 \\ 2;\; n = n' = 0 \end{cases}\;;\; N^{(z)}_v = \begin{cases} 0;\; v \neq v' \\ 1;\; v = v' \neq 0 \\ 2;\; v = v' = 0 \end{cases}$
Mode coupling (I)–(II) in $s = \pi a^2$	$S_{n,v} = \iint_s \cos(\eta_n y)\,\cos(\chi_v z)\,ds = 2\pi\,a^2\,\frac{J_1\left(a\sqrt{\eta_n^2 + \chi_v^2}\right)}{a\sqrt{\eta_n^2 + \chi_v^2}} \xrightarrow[\eta_n = \chi_v = 0]{} \pi\,a^2$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \frac{1 + C_0/B_0}{1 - C_0/B_0} = \frac{1}{\pi a^2}\sum_{n,v\geq 0}\frac{k_0}{\gamma_{n,v}}\,\frac{S^2_{n,v}}{N^{(II)}_{n,v}}\,\frac{j\,\Gamma_i Z_i\,\gamma_{n,v} - \kappa_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right)\;\tanh\left(\kappa_{n,v} s\right)}{\Gamma_i Z_i\,\gamma_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right) + j\,\kappa_{n,v}\tanh\left(\kappa_{n,v} s\right)}$ $\underline{Z}_K = \frac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = \frac{D_{0,0} + E_{0,0}}{D_{0,0} - E_{0,0}} = \frac{Z_i + j\,\tan\left(k_0(t-s)\right)\;\tanh\left(\Gamma_i\,k_0 s\right)}{Z_i\tan\left(k_0(t-s)\right) - j\,\tanh\left(\Gamma_i\,k_0 s\right)}$



Example of axial particle velocity profiles. Parameters: $a/b = 0.3$; $c/b = 0.5$; $t/b = 2.0$; $s/t = 0.3$; $b/\lambda_0 = 1.2$; $R_b = \Xi b/Z_0 = 5.0$

Table 20 Medium-wide square neck and rectangular duct; full analysis

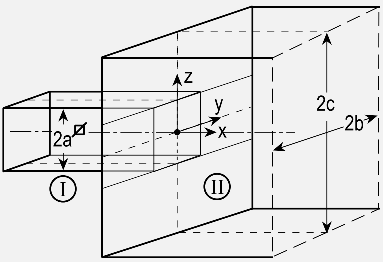
	A square neck with 2a side length ends in a rectangular empty duct with 2b, 2c side lengths. A plane wave with amplitude B_0 is incident in the neck on the orifice; a mode sum is reflected. $\sigma = a^2/(bc)$.
Field formulation in (I)	$p_I(x,y,z) = B_0\, e^{-j\,k_0 x} + \sum_{m,\mu\geq 0,0} C_{m,\mu}\, e^{+\kappa_{m,\mu} x} \cdot \cos(\varepsilon_m y)\, \cos(\varepsilon_\mu z)$ $Z_0 v_{Ix}(x,y,z) = B_0\, e^{-j\,k_0 x} + j \sum_{m,\mu\geq 0,0} C_{m,\mu}\, \frac{\kappa_{m,\mu}}{k_0} e^{+\kappa_{m,\mu} x} \cdot \cos(\varepsilon_m y)\, \cos(\varepsilon_\mu z)$
Eigenvalues $\varepsilon_m a$	$\sin(\varepsilon_m a) = 0 \;\Rightarrow\; \varepsilon_m a = m\pi$; $m = 0, 1, 2, \ldots$; $\kappa_{m,\mu}^2 = \varepsilon_m^2 + \varepsilon_\mu^2 - k_0^2$

Table 20 continued

Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\ e^{-\gamma_{n,\nu}x} \cdot \cos(\eta_n y)\ \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x} = -j \sum_{n,\nu\geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0} e^{-\gamma_{n,\nu}x}$ $\cdot \cos(\eta_n y)\ \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \ \Rightarrow\ \eta_n b = n\pi\ ;\ n = 0, 1, 2, \ldots$ $\sin(\chi_\nu c) = 0 \Rightarrow \chi_\nu c = \nu\pi\ ;\ \nu = 0, 1, 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$
System of equations for $C_{m,\mu}$ $m', \mu' = 0, 1, 2, \ldots$	$\sum_{m,\mu\geq 0} C_{m,\mu} \left[\delta_{m,m'}\ \delta_{\mu,\mu'}\ N^{(I)}_{m',\mu'} + \frac{\kappa_{m,\mu}}{k_0}\sum_{n,\nu\geq 0}\frac{S^{\mu,\nu}_{m,n}\ S^{\mu',\nu}_{m',n}}{(\gamma_{n,\nu}/k_0)\, N^{(II)}_{n,\nu}}\right]$ $= B_0\left[-\delta_{m',0}\ \delta_{\mu',0}\ N^{(I)}_{0,0} + j\sum_{n,\nu\geq 0}\frac{S^{0,\nu}_{0,n}\ S^{\mu',\nu}_{m',n}}{(\gamma_{n,\nu}/k_0)\, N^{(II)}_{n,\nu}}\right]$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \frac{j}{(\gamma_{n,\nu}/k_0)\, N^{(II)}_{n,\nu}}\left(B_0 S^{0,\nu}_{0,n} + j\sum_{m,\mu\geq 0} C_{m,\mu}\frac{\kappa_{m,\mu}}{k_0}\, S^{\mu,\nu}_{m,n}\right)$
Mode norms in (I)	$N^{(I)}_{y,m} = \int_{-a}^{+a} \cos(\varepsilon_m y)\ \cdot\ \cos(\varepsilon_{m'} y)\, dy$ $N^{(I)}_{z,\mu} = \int_{-a}^{+a} \cos(\varepsilon_\mu z)\ \cdot\ \cos(\varepsilon_{\mu'} z)\, dz$ $N^{(I)}_{m,\mu} = N^{(I)}_{y,m} \cdot N^{(I)}_{z,\mu}\ ;\ N^{(I)}_{y,m} = \begin{cases} a\ ;\ m = m' \neq 0 \\ 2a\ ;\ m = m' = 0 \end{cases}\quad ;$ $N^{(I)}_{z,\mu} = \begin{cases} a\ ;\ \mu = \mu' \neq 0 \\ 2a\ ;\ \mu = \mu' = 0 \end{cases}$
Mode norms in (II)	$N^{(II)}_{n,\nu} = N^{(II)}_{y,n} \cdot N^{(II)}_{z,\nu}\ \ ;\ \ N^{(II)}_{y,n} = \begin{cases} b\ \ ;\ n \neq 0 \\ 2b\ ;\ n = 0 \end{cases}\quad ;$ $N^{(II)}_{z,\nu} = \begin{cases} c\ \ ;\ \nu \neq 0 \\ 2c\ ;\ \nu = 0 \end{cases}$

Table 20 continued

Mode coupling (I)–(II)	$S_{m,n}^{\mu,\nu} = \int_{-a}^{+a} \cos(\varepsilon_m y)\cos(\eta_n y)\,dy \int_{-a}^{+a} \cos(\varepsilon_\mu z)\cos(\chi_\nu z)\,dz = S_{m,n}\cdot S^{\mu,\nu}$ $S_{m,n} = \begin{cases} a\left(\dfrac{\sin(a(\varepsilon_m-\eta_n))}{a(\varepsilon_m-\eta_n)} + \dfrac{\sin(a(\varepsilon_m+\eta_n))}{a(\varepsilon_m+\eta_n)}\right) ; & \varepsilon_m \neq \eta_n \\ 2a ; & \varepsilon_m = \eta_n = 0 \\ a\left(1+\dfrac{\sin(a(\varepsilon_m+\eta_n))}{a(\varepsilon_m+\eta_n)}\right) ; & \varepsilon_m = \eta_n \neq 0 \end{cases}$ $S^{\mu,\nu} = \begin{cases} a\left(\dfrac{\sin(a(\varepsilon_\mu-\chi_\nu))}{a(\varepsilon_\mu-\chi_\nu)} + \dfrac{\sin(a(\varepsilon_\mu+\chi_\nu))}{a(\varepsilon_\mu+\chi_\nu)}\right) ; & \varepsilon_\mu \neq \chi_\nu \\ 2a ; & \varepsilon_\mu = \chi_\nu = 0 \\ a\left(1+\dfrac{\sin(a(\varepsilon_\mu+\chi_\nu))}{a(\varepsilon_\mu+\chi_\nu)}\right) ; & \varepsilon_\mu = \chi_\nu \neq 0 \end{cases}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = 1$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_{0,0}\rangle_a}{\langle B_0 - C_{0,0}\rangle_a} = \dfrac{1 + C_{0,0}/B_0}{1 - C_{0,0}/B_0}$

Table 21 Medium-wide square neck and rectangular duct

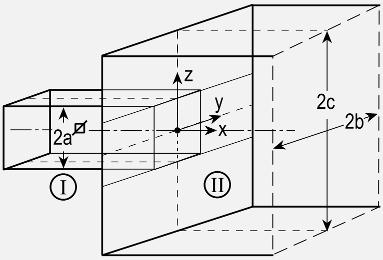
	A square neck with 2a side length ends in a rectangular empty duct with 2b, 2c side lengths. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. $\sigma = a^2/(bc)$.
Simplifications from Table 20	$C_{m,\mu} \to C_0\,;\ \varepsilon_m,\ \varepsilon_\mu \to 0\,;\ \kappa_{m,\mu} \to \kappa_{0,0} = j\,k_0\,;$ $N_{m,\mu}^{(I)} \to N_{0,0}^{(I)} = 4a^2\,;\ S_{m,n}^{\mu,\nu} \to S_{0,n}^{0,\nu} =: S_n^\nu$
Field formulation in (I)	$p_I(x,y,z) = B_0\,e^{-j k_0 x} + C_0\,e^{+j k_0 x}$ $Z_0 v_{Ix}(x,y,z) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial c} = B_0\,e^{-j k_0 x} - C_0\,e^{+j k_0 x}$

Table 21 continued

Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\; e^{-\gamma_{n,\nu}x} \cdot \cos(\eta_n y)\; \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial x}$ $= -j \sum_{n,\nu\geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0}\; e^{-\gamma_{n,\nu}x} \cdot \cos(\eta_n y)\; \cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \;\Rightarrow\; \eta_n b = n\pi \;;\; n = 0,\,1,\,2,\ldots;$ $\sin(\chi_\nu c) = 0 \;\Rightarrow\; \chi_\nu c = \nu\pi \;;\; \nu = 0,\,1,\,2,\ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{j}{4a^2} \sum_{n,\nu\geq 0} \frac{(S_n^\nu)^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{j}{4a^2} \sum_{n,\nu\geq 0} \frac{(S_n^\nu)^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \frac{j\, S_n^\nu}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\,(B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = N_{y,n}^{(II)} \cdot N_{z,\nu}^{(II)} \;;\; N_{y,n}^{(II)} = \begin{cases} b \;;\; n \neq 0 \\ 2b \;;\; n = 0 \end{cases} \;;$ $N_{z,\nu}^{(II)} = \begin{cases} c \;;\; \nu \neq 0 \\ 2c \;;\; \nu = 0 \end{cases}$
Mode coupling (I)–(II)	$S_n^\nu = \int_{-a}^{+a}\int_{-a}^{+a} \cos(\eta_n y)\cos(\chi_\nu z)\; dy\, dz$ $= \left\{\begin{array}{l} 2a \;;\; n = 0 \\ a \;;\; n \neq 0 \end{array}\right\} \times \left\{\begin{array}{l} 2a \;;\; \nu = 0 \\ a \;;\; \nu \neq 0 \end{array}\right\}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K \;;\; \underline{Z}_K = 1$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \frac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 22 Medium-wide square neck and rectangular duct with absorber

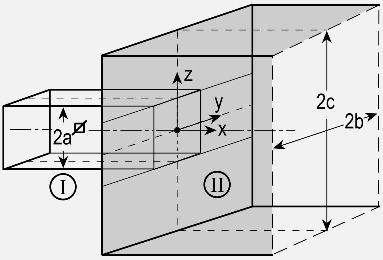
	A square neck with 2a side length ends in a rectangular duct with 2b, 2c side lengths, filled with porous absorber. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = a^2/(bc)$.
Changes rel. to Table 17	$s = \pi a^2 \to 4a^2$; $\sigma = \pi a^2/(4bc) \to a^2/(bc)$; $S_{n,\nu} \to S_n^\nu$
Field formulation in (I)	$p_I(x,y,z) = B_0\, e^{-j k_0 x} + C_0\, e^{+j k_0 x}$ $Z_0 v_{Ix}(x,y,z) = \frac{j}{k_0}\frac{\partial p}{\partial c} = B_0\, e^{-j k_0 x} - C_0\, e^{+j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\; e^{-\gamma_{n,\nu} x} \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{1}{\Gamma_i Z_i} \sum_{n,\nu\geq 0} D_{n,\nu} \frac{\gamma_{n,\nu}}{k_0}\, e^{-\gamma_{n,\nu} x} \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \;\Rightarrow \eta_n b = n\pi$; $n = 0, 1, 2, \ldots$; $\sin(\chi_\nu c) = 0 \;\Rightarrow \chi_\nu c = \nu\pi$; $\nu = 0, 1, 2, \ldots$; $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \frac{\Gamma_i Z_i}{4a^2} \sum_{n,\nu\geq 0} \frac{(S_n^\nu)^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{\Gamma_i Z_i}{4a^2} \sum_{n,\nu\geq 0} \frac{(S_n^\nu)^2}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i \frac{S_n^\nu}{(\gamma_{n,\nu}/k_0)\, N_{n,\nu}^{(II)}} (B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = N_{y,n}^{(II)} \cdot N_{z,\nu}^{(II)}$; $N_{y,n}^{(II)} = \begin{cases} b & ;\; n \neq 0 \\ 2b & ;\; n = 0 \end{cases}$; $N_{z,\nu}^{(II)} = \begin{cases} c & ;\; \nu \neq 0 \\ 2c & ;\; \nu = 0 \end{cases}$

Table 22 continued

Mode coupling (I)–(II)	$S_n^\nu = \int_{-a}^{+a}\int_{-a}^{+a} \cos(\eta_n y)\cos(\chi_\nu z)\; dy\, dz$ $= \left\{ \begin{array}{l} 2a\,;\ n=0 \\ a\ ;\ n\neq 0 \end{array} \right\} \times \left\{ \begin{array}{l} 2a\,;\ \nu=0 \\ a\ ;\ \nu\neq 0 \end{array} \right\}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K \quad ; \quad \underline{Z}_K = Z_i$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 23 Medium-wide square neck and rectangular empty chamber

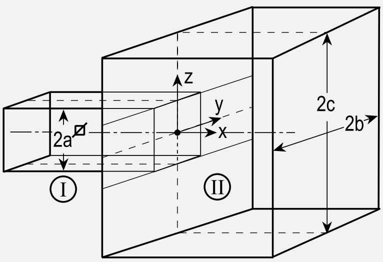
	A square neck with 2a side length ends in a rectangular empty chamber with 2b, 2c side lengths and depth t. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. $\sigma = a^2/(bc)$.
Changes rel. to Table 16	$s = \pi a^2 \to 4a^2\ ;\ \sigma = \pi a^2/(4bc) \to a^2/(bc)\ ;$ $S_{n,\nu} \to S_n^\nu$
Field formulation in (I)	$p_I(x) = B_0 e^{-j k_0 x} + C_0 e^{j k_0 x}$ $Z_0 v_{Ix}(x) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{j k_0 x}$
Field formulation in (II)	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu} \cosh\left(\gamma_{n,\nu}(x-t)\right)\cdot \cos(\eta_n y)\cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = j \sum_{n,\nu\geq 0} D_{n,\nu}\dfrac{\gamma_{n,\nu}}{k_0} \sinh\left(\gamma_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\cos(\chi_\nu z)$
Eigenvalues $\eta_n b, \chi_\nu c$	$\sin(\eta_n b) = 0 \;\Rightarrow \eta_n b = n\pi\,;\ n = 0, 1, 2, \ldots;$ $\sin(\chi_\nu c) = 0 \;\Rightarrow \chi_\nu c = \nu\pi\,;\ \nu = 0, 1, 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$

Table 23 continued

Equation for C_0	$C_0/B_0 = \left[-1 + \frac{j}{4a^2}\sum\limits_{n,\nu\geq 0}\frac{(S_n^\nu)^2\,\coth\left(\gamma_{n,\nu}t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right] \times \left[1 + \frac{j}{4a^2}\sum\limits_{n,\nu\geq 0}\frac{(S_n^\nu)^2\,\coth\left(\gamma_{n,\nu}t\right)}{(\gamma_{n,\nu}/k_0)\,N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = j\,\frac{S_n^\nu}{(\gamma_{n,\nu}/k_0)\,\sinh\left(\gamma_{n,\nu}t\right)\,N_{n,\nu}^{(II)}}\,(B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = N_{y,n}^{(II)}\cdot N_{z,\nu}^{(II)}\ ;\ N_{y,n}^{(II)} = \begin{cases} b & ;\ n\neq 0 \\ 2b & ;\ n=0\end{cases}\ ;$ $N_{z,\nu}^{(II)} = \begin{cases} c & ;\ \nu\neq 0 \\ 2c & ;\ \nu=0\end{cases}$
Mode coupling (I)–(II) in $s = 4a^2$	$S_n^\nu = \int\limits_{-a}^{+a}\int\limits_{-a}^{+a}\cos(\eta_n y)\cos(\chi_\nu z)\ dy\,dz$ $= \begin{Bmatrix} 2a\ ;\ n=0 \\ a\ ;\ n\neq 0\end{Bmatrix} \times \begin{Bmatrix} 2a\ ;\ \nu=0 \\ a\ ;\ \nu\neq 0\end{Bmatrix}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K\ \ ;\ \ \underline{Z}_K = -j\cot(k_0t)$ $Z_{sh} = \frac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \frac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 24 Medium-wide square neck and rectangular chamber with absorber

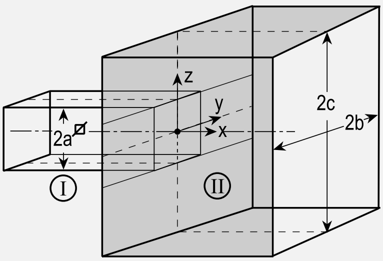
	A square neck with 2a side length ends in a rectangular chamber with 2b, 2c side lengths and depth t, filled with porous absorber material. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = a^2/(bc)$.
Changes rel. to Table 17	$s = \pi a^2 \to 4a^2\ ;\ \sigma = \pi a^2/(4bc) \to a^2/(bc)\ ;$ $S_{n,\nu} \to S_n^\nu$

Table 24 continued

Field formulation in (I)	$p_I(x) = B_0 e^{-j\,k_0 x} + C_0 e^{j\,k_0 x}$ $Z_0 v_{Ix}(x) = \dfrac{j}{k_0}\dfrac{\partial p}{\partial x} = B_0 e^{-j\,k_0 x} - C_0 e^{j\,k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu\geq 0} D_{n,\nu}\ \cosh\left(\gamma_{n,\nu}(x-t)\right)\cdot\cos(\eta_n y)\ \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \dfrac{-1}{\Gamma_i Z_i}\sum_{n,\nu\geq 0} D_{n,\nu}\dfrac{\gamma_{n,\nu}}{k_0}\sinh\left(\gamma_{n,\nu}(x-t)\right)\cdot\cos(\eta_n y)\ \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \ \Rightarrow\ \eta_n b = n\pi\,;\ n = 0,\ 1,\ 2, \ldots;$ $\sin(\chi_\nu c) = 0 \ \Rightarrow \chi_\nu c = \nu\pi\,;\ \nu = 0,\ 1,\ 2, \ldots;$ $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$
Equation for C_0	$C_0/B_0 = \left[-1 + \dfrac{\Gamma_i Z_i}{4a^2}\sum_{n,\nu\geq 0}\dfrac{(S_n^\nu)^2\ \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\ N_{n,\nu}^{(II)}}\right] \times \left[1 + \dfrac{\Gamma_i Z_i}{4a^2}\sum_{n,\nu\geq 0}\dfrac{(S_n^\nu)^2\ \coth\left(\gamma_{n,\nu} t\right)}{(\gamma_{n,\nu}/k_0)\ N_{n,\nu}^{(II)}}\right]^{-1}$
Equations for $D_{n,\nu}$	$D_{n,\nu} = \Gamma_i Z_i \dfrac{S_n^\nu}{(\gamma_{n,\nu}/k_0)\ \sinh\left(\gamma_{n,\nu} t\right)\ N_{n,\nu}^{(II)}}\,(B_0 - C_0)$
Mode norms in (II)	$N_{n,\nu}^{(II)} = N_{y,n}^{(II)}\cdot N_{z,\nu}^{(II)}\ ;\quad N_{y,n}^{(II)} = \begin{cases} b\ ;\ n \neq 0 \\ 2b\ ;\ n = 0\end{cases}\ ;$ $N_{z,\nu}^{(II)} = \begin{cases} c\ ;\ \nu \neq 0 \\ 2c\ ;\ \nu = 0\end{cases}$
Mode coupling (I)–(II) in $s = 4a^2$	$S_n^\nu = \int_{-a}^{+a}\int_{-a}^{+a}\cos(\eta_n y)\cos(\chi_\nu z)\ dy\,dz$ $= \begin{Bmatrix} 2a\,;\ n = 0 \\ a\ ;\ n \neq 0\end{Bmatrix} \times \begin{Bmatrix} 2a\,;\ \nu = 0 \\ a\ ;\ \nu \neq 0\end{Bmatrix}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma\cdot \underline{Z}_K\ ;\quad \underline{Z}_K = Z_i\coth(\Gamma_a t)$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_s}{\langle Z_0 v_{Ix}(0,r)\rangle_s} = \dfrac{1 + C_0/B_0}{1 - C_0/B_0}$

Table 25 Medium-wide square neck and rectangular chamber with front absorber

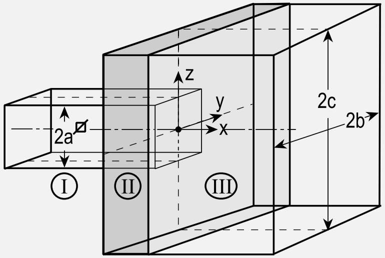
	A square neck with 2a side length ends in a rectangular chamber with 2b, 2c side lengths and depth t, partially filled with a porous absorber layer of thickness s adjacent to the neck. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = a^2/(bc)$.
Changes rel. to Table 18	$s = \pi a^2 \to 4a^2$; $\sigma = \pi a^2/(4bc) \to a^2/(bc)$; $S_{n,\nu} \to S_n^\nu$
Field formulation in (I)	$p_I(x) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x) = \frac{j}{k_0} \frac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$
Field formulation in (II) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{II}(x,y,z) = \sum_{n,\nu \geq 0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu} x} + E_{n,\nu}\, e^{+\gamma_{n,\nu} x}\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $Z_0 v_{IIx}(x,y,z) = \frac{-1}{\Gamma_a Z_{an}} \frac{\partial p}{\partial x} = \frac{1}{\Gamma_{an} Z_{an}} \sum_{n,\nu \geq 0} \frac{\gamma_{n,\nu}}{k_0} \left(D_{n,\nu}\, e^{-\gamma_{n,\nu} x} - E_{n,\nu}\, e^{+\gamma_{n,\nu} x}\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \Rightarrow \eta_n b = n\pi$; $n = 0, 1, 2, \ldots$; $\gamma_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 + \Gamma_a^2$ $\sin(\chi_\nu c) = 0 \Rightarrow \chi_\nu c = \nu\pi$; $\nu = 0, 1, 2, \ldots$;
Field formulation in (III) Eigenvalues $\eta_n b, \chi_\nu c$	$p_{III}(x,y,z) = \sum_{n,\nu \geq 0} F_{n,\nu} \cosh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $Z_0 v_{IIIx}(x,y,z) = j \sum_{n,\nu \geq 0} F_{n,\nu} \frac{\kappa_{n,\nu}}{k_0} \sinh\left(\kappa_{n,\nu}(x-t)\right) \cdot \cos(\eta_n y)\, \cos(\chi_\nu z)$ $\sin(\eta_n b) = 0 \Rightarrow \eta_n b = n\pi$; $n = 0, 1, 2, \ldots$; $\kappa_{n,\nu}^2 = \eta_n^2 + \chi_\nu^2 - k_0^2$ $\sin(\chi_\nu c) = 0 \Rightarrow \chi_\nu c = \nu\pi$; $\nu = 0, 1, 2, \ldots$;

Table 25 continued

Equation for C_0	$C_0/B_0 = \left[-1 + \frac{\Gamma_i Z_i}{4a^2} \sum_{n,v\geq 0} \frac{k_0}{\gamma_{n,v}} \frac{(S_n^v)^2}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\gamma_{n,v} s)\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v} \tanh(\gamma_{n,v} s) + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\kappa_{n,v}(s-t))}\right] \times \left[1 + \frac{\Gamma_i Z_i}{4a^2} \sum_{n,v\geq 0} \frac{k_0}{\gamma_{n,v}} \frac{(S_n^v)^2}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\gamma_{n,v} s)\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v} \tanh(\gamma_{n,v} s) + j\,\Gamma_i Z_i\,\kappa_{n,v} \tanh(\kappa_{n,v}(s-t))}\right]^{-1}$
Equations for $D_{n,v}$	$D_{n,v} = (B_0 - C_0)\,\Gamma_i Z_i \frac{k_0}{\gamma_{n,v}} \frac{S_n^v}{N_{n,v}^{(II)}} \frac{\gamma_{n,v} + j\,\Gamma_i Z_i\,\kappa_{n,v}\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v}\left(1 - e^{-2\gamma_{n,v} s}\right) + j\,\Gamma_i Z_i\,\kappa_{n,v}\left(1 + e^{-2\gamma_{n,v} s}\right)\ \tanh(\kappa_{n,v}(s-t))}$
Equations for $E_{n,v}$	$E_{n,v} = (B_0 - C_0)\,\Gamma_i Z_i \frac{k_0}{\gamma_{n,v}} \frac{S_n^v}{N_{n,v}^{(II)}} e^{-2\gamma_{n,v} s} \frac{\gamma_{n,v} - j\,\Gamma_i Z_i\,\kappa_{n,v}\ \tanh(\kappa_{n,v}(s-t))}{\gamma_{n,v}\left(1 - e^{-2\gamma_{n,v} s}\right) + j\,\Gamma_i Z_i\,\kappa_{n,v}\left(1 + e^{-2\gamma_{n,v} s}\right)\ \tanh(\kappa_{n,v}(s-t))}$
Equations for $F_{n,v}$	$F_{n,v} = \frac{D_{n,v}\, e^{-\gamma_{n,v} s} + E_{n,v}\, e^{+\gamma_{n,v} s}}{\cosh(\kappa_{n,v}(s-t))}$
Mode norms in (II)	$N_{n,v}^{(II)} = N_{y,n}^{(II)} \cdot N_{z,v}^{(II)}\ ;\ N_{y,n}^{(II)} = \begin{cases} b\ ;\ n \neq 0 \\ 2b\ ;\ n = 0 \end{cases}\ ; N_{z,v}^{(II)} = \begin{cases} c\ ;\ v \neq 0 \\ 2c\ ;\ v = 0 \end{cases}$
Mode coupling (I)–(II) in $s = 4a^2$	$S_n^v = \int_{-a}^{+a}\int_{-a}^{+a} \cos(\eta_n y) \cos(\chi_v z)\ dy\,dz = \left\{ \begin{array}{l} 2a\ ;\ n = 0 \\ a\ ;\ n \neq 0 \end{array} \right\} \times \left\{ \begin{array}{l} 2a\ ;\ v = 0 \\ a\ ;\ v \neq 0 \end{array} \right\}$

Table 25 continued

Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \dfrac{\langle p_I(0,r)\rangle_a}{\langle Z_0 v_{Ix}(0,r)\rangle_a} = \dfrac{\langle B_0 + C_0\rangle_a}{\langle B_0 - C_0\rangle_a} = \dfrac{B_0 + C_0}{B_0 - C_0}$ $= \dfrac{\Gamma_i Z_i}{4a^2} \sum_{n,\nu \geq 0} \dfrac{k_0}{\gamma_{n,\nu}} \dfrac{(S_n^\nu)^2}{N_{n,\nu}^{(II)}} \dfrac{\gamma_{n,\nu} + j\,\Gamma_i Z_i\,\kappa_{n,\nu} \tanh(\gamma_{n,\nu} s)\ \tanh(\kappa_{n,\nu}(s-t))}{\gamma_{n,\nu} \tanh(\gamma_{n,\nu} s) + j\,\Gamma_i Z_i\,\kappa_{n,\nu} \tanh(\kappa_{n,\nu}(s-t))}$ $\underline{Z}_K = \dfrac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = Z_i \dfrac{D_{0,0} + E_{0,0}}{D_{0,0} - E_{0,0}} = Z_i \dfrac{1 + j\,Z_i \tanh(\Gamma_i k_0 s)\ \tan(k_0(t-s))}{\tanh(\Gamma_i k_0 s) + j\,Z_i\ \tan(k_0(t-s))}$

Table 26 Medium-wide square neck and rectangular chamber with rear absorber

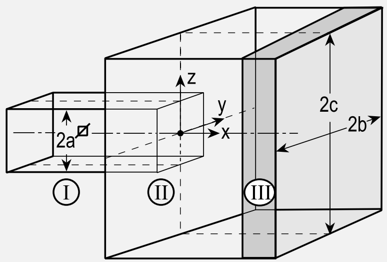
	A square neck with 2a side length ends in a rectangular chamber with 2b, 2c side lengths and depth t, partially filled with a porous absorber layer of thickness s at the back side. A plane wave with amplitude B_0 is incident in the neck on the orifice; a plane wave with amplitude C_0 is reflected. Absorber characteristic values: $\Gamma_i = \Gamma_a/k_0$; $Z_i = Z_a/Z_0$. $\sigma = a^2/(bc)$.
Changes rel. to Table 19	$s = \pi a^2 \to 4a^2$; $\sigma = \pi a^2/(4bc) \to a^2/(bc)$; $S_{n,\nu} \to S_n^\nu$
Field formulation in (I)	$p_I(x) = B_0 e^{-j k_0 x} + C_0 e^{+j k_0 x}$ $Z_0 v_{Ix}(x) = \dfrac{j}{k_0} \dfrac{\partial p}{\partial x} = B_0 e^{-j k_0 x} - C_0 e^{+j k_0 x}$

Table 26 continued

Field formulation in (II)	$p_{II}(x,y,z)=\sum_{n,v\geq 0}\left(D_{n,v}\,e^{-\gamma_{n,v}x}+E_{n,v}\,e^{+\gamma_{n,v}x}\right)\cdot\cos(\eta_n y)\,\cos(\chi_v z)$ $Z_0 v_{IIx}(x,y,z)=\frac{j}{k_0}\frac{\partial p}{\partial x}=-j\sum_{n,v\geq 0}\frac{\gamma_{n,v}}{k_0}\left(D_{n,v}\,e^{-\gamma_{n,v}x}-E_{n,v}\,e^{+\gamma_{n,v}x}\right)\cdot\cos(\eta_n y)\,\cos(\chi_v z)$
Eigenvalues $\eta_n b$, $\chi_v c$	$\sin(\eta_n b)=0\ \Rightarrow \eta_n b=n\pi\,;\ n=0,1,2,\dots;$ $\quad\gamma_{n,v}^2=\eta_n^2+\chi_v^2-k_0^2$ $\sin(\chi_v c)=0\ \Rightarrow\ \chi_v c=v\pi\,;\ v=0,1,2,\dots;$
Field formulation in (III)	$p_{III}(x,y,z)=\sum_{n,v\geq 0}F_{n,v}\,\cosh\left(\kappa_{n,v}(x-t)\right)\cdot\cos(\eta_n y)\,\cos(\chi_v z)$ $Z_0 v_{IIIx}(x,y,z)=\frac{-1}{\Gamma_i Z_i}\sum_{n,v\geq 0}F_{n,v}\frac{\kappa_{n,v}}{k_0}\sinh\left(\kappa_{n,v}(x-t)\right)\cdot\cos(\eta_n y)\,\cos(\chi_v z)$
Eigenvalues $\eta_n b, \chi_v c$	$\sin(\eta_n b)=0\ \Rightarrow \eta_n b=n\pi\,;\ n=0,1,2,\dots;$ $\quad\kappa_{n,v}^2=\eta_n^2+\chi_v^2+\Gamma_a^2$ $\sin(\chi_v c)=0\ \Rightarrow\ \chi_v c=v\pi\,;\ v=0,1,2,\dots;$
Equation for C_0	$C_0/B_0 = \left[-1+\frac{1}{4a^2}\sum_{n,v\geq 0}\frac{k_0}{\gamma_{n,v}}\frac{(S_n^v)^2}{N_{n,v}^{(II)}}\frac{j\,\Gamma_i Z_i\,\gamma_{n,v}-\kappa_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right)\ \tanh\left(\kappa_{n,v}s\right)}{\Gamma_i Z_i\,\gamma_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right)+j\ \kappa_{n,v}\tanh\left(\kappa_{n,v}s\right)}\right]$ $\times\left[1+\frac{1}{4a^2}\sum_{n,v\geq 0}\frac{k_0}{\gamma_{n,v}}\frac{(S_n^v)^2}{N_{n,v}^{(II)}}\frac{j\,\Gamma_i Z_i\,\gamma_{n,v}-\kappa_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right)\ \tanh\left(\kappa_{n,v}s\right)}{\Gamma_i Z_i\,\gamma_{n,v}\tanh\left(\gamma_{n,v}(t-s)\right)+j\ \kappa_{n,v}\tanh\left(\kappa_{n,v}s\right)}\right]^{-1}$
Equations for $D_{n,v}$	$D_{n,v}=(B_0-C_0)\frac{k_0}{\gamma_{n,v}}\frac{S_n^v}{N_{n,v}^{(II)}}\frac{j\,\Gamma_i Z_i\,\gamma_{n,v}-\ \kappa_{n,v}\ \tanh\left(\kappa_{n,v}s\right)}{\Gamma_i Z_i\,\gamma_{n,v}\left(1-e^{-2\gamma_{n,v}(t-s)}\right)+j\,\kappa_{n,v}\left(1+e^{-2\gamma_{n,v}(t-s)}\right)\ \tanh\left(\kappa_{n,v}s\right)}$

Table 26 continued

Equations for $E_{n,v}$	$E_{n,v} = (B_0 - C_0)\,\frac{k_0}{\gamma_{n,v}}\,\frac{S_n^v}{N_{n,v}^{(II)}}\,e^{-2\gamma_{n,v}(t-s)}\,\frac{j\,\Gamma_i Z_i\,\gamma_{n,v} + \kappa_{n,v}\,\tanh(\kappa_{n,v}s)}{\Gamma_i Z_i\,\gamma_{n,v}\left(1-e^{-2\gamma_{n,v}(t-s)}\right) + j\,\kappa_{n,v}\left(1+e^{-2\gamma_{n,v}(t-s)}\right)\,\tanh(\kappa_{n,v}s)}$
Equations for $F_{n,v}$	$F_{n,v} = \frac{D_{n,v}\,e^{-\gamma_{n,v}(t-s)} + E_{n,v}\,e^{+\gamma_{n,v}(t-s)}}{\cosh(\kappa_{n,v}s)}$
Mode norms in (II)	$N_{n,v}^{(II)} = N_{y,n}^{(II)} \cdot N_{z,v}^{(II)}\ ;\ N_{y,n}^{(II)} = \begin{cases} b & ;\ n \neq 0 \\ 2b & ;\ n = 0 \end{cases}\ ;N_{z,v}^{(II)} = \begin{cases} c & ;\ v \neq 0 \\ 2c & ;\ v = 0 \end{cases}$
Mode coupling (I)–(II) in $s = 4a^2$	$S_n^v = \int_{-a}^{+a}\int_{-a}^{+a} \cos(\eta_n y)\cos(\chi_v z)\,dy\,dz = \left\{\begin{array}{l} 2a;\ n = 0 \\ a\ ;\ n \neq 0 \end{array}\right\} \times \left\{\begin{array}{l} 2a;\ v = 0 \\ a\ ;\ v \neq 0 \end{array}\right\}$
Orifice partition impedance	$Z_{Mi} = Z_{sh} - \sigma \cdot \underline{Z}_K$ $Z_{sh} = \frac{1 + C_0/B_0}{1 - C_0/B_0} = \frac{1}{4a^2}\sum_{n,v\geq 0}\frac{k_0}{\gamma_{n,v}}\,\frac{(S_n^v)^2}{N_{n,v}^{(II)}}\,\frac{j\,\Gamma_i Z_i\,\gamma_{n,v} - \kappa_{n,v}\tanh(\gamma_{n,v}(t-s))\,\tanh(\kappa_{n,v}s)}{\Gamma_i Z_i\,\gamma_{n,v}\tanh(\gamma_{n,v}(t-s)) + j\,\kappa_{n,v}\tanh(\kappa_{n,v}s)}$ $\underline{Z}_K = \frac{\langle p_{II}(0,r)\rangle_b}{\langle Z_0 v_{IIx}(0,r)\rangle_b} = \frac{D_{0,0} + E_{0,0}}{D_{0,0} - E_{0,0}} == \frac{Z_i + j\,\tan(k_0(t-s))\,\tanh(\Gamma_i k_0 s)}{Z_i \tan(k_0(t-s)) - j\,\tanh(\Gamma_i k_0 s)}$

References

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