R Ultrasound Absorption in Solids

with W. Arnold

List of symbols used in this Chapter:

a	transducer radius;
A	area, amplitude;
b	Burger's vector;
c	elastic constant;
c_p	specific heat at constant pressure;
d	thickness, grain size;
e	piezoelectric constant;
f	frequency;
D	deformation potential;
E	electric field, Young's modulus;
G	shear modulus;
H	magnetic field;
k	k-vector;
k_B	Boltzmann's constant;
K	piezoelectric coupling factor, compressibility;
l_e	mean free path for electrons;
M	elastic moduli, deformation potential;
n	volume density of scatterers;
P	polarisation, power;
Q	Q-value;
S	ultrasonic energy;
$S_{L,T}$	scattering parameter;
T	temperature;
v	sound velocity;
v_F	Fermi velocity;
V	volume;
W	energy;
x	path length;
Z	acoustic impedance;
α	attenuation, absorption;
β	thermal expansion;
ε	strain;
γ	cross-section, heat conduction, Grüneisen constant;

κ dielectric permittivity, k-vector;
λ wavelength;
λ, μ Lamé constants;
η polarisability;
ξ displacement;
ρ density;
τ relaxation time;
Θ angle, Debye temperature;
σ stress, electrical conductivity;
υ frequency;
ω angular frequency

R.1   Generation of Ultrasound

Surface excitation:

In most cases ultrasound is generated by piezoelectric transducers. The principle can be easily seen by a one-dimensional consideration.

Piezoelectric equations: $\sigma_1 = c_{11}\varepsilon_1 - e_{11}E_1; \quad P_1 = e_{11}\varepsilon_1 + \eta_{11}E_1,$ (1,2)

E is the electric field, ε the strain, e the piezoelectric constant, η the dielectric suszeptibility, σ the stress and P the electric polarisation. Applying an electrical field $E_1(t) = E_0 e^{j\omega t}$ across the surface of a piezoelectric crystal yields for the wave equation:

$$\frac{\partial^2 \xi}{dx^2} - \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2} = d_{11}\frac{\partial E_1}{\partial x}$$ (3)

with $d_{11} = e_{11}/c_{11}$ and the boundary condition that the surface is stress free:

$$\partial \xi / \partial x = \varepsilon_1 = d_{11}E_1(t)$$ (4)

one obtains:

$$\xi = j\,(d_{11}E_0/k)\,e^{j\omega t}e^{-j(x-x_0)}.$$ (5)

Equation (3) shows that the gradient of the E-field is the source of ultrasound. This holds for all piezoelectric transducers.

The radiated ultrasonic energy is, see figure below:

$$S = P_{in}\left[c_{11}d_{11}^2 vAQ/\omega\varepsilon_r\varepsilon_0\left(\varepsilon_r V_g + V_r\right)\right].$$ (6)

A is the cross section of the rod, Q is the quality factor of the resonator, P_{in} is the input electrical intensity. Equation (6) is also valid correspondingly for placing the transducer into a capacitor.
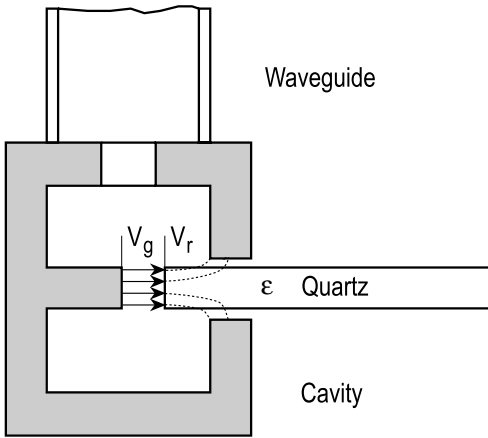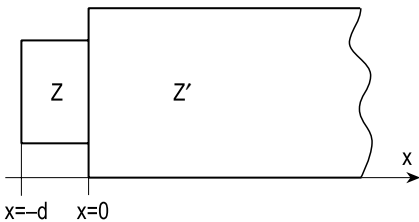
Principle of surface generation of ultrasonic waves due to piezoelectricity

Thin films and piezoelectric discs:

In the case there is a thin film or a piezoelectric disc as a transducer, waves are generated at both surfaces of the transducer:

$$S = \left(\frac{Z'}{Z}\right) \frac{Av_1 \kappa K^2 E_{0^2} (1 - \cos(kd))^2}{2 \left(\sin^2(kd) + (Z'/Z)^2 \cos^2(kd)\right)}. \tag{7}$$

Here, d is the film thickness, Z' and Z are the impedances of the transducer and material, respectively, K^2 is the piezoelectric coupling factor, κ is the dielectric permittivity, and A is the area of the transducer, see figure below.

Principle of generation of ultrasonic waves due to piezoelectricity in a thin film

R.2   Ultrasonic attenuation

General considerations:

Strain wave:

$$\sigma(x, t) = \sigma_0 e^{j(\omega t - kx)} \tag{1}$$

The relation between k-vector k and angular frequency ω is $k^2 v^2 = \omega^2$ with phase velocity v. If there is attenuation, either the velocity v, the k-vector or the frequency is complex:

$$v = v_1 + j\,v_2; \quad k = k_1 - j\,\alpha. \tag{2}$$

Hence:

$$\sigma(x, t) = \sigma_0 e^{-\alpha x} e^{j(\omega t - k_1 x)}, \tag{3}$$

α is the damping coefficient and v_p is the sound velocity as a real quantity; $v_p = \omega/k_1$.

With (2):

$$\alpha = \frac{\omega v_2}{\left(v_1^2 + v_2^2\right)} = \frac{\omega v_2}{|v|^2}; \quad k_1 = \frac{\omega v_1}{\left(v_1^2 + v_2^2\right)} = \frac{\omega v_1}{|v|^2}. \tag{4a,b}$$

Likewise one may start with a complex elastic modulus: $c = c_1 + j\,c_2$ (5)
and obtains:

$$\alpha = \left(\frac{\omega v_2}{v_\rho v_1}\right) = \left(\frac{\omega\,(c_2)^{1/2}}{v\rho\,(c_1)^{1/2}}\right). \tag{6}$$

Less common is the assumption that the frequency is complex and k real, details see [Truell/Elbaum/Chick (1969)].

Definition of attenuation:

$$\sigma(x) = \sigma_0 e^{-\alpha x}; \quad \alpha = \frac{1}{x_2 - x_1}\log_e\left(\frac{\sigma(x_1)}{\sigma(x_2)}\right), \tag{7a,b}$$

then

$$\alpha = \left(\frac{1}{x_2 - x_1}\right) 20\log_{10}\left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \quad [\text{dB/unit length}]; \tag{8a}$$

$$\alpha = \left(\frac{1}{x_2 - x_1}\right) \log_e\left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \quad [\text{nepers/unit length}]. \tag{8b}$$

Conversion factors:

$$\alpha\,[\text{dB/unit length}] = 8.686\,\alpha\,[\text{nepers/unit length}] \tag{9a}$$

$$\alpha\,[\text{dB/unit time}] = \alpha\,[\text{dB/unit length}] \times \text{sound velocity} \tag{9b}$$

Logarithmic decrement: $\quad\quad\quad \delta = \log_e\left(\frac{\sigma_n}{\sigma_{n+1}}\right), \tag{10}$

where two consecutive oscillations are considered. Hence:

$$\delta\,[\text{nepers}] = \alpha\,[\text{nepers/cm}]\,\lambda\,[\text{cm}]; \quad \delta = \frac{\alpha\,[\text{nepers/cm}]}{v\,[1/\sec]}v\,[\text{cm/sec}]. \tag{11a,b}$$

Therefore:

$$\alpha \, [\text{dB}/\mu \sec] = 8.68 \times 10^{-6} v \, [\text{cm}/\sec] \, \alpha \, [\text{nepers/cm}]$$

$$\alpha \, [\text{dB}/\mu \sec] = 8.68 \times 10^{-6} \upsilon \, [\sec^{-1}] \, \delta \, [\text{nepers}] \tag{12}$$

$$\alpha \, [\text{dB}/\mu \sec] = \alpha \, [\text{dB/cm}] \times 10^{-6} v \, [\text{cm}/\sec].$$

Definition of Q-value from the bandwidth Δv and resonance v_r: $Q = v_r/\Delta \upsilon$ $\quad$ (13)

equivalent to:

$$Q = \omega_r \frac{\text{energy in the system}}{\text{energy dissipated per second}} \tag{14a}$$

or $\quad$ $Q = \omega_1 \dfrac{W}{dW/dt}$, $\qquad$ (14b)

where W is the energy stored dissipating with dW/dt per second, hence:

$$W = W_o e^{-(\omega_1/Q)\,t}. \tag{15}$$

There are close similarities to the behaviour of oscillators.

Geometrical losses:

Diffraction losses play an important role, particularly at low frequencies:

$$\alpha_g = \frac{1.8}{1.05 a^2/\lambda} \, [\text{dB/cm}] \quad \text{or} \quad \alpha_d = 1.7 \frac{\upsilon}{a^2 v} \, [\text{dB/cm}] , \tag{16}$$

a = radius of transducer, λ = wavelength, υ = frequency, and v = sound velocity.

Non-parallelism is another source of geometrical attenuation:

$$\alpha \cong 8.7 \times 10^{-5} v a \theta, \tag{17}$$

θ is the angle between non-parallel surfaces of the sample.

Scattering losses:

Scattering at single spheres:

Scattering losses occur if there is a change in local mechanical impedance in poly-crystalline and two- or multiphase materials. In case of cavities, diameter a , in elastic materials (denoted 1), this leads to a scattering cross-section for da $\ll$ 1 (Rayleigh approximation):

$$\gamma_N = \frac{4}{9} g_c (k_1 \, a)^4 , \tag{18a}$$

$$g_c = \frac{4}{3} + 40 \frac{2 + 3 \, (\kappa_1/k_1)^5}{(4 - 9 \, (\kappa_1/k_1)^2)^2} - \frac{3}{2} \left(\frac{\kappa_1}{k_1} \right)^2 + \frac{2}{3} \left(\frac{\kappa_1}{k_1} \right)^3 + \frac{9}{16} \left(\frac{\kappa_1}{k_1} \right)^4 . \tag{18b}$$

Here (and below):

$$k = \omega \left(\frac{\rho}{\lambda + 2\mu} \right)^{1/2}, \qquad \kappa = \omega / \left(\mu/\rho \right)^{1/2} \tag{19}$$

with k = longitudinal wavenumber, κ = transversal wavenumber, ρ = density, λ, μ = Lamé constants.

In case of *elastic spheres* (denoted 2) in matrix 1, incident *longitudinal waves*:

$$\gamma_N = \frac{4}{9} g_{el}(k_1\, a)^4, \tag{20a}$$

$$
g_{el} = \left[\frac{3\,(\kappa_1/k_1)^2}{\left(3\,(\kappa_2/k_2)^2 - 4\right)(\mu_2/\mu_1) + 4} - 1 \right]^2 + \frac{1}{3} \left[1 + 2\left(\frac{\kappa_1}{k_1}\right)^3 \right] \left[\left(\frac{\kappa_2}{\kappa_1}\right)^2 \frac{\mu_2}{\mu_1} - 1 \right]^2
$$
$$
+ 40 \left[2 + 3\left(\frac{\kappa_1}{k_1}\right)^5 \right] \left[\frac{(\mu_2/\mu_1) - 1}{2\left(3\,(\kappa_1/k_1)^2 + 2\right)(\mu_2/\mu_1) + 9\,(\kappa_1/k_1)^2 - 4} \right]^2. \tag{20b}
$$

In case of *elastic spheres* (denoted 2) in matrix 1, incident *transversal waves*:

$$\gamma_N = \frac{4}{9} g_{el,t}(\kappa_1\, a)^4, \tag{21a}$$

$$
g_{el,t} = \frac{8}{3} \left[1 + \frac{1}{2}\frac{k_1^3}{\kappa_1^3} \right] \left\{ \frac{\left[3\frac{\kappa_1^2}{\kappa_2^2} - 3\frac{\kappa_2^2}{\kappa_1^2} - 4\frac{k_2^2 \kappa_1^2}{\kappa_2^4} + 10\frac{k_2^2}{\kappa_2^2} - 6\frac{k_2^2}{\kappa_1^2} \right]}{\left[1 - 10\frac{\kappa_1}{k_1} + 6\frac{k_2}{k_1} - 6\frac{k_2}{k_1}\frac{\kappa_2^2}{\kappa_1^2} + 9\frac{\kappa_2^2}{\kappa_1^2} \right]^2} \right\}. \tag{21b}
$$

Scattering (neglecting multiple scattering) leads to an attenuation coefficient:

$$\alpha_s = \frac{1}{2} n_0 \gamma \tag{22}$$

with n_0 = density of scatterers.

Scattering in polycrystalline materials:

Here, scattering at the polycrystals boundaries arises due the anisotropy of the crystals described by the anisotropy factor:

$$\alpha_L = \frac{8\pi^3}{375} \cdot \frac{Vf^4 A^2}{\rho_0^2 v_L^8} \left[2 + 3\left(\frac{v_L}{v_T}\right)^5 \right]; \qquad \alpha_T = \frac{6\pi^3}{375} \cdot \frac{Vf^4 A^2}{\rho_0^2 v_T^8} \left[3 + 2\left(\frac{v_T}{v_L}\right)^5 \right]. \tag{23a,b}$$
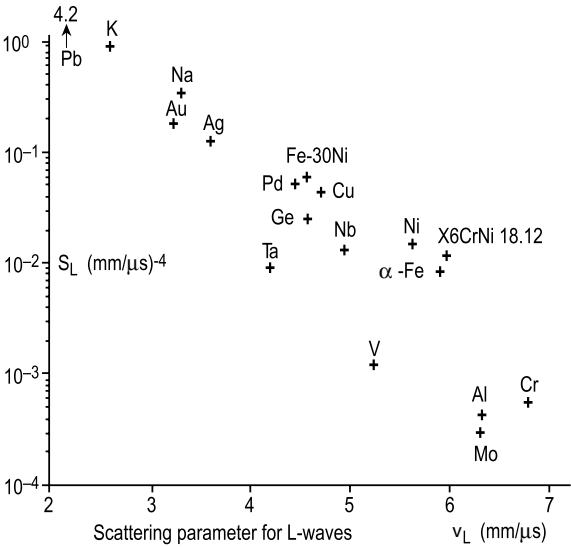
Attenuation can be written as:

$$\alpha_{L,T} = S_{L,T} Vf^4 \approx S_{L,T} d^3 f^4, \tag{24}$$

where: $S_{L,T}$ is the scattering parameter for longitudinal and transverse waves, respectively. The volume of the scatterers is $V \approx d^3$, d grain size. The anisotropy factor is shown in the figures below.
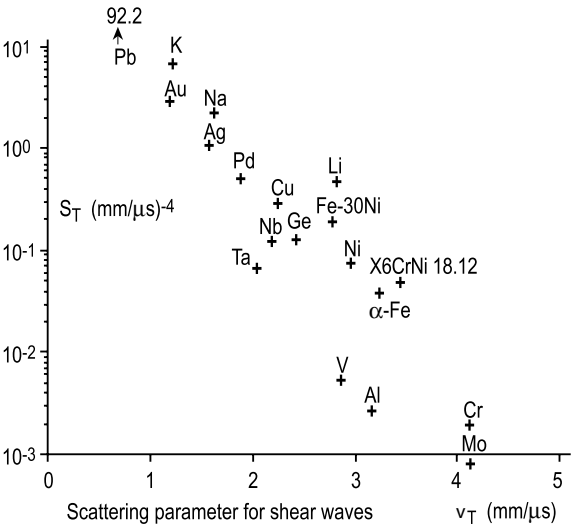
For cubic crystals: $\qquad A^2 = (c_{11} - c_{12} - 2c_{44})^2$

Because $(v_L/v_T)^5 \approx 32$ transverse wave scattering is much stronger.

Scattering parameter for longitudinal waves

Scattering parameter for transverse waves

Ultrasonic backscattering:

Ultrasonic backscattering may be used to characterise microstructures which are influenced by damage of various kinds, see Ref. [Goebbels (1980); Hirsekorn/Andel/Netzelmann (1980); Boyd et al. (1998)]. Neglecting multiple scattering, the backscattered signal is:

Intensity: $\quad I_S(x) = I_0 \cdot 2\alpha_{L,T}\Delta x \exp(-2\alpha x)$ (25a)

Amplitude: $\quad A_S(x) = A_0\,(\alpha_S\Delta x)^i \cdot \exp(-\alpha x)$ (25b)

with $\alpha = \alpha_{L,T} + \alpha_A$ the total attenuation coefficient, and α_A is the absorption coefficient which describes internal friction, see the following Sections.

R.3   Absorption and Dispersion in Solids Due to Dislocations

Equation of motion:

$$\frac{\partial^2\sigma_{ij}}{\partial x_j^2} = \rho\frac{\partial^2}{\partial t^2}\varepsilon_{ij} \tag{1}$$

Strain in the solid:

Elastic strain is determined by Hooke's law. The contribution of the dislocation to the strain is cast into ε_{dis}:

$$\varepsilon = \varepsilon_{el} + \varepsilon_{dis}; \quad \sigma = G\varepsilon_{el}, \tag{2a, b}$$

where G is the shear modulus. The dislocations can undergo oscillations much like a string:

$$A\frac{\partial^2\xi}{\partial t^2} + B\frac{\partial\xi}{\partial t} - C\frac{\partial^2\xi}{\partial y^2} = b\sigma \tag{3}$$

with $\quad A = \pi\rho b^2 \quad$ and $\quad C = 2Gb^2/\left(\pi(1-\upsilon)\right).$

Here, ξ is the amplitude of the oscillating dislocation, A is their effective mass per unit length, B is the damping constant and is determined by viscous drag in the phonon and electron bath, C is the tension in the bowed-out dislocation, b is the Burger's vector, σ is given above, and υ is the Poisson ratio. Combining the above equations leads to:

$$\frac{\partial^2\sigma_{xx}}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2\sigma_{xx}}{\partial t^2} = \frac{\Lambda\rho b}{1}\frac{\partial^2}{\partial t^2}\int_0^1 \xi(y)\,dy. \tag{4}$$

The integral in (4) represents the average amplitude of the dislocation oscillating in y-direction with l being their length. Solving (4) leads for the absorption α and dispersion $v(\omega)$ to:

$$\alpha(\omega) = \frac{1}{v}\left(\frac{4Gb^2}{\pi^4 C}\right)\omega_0^2\Lambda L^2\frac{\omega^2 d}{\left(\omega_0^2 - \omega^2\right)^2 + (\omega d)^2}, \tag{5}$$

$$v(\omega) = v_0 \left[1 - \left(\frac{4Gb^2}{\pi^4 C} \right) \omega_0^2 \Lambda L^2 \frac{\omega_0^2 - \omega^2}{\left(\omega_0^2 - \omega^2 \right)^2 + (\omega d)^2} \right], \tag{6}$$

where $\quad v_0 = \sqrt{G/\rho}; \quad \omega_0 = \left(\pi/L\sqrt{C/A} \right); \quad d = B/A.$ $\hspace{2cm}$ (7)

L is the loop length of the dislocation, Λ their density per area, and ω_0 is the resonance frequency of the dislocation. Two limits arise: resonance ($a \equiv d/\omega_0 \ll 1$) and relaxation ($a \gg 1$). In case of relaxation Eq. (5) reduces to:

$$\alpha = \left[8.68 \times 10^{-6} \left(\frac{4Gb^2}{\pi^4 C} \right) \Lambda L^2 \omega_m \right] \left[\frac{(\omega/\omega_m)^2}{1 + (\omega/\omega_m)^2} \right] \quad \left[dB/\mu s) \right]. \tag{8}$$

For convenience Eq. (8) is written to give units dB/μs. The constant B is of the order $B \approx 10^{-4}$ [s/cm]. There are many other absorption mechanisms possible, based on dislocations dynamics, see [Truell/Elbaum/Chick (1969); Gremaud/Kustov (1999); Gremaud (2001)] and references contained in [Gremaud/Kustov (1999); Gremaud (2001)].
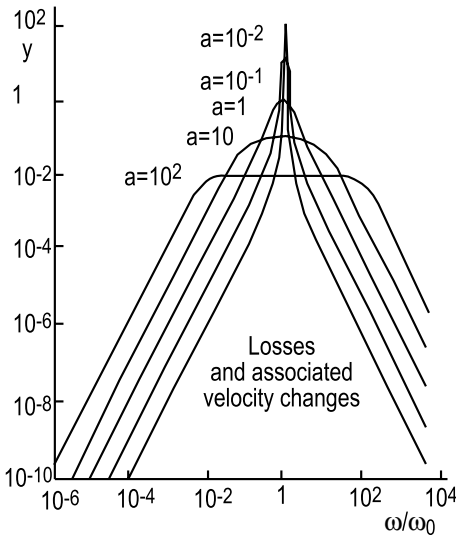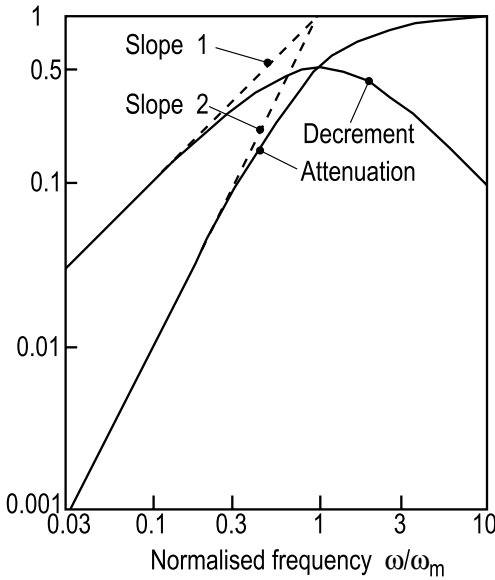

Absorption due to dislocations with transition from resonant ($a \equiv d/\omega_0 \ll 1$) to relaxation absorption ($a \gg 1$).

Dependence of

$$y = \frac{\alpha(\omega)}{(1/v)(4Gb^2/\pi^2 C)\,\Lambda L^2\,\omega_0}$$

on ω/ω_0 for various values of $a = d/\omega_0$

Relaxation absorption from dislocations for a $\gg 1$.
Normalised attenuation and decrement as functions of ω/ω_m for the case of large damping

R.4 Absorption Due to the Thermoelastic Effects, Phonon Scattering and Related Effects

Thermoelastic effect:

In all solids an absorption mechanism arises because a propagating ultrasonic wave entails a temperature modulation for longitudinal waves due to thermoelasticity. This temperature modulation tends to return to equilibrium by thermal conductivity.

Phenomenological description for the stress-strain relation:

$$\tau^{-1}\sigma + \dot{\sigma} = (M_1/\tau)\,\varepsilon + M_0\dot{\varepsilon}, \tag{1}$$

τ is the relaxation time, σ the stress and ε the strain. The modulus defect $\Delta M/M$ describes the microscopic coupling and has to be determined in each individual case:

$$\frac{\Delta M}{M} = \frac{M_0 - M_1}{M_0}. \tag{2}$$

General solution:

$$\alpha' v = \alpha \approx \left[\frac{\Delta M}{2M_0}\left(1 - \frac{\omega^2\tau}{1 + \omega^2\tau^2}\right)\right]; \tag{3}$$

$$v = \frac{\omega}{k} = \left[\frac{M_0}{\rho}\left(1 - \frac{\Delta M/M_0}{1 + \omega^2\tau^2}\right)\right]^{1/2} \equiv \left(\frac{M\omega}{\rho}\right)^{1/2}, \tag{4}$$

$$v_g = \left(\frac{M_0}{\rho}\right)^{1/2} \left\{1 - \frac{\Delta M}{M_0}\frac{(1 - \omega^2\tau^2)}{(1 + \omega^2\tau^2)^2}\right\}. \tag{5}$$

Equations (4) and (5) indicate that there is dispersion, i.e., that the phase velocity v and group velocities v_g are not the same.

For the thermoelastic effect:

$$\frac{\Delta M}{M} = \frac{M_0 - M_1}{M_0} = \frac{E_{ad} - E_T}{E_T} = \frac{\beta^2 T}{\rho c_p}E_{ad} \approx \frac{\beta^2 T}{\rho c_p}E_T, \tag{6}$$

where T is the temperature, β is the thermal expansion, E_{ad} and E_T are the adiabatic Young's modulus and at constant temperature, respectively, and c_P is the specific heat at constant pressure. The relaxation time τ is given by the thermal diffusivity D and the Debye average for the sound velocity v:

$$\tau \equiv \frac{L^2}{D} = \frac{L^2\rho c_p}{\gamma} = \frac{D}{v^2} = \frac{\gamma}{\rho c_p v^2}. \tag{7}$$

L is a length comparable to the wavelength, and γ the heat conductivity and c_p the specific heat. Over the length $L \cong \lambda$ the temperature difference generated by the ultrasonic wave due to the thermoelastic effect is equalised via thermal conductivity.

In polycrystals, heat may flow from one grain to the next because they heat up differently due to their anisotropy. This leads to an absorption, important for all technical materials:

$$\alpha = \frac{C_p - C_v R}{C_v}\frac{R}{2v}\frac{\omega^2\tau}{1 + \omega^2\tau^2} \tag{8}$$

with τ again the relaxation time, C_p, C_v are heat capacities for longitudinal and transversal sound. L is the mean grain diameter. R depends on the anisotropy for the strain energy and extends from $R \approx 10^{-6}$ for tungsten to $R = 6.5 \times 10^{-2}$ for lead [Beyer/Letcher (1969)].

Phonon interactions:

Ultrasonic waves also modulate *locally* the thermal phonon distribution [Beyer/Letcher (1969)]. This holds both for longitudinal and shear waves and hence differs from the thermoelastic effect. For $\omega\tau_{th} \ll 1$, the absorption for longitudinal waves is:

$$\alpha_l = \frac{\Delta c_{jj}}{2\rho v_{long}^3}\frac{\omega^2\tau_{th}}{1 + \omega^2\tau_{th}^2} = \frac{1}{2\rho v_{long}^3}\left[\frac{3U_0}{N}\sum_i \left(\gamma_{j(i)}\right)^2 - \gamma^2 C_v\Theta\right]\frac{\omega^2\tau_{th}}{1 + \omega^2\tau_{th}^2} \tag{9}$$

with the modulus defect:

$$\Delta c_{jj} = \frac{3U_0}{N}\sum_i \left(\gamma_{j(i)}\right)^2 - \gamma^2 C_v\Theta. \tag{10}$$

For transverse wave:

$$\Delta c_{jj} = \frac{3U_0}{N}\sum_i \left(\gamma_{j(i)}\right)^2, \tag{11}$$

$$\alpha_t = \frac{\Delta c_{jj}}{2\rho v_{long}^3}\frac{\omega^2 \tau_{th}}{1 + \omega^2 \tau_{th}^2} = \frac{1}{2\rho v_{long}^3}\left[\frac{3U_0}{N}\sum_i \left(\gamma_{j(i)}\right)^2\right]\frac{\omega^2 \tau_{th}}{1 + \omega^2 \tau_{th}^2}, \tag{12}$$

$$\tau_{th} = \frac{3k}{C_p v_0^2}. \tag{13}$$

Here, $\sum_i \left(\gamma_{j(i)}\right)^2$ is an average Grüneisen constant for transverse and longitudinal phonons and C_v and C_p are the corresponding heat capacities per volume. U_0 is the total thermal energy. Note that this effect occurs in addition to the one described by the thermoelastic effect.

Equations (9), (12) can be simplified for practical purposes:

$$\text{Eq. (9):} \quad \alpha_l = \frac{\Theta \kappa \gamma^2 \omega^2}{2\rho v_{long}^5} \quad \text{and Eq. (12):} \quad \alpha_t = \frac{\Theta \kappa \gamma^2 \omega^2}{2\rho v_t^5}. \tag{14a,b}$$

If $\omega \tau_{th} \gg 1$, a different view-point must be considered. The ultrasonic wave no longer modulates the phonon distribution, but ultrasonic phonon is scattered by the thermal phonons and corresponding conservation of momentums must be taken into account. This mechanism plays a role for low temperatures (< 40 K), i.e. much below the Debye temperature Θ [Tucker/Rampton (1972); Beyer/Letcher (1969); Dransfeld (1967)].

$$\alpha_t = \frac{\pi^3}{60}\frac{k_B^4 \bar{F}_1^2}{\rho^3 v_0^{10}\hbar^3}\omega_1 T^4, \tag{15}$$

F_1 is an average of second and third order elastic constants. Note the strong dependence on sound velocity which was verified experimentally. This mechanism also holds for longitudinal and transverse waves in various crystallographic orientations in crystals and the $\omega_1 T^4$ is retained, however, the pre-factor is different. The pre-factor also depends on the crystal class.

R.5   Interaction of Ultrasound with Electrons in Metals

The ultrasonic wave leads to a spatial separation of the ions from the free electrons at the Fermi surface. This causes an electrical field which eventually leads to a redistribution of the electrons and hence to an absorption coefficient α [Tucker/Rampton (1972)]:

$$\alpha = \frac{nm}{2\rho v_l \tau}\left[\frac{\frac{1}{3}k^2 l_e^2 \tan^{-1}(k l_e)}{k l_e - \tan^{-1}(k l_e)} - 1\right]. \tag{1}$$

Here, n is the number of conduction electrons per volume, m is the electronic mass, l_e is the mean free path for electrons, v_F is the Fermi velocity, k the wave vector of the ultrasonic wave, and v_l the longitudinal sound velocity.

In the limit of $k l_e \leq 1$ this leads to:

$$\alpha = \frac{2}{15}\frac{nmv_F^2\tau}{\rho v_l^3}(1 - 9/35(k_e l)^2 + \ldots). \tag{2}$$

In the limit $kl_e \gg 1$:

$$\alpha = \frac{\pi n m v_F}{12 \rho v_l^2} \omega. \tag{3}$$

The collision time τ is determined by the electrical conductivity $\quad \sigma = ne^2\tau/m$.

Transverse waves also cause absorption because they generate indirectly an internal electrical field via an internal magnetic field. Final results are similar to the equations above.

The absorption is magnetic field dependent because the electrons follow curved trajectories in a magnetic field and are bound to the Fermi surface. For transverse waves and a magnetic field perpendicular to the polarisation vector and k-vector:

$$\frac{\alpha(H)}{\alpha(0)} = \frac{1}{1 + (2\omega_c\tau)^2}, \tag{4}$$

where ω_c is the cyclotron resonance frequency $\quad \omega_c = (eH/mc) \tag{5}$

similarly for H parallel to the polarisation vector and perpendicular to the k-vector:

$$\frac{\alpha(H)}{\alpha(0)} = \frac{1}{1 + (\omega_c\tau)^2}. \tag{6}$$

Various resonance phenomena may occur if the electrons can complete an orbit on the Fermi surface whose size is equal to an integer number of the wavelength. If H is perpendicular to k, geometrical resonance occurs. When the ultrasonic frequency $\omega = n\omega_c$, n integer, and k parallel to H, temporal resonance may be induced. Finally quantum oscillations may be excited. Here the effect is due to the quantisation of the electron energy in a strong magnetic field applied in the z-direction. In all three cases the ultrasonic absorption becomes periodically field dependent with the periodicity (1/H):

$$\Delta(1/H) = e\lambda/\hbar k_y v_l, \tag{7}$$

k_y is the size of the orbit in k-space equivalent to the cross section of the Fermi surface.

$$\Delta(1/H) = e/\omega m v_l, \tag{8}$$

$$\Delta\left(\frac{1}{H}\right) = \frac{e\hbar}{m v_l}\left[\frac{\hbar^2 k_f^2}{2m} - \frac{\hbar^2 k_z^2}{2m}\right]^{-1} = \frac{2\pi e}{\hbar c_l}\frac{1}{A(E_f, k_z)}, \tag{9}$$

m is the effective mass of the electron and A is the cross section of the Fermi surface at E_f and k_z.

The latter case is analogue to the so-called de Haas van Alphen effect. Exploitation of Eqs. (7)–(9) allow to measure the shape of the Fermi surface in metals and its anisotropy.

If the metal becomes superconducting, the absorption drops. This can be exploited to measure the gap function $\Delta(T)$ in the superconductor:

$$\frac{\alpha_s}{\alpha_n} = \frac{2}{\exp(\Delta/kT) + 1}. \tag{10}$$

R.6   Wave Propagation in Piezoelectric Semiconducting Solids

The generation of a stress field in a piezoelectric solids leads to an accompanying electric field which accelerates electrons and leads to absorption and dispersion because the electrons undergo inelastic scattering [Hutson/White (1962)]:

$$v = v_0 \left[1 + \frac{e^2}{2cp} \frac{1 + (\omega_C/\omega_D) + (\omega/\omega_D)^2}{1 + 2(\omega_C/\omega_D) + (\omega/\omega_D)^2 + (\omega_C/\omega)^2} \right], \tag{1}$$

$$\alpha = \frac{\omega}{v_0} \frac{e^2}{2cp} \left[\frac{\omega_C/\omega}{1 + 2(\omega_C/\omega_D) + (\omega/\omega_D)^2 + (\omega_C/\omega)^2} \right]. \tag{2}$$

Here, ω_C is the so-called conductivity frequency and ω_D the diffusion frequency; $\omega_C = \sigma/\varepsilon_d$ (ε_d dielectric constant), $\omega_D \approx (ev^2/\mu kT)$ (μ: mobility of the electrons, k: Boltzmann constant, T: temperature).

R.7   Absorption in Amorphous Solids and Glasses

Disordered materials exhibit additional absorption mechanism, mostly due to relaxing units in the molecular or atomic structure.
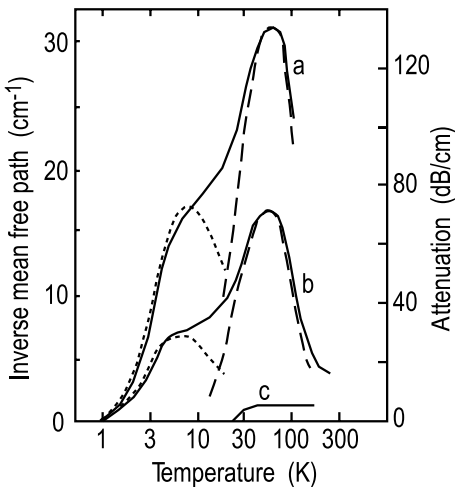
Similar to Eq. R.4.(3), the absorption is described as:

$$\alpha \approx \left[\frac{\Delta M}{2M_0} \left(1 - \frac{\omega^2 \tau}{1 + \omega^2 \tau^2} \right) \right], \tag{1}$$

however, with the modulus defect or relaxation strength $\Delta M/2M_0$, orders of magnitude larger than in the thermoelastic regime, see figure below. The relaxation frequency $1/\tau$ is determined by an Arrhenius process:

$$1/\tau = kT/he^{-\Delta E/kT}$$

with ΔE the activation energy, k the Boltzmann constant and T the temperature.

For low temperatures, amorphous solids behave in a much different way than crystalline solids. Some of the structural units can tunnel between different local spatial co-ordinates leading to a resonant absorption which depends on the ultrasonic intensity S, S_c is a critical intensity, M is the coupling coefficient (M ≈ 0.5 eV) and n is the density of the tunnel units per energy and volume ($n_0 \approx 10^{33}$ erg^{-1} cm^{-3}):

$$\alpha_{res} = -\frac{\pi n_0 \omega M^2}{\rho v^3} \frac{\tanh(\hbar\omega/kT)}{\sqrt{1 + S/S_c}}. \tag{2}$$

The corresponding change of sound velocity is for $S \ll S_c$:

$$\Delta v/v_0 = \frac{n_0 M^2}{\rho v^2} \ln(T/T_0), \tag{3}$$

where T_0 is a reference temperature. There is also a relaxation absorption due to the coupling to the phonon bath of the tunnelling units. An ultrasonic wave modulates the thermal occupation leading to:

$$\alpha_{rel} = \frac{3 n_0 D^2 T^3}{\pi \rho^2 \hbar^4 v^3} \left(\frac{M_l^2}{v_l^5} + \frac{2 M_{tl}^2}{v_t^5} \right). \tag{4}$$

Also this absorption mechanism leads to dispersion, see [Hunklinger/Arnold (1976)] and [Enss/Hunklinger (2000)].

If crystals exhibit a certain disorder or are irradiated by electrons or neutrons, similar phenomena are observed.

R.8   Relation of Ultrasonic Absorption to Internal Friction

Internal friction discusses the absorption mechanism of mechanical waves and oscillations at low frequencies usually below 20 kHz, i.e., in the audible range. The mechanism are mostly relaxation phenomena which are described by equations like R.7.(1), where the relaxation strength is adapted to the corresponding situation. Overviews can be found in [de Batist (1972); Nowick/Berry (1972); Schaller/Fantozzi/Gremaud (2001)].

R.9   Gases and Liquids

Treatments of the ultrasonic absorption due to relaxation phenomena in gases and liquids can be found in [Bhatia (1967)].

R.10   Kramers-Kroning Relation

Kramers-Kroning relation describe the interdependence between absorption and dispersion:

$$K_1 = \frac{2}{\pi} P \int_0^\infty \frac{\omega' K_2(\omega')}{\omega'^2 - \omega^2} d\omega', \tag{1}$$

$$K_2 = \frac{2}{\pi} P \int_0^\infty \frac{\omega' K_1(\omega')}{\omega'^2 - \omega^2} d\omega'. \tag{2}$$

Here, K_1 is the real part and K_2 the imaginary part of the compressibility. Analogue expressions hold for the k-vector k and the absorption coefficient.

Practical simplification of eqs. (1) and (2) are:

$$\frac{dK_1(\omega)}{d\omega} = -\frac{2}{\rho_0 v^3(\omega)} \frac{dv(\omega)}{d\omega}, \tag{3}$$

$$\frac{dv(\omega)}{d\omega} = 2v^2(\omega)\,\alpha(\omega)/\pi\omega^2, \tag{4}$$

$$\frac{dv(\omega)}{v^2(\omega)} = \frac{2\alpha(\omega)}{\omega^2} d\omega, \tag{5}$$

$$\frac{1}{v_0} - \frac{1}{v(\omega)} = \frac{2}{\pi} \int_{\omega_0}^{\omega} \frac{\alpha(\omega')}{\omega'^2} d\omega', \tag{6}$$

$$\alpha(\omega) = \frac{\pi\omega^2}{2v_0^2} \frac{dv(\omega)}{d\omega}, \tag{7}$$

$$\Delta v = v(\omega) - v_0 = \frac{2v_0^2}{\pi} \int_0^{\omega} \frac{\alpha(\omega')}{\omega'^2} d\omega'. \tag{8}$$

Here v_0 is the sound velocity at the frequency ω_0.

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