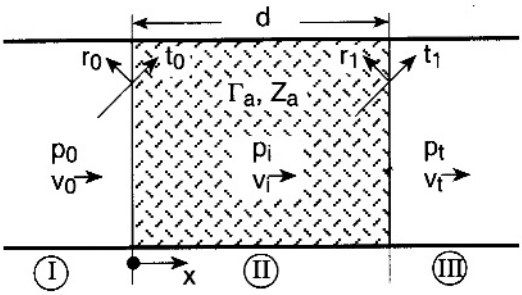
I Sound Transmission

This chapter deals with sound transmission through objects like porous absorber material layers ("noise barriers"), slits and holes in walls, wide passages which nevertheless are sound insulating ("noise sluices"), plates and multiple leaf walls, suspended ceilings, office fences, etc.

I.1 "Noise Barriers"

► *See also:* Mechel, Vol. III, Ch. 7 (1998)

Flanking ducts, e.g. cable ducts or plenum ducts of suspended ceilings, may be the critical path for sound transmission between neighbouring rooms. It may be difficult to install a well-fitting partition wall in the duct, but it is easy to fill the duct to some length d with a "plug" of porous absorber material. Let Γ_a, Z_a be the characteristic propagation constant and wave impedance of the material.



Sound fields for *normal incidence*:

In zone I:
$$\begin{aligned} p_0 &= e^{-j\,k_0 x} + r_0\, e^{+j\,k_0 x}, \\ Z_0 v_0 &= e^{-j\,k_0 x} - r_0\, e^{+j\,k_0 x}. \end{aligned} \qquad (1)$$

In zone II:
$$\begin{aligned} p_i &= A \cdot \cosh\,(\Gamma_a x) + B \cdot \sinh\,(\Gamma_a x), \\ Z_0 v_i &= \frac{-1}{Z_a/Z_0}\,[A \cdot \sinh\,(\Gamma_a x) + B \cdot \cosh\,(\Gamma_a x)\,]. \end{aligned} \qquad (2)$$

In zone III:
$$\begin{aligned} p_t &= t\, e^{-j\,k_0 x}, \\ Z_0 v_t &= t\, e^{-j\,k_0 x}. \end{aligned} \qquad (3)$$

The boundary conditions give:
$$A = 1 + r_0 \quad ; \quad B = -\frac{Z_a}{Z_0}\,(1 - r_0), \qquad (4)$$

and for the sound transmission factor:
$$t e^{-jk_0 d} = \frac{2}{2\cosh\,(\Gamma_a d) + \left(\dfrac{Z_a}{Z_0} + \dfrac{Z_0}{Z_a}\right) \sinh\,(\Gamma_a d)}\,. \qquad (5)$$

The transmission coefficient $\tau = |t\, e^{-j\,k_0 d}|^2$, and from that the transmission loss $R = -10 \cdot \lg\tau$ is:

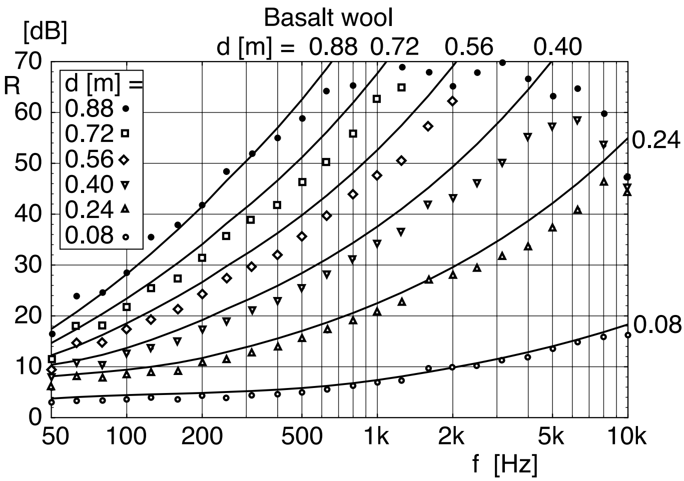
$$R = -20 \cdot \lg \left| \frac{2}{2\ \cosh(\Gamma_a d) + (Z_a/Z_0 + Z_0/Z_a)\ \sinh(\Gamma_a d)} \right| [\mathrm{dB}]. \tag{6}$$

It is advisable to take for the evaluation of Γ_a, Z_a an effective absorber variable E_{eff} which takes the vibration of the material matrix (with bulk density RG) into account:

$$E_{eff} = \rho_0 f/\Xi_{eff} = E - \frac{j}{2\pi}\,\frac{\rho_0}{RG}. \tag{7}$$

The diagram compares sound transmission loss values R for layers of basalt wool with different thickness d, from measurements (points) and from the present evaluation.

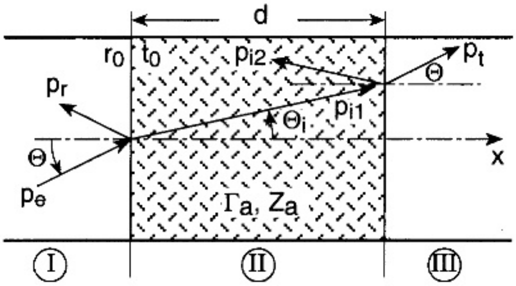
ρ_0	=	density of air;
RG	=	bulk density of porous material;
Ξ	=	flow resistivity of material;
E	=	$\rho_0 f/\Xi$;
f	=	frequency



Oblique sound incidence: (under polar angle Θ)

Internal angle Θ_i:
(with $\Gamma_{an} = \Gamma_a/k_0$; $Z_{an} = Z_a/Z_0$)

$$\begin{aligned} \sin\Theta_i &= \frac{j}{\Gamma_{an}}\ \sin\Theta, \\ \Gamma_{an}\ \cos\Theta_i &= \sqrt{\Gamma_{an}^2 + \sin^2\Theta}. \end{aligned} \tag{8}$$



Sound fields:

$$\begin{aligned}
p_e &= P_e \cdot e^{-jk_0(x\cos\Theta + y\sin\Theta)},\\
p_r &= r_0\, P_e \cdot e^{-jk_0(-x\cos\Theta + y\sin\Theta)},\\
p_i &= P_{i1} \cdot e^{-\Gamma_a(x\cos\Theta_i + y\sin\Theta_i)} + P_{i2} \cdot e^{-\Gamma_a(-x\cos\Theta_i + y\sin\Theta_i)},\\
p_t &= P_t \cdot e^{-jk_0(x\cos\Theta + y\sin\Theta)}.
\end{aligned} \tag{9}$$

Reflection and transmission factors:

$$\begin{aligned}
r_1 &= \frac{1-z}{1+z} \quad ; \quad r_0 = -r_1 \frac{1-e^{-2y}}{1-r_1^2 e^{-2y}} = -\frac{(1-z^2)(1-e^{-2y})}{(1+z)^2-(1-z)^2 e^{-2y}},\\
t_1 &= 1 + r_1 = \frac{2}{1+z} \quad ; \quad t_0 = \frac{1-r_1}{1-r_1^2 e^{-2y}} = \frac{2z^2}{(1+z)^2-(1-z)^2 e^{-2y}},\\
t &= t_0 t_1\, e^{-y} = e^{-y} \frac{4z}{(1+z)^2-(1-z)^2 e^{-2y}}
\end{aligned} \tag{10}$$

with abbreviations: $y := \Gamma_a d\cos\Theta_i = k_0 d \cdot \Gamma_{an} \cdot \cos\Theta_i \quad ; \quad z := Z_{an} \dfrac{\cos\Theta}{\cos\Theta_i}.$ (11)

For a given transmission coefficient $\tau(\Theta) = |t(\Theta)|^2$, the sound transmission coefficient with *diffuse* sound incidence is:

in three dimensions: $$\tau_{3D-diff} = 2 \int_0^{\Theta_{max}} \tau(\Theta)\cos\Theta\sin\Theta\, d\Theta, \tag{12}$$

in two dimensions: $$\tau_{2D-diff} = \int_0^{\Theta_{max}} \tau(\Theta)\cos\Theta\, d\Theta. \tag{13}$$

Transmission loss values for different kinds of excitation (with $\Gamma_a = \Gamma_a' + j \cdot \Gamma_a''$):

Plane wave excitation with normal incidence: $$R_\perp \approx 8.68 \cdot \Gamma_a' d + 20\log\left|\frac{1}{2}\left[1 + \frac{1}{2}\left(\frac{Z_a}{Z_0} + \frac{Z_0}{Z_a}\right)\right]\right|. \tag{14}$$

Conphase excitation with given v_0: $$R_{v_0} \approx 8.68 \cdot \Gamma_a' d + 20\log\left|\frac{1}{2}\left[1 + \frac{Z_0}{Z_a}\right]\right|. \tag{15}$$

Conphase excitation with given p_0: $$R_{p_0} \approx 8.68 \cdot \Gamma_a' d + 20\log\left|\frac{1}{2}\left[1 + \frac{Z_a}{Z_0}\right]\right|. \tag{16}$$

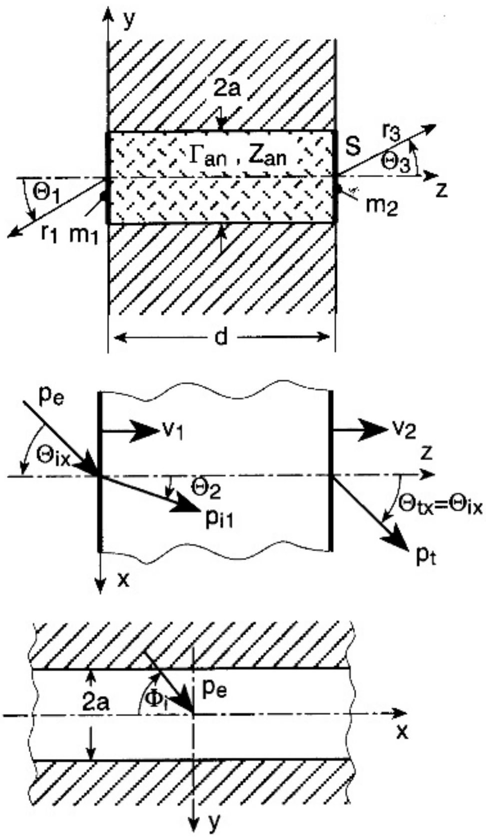
I.2 Sound Transmission through a Slit in a Wall

► *See also:* Mechel, Vol. III, Ch. 8 (1998)

Let a slit of width 2a be in a hard wall of thickness d. A plane wave p_e is incident at the angles indicated in the graph.

The slit is possibly filled with a porous material having the normalised characteristic values Γ_{an}, Z_{an}. For air in the slit: $\Gamma_{an} \to j$; $Z_{an} \to 1$.

The slit orifices may be covered with (poro-elastic) foils with surface mass densities m_1, m_2. These can represent plastic sealing masses.



The sound field on the front side has the components:

$$p_1 = p_e + p_r + p_s \tag{1}$$

p_e	=	incident plane wave;
p_r	=	reflected wave from a hard wall;
p_s	=	scattered wave

with the following formulations:

$$p_e(x, y, z) = P_e \cdot e^{-jk_0(x \cdot \sin\Theta_i \cos\Phi_i + y \cdot \sin\Theta_i \sin\Phi_i + z \cdot \cos\Theta_i)},$$
$$p_r(x, y, z) = P_e \cdot e^{-jk_0(x \cdot \sin\Theta_i \cos\Phi_i + y \cdot \sin\Theta_i \sin\Phi_i - z \cdot \cos\Theta_i)} \tag{2}$$

and

$$p_s(x, y, z) = \frac{-j\omega\rho_0 V_1 \cdot 2a}{2\pi} \cdot e^{-jk_x x} \cdot \int_{-\infty}^{+\infty} \frac{\sin(\alpha a)}{\alpha a} \frac{e^{-j\alpha y + z\sqrt{\alpha^2-\gamma^2}}}{\sqrt{\alpha^2-\gamma^2}}\, d\alpha, \tag{3}$$

where V_1 is the average particle velocity in the entrance orifice and

$$k_x = k_0 \cdot \sin\Theta_i \cdot \cos\Phi_i,$$
$$\gamma^2 = k_0^2 - k_x^2. \tag{4}$$

The waves in the slit are:

$$p_{i1}(x, z) = P_{i1} \cdot e^{-\Gamma_a(x \cdot \sin\Theta_2 + z \cdot \cos\Theta_2)},$$
$$p_{i2}(x, z) = P_{i2} \cdot e^{-\Gamma_a(x \cdot \sin\Theta_2 - z \cdot \cos\Theta_2)} \tag{5}$$

with the internal angle Θ_2 from

$$\frac{\sin\Theta_2}{\sin\Theta_i} = \frac{j k_0 \cos\Phi_i}{\Gamma_a} = \frac{j \cos\Phi_i}{\Gamma_{an}} \quad ; \quad \Gamma_{an} \cos\Theta_2 = \sqrt{\Gamma_{an}^2 + \sin^2\Theta_i \cos^2\Phi_i}. \tag{6}$$

The transmitted wave is:

$$p_t(x, y, z') = \frac{j\omega\rho_0 V_2 \cdot 2a}{2\pi} \cdot e^{-jk_x x} \int_{-\infty}^{+\infty} \frac{\sin(\alpha a)}{\alpha a} \frac{e^{-j\alpha y - z'\sqrt{\alpha^2-\gamma^2}}}{\sqrt{\alpha^2-\gamma^2}}\, d\alpha \tag{7}$$

with V_2 the average particle velocity amplitude in the exit orifice and shifted z co-ordinate z' ($z' = z - d$). Both p_s and p_t satisfy the boundary condition at the hard-wall surfaces.

Let $Z_1 = j\omega m_1 + Z_{r1}$ and $Z_2 = j\omega m_2 + Z_{r2}$ be the sums of the sealing impedance and radiation impedance Z_r of the orifices (see "Radiation Impedance" and End Corrections" in Ch. F, "Radiation of Sound"). Setting the arbitrary amplitude $P_e = 1$, the boundary conditions give the following system of equations:

$$\begin{pmatrix} 1 & 1 & Z_1 & 0 \\ 1 & -1 & -Z_a/\cos\Theta_2 & 0 \\ e^{-\Gamma_a d \cos\Theta_2} & -e^{+\Gamma_a d \cos\Theta_2} & 0 & -Z_a/\cos\Theta_2 \\ e^{-\Gamma_a d \cos\Theta_2} & e^{+\Gamma_a d \cos\Theta_2} & 0 & -Z_2 \end{pmatrix} \cdot \begin{pmatrix} P_{i1} \\ P_{i2} \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2\,\mathrm{si}(k_0 a \sin\Theta_i \sin\Phi_i) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

with $si(z) = (\sin z)/z$. The average particle velocity amplitude in the exit orifice V_2 is:

$$V_2 = \frac{2\,\dfrac{Z_a}{\cos\Theta_2}\cdot si(k_0 a\ \sin\Theta_i\ \sin\Phi_i)}{\dfrac{Z_a}{\cos\Theta_2}(Z_1+Z_2)\ \cosh(\Gamma_a d\ \cos\Theta_2)+\left[\left(\dfrac{Z_a}{\cos\Theta_2}\right)^2+Z_1 Z_2\right]\ \sinh(\Gamma_a d\ \cos\Theta_2)}. \tag{9}$$

The sound transmission coefficient $\tau(\Theta_i,\Phi_i)$ is then:

$$\tau(\Theta_i,\Phi_i) = \frac{Z_0}{\cos\Theta_i}\cdot \mathrm{Re}\{Z_{r2}\}\cdot\left|\frac{V_2}{P_e}\right|^2. \tag{10}$$

The sound transmission coefficient τ_{dif} for diffuse sound incidence follows by integration:

$$\begin{aligned}\tau_{dif} &= \frac{1}{\pi}\int_0^{2\pi} d\Phi \int_0^{\pi/2} \tau(\Theta,\Phi)\ \cos\Theta\ \sin\Theta\ d\Theta,\\ &= \frac{4}{\pi}\int_0^{\pi/2} d\Phi \int_0^{\pi/2} \tau(\Theta,\Phi)\ \cos\Theta\ \sin\Theta\ d\Theta.\end{aligned} \tag{11}$$

The normalised radiation impedance $Z_{rn} = Z_r/Z_0$ is (with $u = 2\gamma a$):

$$Z_{rn} = 2k_0 a\left\{H_0^{(2)}(u)+\frac{\pi}{2}\left[H_1^{(2)}(u)\,S_0(u)-H_0^{(2)}(u)\,S_1(u)\right]-\frac{1}{u}H_1^{(2)}(u)+\frac{2j}{\pi u^2}\right\}, \tag{12}$$

where $H_n^{(2)}(u)$ are Hankel functions of the second kind and $S_n(u)$ are Struve functions. The real and imaginary parts of Z_{rn} are:

$$\begin{aligned}Z'_{rn} &= 2k_0 a\left\{J_0(u)-\frac{J_1(u)}{u}+\frac{\pi}{2}\left[J_1(u)\,S_0(u)-J_0(u)\,S_1(u)\right]\right\},\\ Z''_{rn} &= -2k_0 a\left\{Y_0(u)-\frac{Y_1(u)}{u}-\frac{2}{\pi u^2}+\frac{\pi}{2}\left[Y_1(u)\,S_0(u)-Y_0(u)\,S_1(u)\right]\right\},\end{aligned} \tag{13}$$

and an approximation for small u (with C = 0.577216 Euler's constant) is:

$$\begin{aligned}Z_{rn} = k_0 a\Big\{&\left[1-u^2/24+u^4/960-u^6/64512+u^8/6635520\right]\\ &+\frac{j}{\pi}\Big[3-19\,u^2/144+7\,u^4/1800-353\,u^6/5419008\\ &+413\,u^8/597196800-\left(\ln\frac{u}{2}+C\right)\Big(2-u^2/12+u^4/480-u^6/32256\\ &+u^8/3317760\Big)\Big]\Big\}.\end{aligned} \tag{14}$$

Approximations (for $\Phi_i = 0$):

Use the set of non-dimensional parameters (with $m_1 = m_2 = m$):

$$F = f\,d/c_0 = d/\lambda_0\quad;\quad A = 2a/d\quad;\quad X = \Xi d/Z_0\quad;\quad M = m/\rho_0 d. \tag{15}$$

For the analytical discussion below, $\Gamma_{an} \to j$ and $Z_{an} \to 1$ are used for an empty slit; in numerical evaluations it is better to use the propagation constant and wave impedance of a flat capillary for Γ_a, Z_a (see *Ch. J*, "Duct Acoustics").

Neglecting terms $(FA\cdot\cos\Theta)^n$ with $n > 1$:

$$\tau(\Theta) = \frac{\pi}{\cos\Theta}\left|\left\{\pi + j\left[4 - 2C - 2\ln(\pi FA\cos\Theta) + 2\pi\frac{M}{A\cos\Theta}\right]\right\}\right.$$
$$\cdot\{1 + \sin^2\Theta - 2\pi^2F^2 + j\pi\kappa FX\} + \frac{\pi}{A\cos\Theta}\left\{\frac{1}{2\pi\sigma}\frac{X}{F} + \frac{j}{\kappa\sigma}\right. \tag{16}$$
$$\left.\left. - j4\pi^2\kappa\sigma F^2M^2\right\}\right|^{-2},$$

where κ is the adiabatic exponent of air and σ the porosity of the absorber material. For an empty slit, set $X = 0$ and $\kappa\sigma \to 1$. For a very narrow and empty slit:

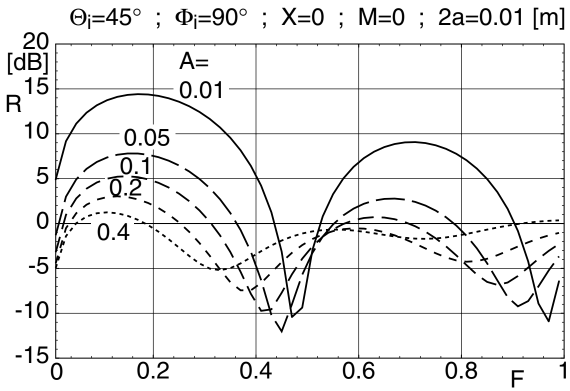
$$\tau(\Theta) = \frac{A}{\pi F}\left|1 + 2M - 4\pi^2F^2M(1+M)\cos^2\Theta\right|^{-2}. \tag{17}$$

For a very narrow, empty slit without sealing: $$\tau(\Theta) = \frac{A}{\pi F}. \tag{18}$$

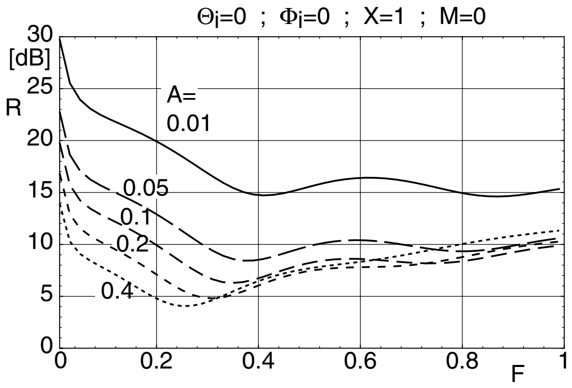
For an empty, narrow slit with sealing, in a thin wall (or at low frequencies, $F \ll 1$): $$\tau(\Theta) = \frac{A}{\pi F(1+2M)^2}. \tag{19}$$

For a narrow slit with porous material, in a thin wall (or at low frequencies, $F \ll 1$): $$\tau(\Theta) = 4\pi\sigma^2 \cdot FA/X^2. \tag{20}$$

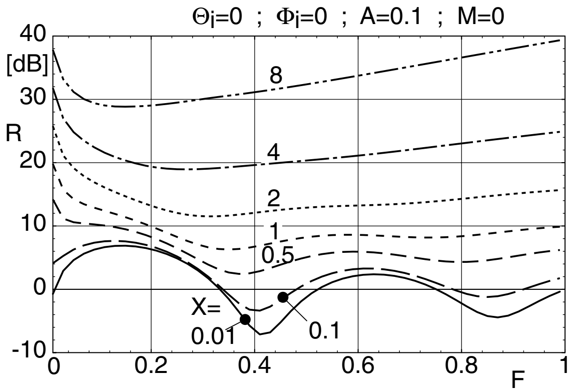
The frequency response curves of $R(\Theta) = -\lg\tau(\Theta)$ evidently can have very different shapes.



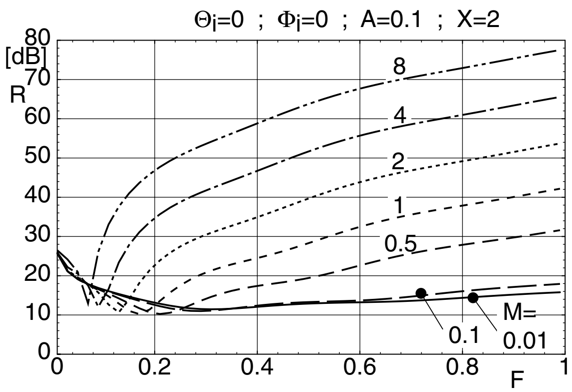
Empty slit, oblique incidence



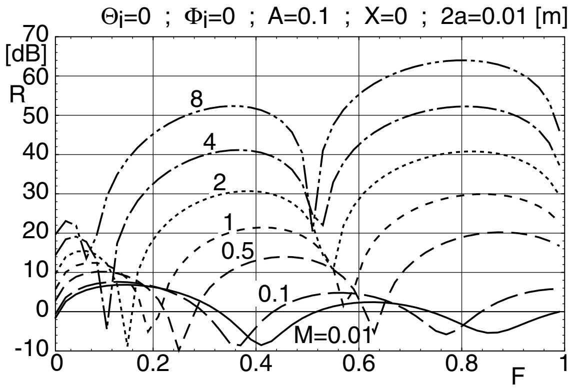
Filled slit, normal incidence



Slits with different absorber material fill; normal incidence



Slits with absorber fill and different sealing

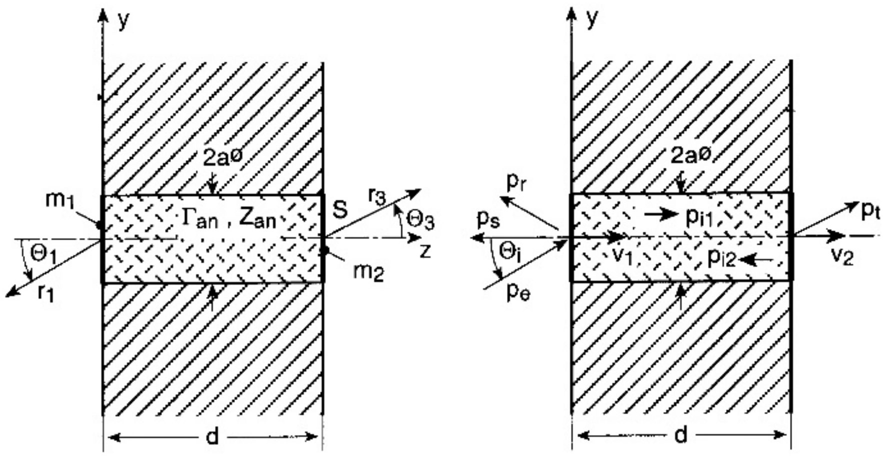


Empty slit with different sealing

I.3 Sound Transmission through a Hole in a Wall

► *See also:* Mechel, Vol. III, Ch. 8 (1998)

A circular hole with diameter 2a is in a wall of thickness d. The hole is possibly filled with a porous material having a normalised propagation constant Γ_{an} and a normalised wave impedance Z_{an}. Possibly poro-elastic foils with effective surface mass densities m_1, m_2 seal the hole orifices. A plane wave p_e is incident at a polar angle Θ_i. The graphs show the co-ordinates and wave components used. The sound field p_1 on the front side is formulated as $p_1 = p_e + p_r + p_s$, where p_r is the plane wave after reflection at a hard wall and p_s is the scattered field.



If the hole is "empty", i.e. filled with air, substitute for analytical discussions $\Gamma_{an} \to j$ and $Z_{an} \to 1$, and for numerical evaluations use the propagation constant and wave impedance for sound propagation in a circular capillary (see *Ch. J*, "Duct Acoustics").

Field formulations:

$$p_e(x,y) = P_e \cdot e^{-j\,k_0(z\cos\Theta_i + y\sin\Theta_i)},$$

$$p_r(x,y) = P_e \cdot e^{-j\,k_0(-z\cos\Theta_i + y\sin\Theta_i)}, \tag{1}$$

$$p_s(r_1,\Theta_1) = j\,(k_0 a)^2 Z_0 \cdot V_1 \cdot \frac{e^{-j\,k_0 r_1}}{k_0 r_1}\,\frac{J_1(k_0 a\ \sin\Theta_1)}{k_0 a\ \sin\Theta_1},$$

where V_1 is the average particle velocity in the front side orifice. Further, in the hole:

$$p_{i1}(z) = P_{i1}\cdot e^{-\Gamma_a z} \quad ; \quad p_{i2}(z) = P_{i2}\cdot e^{+\Gamma_a z}, \tag{2}$$

and for the transmitted wave:

$$p_t(r_3,\Theta_3) = j\,(k_0 a)^2 Z_0 \cdot V_2 \cdot \frac{e^{-j\,k_0 r_3}}{k_0 r_3}\,\frac{J_1(k_0 a\ \sin\Theta_3)}{k_0 a\ \sin\Theta_3}, \tag{3}$$

where V_2 is the average particle velocity in the back side orifice. p_s and p_t satisfy the boundary condition at the wall.

The boundary conditions for matching the field components give the following system of equations:

$$\begin{pmatrix} 1 & 1 & j\omega m_1 + Z_{r1} & 0 \\ 1 & -1 & -Z_a & 0 \\ e^{-\Gamma_a d} & -e^{+\Gamma_a d} & 0 & -Z_a \\ e^{-\Gamma_a d} & e^{+\Gamma_a d} & 0 & -j\omega m_2 - Z_{r2} \end{pmatrix} \cdot \begin{pmatrix} P_{i1} \\ P_{i2} \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{4}$$

where Z_{r1}, Z_{r2} are the radiation impedances of circular piston radiators in a baffle wall (see *Ch. F*, “Radiation of Sound”). The solutions are, with $Z_1 = j\omega m_1 + Z_{r1}$, $Z_2 = j\omega m_2 + Z_{r2}$:

$$\begin{aligned} P_{i1}/P_e &= 2\,Z_a\,(Z_a + Z_2)\,e^{+\Gamma_a d}/D, \\ P_{i2}/P_e &= 2\,Z_a\,(Z_a - Z_2)\,e^{-\Gamma_a d}/D, \\ V_1/P_e &= 4\,(Z_a\ \cosh(\Gamma_a d) + Z_2\ \sinh(\Gamma_a d))\,/D, \\ V_2/P_e &= 4\,Z_a/D \end{aligned} \tag{5}$$

with the determinant of the matrix

$$D = 2\,[\,Z_a(Z_1 + Z_2)\ \cosh(\Gamma_a d) + (Z_a^2 + Z_1 Z_2)\ \sinh(\Gamma_a d)\,]. \tag{6}$$

The transmission loss R of the hole follows from the transmission coefficient τ, which is the ratio of the transmitted effective power Π_t to the incident effective power Π_e:

$$R = -10\ \lg\tau\ \ [\mathrm{dB}] \quad ; \quad \tau(\Theta_i) = \frac{\Pi_t}{\Pi_e(\Theta_i)},$$

$$\Pi_e(\Theta_i) = \frac{1}{2}S\cdot\cos\Theta_i\cdot\frac{|P_e|^2}{Z_0} \quad ; \quad \Pi_t = \frac{1}{2}S\cdot\mathrm{Re}\{Z_{r2}\}\cdot|V_2|^2 \tag{7}$$

(S = orifice area). Thus:

$$\tau(\Theta_i) = \frac{Z_0}{\cos\,\Theta_i} \cdot \mathrm{Re}\{Z_{r2}\} \cdot \left|\frac{V_2}{P_e}\right|^2$$
$$= \frac{Z_0}{\cos\,\Theta_i} \cdot \mathrm{Re}\{Z_{r2}\} \cdot \left|\frac{2\,Z_a}{Z_a\,(Z_1+Z_2)\ \cosh(\Gamma_a d) + (Z_a^2 + Z_1 Z_2)\ \sinh(\Gamma_a d)}\right|^2 . \tag{8}$$

The sound transmission coefficient τ_{dif} for diffuse sound incidence from the whole half-space is simply $\tau_{dif} = \pi \cdot \tau(0)$
because of $\tau(\Theta_i) = \tau(0)/\cos\Theta_i$. (9)

Approximations and special cases:

(normal incidence $\Theta_i = 0$, and $m_1 = m_2$; index n at impedances means normalisation)

Use the set of non-dimensional parameters:

$$F = f\,d/c_0 = d/\lambda_0 \quad ; \quad A = 2a/d \quad ; \quad X = \Xi d/Z_0 \quad ; \quad M = m/\rho_0 d, \tag{10}$$

where Ξ = flow resistivity of the porous fill material (needed for the evaluation of Γ_{an}, Z_{an}).

For $2k_0 a \ll 1$ and $|\Gamma_a d| \ll 1$:

$$\tau(0) \approx \frac{\mathrm{Re}\{Z_{rn}\}}{\left|Z_{1n} + (Z_{an}^2 + Z_{1n}^2)\ k_0 d\ \dfrac{\Gamma_{an}}{2\,Z_{an}}\right|^2}, \tag{11}$$

and with the leading term of the radiation impedance:

$$\tau(0) \approx \frac{1}{2}\left(\frac{\pi A F}{\dfrac{\pi^2}{2}\,A^2F^2 + X/(2\sigma)}\right)^2 \approx 2\,(\pi\sigma\ AF/X)^2, \tag{12}$$

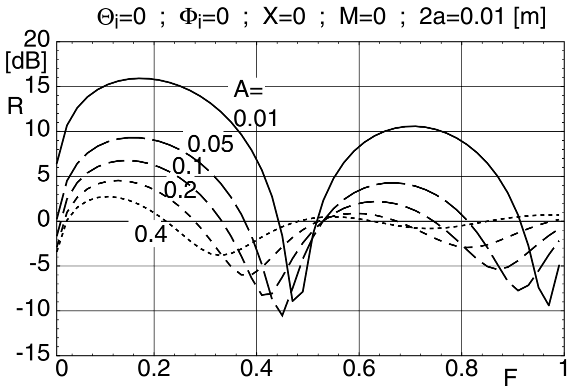
where σ = porosity of the absorber fill. If additionally the hole is empty (X = 0):

$$\tau(0) \approx \frac{1}{8}\left(\frac{A}{\dfrac{4}{3\pi}A + M}\right)^2 . \tag{13}$$

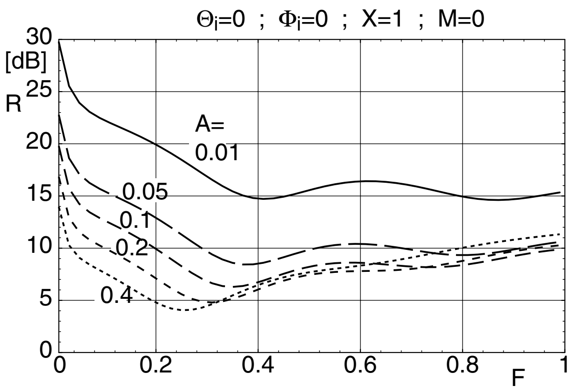
Empty and unsealed hole (M = 0; X = 0) at low frequencies:

$$\tau(0) \approx \frac{9\pi^2}{128} = 0.694. \tag{14}$$

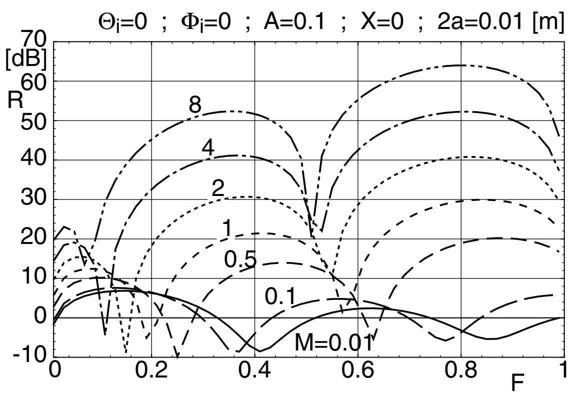
The following diagrams give some examples of R for normal sound incidence and some parameter combinations.



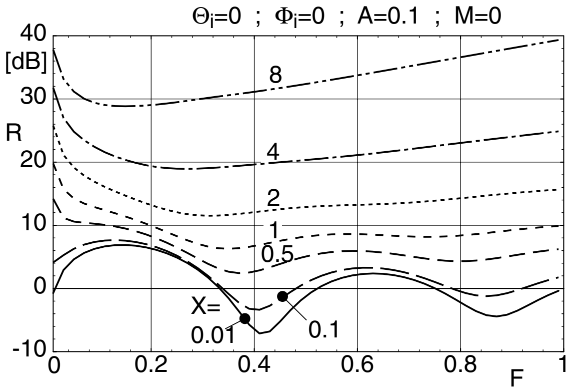
Empty and unsealed holes



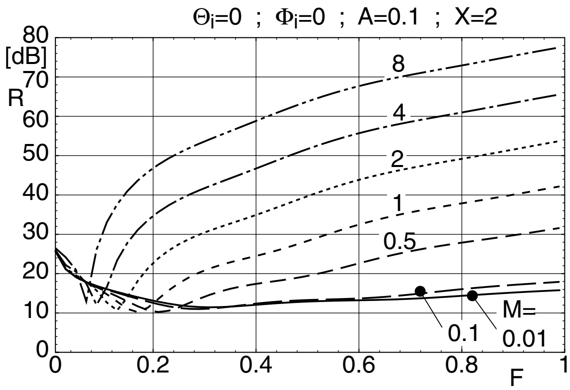
Filled but unsealed holes



Sealed but empty holes



Filled but unsealed holes



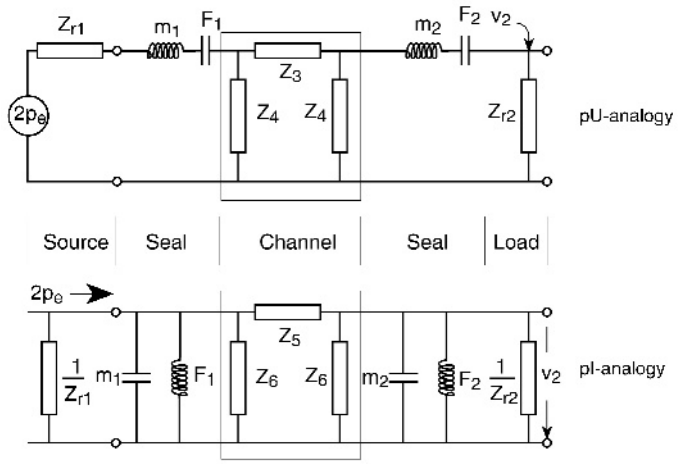
Filled and sealed holes

I.4 Hole Transmission with Equivalent Network

► *See also:* Mechel, Vol. III, Ch. 8 (1998)

Sound transmission through holes and slits in a wall is derived in the previous *Sects. I.2 and I.3* as a boundary value problem. Equal results are obtained by application of the method of equivalent networks.

The equivalent network of a sealed and filled hole (or slit) in a wall is in the pU-analogy and in the pI-analogy:



The elements are:

m_1, F_1; m2, F2	surface mass densities and resilience values of the orifice seals;
Z_{r1}, Z_{r2}	radiation impedances of the orifices;

$$Z_3 = Z_a \cdot \sinh(\Gamma_a d); \qquad \frac{1}{Z_4} = \frac{1}{Z_a}\,\frac{\cosh(\Gamma_a d) - 1}{\sinh(\Gamma_a d)};$$
$$Z_5 = \frac{1}{Z_a} \cdot \sinh(\Gamma_a d); \qquad \frac{1}{Z_6} = Z_a\,\frac{\cosh(\Gamma_a d) - 1}{\sinh(\Gamma_a d)}. \tag{1}$$

Combine

$$Z_1 = Z_{r1} + j\omega m_1 + \frac{1}{j\omega F_1} \quad ; \quad Z_2 = Z_{r2} + j\omega m_2 + \frac{1}{j\omega F_2} \tag{2}$$

and

$$Z_I = Z_1 + \frac{1}{Z_6} = Z_{r1} + j\omega m_1 + \frac{1}{j\omega F_1} + Z_a\,\frac{\cosh(\Gamma_a d) - 1}{\sinh(\Gamma_a d)}.$$
$$Z_{II} = Z_2 + \frac{1}{Z_6} = Z_{r2} + j\omega m_2 + \frac{1}{j\omega F_2} + Z_a\,\frac{\cosh(\Gamma_a d) - 1}{\sinh(\Gamma_a d)}. \tag{3}$$

Then:

$$V_2 = \frac{2P_e}{Z_I + Z_{II} + Z_5 \cdot Z_I \cdot Z_{II}}, \tag{4}$$

and the transmission coefficient of the *hole* is:

$$\tau(\Theta_i) = \frac{Z_0}{\cos\,\Theta_i} \cdot \mathrm{Re}\{Z_{r2}\} \cdot \left| \frac{2}{Z_I + Z_{II} + Z_5 \cdot Z_I \cdot Z_{II}} \right|^2. \tag{5}$$

Insertion of the abbreviations leads to the result of the boundary value problem.

For a *slit* apply the substitutions:

$$\Gamma_a \to \Gamma_a \cos\Theta_2 \quad ; \quad Z_a \to Z_a / \cos\Theta_2;$$
$$Z_{r,circle} \to Z_{r,strip}; \tag{6}$$
$$2P_e \to 2P_e \,\mathrm{si}\,(k_0 a \sin\Theta_i \sin\Phi_i)$$

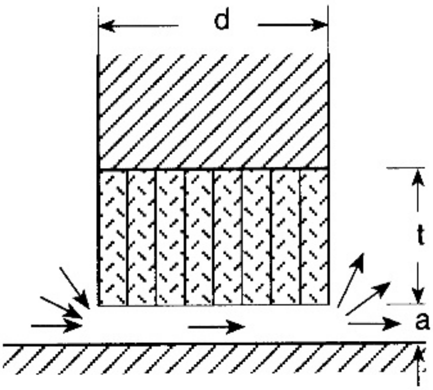
with $\mathrm{si}(z) = \sin(z)/z$. This returns the result of Sect. I.2.

I.5 Sound Transmission through Lined Slits in a Wall

▸ *See also:* Mechel, Vol. III, Ch. 8 (1998)

Sometimes joints of wall elements which form slits in the wall cannot be sealed or filled, e.g. the lower gap at a door.

The quantities and relations are those of Sect. I.2, except that Γ_a, Z_a are the propagation constant and wave impedance of the least attenuated mode in a flat (silencer) duct of height a, which is lined with an absorber having a surface admittance G_y.



The lining is preferably made locally reacting (if necessary with thin partitions). Because $k_0 a \ll 1$ holds in general, low-frequency approximations can be used for the determination of Γ_a.

Channel wave components

(with substitutions $\Gamma_a \to \Gamma_s$; $Z_a \to Z_s$ in order to avoid confusion with material data):

$$\left.\begin{aligned} p_{i1}(x,\ y,\ z) &= P_{i1} \cdot e^{-\Gamma_s\,(x\cdot\sin\Theta_2 + z\cdot\cos\Theta_2)} \cdot \cos\varepsilon_y y \\ p_{i2}(x,\ y,\ z) &= P_{i2} \cdot e^{-\Gamma_s\,(x\cdot\sin\Theta_2 - z\cdot\cos\Theta_2)} \cdot \cos\varepsilon_y y \end{aligned}\right\} \quad ; \quad \varepsilon_y^2 = \Gamma_s^2 + k_0^2. \tag{1}$$

$\varepsilon_y a$ is a solution of the characteristic equation for a *locally reacting lining*:

$$\varepsilon_y a \cdot \tan\varepsilon_y a = j \cdot k_0 a \cdot Z_0 G_y := j \cdot U. \tag{2}$$

The following low-frequency approximation is applicable:

$$(\varepsilon_y a)^2 = \frac{105 + 45jU \pm \sqrt{11\,025 + 5250jU - 1605U^2}}{20 + 2jU}. \tag{3}$$

The sign of the root is chosen so that the real part of $\Gamma_s a = \sqrt{(\varepsilon_y a)^2 - (k_0 a)^2}$ (4)

is a minimum. The wave impedance Z_s of the least attenuated mode is: $\frac{Z_s}{Z_0} = \frac{j}{\Gamma_s/k_0}$. (5)

If the lining is a *bulk reacting* homogeneous, porous absorber layer of thickness t with characteristic values Γ_a, Z_a of the material, the equation to be solved is:

$$\varepsilon_y a \cdot \tan(\varepsilon_y a) = -j \left(\frac{\Gamma_a}{k_0}\frac{Z_a}{Z_0}\right)^{-1} \sqrt{(\varepsilon_y a)^2 - (\eta a)^2} \cdot \tan\left(\frac{t}{a}\sqrt{(\varepsilon_y a)^2 - (\eta a)^2}\right), \quad (\eta a)^2 := (\Gamma_a a)^2 + (k_0 a)^2. \tag{6}$$

A continued fraction approximation for low frequencies leads to a polynomial equation:

$$a_0 + a_1 \cdot (\varepsilon_y a)^2 + a_2 \cdot (\varepsilon_y a)^4 + a_3 \cdot (\varepsilon_y a)^6 + a_4 \cdot (\varepsilon_y a)^8 = 0 \tag{7}$$

with coefficients

$$\begin{aligned} a_0 &= -11025\,(\eta a)^2 - 1050\,(t/a)^2(\eta a)^4, \\ a_1 &= 11025 + (\eta a)^2(4\,725 + 2100\,(t/a)^2) + 450\,(t/a)^2(\eta a)^4 \\ &\quad - A \cdot [11025/(t/a)^2 + 4725\,(\eta a)^2 + 105\,(t/a)^2(\eta a)^4], \\ a_2 &= -\{4725 + 1050\,(t/a)^2 + (\eta a)^2(105 + 900\,(t/a)^2) + 10(t/a)^2(\eta a)^4 \\ &\quad + A \cdot [1050\,/(t/a)^2 + 4725 + (\eta a)^2(450 + 210\,(t/a)^2) + 10\,(t/a)^2(\eta a)^4]\}, \end{aligned} \tag{8}$$

$$\begin{aligned} a_3 &= 105 + 450\,(t/a)^2 + 20\,(t/a)^2(\eta a)^2 - A \cdot [450 + 105\,(t/a)^2 + 20\,(t/a)^2(\eta a)^2], \\ a_4 &= 10\,(t/a)^2(A-1) \end{aligned} \tag{9}$$

and the abbreviation $A := j\,\frac{t}{a}\,\frac{\Gamma_a}{k_0}\,\frac{Z_a}{Z_0}$. (10)

The least attenuated mode in the slit channel has a cosine profile; the matching to the exterior sound fields has to be performed with the average values of sound pressure and axial particle velocity.

The boundary conditions in the orifices lead to the following system of equations:

$$\begin{aligned} &\mathrm{si}(\varepsilon_y a) \cdot (P_{i1} + P_{i2}) + Z_{r1} V_1 + 0 = 2 P_e\, \mathrm{si}(k_y a) \\ &j\beta\, \mathrm{si}(\varepsilon_y a) \cdot (P_{i1} - P_{i2}) + Z_0 V_1 + 0 = 0 \\ &\mathrm{si}(\varepsilon_y a) \cdot (P_{i1} e^{-\gamma} + P_{i2} e^{+\gamma}) + 0 - Z_{r2} V_2 = 0 \\ &j\beta\, \mathrm{si}(\varepsilon_y a) \cdot (P_{i1} e^{-\gamma} - P_{i2} e^{+\gamma}) + 0 + Z_0 V_2 = 0 \end{aligned} \tag{11}$$

with $\mathrm{si}(z) = \sin(z)/z$ and the abbreviations

$$\begin{aligned} \beta &:= \Gamma_s/k_0 \cdot \cos\Theta_2 = \sqrt{(\varepsilon_y/k_0)^2 - 1 + \sin^2\Theta_i\ \cos^2\Phi_i}, \\ \gamma &:= \Gamma_s d \cdot \cos\Theta_2 = k_0 d\,\sqrt{(\varepsilon_y/k_0)^2 - 1 + \sin^2\Theta_i\ \cos^2\Phi_i}. \end{aligned} \tag{12}$$

The matrix determinant is:

$$D = 2\,\mathrm{si}^2(\varepsilon_y a) \cdot [j\beta Z_0(Z_{r1} + Z_{r2}) \cdot \cosh\gamma + (\beta^2 Z_{r1} Z_{r2} - Z_0^2) \cdot \sinh\gamma\,], \tag{13}$$

and the desired average axial particle velocity V_2 in the exit orifice is:

$$\frac{V_2}{P_e} = \frac{2j\,\beta Z_0\ \mathrm{si}\,(k_y a)}{j\,\beta Z_0(Z_{r1} + Z_{r2}) \cdot \cosh\gamma + (\beta^2 Z_{r1} Z_{r2} - Z_0^2) \cdot \sinh\gamma}. \tag{14}$$

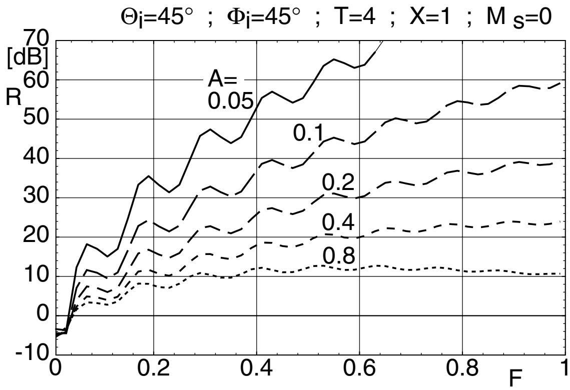
The coefficient of transmission is, with the normalised radiation impedances Z_{r1n}, Z_{r2n} of the orifices (*Sect. I.2*):

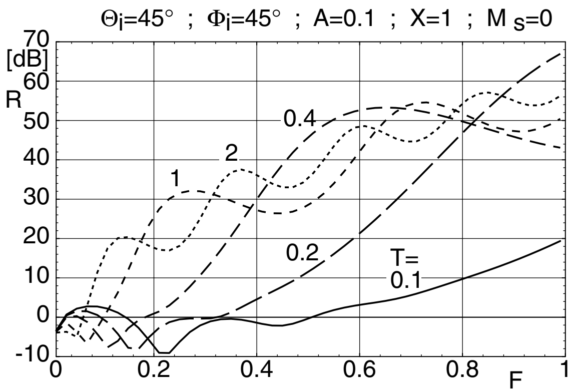
$$\tau(\Theta_i,\ \Phi_i) = \frac{\mathrm{Re}\{Z_{r2n}\}}{\cos\Theta_i} \left| \frac{2j\,\beta \cdot \mathrm{si}\,(k_0 a\ \sin\Theta_i\ \sin\Phi_i)}{j\,\beta\,(Z_{r1n} + Z_{r2n}) \cdot \cosh\gamma + (\beta^2 Z_{r1n} Z_{r2n} - 1) \cdot \sinh\gamma} \right|^2. \tag{15}$$

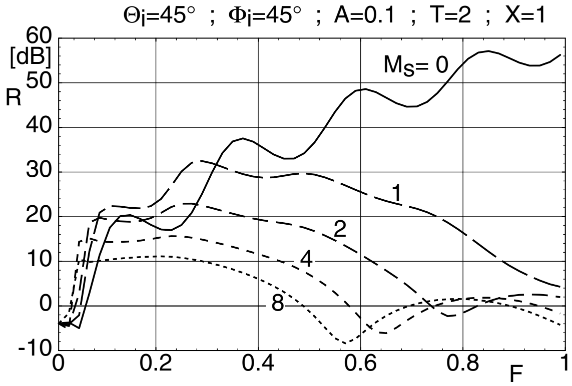
A set of non-dimensional parameters for a slit with a lining consisting of a layer of porous material (flow resistivity Ξ, made locally reacting by partitions) covered with a foil of surface mass density m is:

$$F = f\,d/c_0 = d/\lambda_0\ \ ;\ \ A = 2a/d\ \ ;\ \ X = \Xi\,t/Z_0\ \ ;\ \ M_s = m/\rho_0 d\ \ ;\ \ T = t/d. \tag{16}$$

The frequency response curves of the sound transmission loss $R = -10 \cdot \lg(\tau)$ have a great variety of forms, depending on the parameter values. A few examples will be given below.





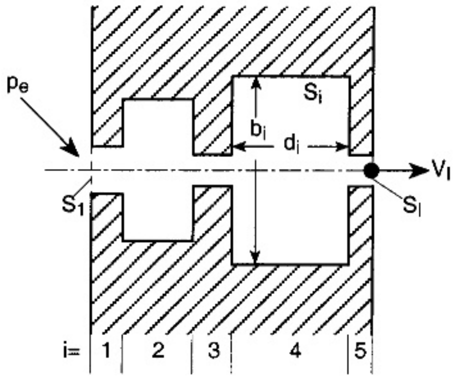


I.6 Chambered Joint

▸ *See also:* Mechel, Vol. III, Ch. 8 (1998)

Some joints between construction elements have a cross section which, in principle, is a sequence of chambers (e.g. joints of facade elements, joints of the window frame, etc.).

Let a plane sound wave p_e be incident with the wave vector in the plane normal to the wall and the length of the joint, at a polar angle Θ_i.



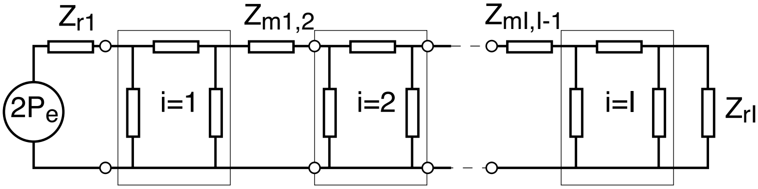
Let S_i; $i = 1, 2, \ldots, I$; be the cross-section areas of the duct elements into which the joint can be subdivided.

The sound transmission coefficient $\tau(\Theta_i)$ will be:

$$\tau(\Theta_i) = \frac{S_I}{S_1}\,\frac{\mathrm{Re}\{Z_{rI}/Z_0\}}{\cos\Theta_i}\cdot\left|\frac{Z_0 V_I}{P_e}\right|^2 = 4\,\frac{S_I}{S_1}\,\frac{\mathrm{Re}\{Z_{rI}/Z_0\}}{\cos\Theta_i}\cdot\left|\left(\frac{Z_0}{S_I}\,S_I V_I\right)/(2P_e)\right|^2, \qquad (1)$$

where Z_{rI} is the radiation impedance of the exit orifice S_I and Z_{r1} the radiation impedance of the entrance orifice $i = 1$.

The composed duct can be described with an equivalent chain network, in which the duct sections are represented by Π-fourpoles separated in the longitudinal branch by mass reactances $Z_{mi,k}$ of an internal orifice i if it enters into a wider duct section k.



Interpret the network as a pq-network, i.e. with volume flows $q_i = S_i v_i$ instead of particle velocities. Then the Π-fourpole elements (longitudinal impedances Z_i, transversal admittances G_i) are:

$$Z_i = \frac{Z_{ai}}{S_i} \sinh(\Gamma_{ai} d_i) \quad ; \quad G_i = \left[\frac{Z_{ai}}{S_i} \tanh(\Gamma_{ai} d_i) \right]^{-1} - \frac{1}{Z_i}, \tag{2}$$

and the longitudinal impedances $Z_{mi,k}$ from the end corrections are:

$$Z_{mi,k} = j \frac{Z_0}{b_i} k_0 b_i \frac{\Delta\ell_{i,k}}{b_i}, \tag{3}$$

where the end corrections are taken from *Ch. F*, or from *Sects. H.4–H.7*. The required quantity $S_I V_I/(2P_e)$ is evaluated with the iterative method of *Sect. C.5*, with p_N, p_0 taken from there, and $$\frac{S_I V_I}{2P_e} = \frac{S_I}{Z_{rI}} \frac{p_N}{p_0}. \tag{4}$$

I.7 "Noise Sluice"

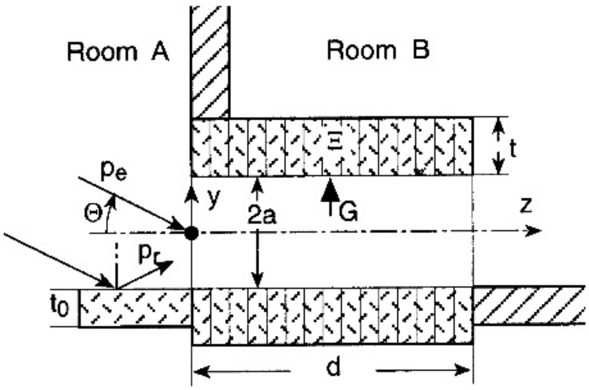
► *See also:* Mechel, Vol. III, Ch. 9 (1998)

Imagine two neighbouring rooms, room A very loud, room B with many working places, and a heavy traffic of fork trucks between the rooms (for material transport), so that sound-insulating doors would be inconvenient.

Design an open passageway as a sufficiently wide, lined duct. It is important that the opening of the duct be placed in room A such that sound incidence on it is predominantly under large polar angles Θ (see below for additional measures to achieve that). If the opening is near a side wall of room A (as shown), that side wall should be made absorbing on a certain length to produce an unsymmetrical sound incidence.

Formulation (in two dimensions) of the directly incident plane wave p_e and of the reflected incident wave p_r:

$$\begin{aligned} p_e(y,z) &= P_e\, e^{jk_0 y \sin\Theta} \cdot e^{-jk_0 z \cos\Theta}, \\ p_r(y,z) &= r_0 \cdot p_e(-a,z) \cdot e^{-jk_0 y \sin\Theta} = r_0 P_e\, e^{-jk_0 a \sin\Theta} \cdot e^{-jk_0 y \sin\Theta} \cdot e^{-jk_0 z \cos\Theta}. \end{aligned} \tag{1}$$



The sound field p(x, y) in the sluice duct is formulated as a sum of symmetrical and anti-symmetrical silencer modes:

$$p(y,z) = \sum_m p_m(y,z) = \sum_m A_m\, e^{-\Gamma_m z} \cdot \begin{cases} \cos(\varepsilon_m y) \\ \sin(\varepsilon_m y) \end{cases} \quad ; \quad \Gamma_m^2 = \varepsilon_m^2 - k_0^2. \tag{2}$$
$$Z_0 v_z(y,z) = -\sum_m j\,\frac{\Gamma_m}{k_0} A_m\, e^{-\Gamma_m z} \cdot \begin{cases} \cos(\varepsilon_m y) \\ \sin(\varepsilon_m y) \end{cases}$$

The transversal wave numbers are solutions of the characteristic equations:

$$\begin{aligned} \varepsilon_{m_{sy}} a \cdot \tan(\varepsilon_{m_{sy}} a) &= \ j\,k_0 a \cdot Z_0 G = \ jU, \\ \varepsilon_{m_{as}} a \cdot \cot(\varepsilon_{m_{as}} a) &= -j\,k_0 a \cdot Z_0 G = -jU \end{aligned} \tag{3}$$

with $U = k_0 a \cdot Z_0 G$, if the lining is locally reacting with a surface admittance G (see *Ch. J*, "Duct Acoustics", for an evaluation of sets of mode wave numbers).

Mode norms:

$$\begin{aligned} N_{m_{sy}} &:= \frac{1}{2a}\int_{-a}^{a} \cos^2(\varepsilon_{m_{sy}} y)\, dy = \frac{1}{2}\left(1 + \frac{\sin(2\varepsilon_{m_{sy}} a)}{2\varepsilon_{m_{sy}} a}\right), \\ N_{m_{as}} &:= \frac{1}{2a}\int_{-a}^{a} \sin^2(\varepsilon_{m_{as}} y)\, dy = \frac{1}{2}\left(1 - \frac{\sin(2\varepsilon_{m_{as}} a)}{2\varepsilon_{m_{as}} a}\right). \end{aligned} \tag{4}$$

Mode-coupling coefficients with a plane wave:

$$\begin{aligned} S_{m_{sy\pm}} &:= \frac{1}{2a}\int_{-a}^{+a} e^{\pm j k_0 y \sin\Theta} \cdot \cos(\varepsilon_{m_{sy}} y)\, dy, \\ S_{m_{as\pm}} &:= \frac{1}{2a}\int_{-a}^{+a} e^{\pm j k_0 y \sin\Theta} \cdot \sin(\varepsilon_{m_{as}} y)\, dy. \end{aligned} \tag{5}$$

Because $S_{m_{sy+}} = S_{m_{sy-}}$ and $S_{m_{as+}} = -S_{m_{as-}}$, only the index + is needed further, so we simplify: $S_{m_{sy}} = S_{m_{sy+}}$, $S_{m_{as}} = S_{m_{as+}}$:

$$S_{m_{sy}} = \frac{1}{2}\left(\frac{\sin(\varepsilon_{m_{sy}} a - k_0 a \sin\Theta)}{\varepsilon_{m_{sy}} a - k_0 a \sin\Theta} + \frac{\sin(\varepsilon_{m_{sy}} a + k_0 a \sin\Theta)}{\varepsilon_{m_{sy}} a + k_0 a \sin\Theta}\right)$$
$$S_{m_{as}} = \frac{j}{2}\left(\frac{\sin(\varepsilon_{m_{as}} a - k_0 a \sin\Theta)}{\varepsilon_{m_{as}} a - k_0 a \sin\Theta} - \frac{\sin(\varepsilon_{m_{as}} a + k_0 a \sin\Theta)}{\varepsilon_{m_{as}} a + k_0 a \sin\Theta}\right) \tag{6}$$

with limit values for $\varepsilon_m a \to \pm k_0 a \cdot \sin\Theta$: $S_{m_{sy}} \to N_{m_{sy}}; S_{m_{as}} \to \pm j\, N_{m_{as}}$,
and if $\varepsilon_m a \to 0$: $N_{m_{sy}} \to 1; N_{m_{as}} \to 0.$ (7)

Field matching at the sluice entrance gives for the mode amplitudes:

$$\frac{A_{m_{sy}}}{P_e} = \frac{S_{m_{sy+}}}{N_{m_{sy}}}\left(1 + r_0\, e^{-j k_0 a \sin\Theta}\right); \frac{A_{m_{as}}}{P_e} = \frac{S_{m_{as+}}}{N_{m_{as}}}\left(1 - r_0\, e^{-j k_0 a \sin\Theta}\right). \tag{8}$$

Effective sound power of the incident wave p_e:

$$\Pi_e(\Theta) = \frac{a}{Z_0}\,|P_e|^2 \cos\Theta. \tag{9}$$

Effective modal sound power incident in the duct on the exit orifice:

$$\Pi_m(d) = \frac{1}{2}\,\mathrm{Re}\left\{\int_{-a}^{a} p_m(y,d)\cdot v_{zm}^*(y,d)\,dy\right\}. \tag{10}$$

Assuming a good radiation efficiency of the modes at the duct exit (because the duct is wide), the transmission coefficient of the noise sluice is:

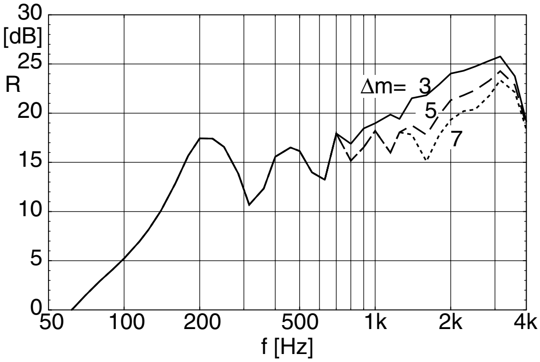
$$\tau(\Theta) = \frac{\sum_m \Pi_m(d)}{\Pi_e(\Theta)} = \frac{1}{2\cos\Theta}\sum_m \frac{\Gamma''_m}{k_0}\, e^{-2\Gamma'_m d}$$
$$\times \left|\frac{S_m}{N_m}\left(1 \pm r_0\, e^{-j k_0 a \sin\Theta}\right)\right|^2 \left[\frac{\sinh(2\varepsilon''_m a)}{2\varepsilon''_m a} \pm \frac{\sin 2(\varepsilon'_m a)}{2\varepsilon'_m a}\right] \tag{11}$$

(with $\Gamma_m = \Gamma'_m + j\cdot\Gamma''_m$; $\varepsilon_m = \varepsilon'_m + j\cdot\varepsilon''_m$). The summation is over the symmetrical (with the upper sign in $\pm$) and anti-symmetrical modes (with the lower sign in $\pm$).

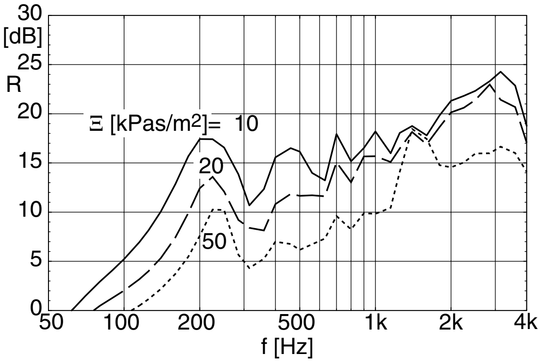
In the following examples of the transmission loss $R = -\lg(\tau(\Theta))$ the lining of the duct and of the wall in front of the duct is assumed to be a layer of glass fibre material, if necessary made locally reacting by internal partitions; the flow resistivity of the material is Ξ.

The mode orders m in the summation are taken in a range $\mathrm{Max}(1, m_0 - \Delta m) \le m \le m_0 + \Delta m$ with an interval width $2\Delta m$ and the central order m_0 selected so that

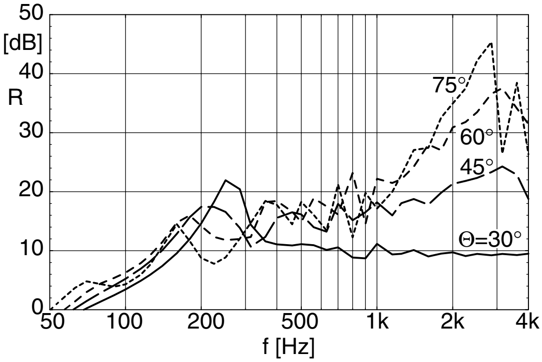
$$|\mathrm{Re}\{\varepsilon_m a - k_0 a \cdot \sin\Theta\}| = \min(m).$$



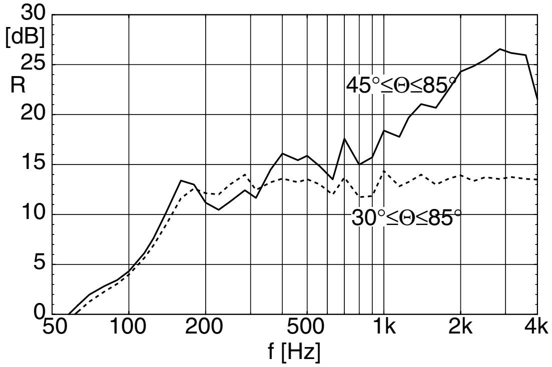
Influence of order interval width Δm on R(Θ).
Input parameters: $\Theta = 45°$; $a = 1[\mathrm{m}]$; $d = 3[\mathrm{m}]$; $t = 0.25[\mathrm{m}]$; $t_0 = 0.25[\mathrm{m}]$; $\Xi = 10[\mathrm{kPa} \cdot \mathrm{s/m^2}]$



Influence of lining resistivity Ξ on R(Θ).
Input parameters: $\Theta = 45°$; $a = 1[\mathrm{m}]$; $d = 3[\mathrm{m}]$; $t = 0.25[\mathrm{m}]$; $t_0 = 0.25[\mathrm{m}]$; $\Delta m = 5$

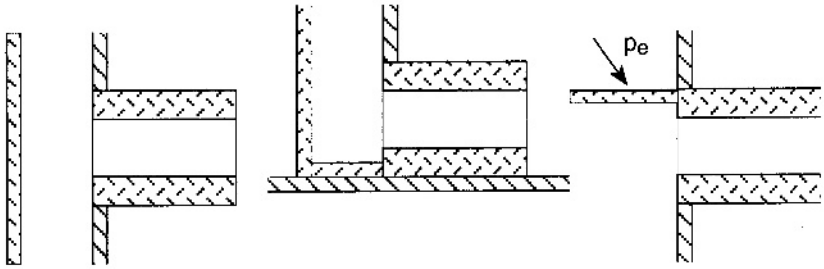


Influence of angle of incidence on R(Θ).
Input parameters: $a = 1[\mathrm{m}]$; $d = 3[\mathrm{m}]$; $t = 0.25[\mathrm{m}]$; $t_0 = 0.25[\mathrm{m}]$; $\Xi = 10[\mathrm{kPa} \cdot \mathrm{s/m^2}]$; $\Delta m = 5$



Sound transmission loss R of a noise sluice for quasi-diffuse sound incidence with two ranges of incidence angle.
Input parameters: $a = 1[\mathrm{m}]; d = 3[\mathrm{m}]; t = 0.25[\mathrm{m}]; t_0 = 0.25[\mathrm{m}]; \Xi = 10[\mathrm{kPa} \cdot \mathrm{s/m^2}];$ $\Delta m = 5$

Measures to avoid sound incidence at small Θ:



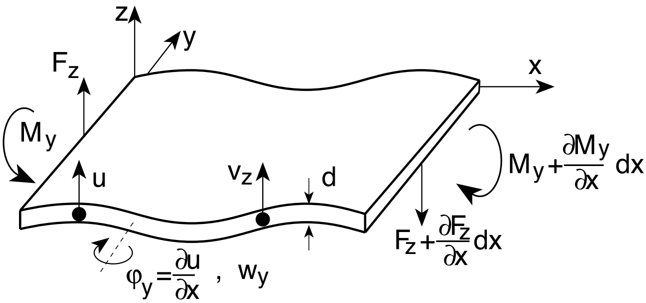
I.8 Sound Transmissionßindexsound transmission through plates through Plates, Some Fundamentals

► *See also:* Mechel, Vol. III, Ch. 10 (1998)

Bending-wave equation: $$[\Delta\Delta - k_B^4]\, v_z = \frac{j\omega}{B}\, \delta p \qquad (1)$$

with Δ = Laplace operator, ω = angular frequency, B = bending stiffness, $\delta p = p_{front} - p_{back}$ = driving sound pressure difference, k_B = free bending wave number.

$$\begin{aligned} k_B &= \frac{\omega}{c_B} = \frac{2\pi}{\lambda_B} = \sqrt{\omega}\,\sqrt[4]{m/B} = \sqrt{\frac{\omega}{c_L d}}\,\sqrt[4]{12\,(1-\sigma^2)} = \frac{1}{d}\sqrt{k_L d}\,\sqrt[4]{12\,(1-\sigma^2)} \\ &= k_0\sqrt{f_{cr}/f} \xrightarrow[\sigma=0.35]{} 4.515\sqrt{\frac{f}{c_L d}} \end{aligned} \qquad (2)$$



Kinematic and dynamic quantities at a plate section, in Cartesian coordinates.

u	=	elongation
v_z	=	velocity
φ	=	rotation angle with y as axis
w_z	=	rotational velocity
F_z	=	transversal force
M_y	=	torsional moment with y as axis

with c_B = bending wave velocity, λ_B = bending wavelength, m = surface mass density, d = plate thickness, k_L = longitudinal wave number, c_L = longitudinal wave velocity, σ = Poisson ratio, f = frequency, f_{cr} = critical coincidence frequency.

$$f_{cr} = \frac{c_0^2}{2\pi}\sqrt{\frac{m}{B}} = \frac{c_0^2}{2\pi d}\sqrt{\frac{12\rho(1-\sigma^2)}{E}} = \frac{c_0^2}{2\pi d\, c_L}\sqrt{12(1-\sigma^2)} \xrightarrow[\sigma=0.35]{} \frac{60\,761}{d\, c_L}. \tag{3}$$

Coincidence frequency for angle Θ of incidence: $$f_c = \frac{f_{cr}}{\sin^2\Theta}. \tag{4}$$

Table 1 Characteristic wave speeds. See Table 4 for S, E, D, B

Wave type	Speed	Remark
Shear wave, torsional wave	$c_S = \sqrt{S/\rho}$	ρ = plate material density
Compressional wave in a bar	$c_{L,St} = \sqrt{E/\rho}$	
Compressional wave, longitudinal wave	$c_D = \sqrt{D/\rho}$ $= c_0 = \gamma P_0$ in a gas	γ = adiabatic exponent P_0 = static pressure
Bar-bending wave	$c_{B,St} = \sqrt[4]{\frac{\omega^2 B_{St}}{m'}}$	m' = mass per length
Plate-bending wave	$c_{B,Pl} = c_B = \sqrt{\omega}\sqrt[4]{\frac{B}{m}}$	m = mass per area (surface mass density)
Rayleigh wave	$c_{Rayl} \approx 0.92 \cdot c_s$	

If a plate is infinite in its lateral extension (in practice: if its smallest lateral dimension is large compared to the bending wavelength λ_B, so that plate resonances can be neglected in their influence on the transmission loss) the most important quantity of a plate is its partition impedance Z_T (see below and *Sect. H.17*).

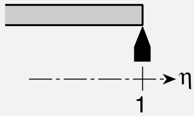
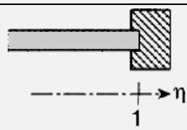
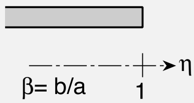
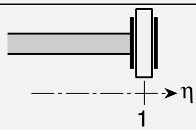
If finite plate dimensions must be considered, the boundary conditions at the plate boundaries must be taken into account. "Classical" boundary conditions are given in Table 2. They can be expressed also with *force impedances* Z_F and *momentum impedances* Z_M at the boundary $\{x_R, y_R\}$. These impedances are defined by:

$$\begin{aligned} F(x_R, y_R) &= Z_K(x_R, y_R) \cdot v_z(x_R, y_R), \\ M_i(x_R, y_R) &= Z_M(x_R, y_R) \cdot w_i(x_R, y_R). \end{aligned} \tag{5}$$

Additional *corner impedances* Z_E apply at plate corners $\{x_E, y_E\}$:

$$F_E(x_E, y_E) = Z_E(x_E, y_E) \cdot v_z(x_E, y_E). \tag{6}$$

Table 2 Classical boundary conditions for plates

Fixation	Condition	Symbol
Simply supported	$v(\xi, 1) = \dfrac{\partial^2 v(\xi, 1)}{\partial \eta^2} = 0$ $1/Z_K = Z_M = 0$	
Clamped	$v(\xi, 1) = \dfrac{\partial v(\xi, 1)}{\partial \eta} = 0$ $1/Z_K = 1/Z_M = 0$	
Free	$\dfrac{\partial^2 v}{\partial \eta^2} + \sigma \beta^2 \dfrac{\partial^2 v}{\partial \xi^2} = 0$ $\dfrac{\partial^3 v}{\partial \eta^3} + (2 - \sigma)\beta^2 \dfrac{\partial^3 v}{\partial \eta \partial \xi^2} = 0$ $Z_K = Z_M = 0$	
Hinged	$Z_K = 1/Z_M = 0$	

Partition impedance Z_T:

Let a thin, i.e. incompressible, plate in an orthogonal co-ordinate system $\{x_1, x_2, x_3\}$ be on the co-ordinate surface $x_3 = \xi$, and let $p_f(x_1, x_2, \xi_3)$ be the sound pressure at the plate on its front side (side of excitation) and $p_b(x_1, x_2, \xi_3)$ the sound pressure at the plate on its back side; further, let $\Delta p = p_f(x_1, x_2, \xi_3) - p_b(x_1, x_2, \xi_3)$ be the driving pressure difference, which generates a plate velocity $v(x_1, x_2, \xi_3)$ (counted positive in the direction front $\rightarrow$ back).

The boundary conditions at the plate are:

$$v_f = v_b = v \quad ; \quad p_f - p_b = Z_T \cdot v, \tag{7}$$

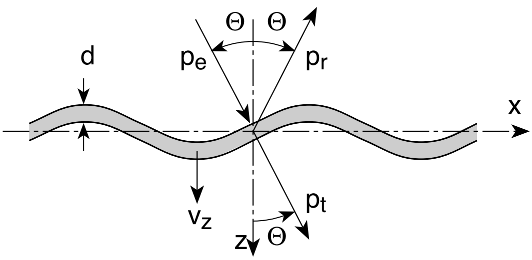
where the last condition demands proportionality between Δp and v with a constant factor Z_T if the plate is homogeneous in x_1, x_2 (i.e. for example, the plate is large or a closed shell with constant curvature). It is assumed that p_f and p_b satisfy the wave equation in air (and possible other boundary conditions at surfaces other than the plate). The bending-wave equation for v

$$\left[\Delta_{x_1,x_2}\Delta_{x_1,x_2} - k_B^4\right] v = \frac{j\omega}{B} \cdot \Delta p \tag{8}$$

and the last boundary condition can be combined into:

$$\frac{\Delta_{x_1,x_2}\Delta_{x_1,x_2} v}{v} - k_B^4 = \frac{j\omega}{B} \cdot \frac{\Delta p}{v} = \frac{j\omega}{B} \cdot Z_T. \tag{9}$$

For stationary sound fields and homogeneous plates the function $v(x_1, x_2)$ is, up to a constant factor, the same as $p_f(x_1, x_2, \xi_3)$, $p_b(x_1, x_2, \xi_3)$. So the first term in the last equation is known for given sound field formulations.



For a plane plate in the plane $z = 0$ and a plane wave p_e incident in the x,z plane at a polar angle Θ, i.e. with a trace wave number $k_x = k_0 \cdot \sin\Theta$, the partition impedance for a plate with surface mass density m is:

$$Z_T = \frac{\Delta p}{v} = \frac{B}{j\omega}\left(k_x^4 - k_B^4\right) = j\omega m\,\frac{k_B^4 - k_x^4}{k_B^4} = j\omega m\left(1 - \left(\frac{f}{f_c}\right)^2\right)$$
$$= j\omega m\left(1 - \left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta\right). \tag{10}$$

If the plate has bending losses, i.e. $B \longrightarrow B(1 + j\eta)$ with the loss factor η, then:

$$\frac{Z_T}{Z_0} = \frac{\omega m}{Z_0}\left[\eta\left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta \;+ j\left(1-\left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta\right)\right]$$

$$= 2\pi\, Z_m F\left[\eta F^2 \sin^4\Theta + j\left(1 - F^2\sin^4\Theta\right)\right] \qquad F := \frac{f}{f_{cr}}\;;\; Z_m := \frac{f_{cr} m}{Z_0} = \frac{f_{cr} d}{Z_0}\rho \qquad (11)$$

$$= 2\pi\, Z_m F\left[\eta\left(\frac{f}{f_c}\right)^2 + j\left(1-\left(\frac{f}{f_c}\right)^2\right)\right];$$

where ρ is the plate material mass density.

As long as the compressibility of the plate can be neglected, the Timoshenko-Mindlin theory of thick plates gives in an analogous way for the partition impedance:

$$Z_T = j\omega m\,\frac{k_B^4 - (k_x^2 - k_L^2)(k_x^2 - k_R^2)}{k_B^4 - k_R^2(k_x^2 - k_L^2)} \quad ; \quad k_R = \frac{\sqrt{12}}{\pi}k_S \qquad (12)$$

with k_x the trace wave number of the exciting field, k_B the free bending wave number, k_L the longitudinal wave number and k_S the shear wave number.

Table 3 Density and elastic constants of materials

Material	Density $\rho[kg/m^3]$	E modulus $E[MN/m^2]$	$f_{cr}d$ [Hz · m]	Z_m [–]	Loss fact. η[–]
Construction materials					
Concrete	2100–2300	$25–40 \cdot 10^3$	15–18.5	77–104	0.05
Lean concrete	2000	15 000	23	112	
Light concrete	800–1400	$1.5–3 \cdot 10^3$	37–48	72–164	0.015
Porous concrete	600–700	$1.4–2 \cdot 10^3$	37–45	54–77	0.01
Cement floor	2200	30 000	17	91	
Xylolith floor	1600	6000	32.5	127	0.03
Asphalt floor	2200	$6–15 \cdot 10^3$	24–42	129–225	0.03–0.3
Plaster floor	1200	20000	15.5–16	45–47	0.006
Gypsum panel	1000–1200	$3.5–7 \cdot 10^3$	24–35	58–102	0.004
Plaster board	1000	3200	31–35	85	0.03
Fibre cement board	2000–2100	$20–30 \cdot 10^3$	16.5–20	80–102	0.01
Brick wall	1700–1800	$9–25 \cdot 10^3$	16–27	66–118	0.04
Glass	2500	$60–80 \cdot 10^3$	11–13	67–79	0.001

Table 3 continued

Material	Density $\rho[kg/m^3]$	E modulus $E[MN/m^2]$	$f_{cr}d$ [Hz · m]	Z_m [–]	Loss fact. η[–]
Chip board	600–1000	$2–5 \cdot 10^3$	23–36	34–88	0.03
Plywood	600–800	$5–12 \cdot 10^3$	14–34.5	20–65	0.02
Oak wood	700	200–1000	18–32	31–55	0.01
Pine wood	480	100–500	20–32	23–37	0.01
Hard board	1000	$3–4.5 \cdot 10^3$	29.5–36.5	72–89	0.015
Plastics					
Acryl glass	1200	5600	29	85	0.06
Polypropylene	1100	3000	38	102	0.1
Polyester	1200	4500	32.5	95	0.14
PVC, hard	1300	2700	43.5	138	0.04
PVC, 30% softener	1250		48	1220	
Polyethylene, hard	950	1700	47	109	0.04
Polyethylene, soft	920	400	95.5	214	0.1
Polystyrene	1070	3000	37.5	98	0.01
Polystyrene+30% glass fibre	1450	8000	27	95	
Polyester+glass fibre	2200	11500	27.5	147	0.02
Metals					
Aluminium	2700	74 000	12	79	$7 \cdot 10^{-5}$
Lead	11400	18000	48.5	1348	0.02
Copper	8900	125000	17	369	
Brass	8500	96000	18.6	386	0.001
Steel, cast steel	7800	200000	12.3	234	$1 \cdot 10^{-4}$
Malleable iron	7500	170000	13.2	241	
Cast iron with spher. graphite	7250	120000	15.4	272	0.01
Cast iron with lamell. graphite	7250	120000	15.4	272	0.02
Zinc	7130	13000	46.5	809	
Tin	7280	4400	81	1438	

Table 3 above gives the required elastic data of materials. Of special interest are the density ρ and the product $f_{cr} \cdot d$ which is a material constant.

Table 4 Relations of elastic constants

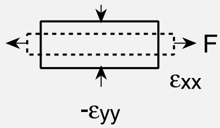
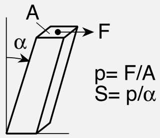
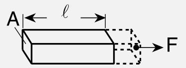
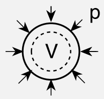
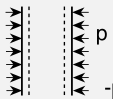
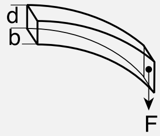
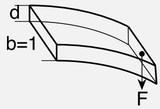
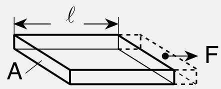
Quantity	Symbol, relations	Remark
Lame constants	λ, μ $-p_{ii} = \lambda \cdot \operatorname{div} \vec{s} + 2\mu \cdot \varepsilon_{ii}$ $-p_{ik} = 2\mu \cdot \varepsilon_{ik}$	
Poisson number	$\sigma = -\varepsilon_{yy}/\varepsilon_{xx} = \dfrac{\lambda}{2(\lambda+\mu)}$ $= \dfrac{E}{2S} - 1$ $\dfrac{\mu}{\lambda} = \dfrac{1-2\sigma}{2\sigma}$; $0 \le \sigma < 0.5$	 incompressible: σ=0.5
Shear modulus	$S = \mu$ [Pa] $= \dfrac{1}{2}\dfrac{1}{1+\sigma}E$; $(\sigma < 0.4)$	
E modulus, Young's modulus	$E = \mu\dfrac{3\lambda+2\mu}{\lambda+\mu}$ [Pa] $= 2\mu/(1+\sigma) = 2S(1+\sigma)$	 -p= f/A= E·Δℓ/ℓ= E·ε_{xx}
Compression modulus	$K = \lambda + \dfrac{2}{3}\mu$ [Pa] $= \left(\dfrac{2}{3} + \dfrac{2\sigma}{1-2\sigma}\right) \cdot S$ $= \dfrac{E}{3(1-2\sigma)} = \dfrac{1}{\kappa}$	 -p= K·dV/V κ = compressibility
Dilation modulus	$D = \lambda + 2\mu$ [Pa] $= 2\dfrac{1-\sigma}{1-2\sigma} \cdot S = \dfrac{1-\sigma}{(1+\sigma)(1-2\sigma)} \cdot E$ $= \dfrac{1}{3}\dfrac{1-\sigma}{1+\sigma} \cdot K$	 -p= D·s_{xx} for gas: $D = \rho_0 c_0^2 = \gamma P$

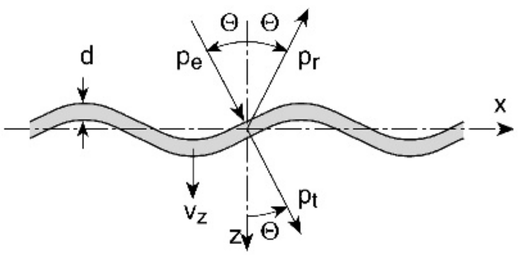
Table 4 continued

Quantity	Symbol, relations	Remark
Bar-bending modulus	$B_{st} = \frac{bd^3}{12} \cdot E\,[\mathrm{Pa \cdot m^4}]$	
Plate-bending modulus	$B = B_{pl} = I \cdot E \quad [\mathrm{Pa \cdot m^3}]$ $= \frac{d^3}{3}\frac{\mu(\lambda+\mu)}{\lambda+2\mu} = \frac{d^3}{12(1-\sigma^2)} \cdot E$ $= \frac{d^3}{6(1-\sigma)} \cdot S$	 moment of inertia $I = \frac{d^3}{12(1-\sigma^2)}$
Plate-dilation modulus	$D_{pl} = \frac{E}{1-\sigma} \quad [\mathrm{Pa}]$	

I.9 Sound Transmission through a Simple Plate

► *See also:* Mechel, Vol. III, Ch. 10 (1998)

A plate is "simple" if it is thin (incompressible), homogeneous (except possibly on a micro-scale), isotropic, and unbounded (or at least with large dimensions, so that boundary effects can be neglected).



Transmission as a boundary value problem:

Field formulations:

$$p_e(x,z) = P_e \cdot e^{-jk_x x} \cdot e^{-jk_z z} \qquad k_x^2 + k_z^2 = k_0^2,$$
$$p_r(x,z) = r \cdot P_e \cdot e^{-jk_x x} \cdot e^{+jk_z z} \qquad k_x = k_0 \cdot \sin\Theta \quad ; \quad k_z = k_0 \cdot \cos\Theta, \tag{1}$$
$$p_t(x,z) = P_t \cdot e^{-jk_x x} \cdot e^{-jk_z z}$$

Plate velocity: $$v(x) = V \cdot e^{-jk_x x}, \tag{2}$$

Plate driving pressure: $$\delta p(x) = p_e(x,0) + p_r(x,0) - p_t(x,0), \tag{3}$$

From the bending-wave equation:

$$\left[\frac{\partial^4}{\partial x^4} - k_B^4\right] v_z(x) = \frac{j\omega}{B}\,\delta p(x)$$
$$(k_x^4 - k_B^4)\cdot V\cdot e^{-jk_x x} = \frac{j\omega}{B}\;(P_e(1+r) - P_t)\cdot e^{-jk_x x}, \tag{4}$$

the partition impedance (boundary condition for pressure) follows:

$$Z_T = \frac{\delta p}{v_z} = \frac{(P_e(1+r) - P_t)}{V} = \frac{B}{j\omega}\;\left(k_x^4 - k_B^4\right) = j\omega m\,\frac{k_B^4 - k_x^4}{k_B^4}$$
$$= j\omega m\left(1 - \left(\frac{f}{f_c}\right)^2\right) = j\omega m\left(1 - \left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta\right). \tag{5}$$

With boundary conditions for particle velocity:

$$v_z \overset{!}{=} v_{tz}(x,0) = \frac{k_z}{k_0 Z_0}\,P_t\cdot e^{-jk_x x}\;\;;\;\; v_z \overset{!}{=} v_{ez}(x,0) + v_{rz}(x,0) = P_e\,\frac{k_z}{k_0 Z_0}\,(1-r)\cdot e^{-jk_x x}, \tag{6}$$

the system of equations follows:

$$\begin{pmatrix} Z_T & -1 & 1 \\ 1 & 0 & -\cos\Theta/Z_0 \\ 1 & \cos\Theta/Z_0 & 0 \end{pmatrix}\cdot\begin{pmatrix} V \\ rP_e \\ P_t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \cos\Theta/Z_0 \end{pmatrix} \tag{7}$$

having the solution: $$\frac{P_t}{P_e} = \left(1 + \frac{1}{2}\,\frac{Z_T\,\cos\Theta}{Z_0}\right)^{-1}. \tag{8}$$

Sound transmission coefficient: $$\tau(\Theta) = \left|\frac{P_t}{P_e}\right|^2 = \left|1 + \frac{1}{2}\,\frac{Z_T}{Z_0}\,\cos\Theta\right|^{-2} \tag{9}$$

After insertion:

$$\frac{1}{\tau(\Theta)} = 1 + \left(\frac{\cos\Theta}{2Z_0}\,\omega m\left[1 - \left(\frac{k_0}{k_B}\,\sin\Theta\right)^2\right]\right)^2$$
$$= 1 + \left(\frac{\cos\Theta}{2Z_0}\,\omega m\left[1 - \left(\frac{f}{f_c}\right)^2\right]\right)^2$$
$$= 1 + \left(\pi\,Z_m F\cdot\cos\Theta\cdot(1 - F^2\sin^4\Theta)\right)^2$$
$$= 1 + \left(\pi\,Z_m\sqrt{F(F-y)}\cdot(1-y^2)\right)^2 \tag{10}$$

with non-dimensional parameters:

$$y := \frac{f}{f_c}\quad;\quad F := \frac{f}{f_{cr}}\quad;\quad Z_m := \frac{f_{cr} m}{Z_0} = \frac{f_{cr} d}{Z_0}\,\rho. \tag{11}$$

Transmission coefficients for diffuse sound incidence in three and two dimensions:

$$\tau_{3-\mathrm{dif}} = \frac{\int_0^{\Theta_{hi}} \tau(\Theta)\ \cos\Theta\ \sin\Theta\ d\Theta}{\int_0^{\Theta_{hi}} \cos\Theta\ \sin\Theta\ d\Theta} = \frac{2}{\sin^2\Theta_{hi}} \int_0^{\Theta_{hi}} \tau(\Theta)\ \cos\Theta\ \sin\Theta\ d\Theta$$

$$\tau_{2-\mathrm{dif}} = \frac{\int_0^{\Theta_{hi}} \tau(\Theta)\ \cos\Theta\ d\Theta}{\int_0^{\Theta_{hi}} \cos\Theta\ d\Theta} = \frac{1}{\sin\Theta_{hi}} \int_0^{\Theta_{hi}} \tau(\Theta)\ \cos\Theta\ d\Theta \tag{12}$$

(Θ_{hi} = upper limit, $\leq \pi/2$, of sound incidence angles).

Approximations and special cases:

Berger's law for $f \ll f_{cr}$:

$$\frac{1}{\tau(\Theta)} \approx \left(\omega m \frac{\cos\Theta}{2Z_0}\right)^2 . \tag{13}$$

$\dfrac{f \cdot \sin\Theta}{f_c} \gg 1$:

$$\frac{1}{\tau(\Theta)} \approx 1 + \left(\frac{\omega m}{2Z_0}\ \cos\Theta\ \sin^2\Theta\ \left(\frac{f}{f_c}\right)^2\right)^2 . \tag{14}$$

Normal or grazing incidence:

$$\tau(0) = \frac{1}{1 + (\pi\, Z_m F)^2} \quad ; \quad \tau(\pi/2) = 1. \tag{15}$$

Diffuse incidence:

$$\tau_{3-\mathrm{dif}} = \frac{2}{F\ \sin^2\Theta_{hi}} \int_0^{F\ \sin^2\Theta_{hi}} \frac{dy}{1 + \pi^2 Z_m^2\, F\,(F-y)(1-y^2)^2} . \tag{16}$$

When $R_{dif} = -10 \cdot \lg(\tau_{dif})$ is plotted over F, it depends on the single parameter F_m (which is a material constant) (besides the parameter Θ_{hi} of the test conditions).

Transmission with equivalent circuit:

The equivalent circuit method is adequate for the sound transmission through a simple plate.

The immediate result is, as above:

$$\tau(\Theta) = \left|\frac{P_t}{P_e}\right|^2 = \left|1 + \frac{1}{2}\frac{Z_T}{Z_0}\cos\Theta\right|^{-2} . \tag{17}$$

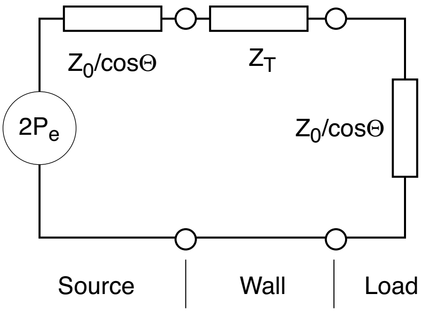


Plate with bending losses:

With bending loss factor η, use above:

$$\frac{Z_T}{Z_0} = \frac{\omega m}{Z_0}\left[\eta\left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta + j\left(1-\left(\frac{f}{f_{cr}}\right)^2 \sin^4\Theta\right)\right]$$
$$= 2\pi\, Z_m F\left[\eta F^2 \sin^4\Theta + j\,\left(1 - F^2 \sin^4\Theta\right)\right] \qquad (18)$$
$$= 2\pi\, Z_m F\left[\eta\left(\frac{f}{f_c}\right)^2 + j\left(1-\left(\frac{f}{f_c}\right)^2\right)\right].$$

Transmission coefficient for diffuse sound incidence:

$$\tau_{3-\mathrm{dif}} = \frac{2}{F\ \sin^2\Theta_{hi}} \int_0^{F\ \sin^2\Theta_{hi}} \frac{dy}{[\,1+\pi\eta Z_m\sqrt{F(F-y)}\,y^2\,]^2 + \pi^2 Z_m^2\, F\,(F-y)(1-y^2)^2}. \qquad (19)$$

Thick plates:

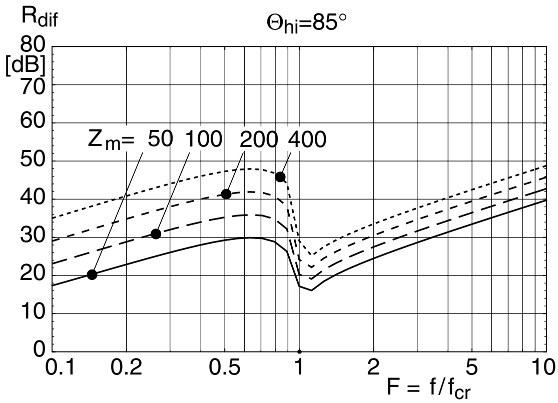
Use in above relations the partition impedance fromßindexTimoshenko-Mindlin theory the Timoshenko-Mindlin theory:

$$Z_T = j\omega m\, \frac{k_B^4 - (k_x^2 - k_L^2)(k_x^2 - k_R^2)}{k_B^4 - k_R^2(k_x^2 - k_L^2)};\, k_R = \frac{\sqrt{12}}{\pi}\, k_S \qquad (20)$$

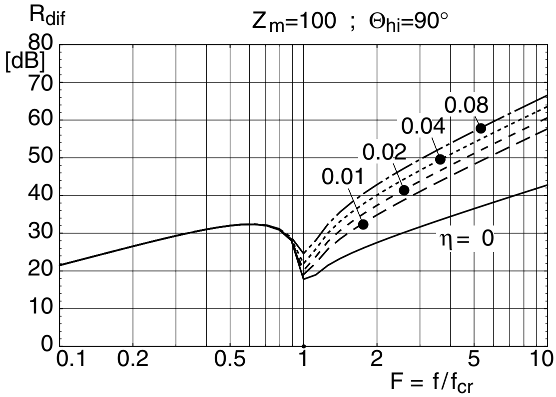
with k_x the trace wave number of the exciting field, k_B the free bending-wave number, k_L the longitudinal wave number and k_S the shear wave number.

Alternatively, use in $Z_T = j\omega m\left[1-\left(\frac{k_s}{k_B}\right)^4\right]$ the "corrected" bending-wave number k_B:

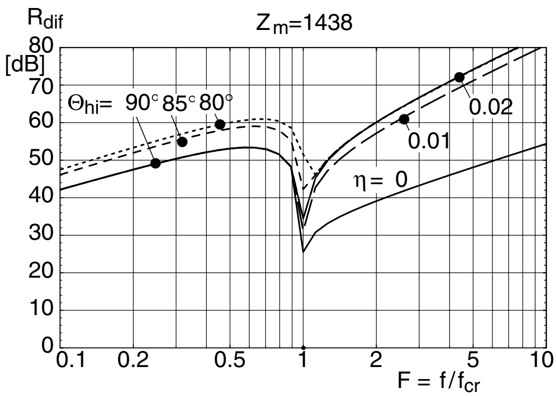
$$k_B^2 = \frac{4.43}{24}\,\frac{\omega^2 m\, d^2}{B} + \sqrt{\left(\frac{4.43}{24}\,\frac{\omega^2 m\, d^2}{B}\right)^2 - \frac{m\omega^2}{B}\,\left(\frac{0.26\,\omega^2 md}{E} - 1\right)}. \qquad (21)$$



Transmission loss for diffuse sound incidence of simple plates with different values of Z_m



Transmission loss for diffuse sound incidence of simple plates with different values of bending loss factor η



Highest possible transmission loss values of a simple plate for diffuse sound incidence; the plate consists of tin

Plane wave transmission through unbounded, thin, homogeneous and anisotropic plate between two different fluids: (▸ *See also:* Maysenhölder, Acustica 84 (1998))

A thin plate with thickness h, mass density ρ_{II} and generalized bending stiffnesses $B_{\alpha\beta\gamma\delta}$ [see *Sect. Q.10.3*, eq. (35)] separates two fluids with densities ρ_I and ρ_{III} and sound speeds c_I and c_{III}. The polar angle of incidence (relative to plate normal) is Θ_I; the azimuthal angle of incidence (in plane of plate) is Φ. The polar angle Θ_{III} of the transmitted wave is given by Snell's law:

$$\frac{\sin\Theta_{III}}{\sin\Theta_I}=\frac{c_{III}}{c_I}. \tag{22}$$

The transmission factor T is:

$$T = 2\left\{\frac{\cos\Theta_{III}}{\rho_{III}\,c_{III}}\left[\left(\frac{\rho_I\,c_I}{\cos\Theta_I}+\frac{\rho_{III}\,c_{III}}{\cos\Theta_{III}}\right)\right]\right. \\ \left. + j\left(\omega\,\rho_{II}\,h-\omega^3\,\hat{B}\left(\frac{\sin\Theta_I}{c_I}\right)^4\right)\right\}^{-1} \tag{23}$$

$$\text{with}\quad \hat{B}=\sum_{\alpha,\beta,\gamma,\delta=1}^{2} B_{\alpha\beta\gamma\delta}\,\varphi_\alpha\,\varphi_\beta\,\varphi_\gamma\,\varphi_\delta\ ;\qquad \varphi_1=\cos\Phi\ ;\quad \varphi_2=\sin\Phi. \tag{24}$$

The transmission coefficient is:

$$\tau(\Theta_I)\ =\ \frac{\rho_I\,c_I}{\rho_{III}\,c_{III}}\,\frac{\cos\Theta_{III}}{\cos\Theta_I}\ |T|^2 \tag{25}$$

(if $\cos\Theta_{III}$ = real, otherwise $\tau = 0$), and the transmission loss is $R(\Theta_I) = -10\cdot\lg\tau$.

In the special case of an isotropic plate between identical fluids one recovers Cremer's result (B the usual bending stiffness) with:

$$T\ =\ \left[1\ +\ j\,\frac{\cos\Theta_I}{2\,\rho_I\,c_I}\left(\omega\,\rho_{II}\,h-\omega^3\,B\left(\frac{\sin\Theta_I}{c_I}\right)^4\right)\right]^{-1}. \tag{26}$$

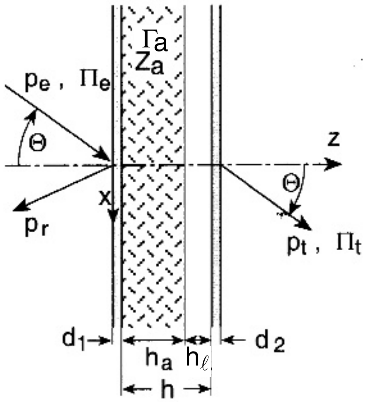
In the special case where there is no plate between two different fluids:

$$T\ =\ \frac{2}{1\ +\ \dfrac{\rho_I\,c_I\,\cos\Theta_{III}}{\rho_{III}\,c_{III}\,\cos\Theta_I}}. \tag{27}$$

I.10 Infinite Double-Shell Wall with Absorber Fill

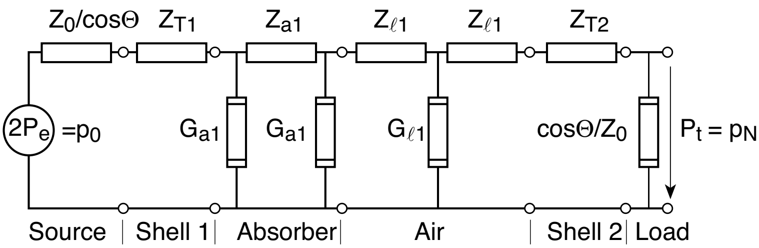
► *See also:* Mechel, Vol. III, Ch. 11 (1998)

Two simple plates with thicknesses d_1, d_2 form an interspace which is filled with a fraction $\beta = h_a/h$ with a bulk reacting porous material having the characteristic values Γ_a, Z_a.



A plane wave p_e is incident at a polar angle of incidence Θ.

The sound transmission is evaluated with the method of equivalent networks.



$Z_{Ti}, i = 1, 2$, are partition impedances of the plates; the network elements of the absorber layer are:

$$\frac{Z_{a1}}{Z_0} = \frac{Z_a}{Z_0} \frac{\sinh(\Gamma_a h_a \cos\Theta_a)}{\cos\Theta_a} \quad ; \quad Z_0 G_{a1} = \frac{\cos\Theta_a}{Z_a/Z_0} \frac{\cosh(\Gamma_a h_a \cos\Theta_a) - 1}{\sinh(\Gamma_a h_a \cos\Theta_a)}, \tag{1}$$

and of the air layer:

$$\frac{Z_{\ell 1}}{Z_0} = j\,\frac{1 - \cos(k_0 h_\ell \cos\Theta)}{\cos\Theta \, \sin(k_0 h_\ell \cos\Theta)} \quad ; \quad Z_0 G_{\ell 1} = j \, \cos\Theta \, \sin(k_0 h_\ell \cos\Theta) \tag{2}$$

with internal angle in the absorber layer from $\cos\Theta_a = \sqrt{1 + \dfrac{\sin^2\Theta}{(\Gamma_a/k_0)^2}}$. (3)

The sound transmission coefficient is: $\tau(\Theta) = \left|\dfrac{P_t}{P_e}\right|^2 = 4\left|\dfrac{p_N}{p_0}\right|^2$. (4)

The numerical evaluation may apply the iterative method of *Sect. C.5*, or analytically with:

$$\frac{p_N}{p_0} = \frac{P_t}{2P_e} = \frac{1}{z_e + (b + g_{a1}\, z_e)\, z_a} \tag{5}$$

and the auxiliary quantities:

$$\begin{aligned}
z_a &= z_{T1} + 1/\cos\Theta \quad ; \quad z_b = z_{\ell 1} + z_{T2} \quad ; \quad z_c = 1 + z_b \cos\Theta; \\
z_d &= z_c + a \cdot z_{\ell 1} \quad ; \quad z_e = z_d + b \cdot z_{a1}; \\
a &= \cos\Theta + g_{\ell 1}\, z_c \quad ; \quad b = a + g_{a1}\, z_d,
\end{aligned} \tag{6}$$

in which z, g are normalised (to Z_0) impedances and admittances.

If the *interspace is full* (with absorber material), then:

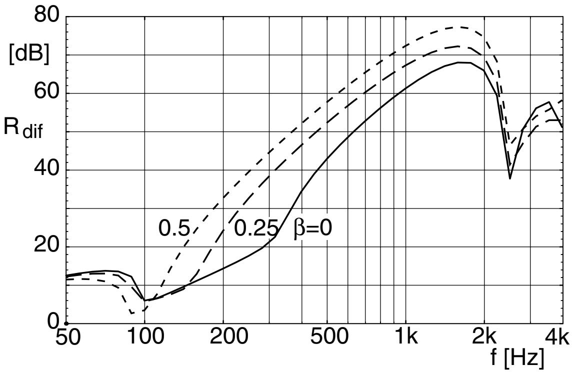
$$\begin{aligned}
\frac{p_N}{p_0} &= \frac{P_t}{2P_e} = \frac{1}{z_c + (a + g_{a1}\, z_c)\, z_a}, \\
z_a &= z_{T1} + 1/\cos\Theta \quad ; \quad z_b = 1 + z_{T2}\cos\Theta \quad ; \quad z_c = z_b + a \cdot z_{a1}, \\
a &= \cos\Theta + g_{a1} \cdot z_b.
\end{aligned} \tag{7}$$

If the *interspace is empty* (i.e. no absorber), then:

$$\begin{aligned}
\frac{p_N}{p_0} &= \frac{P_t}{2P_e} = \frac{1}{z_a + (\cos\Theta + g_{\ell 1}\, z_a)\, z_b}, \\
z_a &= 1 + (z_{\ell 1} + z_{T2})\cos\Theta \quad ; \quad z_b = z_{\ell 1} + z_{T1} + 1/\cos\Theta,
\end{aligned} \tag{8}$$

or after insertion:

$$\begin{aligned}
\frac{p_N}{p_0} &= \frac{P_t}{2P_e} \\
&= \frac{1}{2 + (2\,z_{\ell 1} + z_{T1} + z_{T2})\cos\Theta + g_{\ell 1}\left(\dfrac{1}{\cos\Theta} + z_{\ell 1} + z_{T1}\right)\left(\dfrac{1}{\cos\Theta} + z_{\ell 1} + z_{T2}\right)}.
\end{aligned} \tag{9}$$



Sound transmission loss for diffuse sound incidence of a double-shell wall of plaster board shells, with different fill factors $\beta = h_a/h$ of the absorber layer. Input parameters: $\Theta_{hi} = 85°$; $h = 0.06[m]$; $d_1 = d_2 = 0.0125[m]$: $f_{cr}d_1 = f_{cr}d_2 = 31\ [Hz \cdot m]$; $\rho_1 = \rho_2 = 1000[kg/m^3]$; $\eta_1 = \eta_2 = 0.03$; $\Xi = 10000[Pa \cdot s/m^2]$

The *double-shell resonance* for an empty interspace is at:

$$f_0(\Theta) = \frac{c_0}{2\pi\ \cos\Theta}\sqrt{\frac{\rho_0}{h}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}. \tag{10}$$

I.11 Double-Shell Wall with Thin Air Gap

► *See also:* Mechel, Vol. III, Ch. 11 (1998)

The present object is formally treated as a double-shell wall, completely filled with absorber material, from the previous *Sect. I.10*, but using for Γ_a, Z_a the characteristic values in a flat capillary. Finally, the limit transition $\Gamma_a h_a \to 0$ is applied:

$$\frac{p_N}{p_0} = \frac{P_t}{2P_e} = \frac{1}{z_c + (a + g_{a1}\, z_c)\, z_a},$$

$$z_a = z_{T1} + 1/\cos\Theta\ ; \quad z_b = 1 + z_{T2}\ \cos\Theta\ ; \quad z_c = z_b + a \cdot z_{a1}; \tag{1}$$

$$a = \cos\Theta + g_{a1} \cdot z_b .$$

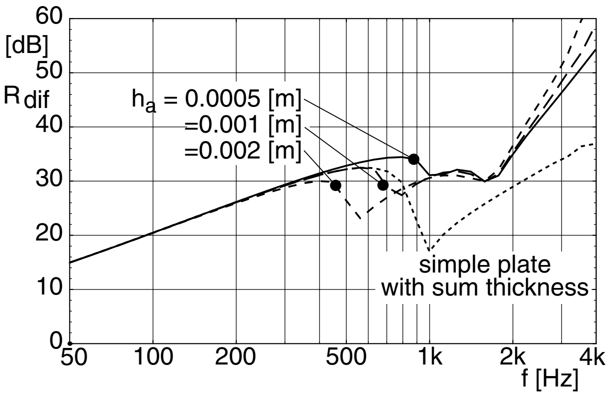
In the limit $\quad z_{a1} \to \dfrac{\Gamma_a Z_a}{k_0 Z_0}\, k_0 h_a\, \dfrac{\cos\Theta_a}{\cos\Theta} \quad ; \quad g_{a1} \to \dfrac{1}{z_{a1}} - \dfrac{1}{z_{a1}} = 0,$ (2)

and therewith the transmission factor:

$$\frac{p_N}{p_0} \approx \frac{1}{2 + (z_{T1} + z_{T2})\ \cos\Theta + \dfrac{\Gamma_a Z_a}{k_0 Z_0}\, k_0 h_a\ \cos\Theta_a}. \tag{3}$$

In the third term of the denominator for very small h_a (with η = dynamic viscosity of air):

$$\frac{\Gamma_a Z_a}{k_0 Z_0} \approx \frac{12\,\eta}{\omega\rho_0\, h_a^2}. \tag{4}$$



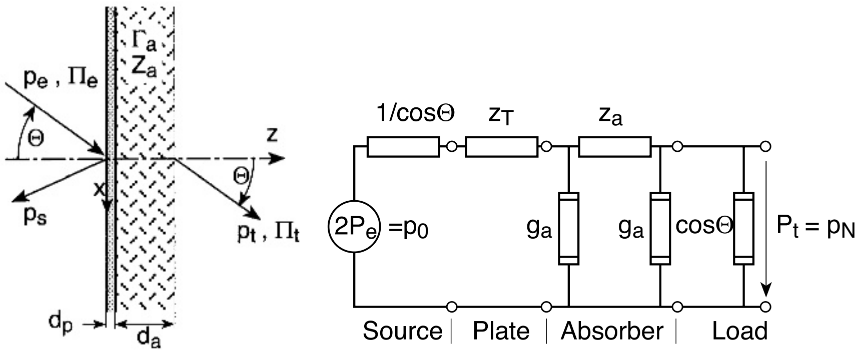
Double-glass pane with thin air gap of different thickness. Input parameters: $\Theta_{hi} = 85°$; $d_1 = 0.004[\mathrm{m}]$; $d_2 = 0.008[\mathrm{m}]$; $f_{cr}d_1 = f_{cr}d_2 = 11[\mathrm{Hzm}]$; $\rho_1 = \rho_2 = 2500[\mathrm{kg/m^3}]$; $\eta_1 = \eta_2 = 0.002$

I.12 Plate with Absorber Layer Behind

► *See also:* Mechel, Vol. III, Ch. 11 (1998)

It is assumed that the structure-borne sound transmission between the plate and the porous absorber layer is negligible (no or only loose contact between them). The evaluation uses the equivalent network method (it produces identical results with the solution of a boundary value problem). Γ_a, Z_a are the characteristic values of the porous material; $\Gamma_{an} = \Gamma_a/k_0,\ Z_{an} = Z_a/Z_0$; z, g are normalised (with Z_0) impedances or admittances.

Positions of the absorber layer in front of or behind the plate give the same transmission coefficients.



The network elements are:

$$z_a = \frac{Z_{an}}{\cos\Theta_a}\ \sinh(\Gamma_{an}k_0 d_a\ \cos\Theta_a),$$
$$g_a = \frac{\cos\Theta_a}{Z_{an}}\ \frac{\cosh(\Gamma_{an}k_0 d_a\ \cos\Theta_a) - 1}{\sinh(\Gamma_{an}k_0 d_a\ \cos\Theta_a)} \qquad (1)$$

$$g_a z_a = \cosh(\Gamma_{an} k_0 d_a \cos\Theta_a) - 1 \quad ; \quad \cos\Theta_a = \sqrt{1 + \frac{\sin^2\Theta}{\Gamma_{an}^2}}$$

$$z_T = 2\pi Z_m F\,[\eta F^2 \cdot \sin^4\Theta + j(1 - F^2 \cdot \sin^4\Theta)] \quad ; \quad Z_m = \frac{f_{cr} d_p}{Z_0}\rho \,; \quad F = \frac{f}{f_{cr}}$$

(f_{cr} = critical frequency of the plate ; ρ = its density ; η = its bending loss factor).

Transmission loss: $R(\Theta) = 10\,\lg(1/\tau(\Theta)) = 10 \cdot \lg(0.25\,|p_0/p_N|^2)$ (2)

with

$$\frac{p_N}{p_0} = \frac{P_t}{2P_e} = \frac{1}{z_2 + (a + g_a \cdot z_2)/\cos\Theta} \tag{3}$$

$$z_1 = 1 + z_T \cos\Theta \,; \quad a = \cos\Theta + g_a \cdot z_1 \,; \quad z_2 = z_1 + a \cdot z_a ,$$

and after insertion:

$$\begin{aligned}\frac{p_0}{p_N} &= \frac{2\,P_e}{P_t} \\ &= 2 + g_a \cdot (2\,z_a + 2\,z_T + g_a\,z_a\,z_T) + (z_a + z_T + g_a\,z_a\,z_T) \cdot \cos\Theta \\ &\quad + g_a \cdot (2 + g_a\,z_a)/\cos\Theta .\end{aligned} \tag{4}$$

I.13 Sandwich Panels

► *See also:* Mechel, Vol. III, Ch. 12 (1998)

Sandwich panels are combinations of sheets with high E and shear modulus G (index 2) with boards having lower E and shear moduli (index 1). The layers are combined with an adhesive layer, either very thin (or at least with no shear) or of thickness δ, the adhesive having a shear modulus G (without index).

One must distinguish between boards which are tight and boards which are porous.

Tight boards:

It is sufficient to know the effective bending stiffness B of the sandwich. The required partition impedance Z_T is then obtained from *Sect. I.8*.

Table 1 Sandwich panels

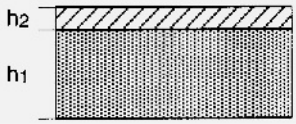
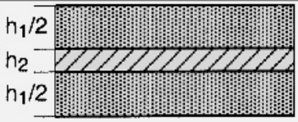
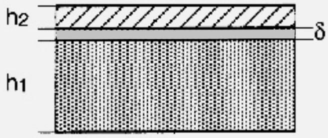
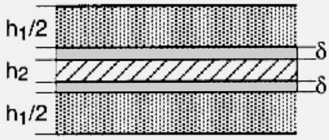
No.	Sandwich	Connection
1		tight joint
2		tight joint
3		connection with shear $\delta \geq \frac{3.5 \cdot 10^{-3}}{h_1} \frac{G}{E_1} - 1.3 \cdot 10^{-12}[\mathrm{m}]$ $E_2 h_2 \geq 2 \cdot 10^7 [\mathrm{Pa\ m}]$
4		connection with shear $\delta \geq \frac{0.25 \cdot 10^{-3}}{h_1} \frac{G}{E_1} [\mathrm{m}]$ h_i in [m]; G, E_i in [Pa]

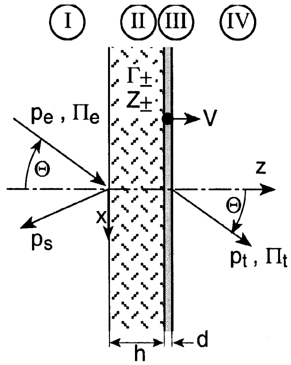
Table 2 Effective bending moduli B for sandwiches from Table 1

No.	Effective bending modulus	Remark
1	$B = B_2 \dfrac{1 + 2\frac{E_1}{E_2}\left[2\left(\frac{h_1}{h_2}\right) + 3\left(\frac{h_1}{h_2}\right)^2 + 2\left(\frac{h_1}{h_2}\right)^3\right] + \left(\frac{h_1}{h_2}\right)^2 \left(\frac{E_1}{E_2}\right)^4}{1 + \frac{h_1}{h_2}\frac{E_1}{E_2}}$	
2	$B \approx B_1 \left(1 + \frac{h_2}{h_1}\right)^3 + B_2$	
3	$B \approx B_1 + B_2 + 3G\delta \frac{h_1^2}{4} + \frac{E_1 h_1 E_2 h_2 (h_1/2 + h_2/2)^2 \cdot g}{E_1 h_1 + g \cdot (E_1 h_1 + E_2 h_2)}$ $- 3G\delta \frac{h_1 + h_2}{4} \frac{E_1 h_1^2 + 2gE_2 h_1 h_2}{E_1 h_1 + g \cdot (E_1 h_1 + E_2 h_2)}$	$\delta \ll h_1$ $g = \frac{G}{\delta E_1 h_1 \omega}\sqrt{B/m}$
4	$B = B_2 + 2E_1 \frac{(h_1/2)^3}{12(1 - \sigma_1^2)}$	

Sandwich with porous board on front side:

A porous layer of thickness h propagates both sound waves in the pores (index 2) and dilatational waves in the matrix (index 1). Their coupling with each other leads to two characteristic propagation constants $\Gamma_{\pm}$.

The boundary conditions between board and sheet neglect shear stresses at the boundary II–III.



Field formulations (without common factor $e^{-j k_x x}$):

$$
\begin{aligned}
&p_e(z) = P_e \cdot e^{-j k_z z} \\
&p_s(z) = P_s \cdot e^{+j k_z z} \quad ; \quad P_s = r \cdot P_e \qquad k_x^2 + k_z^2 = k_0^2 \\
&v_{ez}(0) + v_{sz}(0) = P_e \frac{k_z}{k_0 Z_0}(1-r) \qquad k_x = k_0 \sin\Theta \quad ; \quad k_z = k_0 \cos\Theta,
\end{aligned} \tag{1}
$$

$$
p_t(z) = P_t \cdot e^{-j k_z (z-h-d)} \quad ; \quad v_{tz}(z) = \frac{k_z}{k_0 Z_0} P_t \cdot e^{-j k_z (z-h-d)}. \tag{2}
$$

Sound waves in the matrix (index 1) and in the pores (index 2) of the board:

$$
\begin{aligned}
p_1(z) &= \frac{\Gamma_{z+}}{\Gamma_+} P_{12+}[A \cdot e^{-\Gamma_{z+} z} + B \cdot e^{+\Gamma_{z+} z}] + \frac{\Gamma_{z-}}{\Gamma_-} P_{12-}[C \cdot e^{-\Gamma_{z-} z} + D \cdot e^{+\Gamma_{z-} z}] \\
Z_0 v_{1z}(z) &= \frac{\Gamma_{z+}}{\Gamma_+} \frac{V_{12+}}{Z_{22+}} [A \cdot e^{-\Gamma_{z+} z} - B \cdot e^{+\Gamma_{z+} z}] + \frac{\Gamma_{z-}}{\Gamma_-} \frac{V_{12-}}{Z_{22-}} [C \cdot e^{-\Gamma_{z-} z} - D \cdot e^{+\Gamma_{z-} z}] \\
p_2(z) &= A \cdot e^{-\Gamma_{z+} z} + B \cdot e^{+\Gamma_{z+} z} + C \cdot e^{-\Gamma_{z-} z} + D \cdot e^{+\Gamma_{z-} z} \\
Z_0 v_{2z}(z) &= \frac{\Gamma_{z+}/\Gamma_+}{Z_{22+}} [A \cdot e^{-\Gamma_{z+} z} - B \cdot e^{+\Gamma_{z+} z}] + \frac{\Gamma_{z-}/\Gamma_-}{Z_{22-}} [C \cdot e^{-\Gamma_{z-} z} - D \cdot e^{+\Gamma_{z-} z}]
\end{aligned} \tag{3}
$$

with $\quad \Gamma_{\pm}^2 = \Gamma_{x\pm}^2 + \Gamma_{z\pm}^2 \quad ; \quad \Gamma_{x\pm} = j k_x \quad ; \quad \Gamma_{z\pm}^2 = \Gamma_{\pm}^2 + k_x^2 = \Gamma_{\pm}^2 + k_0^2 \sin^2\Theta$

The free field propagation constants $\Gamma_\pm$ are solutions of the characteristic equation:

$$\left(\frac{\Gamma}{k_0}\right)^4 \cdot \frac{K_1}{K_0}\frac{K_2}{K_0} + \left(\frac{\Gamma}{k_0}\right)^2 \cdot \left[\chi\frac{K_1}{K_0} + \frac{K_2}{K_0}\left(\chi - 1 + \frac{\rho_1}{\rho_0}\right) - \frac{j\sigma}{2\pi E}\left(\frac{K_1}{K_0} + \frac{K_2}{K_0}\right)\right] + \sigma(\chi - 1) + \chi\frac{\rho_1}{\rho_0} - \frac{j\sigma}{2\pi E}\left(\frac{\rho_1}{\rho_0} + \sigma\right) = 0 \quad (4)$$

($\pm$ indicates the sign of the root in the solution Γ^2), or, in an approximation for $\sigma \approx 1$; $\chi \approx 1$; $\rho_1/\rho_0 \ll 1$:

$$\left(\frac{\Gamma}{k_0}\right)^4 \cdot \frac{K_1}{K_0}\frac{K_2}{K_0} + \left(\frac{\Gamma}{k_0}\right)^2 \cdot \left[\chi\frac{K_1}{K_0} + \frac{K_2}{K_0}\frac{\rho_1}{\rho_0} - \frac{j\sigma}{2\pi E}\left(\frac{K_1}{K_0} + \frac{K_2}{K_0}\right)\right] + \frac{\rho_1}{\rho_0}\left(\chi - \frac{j\sigma}{2\pi E}\right) = 0. (5)$$

The coefficients $Z_{22\pm}, P_{12\pm}, V_{12\pm}$ are:

$$Z_{22\pm} = \frac{p_2}{Z_0 v_2} = -j\frac{\Gamma_\pm}{k_0}\frac{K_2}{K_0}\frac{2\pi E \cdot (\chi - \sigma) - j\sigma}{2\pi E \cdot \left[\chi - 1 + \left(\frac{\Gamma_\pm}{k_0}\right)^2 \frac{K_2}{K_0}\frac{\sigma - 1}{\sigma}\right] - j\sigma}$$

$$P_{12\pm} = \frac{p_1}{p_2} = \frac{2\pi E \cdot \left(\chi + \left(\frac{\Gamma_\pm}{k_0}\right)^2 \frac{K_2}{K_0}\right) - j\,\sigma}{2\pi E \cdot (\chi - \sigma) - j\,\sigma} \quad (6)$$

$$V_{12\pm} = \frac{v_1}{v_2} = \frac{2\pi E \cdot \left[\chi + \left(\frac{\Gamma_\pm}{k_0}\right)^2 \frac{K_2}{K_0}\right] - j\sigma}{2\pi E \cdot \left[\chi - 1 + \left(\frac{\Gamma_\pm}{k_0}\right)^2 \frac{K_2}{K_0}\frac{\sigma - 1}{\sigma}\right] - j\sigma}$$

with the compression modulus of the air in the pores:

$$\frac{K_2}{K_0} = \frac{1 + jE/E_0}{\kappa + jE/E_0} \quad ; \quad E_0 = \frac{\rho_0 f_0}{\Xi} \quad ; \quad 2\pi f_0 \cdot \tau_0 = 1. \quad (7)$$

See the inset frame for the meaning of other symbols.
The velocity V of the sheet is defined with its partition impedance Z_T and the driving pressure difference Δp:

$$V \cdot Z_T = p_1(h) + p_2(h) - p_t(h + d). \quad (8)$$

The seven unknown amplitudes P_s (or r), P_t, A, B, C, D, V are determined from the boundary conditions (one boundary condition is contained in the definition of V):

$$\text{at I–II:} \quad \begin{aligned} &p_1(0) = (1 - \sigma)\, P_e\,(1 + r), \\ &p_2(0) = \sigma\, P_e\,(1 + r), \\ &Z_0[(1 - \sigma)\, v_{1z}(0) + \sigma\, v_{2z}(0)] = Z_0\,(v_e(0) + v_s(0)) = \frac{k_z}{k_0} P_e\,(1 - r), \end{aligned} \quad (9)$$

$$\text{at II–III:} \quad v_{1z}(h) = V \quad ; \quad v_{2z}(h) = V, \quad (10)$$

at III–IV: $v_{tz}(d+h) = V.$ (11)

ρ_0	=	density of air;
c_0	=	adiabatic sound velocity;
κ	=	adiabatic exponent of air;
k_0	=	ω / c_{O_2};
K_0	=	$\rho_0 c_0$ = adiabatic compression modulus of air;
ρ_1	=	density of matrix material;
K_1	=	dilation modulus of matrix material;
K_2	=	compression modulus of air in the pores;
σ	=	porosity of porous material;
χ	=	tortuosity of porous material;
Ξ	=	flow resistivity of porous material;
E	=	$\rho_0 f / \Xi$ = absorber variable of porous material;
E_0	=	value of E at relaxation frequency f_0 in the pores

The system of equations

$$(\text{Matrix}) \bullet \begin{pmatrix} P_s/P_e \\ P_t/P_e \\ A/P_e \\ B/P_e \\ C/P_e \\ D/P_e \\ Z_0 V/P_e \end{pmatrix} = \begin{pmatrix} 1-\sigma \\ \sigma \\ k_z/k_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

has a matrix with the following columns:

$$\{P_s/P_e\} = \{-(1-\sigma), -\sigma, k_z/k_0, 0, 0, 0, 0\}\,,$$

$$\{P_t/P_e\} = \{0, 0, 0, 0, 0, k_z/k_0, 1\}\,,$$

$$\begin{aligned} \{A/P_e\} = \{&\Gamma_{z+} P_{12+}/\Gamma_+, 1, \Gamma_{z+}(\sigma + V_{12+}(1-\sigma))/(\Gamma_+ Z_{22+}), \\ &\Gamma_{z+} V_{12+}/(e^{\Gamma_{z+}h}\,\Gamma_+ Z_{22+}), \Gamma_{z+}/(e^{\Gamma_{z+}h}\,\Gamma_+ Z_{22+}), 0, \\ &-(1+\Gamma_{z+} P_{12+}/\Gamma_+)/e^{\Gamma_{z+}h}\}\,, \end{aligned} \tag{13}$$

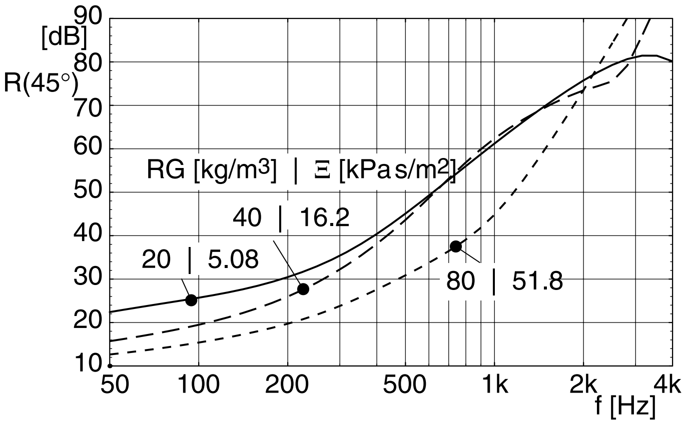
$$\begin{aligned} \{B/P_e\} = \{&\Gamma_{z+} P_{12+}/\Gamma_+, 1, -\Gamma_{z+}(\sigma + V_{12+}(1-\sigma))/(\Gamma_+ Z_{22+}), \\ &-\Gamma_{z+} V_{12+} e^{\Gamma_{z+}h}/(\Gamma_+ Z_{22+}), -\Gamma_{z+} e^{\Gamma_{z+}h}/(\Gamma_+ Z_{22+}), 0, \\ &-(1+\Gamma_{z+} P_{12+}/\Gamma_+)\cdot e^{\Gamma_{z+}h}\}\,, \end{aligned} \tag{14}$$

$$\{C/P_e\} = \{\Gamma_{z-}P_{12-}/\Gamma_-, 1, \Gamma_{z-}(\sigma + V_{12-}(1-\sigma))/(\Gamma_- Z_{22-}),$$
$$\Gamma_{z-}V_{12-}/(e^{\Gamma_{z-}h}\,\Gamma_- Z_{22-}), \Gamma_{z-}/(e^{\Gamma_{z-}h}\,\Gamma_- Z_{22-}), 0,$$
$$-(1+\Gamma_{z-}P_{12-}/\Gamma_-)/e^{\Gamma_{z-}h}\Big\},$$
$$\{D/P_e\} = \{\Gamma_{z-}P_{12-}/\Gamma_-, 1, -\Gamma_{z-}(\sigma + V_{12-}(1-\sigma))/(\Gamma_- Z_{22-}), \tag{15}$$
$$-\Gamma_{z-}V_{12-}e^{\Gamma_{z-}h}/(\Gamma_- Z_{22-}), -\Gamma_{z-}e^{\Gamma_{z-}h}/(\Gamma_- Z_{22-}), 0,$$
$$-(1+\Gamma_{z-}P_{12-}/\Gamma_-)\cdot e^{\Gamma_{z-}h}\Big\},$$
$$\{Z_0V/P_e\} = \{0, 0, 0, -1, -1, -1, Z_T/Z_0\}.$$

The ultimately desired transmission coefficient $\tau(\Theta)$ follows from

$$\tau(\Theta) = \left|P_t/P_e\right|^2. \tag{16}$$

The following example is for a sandwich with a front-side glass fibre board, having different bulk densities $RG = (1-\sigma)\cdot\rho_1$ (with $\rho_1 = 2500$ [kg/m^3]) and flow resistivities Ξ, and a back-side plaster board, d = 12.5 [mm] thick.



Sound transmission loss for oblique sound incidence of a sandwich with a front-side glass fibre board of different bulk densities RG and flow porosities Ξ, and a back-side plaster board, d = 12.5 [mm] thick.
Parameters: $h = 0.1$ [m]; $d = 0.0125$ [m]; $f_{cr}d = 31$ [Hzm]; $\rho_p = 1000$ [kg/m^3]; $\eta_p = 0.03$; $\chi = 1.35$; $\eta_a = 0.25$

Sandwich with porous board on back side:

The field definitions remain as in the previous arrangement.

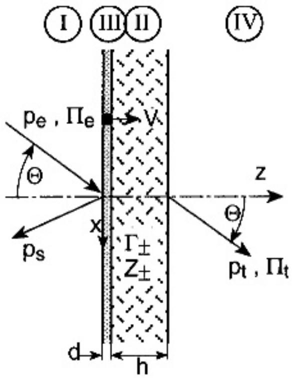
The boundary conditions now are as follows:

plate: $$V \cdot Z_T = p_e(0) + p_s(0) - \big(p_1(d) + p_2(d)\big) \tag{17}$$

$$\text{at I–III:} \qquad v_{ez}(0) + v_{sz}(0) = \frac{k_z}{k_0 Z_0}(P_e - P_s) \stackrel{!}{=} V, \tag{18}$$

$$\text{at III–II:} \qquad v_{1z}(d) = V \quad ; \quad v_{2z}(d) = V, \tag{19}$$

$$\text{at II–IV:} \qquad \begin{aligned} & p_1(d+h) = (1-\sigma)\,P_t, \\ & p_2(d+h) = \sigma\,P_t, \\ & Z_0[(1-\sigma)\,v_{1z}(d+h) + \sigma\,v_{2z}(d+h)\,] = Z_0 v_{tz}(d+h) = \frac{k_z}{k_0}P_t. \end{aligned} \tag{20}$$



The system of equations to be solved is:

$$(\text{Matrix}) \bullet \begin{pmatrix} P_s/P_e \\ P_t/P_e \\ A/P_e \\ B/P_e \\ C/P_e \\ D/P_e \\ Z_0V/P_e \end{pmatrix} = \begin{pmatrix} -\cos\Theta \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \tag{21}$$

with the following matrix columns:

$$\begin{aligned} \{P_s/P_e\} &= \{-\cos\Theta,\ \ 0,\ \ 0,\ \ 0,\ \ 0,\ \ 0,\ \ 1\} \\ \{P_t/P_e\} &= \{0,\ \ 0,\ \ 0,\ \ -(1-\sigma),\ \ -\sigma,\ \ -\cos\Theta,\ \ 0\} \\ \{A/P_e\} &= \Big\{0,\ \ \Gamma_{z+}V_{12+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+}d}),\ \ \Gamma_{z+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+}d}), \\ &\qquad \Gamma_{z+}P_{12+}/(\Gamma_+ e^{\Gamma_{z+}(d+h)}),\ \ 1/e^{\Gamma_{z+}(d+h)}, \\ &\qquad \Gamma_{z+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+}(d+h)}) \cdot [\sigma +\ (1-\sigma)V_{12+}],\ \ -(1+\Gamma_{z+}P_{12+}/\Gamma_+)/e^{\Gamma_{z+}d}\Big\}, \end{aligned} \tag{22}$$

$$\begin{aligned} \{B/P_e\} &= \Big\{0,\ \ -\Gamma_{z+}V_{12+}e^{\Gamma_{z+}d}/(\Gamma_+ Z_{22+}),\ \ -\Gamma_{z+}e^{\Gamma_{z+}d}/(\Gamma_+ Z_{22+}), \\ &\qquad \Gamma_{z+}P_{12+}e^{\Gamma_{z+}(d+h)}/\Gamma_+,\quad e^{\Gamma_{z+}(d+h)}, \\ &\qquad -\Gamma_{z+}e^{\Gamma_{z+}(d+h)}/(\Gamma_+ Z_{22+}) \cdot [\sigma + V_{12+}(1-\sigma)],\quad -\ e^{\Gamma_{z+}d} \cdot (1+\Gamma_{z+}P_{12+}/\Gamma_+)\Big\}, \end{aligned} \tag{23}$$

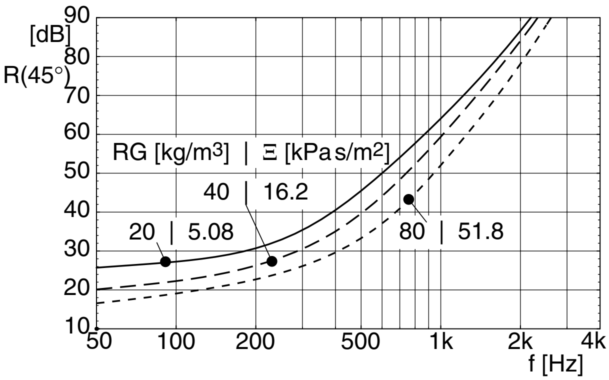
$$\begin{aligned}\{C/P_e\} = \Big\{&0,\quad \Gamma_{z-}V_{12-}/(\Gamma_- Z_{22-}e^{\Gamma_{z-}d}),\quad \Gamma_{z-}/(\Gamma_- Z_{22-}e^{\Gamma_{z-}d}),\\ &\Gamma_{z-}P_{12-}/(\Gamma_- e^{\Gamma_{z-}(d+h)}),\quad 1/e^{\Gamma_{z-}(d+h)},\\ &\Gamma_{z-}/(\Gamma_- Z_{22-}e^{\Gamma_{z-}(d+h)})\cdot[\sigma+(1-\sigma)V_{12-}],\quad -(1+\Gamma_{z-}P_{12-}/\Gamma_-)/e^{\Gamma_{z-}d}\Big\},\end{aligned} \tag{24}$$

$$\begin{aligned}\{D/P_e\} = \Big\{&0,\quad -\Gamma_{z-}V_{12-}e^{\Gamma_{z-}d}/(\Gamma_- Z_{22-}),\quad -\Gamma_{z-}e^{\Gamma_{z-}d}/(\Gamma_- Z_{22-}),\\ &\Gamma_{z-}P_{12-}e^{\Gamma_{z-}(d+h)}/\Gamma_-,\quad e^{\Gamma_{z-}(d+h)},\\ &-\Gamma_{z-}e^{\Gamma_{z-}(d+h)}/(\Gamma_- Z_{22-})\cdot[\sigma+(1-\sigma)V_{12-}],\ -e^{\Gamma_{z-}d}\cdot(1+\Gamma_{z-}P_{12-}/\Gamma_-)\Big\},\end{aligned} \tag{25}$$

$$\{Z_0V/P_e\} = \{-1,\quad -1,\quad -1,\quad 0,\quad 0,\quad 0,\quad -Z_T\}\,. \tag{26}$$

The ultimately desired transmission coefficient $\tau(\Theta)$ follows from $\tau(\Theta) = |P_t/P_e|^2$.(27)

The following example is for the same object as above. Both the example and the equations show that the position of the absorber board modifies the sound transmission loss.



Same sandwich as above, but with a reversed arrangement of the glass fibre board and the plaster board

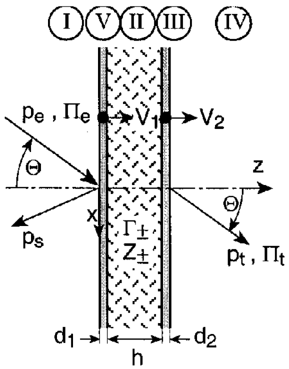
Sandwich with a porous layer asßindexsandwich panel!with porous board as core core:

The transmitted wave p_t is modified as follows:

$$\begin{aligned} p_t(z) &= P_t\cdot e^{-j\,k_z(z-h-d_1-d_2)}\\ v_{tz}(z) &= \frac{k_z}{k_0Z_0}\,P_t\cdot e^{-j\,k_z(z-h-d_1-d_2)}.\end{aligned} \tag{28}$$

The equations for the cover plates are:

$$\begin{aligned} V_1\cdot Z_{T1} &= p_e(0)+p_s(0)-\big(p_1(d_1)+p_2(d_1)\big)\\ V_2\cdot Z_{T2} &= p_1(d_1+h)+p_2(d_1+h)-p_t(d_1+d_2+h).\end{aligned} \tag{29}$$



The boundary conditions are now as follows:

$$\text{at I–V:}\qquad v_{ez}(0) + v_{sz}(0) = \frac{k_z}{k_0 Z_0}(P_e - P_s) \overset{!}{=} V_1 \tag{30}$$

$$\text{at V–II:}\qquad v_{1z}(d_1) = V_1 \quad ; \quad v_{2z}(d_1) = V_1 \tag{31}$$

$$\text{at II–III:}\qquad v_{1z}(d_1 + h) = V_2 \quad ; \quad v_{2z}(d_1 + h) = V_2 \tag{32}$$

$$\text{at III–V:}\qquad v_{tz}(d_1 + d_2 + h) = \frac{k_z}{k_0 Z_0} P_t \overset{!}{=} V_2. \tag{33}$$

The system of equations to be solved is:

$$(\text{Matrix}) \bullet \begin{pmatrix} P_s/P_e \\ P_t/P_e \\ A/P_e \\ B/P_e \\ C/P_e \\ D/P_e \\ Z_0V_1/P_e \\ Z_0V_2/P_e \end{pmatrix} = \begin{pmatrix} -\cos\Theta \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} \tag{34}$$

with the following matrix columns:

$$\begin{aligned} \{P_s/P_e\} &= \{-\cos\Theta, 0, 0, 0, 0, 0, 1, 0\} \\ \{P_t/P_e\} &= \{0, 0, 0, 0, 0, \cos\Theta, 0, -1\} \end{aligned} \tag{35}$$

$$\begin{aligned} \{A/P_e\} = \big\{&0, \Gamma_{z+}V_{12+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+} d_1}), \Gamma_{z+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+} d_1}), \\ &\Gamma_{z+}V_{12+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+}(d_1+h)}), \Gamma_{z+}/(\Gamma_+ Z_{22+} e^{\Gamma_{z+}(d_1+h)}), 0, \\ &-(1 + \Gamma_{z+}P_{12+}/\Gamma_+)/e^{\Gamma_{z+} d_1}, (1 + \Gamma_{z+}P_{12+}/\Gamma_+)/e^{\Gamma_{z+}(d_1+h)}\big\}, \end{aligned} \tag{36a}$$

$$\begin{aligned} \{B/P_e\} = \big\{&0, -\Gamma_{z+}V_{12+} e^{\Gamma_{z+} d_1}/(\Gamma_+ Z_{22+}), -\Gamma_{z+} e^{\Gamma_{z+} d_1}/(\Gamma_+ Z_{22+}), \\ &-\Gamma_{z+}V_{12+} e^{\Gamma_{z+}(d_1+h)}/(\Gamma_+ Z_{22+}), -\Gamma_{z+} e^{\Gamma_{z+}(d_1+h)}/(\Gamma_+ Z_{22+}), 0, \\ &-e^{\Gamma_{z+} d_1} \cdot (1 + \Gamma_{z+}P_{12+}/\Gamma_+), e^{\Gamma_{z+}(d_1+h)} \cdot (1 + \Gamma_{z+}P_{12+}/\Gamma_+)\big\}, \end{aligned} \tag{36b}$$

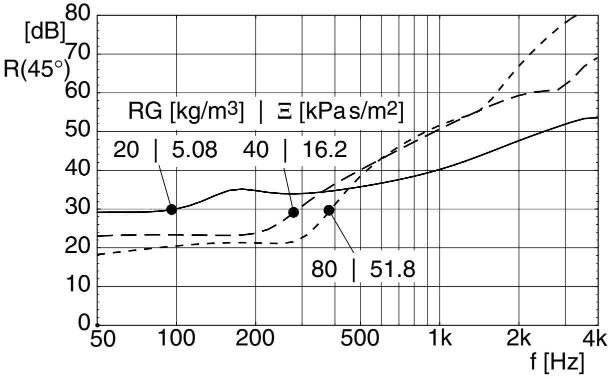
$$\{C/P_e\} = \Big\{0,\ \Gamma_{z-}V_{12-}/(\Gamma_- Z_{22-} e^{\Gamma_{z-} d_1}),\ \Gamma_{z-}/(\Gamma_- Z_{22-} e^{\Gamma_{z-} d_1}),\\ \Gamma_{z-}V_{12-}/(\Gamma_- Z_{22-} e^{\Gamma_{z-}(d_1+h)}),\ \Gamma_{z-}/(\Gamma_- Z_{22-} e^{\Gamma_{z-}(d_1+h)}),\ 0,\\ -(1+\Gamma_{z-}P_{12-}/\Gamma_-)/e^{\Gamma_{z-} d_1},\ (1+\Gamma_{z-}P_{12-}/\Gamma_-)/e^{\Gamma_{z-}(d_1+h)}\Big\}, \tag{37}$$

$$\{D/P_e\} = \Big\{0,\ -\Gamma_{z-}V_{12-}e^{\Gamma_{z-} d_1}/(\Gamma_- Z_{22-}),\ -\Gamma_{z-}e^{\Gamma_{z-} d_1}/(\Gamma_- Z_{22-}),\\ -\Gamma_{z-}V_{12-}e^{\Gamma_{z-}(d_1+h)}/(\Gamma_- Z_{22-}),\ -\Gamma_{z-}e^{\Gamma_{z-}(d_1+h)}/(\Gamma_- Z_{22-}),\ 0,\\ -e^{\Gamma_{z-} d_1}\cdot(1+\Gamma_{z-}P_{12-}/\Gamma_-),\ e^{\Gamma_{z-}(d_1+h)}\cdot(1+\Gamma_{z-}P_{12-}/\Gamma_-)\Big\}, \tag{38}$$

$$\begin{aligned}\{Z_0V_1/P_e\} &= \{-1,-1,-1,0,0,0,-Z_{T1},0\}\\ \{Z_0V_2/P_e\} &= \{0,0,0,-1,-1,-1,0,-Z_{T2}\}.\end{aligned} \tag{39}$$

The ultimately desired transmission coefficient $\tau(\Theta)$ follows from $\tau(\Theta) = \left|P_t/P_e\right|^2$. (40)

The example is for a sandwich with a glass fibre core for different bulk densities RG and flow resistivities Ξ of the glass fibre material.



Sandwich with a glass fibre core and plaster board cover plates, for different bulk densities RG and flow resistivities Ξ of the core material.
Parameters: $h = 0.1$ [m]; $d_1 = 0.0125$ [m]; $d_2 = 0.0095$ [m];
$f_{cr}d_1 = 31$ [Hzm]; $\rho_{p1} = 1000 [kg/m^3]$; $\eta_{p1} = 0.03$; $f_{cr}d_2 = 31$ [Hzm]; $\rho_{p2} = 1000$ [kg/m^3];
$\eta_{p2} = 0.03$; $c = 1.35$; $h_a = 0.25$

I.14 Finite-Size Plate

► *See also:* Mechel, Vol. III, Ch. 14 (1998)

Let a plate be two-dimensional (for ease of writing, a three-dimensional plate is treated similarly), infinite in the y direction, and with supported borders at $x = \pm h$. The non-dimensional co-ordinate $\xi = x/h$ will be used. In this section solutions $v_n(\xi) = v_n(\gamma_n\xi)$ of the homogeneous bending wave equation

$$\left(\frac{\partial^4}{\partial\xi^4} - \gamma_n^4\right) v_n(\xi) = 0 \tag{1}$$

are given which satisfy the boundary conditions of different kinds of boundary support. These solutions are *plate modes* which will be used to synthesise plate velocity patterns:

$$V(\xi) = \sum_n V_n \cdot v_n(\xi). \qquad (2)$$

The bending wave equation

$$\left(\frac{\partial^4}{\partial \xi^4} - (k_B h)^4\right) V(\xi) = h^4 \frac{j\omega}{B}\, \delta p(\xi) \qquad (3)$$

then gives:

$$\sum_n V_n \left(\gamma_n^4 - (k_B h)^4\right) v_n(\xi) = h^4 \frac{j\omega}{B}\, \delta p(\xi), \qquad (4)$$

which can be written as:

$$\sum_n V_n Z_{Tn} \cdot v_n(\xi) = \delta p(\xi), \qquad (5)$$

thereby defining the *modal partition impedances* Z_{Tn}:

$$Z_{Tn} = \frac{B}{j\omega}\left((\gamma_n/h)^4 - k_B^4\right) = j\omega m \left[1 - \left(\frac{\gamma_n}{k_B h}\right)^4\right] = j\omega m \left[1 - \left(\frac{k_{bn}}{k_B}\right)^4\right]. \qquad (6)$$

In the last expression $k_{bn} = \gamma/h$ is the modal bending-wave number. For a plate with bending losses (η = bending loss factor) correspondingly:

$$\begin{aligned} Z_{Tn} &= \omega m \left[\eta \left(\frac{\gamma_n}{k_B h}\right)^4 + j\left(1 - \left(\frac{\gamma_n}{k_B h}\right)^4\right)\right], \\ \frac{Z_{Tn}}{Z_0} &= 2\pi\, Z_m F \left[\eta\, F^2 \left(\frac{\gamma_n}{k_0 h}\right)^4 + j\left(1 - F^2\left(\frac{\gamma_n}{k_0 h}\right)^4\right)\right], \end{aligned} \quad F = \frac{f}{f_{cr}}\,;\; Z_m = \frac{f_{cr} m}{Z_0} = \frac{f_{cr} d}{Z_0}\rho. \qquad (7)$$

Depending on the pattern of excitation, symmetrical modes $v_n^{(s)}(\xi)$ and anti-symmetrical modes $v_n^{(a)}(\xi)$ will be excited, even for symmetrical support at $\xi = \pm 1$ (symmetry defined with respect to $\xi = 0$). With different supports at both sides, the plate velocity pattern can be written as a mode sum over symmetrical and anti-symmetrical modes.

For the "classical" supports (*Sect. I.8*) the plate modes are orthogonal:

$$\int_{-1}^{+1} v_m^{(\beta)}(\xi) \cdot v_n^{(\beta)}(\xi)\, d\xi = \delta_{mn} \cdot N_{Pn}^{(\beta)} \quad ; \quad \delta_{mn} = \begin{cases} 1 & ; \quad m = n \\ 0 & ; \quad m \neq n \end{cases}, \qquad (8)$$

where δ_{mn} is a Kronecker symbol and $N_{Pn}^{(\beta)}$ the norm of the plate mode. Modes with different types β of symmetry are evidently always orthogonal to each other.

Simply supported plate

(the most probable classical support for many technical fixations of construction panels):

Boundary conditions: $$v_n(\xi) = \frac{\partial^2 v_n(\xi)}{\partial \xi^2} = 0 \quad ; \quad \xi = \pm 1. \tag{9}$$

Solutions:

$$v_n^{(\beta)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi)\ ; & \gamma_n^{(s)} = n_o \dfrac{\pi}{2}\ ; \quad n_o = 1,\ 3,\ 5, \ldots \\ \sin(\gamma_n^{(a)}\xi)\ ; & \gamma_n^{(a)} = n_e \dfrac{\pi}{2}\ ; \quad n_e = 2,\ 4,\ 6, \ldots . \end{cases} \tag{10}$$

Polynomial solutions are excluded. Mode norms: $$N_{Pn}^{(\beta)} = \int_{-1}^{+1} [v_n^{(\beta)}(\xi)]^2 \, d\xi = 1. \tag{11}$$

Clamped plate: $\beta = s,a$ or $\beta = 0, 1$.

Boundary conditions: $$v_n(\pm 1) = \frac{\partial v_n(\pm 1)}{\partial \xi} = 0 \tag{12}$$

General solutions:

$$v_n^{(\beta)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi) + C_n^{(s)} \cosh(\gamma_n^{(s)}\xi) \\ \sin(\gamma_n^{(a)}\xi) + C_n^{(a)} \sinh(\gamma_n^{(a)}\xi) \end{cases}, \tag{13}$$

where $C_n^{(\beta)}$ are solutions of

$$C_n^{(\beta)} = (-1)^\beta \frac{\sin \gamma_n^{(\beta)}}{\sinh \gamma_n^{(\beta)}} = -\frac{\cos \gamma_n^{(\beta)}}{\cosh \gamma_n^{(\beta)}}. \tag{14}$$

The second equation gives the characteristic equation for $\gamma_n^{(\beta)}$:

$$\tan \gamma_n^{(\beta)} = \mp \tanh \gamma_n^{(\beta)}\ ; \quad \beta = \begin{cases} s \\ a \end{cases} \tag{15}$$

with approximate solutions: $$\gamma_n^{(\beta)} \approx \pi\,(n \mp 1/4)\ ; \quad n = 1,\ 2,\ 3, \ldots . \tag{16}$$

Table 1 Characteristic valuess γ_n (exact and approximations) for a clamped plate

n	$\gamma_{n,ex}^{(s)}$	$\gamma_{n,apr}^{(s)}$	$\gamma_{n,ex}^{(a)}$	$\gamma_{n,apr}^{(a)}$
1	2.36502	2.35619	3.92660	3.92699
2	5.49780	5.49779	7.06858	7.06858
3	8.63938	8.63938	10.21018	10.21018
4	11.78097	11.78097	13.35177	13.35177
≥ 5	14.92257	$\gamma_n^{(s)} \approx \pi(n - 1/4)$	16.49336	$\gamma_n^{(a)} \approx \pi(n + 1/4)$

Mode norms:

$$\begin{aligned} N_{Pn}^{(\beta)} &= \int_{-1}^{+1} [v_n^{(\beta)}(\gamma_n^{(\beta)}\xi)]^2 \, d\xi = 1 \pm \frac{\sin(2\gamma_n^{(\beta)})}{2\gamma_n^{(\beta)}} \\ &\pm [C_n^{(\beta)}]^2 \left(1 \pm \frac{\sinh(2\gamma_n^{(\beta)})}{2\gamma_n^{(\beta)}}\right) \\ &+ 2\frac{C_n^{(\beta)}}{\gamma_n^{(\beta)}} \left(\sin\gamma_n^{(\beta)} \cdot \cosh\gamma_n^{(\beta)} \pm \cos\gamma_n^{(\beta)} \cdot \sinh\gamma_n^{(\beta)}\right) \\ &\approx 1 \pm \frac{\sin(2\gamma_n^{(\beta)})}{2\gamma_n^{(\beta)}} \pm [C_n^{(\beta)}]^2 \left(1 \pm \frac{\sinh(2\gamma_n^{(\beta)})}{2\gamma_n^{(\beta)}}\right) \end{aligned} \tag{17}$$

Table 2 Mode norms (exact and approximate) for a clamped plate

n	$N_{Pn,ex}^{(s)}$	$N_{Pn,apr}^{(s)}$	$N_{Pn,ex}^{(a)}$	$N_{Pn,apr}^{(a)}$
1	1.017651	1.029835	0.9992230	0.9995197
2	1.000034	1.000043	0.9999986	0.9999989
3	1.000000	1.000000	1.000000	1.000000
4	1.000000	1.000000	1.000000	1.000000

Free plate: $\beta = s,a$ or $\beta = 0, 1$

Boundary conditions: $$\frac{\partial^2 v_n(\xi)}{\partial \xi^2} = \frac{\partial^3 v_n(\xi)}{\partial \xi^3} = 0 \quad ; \quad \xi = \pm 1. \tag{18}$$

General solutions: $$v_n^{(\beta)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi) + C_n^{(s)} \cosh(\gamma_n^{(s)}\xi) \\ \sin(\gamma_n^{(a)}\xi) + C_n^{(a)} \sinh(\gamma_n^{(a)}\xi), \end{cases} \tag{19}$$

where $C_n^{(\beta)}$ are solutions of: $$C_n^{(\beta)} = -(-1)^\beta \frac{\sin\gamma_n^{(\beta)}}{\sinh\gamma_n^{(\beta)}} = \frac{\cos\gamma_n^{(\beta)}}{\cosh\gamma n^\beta}. \tag{20}$$

Characteristic equation for the $\gamma_n^{(\beta)}$: $$\tan\gamma_n^{(\beta)} = \mp\tanh\gamma_n^{(\beta)} \quad ; \quad \beta = \begin{cases} s \\ a \end{cases} \tag{21}$$

with the same solutions and approximations as for the clamped plate. Additionally a piston-like oscillation (with index $n = 0$) of the form $v_0^{(s)}(\xi) = 1 + C_0^{(s)} = \text{const}$ is possible with the characteristic value $\gamma_0^{(s)} = 0$. With the choice $C_0^{(s)} = 1$, this mode has a norm with unit value. Further, an anti-symmetrical mode is possible with:

$$v_0^{(a)}(\xi) = C_0^{(a)} \cdot \xi \quad ; \quad \gamma_0^{(a)} = 0 \quad ; \quad C_0^{(a)} = 1 \quad ; \quad n = 0 \tag{22}$$

(the choice $C_0^{(s)} = 1$ is arbitrary). These additional modes are called *polynomial modes*.

The mode norms are the same as for the clamped plate, except for the polynomial modes:

$$N_{P0}^{(s)} = 2\,(1 + C_0^{(s)})^2 = 8 \quad ; \quad N_{P0}^{(a)} = \frac{2}{3}\,C_0^{(a)\,2} = \frac{2}{3}. \tag{23}$$

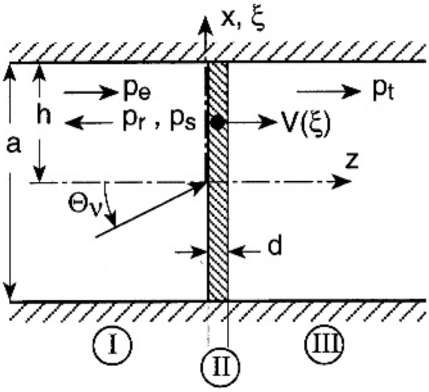
The polynomial modes are orthogonal to the other modes.

I.15 Single Plate across a Flat Duct

► *See also:* Mechel, Vol. III, Ch. 14 (1998)

A flat (two-dimensional) duct of width a = 2h with hard walls is subdivided by a plate of thickness d; the plate will have different "classical" fixations at the duct walls in $x = \pm h$, i.e. in $\xi = x/h = 1$.

The excitation will first be by a single duct mode of order μ and (arbitrary) amplitude P_e. Later "mode mixtures" will be considered.



The field in front of the plate (i.e. in zone I) is formulated as a sum:

$$p_I(\xi, z) = p_e(\xi, z) + p_r(\xi, z) + p_s(\xi, z), \tag{1}$$

where $p_e(\xi, z)$ is the incident duct mode, $p_r(\xi, z)$ the hardly reflected duct mode and $p_s(\xi, z)$ the scattered field. The scattered field p_s and the transmitted wave p_t are formulated as sums of duct modes. The velocity pattern of the plate is formulated as a sum of plate modes (see previous *Sect. I.14*, especially for γ_n and $C_n^{(\beta)}$).

$$\begin{aligned} p_e(\xi, z) &= P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{-j\,k_{\mu z} z} \\ Z_0 v_{ez}(\xi, z) &= \cos\Theta_\mu P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{-j\,k_{\mu z} z} \end{aligned} \tag{2}$$

$$\begin{aligned} p_r(\xi, z) &= P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{+j\,k_{\mu z} z} \\ Z_0 v_{rz}(\xi, z) &= -\cos\Theta_\mu P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{+j\,k_{\mu z} z} \end{aligned} \tag{3}$$

$$\begin{aligned} p_s(\xi, z) &= \sum_\nu P_{s\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{+j\,k_{\nu z} z} \\ Z_0 v_{sz}(\xi, z) &= -\sum_\nu \cos\Theta_\nu P_{s\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{+j\,k_{\nu z} z} \end{aligned} \tag{4}$$

$$\begin{aligned}
p_t(\xi, z) &= \sum_\nu P_{t\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{-j\,k_{\nu z}(z-d)} \\
Z_0 v_{tz}(\xi, z) &= \sum_\nu \cos\,\Theta_\nu P_{t\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{-j\,k_{\nu z}(z-d)} \\
V(\xi) &= \sum_n V_n \cdot v_n^{(\beta)}(\xi)
\end{aligned} \tag{5}$$

with the duct mode lateral profiles for symmetrical ($\beta = s$) and anti-symmetrical ($\beta = a$) modes:

$$q_\nu^{(\beta)}(\xi) = \begin{cases} \cos(\kappa_\nu^{(s)}\xi) & \kappa_\nu^{(s)} = \nu_e \dfrac{\pi}{2} \quad ; \quad \nu_e = 0,\ 2,\ 4, \ldots \\ \sin(\kappa_\nu^{(a)}\xi) & \kappa_\nu^{(a)} = \nu_o \dfrac{\pi}{2} \quad ; \quad \nu_o = 1,\ 3,\ 5, \ldots \end{cases} \tag{6}$$

having the duct mode norms

$$N_\nu^{(\beta)} = \int_{-1}^{+1} [q_\nu^{(\beta)}(\xi)]^2\, d\xi = 1 \pm \frac{\sin(2\kappa_\nu^{(\beta)})}{2\kappa_\nu^{(\beta)}} = \frac{2}{\delta_\nu} \quad \left\{ \begin{array}{l} \text{symm.} \\ \text{anti-symm.} \end{array} \right. \quad ;\ \delta_i = \begin{cases} 1 ; i = 0 \\ 2 ; i > 0 \end{cases} . \tag{7}$$

Setting $k_{\nu x}^{(\beta)} = \kappa_\nu^{(\beta)}/h$, one can introduce duct mode angles $\Theta_\nu^{(\beta)}$, using the wave equation, by:

$$\begin{aligned}
k_0^2 &= [k_{\nu x}^{(\beta)}]^2 + [k_{\nu z}^{(\beta)}]^2 = k_0^2\,[\sin^2\Theta_\nu^{(\beta)} + \cos^2\Theta_\nu^{(\beta)}], \\
k_{\nu x}^{(\beta)} &= k_0 \sin\,\Theta_\nu^{(\beta)}, \\
k_{\nu z}^{(\beta)} &= k_0 \cos\,\Theta_\nu^{(\beta)} = k_0\sqrt{1-\sin^2\Theta_\nu^{(\beta)}} \quad ; \quad \left\{ \begin{array}{l} \mathrm{Re}\{\sqrt{\ldots}\} \geq 0 \\ \mathrm{Im}\{\sqrt{\ldots}\} \geq 0 \end{array} \right. .
\end{aligned} \tag{8}$$

They represent the angle with the z axis of plane waves, from which the duct modes can be composed.

Coupling coefficients between duct modes and plate modes:

$$S_{\nu n}^{(\beta)} = \int_{-1}^{+1} q_\nu^{(\beta)}(\xi) \cdot v_n^{(\beta)}(\xi)\, d\xi. \tag{9}$$

The remaining boundary conditions of zone matching are:

$$\begin{aligned}
P_{s\nu} &= -P_{t\nu}, \\
\sum_\nu P_{t\nu} \cos\,\Theta_\nu \cdot q_\nu(\xi) &= \sum_n Z_0 V_n \cdot v_n(\xi) \\
2\sum_\nu [\delta_{\mu\nu} P_e - P_{t\nu}] \cdot q_\nu(\xi) &= \sum_n Z_{Tn} V_n \cdot v_n(\xi).
\end{aligned} \tag{10}$$

They lead to the following system of equations for $Z_0 V_n$:

$$\sum_{n(\beta_\mu)} Z_0 V_n \cdot \left[\delta_{mn} N_{Pm} \frac{Z_{Tm}}{Z_0} + \sum_{\nu(\beta_\nu)} \delta_\nu \frac{S_{\nu m} S_{\nu n}}{\cos\,\Theta_\nu} \right] = 2\,S_{\mu m} \cdot P_e. \tag{11}$$

$n(\beta_\mu)$, $\nu(\beta_\mu)$ means: the summation over n and m are in the range of indices belonging to the symmetry type of the incident duct mode, and ν is in the range of plate modes for the type of plate fixation used. The transmitted duct mode amplitudes $P_{t\nu}$ are evaluated with:

$$P_{t\nu} = \frac{\delta_\nu}{2\,\cos\,\Theta_\nu} \sum_{n(\beta_\mu)} S_{\nu n} \cdot Z_0 V_n \tag{12}$$

and the scattered duct mode amplitudes $P_{s\nu}$ from the first boundary condition. The sound transmission coefficient τ_μ for the incident μ-th duct mode is:

$$\tau_\mu = \frac{\delta_\mu}{4\,\cos\,\Theta_\mu} \sum_{\nu(\beta_\mu)}^{\nu_{gr}} \frac{\delta_\nu}{\cos\,\Theta_\nu} \cdot \left| \sum_{n(\beta_\mu)}^{n_{ob}} S_{\nu n} \frac{Z_0 V_n}{P_e} \right|^2 . \tag{13}$$

The range of duct mode indices used is the range of cut-on modes, with the limit:

$$n_{con} = 1 - \beta/2 + \frac{k_0 a}{2\pi} \tag{14}$$

($\beta = 0$ for symmetrical duct modes, $\beta = 1$ for anti-symmetrical duct modes).
Simply supported plate:

$$\begin{aligned} S^{(s)}_{\nu n} &= \int_{-1}^{+1} \cos\left(\nu_e \frac{\pi}{2}\xi\right) \cdot \cos\left(n_o \frac{\pi}{2}\xi\right)\, d\xi \\ &= \frac{\sin(\,(\nu_e - n_o)\pi/2)}{(\nu_e - n_o)\pi/2} + \frac{\sin(\,(\nu_e + n_o)\pi/2)}{(\nu_e + n_o)\pi/2} = \frac{4}{\pi}\,(-1)^{(\nu_e + n_o - 1)/2} \frac{n_o}{n_o^2 - \nu_e^2}, \\ S^{(a)}_{\nu n} &= \int_{-1}^{+1} \sin\left(\nu_o \frac{\pi}{2}\xi\right) \cdot \sin\left(n_e \frac{\pi}{2}\xi\right)\, d\xi \\ &= \frac{\sin(\,(\nu_o - n_e)\pi/2)}{(\nu_o - n_e)\pi/2} - \frac{\sin\,((\nu_o + n_e)\pi/2)}{(\nu_o + n_e)\pi/2} = \frac{4}{\pi}\,(-1)^{(n_e + \nu_o - 1)/2} \frac{n_e}{\nu_o^2 - n_e^2}. \end{aligned} \tag{15}$$

Clamped plate:

$$S_{\nu n}^{(\beta)} = \int_{-1}^{+1} \left\{ \begin{array}{c} \cos(\nu_e \frac{\pi}{2}\xi) \\ \\ \sin(\nu_o \frac{\pi}{2}\xi) \end{array} \right\} \cdot v_n^{(\beta)}(\gamma_n^{(\beta)}\xi)\ d\xi; \left\{ \begin{array}{l} (\beta) = (s); \nu = \nu_e = 0,\ 2,\ 4,\ldots \\ (\beta) = (a); \nu = \nu_o = 1,\ 3,\ 5,\ldots \end{array} \right. \tag{16}$$

$$S_{\nu n}^{(\beta)} = \left\{ \begin{array}{l} 8\,(-1)^{\nu_e/2}\,\gamma_n^{(s)} \left(\dfrac{\sin\gamma_n^{(s)}}{16\,(\gamma_n^{(s)})^4 - (\nu_e\pi)^4} + C_n^{(s)} \dfrac{\sinh\gamma_n^{(s)}}{16\,(\gamma_n^{(s)})^4 + (\nu_e\pi)^4} \right) \\ \\ -\,8\,(-1)^{(\nu_o-1)/2}\,\gamma_n^{(a)} \left(\dfrac{\cos\gamma_n^{(a)}}{16\,(\gamma_n^{(a)})^4 - (\nu_o\pi)^4} - C_n^{(a)} \dfrac{\cosh\gamma_n^{(a)}}{16\,(\gamma_n^{(a)})^4 + (\nu_o\pi)^4} \right) \end{array} \right.$$

$$= \left\{ \begin{array}{l} 64\,(-1)^{\nu_e/2}\, \dfrac{(\gamma_n^{(s)})^3\ \sin\ \gamma_n^{(s)}}{16\,(\gamma_n^{(s)})^4 - (\nu_e\pi)^4} \\ \\ -\,64\,(-1)^{(\nu_o-1)/2}\, \dfrac{(\gamma_n^{(a)})^3\ \cos\ \gamma_n^{(a)}}{16\,(\gamma_n^{(a)})^4 - (\nu_o\pi)^4} \end{array} \right. . \tag{17}$$

Free plate:

$$S_{\nu n}^{(\beta)} = \left\{ \begin{array}{l} 16\,(-1)^{\nu_e/2}(\nu_e\pi)^2\, \dfrac{\gamma_n^{(s)}\ \sin\ \gamma_n^{(s)}}{16\,(\gamma_n^{(s)})^4 - (\nu_e\pi)^4} \\ \\ -\,16\,(-1)^{(\nu_o-1)/2}(\nu_o\pi)^2\, \dfrac{\gamma_n^{(a)}\ \cos\ \gamma_n^{(a)}}{16\,(\gamma_n^{(a)})^4 - (\nu_o\pi)^4} \end{array} \right. . \tag{18}$$

Coupling coefficients of polynomial plate modes:

$$\begin{array}{l} S_{\nu_e 0}^{(s)} = 2\,(1 + C_0^{(s)})\, \dfrac{\sin(\nu_e\pi/2)}{\nu_e\pi/2} = \left\{ \begin{array}{lll} 2\,(1 + C_0^{(s)}) = 4 & ; & \nu_e = 0, \\ 0; & & \nu_e > 0 \end{array} \right. \\ S_{\nu_o 0}^{(a)} = C_0^{(a)}\, \dfrac{8}{\nu_o\pi}(-1)^{(\nu_o-1)/2} \quad = \dfrac{8}{\nu_o\pi}(-1)^{(\nu_o-1)/2}. \end{array} \tag{19}$$

Relation between coupling coefficients for free and clamped plates:

$$S_{\nu n,\,\mathrm{free}}^{(\beta)} = \frac{\pi^2}{4} \left(\frac{\nu^{(\beta)}}{\gamma_n^{(\beta)}} \right)^2 \cdot S_{\nu n,\,\mathrm{clamped}}^{(\beta)} \approx \left(\frac{\nu^{(\beta)}/2}{n \mp 1/4} \right)^2 \cdot S_{\nu n,\,\mathrm{clamped}}^{(\beta)}. \tag{20}$$

Mixture of incident duct modes

(written for a three-dimensional, rectangular duct with mode angles $\Theta_{m,n}$ and mode amplitudes $A_{m,n}$ of incident modes, with an arbitrary reference pressure p_0):

Sound transmission coefficient τ for an arbitrary mixture of incident duct modes, each of which has a modal transmission coefficient $\tau(\Theta_{m,n})$:

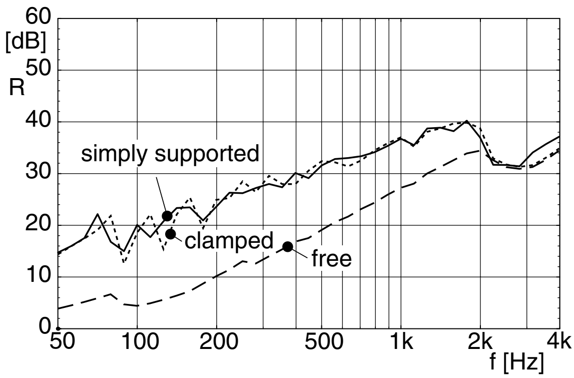
$$\tau = \frac{\sum\limits_{m,n} \frac{\tau(\Theta_{mn})}{\delta_m \delta_n} \cdot \left|\frac{A_{mn}}{p_0}\right|^2 \cdot \cos\Theta_{mn}}{\sum\limits_{m,n} \frac{1}{\delta_m \delta_n} \cdot \left|\frac{A_{mn}}{p_0}\right|^2 \cdot \cos\Theta_{mn}} \quad ; \quad \delta_m = \begin{cases} 1\,;\; m=0 \\ 2\,;\; m>0. \end{cases} \tag{21}$$

In the special case where all incident duct modes have the same energy density (a model that corresponds best to the diffuse sound incidence of room acoustics):

$$\left|\frac{A_{mn}}{p_0}\right|^2 = \frac{\delta_m \delta_n}{\sum\limits_{m,n} \mathrm{Re}\{\cos\Theta_{mn}\}}. \tag{22}$$

Therefore: $$\tau = \frac{\sum\limits_{m,n} \tau(\Theta_{mn}) \cos\Theta_{mn}}{\sum\limits_{mn} \cos\Theta_{mn}} \tag{23}$$

(this is, up to the discretisation of the angle of incidence, the relation for diffuse sound incidence).



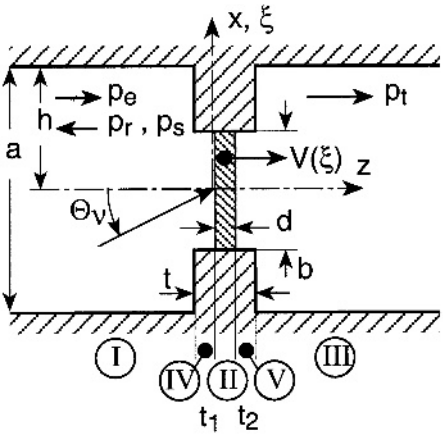
Sound transmission loss R of a plaster board across a duct, $a = 2$ [m] wide, for all propagating duct modes incident with same energy density, and three different boundary fixations of the plate.
Parameters: $a = 2$ [m]; $d = 0.0125$ [m]; $f_{cr}d = 31$ [Hzm]; $\rho_p = 1000$ [kg/m^3]; $\eta_p = 0.03$

I.16 Single Plate in a Wall Niche

► *See also:* Mechel, Vol. III, Ch. 15 (1998)

The influence of mounting a wall in a niche of the partition wall (baffle wall) between an emission and a receiving room has played for a while as a "*niche effect*" some role in the discussion of sound transmission tests.

The object and the sound field formulations here are similar to those in the previous Sect. I.15, except for the new fields in the niches with depths t_1, t_2, which are formulated as sums of "niche modes". The plate and the niche have a width $b = 2h'$.



The niche is centred in the baffle wall. The baffle wall is hard and rigid. Only a freely supported plate will be considered below (most parts of the formulas below can be used for other types of fixation also; then the pertinent values for γ_n and $S_{i,n}^{(\beta)}$ must be used). The incident wave p_e is the μ-th (propagating) duct mode (symmetrical or anti-symmetrical). Because of the central position of the niche, all other modes have the same symmetry as the incident mode.

The field on the front side is composed as $p_I = p_e + p_r + p_s$ (1)

with: p_e = incident duct mode,
p_r = incident duct mode after hard reflection,
p_s = scattered field.

p_r, p_s and the transmitted wave p_t are formulated as sums of duct modes. The plate vibration is formulated as a sum of plate modes. So far the field formulations are similar to those in Sect. I.15; here additional mode sums of "niche modes" will be used for the fields p_1, p_2 in the front and back niche, respectively.

Non-dimensional lateral co-ordinates: $\xi = x/h;\ \xi' = x/b = x/(2h')$ (2)

Field formulations:

Zone I (emission side):

$$\begin{aligned} p_e(\xi, z) &= P_e \cdot q_\mu(\xi) \cdot e^{-j\,k_{\mu z} z}, \\ Z_0 v_{ez}(\xi, z) &= \cos\Theta_\mu\, P_e \cdot q_\mu(\xi) \cdot e^{-j\,k_{\mu z} z}, \end{aligned} \tag{3}$$

$$\begin{aligned} p_r(\xi, z) &= P_e \cdot q_\mu(\xi) \cdot e^{+j\,k_{\mu z} z}, \\ Z_0 v_{rz}(\xi, z) &= -\cos\Theta_\mu\, P_e \cdot q_\mu(\xi) \cdot e^{+j\,k_{\mu z} z}, \end{aligned} \tag{4}$$

$$\begin{aligned} p_s(\xi, z) &= \sum_\nu P_{s\nu} \cdot q_\nu(\xi) \cdot e^{+j\,k_{\nu z} z}, \\ Z_0 v_{sz}(\xi, z) &= -\sum_\nu \cos\Theta_\nu\, P_{s\nu} \cdot q_\nu(\xi) \cdot e^{+j\,k_{\nu z} z}. \end{aligned} \tag{5}$$

Zone III (transmission side):

$$\begin{aligned} p_t(\xi, z) &= \sum_\nu P_{t\nu} \cdot q_\nu(\xi) \cdot e^{-j\,k_{\nu z}(z-t)}, \\ Z_0 v_{tz}(\xi, z) &= \sum_\nu \cos\,\Theta_\nu\, P_{t\nu} \cdot q_\nu(\xi) \cdot e^{-j\,k_{\nu z}(z-t)}. \end{aligned} \tag{6}$$

Zone IV (front side niche):

$$\begin{aligned} p_1(\xi', z) &= \sum_i \left(A_i\, e^{-j\,g_{iz}(z-t_1)} + B_i\, e^{+j\,g_{iz}(z-t_1)}\right) \cdot \varphi_i^{(1)}(\xi'), \\ Z_0 v_{1z}(\xi', z) &= \sum_i \cos\,\Phi_i\ \left(A_i\, e^{-j\,g_{iz}(z-t_1)} - B_i\, e^{+j\,g_{iz}(z-t_1)}\right) \cdot \varphi_i^{(1)}(\xi'). \end{aligned} \tag{7}$$

Zone V
(back side niche, if its width h'' is different from that of the front side niche; $\xi'' = x/h''$):

$$\begin{aligned} p_2(\xi'', z) &= \sum_i \left(C_i\, e^{-j\,g_{iz}^{(2)}(z-t_1-d)} + D_i\, e^{+j\,g_{iz}^{(2)}(z-t_1-d)}\right) \cdot \varphi_i^{(2)}(\xi''), \\ Z_0 v_{2z}(\xi'', z) &= \sum_i \cos\,\Phi_i^{(2)}\ \left(C_i\, e^{-j\,g_{iz}^{(2)}(z-t_1-d)} - D_i\, e^{+j\,g_{iz}^{(2)}(z-t_1-d)}\right) \cdot \varphi_i^{(2)}(\xi''). \end{aligned} \tag{8}$$

For $h' = h''$ is $\xi'' = \xi'$ and the upper niche indices (1),(2) are not needed.

Zone II (plate):

$$Z_0 V(\xi') = \sum_n Z_0 V_n \cdot v_n(\xi'). \tag{9}$$

Mode profiles

Duct modes:

$$q_\nu^{(\beta)}(\xi) = \begin{cases} \cos(\kappa_\nu^{(s)}\xi)\ ;\ \text{symmetrical} & \kappa_\nu^{(s)} = \nu_e \dfrac{\pi}{2} \quad ; \quad \nu_e = 0,\ 2,\ 4, \ldots \\ \sin(\kappa_\nu^{(a)}\xi)\ ;\ \text{anti-symmetrical} & \kappa_\nu^{(a)} = \nu_o \dfrac{\pi}{2} \quad ; \quad \nu_o = 1,\ 3,\ 5, \ldots \end{cases} \tag{10}$$

having the duct mode norms

$$N_{K\nu}^{(\beta)} = \int_{-1}^{+1} [q_\nu^{(\beta)}(\xi)]^2\, d\xi = 1 \pm \frac{\sin(2\kappa_\nu^{(\beta)})}{2\kappa_\nu^{(\beta)}} = \frac{2}{\delta_\nu} \begin{cases} \text{symm.} \\ \text{anti-symm.} \end{cases} ;\ \delta_n = \begin{cases} 1; n = 0 \\ 2; n > 0. \end{cases} \tag{11}$$

Niche modes:

$$\varphi_i(\xi') = \begin{cases} \cos(i_e \dfrac{\pi}{2}\xi') & ; \quad \beta = s = 0 \quad ; \quad i_e = 0,\ 2,\ 4 \ldots \\ \sin(i_o \dfrac{\pi}{2}\xi') & ; \quad \beta = a = 1 \quad ; \quad i_o = 1,\ 3,\ 5 \ldots \end{cases} \tag{12}$$

with axial wave numbers and mode angles:

$$g_{iz} = k_0 \cos \Phi_i = k_0\sqrt{1-\sin^2 \Phi_i}; \begin{cases} \mathrm{Re}\sqrt{\dots} \geq 0 \\ \mathrm{Im}\sqrt{\dots} \leq 0 \end{cases} \quad ; \quad \sin \Phi_i = i^{(\beta)} \frac{\lambda_0}{2b} \tag{13}$$

having niche mode norms: $$N_{Ni}^{(\beta)} = \frac{2}{\delta_i}. \tag{14}$$

Plate modes:

$$v_n^{(\beta)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi) \quad ; \quad \gamma_n^{(s)} = n_o \frac{\pi}{2} \quad ; \quad n_o = 1,\ 3,\ 5, \dots \\ \sin(\gamma_n^{(a)}\xi) \quad ; \quad \gamma_n^{(a)} = n_e \frac{\pi}{2} \quad ; \quad n_e = 2,\ 4,\ 6, \dots \end{cases} \tag{15}$$

having plate mode norms: $$N_{Pn}^{(\beta)} = \int_{-1}^{+1} [v_n^{(\beta)}(\xi)]^2 \, d\xi = 1. \tag{16}$$

Auxiliary amplitudes:

$$\begin{aligned} X_{i\pm} &= A_i \pm B_i \quad ; & X_{i+} &= \frac{j}{\tan(g_{iz}t_1)} \cdot X_{i-} - \frac{j}{\sin(g_{iz}t_1)} \cdot Y_{i-}, \\ Y_{i\pm} &= A_i\, e^{+j\, g_{iz}t_1} \pm B_i\, e^{-j\, g_{iz}t_1} \quad ; & Y_{i+} &= \frac{j}{\sin(g_{iz}t_1)} \cdot X_{i-} - \frac{j}{\tan(g_{iz}t_1)} \cdot Y_{i-}, \end{aligned} \tag{17}$$

$$\begin{aligned} U_{i\pm} &= C_i \pm D_i \quad ; & U_{i+} &= \frac{-j}{\tan(g_{iz}t_2)} \cdot U_{i-} + \frac{j}{\sin(g_{iz}t_2)} \cdot W_{i-}, \\ W_{i\pm} &= C_i\, e^{-j\, g_{iz}t_2} \pm D_i\, e^{+j\, g_{iz}t_2} \quad ; & W_{i+} &= \frac{-j}{\sin(g_{iz}t_2)} \cdot U_{i-} + \frac{j}{\tan(g_{iz}t_2)} \cdot W_{i-}. \end{aligned} \tag{18}$$

The boundary conditions of field matching at the zone limits lead to two coupled systems of equations for Y_{1-}, W_{1-}:

$$\begin{aligned} \{M_{11}\} \circ \{Y_{\iota-}\} + \{M_{12}\} \circ \{W_{\iota-}\} &= 2\,P_e \cdot \left\{Q_{\mu i}\right\}_{\mu}, \\ \{M_{21}\} \circ \{Y_{\iota-}\} + \{M_{22}\} \circ \{W_{\iota-}\} &= 0, \end{aligned} \tag{19}$$

with matrices

$$\begin{aligned}
\{M_{11}\} &= -j\ \{I_{ii}\} * \left\{N_{Ni}\cos(g_{iz}t_1)\right\} \\
&\quad + \frac{b}{a}\left\{\sin(g_{iz}t_1)\right\} * \{T_{\nu i}\}^t \circ \{T_{\nu\iota}\} * \left\{\frac{\cos\Phi_\iota}{N_{K\nu}\cos\Theta_\nu}\right\}, \\
\{M_{12}\} &= +j\ \{I_{ii}\} * \left\{N_{Ni}\cos(g_{iz}t_2)\right\} \\
&\quad - \frac{b}{a}\left\{\sin(g_{iz}t_2)\right\} * \{T_{\nu i}\}^t \circ \{T_{\nu\iota}\} * \left\{\frac{\cos\Phi_\iota}{N_{K\nu}\cos\Theta_\nu}\right\}, \\
\{M_{21}\} &= j\ \{I_{ii}\} * \left\{\frac{N_{Ni}}{\sin(g_{iz}t_1)}\right\}, \\
\{M_{22}\} &= j\ \{I_{ii}\} * \left\{\frac{N_{Ni}}{\sin(g_{iz}t_2)}\right\} - \{G_{ii'}\} \circ \{H_{i'\iota}\}
\end{aligned} \tag{20}$$

and right-side vector

$$\left\{Q_{\mu i}\right\}_\mu = \left\{\sin(g_{iz}t_1)T_{\mu i}\right\}_\mu, \tag{21}$$

where $\circ$ indicates a matrix multiplication; $\{a_m\} * \{c_m\} = \{a_m c_m\}$ indicates a term-wise multiplication of two vectors, and correspondingly $\{c_m x_{mn}\} = \{c_m\}*\{x_{mn}\}$ indicates the multiplication of the *m*th row of the matrix $\{x_{mn}\}$ with the element c_m of the vector $\{c_m\}$, and $\{d_n x_{mn}\} = \{d_n\}*\{x_{mn}\}$ $\{\{d_n\}*\{x_{mn}\}^t\}^t$ indicates the multiplication of the n-th column of $\{x_{mn}\}$ with d_n ; $\{x_{mn}\}^t$ is the transposed matrix; and $\{I_{ii}\}$ is the unit matrix.

The coupling coefficients $T_{\nu i} = T_{\nu i}^{(\beta)}$ between duct modes and niche modes (where the second form indicates the symmetry type $\beta = s, a$) are:

$$\begin{aligned}
T_{\nu i} &= \int_{-1}^{+1} q_\nu\left(\frac{b}{a}\xi'\right) \cdot \varphi_i(\xi')\, d\xi', \\
T_{\nu i}^{(\beta)} &= \int_{-1}^{+1} \begin{matrix}\cos(\nu_e\pi/2 \cdot b/a \cdot \xi') \cdot \cos(i_e\pi/2 \cdot \xi') \\ \sin(\nu_o\pi/2 \cdot b/a \cdot \xi') \cdot \sin(i_o\pi/2 \cdot \xi')\end{matrix} \cdot\ d\xi' \quad ; \quad \beta = \left\{\begin{matrix} s \\ a \end{matrix}\right. \\
&= \frac{\sin((i-\nu b/a)\pi/2)}{(i-\nu b/a)\pi/2} \pm \frac{\sin((i+\nu b/a)\pi/2)}{(i+\nu b/a)\pi/2} \quad ; \quad \beta = \left\{\begin{matrix} s;\nu_e,\ i_e \\ a;\nu_o,\ i_o \end{matrix}\right. .
\end{aligned} \tag{22}$$

Abbreviation used:

$$\{G_{i\iota}\} := \left\{j\ \{I_{ii}\} * \left\{N_{Ni}\frac{\sin(g_{iz}(t_1+t_2))}{\sin(g_{iz}t_1)\cdot\sin(g_{iz}t_2)}\right\} - \{S_{in}\} \circ \{S_{\iota n}\}^t * \left\{\frac{Z_{Tn}\cdot\cos\Phi_\iota}{N_{Pn}}\right\}\right\}, \tag{23}$$

with which

$$\{G_{i\iota}\} \circ \{U_{\iota -}\} = j\left\{\frac{N_{Ni}Y_{i-}}{\sin(g_{iz}t_1)} + \frac{N_{Ni}W_{i-}}{\sin(g_{iz}t_2)}\right\}. \tag{24}$$

The symbol Z_{Tn} denotes the n-th modal partition impedance of the plate (*Sect. I.15*).

With the solutions Y_{1-}, W_{1-} the other mode amplitudes in the duct are evaluated from:

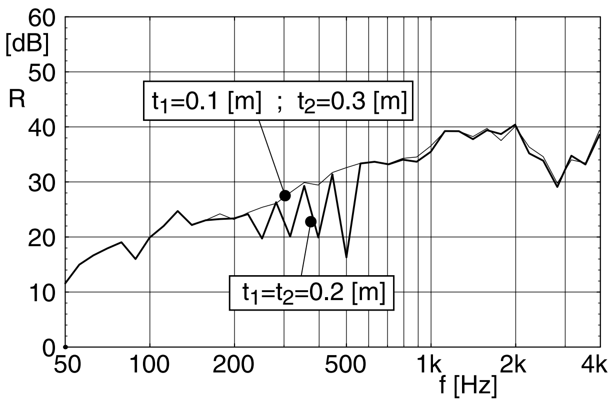
$$P_{s\nu} = \frac{-b/a}{N_{K\nu}\ \cos\Theta_\nu} \sum_i \cos\Phi_i T_{\nu i} \cdot Y_{i-} \ ; \ P_{t\nu} = \frac{b/a}{N_{K\nu}\ \cos\Theta_\nu} \sum_i \cos\Phi_i T_{\nu i} \cdot W_{i-}. \tag{25}$$

If one is only interested in the $P_{t\nu}$, a simplified system of equations is:

$$\left\{-\{M_{11}\} \circ \{M_{21}\}^{-1} \circ \{M_{22}\} + \{M_{12}\}\right\} \circ \{W_{\iota-}\} = 2\,P_e \cdot \left\{Q_{\mu i}\right\}_\mu . \tag{26}$$

The transmission coefficient τ_μ for a single incident (propagating) duct mode is:

$$\begin{aligned}
\tau_\mu = \frac{\sum_\nu \Pi'_{t\nu}}{b/a \cdot \Pi'_{e\mu}} &= \frac{1}{b/a \cdot N_{K\mu} \cos\Theta_\mu} \sum_\nu N_{K\nu} \cos\Theta_\nu \left|\frac{P_{t\nu}}{P_e}\right|^2 \\
&= \frac{b/a}{N_{K\mu} \cos\Theta_\mu} \sum_\nu \frac{1}{N_{K\nu} \cos\Theta_\nu} \left| \{T_{\nu i}\} \circ \left\{\cos\Phi_i \frac{W_{i-}}{P_e}\right\} \right|^2 \\
&= \frac{b/a}{N_{K\mu} \cos\Theta_\mu} \sum_\nu \frac{1}{N_{K\nu} \cos\Theta_\nu} \left| \sum_i T_{\nu i} \cos\Phi_i \frac{W_{i-}}{P_e} \right|^2 .
\end{aligned} \tag{27}$$



Transmission loss R through a plaster board plate in a niche in partition wall between test rooms. All propagating modes of the emission side duct are incident with equal energy density. The example shows the singularity of a central position of the test object in a niche

I.17 Strip-Shaped Wall in Infinite Baffle Wall

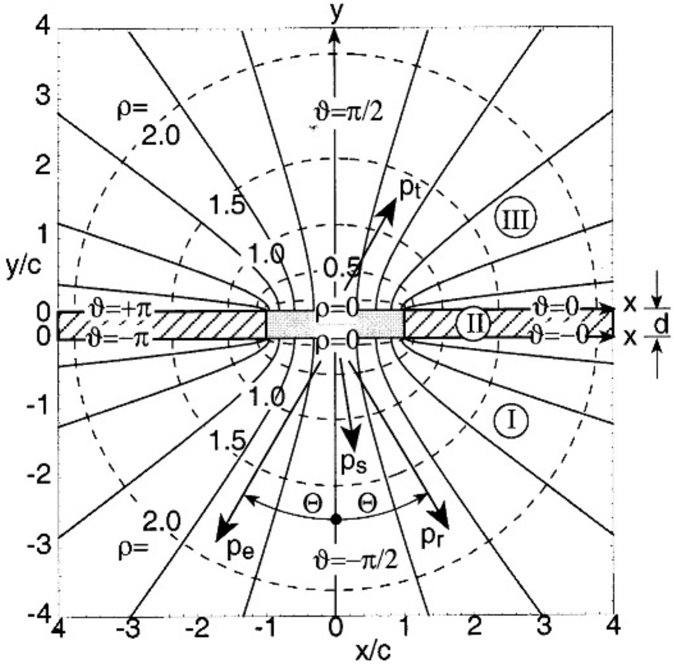
► *See also:* Mechel, Vol. III, Ch. 16 (1998)

A wall of thickness d and width $a = 2c$ is placed in a hard baffle wall, also of thickness d.

Elliptic-hyperbolic cylindrical systems of co-ordinates ρ, ϑ are used with focus positions at $x = \pm c$.

A plane wave p_e is assumed to be incident at a polar angle Θ; it becomes p_r after hard reflection at the front side surface. An additional scattered wave p_s is needed to formulate the front side sound field:

$$p_I = p_e + p_r + p_s. \tag{1}$$



The transmitted field is p_t. Both p_s and p_t are formulated as sums of Mathieu functions (► *See also:* Mechel (1997) for notations, formulas and generation of Mathieu functions). The velocity pattern of the plate is formulated as a sum of plate modes in the normalised co-ordinate $\xi = x/c$.

The sound-transmitting wall here is assumed to be a simply supported single plate. Other types of walls are treated correspondingly.

Transformation between Cartesian and elliptic-hyperbolic co-ordinates:

$$\left.\begin{aligned} x &= c\cdot\cosh\,\rho\cdot\cos\,\vartheta \\ z &= c\cdot\sinh\,\rho\cdot\sin\,\vartheta \end{aligned}\right\} \quad ; \quad 0 \le \rho < \infty\,; \quad -\pi \le \vartheta \le +\pi. \tag{2}$$

Field formulations:

$$p_e(\rho,\ \vartheta) + p_r(\rho,\ \vartheta) = 4P_e \sum_{m=0}^{\infty} (-j)^m ce_m(\alpha)\cdot Jc_m(\rho)\cdot ce_m(\vartheta), \tag{3}$$

$$p_t(\rho,\ \vartheta) = -p_s(\rho,\ \vartheta) = 2\sum_{m=0}^{\infty} D_m(-j)^m ce_m(\alpha)\cdot Hc_m^{(2)}(\rho)\cdot ce_m(\vartheta), \tag{4}$$

$$\left.\begin{array}{l} Z_0 v_{t\rho}(0,\ \vartheta > 0) \\ Z_0 v_{s\rho}(0,\ \vartheta < 0) \end{array}\right\} = \frac{\pm j}{\beta\ \sin\vartheta} \sum_{m=0} D_m (-j)^m ce_m(\alpha)\ Hc_m^{\prime(2)}(0) \cdot ce_m(\vartheta), \tag{5}$$

where P_e is the amplitude of the incident plane wave, $\alpha = \pi/2 - \Theta$, $\beta = k_0 c/2$, $ce_m(\vartheta)$ are the even azimuthal Mathieu functions, $Hc_m^{(2)}(\rho)$ are the radial Hankel-Mathieu functions of the second kind (associated with the $ce_m(\vartheta)$), and D_m are the mode amplitudes.

Plate velocity:

$$V(\xi) = \sum_n V_n \cdot v_n^{(s)}(\xi) \tag{6}$$

with (symmetrical, $\sigma = s$, and anti-symmetrical, $\sigma = a$) plate modes:

$$v_n^{(\sigma)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi)\ ;\ \ \gamma_n^{(s)} = n_o\pi/2\ ;\quad n_o = 1,\ 3,\ 5,\ \ldots\ ;\ (\sigma) = (s) = \text{symmetrical} \\ \sin(\gamma_n^{(a)}\xi)\ ;\ \ \gamma_n^{(a)} = n_e\pi/2 ; n_e = 2,\ 4,\ 6,\ \ldots\ ;\ (\sigma) = (a) = \text{anti-symmetrical} \end{cases} \tag{7}$$

having mode norms: $$N_{Pn} = \int_{-1}^{+1} v_n^{(\sigma)2}(\xi)\ d\xi = 1, \tag{8}$$

and modal plate partition impedances:

$$\frac{Z_{Tn}}{Z_0} = 2\pi Z_m F\left[\eta F^2\left(\frac{\gamma_n}{k_0 c}\right)^4 + j\left(1 - F^2\left(\frac{\gamma_n}{k_0 c}\right)^4\right)\right]; F = \frac{f}{f_{cr}}; Z_m = \frac{f_{cr} m}{Z_0} = \frac{f_{cr} d}{Z_0}\rho_P, \tag{9}$$

where: f = frequency; f_{cr} = critical frequency; $m = d \cdot \rho_P$ = plate surface mass density; η = plate loss factor.

The boundary conditions give a linear system of equations for the plate mode amplitudes V_n:

$$\sum_n Z_0 V_n \left[\delta_{n,\nu} \cdot N_{P\nu}\frac{Z_{T\nu}}{Z_0} + \frac{8\beta}{j\pi}\sum_{m\geq 0}\frac{Hc_m^{(2)}(0)}{Hc_m^{\prime(2)}(0)} Q_{m,\nu}\, Q_{m,n}\right]$$

$$= 4\,P_e \sum_{m\geq 0} (-j)^m ce_m(\alpha)\ Jc_m(0)\ Q_{m,\nu} \quad ; \quad \nu = 1,\ 2,\ 3, \ldots, \tag{10}$$

and with its solutions the mode amplitudes D_m of the transmitted wave:

$$D_m = \frac{2\beta\,(j)^{m-1}}{\pi\ ce_m(\alpha)\, Hc_m^{\prime(2)}(0)} \sum_n Q_{m,n} \cdot Z_0 V_n. \tag{11}$$

Therein: $\delta_{n,\nu}$ = Kronecker symbol, and mode coupling coefficients:

$$\begin{aligned} Q_{m,n} &:= \int_{-1}^{+1} ce_m(\arccos\xi;\ \beta^2) \cdot v_n^{(\sigma)}(\gamma_n \xi)\ d\xi \\ &= \int_0^{\pi} \sin\vartheta \cdot ce_m(\vartheta;\ \beta^2) \cdot v_n^{(\sigma)}(\gamma_n \cos\vartheta)\ d\vartheta, \end{aligned} \tag{12}$$

which are evaluated for symmetrical plate modes, for which m = 2r, by:

$$Q_{2r,n} = \sqrt{\frac{2\pi}{\gamma_n}} \sum_{s=0} A_{2s} \left[J_{1/2}(\gamma_n) + (1-\delta_{0,s}) \sum_{i=1}^{s} (-1)^i \frac{i!}{(2i)!} \left(\frac{2}{\gamma_n}\right)^i \cdot J_{i+1/2}(\gamma_n) \prod_{k=0}^{i-1} \left(4s^2 - 4k^2\right) \right], \tag{13}$$

and for anti-symmetrical plate modes, for which m = 2r + 1:

$$Q_{2r+1,n} = \sqrt{\frac{2\pi}{\gamma_n}} \sum_{s=0} A_{2s+1} \left[J_{3/2}(\gamma_n) + (1-\delta_{0,s}) \sum_{i=1}^{s} (-1)^i \frac{i!}{(2i)!} \prod_{k=1}^{i} \left((2s+1)^2 - (2k-1)^2\right) \cdot \left(\frac{2}{\gamma_n}\right)^i \cdot J_{i+3/2}(\gamma_n) \right]. \tag{14}$$

Here $J_n(z)$ are Bessel functions and A_n are the Fourier series components needed for the evaluation of the azimuthal Mathieu functions.

The sound transmission coefficient $\tau(\Theta)$ for oblique incidence finally is:

$$\tau(\Theta) = \frac{1}{\beta \cos\Theta} \sum_{m\geq 0} \left| \frac{D_m}{P_e} \right|^2 ce_m^2(\pi/2 - \Theta), \tag{15}$$

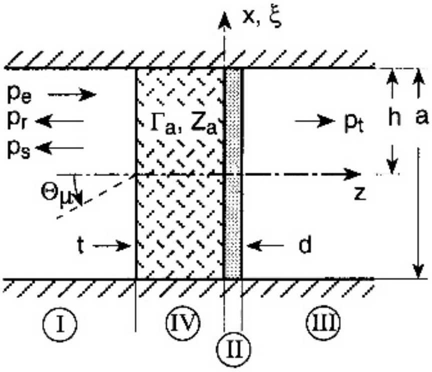
and for diffuse sound incidence (two-dimensional):

$$\tau_{2-dif} = \frac{2}{\beta} \int_{-\pi/2}^{+\pi/2} \tau(\Theta) \cos\Theta \, d\Theta. \tag{16}$$

I.18 Finite-Size Plate with a Front Side Absorber Layer

► *See also:* Mechel, Vol. III, Ch. 17 (1998)

A simply supported plate of thickness d with a porous layer of thickness t and characteristic values Γ_a, Z_a of the material on its front side is placed across a flat duct with lateral dimension a = 2h. There is no (or only loose) mechanical contact between the plate and the layer.



The μ-th (propagating) duct mode is assumed to be the incident wave; the index $\beta = s, a$ indicates whether the incident duct mode is symmetrical or anti-symmetrical. All other fields are of the same symmetry type. The field p_I on the front side is composed as $p_I = p_e + p_r + p_s$ of the incident wave p_e, of its hard reflection p_r (at $x = 0$) and of a scattered wave. The scattered wave p_s and the transmitted wave p_t, as well as the field p_a in the absorber layer, are formulated as duct mode sums. The plate vibration $V(\xi)$ is a plate mode sum with $\xi = x/h$.

Field formulations:

$$p_e(\xi, z) = P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{-j\,k_{\mu z} z},$$
$$Z_0 v_{ez}(\xi, z) = \cos\Theta_\mu P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{-j\,k_{\mu z} z}, \tag{1}$$

$$p_r(\xi, z) = P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{+j\,k_{\mu z} z},$$
$$Z_0 v_{rz}(\xi, z) = -\cos\Theta_\mu P_e \cdot q_\mu^{(\beta)}(\xi) \cdot e^{+j\,k_{\mu z} z}, \tag{2}$$

$$p_s(\xi, z) = \sum_\nu P_{s\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{+j\,k_{\nu z} z},$$
$$Z_0 v_{sz}(\xi, z) = -\sum_\nu \cos\Theta_\nu P_{s\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{+j\,k_{\nu z} z}, \tag{3}$$

$$p_t(\xi, z) = \sum_\nu P_{t\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{-j\,k_{\nu z}(z-d)},$$
$$Z_0 v_{tz}(\xi, z) = \sum_\nu \cos\Theta_\nu P_{t\nu} \cdot q_\nu^{(\beta)}(\xi) \cdot e^{-j\,k_{\nu z}(z-d)}, \tag{4}$$

$$V(\xi) = \sum_n V_n \cdot v_n^{(\beta)}(\xi), \tag{5}$$

$$p_a(\xi, z) = \sum_\nu P_{a\nu} \cdot q_\nu(\xi) \cdot \left[e^{-\Gamma_\nu z} + r_\nu\, e^{+\Gamma_\nu z}\right],$$
$$Z_0 v_{az}(\xi, z) = \sum_\nu \frac{\Gamma_\nu}{\Gamma_a Z_{an}} P_{a\nu} \cdot q_\nu(\xi) \cdot \left[e^{-\Gamma_\nu z} - r_\nu\, e^{+\Gamma_\nu z}\right] \tag{6}$$

with

$$q_\nu^{(\beta)}(\xi) = \begin{cases} \cos(\kappa_\nu^{(s)}\xi) \\ \sin(\kappa_\nu^{(a)}\xi) \end{cases} ; \quad \kappa_\nu^{(\beta)} = k_{\nu x}^{(\beta)} h = \left.\begin{matrix} \nu_e \\ \nu_o \end{matrix}\right\} \frac{\pi}{2} \quad ; \quad \begin{cases} \nu_e = 0,\ 2,\ 4\ldots \\ \nu_o = 1,\ 3,\ 5,\ldots, \end{cases}$$

$$\frac{k_{\nu z}}{k_0} = \cos\Theta_\nu = \sqrt{1 - \left(\frac{\nu\pi}{2k_0 h}\right)^2} \quad ; \quad \begin{cases} \mathrm{Re}\{\surd\} \geq 0 \\ \mathrm{Im}\{\surd\} \leq 0, \end{cases} \tag{7}$$

$$\frac{\Gamma_\nu}{k_0} = \sqrt{\left(\frac{\Gamma_a^2}{k_0}\right) + \left(\frac{\nu\pi}{2k_0 h}\right)^2} \quad ; \qquad \mathrm{Re}\{\surd\} \geq 0 \quad ; \quad N_{K\nu}^{(\beta)} = 2/\delta_\nu,$$

and

$$v_n^{(\beta)}(\xi) = \begin{cases} \cos(\gamma_n^{(s)}\xi)\ ; \quad \gamma_n^{(s)} = n_o \dfrac{\pi}{2}\ ; \quad n_o = 1,\ 3,\ 5,\ldots \\[2ex] \sin(\gamma_n^{(a)}\xi)\ ; \quad \gamma_n^{(a)} = n_e \dfrac{\pi}{2}\ ; \quad n_e = 2,\ 4,\ 6,\ldots. \end{cases} \tag{8}$$

The boundary conditions give a linear system of equations for the plate mode amplitudes V_n:

$$\sum_n Z_0 V_n \cdot \left[\delta_{m,n} N_{Pm} \frac{Z_{Tm}}{Z_0} + \sum_\nu \frac{\delta_\nu}{2} \frac{S_{\nu m} S_{\nu n}}{\cos\Theta_\nu} \frac{(1+C_\nu)^2 - (1-C_\nu)^2 e^{-2\Gamma_\nu t}}{(1+C_\nu) - (1-C_\nu)\, e^{-2\Gamma_\nu t}}\right]$$
$$= \frac{4\, C_\mu S_{\mu m}\, e^{-\Gamma_\mu t}}{(1+C_\mu) - (1-C_\mu)\, e^{-2\Gamma_\mu t}} \cdot P_e\ ;\ m \tag{9}$$

with the plate mode norms $N_{Pm} = 1$, the abbreviations:

$$C_\nu := \Gamma_{an} Z_{an} \cos\Theta_\nu \frac{k_0}{\Gamma_\nu} \quad ; \quad \Gamma_{an} = \Gamma_a / k_0 \quad ; \quad Z_{an} = Z_a / Z_0 \tag{10}$$

and the mode coupling coefficients:

$$\begin{aligned} S_{\nu n}^{(s)} &= \int_{-1}^{+1} \cos\left(\nu_e \frac{\pi}{2}\xi\right) \cdot \cos\left(n_o \frac{\pi}{2}\xi\right)\, d\xi \\ &= \frac{\sin((\nu_e - n_o)\pi/2)}{(\nu_e - n_o)\pi/2} + \frac{\sin((\nu_e + n_o)\pi/2)}{(\nu_e + n_o)\pi/2} = \frac{4}{\pi}(-1)^{(\nu_e + n_o - 1)/2} \frac{n_o}{n_o^2 - \nu_e^2}, \end{aligned} \tag{11}$$

$$\begin{aligned} S_{\nu n}^{(a)} &= \int_{-1}^{+1} \sin\left(\nu_o \frac{\pi}{2}\xi\right) \cdot \sin\left(n_e \frac{\pi}{2}\xi\right)\, d\xi \\ &= \frac{\sin((\nu_o - n_e)\pi/2)}{(\nu_o - n_e)\pi/2} - \frac{\sin((\nu_o + n_e)\pi/2)}{(\nu_o + n_e)\pi/2} = \frac{4}{\pi}(-1)^{(n_e + \nu_o - 1)/2} \frac{n_e}{\nu_o^2 - n_e^2}. \end{aligned} \tag{12}$$

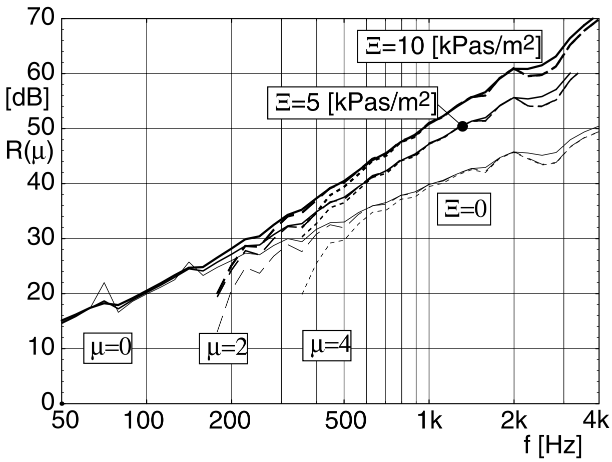
One computes, with the solutions $Z_0 V_n$, the transmitted duct mode amplitudes $P_{t\nu}$ by:

$$P_{t\nu} = \frac{\delta_\nu}{2\cos\Theta_\nu} \sum_n S_{\nu n} \cdot Z_0 V_n, \tag{13}$$

or, directly, the transmission coefficient:

$$\tau_\mu = \frac{\delta_\mu}{4\cos\Theta_\mu}\sum_{\nu}^{\nu_{lim}}\frac{\delta_\nu}{\cos\Theta_\nu}\cdot\left|\sum_{n}^{n_{hi}} S_{\nu n}\,\frac{Z_0 V_n}{P_e}\right|^2 , \tag{14}$$

where the upper summation limit ν_{lim} is the index limit for propagating duct modes and n_{hi} is the upper index limit for the plate modes used, which is set by the convergence of the system of equations for Z_0V_n.



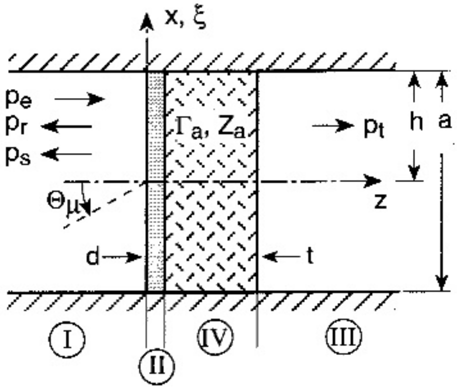
Sound transmission loss R(μ) for the μ-th duct mode through a plaster board with a front side glass fibre layer (if $\Xi > 0$) having flow resistivity values Ξ. Parameters: $a = 2$ [m]; $d_p = 0.0125$ [m]; $t = 0.1$ [m]; $f_{cr}d = 31$ [Hzm]; $\rho_p = 1000$ [kg/m^3]; $\eta = 0.03$

I.19 Finite-Size Plate with a Back Side Absorber Layer

► *See also:* Mechel, Vol. III, Ch. 17 (1998)

See the previous *Sect. I.18* for the duct, the plate, its mounting and the field composition.

The incident wave is again the μ-th propagating duct mode.



The system of equations for the plate mode amplitudes V_n now reads:

$$\sum_n Z_0 V_n \cdot \left[\delta_{m,n} N_{Pn} \frac{Z_{Tm}}{Z_0} + \sum_\nu \frac{\delta_\nu}{2\cos\Theta_\nu} \left(1 + C_\nu \frac{1+r_\nu}{1-r_\nu} \right) \cdot S_{\nu n} S_{\nu m} \right] = 2\, S_{\mu m} \cdot P_e \tag{1}$$

with the modal reflection factors at the back side of the absorber layer:

$$r_\nu = e^{-2\Gamma_\nu t} \frac{1-C_\nu}{1+C_\nu}. \tag{2}$$

After solving for $Z_0 V_n$, the transmitted duct mode amplitudes are evaluated from:

$$P_{t\nu} = \delta_\nu \frac{C_\nu}{1+C_\nu} \; \frac{e^{-\Gamma_\nu t}}{(1-r_\nu)\cos\Theta_\nu} \sum_n S_{\nu n} \cdot Z_0 V_n, \tag{3}$$

or the sound transmission coefficient directly from:

$$\tau_\mu = \frac{\delta_\mu}{\cos\Theta_\mu} \sum_\nu^{\nu_{lim}} \frac{\delta_\nu}{\cos\Theta_\nu}\, e^{-2\Gamma'_\nu t} \left| \frac{C_\nu}{(1-r_\nu)(1+C_\nu)} \right|^2 \cdot \left| \sum_n^{n_{hi}} S_{\nu n} \cdot \frac{Z_0 V_n}{P_e} \right|^2 \tag{4}$$

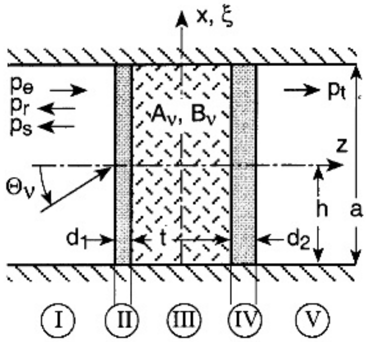
with $\Gamma' = \mathrm{Re}\{\Gamma\}$; see the previous section for the mode order limits ν_{lim} and n_{hi}.

The numerical results are the same as with a front side absorber layer (see previous *Sect. I.18*), except for some details around the coincidence frequency of the plate.

I.20 Finite-Size Double Wall with an Absorber Core

► *See also:* Mechel, Vol. III, Ch. 18 (1998)

A double wall with a porous absorber layer as a core is mounted in a flat, hard duct. The plate borders are simply supported at the duct walls (as an example; other supports only need other plate mode wave numbers γ_n, mode norms N_{Pn}, and modal partition impedance Z_{Tn}). There is no (or only a loose) mechanical contact between the plates and the absorber layer. The characteristic values of the porous material are Γ_a, Z_a, or, in normalised forms, $\Gamma_{an} = \Gamma_a/k_0$, $Z_{an} = Z_a/Z_0$.



The incident wave p_e is the μ-th (symmetrical or anti-symmetrical) propagating duct mode. The sound field on the front side is composed as:

$$p_I = p_e + p_r + p_s, \tag{1}$$

where p_r is the incident mode after hard reflection at $z = -(t/2 + d_1)$ and p_s is the scattered field. The scattered wave and the transmitted wave p_t are formulated as duct mode sums, as well as the sound field p_a in the absorber layer. The plate velocity patterns $V^{(i)}(\xi)$; i = 1, 2; are plate mode sums with $\xi = x/h$.

Field formulations:

$$\begin{aligned} p_e(\xi, z) &= P_e \cdot q_\mu(\xi) \cdot e^{-j\,k_{\mu z}(z+t/2+d_1)}, \\ Z_0 v_{ez}(\xi, z) &= \cos\Theta_\mu \cdot p_e(\xi, z), \end{aligned} \tag{2}$$

$$\begin{aligned} p_r(\xi, z) &= P_e \cdot q_\mu(\xi) \cdot e^{+j\,k_{\mu z}(z+t/2+d_1)}, \\ Z_0 v_{rz}(\xi, z) &= -\cos\Theta_\mu \cdot p_r(\xi, z), \end{aligned} \tag{3}$$

$$\begin{aligned} p_s(\xi, z) &= \sum_\nu P_{s\nu} \cdot q_\nu(\xi) \cdot e^{+j\,k_{\nu z}(z+t/2+d_1)}, \\ Z_0 v_{sz}(\xi, z) &= -\sum_\nu \cos\Theta_\nu\; P_{s\nu} \cdot q_\nu(\xi) \cdot e^{+j\,k_{\nu z}(z+t/2+d_1)}, \end{aligned} \tag{4}$$

$$\begin{aligned} p_t(\xi, z) &= \sum_\nu P_{t\nu} \cdot q_\nu(\xi) \cdot e^{-j\,k_{\nu z}(z-t/2-d_2)}, \\ Z_0 v_{tz}(\xi, z) &= \sum_\nu \cos\Theta_\nu\; P_{t\nu} \cdot q_\nu(\xi) \cdot e^{-j\,k_{\nu z}(z-t/2-d_2)}, \end{aligned} \tag{5}$$

$$\begin{aligned} p_a(\xi, z) &= \sum_\nu \left(A_\nu e^{-\Gamma_\nu z} + B_\nu e^{+\Gamma_\nu z}\right) \cdot q_\nu(\xi), \\ Z_0 v_{az}(\xi, z) &= \sum_\nu \frac{\Gamma_\nu}{\Gamma_a Z_{an}} \left(A_\nu e^{-\Gamma_\nu z} - B_\nu e^{+\Gamma_\nu z}\right) \cdot q_\nu(\xi), \end{aligned} \tag{6}$$

$$V^{(i)}(\xi) = \sum_n V_n^{(i)} \cdot v_n^{(i)}(\xi) \quad ; \quad i = 1,\ 2. \tag{7}$$

Duct modes:

$$q_\nu(\xi) = \begin{cases} \cos\left(\dfrac{\nu\pi}{2}\xi\right) & ; \quad \nu = 0,\ 2,\ 4, \ldots \quad ; \quad \text{symmetrical} \\ \sin\left(\dfrac{\nu\pi}{2}\xi\right) & ; \quad \nu = 1,\ 3,\ 5, \ldots \quad ; \quad \text{anti-symmetrical,} \end{cases} \tag{8}$$

$$\begin{aligned} \frac{k_{\nu z}}{k_0} &= \cos\,\Theta_\nu = \sqrt{1 - \left(\frac{\nu\pi}{2k_0 h}\right)^2}\,; \begin{cases} \operatorname{Re}\{\sqrt{\ldots}\} \geq 0 \\ \operatorname{Im}\{\sqrt{\ldots}\} \leq 0, \end{cases} \\ \frac{\Gamma_\nu}{k_0} &= \sqrt{\Gamma_{an}^2 + \left(\frac{\nu\pi}{2k_0 h}\right)^2} \quad ; \quad \operatorname{Re}\{\sqrt{\ldots}\} \geq 0 \quad ; \quad \Gamma_{an} = \Gamma_a / k_0. \end{aligned} \tag{9}$$

Plate modes (for simplyßindexsound transmission!through simly supported plates supported plates; see Sect. I.14 for other supports):

$$v_n^{(\beta)}(\xi) = \begin{cases} \cos\left(\gamma_n^{(s)}\xi\right) & ; \quad \gamma_n^{(s)} = n_o \dfrac{\pi}{2} \quad ; \quad n_o = 1,\ 3,\ 5, \ldots \\ \sin\left(\gamma_n^{(a)}\xi\right) & ; \quad \gamma_n^{(a)} = n_e \dfrac{\pi}{2} \quad ; \quad n_e = 2,\ 4,\ 6, \ldots, \end{cases} \tag{10}$$

$$N_{Pn}^{(\beta)} = \int_{-1}^{+1} \left[v_n^{(\beta)}(\xi)\right]^2 \, d\xi = \frac{2}{\delta_n} = 1, \tag{11}$$

$$\begin{aligned} Z_{Tn} &= \omega m \left[\eta \left(\frac{\gamma_n}{k_B h}\right)^4 + j\left(1 - \left(\frac{\gamma_n}{k_B h}\right)^4\right)\right], \\ \frac{Z_{Tn}}{Z_0} &= 2\pi\, Z_m F \left[\eta\, F^2 \left(\frac{\gamma_n}{k_0 h}\right)^4 + j\left(1 - F^2 \left(\frac{\gamma_n}{k_0 h}\right)^4\right)\right]; \end{aligned} \qquad F = \frac{f}{f_{cr}}; Z_m = \frac{f_{cr} m}{Z_0} = \frac{f_{cr} d}{Z_0}\rho \tag{12}$$

Mode norms:

$$N_{K\nu} := \int_{-1}^{1} \left[q_\nu(\xi)\right]^2 \, d\xi = \frac{2}{\delta_\nu} \quad ; \quad N_{Pn}^{(\beta)} := \int_{-1}^{1} \left[v_n^{(\beta)}(\xi)\right]^2 \, d\xi. \tag{13}$$

Abbreviation:

$$C_\nu = \Gamma_{an} Z_{an} \cos\,\Theta_\nu \frac{k_0}{\Gamma_\nu}. \tag{14}$$

Auxiliary amplitudes:

$$\begin{aligned} X_{\nu\pm} &:= A_\nu e^{+\Gamma_\nu t/2} \pm B_\nu e^{-\Gamma_\nu t/2}, \\ Y_{\nu\pm} &:= A_\nu e^{-\Gamma_\nu t/2} \pm B_\nu e^{+\Gamma_\nu t/2} \end{aligned} \tag{15}$$

with intrinsic relations:

$$X_{\nu+} = \frac{X_{\nu-}}{\tanh(\Gamma_\nu t)} - \frac{Y_{\nu-}}{\sinh(\Gamma_\nu t)}\,; \quad Y_{\nu+} = \frac{X_{\nu-}}{\sinh(\Gamma_\nu t)} - \frac{Y_{\nu-}}{\tanh(\Gamma_\nu t)}. \tag{16}$$

The boundary conditions for them give the following coupled systems of linear equations:

$$\sum_{\nu}\left\{\left[X_{\nu-}\left(\frac{1+\delta_{\nu,\nu'}N_{K\nu'}\cos\Theta_{\nu'}}{C_\nu}+\frac{1}{\tanh(\Gamma_\nu t)}\right)-\frac{Y_{\nu-}}{\sinh(\Gamma_\nu t)}\right]\sum_n \frac{S^{(1)}_{\nu' n}S^{(1)}_{\nu n}}{N^{(1)}_{Pn}\ Z^{(1)}_{Tn}/Z_0}\right\}$$
$$= 2\,P_e\sum_n \frac{S^{(1)}_{\nu' n}S^{(1)}_{\mu n}}{N^{(1)}_{Pn}\ Z^{(1)}_{Tn}/Z_0}, \tag{17}$$

$$\sum_{\nu}\left\{\left[\frac{-X_{\nu-}}{\sinh(\Gamma_\nu t)}+Y_{\nu-}\left(\frac{1+\delta_{\nu,\nu'}N_{K\nu'}\cos\Theta_{\nu'}}{C_\nu}+\frac{1}{\tanh(\Gamma_\nu t)}\right)\right]\sum_n \frac{S^{(2)}_{\nu' n}S^{(2)}_{\nu n}}{N^{(2)}_{Pn}\ Z^{(2)}_{Tn}/Z_0}\right\}=0$$

with the mode coupling coefficients:

$$S^{(i)}_{\nu n} := \int_{-1}^{1} q_\nu(\xi)\cdot v^{(i)}_n(\xi)\,d\xi. \tag{18}$$

The scattered and transmitted mode amplitudes are:

$$P_{s\nu}=\frac{-1}{C_\nu}\cdot X_{\nu-}\quad ;\quad P_{t\nu}=\frac{1}{C_\nu}\cdot Y_{\nu-}, \tag{19}$$

and the transmission coefficient τ_μ for a single incident mode is:

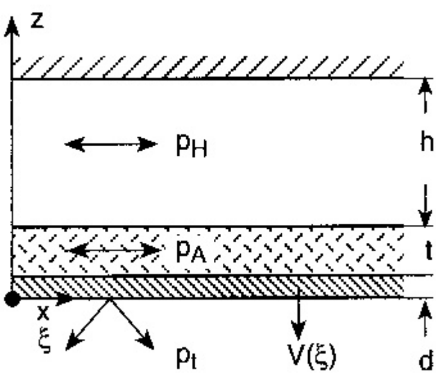
$$\tau_\mu=\frac{\delta_\mu}{\cos\Theta_\mu}\sum_{\nu}^{\nu_{lim}}\frac{\cos\Theta_\nu}{\delta_\nu}\cdot\left|\frac{Y_{\nu-}}{C_\nu}\right|^2, \tag{20}$$

where ν_{lim} is the mode order limit for propagating duct modes.

I.21 Plenum Modes

Mechel, Vol. III, Ch. 19 (1998)

This section serves as preparation for the next *Sect. I.22*. It deals with characteristic solutions in the plenum of suspended ceilings.



The object is a flat (two-dimensional) duct with one hard wall (at $z = d + t + h$), an air space of height h ($d + t \le z \le d + t + h$), a porous absorber layer of thickness t ($d \le z \le d + t$), and an elastic plate of thickness d ($0 \le z \le d$). The free space $z \le 0$ belongs to the object.

The porous absorber material is described by its characteristic propagation constant Γ_a and wave impedance Z_a (or, in normalised form, $\Gamma_{an} = \Gamma_a/k_0$, $Z_{an} = Z_a/Z_0$). The elastic plate is described by its partition impedance Z_T for a polar angle of incidence Φ.

Elementary solutions are sought which obey the wave equations in air and in the absorber material :

$$\left(\Delta + k_0^2\right)\ p_H = 0 \quad ; \quad \left(\Delta + k_0^2\right)\ p_t = 0 \quad ; \quad \left(\Delta - \Gamma_a^2\right)\ p_A = 0 \tag{1}$$

and the bending wave equation of the plate (which is guaranteed by using the partition impedance), as well as the boundary conditions. The solutions are called "plenum modes", with mode index n. A normalised longitudinal co-ordinate may be used with some reference length a: $\xi = x/a$.

Formulations of the component fields:

$$\begin{aligned} p_{Hn}(\xi, z) &= P_{Hn} \cdot e^{\pm\Gamma_n a\,\xi} \cdot \cos\,(\varepsilon_n\,(z - h - t - d))\,, \\ Z_0 v_{Hnz}(\xi, z) &= -j\,\frac{\varepsilon_n}{k_0} P_{Hn} \cdot e^{\pm\Gamma_n a\,\xi} \cdot \sin\,(\varepsilon_n\,(z - h - t - d))\,, \end{aligned} \tag{2}$$

$$\begin{aligned} p_{tn}(\xi, z) &= P_{tn} \cdot e^{\pm\Gamma_n a\,\xi} \cdot e^{-\Gamma_n a\,\xi} \cdot e^{j\,\kappa_n z}, \\ Z_0 v_{tnz}(\xi, z) &= -\frac{\kappa_n}{k_0} p_{tn}(\xi, z), \end{aligned} \tag{3}$$

$$p_{An}(\xi, z) = e^{\pm\Gamma_n a\,\xi}\left[C_n \cdot e^{-\gamma_n\,(z-d)} + D_n \cdot e^{+\gamma_n\,(z-d)}\right]. \tag{4}$$

From the wave equation in the plenum space it follows that:

$$\begin{aligned} &\Gamma_n^2 - \varepsilon_n^2 + k_0^2 = 0, \\ &\left(\frac{\Gamma_n}{j\,k_0}\right)^2 + \left(\frac{\varepsilon_n}{k_0}\right)^2 = 1 = \sin^2\Phi_n + \cos^2\Phi_n \quad ; \quad \left\{ \begin{aligned} \sin\ \Phi_n &= \Gamma_n/j\,k_0, \\ \cos\ \Phi_n &= \varepsilon_n/k_0 \end{aligned} \right. \\ &\Gamma_n h = \sqrt{(\varepsilon_n h)^2 - (k_0 h)^2};\ \mathrm{Re}\{\Gamma_n h\} \ge 0, \end{aligned} \tag{5}$$

which defines a modal angle of incidence Φ_n. The corresponding equations in the free space $z \le 0$ lead to $\varepsilon_n = \pm\kappa_n$. The wave equation in the absorber material is satisfied with:

$$\frac{\gamma_n}{k_0} = \sqrt{\Gamma_{an}^2 + 1 - (\varepsilon_n/k_0)^2}\ ; \quad \mathrm{Re}\{\sqrt{\ldots}\} \ge 0. \tag{6}$$

An abbreviation used later is: $\quad G_n := \Gamma_{an} Z_{an} \dfrac{\varepsilon_n}{\gamma_n}\ \tan\,(\varepsilon_n h).$ (7)

The modal partition impedance of the plate is (Sect. I.9):

$$\frac{Z_{Tn}}{Z_0} = 2\pi\, Z_m F\,\left[\eta F^2\,\sin^4\Phi_n + j\,\left(1-F^2\sin^4\Phi_n\right)\right] \quad ; \quad F:=\frac{f}{f_{cr}};\, Z_m:=\frac{f_{cr}m}{Z_0}=\frac{f_{cr}d}{Z_0}\,\rho \tag{8}$$

$$\sin^4\,\Phi_n = \left(1-(\varepsilon_n/k_0)^2\right)^2 .$$

The remaining boundary conditions give the following homogeneous linear system of equations for the mode amplitudes C_n, D_n:

$$\begin{aligned} &C_n e^{-\gamma_n t}\cdot\left[j\,\frac{\varepsilon_n}{k_0}\,\tan(\varepsilon_n h) - \frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\right] + D_n e^{+\gamma_n t}\cdot\left[j\,\frac{\varepsilon_n}{k_0}\,\tan(\varepsilon_n h) + \frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\right] = 0,\\ &C_n\cdot\left[1+\frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\left(\frac{k_0}{\kappa_n}+\frac{Z_{Tn}}{Z_0}\right)\right] + D_n\cdot\left[1-\frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\left(\frac{k_0}{\kappa_n}+\frac{Z_{Tn}}{Z_0}\right)\right] = 0. \end{aligned} \tag{9}$$

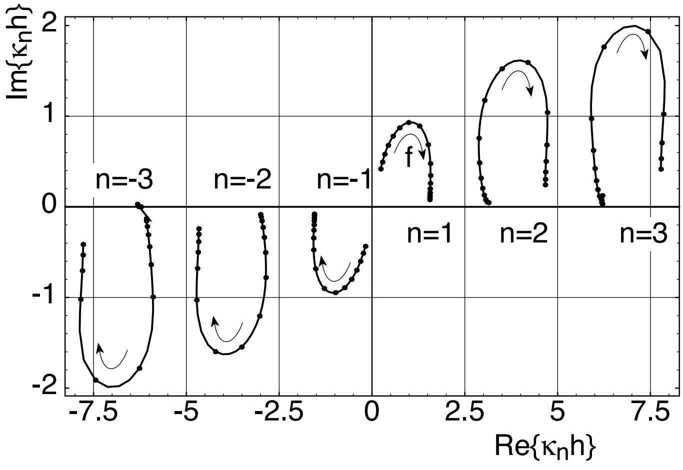
A non-trivial solution exists if the determinant of the coefficient matrix vanishes; this gives the following characteristic equation for the wave numbers of the plenum modes:

$$\begin{aligned} &\cosh(\gamma_n t)\cdot\\ &\left\{j\,\frac{\varepsilon_n}{k_0}\,\tan(\varepsilon_n h)\cdot\left[\tanh(\gamma_n t)+\frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\left(\frac{k_0}{\kappa_n}+\frac{Z_{Tn}}{Z_0}\right)\right]\right.\\ &\qquad\left.+\frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\cdot\left[1+\frac{\gamma_n/k_0}{\Gamma_{an}Z_{an}}\left(\frac{k_0}{\kappa_n}+\frac{Z_{Tn}}{Z_0}\right)\right]\right\} = 0. \end{aligned} \tag{10}$$

The leading factor can be assumed to be $\cosh(\gamma_n t) \neq 0$; thus the expression in the curled brackets must be zero. Taking $z = \kappa_n h$ as the quantity for which solutions shall be found, the equation reads:

$$\begin{aligned} &j\,z\cdot\tan z\cdot\left[z\cdot\tanh\left(k_0 t\,\frac{\gamma_n}{k_0}\right)+\frac{1}{\Gamma_{an}Z_{an}}\,\frac{\gamma_n}{k_0}\left(1+z\,\frac{Z_{Tn}}{Z_0}\right)\right]\\ &\qquad+\frac{1}{\Gamma_{an}Z_{an}}\,\frac{\gamma_n}{k_0}\left[z+\frac{1}{\Gamma_{an}Z_{an}}\,\frac{\gamma_n}{k_0}\left(1+z\,\frac{Z_{Tn}}{Z_0}\right)\right]=0 \end{aligned} \tag{11}$$

(γ_n and Z_{Tn} are functions of z). A method of solving for a set z_n of modes is described in [Mechel, Vol. III, Ch. 19 (1998)].



Example of plenum mode solutions $\kappa_n h$ for a plaster board as the elastic plate, and with a t = 4 [cm] thick glass fibre mat as absorber layer.
Parameters: $h = 0.4$ [m]; $d = 0.0095$ [m]; $t = 0.04$ [m]; $f_{cr}d = 31$ [Hz · m]; $\rho_p = 1000$ [kg/m^3]; $\eta = 0.1$; $\Xi = 10$ [kPa s/m^2]

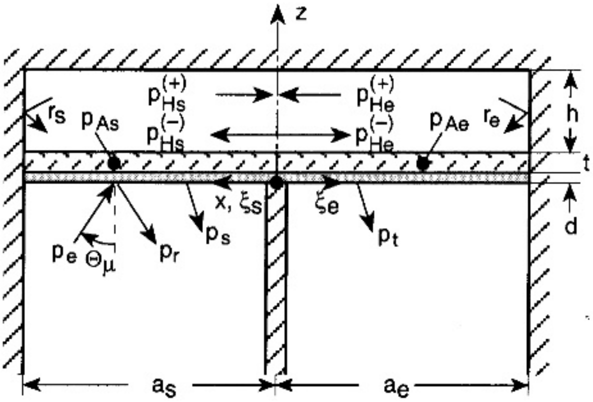
I.22 Sound Transmission through Suspended Ceilings

► *See also:* Mechel, Vol. III, Ch. 19 (1998)

A typical set-up of a suspended ceiling is taken from the previous *Sect. I.21*, from where notations for the component fields are also used. The next graph shows the arrangement of an emission room of width a_s (index s on the emission side from the German *Sendeseite*) and a receiving room of width a_e (index e on the receiver side from the German *Empfangsseite*). The suspended ceiling spans over both rooms; the partition wall between the two rooms is rigid. The target quantity is the flanking transmission loss $R_f = -10 \cdot \lg(\tau)$ of sound transmission through the suspended ceiling. First, the transmission coefficient τ_μ for the μ-th propagating mode of the emission room as incident wave p_e will be given; then the transmission loss for all propagating emission room modes (with equal energy densities) follows.

The non-dimensional axial coordinates $\xi_s = x/a_s$; $\xi_e = -x/a_e$ will be used. The back walls of the plenum are assumed to have reflection factors r_s, r_e.

The sound field in the emission room is composed as $p_e + p_r + p_s$, where p_e is the μ-th propagating room mode of the emission room, p_r is this mode after hard reflection at the lower surface of the suspended ceiling and p_s is a (back-)scattered field. Both p_s and the transmitted sound p_t are composed as mode sums of the room modes in the relevant rooms.



The sound fields in the plenum spaces and in the absorber layer are composed as sums of plenum modes. A lower index $\beta = s, e$ indicates to which side the sound wave belongs; an upper index $(\pm)$ indicates the direction of a mode.

The incident and reflected emission room modes are ($\mu = 0, 1, 2, \ldots$):

$$\begin{aligned} p_e(\xi_s, z) &= P_e \cdot \cos(\mu\pi\xi_s) \cdot e^{-j\,k_{\mu z} z} \quad ; \quad Z_0 v_{ez}(\xi_s, z) = \cos\Theta_\mu \cdot p_e(\xi_s, z), \\ p_r(\xi_s, z) &= P_e \cdot \cos(\mu\pi\xi_s) \cdot e^{+j\,k_{\mu z} z} \quad ; \quad Z_0 v_{rz}(\xi_s, z) = -\cos\Theta_\mu \cdot p_r(\xi_s, z) \end{aligned} \tag{1}$$

with modal angles of incidence:

$$\sin\Theta_\mu = \frac{k_{\mu x}}{k_0} = \frac{\mu\pi}{k_0 a_s} \quad ; \quad \cos\Theta_\mu = \frac{k_{\mu z}}{k_0} = \sqrt{1 - \left(\frac{\mu\pi}{k_0 a_s}\right)^2} \quad ; \quad \left\{ \begin{array}{l} \mathrm{Re}\{\sqrt{\ldots}\} \geq 0 \\ \mathrm{Im}\{\sqrt{\ldots}\} \leq 0 \end{array} \right. \tag{2}$$

and similarly modal angle Θ_ν in the receiving room.

The transmitted sound field is ($\nu = 0, 1, 2, \ldots$):

$$\begin{aligned} p_t(\xi_e, z) &= \sum_\nu P_{t\nu} \cdot \cos(\nu\pi\xi_e) \cdot e^{j\,k_{\nu z} z}, \\ Z_0 v_{tz}(\xi_e, z) &= -\sum_\nu P_{t\nu} \cos\Theta_\nu \cdot \cos(\nu\pi\xi_e) \cdot e^{j\,k_{\nu z} z}. \end{aligned} \tag{3}$$

The desired sound transmission coefficient is:

$$\tau_\mu = \frac{\delta_\mu}{\cos\Theta_\mu} \sum_\nu^{\nu_{lim}} \frac{\cos\Theta_\nu}{\delta_\nu} \left| \frac{P_{t\nu}}{P_e} \right|^2 \tag{4}$$

with ν_{lim} the mode order limit for propagating modes,

given by the condition $\quad \nu \leq \dfrac{k_0 a_e}{\pi} = \dfrac{2a_e}{\lambda_0}.$ (5)

The modal components in the plenum space are formulated as ($\beta = s, e$):

$$p_{Hn}(\xi_\beta, z) = \left[P_{H\beta n}^{(+)} \cdot e^{+\Gamma_n a\,\xi_\beta} + P_{H\beta n}^{(-)} \cdot e^{-\Gamma_n a\,\xi_\beta} \right] \cdot \cos\left(\varepsilon_n (z - h - t - d)\right), \tag{6}$$

$$Z_0 v_{Hnz}(\xi_\beta, z) = -j\,\frac{\varepsilon_n}{k_0}\left[P^{(+)}_{H\beta n}\cdot e^{+\Gamma_n a\,\xi_\beta} + P^{(-)}_{H\beta n}\cdot e^{-\Gamma_n a\,\xi_\beta}\right]\cdot \sin\,(\varepsilon_n\,(z-h-t-d))\,, \tag{7}$$

and in the absorber layer:

$$\begin{aligned} p_{An}(\xi_\beta, z) = {} & e^{+\Gamma_n a\,\xi_\beta}\,\left[C^{(+)}_n\cdot e^{-\gamma_n\,(z-d)} + D^{(+)}_n\cdot e^{+\gamma_n\,(z-d)}\right] \\ & + e^{-\Gamma_n a\,\xi_\beta}\,\left[C^{(-)}_n\cdot e^{-\gamma_n\,(z-d)} + D^{(-)}_n\cdot e^{+\gamma_n\,(z-d)}\right]. \end{aligned} \tag{8}$$

The reflection at the back walls of the plenum with given reflection factors r_β can serve to eliminate some sets of amplitudes:

$$P^{(+)}_{H\beta n} = r_\beta\cdot e^{-2\,\Gamma_n a_\beta}\cdot P^{(-)}_{H\beta n}, \tag{9}$$

and from the matching of fields at the surface between the plenum space and absorber layer:

$$\begin{aligned} C^{(\pm)}_{\beta n} &= \frac{1}{2}P^{(\pm)}_{H\beta n}\cdot e^{+\gamma_n t}\cdot\cos\,(\varepsilon_n h)\cdot\left(1+j\,G_n\right); \\ D^{(\pm)}_{\beta n} &= \frac{1}{2}P^{(\pm)}_{H\beta n}\cdot e^{-\gamma_n t}\cdot\cos\,(\varepsilon_n h)\cdot\left(1-j\,G_n\right). \end{aligned} \tag{10}$$

The field matching in the plane $x = 0$ leads to two coupled systems of linear equations for the $P^{(-)}_{H\beta n}$:

$$\begin{aligned} &\sum_n \left[P^{(-)}_{Hsn}\,\left(1+r_s\,e^{-2\,\Gamma_n a_s}\right) - P^{(-)}_{Hen}\,\left(1+r_e\,e^{-2\,\Gamma_n a_e}\right)\right]\cdot M_{mn} \\ &\qquad = j\,h\,S_{\mu m}\cdot P^{(h)}_{Hs\mu} - \frac{t}{\Gamma_{an} Z_{an}}\left(R^{(-)}_{\mu m}\cdot A_{s\mu} + R^{(+)}_{\mu m}\cdot B_{s\mu}\right), \end{aligned} \tag{11}$$

$$\sum_n \frac{\Gamma_n}{k_0}\left[P^{(-)}_{Hsn}\,\left(1-r_s\,e^{-2\,\Gamma_n a_s}\right) + P^{(-)}_{Hen}\,\left(1-r_e\,e^{-2\,\Gamma_n a_e}\right)\right]\cdot M_{mn} = 0. \tag{12}$$

With the solutions one evaluates $P^{(+)}_{H\beta n}$, and with these $C^{(\pm)}_{\beta n}$, $D^{(\pm)}_{\beta n}$. The amplitudes $P_{t\nu}$, which are needed for the transmission coefficient, are evaluated from:

$$P_{t\nu} = -\frac{\delta_\nu}{\cos\,\Theta_\nu}\sum_n \frac{\gamma_n/k_0}{\Gamma_{an} Z_{an}}\left[\left(C^{(+)}_{en} - D^{(+)}_{en}\right)\,T^{(+)}_{\nu n} + \left(C^{(-)}_{en} - D^{(-)}_{en}\right)\,T^{(-)}_{\nu n}\right]. \tag{13}$$

One has in the above equations:

$$\begin{aligned} A_\mu &= \frac{1}{2}\,e^{+\chi_\mu t}\left[\cos\,(k_{\mu z}h) + j\,C_\mu\,\sin\,(k_{\mu z}h)\right]\cdot P_H, \\ B_\mu &= \frac{1}{2}\,e^{-\chi_\mu t}\left[\cos\,(k_{\mu z}h) - j\,C_\mu\,\sin\,(k_{\mu z}h)\right]\cdot P_H, \end{aligned} \tag{14}$$

and

$$P_H = \frac{4\,P_e \cdot e^{-\chi_\mu t}}{\cos(k_{\mu z}h)\,\left[1 + e^{-2\chi_\mu t} + 1/C_\mu \cdot \left(1 + Z_{T\mu}/Z_0 \cdot \cos\Theta_\mu\right)\,\left(1 - e^{-2\chi_\mu t}\right)\right] + \ldots} \atop \overline{\ldots + j\,\sin(k_{\mu z}h)\,\left[C_\mu\left(1 - e^{-2\chi_\mu t}\right) + \left(1 + Z_{T\mu}/Z_0 \cdot \cos\Theta_\mu\right)\,\left(1 + e^{-2\chi_\mu t}\right)\right]} \tag{15}$$

with

$$k_{\mu z}h = k_0 h \cdot \cos\Theta_\mu \quad ; \quad \frac{\chi_\mu}{k_0} = \sqrt{\Gamma_{an}^2 + \left(\frac{\mu\pi}{k_0 a}\right)^2} \quad ; \quad \mathrm{Re}\{\sqrt{\ldots}\} \geq 0,$$
$$C_\mu := \Gamma_{an} Z_{an}\,\cos\Theta_\mu \frac{k_0}{\chi_\mu}. \tag{16}$$

Further, the weight factors $M_{m,n}$ of the inter-orthogonality of plenum mode factors are used; they are defined by:

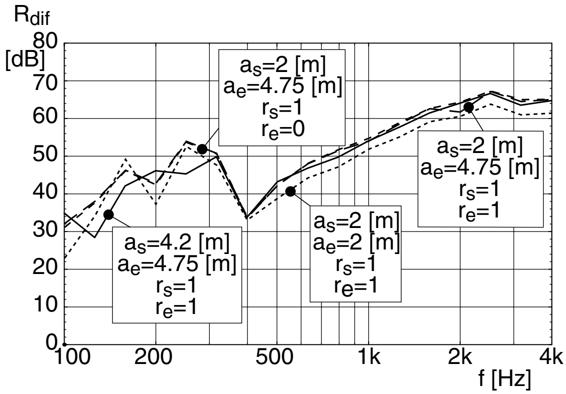
$$\frac{1}{j\,k_0 Z_0}\int\limits_H p_{Hm}(z)\cdot p_{Hn}(z)\,dz + \frac{1}{\Gamma_a Z_a}\int\limits_A p_{Am}(z)\cdot p_{An}(z)\,dz := \frac{M_{m,n}}{k_0 Z_0} \tag{17}$$

(if the plate of the suspended ceiling is rigid, then $M_{m,n} = \delta_{m,n}$ = Kronecker symbol) with evaluation by:

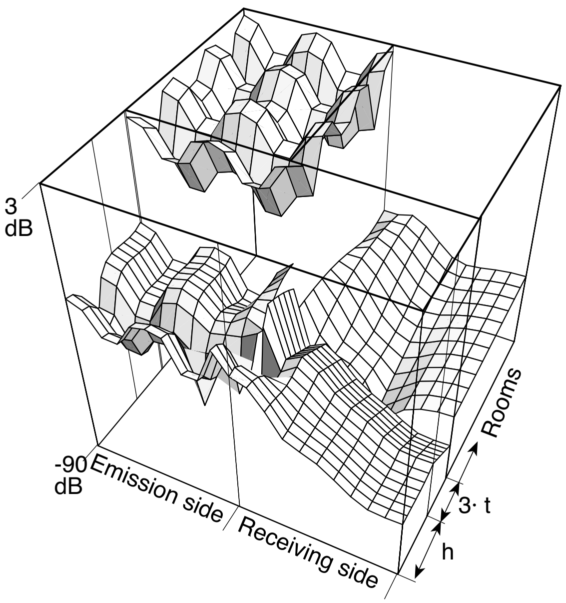
$$\begin{aligned} M_{m,n} = {} & \frac{h}{2j}\left[\frac{\sin((\varepsilon_m - \varepsilon_n)\,h)}{(\varepsilon_m - \varepsilon_n)\,h} + \frac{\sin((\varepsilon_m + \varepsilon_n)\,h)}{(\varepsilon_m + \varepsilon_n)\,h}\right] + \frac{\cos(\varepsilon_m h)\,\cos(\varepsilon_n h)}{4\Gamma_{an} Z_{an}}\,t \\ & + \left[\left(1+jG_m\right)\left(1+jG_n\right)\frac{1 - e^{-(\gamma_m+\gamma_n)\,t}}{(\gamma_m+\gamma_n)\,t} - \left(1-jG_m\right)\left(1-jG_n\right)\frac{1-e^{+(\gamma_m+\gamma_n)\,t}}{(\gamma_m+\gamma_n)\,t}\right. \\ & \left. + \left(1+jG_m\right)\left(1-jG_n\right)\frac{1 - e^{-(\gamma_m-\gamma_n)\,t}}{(\gamma_m-\gamma_n)\,t} - \left(1 - jG_m\right)\left(1 + jG_n\right)\frac{1 - e^{+(\gamma_m-\gamma_n)\,t}}{(\gamma_m - \gamma_n)\,t}\right]. \end{aligned} \tag{18}$$

Other factors are mode coupling factors, between directly transmitted field and plenum mode field in the absorber layer:

$$\begin{aligned} R_{\mu n}^{(\pm)} & := \frac{1}{t}\int\limits_d^{d+t} e^{\pm\chi_n(z-d)}\cdot p_{An}^{(\pm)}(z)\,dz \\ & = \frac{\cos(\varepsilon_n h)}{2t\,(\gamma_n^2 - \chi_n^2)}\left[e^{\gamma_n t}\cdot\left(1 + j\,G_n\right)\left(\gamma_n \pm \chi_n\right) - e^{-\gamma_n t}\cdot\left(1 - j\,G_n\right)\left(\gamma_n \mp \chi_n\right)\right. \\ & \quad \left. + \;2\,e^{\pm\chi_n t}\left(j\,\gamma_n G_n \pm \chi_n\right)\right] \end{aligned} \tag{19}$$



Sound transmission loss for diffuse sound incidence through a suspended ceiling with a plaster board plate, a $t = 4$ [cm] glass fibre layer on it, a plenum height of $h = 39$ [cm], for different situations of room sizes a_s, a_e and plenum back wall reflections factors r_s, r_e. Parameters: $h = 0.39$ [m]; $t = 0.04$ [m]; $d = 0.0095$ [m]; $1 \le n \le 5$; $f_{cr}d = 31$ [Hz · m]; $\rho_p = 1000$ [kg/m^3]; $\eta = 0.1$; $\Xi = 10$ [kPa s/m^2], $\rho_a = 20$ [kg/m^3]



Sound pressure level profile below and in the plenum of a suspended ceiling for the first higher room mode as incident mode. Left rear: emission room; right rear: receiving room; left front: plenum above emission, right front: plenum above receiving room. The space occupied by the absorber layer is drawn with a 3-fold magnification. The suspended ceiling consists of a $d = 9.5$ [mm] plaster board covered with a $t = 8$ [cm] glass fibre mat; the plenum is $h = 35$ [cm] high; the room sizes are $a_s = a_e = 4$ [m]. The plenum back walls are hard between the room modes and the absorber field along the common surface:

$$T_{\nu n}^{(\pm)} := \int_0^1 q_\nu(\xi) \cdot e^{\pm\Gamma_n a \xi}\, d\xi = \frac{\pm\Gamma_n a}{(\Gamma_n a)^2 + (\nu\pi)^2} \left[(-1)^\nu \cdot e^{\pm\Gamma_n a} - 1\right], \tag{20}$$

and between directly transmitted and plenum mode field in the plenum space:

$$\begin{aligned} S_{\mu n} &:= \frac{1}{h} \int_{t+d}^{h+t+d} \cos\left(k_{\mu z}(z-h-t-d)\right) \cdot \cos\left(\varepsilon_n(z-h-t-d)\right)\, dz \\ &= \frac{1}{2}\left[\frac{\sin\left((k_{\mu z}-\varepsilon_n)\,h\right)}{(k_{\mu z}-\varepsilon_n)\,h} + \frac{\sin\left((k_{\mu z}+\varepsilon_n)\,h\right)}{(k_{\mu z}+\varepsilon_n)\,h}\right]. \end{aligned} \tag{21}$$

The simulation of diffuse sound incidence assumes that all propagating room modes have the same sound energy density and are incident modes. The sound transmission coefficient for diffuse sound incidence is evaluated by:

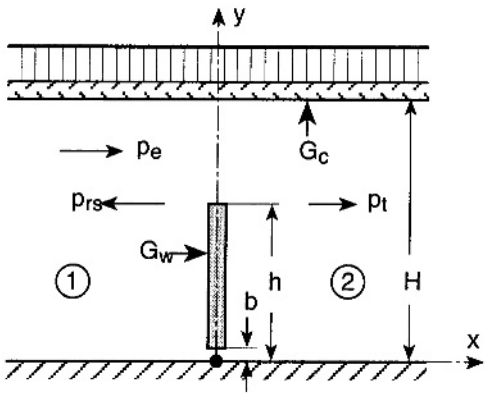
$$\tau_{dif} = \sum_{\mu=0}^{\mu_{lim}} \tau_\mu \, \cos\Theta_\mu \Big/ \sum_{\mu=0}^{\mu_{lim}} \cos\Theta_\mu . \tag{22}$$

I.23 Office Fences

▶ *See also:* Mechel, Vol. III, Ch. 24 (1998)

In a two-dimensional room of height H with a hard floor and an absorbent ceiling is placed a (thin) absorbent and sound transmitting fence with its upper corner at $y = h$ and (possibly) a gap of height b towards the floor. The rooms are anechoic in the $\pm x$ direction.

The ceiling is assumed (as an approximation) to be locally reacting with an admittance G_c. The surface admittance G_w of the fence may be different on both sides, G_{w1}, G_{w2}.



First, a non-transmissive fence without a lower gap ($b = 0$) will be considered; the modifications of the results for other conditions will be explained below.

Let the incident wave be the μ-th mode of the room:

$$p_e(x,y) = P_e \cdot \cos(\varepsilon_\mu y) \cdot e^{-\Gamma_\mu x}, \tag{1}$$

where $\varepsilon_\mu H$ is a solution of the characteristic equation

$$\varepsilon_\mu H \cdot \tan(\varepsilon_\mu H) = j\, k_0 H \cdot Z_0 G_c \tag{2}$$

and $\quad \Gamma_\mu^2 = \varepsilon_\mu^2 - k_0^2. \tag{3}$

The field on the emission side (1) is composed as $p_1 = p_e + p_{rs}$ with the backscattered field formulated as a sum of room modes:

$$p_{rs}(x,y) = \sum_n B_n \cdot \cos(\varepsilon_n y) \cdot e^{+\Gamma_n x}. \tag{4}$$

Similarly the transmitted field p_t is a sum of room modes:

$$p_t(x,y) = \sum_n D_n \cdot \cos(\varepsilon_n y) \cdot e^{-\Gamma_n x}. \tag{5}$$

The matching of the fields to the fence admittance and to each other leads to two coupled systems of linear equations for B_n, D_n:

$$\begin{aligned} &\sum_n B_n \cdot j\frac{\Gamma_n}{k_0}\left(\delta_{m,n} N_m - S_{m,n}\right) - \sum_n D_n \cdot \left(Z_0 G_{w2} - \delta_{m,n}\, j\frac{\Gamma_m}{k_0} N_m\right) \\ &\quad = j\, P_e \frac{\Gamma_\mu}{k_0}\left(\delta_{\mu,m} N_m - S_{\mu,m}\right), \end{aligned} \tag{6}$$

$$\begin{aligned} &\sum_n B_n \cdot \left(j\frac{\Gamma_n}{k_0} S_{m,n} - \delta_{m,n} N_m Z_0 G_{w1}\right) - Z_0 G_{w1} \sum_n D_n \cdot \left(S_{m,n} - \delta_{m,n} N_m\right) \\ &\quad = P_e \left(j\frac{\Gamma_\mu}{k_0} S_{\mu,m} + \delta_{\mu,m} N_\mu Z_0 G_{w1}\right), \end{aligned} \tag{7}$$

where $\delta_{m,n}$ is the Kronecker symbol, N_m the mode norms:

$$\begin{aligned} &\frac{1}{H}\int_0^H \cos(\varepsilon_m y) \cdot \cos(\varepsilon_n y)\, dy = \delta_{m,n} \cdot \frac{1}{2}\left(1 + \frac{\sin(2\varepsilon_m H)}{2\varepsilon_m H}\right) \\ &\quad = \delta_{m,n} \cdot N_m \xrightarrow[\varepsilon_m=\varepsilon_n=0]{} 1 \end{aligned} \tag{8}$$

and $S_{m,n}$ the mode-coupling coefficients

$$\begin{aligned} &\frac{1}{H}\int_0^h \cos(\varepsilon_m y) \cdot \cos(\varepsilon_n y)\, dy = \frac{h}{2H}\left(\frac{\sin((\varepsilon_m - \varepsilon_n)h)}{(\varepsilon_m - \varepsilon_n)h} + \frac{\sin((\varepsilon_m + \varepsilon_n)h)}{(\varepsilon_m + \varepsilon_n)h}\right) \\ &\quad =: S_{m,n} \xrightarrow[\varepsilon_m=\varepsilon_n]{} \frac{h}{2H}\left(1 + \frac{\sin(2\varepsilon_m h)}{2\varepsilon_m h}\right). \end{aligned} \tag{9}$$

If the fence is sound transmissive, its surface admittances G_{w1}, G_{w2} are determined with a free space termination of the fence. If there is a bottom gap ($b \neq 0$), the mode-coupling coefficients $S_{m,n}$ are evaluated for the interval (b, h) of integration, i.e. everywhere $S_{m,n}(0,h)$ is replaced by $S_{m,n}(0,h) \rightarrow S_{m,n}(b,h)$.

I.24 Office Fences, with Second Principle of Superposition

► *See also:* Mechel, Vol. III, Ch. 24 (1998)

The object is the same as in the previous *Sect. I.23*, but it will be treated here with the second principle of superposition (PSP) from *Sect. B.10*. The advantage of the PSP is that it halves the size of the system of equations to be solved and it makes the field formulations more plausible. The PSP can be applied if the object has a plane of symmetry S (which is $x = 0$ in our case). It splits the task into two subtasks: first the sound transmissive parts of S are assumed to be hard (upper index $\beta = (h)$), and second the sound transmissive parts of S are assumed to be soft (upper index $\beta = (w)$). The surface of the fence has the admittances $G_w^{(\beta)}$ in both subtasks (i.e. with hard or soft termination at $x = 0$), respectively.

The incident wave p_e in both subtasks is assumed to be the μ-th mode of the emission room, which is lined on one side with a locally reacting ceiling:

$$p_e(x, y) = P_e \cdot \cos(\varepsilon_\mu y) \cdot e^{-\Gamma_\mu x}. \tag{1}$$

It is associated on the front side (1) (see graph in *Sect. I.23*) with the mode field $p_r^{(\beta)}(x, y)$ after hard or soft reflection, respectively, at $x = 0$:

$$p_r^{(\beta)}(x, y) = \pm P_e \cdot \cos(\varepsilon_\mu y) \cdot e^{+\Gamma_\mu x} \quad ; \quad (\beta) = (h),\ (w), \tag{2}$$

and a scattered wave

$$p_s^{(\beta)}(x, y) = \sum_n C_n^{(\beta)} \cdot \cos(\varepsilon_n y) \cdot e^{+\Gamma_n x}, \tag{3}$$

which is a sum of duct modes. The sound fields in front of and behind the screen are then:

$$\begin{aligned} p_1(x < 0, y) &= p_e(x, y) + \frac{1}{2}\left[p_s^{(h)}(x, y) + p_s^{(w)}(x, y)\right], \\ p_2(x > 0, y) &= p_e(x, y) + \frac{1}{2}\left[p_s^{(h)}(-x, y) - p_s^{(w)}(-x, y)\right]. \end{aligned} \tag{4}$$

Application of the boundary conditions in the surface plane of the screen gives two systems of linear equations for the $C_n^{(\beta)}$:

$$\begin{aligned} &\sum_n C_n^{(h)} \cdot \left[Z_0 G_w^{(h)} \cdot S_{m,n} - \delta_{m,n} \cdot j\frac{\Gamma_m}{k_0} N_m\right] = -2\, Z_0 G_w^{(h)} S_{\mu,m} \cdot P_e, \\ &\sum_n C_n^{(w)} \cdot \left[j\frac{\Gamma_n}{k_0} \cdot S_{m,n} - \delta_{m,n} \cdot Z_0 G_w^{(s)} N_m\right] = 2\, j\frac{\Gamma_\mu}{k_0} S_{\mu,m} \cdot P_e. \end{aligned} \tag{5}$$

Here the N_m are norms of the duct modes:

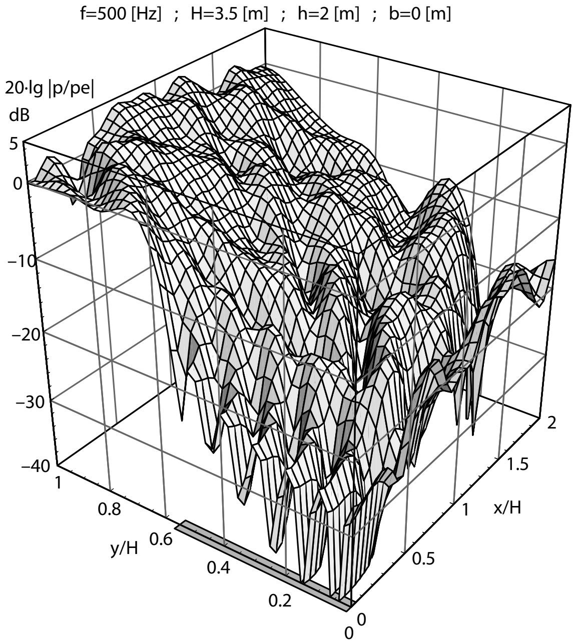
$$\frac{1}{H}\int_0^H \cos(\varepsilon_m y) \cdot \cos(\varepsilon_n y)\, dy = \delta_{m,n} \cdot \frac{1}{2}\left(1 + \frac{\sin(2\varepsilon_m H)}{2\varepsilon_m H}\right) = \delta_{m,n} \cdot N_m \xrightarrow[\varepsilon_m=\varepsilon_n=0]{} 1, \tag{6}$$

and the $S_{m,n} = S_{m,n}(b, h)$ are mode coupling coefficients:

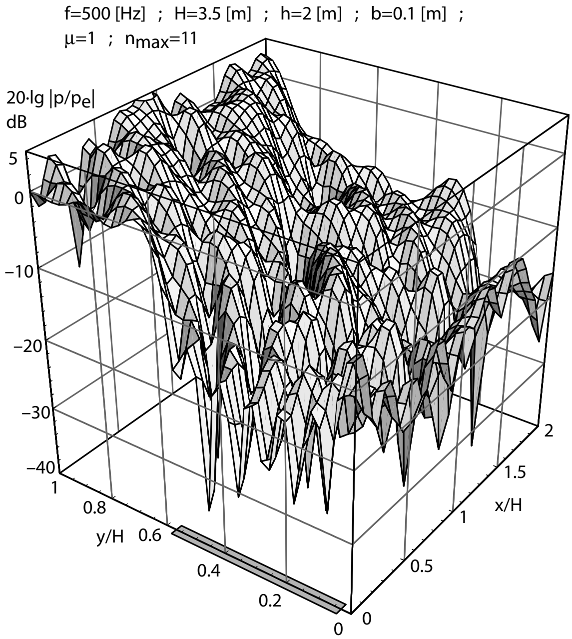
$$\frac{1}{H}\int_{b}^{h} \cos(\varepsilon_m y) \cdot \cos(\varepsilon_n y)\; dy =: S_{m,n}(b, h). \qquad (7)$$

The method can be applied for any sound source on the emission side if its source profile (either a given pressure or a given velocity) can be synthesised with room modes. Then the above evaluation will be performed mode-wise and the results superimposed.

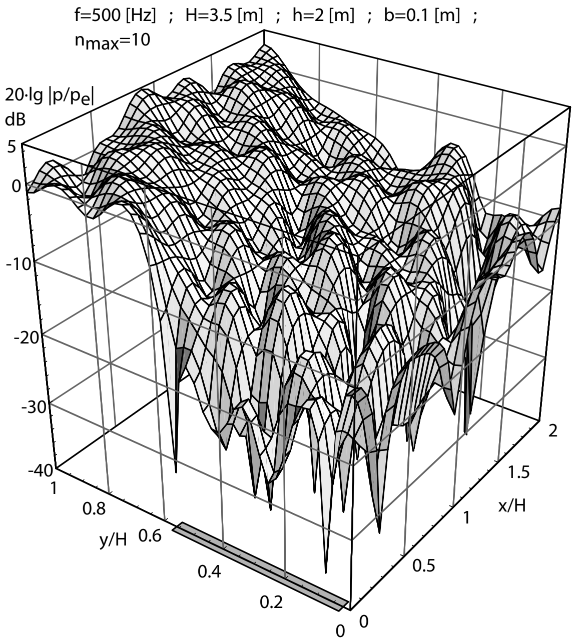
The numerical examples below for the sound pressure level change $20 \cdot \lg(|p/p_e|)$ due to the fence assume the following parameter values: frequency $f = 500$ [Hz]; $H = 3.5$ [m]; $h = 2$ [m]; $b = 0.1$ [m] (if there is a gap); *ceiling* (from low to high): d= 2 [cm] boards of compressed mineral fibre; bulk density RG = 400 [kg/m^3]; flow resistance $5 \cdot Z_0$; elastic constant $f_{cr} \cdot d = 70$; plus a 5 [cm] thick mineral fibre felt with flow resistivity $\Xi = 10$ [kPa $\cdot$ s/m^2] and bulk density RG = 15 [kg/m^3] and a locally reacting air layer 40 [cm] thick below the hard construction ceiling; *fence*: a mineral fibre board, 10 [cm] thick, flow resistivity $\Xi = 10$ [kPa $\cdot$ s/m^2], covered (on both sides) with a perforated metal sheet, 1.5 [mm] thick, porosity 36%, round perforations 4 [mm] wide (with no, or only loose, mechanical contact between the metal sheet and the mineral fibre board). If the fence is assumed to be non-transmissive, a heavy metal sheet may be assumed in its centre. The incident wave is the fundamental room mode $\mu = 1$.



Sound pressure level change behind the fence; the fence is non-transmissive and has no gap at its foot



As above, but fence has a gap between foot and floor, $b = 10$ [cm] wide

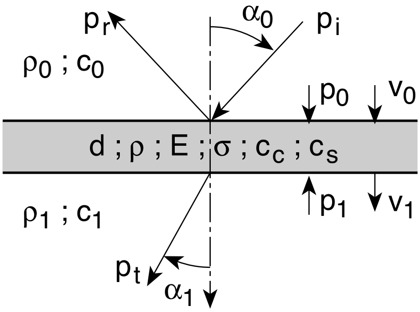


As above (i.e. transmissive fence with gap), but the incident wave is a plane wave

I.25 Infinite Plate Between Two Different Fluids

► *See also:* Alekseev/Dianov (1976)

An infinite plate of thickness d is placed between two fluids with index 0 on the side of incidence of a plane wave p_i, and index 1 on the side of the transmitted wave p_t.



The fluids are characterised by their densities ρ_0, ρ_1 and their sound velocities c_0, c_1.

The plate material has a density ρ, Young's modulus E, Poisson number σ, compressional wave speed c_c and shear wave speed c_s.

Sound pressures at the plate surfaces: $$p_0 = p_i + p_r \quad ; \quad p_1 = p_t, \tag{1}$$

v_0, v_1 are the corresponding plate surface velocities.

Definitions:

Normal components of the fluid wave impedances: $$Z_{0n} = \frac{\rho_0 c_0}{\cos\alpha_0} \; ; \; Z_{1n} = \frac{\rho_1 c_1}{\cos\alpha_1}. \tag{2}$$

Reflection factor: $$R = \frac{p_r}{p_i}. \tag{3}$$

Transmission factor: $$T = \frac{p_t}{p_i}. \tag{4}$$

Pressure ratio: $$K = \frac{p_t}{p_i + p_r} = \frac{p_1}{p_0} = \frac{T}{1+R}. \tag{5}$$

Plate input impedance: $$Z = \frac{p_0}{v_0} = Z_{0n}\frac{1+R}{1-R}. \tag{6}$$

Symmetrical impedance
(impedance for plate compression): $$Z_s = \frac{p_0 + p_1}{v_0 - v_1}. \tag{7}$$

Anti-symmetrical impedance
(impedance for plate bending): $$Z_a = \frac{p_0 - p_1}{v_0 + v_1}. \tag{8}$$

From the relation of the effective powers for a plate without losses $\mathrm{Re}\{p_0 v_0^*\} = \mathrm{Re}\{p_1 v_1^*\}$:

$$|K|^2 = Z_{1n} \cdot \mathrm{Re}\{1/Z\}. \tag{9}$$

In the special case $Z_{0n} \ll |Z|$ & $|Z_s| \gg |Z_a|$ & $|Z_s| \gg Z_{1n}$ is $K \cdot Z \approx Z_{1n}$.

If the medium on the side of incidence is nearly soft, i.e. Z_{0n} is negligible:

$$\begin{aligned} K(\alpha_1) &= \frac{Z_{1n}(Z_s - Z_a)}{2\,Z_s Z_a + Z_{1n}(Z_s + Z_a)} \approx D(\alpha_1)/2, \\ Z(\alpha_1) &= \frac{2\,Z_s Z_a + Z_{1n}(Z_s + Z_a)}{2\,Z_{1n} + Z_s + Z_a} \approx Z_{0n}\frac{1}{1 - R(\alpha_1)}. \end{aligned} \tag{10}$$

The symmetrical and anti-symmetrical impedances Z_s, Z_a follow from:

$$Z_s = -j\left[W_c \cdot \cot\left(\frac{a_1}{2}\right) + W_s \cdot \cot\left(\frac{b_1}{2}\right)\right] \; ; \; Z_a = +j\left[W_c \cdot \tan\left(\frac{a_1}{2}\right) + W_s \cdot \tan\left(\frac{b_1}{2}\right)\right] \tag{11}$$

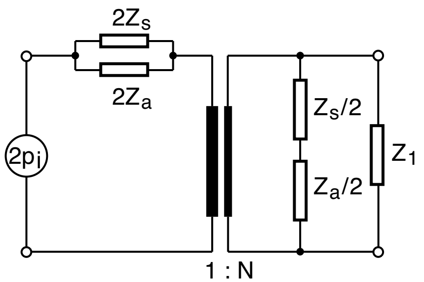
with

$$\begin{aligned} &W_c = \frac{\rho\, c_c}{\cos\alpha} \cdot \cos^2(2\beta) \quad ; \quad W_s = \frac{\rho\, c_s}{\cos\beta} \cdot \sin^2(2\beta) \quad ; \\ &\sin\alpha = \frac{c_c}{c_1} \cdot \sin\alpha_1 \quad ; \quad \sin\beta = \frac{c_s}{c_1} \cdot \sin\alpha_1 \quad ; \\ &a_1 = \frac{\omega d}{c_c} \cdot \cos\alpha \quad ; \quad b_1 = \frac{\omega d}{c_s} \cdot \cos\beta \quad ; \\ &c_c = \sqrt{\frac{E\,(1-\sigma)}{\rho\,(1+\sigma)\,(1-2\sigma)}} \quad ; \quad c_s = \sqrt{\frac{E}{2\rho\,(1+\sigma)}}, \end{aligned} \tag{12}$$

where α is the diffracted angle of the compressional wave in the half-infinite plate material and β is the diffracted angle of the shear wave in the half-infinite plate material.

The sound transmission can be represented by an equivalent circuit. The transformer has the transformer ratio:

$$N = \frac{Z_s + Z_a}{Z_s - Z_a}. \tag{13}$$



Special cases:

a) *Normal radiation* $\alpha_1 = 0$:

$$K(0) = \frac{1}{\cos\left(\frac{\omega d}{c_c}\right) + j\left(\frac{\rho\, c_c}{\rho_1 c_1}\right)\sin\left(\frac{\omega d}{c_c}\right)} \quad ; \quad N = \cos\left(\frac{\omega d}{c_c}\right). \tag{14}$$

b) *Coincidence of the incident wave with the anti-symmetrical wave in the free plate,* i.e. $Z_a = 0$:

$$K(\alpha_1) = 1 \quad ; \quad Z(\alpha_1) = \frac{Z_1 \cdot Z_s}{2Z_1 + Z_s} \quad ; \quad N = 1. \tag{15}$$

c) *Coincidence of the incident wave with the symmetrical wave in the free plate,* i.e. $Z_s = 0$:

$$K(\alpha_1) = -1 \quad ; \quad Z(\alpha_1) = \frac{Z_1 \cdot Z_a}{2Z_1 + Z_a} \quad ; \quad N = -1. \tag{16}$$

d) *Inpermeable plate,* i.e. $Z_s = Z_a$:

$$K(\alpha_1) = 0 \quad ; \quad Z(\alpha_1) = Z_s = Z_a \quad ; \quad N \to \infty. \tag{17}$$

e) *Incidence at first critical angle,* i.e. $Z_s \to \infty$:

$$K(\alpha_1) = \frac{Z_{1n}}{Z_{1n} + 2Z_a} \quad ; \quad Z(\alpha_1) = Z_{1n} + 2Z_a \quad ; \quad N = 1. \tag{18}$$

f) *Only near field on the receiver side,* i.e. $\sin\alpha_1 \to \infty$; $\cos\alpha_1 \to j\infty$:

$$K(\alpha_1) = 0 \quad ; \quad Z(\alpha_1) \to -2j\,\rho c_c \left(\frac{c_s}{c_c}\right)^2 \frac{c_c^2 - c_s^2}{c_c \cdot c_1} \sin\alpha_1 \to -j\,\infty. \tag{19}$$

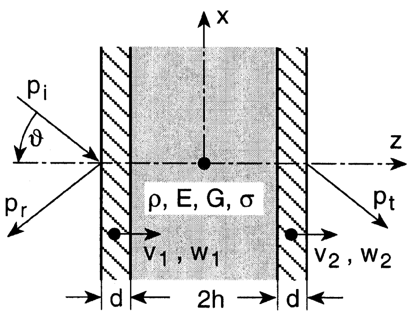
If, additionally, the plate is thin:

$$Z(\alpha_1) \to -16j\,\frac{c_s}{\omega d}\left[1 - \left(c_s/c_c\right)^2\right]. \tag{20}$$

I.26 Sandwich Plate with an Elastic Core

► *See also:* Beshenkov (1974)

An elastic layer of thickness 2h and ρ = density, E = Young's modulus, G = shear modulus, σ = Poisson ratio of its material when it is covered on both sides with identical thin sheets of thickness d and ρ_1 = density, E_1 = Young's modulus, σ_1 = Poisson ratio of the sheet material.



A plane wave p_i is incident at a polar angle ϑ. The moduli may be complex with loss factors η_E , η_G for the core, and η_{E1} for the cover sheets.

v_1, w_1 and v_2, w_2 are the velocities and elongations of the sheets; p_r is the reflected wave; p_t is the transmitted wave. The trace wave number and normal field impedance are:

$$k_\vartheta = k_0 \cdot \sin\vartheta \quad ; \quad Z_\vartheta = Z_0 / \cos\vartheta. \tag{1}$$

Decomposition in compressional (symmetrical, index s) and translational (anti-symmetrical, index a) parts:

$$\begin{aligned} v_s &= \frac{1}{2}(v_1 - v_2) \quad ; \quad v_a = \frac{1}{2}(v_1 + v_2), \\ w_s &= \frac{1}{2}(w_1 - w_2) \quad ; \quad w_a = \frac{1}{2}(w_1 + w_2), \quad ^{*)} \\ p_s &= \frac{1}{2}\left[(p_i + p_r)_{z=-(h+d)} + (p_t)_{z=+(h+d)}\right] \quad ; \quad p_a = \frac{1}{2}\left[(p_i + p_r)_{z=-(h+d)} - (p_t)_{z=+(h+d)}\right]. \end{aligned} \tag{2}$$

Impedances:

$$Z_s = \frac{p_s}{v_s} = Z'_s + j\,Z''_s \quad ; \quad Z_a = \frac{p_a}{v_a} = Z'_a + j\,Z''_a. \tag{3}$$

Transmission coefficient τ_ϑ:

$$\tau_\vartheta = Z_\vartheta^2 \frac{(Z'_s - Z'_a)^2 + (Z''_s - Z''_a)^2}{\left[(Z_\vartheta + Z'_s)^2 + Z''^2_s\right] \cdot \left[(Z_\vartheta + Z'_a)^2 + Z''^2_a\right]}. \tag{4}$$

The impedances Z_s, Z_a are evaluated from:

$$\begin{aligned} Z_s &= \frac{j}{2}\left[\frac{k_\vartheta^2}{\omega}\frac{A_1}{A_2} + \frac{a_7}{\omega} - \omega \cdot a_6\right] \quad ; \quad Z_a = \frac{j}{2}\left[\frac{k_\vartheta^2}{\omega}\frac{B_1}{B_2} - \omega \cdot c_3\right], \\ A_1 &= k_\vartheta^4\left(c_1 \cdot a_4 + a_1^2\right) - k_\vartheta^2\,(a_4 \cdot a_5 + 2\,a_1 \cdot a_3) + k_\vartheta^2\omega^2\,(c_2 \cdot a_4 + c_1 \cdot c_3 + 2\,a_1 \cdot a_2) \\ &\quad + \omega^4\left(c_2 \cdot c_3 + a_2^2\right) - \omega^2\,(c_3 \cdot a_5 + 2\,a_2 \cdot a_3) + a_3^2, \\ A_2 &= k_\vartheta^2 \cdot a_4 + \omega^2 c_3, \\ B_1 &= k_\vartheta^4\left(c_1 \cdot b_4 - b_1^2\right) - k_\vartheta^2\,(b_3 \cdot (b_4 + c_1 + 2\,b_1)) + k_\vartheta^2\omega^2\,(c_2 \cdot b_4 + c_1 \cdot b_5 - 2\,b_1 \cdot b_2) \\ &\quad + \omega^4\left(c_2 \cdot b_5 - b_2^2\right) - \omega^2\,(b_3 \cdot (b_5 + c_2 + 2\,b_2)), \\ B_2 &= k_\vartheta^2 \cdot b_4 + b_3 + \omega^2 b_5. \end{aligned} \tag{5}$$

The coefficients a_i, b_i, c_i are evaluated with the dilatation moduli D, D_1 of the core and cover sheets, respectively:

$$D = E\frac{1-\sigma}{(1+\sigma)(1-2\sigma)} \quad ; \quad D_1 = E_1\frac{1}{1-\sigma_1^2} \tag{6}$$

*) see Preface to the 2nd edition.

from the relations:

$$a_1 = D_1 \cdot d^2 \quad ; \quad a_2 = -\rho_1 \cdot d^2 \quad ; \quad a_3 = -2D\frac{\sigma}{1-\sigma},$$
$$a_4 = 2D_1 \cdot d + 2D \cdot h \quad ; \quad a_5 = \frac{2}{3}G \cdot h \quad ; \quad a_6 = -2\left(\rho_1 d + \frac{1}{3}\rho h\right) \quad ; \quad a_7 = -\frac{2D}{h}, \tag{7}$$

$$b_1 = -a_1 \cdot h \quad ; \quad b_2 = -a_2 \cdot h \quad ; \quad b_3 = -2Gh,$$
$$b_4 = -2\,h^2\left(E_1 d + \frac{1}{3}Eh\right) \quad ; \quad b_5 = -a_6 \cdot h^2,$$

$$c_1 = -\frac{2}{3}E_1 d^3 \quad ; \quad c_2 = \frac{2}{3}\rho_1 d^3 \quad ; \quad c_3 = -2\left(\rho h + \rho_1 d\right).$$

If the core is a viscous fluid with density ρ, sound speed c, and kinematic viscosity ν, then the equivalent elastic constants are:

$$D \to \rho c^2 \quad ; \quad G \to j\,\omega\,\nu \quad ; \quad \sigma \to \frac{1}{2}\left(1 - j\frac{\omega\,\nu}{\rho c^2}\right), \tag{8}$$

and the coefficients become (if the mass ρh of the core can be neglected compared to the surface mass density $m_1 = \rho_1 d$ of the cover sheet):

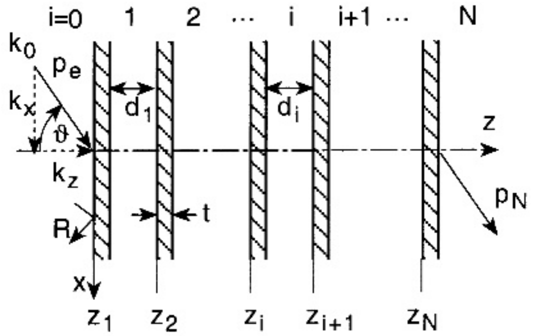
$$a_1 = D_1 \cdot d^2 \quad ; \quad a_2 = -\rho_1 \cdot d^2 \quad ; \quad a_3 = -2\,\rho c^2\left(1 - j\frac{\omega\,\nu}{\rho c^2}\right)\Big/\left(1 + j\frac{\omega\,\nu}{\rho c^2}\right),$$
$$a_4 = 2D_1 \cdot d + 2\rho c^2 \cdot h \quad ; \quad a_5 = \frac{2}{3}j\,\omega\nu \cdot h \quad ; \quad a_6 = -2m_1 \quad ; \quad a_7 = -2\frac{\rho c^2}{h},$$
$$b_1 = -a_1 \cdot h \quad ; \quad b_2 = -a_2 \cdot h \quad ; \quad b_3 = -2j\,\omega\nu\,h, \tag{9}$$
$$b_4 = -2D_1 \cdot h^2 - \frac{2}{3}\rho c^2 \cdot h^2 \quad ; \quad b_5 = -a_6 \cdot h^2,$$
$$c_1 = -\frac{2}{3}E_1 d^3 \quad ; \quad c_2 = \frac{2}{3}m_1 d^2 \quad ; \quad c_3 = -2m_1.$$

I.27 Wall of Multiple Sheets with Air Interspaces

▸ *See also:* Sharp/Beauchamps (1969)

N elastic plates $i = 1, \ldots, N$ with thicknesses t_i and possibly different materials are at mutual distances d_i.

A plane wave p_e is incident at a polar angle ϑ.



Incident wave p_e:

$$p_e(x, z) = p_x(x) \cdot e^{-j k_z z},$$
$$p_x(x) = P \cdot e^{-j k_x x} \quad ; \quad k_x = k_0 \cdot \sin \vartheta \quad ; \quad k_z = k_0 \cdot \cos \vartheta. \tag{1}$$

Sound pressure fields:

$$p_0 = p_x(x) \cdot \left(e^{-j k_z z} + R \cdot e^{+j k_z z}\right) \quad ; \quad z < 0,$$
$$\vdots$$
$$p_i = p_x(x) \cdot \left(A_i \cdot e^{-j k_z (z - z_i)} + B_i \cdot e^{+j k_z (z - z_i)}\right) \quad ; \quad z_i < z < z_{i+1}, \tag{2}$$
$$\vdots$$
$$p_N = p_x(x) \cdot T \cdot e^{-j k_z (z - z_N)} \quad ; \quad z > z_N,$$

where R is the front side reflection factor, T the (total) transmission factor, and R_i and T_i the reflection and transmission factors at the i-th (single) plate.

Boundary conditions:

$\underline{i = 1}$:

$$A_1 = T_1 + B_1 \cdot R_1 \quad ; \quad R = R_1 + B_1 \cdot T_1, \tag{3}$$

$\underline{i = 2, \ldots, N - 1}$:

$$A_i = T_i \cdot A_{i-1} \cdot e^{-j k_z d_{i-1}} + R_i \cdot B_i \quad ; \quad B_{i-1} \cdot e^{+j k_z d_{i-1}} = B_i \cdot T_i + R_i \cdot A_{i-1} \cdot e^{-j k_z d_{i-1}}, \tag{4}$$

$\underline{i = N}$:

$$T = T_N \cdot A_{N-1} \cdot e^{-j k_z d_{N-1}} \quad ; \quad B_{N-1} \cdot e^{+j k_z d_{N-1}} = R_N \cdot A_{N-1} \cdot e^{-j k_z d_{N-1}}. \tag{5}$$

From this a matrix equation follows:

$$\begin{Bmatrix} T \\ 0 \end{Bmatrix} = C \bullet \begin{Bmatrix} 1 \\ R \end{Bmatrix} \quad ; \quad C = \prod_{i=1}^{N} \frac{1}{T_i} \cdot C^{(i)} \quad ; \quad C^{(i)} = \begin{Bmatrix} c_{11}^{(i)} & c_{12}^{(i)} \\ c_{21}^{(i)} & c_{22}^{(i)} \end{Bmatrix} \tag{6}$$

with coefficients:

$\underline{i = 1}$:

$$c_{11}^{(1)} = T_1^2 - R_1^2 \quad ; \quad c_{12}^{(1)} = R_1 \quad ; \quad c_{21}^{(1)} = -R_1 \quad ; \quad c_{22}^{(1)} = 1, \tag{7}$$

$\underline{i = 2, \ldots, N-1}$:

$$c_{11}^{(i)} = (T_i^2 - R_i^2)e^{-j\,k_z d_{i-1}} \;;\quad c_{12}^{(i)} = R_i e^{+j\,k_z d_{i-1}} \;;\quad c_{21}^{(i)} = -R_i e^{-j\,k_z d_{i-1}} \;;\quad c_{22}^{(i)} = e^{+j\,k_z d_{i-1}}, \tag{8}$$

$\underline{i = N}$:

$$c_{11}^{(N)} = T_N^2 e^{+j\,k_z d_{N-1}} \quad ; \quad c_{12}^{(N)} = 0 \quad ; \quad c_{21}^{(N)} = -R_N e^{-j\,k_z d_{N-1}} \quad ; \quad c_{22}^{(N)} = e^{+j\,k_z d_{N-1}}. \tag{9}$$

The required single plate reflection and transmission factors R, T (index i is dropped) are:

$$R = -F \cdot \left[1 + Z_B \cdot Z_E \frac{\cos^2\vartheta}{4Z_0^2}\right] \quad ; \quad T = -F \cdot (Z_B + Z_E)\,\frac{\cos\vartheta}{2Z_0} \tag{10}$$

with the auxiliary quantity:

$$F = \frac{1}{\left(1 + Z_B \dfrac{\cos\vartheta}{2Z_0}\right)\left(1 - Z_E \dfrac{\cos\vartheta}{2Z_0}\right)} \tag{11}$$

and the bending-wave impedance Z_B and the longitudinal-wave impedance Z_E of the plate, which are, *for thin plates*:

$$Z_B = j\left(\omega m'' - \frac{B \cdot k_x^4}{\omega}\right) \quad ; \quad Z_E = j\,\frac{4\,m'' \cdot c_s^2}{\omega\,t^2(1-\sigma)}\;\frac{2k_x^2 - (1-\sigma)\cdot\omega^2/c_s^2}{k_x^2 - \omega^2/c_d^2}, \tag{12}$$

for thick plates:

$$\begin{aligned} Z_B &= -j\,\frac{8\,m'' c_s^4}{t\,\omega^3}\left[\beta\,k_x^2 \tanh(\beta t/2) - \left(k_x^2 - \omega^2/(2c_s^2)\right)\tanh(\alpha t/2)/\alpha\right], \\ Z_E &= -j\,\frac{8\,m'' c_s^4}{t\,\omega^3}\left[-\beta\,k_x^2 \coth(\beta t/2) + \left(k_x^2 - \omega^2/(2c_s^2)\right)\coth(\alpha t/2)/\alpha\right], \\ \alpha^2 &= k_x^2 - \omega^2/c_d^2 \quad ; \quad \beta^2 = k_x^2 - \omega^2/c_s^2, \end{aligned} \tag{13}$$

where $m'' = \rho_p t$ is the surface mass density, ρ_p the plate material density, E the Young's modulus, B the bending modulus, σ the Poisson ratio, and c_s, c_d the velocities of shear and dilatational waves:

$$B = \frac{E\,t^3}{12\,(1-\sigma^2)} \quad ; \quad c_s = \sqrt{\frac{E}{2\,\rho_p(1+\sigma)}} \quad ; \quad c_d = \sqrt{\frac{E\,(1-\sigma)}{\rho_p(1+\sigma)(1-2\sigma)}}. \tag{14}$$

The transmission loss with oblique incidence is $R_\vartheta = -10 \cdot \log|T|^2$; the transmission loss for diffuse incidence is:

$$R = -10 \cdot \log\tau \quad ; \quad \tau = \frac{\displaystyle\int_0^{\vartheta_{\lim}} |T|^2 \sin(2\vartheta)\,d\vartheta}{\displaystyle\int_0^{\vartheta_{\lim}} \sin(2\vartheta)\,d\vartheta}. \tag{15}$$

Special case: *double sheet*

$$T = \frac{T_1 T_2\, e^{-j\,k_z d}}{1 - R_1 R_2\, e^{-2j\,k_z d}}. \tag{16}$$

Special case: *triple sheet*

$$T = \frac{-T_1 T_3\, e^{+j\,k_z (d_1 + d_2)}}{R_1 R_3 \left\{ T_2 - \frac{1}{T_2} \left[\frac{e^{+2j\,k_z d_2}}{R_3} - R_2 \right] \left[\frac{e^{+2j\,k_z d_1}}{R_1} - R_2 \right] \right\}}. \tag{17}$$

References

Alekseev; Dianov: Sovj. Phys. Acoust. **22**, 181–183 (1976)

Beshenkov, S.N.: Sovj. Phys. Acoust. **29**, 115–117 (1974)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 7: The absorber barrier. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 8: Sound transmission through gaps and holes (openings) in a wall. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 9: The noise sluice. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 10: Sound transmission through infinite, simple plates. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 11: Sound transmission through infinite, multiple plates. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 12: Infinite sandwich plate. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 14: Sound transmission through a finite wall in a duct. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 15: Simple plate in a niche of the baffle wall. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 16: A strip of a plate in an infinite baffle wall. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 17: Sound transmission through a finite plate combined with an absorber layer. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 18: Multilayer finite walls. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 19: Flanking sound transmission through suspended ceilings. Hirzel, Stuttgart (1998)

Mechel, F.P.: Schallabsorber, Vol. III, Ch. 24: Fences in a room. Hirzel, Stuttgart (1998)

Sharp; Beauchamps: J. Sound Vibr. **9**, 383 (1969)