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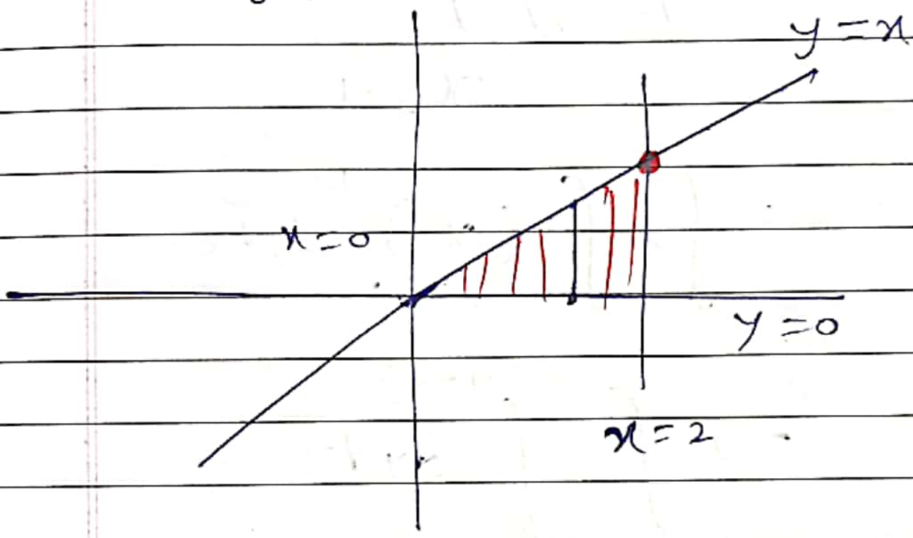
Q.10 (ii)

$$I = \int_0^2 \int_0^x y\,dy\,dx$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=x} y\,dy\,dx.$$



$$= \int_{x=0}^{2} \left[\frac{y^2}{2}\right]_{y=0}^{y=x} dx$$

$$= \frac{1}{2}\int_{x=0}^{2} x^2\,dx$$

$$= \frac{1}{2}\left(\frac{x^3}{3}\right)_0^2$$

$$= \frac{1}{\cancel{2}}\,\frac{\cancel{8}\,4}{3}$$

$$= \boxed{\frac{4}{3}}$$

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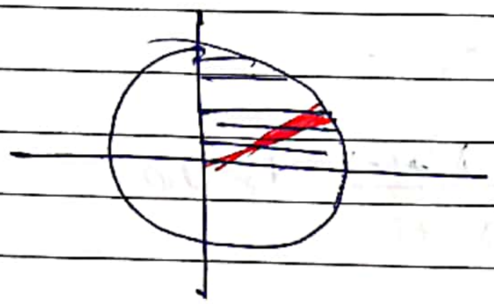
Q.(10) (iv) $I = \int_{0}^{1} \int_{x=0}^{x=\sqrt{1-y^2}} e^{-(x^2+y^2)} \, dx \, dy$



$$= \int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{1} e^{-r^2} r \, dr \, d\theta$$

$$= -\frac{1}{2} \int_{\theta=0}^{\frac{\pi}{2}} \left[e^{-r^2}\right]_{r=0}^{1} d\theta$$

$$= -\frac{1}{2} \int_{\theta=0}^{\frac{\pi}{2}} \left[e^{-1} - e^{-0}\right] d\theta$$

$$= -\frac{1}{2} \left[e^{-1} - 1\right] \frac{\pi}{2}$$

$$= -\frac{\pi}{4} \left[e^{-1} - 1\right]$$

$$= \frac{\pi}{4} \left[1 - e^{-1}\right]$$

Q.(10)(vi) $I = \int_{y=0}^{y=2} \int_{x=-\sqrt{1-(y-1)^2}}^{x=0} xy^2 \, dx \, dy$

$x = -\sqrt{1-(y-1)^2}$

$\cancel{x^2 = 1^2}$

$x^2 + (y-1)^2 = 1$

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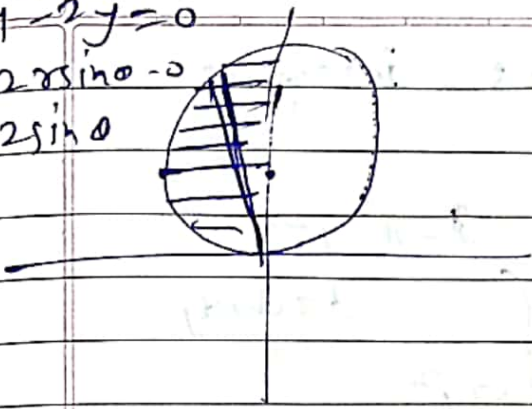
$x^2+y^2-2y=0$

$r^2-2r\sin\theta=0$

$r=2\sin\theta$



$$I=\int_{y=0}^{y=2}\int$$

$$=\int_{\theta=\frac{\pi}{2}}^{\pi}\int_{r=0}^{2\sin\theta} r\cos\theta\, r^2\sin^2\theta\, r\,dr\,d\theta$$

$$=\int_{\theta=\frac{\pi}{2}}^{\pi}\left[\frac{r^5}{5}\right]_{r=0}^{2\sin\theta}\sin^2\theta\cos\theta\,d\theta$$

$$=\int_{\theta=\frac{\pi}{2}}^{\pi}\frac{1}{5}\,32\sin^7\theta\cos\theta\,d\theta$$

$$=\frac{32}{5}\int_{\theta=\frac{\pi}{2}}^{\pi}\sin^7\theta\cos\theta\,d\theta$$

Put $\sin\theta=t$

at $\theta=\frac{\pi}{2},\ t=1$

$\theta=\pi,\ t=0$

$$=\frac{32}{5}\int_{t=1}^{t=0}t^7\,dt$$

$$=\frac{32}{5}\left[\frac{t^8}{8}\right]_1^0$$

$$=\frac{32}{5}\left[-\frac{1}{8}\right]=\boxed{-\frac{4}{5}}$$

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(11) Evaluate the triple integrals

(i)
$$I = \int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz\,dx\,dy$$

$$= \int_0^{\sqrt{2}} \int_0^{3y} \left[8 - x^2 - y^2 - x^2 - 3y^2\right] dx\,dy$$

$$= \int_0^{\sqrt{2}} \int_0^{3y} \left[8 - 2x^2 - 4y^2\right] dx\,dy$$

$$= \int_0^{\sqrt{2}} \left[8x - \frac{2x^3}{3} - 4xy^2\right]_{x=0}^{x=3y} dy$$

$$= \int_0^{\sqrt{2}} 8(3y) - 2\frac{(3y)^3}{3} - 4(3y)y^2 \, dy$$

$$= \int_0^{\sqrt{2}} 24y - \frac{2 \times 27 y^3}{3} - 12y^3 \, dy$$

$$= \left[24\frac{y^2}{2} - 18\frac{y^4}{4} - 12\frac{y^4}{4}\right]_0^{\sqrt{2}}$$

$$= \left[12y^2 - \frac{9}{2}y^4 - 3y^4\right]_0^{\sqrt{2}}$$

$$= 12(2) - \frac{9}{2}(4) - 3(4) \quad = 24 - 18 - 12 = -6$$

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Q.(11)(ii)

$$I = \int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\,dy\,dx$$

$$= \int_0^1 \int_0^{3-3x} (3-3x-y)\,dy\,dx$$

$$= \int_0^1 \left[3y - 3xy - \frac{y^2}{2}\right]_{y=0}^{y=3-3x} dx$$

$$= \int_0^1 3(3-3x) - 3x(3-3x) - \frac{(3-3x)^2}{2}\,dx$$

$$= \int_0^1 9 - 9x - 9x + 9x^2 - \frac{(3-3x)^2}{2}\,dx$$

$$= \int_0^1 9 - 18x + 9x^2 - \frac{(9-18x+9x^2)}{2}\,dx$$

$$= \int_0^1 \frac{18 - 36x + 18x^2 - 9 + 18x - 9x^2}{2}\,dx$$

$$= \frac{1}{2}\int_0^1 9 - 18x + 9x^2\,dx$$

$$= \frac{1}{2}\left[9x - 18\frac{x^2}{2} + 9\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2}\left[\cancel{9} - \cancel{9} + 3\right] = \frac{3}{2}$$

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Q. (iii) $I = \int_0^7 \int_0^2 \int_{p=0}^{p=\sqrt{4-q^2}} \frac{q}{r+1}\, dp\, dq\, dr$

$= \int_0^7 \int_0^2 \left[p\right]_{p=0}^{p=\sqrt{4-q^2}} \frac{q\, dq\, dr}{r+1}$

$= \int_0^7 \int_0^2 \sqrt{4-q^2}\; q\, dq\, \frac{1}{r+1}\, dr$

$= \int_0^7$ Put $4-q^2 = t$

$-2q\, dq = dt$

$q\, dq = \frac{dt}{-2}$

at $q = 0,\ t = 4$

$q = 2,\ t = 0$

$= \int_0^7 \int_4^0 \sqrt{t} \left(-\frac{dt}{2}\right) \frac{1}{r+1}\, dr$

$= \frac{1}{2} \int_0^7 \int_0^4 t^{1/2}\, dt\, \frac{1}{r+1}\, dr$

$= \frac{1}{\cancel{2}} \left[\log(r+1)\right]_0^7 \left[\frac{t^{3/2}}{3/\cancel{2}}\right]_0^4$

$= \frac{1}{3} \log 8 \left[4^{3/2}\right]$

$= \frac{1}{3} \log 8\, (2)^3 = \frac{8}{3} \log 2^3 = 8 \log 2$

(12)

(i) find the volume bounded by $z=0$, $z=1-y$, $y=x^2$, $y=1$, $x=-1$, $x=1$

→ Volume $= \int\int\int_{x=-1}^{x=1} dx\,dy\,dz$

$= \int\int (x)_{-1}^{1}\, dy\,dz$

$= 2\int\int_{y=1}^{y=x^2} dy\,dz$

Volume $= \int\int\int_{z=0}^{z=1-y} dx\,dy\,dz$

$= \int\int [z]_{z=0}^{z=1-y}\, dy\,dz$

$= \int\int (1-y)\, dy\,dx$

$= \int\int_{y=1}^{y=x^2} (1-y)\, dy\,dx$

$$= \int \left[y - \frac{y^2}{2} \right]_{y=1}^{y=x^2} dx$$

$$= \int x^2 - \frac{x^4}{2} - \left(1 - \frac{1}{2}\right) dx$$

$$= \int x^2 - \frac{x^4}{2} - \frac{1}{2}\, dx$$

$$= \left[\frac{x^3}{3} - \frac{1}{2}\cdot\frac{x^5}{5} - \frac{1}{2}x \right]_{x=-1}^{1}$$

$$= \left(\frac{1}{3} - \frac{1}{10} - \frac{1}{2}\right) - \left(-\frac{1}{3} - \frac{(-1)}{10} + \frac{1}{2}\right)$$

$$= \frac{1}{3} - \frac{1}{10} - \frac{1}{2} + \frac{1}{3} - \frac{1}{10} - \frac{1}{2}$$

$$= \frac{2}{3} - \frac{2}{10} - 1$$

$$= \frac{20-6}{30} - 1$$

$$= \frac{14}{30} - 1$$

$$= \frac{7}{15} - 1$$

$$= \frac{-8}{15}$$

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