



WINTER- 17 EXAMINATION

Subject Name: Theory of Structures

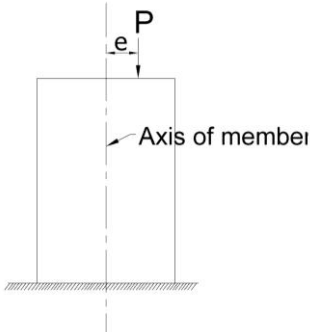
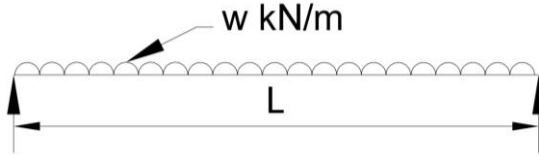
Model Answer

Subject Code:


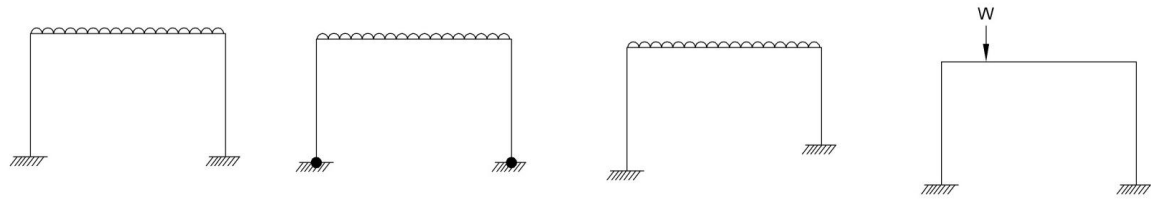
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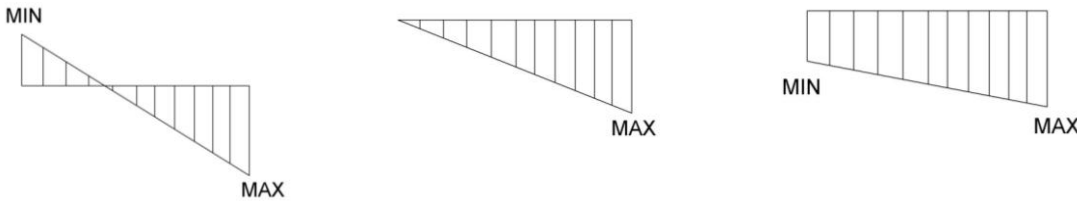
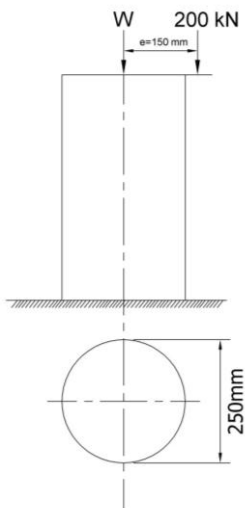
Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

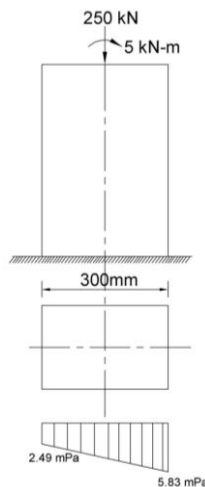
Q. No.	Sub Q. N.	Answer	Marking Scheme
Q.1	(A)i) Ans	<p>Define "Eccentric load with neat sketch. Eccentric loading: It is defined as load whose line of action does not coincide with the longitudinal axis of the member.</p> 	<p>01 Mark</p> <p>01 Mark</p>
Q.1	(A)ii) Ans	<p>Write the values of maximum slope and deflection in case of simply supported beam with u.d.L. over the entire span in terms of EI.</p>  <p>Maximum slope = $(w \times L^3) / (24 \times EI)$ at supports. -----</p> <p>Maximum deflection = $(5 \times w \times L^4) / (384 \times EI)$ at midspan. -----</p>	<p>01 Mark</p> <p>01 Mark</p>
Q.1	(A)iii) Ans	<p>Write the differential equation for slope and deflection and state terms used in the equation. In the theory of simple bending, curvature of beam is expressed as – $1 / R = d^2y/dx^2$ For bending equation, $1 / R = M / EI$</p>	

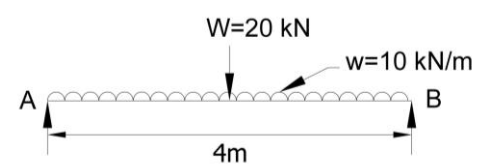
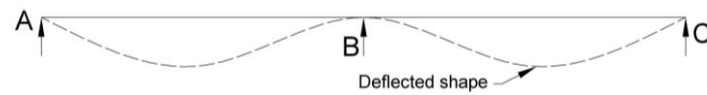
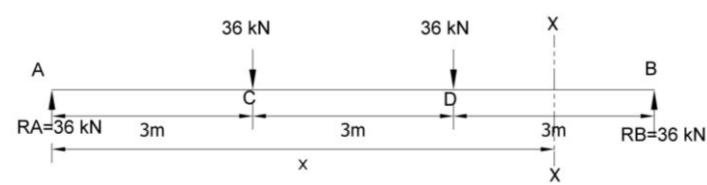


		$M / EI = d^2y/dx^2$ $EI(d^2y/dx^2) = M_x$ --- differential equation Where, E = Modulus of elasticity. I = Moment of inertia of C/S. M_x = Bending moment at the section X-X in beam.	01 Mark
			01 Mark
Q.1	(A)iv) Ans	<p>State values of maximum slope and deflection for cantilever beam of span L carrying a point load at free end with meaning of each term.</p>  <p>Maximum slope = $(WL^2) / 2EI$ at free end -----</p> <p>Maximum deflection = $(WL^3) / 3EI$ at free end -----</p>	01 Mark 01 Mark
Q.1	(A)v) Ans	<p>State any two disadvantage of fixed beam.</p> <ol style="list-style-type: none"> A little sinking of one support over the other induces additional moment at each end. Due to end fixity, temperature stresses are induced due to variation in temperature. Extra care has to be taken to achieve perfect fixity at ends. Frequent fluctuations in loading (moving loads) are likely to disturb end fixity. Practically it is difficult to produce 100% fixity. 	Any Two 01 Mark Each
Q.1	(A)vi) Ans	<p>With sketch state the different types of portal frame.</p> <ol style="list-style-type: none"> Symmetrical portal frame (Non sway type) Unsymmetrical portal frame (Sway type) 	01 Mark each
Q.1	(A)vii) Ans	<p>Define carry over moment and carry over factor.</p> <ol style="list-style-type: none"> Carry over moment: It is defined as the moment induced at the far fixed end of beam by the action of the moment applied at the near simply supported or hinged end. Carry over factor: It is the ratio of moment induced at far end to the moment applied at near end without displacing it. 	01 Mark 01 Mark
Q.1	(A)viii) Ans	<p>List out different types of roof trusses any four.</p> <ol style="list-style-type: none"> King post truss Queen post truss Simple fink truss Compound fink truss Fan truss Pratt truss Howe truss North light truss 	Any Four ½ Mark each
Q.1	(B)i) Ans	<p>Define core of a section and state middle third rule.</p> <ol style="list-style-type: none"> Core of a section: It is defined as the region or area within which if load is applied, produces only compressive resultant stress. Middle third rule: In case of rectangular cross section, if the load is applied at location along the middle third part of both mutually perpendicular axes then the stresses produced are wholly of compressive nature. 	02 Marks 02 Marks

Q.1	(B)ii)	<p>Draw resultant stress diagram for $\sigma_0 < \sigma_b$, $\sigma_0 = \sigma_b$, $\sigma_0 > \sigma_b$.</p> <p>i) $\sigma_0 < \sigma_b$ ii) $\sigma_0 = \sigma_b$ iii) $\sigma_0 > \sigma_b$</p>  <p>Where, σ_0 = Direct stress and σ_b = Bending stress</p>	<p>01 Mark each for dia.</p> <p>01 Mark</p>
Q.1	(B)iii)	<p>a) State the assumptions in the analysis of frame.</p> <ol style="list-style-type: none"> 1. All joints in frame are pinned or hinged. 2. Loads are applied at joints only. 3. Self-weight of members of frame is neglected. 4. Only axial forces (tensile and compressive) are induced in the member. <p>b) Define redundant frame and state its condition.</p> <p>Redundant frame: It is the frame which cannot be analysed internally using basic equations of equilibrium ($\sum M_A = 0$, $\sum F_x = 0$ and $\sum F_y = 0$).</p> <p>Condition: $m > (2j - 3)$ where, m = Number of members and j = No. of joints.</p>	<p>½ mark for each</p> <p>01 Mark</p> <p>01 Mark</p>
Q.2	a)	<p>A solid circular column of diameter 250 mm carries an axial load 'W' kN and a load of 200 kN at an eccentricity of 150mm. Calculate minimum value of 'W' so as to avoid the tensile stresses at base.</p> <p>$A = (\pi \times 250^2) / 4 = 49087.38 \text{ mm}^2$</p> <p>$M = P \times e = 200 \times 10^3 \times 150 = 3 \times 10^7 \text{ N-mm}$</p> <p>$I = (\pi \times 250^4) / 64 = 1.917 \times 10^8 \text{ mm}^4$</p> <p>$y = D / 2 = 250 / 2 = 125 \text{ mm}$.</p> <p>$\sigma_0 = (W + P) / A = (W + 200 \times 10^3) / 49087.38$</p> <p>$\sigma_b = (M \times y) / I = 3 \times 10^7 \times 125 / 1.917 \times 10^8 = 19.562 \text{ N/mm}^2$</p> <p>For no tension, $\sigma_0 = \sigma_b$</p> <p>$(W + 200 \times 10^3) / 49087.38 = 19.562$</p> <p>$W = 760238.3 \text{ N} = 760.24 \text{ kN}$.</p> 	<p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p>

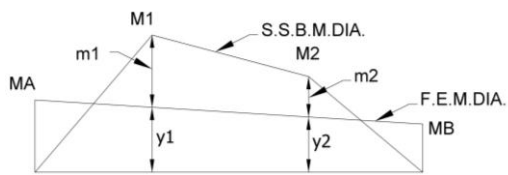
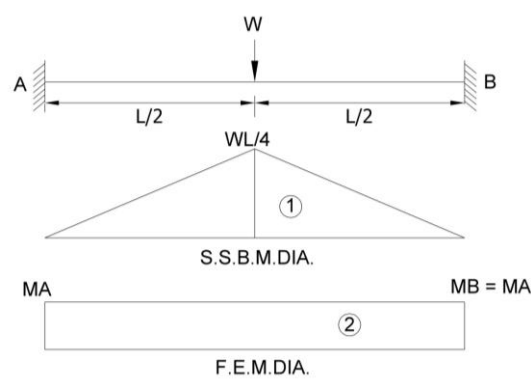


Q.2	b)	<p>A rectangular column 300 mm wide and 200 mm thick carries an axial load of 250 kN and a clockwise moment of 5 kN m in plane bisecting 200 mm side, calculate resultant stresses induced at the base.</p> <p>Ans</p> <p>Axial load = P 250 kN = 250×10^3 N $B = 200$ mm, $d = 300$ mm $A = 200 \times 300 = 60000 \text{ mm}^2$ $I = 200 \times 300^3 / 12 = 4.5 \times 10^8 \text{ mm}^4$ $y = d / 2 = 300 / 2 = 150$ mm $M = 5 \text{ kN-m} = 5 \times 10^6 \text{ N-mm}$ $\sigma_0 = P / A = 250 \times 10^3 / 60000 = 4.16 \text{ N/mm}^2$</p> <p>$\sigma_b = (M \times y) / I = 5 \times 10^6 \times 150 / 4.5 \times 10^8$ $= 1.67 \text{ N/mm}^2$</p> <p>$\sigma_{\max} = \sigma_0 + \sigma_b = 4.16 + 1.67 = 5.83 \text{ N/mm}^2$ $\sigma_{\min} = \sigma_0 - \sigma_b = 4.16 - 1.67 = 2.49 \text{ N/mm}^2$</p>	 <p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p> <p>½ Mark</p> <p>½ Mark</p>
Q.2	c)	<p>A masonry wall 10 m high, 3 m wide and 1.5m thick is subjected to a wind pressure of 1.2 kN/m². Find maximum and minimum intensity induced on the base if the unit weight of masonry is 22kN/m³.</p> <p>Ans</p> <p>Area at base of wall = $3 \times 1.5 = 4.5 \text{ m}^2$ Height of wall (h) = 10 m, Unit weight of material (σ) = 22 kN/m³ Weight of wall (W) = $22 \times 4.5 \times 10 = 990$ kN.</p> <p>$\sigma_0 = \sigma h$ OR $\sigma_0 = W / A$ $= 22 \times 10 = 220 \text{ kN/m}^2$ $= 990 / 4.5 = 220 \text{ kN/m}^2$ -----</p> <p>$I = 3 \times 1.5^3 / 12 = 0.84375 \text{ m}^4$ $y = 1.5 / 2 = 0.75 \text{ m}$ -----</p> <p>Wind force (P) = Wind pressure \times b \times h $= 1.2 \times 3 \times 10 = 36 \text{ kN}$ -----</p> <p>-</p> <p>Moment @ base (M) = P \times h/2 $= 36 \times 10 / 2 = 180 \text{ kN-m}$ -----</p> <p>$\sigma_b = (M \times y) / I = 180 \times 0.75 / 0.84375 = 160 \text{ kN/m}^2$ -----</p> <p>$\sigma_{\max} = \sigma_0 + \sigma_b = 220 + 160 = 380 \text{ kN/m}^2$ -----</p> <p>$\sigma_{\min} = \sigma_0 - \sigma_b = 220 - 160 = 60 \text{ kN/m}^2$ -----</p>	<p>01 Mark</p> <p>½ Mark</p> <p>½ Mark</p> <p>½ Mark</p> <p>½ Mark</p> <p>½ Mark</p> <p>½ Mark</p> <p>½ Mark</p>
Q.2	d)	<p>A wooden cantilever beam of span 2.5 m has a cross section 130 mm wide and 240 mm deep. A load of 6 kN is acting at free end, calculate the deflection and slope at free end take $E = 1 \times 10^5 \text{ N/mm}^2$.</p> <p>Ans</p> <p>$b = 130$ mm, $d = 240$ mm, Span = 2.5 m. = 2500 mm $E = 1 \times 10^5 \text{ N/mm}^2$ $I = 130 \times 240^3 / 12 = 1.4976 \times 10^8 \text{ mm}^4$ $W = 6 \text{ kN} = 6000 \text{ N}$</p> <p>Slope at free end = $WL^2 / 2EI$ $= 6000 \times 2500^2 / (2 \times 1 \times 10^5 \times 1.4976 \times 10^8) = 1.252 \times 10^{-3} \text{ rad.}$</p> <p>Deflection at free end = $WL^3 / 3EI$ $= 6000 \times 2500^3 / (3 \times 1 \times 10^5 \times 1.4976 \times 10^8) = 2.087 \text{ mm.}$</p>	<p>01 Mark</p> <p>½ Mark</p> <p>01 Mark</p> <p>½ Mark</p> <p>01 Mark</p>

Q.2	<p>e)</p> <p>Ans</p>	<p>A simply supported beam of span 4 m carries a central point load of 20 kN and u.d.L. of 10 kN/m over entire span. Find maximum slope and maximum deflection of the beam $I_{xx}=2 \times 10^8 \text{ mm}^4$ $E= 2 \times 10^5 \text{ N/mm}^2$.</p>  <p> $EI = 2 \times 10^8 \times 2 \times 10^5 = 4 \times 10^{13} \text{ N-mm}^2$ Maximum slope = $(W \times L^2 / 16EI)_{P.L.} + (w \times L^3 / 24EI)_{U.D.L.}$ at supports ----- $= (20000 \times 4000^2) / (16 \times 4 \times 10^{13}) + (10 \times 4000^3) / (24 \times 4 \times 10^{13})$. $= 5 \times 10^{-4} + 6.66 \times 10^{-4}$ $= 1.166 \times 10^{-3} \text{ rad.}$ ----- Maximum deflection = $(W \times L^3 / 48EI)_{P.L.} + (5 \times w \times L^4 / 384EI)_{U.D.L.}$ at mid-span ----- $= (20000 \times 4000^3 / 48 \times 4 \times 10^{13}) + (5 \times 10 \times 4000^4 / 384 \times 4 \times 10^{13})$ $= 0.66 + 0.83 = 1.49 \text{ mm}$ ----- </p>	<p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p>
Q.2	<p>f)</p> <p>Ans</p>	<p>State the effect of continuity on the continuous beam. Explain with sketch.</p>  <p>Effects of continuity are as follows.</p> <ol style="list-style-type: none"> Produces support moment of hogging nature. Reduces bending moment along the span. Reduces deflection and increases load carrying capacity. Sagging moment occurs at mid span. 	<p>02 Marks</p> <p>02 Marks</p>
Q.3	<p>a)</p> <p>Ans</p>	<p>A simply supported beam of span 9 m carries two point loads of equal magnitude 36 kN at 3 m from both ends. Calculate values of integration constant and write Macaulay's slope and deflection equation.</p>  <p>Calculation of Reactions:</p> $\Sigma M_A = 0$ $36 \times 3 + 36 \times 6 - R_B \times 9 = 0$ $R_B = (108 + 216) / 9$ $= 36 \text{ kN.}$ $R_A = 72 - 36 = 36 \text{ kN.}$ <p style="text-align: center;">OR</p> <p style="text-align: right;">Due to symmetry,</p> $R_A = R_B = 72 / 2$ $= 36 \text{ kN.}$ <p>Taking section X-X at distance 'x' from A.</p> $M_x = 36 \times x - 36(x - 3) - 36(x - 6)$ $EI d^2y/dx^2 = - M_x$ $= - 36 \times x + 36(x - 3) + 36(x - 6)$ <p>Integrating</p> $EI dy/dx = (- 36 \times x^2)/2 + [36(x - 3)^2]/2 + [36(x - 6)^2]/2 + C_1 \text{ ----- } 1$	<p>01 Mark</p>



		<p>Integrating</p> $Ely = (-18x^3)/3 + [18(x-3)^3]/3 + [18(x-6)^3]/3 + C_1x + C_2 \text{ ----- 2}$ <p>At $x = 0$; $y = 0$ in eqⁿ. 2</p> $0 = 0 + C_2$ $C_2 = 0$ <p>At $x = 9$; $y = 0$ in eqⁿ. 2</p> $0 = (-18 \times 9^3)/3 + [18(9-3)^3]/3 + [18(9-6)^3]/3 + C_1 \times 9 + 0$ $C_1 = 324$ <p>Hence $C_1 = 324$ and $C_2 = 0$</p> <p>Slope equation:</p> $dy/dx = 1/EI[(-18 \times x^2) + 18(x-3)^2 + 18(x-6)^2 + 324]$ <p>Deflection equation:</p> $y = 1/EI[(-6 \times x^3) + 6(x-3)^3 + 6(x-6)^3] + 324x$	01 Mark
			01 Mark
			01 Mark
Q.3	b)	A simply supported beam of 6 m span carries a point load of 60 kN at 2m from left support.	
	Ans	Calculate deflection below point load in terms of EI use Macaulay's method.	
		<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>OR</p> </div> <div style="text-align: center;"> <p>Reactions:</p> $\Sigma M_A = 0$ $= 60 \times 2 - R_B \times 6$ $R_B = 120 / 6 = 20 \text{ kN.}$ $R_A = 60 - 20 = 40 \text{ kN.}$ </div> </div>	01 Mark
		<div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <p>Taking section X-X at distance 'x' from A</p> $M_x = 40 \times x - 60(x-2)$ $EI d^2y/dx^2 = -M_x$ $= -40x + 60(x-2)$ <p>Integrating</p> $EI dy/dx = (-40 \times x^2)/2 + [60(x-2)^2]/2 + C_1$ <p>Integrating</p> $Ely = (-20 \times x^3)/3 + [30(x-2)^3]/3 + C_1x + C_2$ <p>At $x = 0$; $y = 0$ in Ely eqⁿ.</p> $0 = 0 + C_2$ $C_2 = 0$ <p>At $x = 6$; $y = 0$ in Ely eqⁿ.</p> $0 = (-20 \times 6^3)/3 + [10(9-2)^3] + C_1 \times 6 + 0$ $C_1 = 133.33$ <p>Hence $C_1 = 133.33$ and $C_2 = 0$</p> </div> <div style="width: 48%;"> <p>Taking section X-X at distance 'x' from B</p> $M_x = 20 \times x - 60(x-4)$ $EI d^2y/dx^2 = -M_x$ $= -20x + 60(x-4)$ <p>Integrating</p> $EI dy/dx = (-20 \times x^2)/2 + [60(x-4)^2]/2 + C_1$ <p>Integrating</p> $Ely = (-10 \times x^3)/3 + [30(x-4)^3]/3 + C_1x + C_2$ <p>At $x = 0$; $y = 0$ in Ely eqⁿ.</p> $0 = 0 + C_2$ $C_2 = 0$ <p>At $x = 6$; $y = 0$ in Ely eqⁿ.</p> $0 = (-10 \times 6^3)/3 + [10(9-4)^3] + C_1 \times 6 + 0$ $C_1 = 106.67$ <p>Hence $C_1 = 106.67$ and $C_2 = 0$</p> </div> </div>	01 Mark

		<p>Deflection equation- $y = (1/EI)[(- 20 \times x^3)/3 + 10(x - 2)^3 + 133.33x]$ For deflection under load Put $x = 2$ in eqⁿ. $y_c = (1/EI)[(- 20 \times 2^3)/3 + 0 + 133.33 \times 2]$ $= 213.33 / EI$</p>	<p>Deflection equation- $y = (1/EI)[(- 10 \times x^3)/3 + 10(x - 4)^3 + 106.67x]$ For deflection under load Put $x = 4$ in eqⁿ. $y_c = (1/EI)[(- 10 \times 4^3)/3 + 0 + 106.67 \times 4]$ $= 213.33 / EI$</p>	01 Mark
				01 Mark
Q.3	c) Ans	<p>State how B. M. is find out for a fixed beam using super position theorem. Explain it with sketch.</p> <p>After calculating fixed end moments and simply supported bending moments, draw fixed end moment diagram and superimpose simply supported bending moment diagram over it as shown in fig below.</p> <div></div> <p>Let net bending moment at 1 and 2 are m_1 and m_2 respectively. Calculate y_1 and y_2 by interpolation. Then $m_1 = M_1 - y_1$ and $m_2 = M_2 - y_2$</p>		02 Marks
				02 Marks
Q.3	d) Ans	<p>Using first principle find fixed end moment for a fixed beam carrying point load at mid span.</p> <div></div> <p>Due to symmetry, $M_A = M_B$ Area of S. S. B. M. Dia. = $a_1 = 0.5 \times L \times WL/4 = WL^2 / 8$ Area of F. E. M. Dia. = $M_A \times L$ Area of simply supported bending moment diagram = Area of fixed end moment diagram $a_1 = a_2$ $WL^2 / 8 = M_A \times L$ Hence $M_A = WL / 8$ And $M_B = WL / 8$</p>		01 Mark
				01 Mark
				01 Mark
				01 Mark
Q.3	e) Ans	<p>Explain imperfect and perfect frame in detail.</p> <p>Perfect frame: It is the simple frame in which number of joints (j) and number of members (m) satisfies the equation $m = 2j - 3$. Such frames are internally determinate i.e. can be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$).</p>		02 Marks

Imperfect frame: It is the simple frame in which number of joints (j) and number of members (m) does not satisfy the equation $m = 2j - 3$. Such frames are internally indeterminate/redundant or deficient.

If $m > 2j - 3$; then frame is called as indeterminate/redundant frame and cannot be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$).

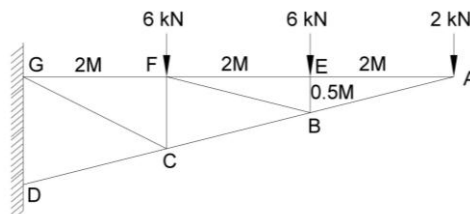
If $m < 2j - 3$; then frame is called as deficient frame and it is unstable frame.

02 Marks

Q.3

f)

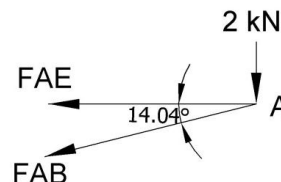
Determine the forces along with nature in the members AB, AE, EB and EF for frame subjected to a load as shown is Fig. using method of joints.



Ans

Consider joint A.

$$\theta = \tan^{-1}(0.5/2) = 14.04^\circ$$



Assuming F_{AE} and F_{AB} both tensile.

$$\Sigma F_y = 0; 2 + F_{AB} \sin 14.04 = 0$$

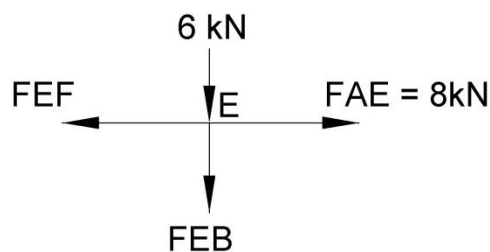
$$F_{AB} = -2 / \sin 14.04 = -8.24 \text{ i.e. } 8.24 \text{ kN (Compressive)}$$

$$\Sigma F_x = 0; -F_{AE} - F_{AB} \cos 14.04 = 0$$

$$-F_{AE} - (-8.24) \cos 14.04 = 0$$

$$F_{AE} = 8.0 \text{ kN (Tensile)}$$

Consider joint E.



Assuming F_{EF} and F_{EB} both tensile.

$$\Sigma F_x = 0; -F_{EF} + 8.0 = 0$$

$$F_{EF} = 8.0 \text{ kN (Tensile)}$$

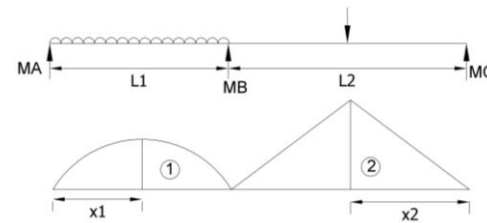
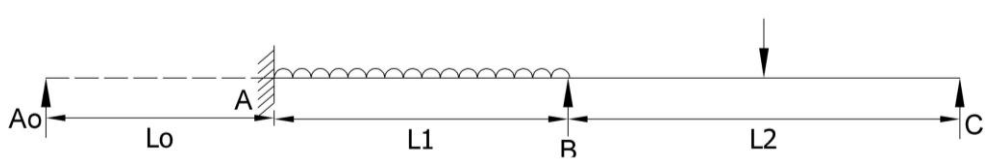
$$\Sigma F_y = 0; -6.0 - F_{EB} = 0$$

$$F_{EB} = -6.0 \text{ i.e. } 6.0 \text{ kN (Compressive)}$$

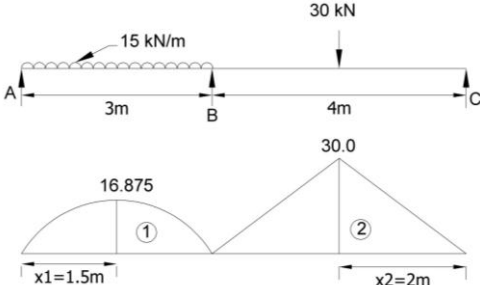
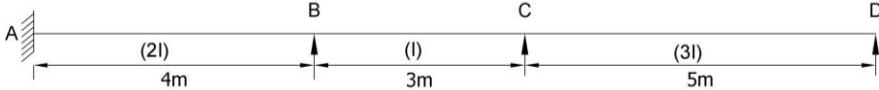
02 Marks

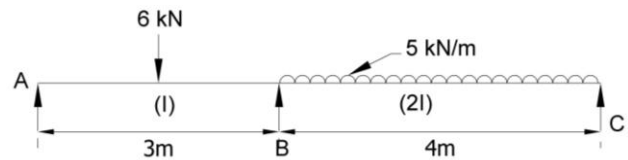
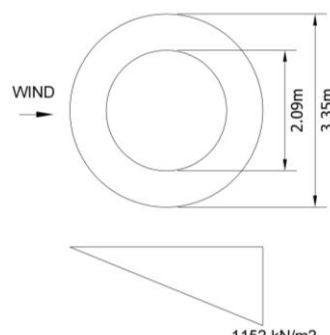
02 Marks

Member	Force	Nature
AB	8.24 kN	Compressive
AE	8.0 kN	Tensile
EB	6.0 kN	Compressive
EF	8.0 kN	Tensile

<p>Q.4</p>	<p>a)</p> <p>Ans.</p>	<p>State Clapeyron's theorem and also write the Clapeyron's three moment theorem for beam with different moment of inertia giving meaning of each term.</p> <p>Clapeyrons theorem: For two span continuous beam having uniform moment of inertia supported at A, B, and C and subjected to any external loading, the support moments M_A, M_B and M_C at the supports A, B and C respectively are given by the relation,</p> $M_A \times L_1 + 2M_B(L_1 + L_2) + M_C \times L_2 = - [(6 \times a_1 \times x_1/L_1) + (6 \times a_2 \times x_2/L_2)]$  <p>If moment of inertia is not constant then Clapeyrons theorem can be stated in form of following equation.</p> $M_A \times (L_1/I_1) + 2M_B[(L_1/I_1) + (L_2/I_2)] + M_C \times (L_2/I_2) = - [(6 \times a_1 \times x_1/L_1 I_1) + (6 \times a_2 \times x_2/L_2 I_2)]$ <p>Where,</p> <p>L_1 & L_2 are length of span AB & BC resp.</p> <p>I_1 & I_2 are Moment of inertia of span AB & BC resp.</p> <p>a_1 & a_2 are area of simply supported BMD of span AB & BC resp.</p> <p>x_1 & x_2 are distances of centroid of simply supported BMD from A & C resp.</p>	<p>02 Marks</p> <p>01 Mark</p> <p>01 Mark</p>
<p>Q.4</p>	<p>b)</p> <p>Ans</p>	<p>Explain the concept of imaginary zero span in case of Clapeyron's theorem.</p> <p>When the ends of continuous beam are fixed, then an imaginary span is considered to the left or right of the fixed support as the case may be and Clapeyrons theorem is applied to the imaginary span and its adjescent span as per regular procedure.</p> <p>If left end is fixed then consider imaginary span left of this support and If right end is fixed then consider imaginary span on right side of that support.</p> <p>Clapeyrons theorem is applied as below.</p>  <p>A_0-A is imaginary span left to fixed end A.</p> <p>For span A_0-A and AB</p> $M_{A_0} \times L_0 + 2M_A(L_0 + L_1) + M_B \times L_1 = - [(6 \times a_0 \times x_0/L_0) + (6 \times a_1 \times x_1/L_1)]$ $0 + 2M_A(L_1) + M_B \times L_1 = - [0 + (6 \times a_1 \times x_1/L_1)]$ <p>Where M_{A_0}, L_0 and x_0 are terms related to imaginary span.</p>	<p>02 Marks</p> <p>01 Mark</p> <p>01 Mark</p>
<p>Q.4</p>	<p>c)</p>	<p>A beam ABC is supported at A, B and C span AB and BC are of lengths 3 m and 4 m respectively. AB carries a u.d.L. of 15 kN/m over entire span and BC carries central point load of 30 kN. Calculate support moment at B using three moment theorem.</p>	<p>01 Mark</p>



	Ans	<div></div> <div><div>$M_1 = 15 \times 3^2/8 = 16.875 \text{ kN-m}$</div><div>$M_2 = 30 \times 4/4 = 30.0 \text{ kN-m}$</div></div> <div>Using three moment theorem;</div> <div>$M_A \times l_1 + 2M_B (l_1 + l_2) + M_C \times l_2 = - [(6 \times a_1 \times x_1/l_1) + 6 \times a_2 \times x_2/l_2]$</div> <div>$M_A = M_C = 0$ (End simple supports)</div> <div>$2M_B(3 + 4) = - [(6 \times 33.75 \times 1.5/3) + 6 \times 60.0 \times 2.0/4]$</div> <div>$14.0M_B = -101.25 - 180$</div> <div>$M_B = -281.25/14 = -20.09 \text{ i.e. } 20.09 \text{ kN-m Hogging}$</div>	01 Mark 01 Mark 01 Mark																					
Q.4	d) Ans	<div>Define stiffness of beam and state stiffness factor for beam with far end fixed and simply supported end.</div> <div>Stiffness of beam: The moment required to produce unit rotation at the near end is called as stiffness of beam.</div> <div>Stiffness factor for beam with far end fixed = $4EI/L$</div> <div>Stiffness factor for beam with far end simply supported = $3EI/L$</div> <div>EI = Flexural rigidity of beam.</div> <div>L = Length of beam.</div>	02 Marks 01 Mark 01 Mark																					
Q.4	e) Ans	<div>Determine distribution factors at continuity for a continuous beam ABCD which is fixed at A and supported at B, C and D. Take $AB = 4 \text{ m}$, $BC = 3 \text{ m}$ and $CD = 5 \text{ m}$ if M.I. for the spans is $I_{AB} = 2I$, $I_{BC} = I$, $I_{CD} = 3I$.</div> <div></div> <table><thead><tr><th>Joint</th><th>Member</th><th>Stiffness (k)</th><th>Σk</th><th>D.F. = $k/\Sigma k$</th></tr></thead><tbody><tr><td rowspan="2">B</td><td>BA</td><td>$4 \times 2EI/4 = 2EI$</td><td rowspan="2">$3EI$</td><td>$2EI/3EI = 0.67$</td></tr><tr><td>BC</td><td>$3 \times EI/3 = EI$</td><td>$EI/3EI = 0.33$</td></tr><tr><td rowspan="2">C</td><td>CB</td><td>$3 \times EI/3 = EI$</td><td rowspan="2">$2.8EI$</td><td>$EI/2.8EI = 0.36$</td></tr><tr><td>CD</td><td>$3 \times 3EI/5 = 1.8EI$</td><td>$1.8EI/2.8EI = 0.64$</td></tr></tbody></table>	Joint	Member	Stiffness (k)	Σk	D.F. = $k/\Sigma k$	B	BA	$4 \times 2EI/4 = 2EI$	$3EI$	$2EI/3EI = 0.67$	BC	$3 \times EI/3 = EI$	$EI/3EI = 0.33$	C	CB	$3 \times EI/3 = EI$	$2.8EI$	$EI/2.8EI = 0.36$	CD	$3 \times 3EI/5 = 1.8EI$	$1.8EI/2.8EI = 0.64$	01 Mark for each
Joint	Member	Stiffness (k)	Σk	D.F. = $k/\Sigma k$																				
B	BA	$4 \times 2EI/4 = 2EI$	$3EI$	$2EI/3EI = 0.67$																				
	BC	$3 \times EI/3 = EI$		$EI/3EI = 0.33$																				
C	CB	$3 \times EI/3 = EI$	$2.8EI$	$EI/2.8EI = 0.36$																				
	CD	$3 \times 3EI/5 = 1.8EI$		$1.8EI/2.8EI = 0.64$																				

Q.4	f)	<p>Calculate support moments by moment distribution method for given continuous as shown in fig.</p>  <p> $M_{AB} = -6 \times 3/8 = -2.25 \text{ kN-m}$ $M_{BA} = 6 \times 3/8 = 2.25 \text{ kN-m}$ $M_{BC} = -5 \times 4^2/12 = -6.67 \text{ kN-m}$ $M_{CB} = 5 \times 4^2/12 = 6.67 \text{ kN-m}$ </p> <table border="1"> <tr> <th>Joint</th><th>Member</th><th>Stiffness (k)</th><th>Σk</th><th>D.F. = $k/\Sigma k$</th></tr> <tr> <td rowspan="2">B</td><td>BA</td><td>$3 \times EI/3 = EI$</td><td rowspan="2">$2.5EI$</td><td>$EI/2.5EI = 0.4$</td></tr> <tr> <td>BC</td><td>$3 \times 2EI/4 = 1.5EI$</td><td>$1.5EI/2.5EI = 0.6$</td></tr> </table> <table border="1"> <tr> <th>Joint</th><th>A</th><th>B</th><th>C</th></tr> <tr> <td>Members</td><td>AB</td><td>BA BC</td><td>CB</td></tr> <tr> <td>Distⁿ. factor</td><td>1.0</td><td>0.4 0.6</td><td>1.0</td></tr> <tr> <td>F.E.M.</td><td>- 2.25</td><td>2.25 - 6.67</td><td>6.67</td></tr> <tr> <td>Balancing</td><td>2.25</td><td>1.768 2.652</td><td>- 6.67</td></tr> <tr> <td>Carry over</td><td></td><td>1.125 - 3.33</td><td></td></tr> <tr> <td>Balancing</td><td></td><td>0.882 1.323</td><td></td></tr> <tr> <td>Final moments</td><td>0.0</td><td>6.025 - 6.025</td><td>0.0</td></tr> </table> <p>Support moment at B = $M_B = 6.025 \text{ kN-m}$ (Hogging)</p>	Joint	Member	Stiffness (k)	Σk	D.F. = $k/\Sigma k$	B	BA	$3 \times EI/3 = EI$	$2.5EI$	$EI/2.5EI = 0.4$	BC	$3 \times 2EI/4 = 1.5EI$	$1.5EI/2.5EI = 0.6$	Joint	A	B	C	Members	AB	BA BC	CB	Dist ⁿ . factor	1.0	0.4 0.6	1.0	F.E.M.	- 2.25	2.25 - 6.67	6.67	Balancing	2.25	1.768 2.652	- 6.67	Carry over		1.125 - 3.33		Balancing		0.882 1.323		Final moments	0.0	6.025 - 6.025	0.0	<p>01 Mark</p> <p>02 Marks</p> <p>01 Mark</p>
Joint	Member	Stiffness (k)	Σk	D.F. = $k/\Sigma k$																																												
B	BA	$3 \times EI/3 = EI$	$2.5EI$	$EI/2.5EI = 0.4$																																												
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Q.5	a)	<p>A circular chimney has external diameter 60% more than internal diameter. The height of chimney is 32 m and is subjected to a horizontal wind pressure of 1.75 kN/m^2. Find out the diameter of the chimney so as to avoid tension at the base of chimney and also draw stress distribution diagram. Unit wt. of chimney material is 18 kN/m^3 and $C = 0.6$.</p> <p>Data: External diameter (D) = $1.6 \times$ internal diameter (d)</p> <p>Height of chimney (h) = 32 m</p> <p>Horizontal wind pressure (p) = 1.75 kN/m^2</p> <p>Unit weight of material (σ) = 18 kN/m^3 $C = 0.6$</p> <p>$\sigma_d = \sigma h = 18 \times 32 = 576 \text{ kN/m}^2$ -----</p> <p>Horizontal wind force (P) = $p \times h \times D \times C$ $= 1.75 \times 32 \times 1.6d \times 0.6$ $= 53.76d$ -----</p> <p>Moment about base (M) = $P \times h/2$ $= 53.76d \times 32/2$ $= 860.16d$ -----</p> <p> $I = (\pi / 64)(D^4 - d^4)$ $= (\pi / 64)[(1.6d)^4 - d^4]$ $= 0.273d^4$ </p> <p>$y_{\max} = 0.8d$ $\sigma_b = M \times y_{\max}/I$ $= 860.16d \times 0.8d / 0.273d^4$ $= 2520.6 / d^2$ -----</p> <p>For no tension;</p> 	<p>01 Mark</p> <p>01 Mark</p> <p>01 Mark</p> <p>01 Mark for stress dia.</p> <p>01 Mark</p>																																													



$$\begin{aligned} \bar{\sigma}_d &= \bar{\sigma}_b \\ 576 &= 2520.6 / d^2 \\ d^2 &= 4.38 \\ d &= 2.092 \text{ m} \\ D &= 1.6 \times 2.092 = 3.35 \text{ m} \\ \bar{\sigma}_{\max} &= 2 \bar{\sigma}_d = 2 \times 576 = 1152 \text{ kN/m}^2 \end{aligned}$$

01 Mark

01 Mark

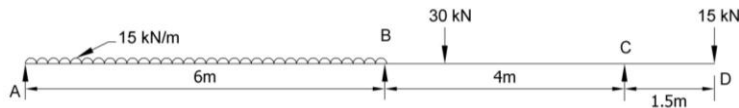
01 Mark

Q.5

b)

Ans

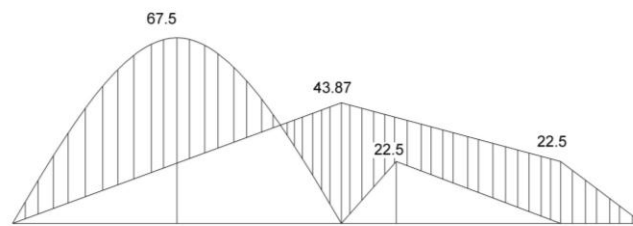
A beam ABCD is supported at A, B and C span CD is having overhang AB = 6 m BC = 4 m and CD = 1.5 m span AB carries UDL of 15 kN/m over entire span and BC carries point load of 30 kN at 1 m from support B and a point load of 15 kN acts at free end at D. Determine support moments using moment distribution method and draw BMD.



$$\begin{aligned} M_{AB} &= -15 \times 6^2 / 12 = -45.0 \text{ kN-m} & M_{BA} &= 15 \times 6^2 / 12 = 45.0 \text{ kN-m} \\ M_{BC} &= -30 \times 1 \times 3^2 / 4^2 = -16.875 \text{ kN-m} & M_{CB} &= 30 \times 1^2 \times 3 / 4^2 = 5.625 \text{ kN-m} \\ M_{CD} &= -15 \times 1.5 = -22.5 \text{ kN-m} \end{aligned}$$

Joint	Member	Stiffness (k)	Σk	D.F. = $k / \Sigma k$
B	BA	$3 \times EI / 6 = 0.5EI$	1.25EI	$0.5EI / 1.25EI = 0.4$
	BC	$3 \times EI / 4 = 0.75EI$		$0.75EI / 1.25EI = 0.6$

Joint	A		B		C
Members	AB	BA	B	CB	CD
Dist ⁿ . factor	1.0	0.4	0.6	1.0	0.0
F.E.M.	-45.0	45.0	-16.875	5.625	-22.5
Balancing	45.30	-11.25	-16.875	16.875	0.0
Carry over		22.5	8.44		
Balancing		-12.38	-18.56		
Final moments	0.0	43.87	-43.87	22.5	-22.5



B. M. D.

02 marks

02 Marks

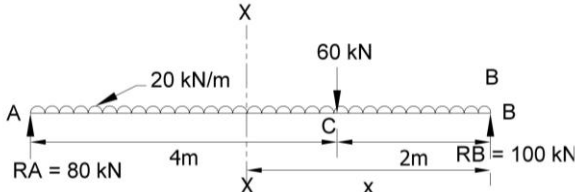
02 Marks

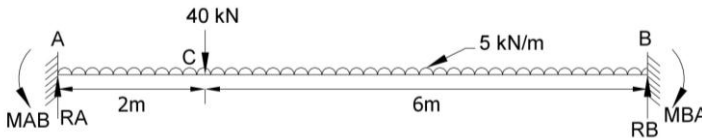
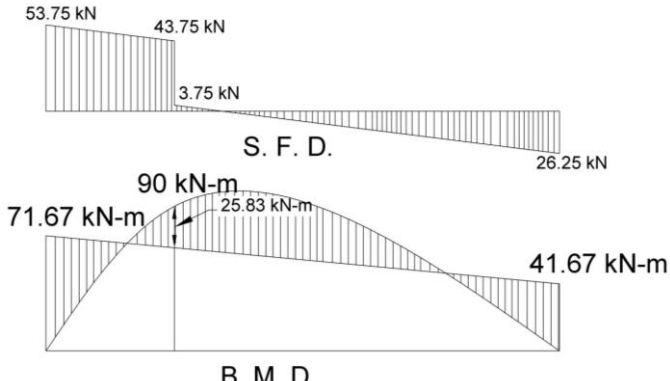
02 Marks

Q.5	c)	<p>Using method of section find forces in members BC, BE and EF and EC for truss shown in Fig.State nature of forces tabulate results.</p>	
Ans		<div data-bbox="461 226 1148 556"> </div> <p>Reactions:</p> $\sum M_A = 0 = 50 \times 3 - 10 \times 3 - R_D \times 9$ $R_D = 13.33 \text{ kN}$ $R_{AV} = 50 - 13.33 = 36.67 \text{ kN}$ $\sum F_V = 0$ $R_{AH} - 10 = 0$ $R_{AH} = 10 \text{ kN}$ <p>Taking section along EF, EC and BC Assuming all forces Tensile</p> <div data-bbox="647 867 1053 1178"> </div> <p>Taking moment @ C;</p> $- 13.33 \times 3 - 10 \times 10 - F_{EF} \times 3 = 0$ $F_{EF} = -23.33 \text{ i.e. } 23.33 \text{ kN (Compressive)}$ <p>Taking moment @ E;</p> $F_{CB} \times 3 - 13.33 \times 6 = 0$ $F_{CB} = 26.67 \text{ kN (Tensile)}$ $\sum F_V = 0 = 13.33 + F_{CE} \sin 45$ $F_{CE} = -18.85 \text{ i.e. } 18.85 \text{ kN (Compressive)}$ <p>Taking section along EF, EC, EB and BA</p> <div data-bbox="633 1507 1148 1782"> </div> $\sum F_V = 0 = 13.33 - 18.85 \sin 45 + F_{BE}$ $F_{BE} = 0$	01 Mark
			01 Mark
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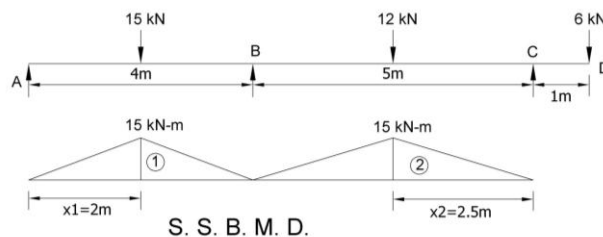


			Member	Force	Nature		
			BC	26.67 kN	Tensile		
			BE	0	--		
			EF	23.33 kN	Compressive		
			EC	18.85 kN	Compressive		02 Marks

Q.6	a)	A simply supported beam of 6 m span carries an u.d.l. of 20 kN/m over entire beam and a point load of 60 kN at 2 m from right hand support using Macaulay's method, locate the point of maximum deflection and find its value in terms of EI.	
	Ans	 <p>Reactions:</p> $\sum M_A = 0$ $60 \times 4 + 20 \times 6 \times 3 - R_B \times 6 = 0$ $R_B = (240 + 360) / 6$ $= 100 \text{ kN.}$ $R_A = 20 \times 6 + 60 - 100 = 80 \text{ kN.}$ <p>Taking section X-X at distance 'x' from B.</p> $M_x = 100 \times x - 60(x - 2) - 20 \times x^2/2$ $EI d^2y/dx^2 = -M_x$ $= -100 \times x + 60(x - 2) + 20 \times x^2/2$ <p>Integrating</p> $EI dy/dx = (-100 \times x^2/2) + [60(x - 2)^2/2] + 10 \times x^3/3 + C_1 \text{ ----- 1}$ <p>Integrating</p> $EI y = (-50 \times x^3/3) + [30(x - 2)^3/3] + 10 \times x^4/12 + C_1 x + C_2 \text{ ----- 2}$ <p>At $x = 0$; $y = 0$ in eqⁿ. 2</p> $0 = 0 + C_2$ $C_2 = 0$ <p>At $x = 6$; $y = 0$ in eqⁿ. 2</p> $0 = (-50 \times 6^3/3) + [30(6 - 2)^3/3] + 10 \times 6^4/12 + C_1 \times 6 + C_2$ $C_1 = 313.33$ <p>Hence $C_1 = 313.33$ and $C_2 = 0$</p> <p>Slope equation:</p> $dy/dx = 1/EI(-50 \times x^2) + [30(x - 2)^2] + 10 \times x^3/3 + 313.33 \text{ ----- I}$ <p>Deflection equation:</p> $y = 1/EI(-50 \times x^3/3) + [10(x - 2)^3] + 10 \times x^4/12 + 313.33x \text{ ----- II}$ <p>Deflection is maximum where slope changes the sign i.e. slope = 0</p> <p>Maximum deflection will be in between A and C</p> <p>Hence, $6 > x > 2$</p> <p>Equating Equⁿ I with zero.</p> $0 = 1/EI(-50 \times x^2) + [30(x - 2)^2] + 10 \times x^3/3 + 313.33$ $= -50 \times x^2 + 30x^2 - 120x + 120 + 10 \times x^3/3 + 313.33$	01 Mark 01 Mark 01 Mark 01 Mark 01 Mark 01 Mark

		$0 = -20x^2 - 120x + 3.33x^3 + 433.33$ <p>Solving this equation by trial and error method.</p> $x = 2.89 \text{ m}$ <p>Hence deflection is maximum at distance 2.89 m from B</p> <p>For y_{\max}, put $x = 2.89$ in equⁿ II</p> $y_{\max} = \frac{1}{EI}(-50 \times 2.89^3/3) + [10(2.89 - 2)^3] + 10 \times 2.89^4/12 + 313.33 \times 2.89$ $= 568.4 / EI$	01 Mark
			01 Mark
Q.6	b)	<p>A fixed beam of span 8 m carries 5 kN/m udl over entire length along with a point load of 40 kN at 2m from left hand support. Find net BM at point load and draw BMD and SFD.</p> 	
	Ans	$M_{AB} = (40 \times 2 \times 6^2 / 8^2) + (5 \times 8^2 / 12)$ $= 71.67 \text{ kN-m}$ $M_{BA} = (40 \times 2^2 \times 6 / 8^2) + (5 \times 8^2 / 12)$ $= 41.67 \text{ kN-m}$ <p>Reactions:</p> $\Sigma M_A = 0$ $40 \times 2 + 5 \times 8 \times 4 + 41.67 - 71.67 - R_B \times 8 = 0$ $R_B = (80 + 160 - 30) / 8$ $= 26.25 \text{ kN.}$ $R_A = 5 \times 8 + 40 - 26.25 = 53.75 \text{ kN.}$ <p>Bending moment at point load</p> $M_C = -71.67 + 53.75 \times 2 - 5 \times 2 \times 1$ $= 25.83 \text{ kN-m}$ <p>Shear force calculations:</p> <p>At B = -26.25 kN</p> <p>At C, just right = -26.25 + 5 × 6 = 3.75 kN</p> <p>At C, just left = -26.25 + 5 × 6 + 40 = 43.75 kN</p> <p>At D = 53.75 kN</p> 	01 Mark
			01 Mark
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			02 Mark
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			01 mark
Q.6	c)	<p>A beam ABCD is supported at A, B and C, CD being overhang AB = 4 m, BC = 5 m and CD = 1m AB and BC carries a central point of 15kN and 12kN respectively and a point load Of 6 kN at D. Calculate support moments using three moment theorem and draw SFD and BMD giving net BM.</p>	

Ans



01 Mark

$$M_1 = 15 \times 4/4 = 15.0 \text{ kN-m}$$

$$a_1 = 0.5 \times 4 \times 15 = 30.0$$

$$x_1 = 4/2 = 2.0 \text{ m}$$

$$M_2 = 12 \times 5/4 = 15.0 \text{ kN-m}$$

$$a_2 = 0.5 \times 5 \times 15 = 37.5$$

$$x_2 = 5/2 = 2.5 \text{ m}$$

Using three moment theorem;

$$M_A \times l_1 + 2M_B (l_1 + l_2) + M_C \times l_2 = - [(6 \times a_1 \times x_1/l_1) + 6 \times a_2 \times x_2/l_2]$$

$$M_A = (\text{End simple supports})$$

$$M_C = -6 \times 1 = -6.0 \text{ kN-m}$$

$$2M_B(4 + 5) - 6 \times 5 = - [(6 \times 30.0 \times 2.0/4) + 6 \times 37.5 \times 2.5/5]$$

$$18.0M_B = -90.0 - 112.5 + 30$$

$$M_B = -172.5/18 = -9.583 \text{ i.e. } 9.583 \text{ kN-m Hogging}$$

Reactions:

$$R_A = (15 \times 2 - 9.583) / 4 = 5.1 \text{ kN.}$$

$$R_C = (12 \times 2.5 + 6 \times 6 - 9.583) / 5 = 11.28 \text{ kN.}$$

$$R_B = 15 + 12 + 6 - 11.28 - 5.1 = 16.62 \text{ kN.}$$

Net bending moments:

$$\text{Under } 15 \text{ kN load} = 15 - (9.583/2) = 10.21 \text{ kN-m}$$

$$\text{Under } 12 \text{ kN load} = 15 - [(9.583 + 6)/2] = 7.21 \text{ kN-m.}$$

Shear force calculations:

$$\text{At D} = 6 \text{ kN}$$

$$\text{At C, right} = 6.0 \text{ kN}$$

$$\text{At C, left} = 6.0 - 11.28 = -5.28 \text{ kN}$$

$$\text{At 12 kN load, right} = -5.28 \text{ kN}$$

$$\text{At 12 kN load, left} = -5.28 + 12 = 6.72 \text{ kN}$$

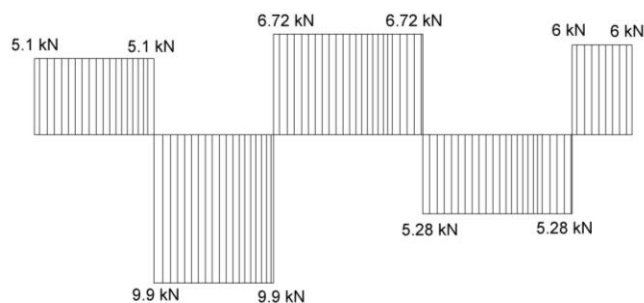
$$\text{At B, right} = 6.72 \text{ kN}$$

$$\text{At B, left} = 6.72 - 16.62 = -9.9 \text{ kN}$$

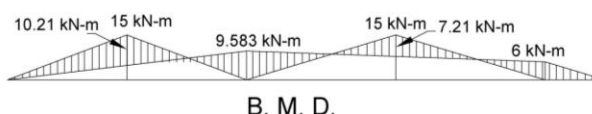
$$\text{At 15 kN load, right} = -9.9 \text{ kN}$$

$$\text{At 15 kN load, left} = -9.9 + 15 = 5.1 \text{ kN}$$

$$\text{At A} = 5.1 \text{ kN}$$



01 Mark



01 Mark