

Subject& Code: BMS (17105)

MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

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(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

Summer - 2016 Examination Model Answer

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		Important Instructions to the Examiners:		
		1) The answers should be examined by key words and not as		
		word-to-word as given in the model answer scheme.		
		2) The model answer and the answer written by candidate may		
		vary but the examiner may try to assess the understanding		
		level of the candidate.		
		3) The language errors such as grammatical, spelling errors		
		should not be given more importance. (Not applicable for		
		subject English and Communication Skills.)		
		4) While assessing figures, examiner may give credit for		
		principal components indicated in the figure. The figures		
		drawn by the candidate and those in the model answer may		
		vary. The examiner may give credit for any equivalent		
		figure drawn.		
		5) Credits may be given step wise for numerical problems. In		
		some cases, the assumed constant values may vary and there		
		may be some difference in the candidate's answers and the		
		model answer.		
		6) In case of some questions credit may be given by judgment		
		on part of examiner of relevant answer based on candidate's		
		understanding.		
		7) For programming language papers, credit may be given to		
		any other program based on equivalent concept.		



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Que.	Sub.	Model Answers	Marks	Total
No. 1.	Que.	Attempt any <u>TEN</u> of the following:	1,141118	Marks 20
1.	a)	Evaluate $\begin{vmatrix} -2 & 1 & 3 \\ 7 & -5 & 8 \\ 1 & 0 & 4 \end{vmatrix}$		20
	Ans.	$\begin{vmatrix} 7 & -5 & 8 \\ 1 & 0 & 4 \end{vmatrix}$ $= -2(-20-0)-1(28-8)+3(0+5)$ $= 35$	1 1	02
	b)	Solve: $\begin{vmatrix} 1 & -2 & 4 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{vmatrix} = \begin{vmatrix} 3 & 6 \\ -2 & -4 \end{vmatrix}$		
	Ans.	$1(9x-6x^2)+2(9-4x^2)+4(6-4x)=-12+12$	1/2	
		$9x - 6x^2 + 18 - 8x^2 + 24 - 16x = 0$	1/2	
		$-14x^2 - 7x + 42 = 0$	1/2	
		$2x^2 + x - 6 = 0$,-	
		(2x-3)(x+2) = 0 $x = \frac{3}{2} \text{ or } x = -2$	1/2	02
	c)	If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 7 \\ 1 & 9 \end{bmatrix}$, find $A + B$ and $A - B$		
	Ans	$A + B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 1 & 9 \end{bmatrix}$		
		$= \begin{bmatrix} 5 & 10 \\ 5 & 14 \end{bmatrix}$	1	
		$A - B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ 1 & 9 \end{bmatrix}$ $- \begin{bmatrix} -1 & -4 \end{bmatrix}$		00
		$= \begin{bmatrix} -1 & -4 \\ 3 & -4 \end{bmatrix}$	1	02
	d)	If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ find AB		



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	Ans	$AB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ $= \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix}$ $= \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$	1	02
	e)	If $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$ find adj A		
		Matrix of minors $=\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$	1	
		Cofactor Matrix $=\begin{bmatrix} 1 & -2 \\ -5 & 6 \end{bmatrix}$	1/2	
		$adjA = \begin{bmatrix} 1 & -5 \\ -2 & 6 \end{bmatrix}$	1/2	02
	f) Ans	Resolve into partial fraction $\frac{1}{x^2 + 5x + 6}$ $\frac{1}{x^2 + 5x + 6} = \frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ $\therefore 1 = A(x+3) + B(x+2)$	1/2	
		Put $x = -2$ $A = 1$	1/2	
		Put $x = -3$ B = -1	1/2	
		$\therefore \frac{1}{(x+2)(x+3)} = \frac{1}{x+2} + \frac{-1}{x+3}$	1/2	02
	g) Ans	Prove that $\sin 2A = 2\sin A \cos A$ $\sin 2A$		
		$=\sin(A+A)$	1/2	
		$= \sin A \cos A + \cos A \sin A$ $= 2 \sin A \cos A$	1	
			1/2	02



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Que.	Sub.	Madal Americans	Martin	Total
No.	Que.	Model Answers	Marks	Marks
1.	h) Ans	Define compound angle. If A and B are any two angle then $A + B$ or $A - B$ are called as compound angle.	02	02
	i)	Find the principal values of $\tan^{-1}(\sqrt{3})$		
	Ans	Let $\theta = \tan^{-1}\left(\sqrt{3}\right)$		
		$\therefore \tan \theta = \sqrt{3}$	1	
		$\therefore \theta = \frac{\pi}{3}$	1	02
	j)	Find tan 75 ⁰		
	Ans	$\tan 75^0 = \tan \left(45^0 + 30^0 \right)$		
		$=\frac{\tan 45^{0} + \tan 30^{0}}{1 - \tan 45^{0} \tan 30^{0}}$	1	
		$=\frac{1+\frac{1}{\sqrt{3}}}{1-(1)\left(\frac{1}{\sqrt{3}}\right)}$	1/2	
		$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$	1/2	02
	k)	Find $\sin 3\alpha$, if $\sin \alpha = 0.4$		
	Ans	$\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$		
	Alis	$=3(0.4)-4(0.4)^3$	1	
		= 0.944	1/2	
			1/2	02
	1)	If the straight line $3y+4px+8=0$ and $3px-9y+10=0$ are perpendicular to each other ,find the value of p .		

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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
1.	Ans	$m_1 = \frac{-4p}{3}, m_2 = \frac{-3p}{-9}$	1	
		$\therefore m_1 \cdot m_2 = -1$ $\therefore \frac{-4p}{3} \times \frac{-3p}{-9} = -1$ $\therefore p^2 = \frac{9}{4}$	1/2	
		$\therefore p = \frac{3}{2} \text{ or } p = -\frac{3}{2}$	1/2	02
2.		Attempt any <u>FOUR</u> of the following:		16
	a)	Solve by using determinant method:		
		x+y+z-6=0 , $2x+y-2z+2=0$, $x+y-3z+6=0$		
	Ans	$\therefore x + y + z = 6$, $2x + y - 2z = -2$, $x + y - 3z = -6$		
		$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ 1 & 1 & -3 \end{vmatrix} = 1(-3+2)-1(-6+2)+1(2-1)=4$	1	
		$D_x = \begin{vmatrix} 6 & 1 & 1 \\ -2 & 1 & -2 \\ -6 & 1 & -3 \end{vmatrix} = 6(-3+2)-1(6-12)+1(-2+6) = 4$		
		$\therefore x = \frac{D_x}{D} = \frac{4}{4} = 1$	1	
		$D_{y} = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -2 & -2 \\ 1 & -6 & -3 \end{vmatrix} = 1(6-12) - 6(-6+2) + 1(-12+2) = 8$		
		$\therefore y = \frac{D_y}{D} = \frac{8}{4} = 2$	1	
		$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & -2 \\ 1 & 1 & -6 \end{vmatrix} = 1(-6+2)-1(-12+2)+6(2-1)=12$		
		$\therefore z = \frac{D_z}{D} = \frac{12}{4} = 3$	1	04
	b)	Find x , if $x + z = 4$, $y + z = 2$, $x + y = 0$ by using Cramer's Rule.		
	Ans	x + z = 4 , $y + z = 2$, $x + y = 0$		

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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		IVIAINS	Marks
2.		$D = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1(0-1) - 0 + 1(0-1) = -2$	1½	
		$D_{x} = \begin{vmatrix} 4 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 4(0-1)-0+1(2-0) = -2$	1½	
		$\therefore x = \frac{D_x}{D} = \frac{-2}{-2} = 1$	1	04
	c)	If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$, find $2A + 3B - 4I$,		
	Ans	Where I is the unit matrix of order 2. $2A + 3B - 4I = 2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1	
		$= \begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	1	
		$= \begin{bmatrix} 3 & 15 \\ 20 & 28 \end{bmatrix}$	2	04
	d)	Find x, y, z if $\begin{cases} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix} \} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$		04
	Ans	$\left\{ \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$		
		$ \left\{ \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 4 \\ 2 & 8 & 10 \\ 4 & 2 & 0 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} $	1	
		$\begin{bmatrix} 7 & 3 & 6 \\ 4 & 8 & 11 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$	1	
		$\begin{bmatrix} 7 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 \end{bmatrix} \begin{bmatrix} z \end{bmatrix}$ $\begin{bmatrix} 7+6+18 \\ 4+16+33 \\ 7+6+6 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	2.001	$\begin{bmatrix} 31 \\ 53 \\ 19 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\therefore x = 31, \ y = 53, \ z = 19$	1	04
	e)	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find the value of $A^2 - 5A + 7I$, Where I is the unit matrix of order 2.		
	Ans	$A^{2} - 5A + 7I = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1	
		$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \end{bmatrix}$	2	
		$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	1	04
	f)	Find the adjoint of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$		
	Ans	$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$		
		Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 3 & 5 & & 1 & 5 & & 1 & 3 \\ 5 & 12 & & 1 & 12 & & 1 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 & & 1 & 3 & & 1 & 2 \\ 5 & 12 & & 1 & 12 & & 1 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 & & 1 & 3 & & 1 & 2 \\ 3 & 5 & & 1 & 5 & & 1 & 3 \end{bmatrix}$		
		$= \begin{bmatrix} 11 & 7 & 2 \\ 9 & 9 & 3 \\ 1 & 2 & 1 \end{bmatrix}$	2	
		Matrix of cofactors = $\begin{bmatrix} 11 & -7 & 2 \\ -9 & 9 & -3 \\ 1 & -2 & 1 \end{bmatrix}$	1	

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Que.	Sub.	Model Answers	Marks	Total
No. 2.	Que.		1 - 2 - 2 - 3	Marks
2.		$Adj.A = \begin{bmatrix} 11 & -9 & 1 \\ -7 & 9 & -2 \\ 2 & -3 & 1 \end{bmatrix}$	1	04
3.		Attempt any FOUR of the following:		16
	a)	If $A = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ then verify that $(AB)' = B'A'$		
	Ans	$AB = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -8 \\ 0 & 1 & -1 \end{bmatrix}$	1	
			1	
		$B'A' = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$	1	
		$= \begin{bmatrix} -5 & 0 \\ 3 & 1 \\ -8 & -1 \end{bmatrix}$	1	04
	b)	Find the inverse of the matrix of the equations $2x+5y=9$, $x+3y=5$ and hence solve the equations.		
	Ans	Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ A = 6 - 5 $\therefore A = 1 \neq 0$ $\therefore A^{-1}$ exists $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$	1	
		$\begin{bmatrix} 1 & 3 \end{bmatrix}$ Matrix of minors = $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$	1/2	
		Cofactor Matrix $=\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$	1/2	
		$adjA = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	1/2	
		L ⁻¹ 2]		

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$A^{-1} = \frac{1}{ A } adjA$ $= \frac{1}{1} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ $\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	1/2	
		$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\therefore x = 2, y = 1$	1	04
	c) Ans	Resolve into partial fraction $\frac{x+4}{x(x+1)(x+2)}$ $\frac{x+4}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$		
		x + 4 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1) Put $x = 0$	1/2	
		$4 = 2A$ $\therefore A = 2$ Put $x = -1$	1	
		$3 = B(-1)(1)$ $\therefore B = -3$ Put $x = -2$ $2 = C(-2)(-1)$	1	
		$2 = 2C$ $\therefore C = 1$	1	
		$\frac{x+4}{x(x+1)(x+2)} = \frac{2}{x} + \frac{-3}{x+1} + \frac{1}{x+2}$	1/2	04
	ď)	Resolve into partial fraction $\frac{x-5}{x^3 + x^2 - 5x}$		
	Ans	$\frac{x-5}{x^3+x^2-5x} = \frac{x-5}{x(x^2+x-5)}$		



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.	f)	Resolve into partial fraction $\frac{3x^2 + 17x + 14}{x^3 - 8}$		
	-/			
	Ans	$\frac{3x^2 + 17x + 14}{x^3 - 8} = \frac{3x^2 + 17x + 14}{(x - 2)(x^2 + 2x + 4)}$		
		$\frac{3x^2 + 17x + 14}{(x-2)(x^2 + 2x + 4)} = \frac{A}{(x-2)} + \frac{Bx + C}{(x^2 + 2x + 4)}$		
		$\therefore 3x^2 + 17x + 14 = A(x^2 + 2x + 4) + (Bx + C)(x - 2)$	1/2	
		Put $x = 2$		
		$\therefore 3(2)^2 + 17(2) + 14 = A(2^2 + 2(2) + 4)$		
		$\therefore 60 = A(12)$		
		$\therefore A = 5$	1	
		Put $x = 0$		
		$\therefore 14 = A(4) + C(-2)$		
		$\therefore 14 = 5(4) + C(-2)$	1	
		C=3		
		Put $x = 1$ $x = 2(1)^2 + 17(1) + 14 = A(7) + (B + C)(-1)$		
		$\therefore 3(1)^{2} + 17(1) + 14 = A(7) + (B+C)(-1)$ $\therefore 34 = 7A - B - C$		
		34 = 7A - B - C $34 = 7(5) - B - 3$		
		$\therefore 34 = 32 - B$		
		$\therefore B = -2$	1	
		$\frac{3x^2 + 17x + 14}{(x-2)(x^2 + 2x + 4)} = \frac{5}{(x-2)} + \frac{-2x + 3}{(x^2 + 2x + 4)}$	1/2	04
			-	
4.		Attempt any FOUR of the following:		16
	a)	Without using calculator find the value of		
	,	$\cos 570^{\circ} \sin 510^{\circ} + \sin(-330^{\circ})\cos(-390^{\circ})$	1/2	
	Ans	$\cos 570^{\circ} = \cos \left(6 \times 90^{\circ} + 30^{\circ} \right)$		
		$=-\cos 30^{\circ}=-\frac{\sqrt{3}}{2}$		
			1/2	
		$\sin 510^0 = \sin \left(6 \times 90^0 - 30^0 \right)$	4/	
		$=\sin 30^{\circ} = \frac{1}{2}$	1/2	
		$\sin(-330^{\circ}) = -\sin(330^{\circ}) \qquad \qquad \dots \text{ Since } \sin(-\theta) = -\sin\theta$		
		$\lim_{n \to \infty} \frac{1}{n} \int_{-\infty}^{\infty} \frac$		



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Que.	Sub.	Model Answers	Marks	Total
No. 4.	Que.	1	1/2	Marks
1.		$= -\sin(4 \times 90^{0} - 30^{0}) = -\sin(-30^{0}) = \frac{1}{2}$	/2	
		$\cos(-390^{\circ}) = \cos 390^{\circ} \qquad \qquad \dots \text{ Since } \cos(-\theta) = \cos \theta$	1/2	
		$=\cos(4\times90^{\circ}+30^{\circ})=\cos 30^{\circ}=\frac{\sqrt{3}}{2}$	1/2	
		$\therefore \cos 570^{\circ} \sin 510^{\circ} + \sin(-330^{\circ}) \cos(-390^{\circ})$		
		$= \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right)$ $= 0$	1	04
	b)	Prove that		
		$\sin\left(\frac{\pi}{3} + A\right)\cos\left(\frac{\pi}{3} + B\right) - \cos\left(\frac{\pi}{3} + A\right)\sin\left(\frac{\pi}{3} + B\right) = \sin\left(A - B\right)$		
	Ans	we know that,		
		$\sin(A-B) = \sin A \cos B - \cos A \sin B$		
		$\therefore L.H.S. = \sin\left(\frac{\pi}{3} + A\right)\cos\left(\frac{\pi}{3} + B\right) - \cos\left(\frac{\pi}{3} + A\right)\sin\left(\frac{\pi}{3} + B\right)$		
		$= \sin\left(\left(\frac{\pi}{3} + A\right) - \left(\frac{\pi}{3} + B\right)\right)$	2	
		$=\sin(A-B)$	2	04
		= R.H.S.		04
	c)	Prove that $\tan 70^{\circ} - \tan 50^{\circ} - \tan 20^{\circ} = \tan 70^{\circ} \tan 50^{\circ} \tan 20^{\circ}$		
	Ans	We have, $\tan 70^{\circ} = \tan (50^{\circ} + 20^{\circ})$		
		,	1/2	
		$\therefore \tan 70^0 = \frac{\tan 50^0 + \tan 20^0}{1 - \tan 50^0 \tan 20^0}$	1	
		$\therefore \tan 70^{\circ} \left(1 - \tan 50^{\circ} \tan 20^{\circ} \right) = \tan 50^{\circ} + \tan 20^{\circ}$	1/2	
		$\therefore \tan 70^{0} - \tan 70^{0} \tan 50^{0} \tan 20^{0} = \tan 50^{0} + \tan 20^{0}$	1	04
		$\therefore \tan 70^{0} - \tan 50^{0} - \tan 20^{0} = \tan 70^{0} \tan 50^{0} \tan 20^{0}$	1	01
	d)	Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$		
	Ans	Let $\cos^{-1}\left(\frac{4}{5}\right) = A$		



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4.	Que.	$\therefore \cos A = \frac{4}{5}$	1/2	Marks
		$\therefore \sin^2 A = 1 - \cos^2 A$		
		$=1-\frac{16}{25}$		
		$=\frac{9}{25}$		
		$\therefore \sin A = \frac{3}{5}$	1/2	
		$\cos^{-1}\left(\frac{12}{13}\right) = B$	1/	
		$\therefore \cos B = \frac{12}{13}$	1/2	
		$\therefore \sin^2 B = 1 - \cos^2 B$		
		$=1-\frac{144}{169}$		
		$=\frac{25}{169}$		
			1/2	
		$\therefore \sin B = \frac{5}{13}$		
		$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$ $= (4)(12) (3)(5)$		
		$= \left(\frac{4}{5}\right) \left(\frac{12}{13}\right) - \left(\frac{3}{5}\right) \left(\frac{5}{13}\right)$ $48 15$	1	
		$=\frac{1}{65}-\frac{1}{65}$	1/2	
		$\therefore \cos\left(A+B\right) = \frac{33}{65}$,2	
		$\therefore A + B = \cos^{-1}\left(\frac{33}{65}\right)$		
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	1/2	04
		OR		
		Let $\cos^{-1}\left(\frac{4}{5}\right) = A$		
		$\therefore \cos A = \frac{4}{5}$ $\therefore \tan A = \frac{3}{4}$		
		$\therefore \tan A = \frac{3}{4}$		



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~	Sub. Que.	Model Answers	Marks	Total Marks
4.		$B = \tan^{-1}\left(\frac{3}{4}\right)$ A B 13 B 12		
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$		
		$\cos^{-1}\left(\frac{12}{13}\right) = B$ $\therefore \cos B = \frac{12}{13}$		
		$\therefore \tan B = \frac{5}{12}$	1/2	
		$B = \tan^{-1}\left(\frac{5}{12}\right)$ $\therefore \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$	1/2	
		$L.H.S. = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$		
		$= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \frac{5}{12}} \right)$	1	
		$= \tan^{-1} \left(\frac{\frac{36+20}{48}}{1-\frac{15}{48}} \right)$		
		$= \tan^{-1} \left(\frac{\frac{56}{48}}{\frac{48-15}{48}} \right)$		
		$= \tan^{-1}\left(\frac{56}{33}\right)$	1/2	
		Let $\tan^{-1}\left(\frac{56}{33}\right) = C$ $\tan C = \frac{56}{33}$		
		$\therefore \tan C = \frac{56}{33}$ $\therefore \cos C = \frac{33}{65}$	1/2	



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(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

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Que.	Sub.	Model Answers	Marks	Total
No.	Que.			Marks
4.		$\therefore C = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore R.H.S. = \cos^{-1}\left(\frac{33}{65}\right)$ 56 33	1/2	04
	e)	If $x > 0$ $y > 0$ and $1 - xy > 0$ then prove that		
	Ans	$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[\frac{x+y}{1-xy} \right]$		
		Put $\tan^{-1} x = A$ and $\tan^{-1} y = B$ $\therefore x = \tan A \text{ and } y = \tan B$ $\therefore \tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	1 1	
		$= \frac{x+y}{1-xy}$ $\therefore A+B = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$ $\therefore \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$	1/ ₂	04
	f) Ans	Prove that $\frac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta} = \tan 5\theta$ $L.H.S. = \frac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta}$ $(8\theta + 2\theta) (8\theta - 2\theta)$		
		$= \frac{2\sin\left(\frac{8\theta + 2\theta}{2}\right)\cos\left(\frac{8\theta - 2\theta}{2}\right)}{2\cos\left(\frac{8\theta + 2\theta}{2}\right)\cos\left(\frac{8\theta - 2\theta}{2}\right)}$ $= \frac{2\sin 5\theta \cos 3\theta}{2\cos 5\theta \cos 3\theta}$	2	
		$= \tan 5\theta$ $= R.H.S.$	1	04



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		IVIALKS	Marks
5.		Attempt any FOUR of the following:		16
	a)	Prove that $\frac{\sin 5A + 2\sin 7A + \sin 9A}{\cos 3A + 2\cos 5A + \cos 7A} = \sin 2A + \cos 2A \tan 5A$		
	Ans	$\sin 5A + 2\sin 7A + \sin 9A$		
		$\cos 3A + 2\cos 5A + \cos 7A$		
		$=\frac{\sin 5A + \sin 9A + 2\sin 7A}{2}$		
		$\cos 3A + \cos 7A + 2\cos 5A$		
		$2\sin\left(\frac{5A+9A}{2}\right)\cos\left(\frac{5A-9A}{2}\right)+2\sin 7A$		
		$= \frac{2}{2\cos\left(\frac{5A+9A}{2}\right)\cos\left(\frac{5A-9A}{2}\right) + 2\cos 5A}$	1	
		$= \frac{2\sin 7A\cos(-2A) + 2\sin 7A}{2\cos 5A\cos(-2A) + 2\cos 5A}$	1/2	
		$= \frac{2\sin 7A \left[\cos(-2A) + 1\right]}{2\cos 5A \left[\cos(-2A) + 1\right]}$	1/2	
		$-2\cos 5A\left[\cos\left(-2A\right)+1\right]$		
		$=\frac{\sin 7A}{\cos x}$		
		$\cos 5A$	1/2	
		$=\frac{\sin(2A+5A)}{5A}$	/2	
		$\cos 5A$ $\sin 2A\cos 5A + \cos 2A\sin 5A$	1	
		$=\frac{\sin 2A\cos 3A + \cos 2A\sin 3A}{\cos 5A}$	1	
		$= \sin 2A + \cos 2A \tan 5A$	1/2	04
	b)	Prove that $\frac{\cos A}{1-\sin A} = \frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}}$		
	Ans	$L.H.S. = \frac{\cos A}{1 - \sin A}$		
		$= \frac{\cos^{2} \frac{A}{2} - \sin^{2} \frac{A}{2}}{\cos^{2} \frac{A}{2} + \sin^{2} \frac{A}{2} - 2\cos \frac{A}{2}\sin \frac{A}{2}}$ $= \frac{\left(\cos \frac{A}{2} - \sin \frac{A}{2}\right) \left(\cos \frac{A}{2} + \sin \frac{A}{2}\right)}{\left(\cos \frac{A}{2} - \sin \frac{A}{2}\right)^{2}}$	1	

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Que.	Sub.	Model Answers	Marks	Total
No. 5.	Que.	$= \frac{\cos\frac{A}{2} + \sin\frac{A}{2}}{\cos\frac{A}{2} - \sin\frac{A}{2}} \qquad \text{divide by } \cos\frac{A}{2} \text{ to N & D}$	1	Marks
		$= \frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}} = R.H.S.$	1	04
	c)	In a triangle ABC , if $A + B + C = \pi$ then prove that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$		
	Ans	In a triangle ABC , $A+B+C=\pi$		
		$\therefore A + B = \pi - C$ $\therefore \tan(A + B) = \tan(\pi - C)$	1	
		$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$	1	
		$\therefore \tan A + \tan B = -\tan C (1 - \tan A \tan B)$	1/ ₂ 1/ ₂	
		$\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C$ $\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$	1	04
	d) Ans	Prove that $\tan^{-1} 1 + \tan^{-2} 2 + \tan^{-3} 3 = \pi$ $L.H.S. = \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$		
	71110	$= \tan^{-1} \left(\frac{1+2}{1-(1)(2)} \right) + \pi + \tan^{-1} 3$	1	
		$= \tan^{-1}(-3) + \pi + \tan^{-1} 3$ $= -\tan^{-1} 3 + \pi + \tan^{-1} 3$	1 1	
		$=\pi = R.H.S.$	1	04
	e)	Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$		
	Ans	$L.H.S. = \frac{\sec 8A - 1}{\sec 4A - 1}$		
		$=\frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$	1/2	

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Que.	Sub.	Model Answers	Marks	Total
No.	Que.			Marks
5.		$\frac{1-\cos 8A}{\cos A}$	1/2	
		$=\frac{\cos 8A}{1-\cos 4A}$		
		$\cos 4A$		
		$= \frac{2\sin^2 4A\cos 4A}{\sin^2 4A\cos 4A}$	1	
		$2\sin^2 2A\cos 8A$		
		$=\frac{\sin 8A\sin 4A}{2\sin^2 2A\cos 8A}$	1	
		$2\sin 2A\cos 8A$ $2\tan 8A\sin 2A\cos 2A$	1/2	
		$=\frac{2\sin(3/3)\sin(2/4\cos(2/4))}{2\sin^2(2/4)}$		
		$=\frac{\tan 8A}{\tan 2A}=R.H.S.$	1/	
		$-\frac{1}{\tan 2A} = KH.5.$	1/2	04
	f)	Prove that $\cos 15^{\circ} \cos 30^{\circ} \cos 60^{\circ} \cos 75^{\circ} = \frac{\sqrt{3}}{16}$		
	1)			
	Ans	$L.H.S. = \cos 15^{\circ} \cos 30^{\circ} \cos 60^{\circ} \cos 75^{\circ}$		
		$=\cos 15^{0} \frac{\sqrt{3}}{2} \times \frac{1}{2} \cos 75^{0}$	1/2	
		$=\frac{\sqrt{3}}{4}\left(\frac{1}{2}\times2\cos 15^{0}\cos 75^{0}\right)$	1/2	
		$=\frac{\sqrt{3}}{8}\left(\cos\left(75^{0}+15^{0}\right)+\cos\left(75^{0}-15^{0}\right)\right)$	1	
		$=\frac{\sqrt{3}}{8}(\cos 90^{\circ} + \cos 60^{\circ})$	1/2	
		$=\frac{\sqrt{3}}{8}\left(0+\frac{1}{2}\right)$	1	
		$=\frac{\sqrt{3}}{16}=R.H.S.$	1/2	04
6.				
	a)	Attempt any FOUR of the following:		16
		If m_1 and m_2 are the slope of two lines then prove that angle between two		
		lines is $\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $		
	Ans	Let θ_1 = Inclination of L_1		



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		1,10110	Marks
6.	Ans	θ_2 =Inclination of L_2		
		$\therefore \text{ Slope of } L_1 \text{ is } m_1 = \tan \theta_1$	1/2	
		Slope of L_2 is $m_2 = \tan \theta_2$		
		$\begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	1	
		∴ from figure,		
		$ heta = heta_1 - heta_2$		
		$\therefore \tan \theta = \tan \left(\theta_1 - \theta_2\right)$		
		$= \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$	1	
		$\therefore \tan \theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$	1/2	
		Since θ is acute $\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $	1/2	
		$\therefore \theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $	1/2	04
	b)	Prove that distance between two parallel lines $ax + by + c = 0$ and		
		$ax + by + c' = 0$ is $d = \left \frac{c' - c}{\sqrt{A^2 + B^2}} \right $		
	Ans	I : ax + by + c = 0		
		$L_1: ax + by + c = 0$ $L_2: ax + by + c' = 0$		
		$D_2 \cdot uu + vy + c = 0$		
			1	l



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		Let $P(x_1, y_1)$ be any point on the line L_1 $\therefore ax_1 + by_1 + c = 0$ $\therefore ax_1 + by_1 = -c$ $P(x_1, y_1)$ $PM \text{ is perpendicular on the line } L_2$	1/2	
		$\therefore PM = \left \frac{ax_1 + by_1 + c'}{\sqrt{A^2 + B^2}} \right $	1	
		$\therefore PM = \left \frac{-c + c'}{\sqrt{A^2 + B^2}} \right $ $\therefore PM = \left \frac{c' - c}{\sqrt{A^2 + B^2}} \right $	1 1/2	
	c)	Find the equation of the line joining the point $(2,-3)$ with the point of intersection of $4x+3y+2=0$ and $6x+5y+6=0$.		04
	Ans	$4x+3y+2=0, 6x+5y+6=0.$ $4x+3y=-2 \times 3$ $6x+5y=-6 \times 2$ $12x+9y=-6$ $12x+10y=-12$ $-y=6 \Rightarrow y=-6$		
		∴ $x = 4$ ∴ point of intersection $= (4, -6) = (x_1, y_1)$ and given point $= (2, -3) = (x_2, y_2)$ its equation in two points form is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$	1	
		$\therefore \frac{y - (-6)}{-3 - (-6)} = \frac{x - 4}{2 - 4}$	1	
		$\therefore \frac{y+6}{3} = \frac{x-4}{-2}$ $\therefore -2(y+6) = 3(x-4)$ $\therefore -2y-12 = 3x-12$	1/ ₂ 1/ ₂ 1/ ₂ 1/ ₂ 1/ ₂	
		$\therefore 3x + 2y = 0$	72	04



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0110	Sub.			Total
Que. No.	Que.	Model Answers	Marks	Marks
6.	d)	Find the length of perpendicular on the line $\sqrt{3} x - y - 14 = 0$		
	Ans	from the origin.		
		L: $\sqrt{3} x - y - 14 = 0$, $P(x_1, y_1) = (0, 0)$ $A = \sqrt{3}$, $B = -1$, $C = -14$ $p = \left \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right $	1	
		$= \left \frac{\sqrt{3}(0) + (-1)(0) + (-14)}{\sqrt{(\sqrt{3})^2 + (-1)^2}} \right $	1	
		$=\frac{14}{2}=7$	2	04
	e) Ans	Find the angle between the lines $y = 5x + 6$ and $y = x$ 5x - y + 6 = 0 and $x - y = 0$		
		$m_1 = 5$ $m_2 = 1$ $\tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $	1	
		$= \left \frac{5-1}{1+5\times 1} \right $	1	
		$=\frac{2}{3}$ or 0.66	1	
		$\theta = \tan^{-1}\left(\frac{2}{3}\right)$	1	04
	f)	Find the equation of the straight line passing through the point of intersection of the lines $4x + 3y = 8$ and $x + y = 1$ and perpendicular to the line $7x + 5y = 9$		
	Ans	4x+3y=8 $x+y=1$ $4x+3y=8$ $3x+3y=3$		
		$\therefore x = 5$ $\therefore y = -4$	1	



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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
6.		\therefore Point of intersection = $(5,-4)$		
		Slope of the line $7x + 5y = 9$ is,		
		$m_0 = -\frac{a}{b} = -\frac{7}{5}$	1/2	
		∴ Slope of the required line is, $m = -\frac{1}{m_0} = \frac{5}{7}$	1/2	
		$\therefore y - y_1 = m(x - x_1)$		
		$y - (-4) = \frac{5}{7}(x - 5)$ $7(y + 4) = 5x - 25$	1	
		7(y+4) = 5x-25	1/2	
		$\therefore 5x - 7y - 53 = 0$	1/2	04
		Important Note		
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.		