Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	a)	$\int \frac{(x-2)^2}{x} dx = \int \frac{x^2-4x+4}{x} dx$		
		$= \int \left(x - 4 + \frac{4}{x} \right) dx$	1	
		$= \frac{x^2}{2} - 4x + 4\log x + c$	1	**2**
		Note: In solution of integration problems, if the constant 'c' is not added, ½ mark **may be** deducted.		
	b)	$\int x \log x dx$		
		$= \log x \int x dx - \int \left(\int x dx \right) \frac{d}{dx}(\log x) dx + c$	½	
		$= \log x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} . \frac{1}{x} dx + c$	½	
		$= \frac{x^2 \log x}{2} - \frac{1}{2} \int x dx + c$		
		$= \frac{x^2 \log x}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + c$	½	
		$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + c$	½	**2**
	c)	$\int \frac{1-\tan x}{1+\tan x} dx$		
		$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ Put $\cos x + \sin x = t$, $(\cos x - \sin x) dx = dt$	1	
		$= \int \frac{1}{t} dt + c$		
		$= \log t + c$	½	
		$= \log(\cos x + \sin x) + c$	½	**2**
		OR		
		$\int \frac{1-\tan x}{1+\tan x} dx$		
		$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ $\frac{d}{dx}(\cos x + \sin x) = \cos x - \sin x$	1	
		$= \log(\cos x + \sin x) + c$	1	**2**
		OR		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$\int \frac{1-\tan x}{1+\tan x}dx = \int \tan\left(\frac{\pi}{4}-x\right)dx$	½	
		$= \frac{\log\sec\left(\frac{\pi}{4}-x\right)}{-1}+c$	1	
		$= -\log\sec\left(\frac{\pi}{4}-x\right)+c$	½	**2**
	d)	The given problem cannot be solved within the given limits because for the integrating the given function within the prescribed limits the function must be well defined on the given interval. For example at $x=\frac{\pi}{4},\ \frac{1}{\sqrt{1-x^2}}$ is a non-real number and hence the function is not defined on the interval $\left[0,\ \pi/4\right]$.	2	**2**
	e)	$\int_1^e \log x dx = \left[\log x\int dx - \int\left(\int 1dx\right)\frac{d}{dx}(\log x)dx\right]_1^e$	½	
		$= \left[\log x.x - \int x.\frac{1}{x}dx\right]_1^e$		
		$= \left[x\log x - \int 1.dx\right]_1^e$		
		$= [x\log x - x]_1^e$	½	
		$= [e\log e - e] - [1\log 1 - 1]$	½	
		$= 0 - [-1]$		
		$= 1$	½	**2**
		OR		
		$\int_1^e \log x dx = [x\log x - x]_1^e$	1	
		$= [e\log e - e] - [1\log 1 - 1]$		
		$= 0 - [-1]$	½	
		$= 1$	½	**2**
	f)	$\frac{d^2y}{dx^2} = \sqrt[4]{y+\left(\frac{dy}{dx}\right)^2}$		
		Order = 2	1	
		$\left(\frac{d^2y}{dx^2}\right)^4 = y+\left(\frac{dy}{dx}\right)^2$		
		Degree = 4	1	**2**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
g)		$y = A\cos 3x + B\sin 3x$		
		$\therefore \frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$	1	
		$\therefore \frac{d^2y}{dx^2} = -9A\cos 3x - 9B\sin 3x$		
		$= -9y$		
		$\therefore \frac{d^2y}{dx^2} + 9y = 0$	1	**2**
h)		$\frac{dy}{dx} = \frac{1+x^2}{y}$		
		$ydy = \left(1+x^2\right)dx$		
		$\int ydy = \int \left(1+x^2\right)dx$	1	
		$\frac{y^2}{2} = x + \frac{x^3}{3} + c$	1	**2**
i)		$L\left[3+2t^2-e^{-t}\right]$		
		$= \frac{3}{s} + \frac{4}{s^3} - \frac{1}{s+1}$	2	**2**
		Note: In the above solution, each term carries ½ mark and if all the terms are correct, the whole answer carries full marks.		
j)		$L\left[\cos 3t\right] = \frac{s}{s^2+9}$	1	
		$\therefore L\left[e^{-2t}.\cos 3t\right] = \frac{s+2}{(s+2)^2+9}$	1	**2**
k)		$L^{-1}\left[\frac{3s-7}{s^2+9}\right]$		
		$= L^{-1}\left[\frac{3s}{s^2+9}\right] - L^{-1}\left[\frac{7}{s^2+9}\right]$		
		$= 3L^{-1}\left[\frac{s}{s^2+9}\right] - \frac{7}{3}L^{-1}\left[\frac{3}{s^2+3^2}\right]$	1	
		$= 3\cos 3t - \frac{7}{3}\sin 3t$	½ + ½	**2**
		Note: ½ + ½ means each term carries ½ marks.		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	*l*)	$\dfrac{1}{s(s+2)}=\dfrac{1/2}{s}+\dfrac{-1/2}{s+2}$ or $\dfrac{1}{2}\left(\dfrac{1}{s}+\dfrac{-1}{s+2}\right)$	1	
		$L^{-1}\left[\dfrac{1}{s(s+2)}\right]$		
		$=L^{-1}\left[\dfrac{1/2}{s}+\dfrac{-1/2}{s+2}\right]$		
		$=\dfrac{1}{2}-\dfrac{1}{2}e^{-2t}$	½ + ½	**2**
2)	a)	$\dfrac{dx}{dt}=6-3x$		
		$\therefore \dfrac{dx}{6-3x}=dt$	1	
		$\therefore \int\dfrac{dx}{6-3x}=\int dt$		
		$\therefore \dfrac{\log(6-3x)}{-3}=t+c$	1	
		$\therefore$ at $x=0,\ t=0$		
		$\dfrac{\log 6}{-3}=0+c$		
		$c=\dfrac{\log 6}{-3}$	1	
		$\therefore \dfrac{\log(6-3x)}{-3}=t-\dfrac{\log 6}{3}$	1	**4**
	b)	$x^2ydx-\left(x^3+y^3\right)dy=0$		
		$\therefore \dfrac{dy}{dx}=\dfrac{x^2y}{x^3+y^3}$		
		Put $y=vx$		
		$\therefore \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$		
		$\therefore v+x\dfrac{dv}{dx}=\dfrac{x^2vx}{x^3+(vx)^3}=\dfrac{v}{1+v^3}$	1	
		$\therefore x\dfrac{dv}{dx}=\dfrac{v}{1+v^3}-v$		
		$\therefore x\dfrac{dv}{dx}=-\dfrac{v^4}{1+v^3}$	1	
		$\therefore \dfrac{1+v^3}{v^4}dv=-\dfrac{1}{x}dx$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$\therefore \int\left(\frac{1}{v^4}+\frac{1}{v}\right)dv=-\int\frac{1}{x}dx$		
		$\therefore \frac{v^{-3}}{-3}+\log v=-\log x+c$	1	
		$\therefore \frac{1}{-3v^3}+\log v=-\log x+c$		
		$\therefore \frac{x^3}{-3y^3}+\log\left(\frac{y}{x}\right)=-\log x+c$	1	**4**
	c)	$\frac{dy}{dx}=\frac{x-y+3}{2x-2y+5}$		
		Put $x-y=t$		
		$\therefore 1-\frac{dy}{dx}=\frac{dt}{dx}$	1	
		$\therefore 1-\frac{dt}{dx}=\frac{t+3}{2t+5}$		
		$\therefore \frac{dt}{dx}=1-\frac{t+3}{2t+5}=\frac{t+2}{2t+5}$		
		$\therefore \frac{2t+5}{t+2}dt=dx$		
		$\therefore \int\frac{2t+5}{t+2}dt=\int dx$		
		$\therefore \int\left(2+\frac{1}{t+2}\right)dt=\int dx$	1	
		$\therefore 2t+\log(t+2)=x+c$	1	
		$\therefore 2(x-y)+\log(x-y+2)=x+c$	1	**4**
	d)	$\cos^2 x\frac{dy}{dx}+y=\tan x$		
		$\therefore \frac{dy}{dx}+\sec^2 x\cdot y=\tan x\cdot\sec^2 x$		
		$\therefore P=\sec^2 x$ and $Q=\tan x\cdot\sec^2 x$		
		$\therefore IF=e^{\int pdx}=e^{\int\sec^2 xdx}=e^{\tan x}$	1	
		$\therefore y\cdot IF=\int Q\cdot IF\cdot dx+c$		
		$\therefore y\cdot e^{\tan x}=\int\tan x\cdot\sec^2 x\cdot e^{\tan x}\cdot dx+c$	1	
		Put $\tan x=t$ $\therefore \sec^2 x\cdot dx=dt$	½	
		$\therefore y\cdot e^{\tan x}=\int \mathrm{t}\,e^{\mathrm{t}}\cdot dt+c$		
		$\therefore y\cdot e^{\tan x}=te^t-e^t+c$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$\therefore y\cdot e^{\tan x}=\tan x e^{\tan x}-e^{\tan x}+c$	1	
		$\therefore y\cdot e^{\tan x}=e^{\tan x}\left(\tan x-1\right)+c$		**4**
	e)	$\frac{dy}{dx}+\frac{y}{x}=y^3$		
		$\therefore \frac{1}{y^3}\frac{dy}{dx}+\frac{1}{x}\cdot\frac{1}{y^2}=1$		
		Put $\frac{1}{y^2}=t$		
		$\therefore -2\cdot\frac{1}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$	1	
		$\therefore \frac{1}{-2}\frac{dt}{dx}+\frac{1}{x}\cdot t=1$		
		$\therefore \frac{dt}{dx}+\frac{-2}{x}\cdot t=-2$		
		$P=\frac{-2}{x}$ and $Q=-2$		
		$\therefore IF=e^{\int pdx}=e^{\int\frac{-2}{x}dx}=e^{-2\log x}=e^{\log\left(x^{-2}\right)}=x^{-2}=\frac{1}{x^2}$	1	
		$\therefore$ the solution is,		
		$t\cdot IF=\int Q\cdot IF\cdot dx+c$		
		$\therefore \quad t\cdot\frac{1}{x^2}=\int -2\cdot\frac{1}{x^2}\cdot dx+c$	1	
		$\therefore \quad t\cdot\frac{1}{x^2}=-2\cdot\frac{-1}{x}+c$	½	
		$\therefore \quad \frac{1}{x^2y^2}=\frac{2}{x}+c$	½	**4**
	f)	$L\cdot\frac{di}{dt}=30\sin\left(10\pi t\right)$		
		$\therefore Ldi=30\sin\left(10\pi t\right)dt$		
		$\therefore \int Ldi=\int 30\sin\left(10\pi t\right)dt$		
		$\therefore Li=30\frac{-\cos\left(10\pi t\right)}{10\pi}+c$	1	
		$\therefore Li=\frac{-3}{\pi}\cos\left(10\pi t\right)+c$	1	
		Given $L=2,\ i=0,\ t=0$		
		$\therefore 0=\frac{-3}{\pi}+c$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$\therefore c=\frac{3}{\pi}$	1	
		$\therefore Li=\frac{-3}{\pi}\cos(10\pi t)+\frac{3}{\pi}$ OR		
		$i=\frac{3}{\pi L}\left[-\cos(10\pi t)+1\right]$	1	**4**
3)	a)	Put $\tan\frac{x}{2}=t$		
		$\therefore dx=\frac{2dt}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$	1	
		$\therefore\int\frac{dx}{5+4\cos x}=\int\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$		
		$=2\int\frac{1}{t^2+9}dt$	1	
		$=2\int\frac{1}{t^2+3^2}dt$		
		$=2\times\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1	
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	1	**4**
	b)	$\int x^2\tan^{-1}x\,dx$		
		$=\tan^{-1}x\int x^2dx-\int\left(\int x^2dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx+c$	1	
		$=\tan^{-1}x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{1+x^2}dx+c$	1	
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\frac{x^3}{1+x^2}dx+c$		
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\left(x-\frac{x}{1+x^2}\right)dx+c$	1	
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\left(x-\frac{1}{2}\cdot\frac{2x}{1+x^2}\right)dx+c$		
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\left[\frac{x^2}{2}-\frac{1}{2}\cdot\log\left(1+x^2\right)\right]+c$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	c)	$\frac{1}{(x^2+4)(x+1)}=\frac{Ax+B}{x^2+4}+\frac{C}{x+1}$		
		$\therefore$ we get,		
		$A=-\frac{1}{5}$	½	
		$B=\frac{1}{5}$	½	
		$C=\frac{1}{5}$	½	
		$\frac{1}{(x^2+4)(x+1)}=\frac{-\frac{1}{5}x+\frac{1}{5}}{x^2+4}+\frac{\frac{1}{5}}{x+1}$	½	
		$\therefore \int\frac{1}{(x^2+4)(x+1)}dx=\int\left(\frac{-\frac{1}{5}x+\frac{1}{5}}{x^2+4}+\frac{\frac{1}{5}}{x+1}\right)dx$		
		$=-\frac{1}{5}\int\frac{x}{x^2+4}dx+\frac{1}{5}\int\frac{1}{x^2+4}dx+\frac{1}{5}\int\frac{1}{x+1}dx$		
		$=-\frac{1}{5}\cdot\frac{1}{2}\log\left(x^2+4\right)+\frac{1}{5}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{5}\log\left(x+1\right)+c$	1½	
		$=-\frac{1}{10}\log\left(x^2+4\right)+\frac{1}{10}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{5}\log\left(x+1\right)+c$	½	**4**
		Note: In the above example, the partial fractions may be carried out as, $\frac{1}{(x^2+4)(x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$ $\therefore$ we get, $A=\frac{1}{5}$, $B=-\frac{1}{5}$, $C=\frac{1}{5}$ Further note that, if one of the values (A, B or C) in the partial fraction is wrong but other values are correct and all the further solution is correct, it is adviced to give appropriate marks.		
	d)	$\int_0^{\pi/2}\sin 5x\cos 3x dx$		
		$=\int_0^{\pi/2}\frac{\sin 8x+\sin 2x}{2}dx$	1	
		$=\frac{1}{2}\left[-\frac{\cos 8x}{8}-\frac{\cos 2x}{2}\right]_0^{\pi/2}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$=\frac{1}{2}\left[-\frac{\cos 4\pi}{8}-\frac{\cos\pi}{2}\right]-\frac{1}{2}\left[-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$	1	
		$=\frac{1}{2}\left[-\frac{1}{8}-\frac{-1}{2}\right]-\frac{1}{2}\left[-\frac{1}{8}-\frac{1}{2}\right]$		
		$=\frac{1}{2}$	1	**4**
	e)	$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$		
		$=\int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	½	
		$I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	1	
		$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½	
		$=\int_0^{\pi/2}1\cdot dx$	½	
		$=[x]_0^{\pi/2}$	½	
		$=\frac{\pi}{2}$	½	
		$\therefore I=\frac{\pi}{4}$	½	**4**
		OR		
		$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ Replace $x\to\frac{\pi}{2}-x$ $\therefore \sin x\to\cos x$ $\&\cos x\to\sin x$	½ 1	
		$\therefore I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	½	
		$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½	
		$=\int_0^{\pi/2}1\cdot dx$	½	
		$=[x]_0^{\pi/2}$	½	
		$=\frac{\pi}{2}$	½	
		$\therefore I=\frac{\pi}{4}$	½	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	f)	Given $y=x^2+1$ and $y=2x+1$		
		$\therefore x^2+1=2x+1$		
		$\therefore x=0, \quad x=2$	1	
		$A=\int_a^b (y_2-y_1)\,dx$		
		$=\int_0^2 \left[(2x+1)-(x^2+1)\right]dx$		
		$=\int_0^2 (2x-x^2)\,dx$	1	
		$=\left[x^2-\frac{x^3}{3}\right]_0^2$	1	
		$=\left[2^2-\frac{2^3}{3}\right]-[0]$		
		$=\frac{4}{3}$ or 1.333	1	**4**
4)	a)	$L[\cos 2t.\cos 4t]=\frac{1}{2}L[2\cos 2t.\cos 4t]$		
		$=\frac{1}{2}L[\cos 6t+\cos(-2t)]$	1	
		$=\frac{1}{2}\left[\frac{s}{s^2+36}+\frac{s}{s^2+4}\right]$	1½+1½	**4**
	b)	$L[\sin 2t]=\frac{2}{s^2+4}$	1	
		$\therefore L\left[e^{-t}\sin 2t\right]=\frac{2}{(s+1)^2+4}$		
		$=\frac{2}{s^2+2s+5}$	1	
		$\therefore L\left[te^{-t}\sin 2t\right]=(-1)\frac{d}{ds}\left[\frac{2}{s^2+2s+5}\right]$	1	
		$=-2\cdot\frac{d}{ds}\left[\frac{1}{s^2+2s+5}\right]$		
		$=-2\cdot\frac{-1}{(s^2+2s+5)^2}\cdot\frac{d}{ds}\left[s^2+2s+5\right]$		
		$=2\cdot\frac{2s+2}{(s^2+2s+5)^2}$		
		$=\frac{4s+4}{(s^2+2s+5)^2}$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		**OR**		
		$L\left[\sin 2t\right]=\dfrac{2}{s^2+4}$	1	
		$\therefore L\left[e^{-t}\sin 2t\right]=\dfrac{2}{\left(s+1\right)^2+4}$		
		$=\dfrac{2}{s^2+2s+5}$	1	
		$\therefore L\left[te^{-t}\sin 2t\right]=\left(-1\right)\dfrac{d}{ds}\left[\dfrac{2}{s^2+2s+5}\right]$	1	
		$=-2\cdot\dfrac{d}{ds}\left[\dfrac{1}{s^2+2s+5}\right]$		
		$=-2\cdot\dfrac{\left(s^2+2s+5\right)\dfrac{d}{ds}\left[1\right]-1.\dfrac{d}{ds}\left[s^2+2s+5\right]}{\left(s^2+2s+5\right)^2}$		
		$=-2\cdot\dfrac{\left(s^2+2s+5\right)\left[0\right]-1.\left[2s+2\right]}{\left(s^2+2s+5\right)^2}$		
		$=\dfrac{4s+4}{\left(s^2+2s+5\right)^2}$	1	**4**
	c)	$\dfrac{1}{\left(s^2+1\right)\left(s+3\right)}=\dfrac{As+B}{s^2+1}+\dfrac{C}{s+3}$		
		Then we get,		
		$A=\dfrac{-1}{10},\quad B=\dfrac{3}{10},\quad C=\dfrac{1}{10}$	½+½+ ½	
		$\therefore \dfrac{1}{\left(s^2+1\right)\left(s+3\right)}=\dfrac{\frac{-1}{10}s+\frac{3}{10}}{s^2+1}+\dfrac{\frac{1}{10}}{s+3}$	½	
		$\therefore \dfrac{1}{\left(s^2+1\right)\left(s+3\right)}=\dfrac{1}{10}\left[\dfrac{-s+3}{s^2+1}+\dfrac{1}{s+3}\right]$		
		$\therefore L\left[\dfrac{1}{\left(s^2+1\right)\left(s+3\right)}\right]=\dfrac{1}{10}L\left[\dfrac{-s+3}{s^2+1}+\dfrac{1}{s+3}\right]$		
		$=\dfrac{1}{10}L\left[\dfrac{-s}{s^2+1}+\dfrac{3}{s^2+1}+\dfrac{1}{s+3}\right]$		
		$=\dfrac{1}{10}\left[-\cos t+3\sin t+e^{-3t}\right]$	2	**4**
		Note: In the last step, each term carries ½ marks and if all the terms are correct, the whole step carries 2 marks.		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		**Note:** In the above example, the partial fraction can be carried out as, $\frac{1}{(s^2+1)(s+3)}=\frac{A}{s+3}+\frac{Bs+C}{s^2+1}$ and we then get $$\frac{1}{(s^2+1)(s+3)}=\frac{\frac{1}{10}}{s+3}+\frac{\frac{-1}{10}s+\frac{3}{10}}{s^2+1}$$		
	d)	Let $f(s)=\frac{1}{s+1}$ and $g(s)=\frac{1}{s-2}$		
		$\therefore L^{-1}[f(s)]=e^{-t}$	½	
		and $L^{-1}[g(s)]=e^{2t}$	½	
		$\therefore F(u)=e^{-u}$ and $G(t-u)=e^{2(t-u)}$	½	
		$\therefore L^{-1}\left[\frac{1}{(s+1)(s-2)}\right]=\int_0^t F(u)\cdot G(t-u)\,du$		
		$=\int_0^t e^{-u}\cdot e^{2(t-u)}du$	½	
		$=e^{2t}\int_0^t e^{-3u}\cdot du$	½	
		$=e^{2t}\left[\frac{e^{-3u}}{-3}\right]_0^t$	½	
		$=\frac{e^{2t}}{-3}\left[e^{-3t}-e^0\right]$	½	
		$=\frac{e^{2t}}{-3}\left[e^{-3t}-1\right]$	½	
		Note: Students may solve the problem by taking $f(s)=\frac{1}{s-2}$ and $g(s)=\frac{1}{s+1}$ Appropriate marks are to be given. But if the problem is solved by any another method e.g., partial fraction method but not by above method of convolution, no marks to be given.		4
	e)	$L^{-1}\left[\frac{2}{(s-3)^4}+\frac{s}{s^2-9}\right]=2L^{-1}\left[\frac{1}{(s-3)^4}\right]+L^{-1}\left[\frac{s}{s^2-9}\right]$		
		$=2\cdot e^{3t}\cdot\frac{t^3}{6}+\cosh 3t$	2+1	
		$=\frac{t^3e^{3t}}{3}+\cosh 3t$	1	4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		**OR**		
		$L^{-1}\left[\frac{1}{s^4}\right]=\frac{t^3}{6}$	1	
		$\therefore L^{-1}\left[\frac{1}{(s-3)^4}\right]=e^{3t}\cdot\frac{t^3}{6}$	1	
		and $L^{-1}\left[\frac{s}{s^2-9}\right]=\cosh 3t$	1	
		$\therefore L^{-1}\left[\frac{2}{(s-3)^4}+\frac{s}{s^2-9}\right]=2\cdot e^{3t}\cdot\frac{t^3}{6}+\cosh 3t$		
		$=\frac{t^3e^{3t}}{3}+\cosh 3t$	1	**4**
	f)	$\frac{d^2y}{dt^2}=-y+t$		
		$\therefore L\left[\frac{d^2y}{dt^2}\right]=L[-y+t]$		
		$\therefore s^2L[y]-s\cdot y(0)-y'(0)=-L[y]+\frac{1}{s^2}$	1	
		$\therefore s^2L[y]+L[y]-s\cdot 1-(-2)=\frac{1}{s^2}$	½	
		$\therefore (s^2+1)L[y]-s+2=\frac{1}{s^2}$		
		$\therefore (s^2+1)L[y]=\frac{1}{s^2}+s-2$		
		$\therefore L[y]=\frac{1}{s^2(s^2+1)}+\frac{s-2}{s^2+1}$		
		$\therefore L[y]=\frac{1}{s^2}+\frac{-1}{s^2+1}+\frac{s-2}{s^2+1}$		
		$\therefore L[y]=\frac{1}{s^2}+\frac{s-3}{s^2+1}$	1	
		$\therefore y=L^{-1}\left[\frac{1}{s^2}+\frac{s-3}{s^2+1}\right]$		
		$=L^{-1}\left[\frac{1}{s^2}+\frac{s}{s^2+1}-\frac{3}{s^2+1}\right]$		
		$=t+\cos t-3\sin t$	1 ½	**4**
		Note: In the above example, in place $L[y]$, the symbol $\bar{y}$ is also used for convenience and the solution turn outs as illustrated further:		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$\frac{d^2y}{dt^2}=-y+t$		
		$\therefore L\left[\frac{d^2y}{dt^2}\right]=L\left[-y+t\right]$		
		$\therefore s^2\bar{y}-s\cdot y(0)-y'(0)=-\bar{y}+\frac{1}{s^2}$	1	
		$\therefore s^2\bar{y}+\bar{y}-s\cdot 1-(-2)=\frac{1}{s^2}$	½	
		$\therefore (s^2+1)\bar{y}-s+2=\frac{1}{s^2}$		
		$\therefore (s^2+1)\bar{y}=\frac{1}{s^2}+s-2$		
		$\therefore \bar{y}=\frac{1}{s^2(s^2+1)}+\frac{s-2}{s^2+1}$		
		$\therefore \bar{y}=\frac{1}{s^2}+\frac{-1}{s^2+1}+\frac{s-2}{s^2+1}$		
		$\therefore \bar{y}=\frac{1}{s^2}+\frac{s-3}{s^2+1}$	1	
		$\therefore y=L^{-1}\left[\frac{1}{s^2}+\frac{s-3}{s^2+1}\right]$		
		$=L^{-1}\left[\frac{1}{s^2}+\frac{s}{s^2+1}-\frac{3}{s^2+1}\right]$		
		$=t+\cos t-3\sin t$	1 ½	**4**
5)	a)	$f(x)=x^3+2x-1$		
		$\therefore f(0)=-1$	1	
		$f(1)=2$		
		$\therefore$ the root is in $(0,\ 1)$.		
		$\therefore x_1=\frac{0+1}{2}=0.5$	1	
		$\therefore f(0.5)=0.125$		
		$\therefore$ the root is in $(0,\ 0.5)$.	1	
		$\therefore x_2=\frac{0+0.5}{2}=0.25$		
		$\therefore f(0.25)=-0.484$		
		$\therefore$ the root is in $(0.25,\ 0.5)$.	1	
		$\therefore x_3=\frac{0.25+0.5}{2}=0.375$		**4**
		OR		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$f(x)=x^3+2x-1$ $\therefore f(0)=-1$ $f(1)=2$ $\therefore$ the root is in $(0,\ 1)$.	1	

a	b	$x=\dfrac{a+b}{2}$	$f(x)$	Marks
0	1	0.5	0.125	1
0	0.5	0.25	-0.484	1
0.25	0.5	0.375	---	1

Total Marks: **4**

b)

$x=\sqrt{6}$

$\therefore x^2-6=0$

Put $f(x)=x^2-6$

$\therefore f(2)=-2$

$f(3)=3$

$\therefore$ the root is in $(2,\ 3)$. — 1

$\therefore x_1=\dfrac{af(b)-bf(a)}{f(b)-f(a)}=2.4$ — 1 ½

$\therefore f(2.4)=-0.24$

$\therefore$ the root is in $(2.4,\ 3)$.

$\therefore x_2=2.44$ — 1 ½ — **4**

OR

$x=\sqrt{6}$ $\therefore x^2-6=0$

Put $f(x)=x^2-6$

$\therefore f(2)=-2$

$f(3)=3$

$\therefore$ the root is in $(2,\ 3)$. — 1

a	b	$f(a)$	$f(b)$	$x=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$	Marks
2	3	-2	3	2.4	-0.24	1 ½
2.4	3	-0.24	3	2.44		1 ½

Total Marks: **4**

Note: If the problem is solved by taking $f(x)=x-\sqrt{6}$, no marks to be given since to find various values of $f(x)$ for different values of x, it is required to use the value of $\sqrt{6}$ and it is not permissible in this example as here given task is to find its approximate value.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	c)	$f(x)=x^3-5x+3$		
		$f'(x)=3x^2-5$	½	
		$f(0)=3$	½	
		$f(1)=-1$		
		$x-\dfrac{f(x)}{f'(x)}=x-\dfrac{x^3-5x+3}{3x^2-5}$		
		$=\dfrac{2x^3-3}{3x^2-5}$	1	
		Start with $x_0=1$,		
		$\therefore x_1=0.5$	1	
		$x_2=0.647$	1	**4**
		OR		
		$f(x)=x^3-5x+3$		
		$f'(x)=3x^2-5$	½	
		$f(0)=3$		
		$f(1)=-1$	½	
		Start with $x_0=1$,		
		$\therefore x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}$		
		$=1-\dfrac{f(1)}{f'(1)}$		
		$=1-\dfrac{-1}{-2}$	1	
		$=0.5$	½	
		$x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}$		
		$=0.5-\dfrac{f(0.5)}{f'(0.5)}$		
		$=0.5-\dfrac{0.625}{-4.25}$	1	
		$=0.647$	½	**4**
	d)	$x+2y+3z=14$		
		$3x+y+2z=11$		
		$2x+3y+z=11$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\begin{array}{r} 3x+6y+9z=42 \\ 3x+y+2z=11 \\ -\quad -\quad -\quad - \\ \hline 5y+7z=31 \end{array}$ and $\begin{array}{r} 2x+4y+6z=28 \\ 2x+3y+z=11 \\ -\quad -\quad -\quad - \\ \hline y+5z=17 \end{array}$	1	
		$\begin{array}{r} 5y+7z=31 \\ 5y+25z=85 \\ \hline -18z=-54 \end{array}$		
		$\therefore z=3$ $y=2$ $x=1$	1 1 1	**4**
		Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking. Let us see, how the solution becomes by eliminating first y and then z to get the value of x, as illustrated below:		
		$\begin{array}{r} x+2y+3z=14 \\ 6x+2y+4z=22 \\ -\quad -\quad -\quad - \\ \hline -5x-z=-8 \end{array}$ and $\begin{array}{r} 9x+3y+6z=33 \\ 2x+3y+z=11 \\ -\quad -\quad -\quad - \\ \hline 7x+5z=22 \end{array}$	1	
		$\begin{array}{r} -25x-5z=-40 \\ 7x+5z=22 \\ \hline -18x=-18 \end{array}$		
		$\therefore x=1$ $y=2$ $z=3$	1 1 1	**4**
	e)	$20x+y-2z=17$ $3x+20y-z=-18$ $2x-3y+20z=25$		
		$\therefore x=\dfrac{17-y+2z}{20}$ $y=\dfrac{-18-3x+z}{20}$ $z=\dfrac{25-2x+3y}{20}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		Starting with $x_0 = 0 = y_0 = z_0$		
		$x_1 = 0.85$ $y_1 = -0.9$ $z_1 = 1.25$	1	
		$x_2 = 1.02$ $y_2 = -0.965$ $z_2 = 1.03$	1	
		$x_3 = 1.001$ $y_3 = -1.001$ $z_3 = 1.003$	1	**4**
	f)	$5x - 2y + 3z = 18$ $x + 7y - 3z = -22$ $2x - y + 6z = 22$		
		$\therefore x = \frac{18 + 2y - 3z}{5}$ $y = \frac{-22 - x + 3z}{7}$ $z = \frac{22 - 2x + y}{6}$	1	
		Starting with $x_0 = 0 = y_0 = z_0$		
		$x_1 = 3.6$ $y_1 = -3.657$ $z_1 = 1.857$	1	
		$x_2 = -4.0058$ $y_2 = -1.77$ $z_2 = 4.71$	1	
		$x_3 = 5.718$ $y_3 = -0.55$ $z_3 = 4.71$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	a) i)	$f(x)=x$		
		$\therefore f(x)$ is odd function.	1	
		$\therefore$ Fourier expansion is $f(x)=\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$		
		Where $b_n=\frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$	1	
		Here, $l=\pi$.		
		$\therefore b_n=\frac{2}{\pi}\int_0^{\pi} x\sin nx\cdot dx$	1	
		$=\frac{2}{\pi}\left[\frac{-x\cos nx}{n}+\frac{\sin nx}{n^2}\right]_0^{\pi}$	1	
		$=\frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}+\frac{\sin n\pi}{n^2}\right]-\frac{2}{\pi}\left[\frac{-0\cos 0}{n}+\frac{\sin 0}{n^2}\right]$	1	
		$=\frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n}+\frac{0}{n^2}\right]-0$	1	
		$=\frac{-2(-1)^n}{n}$ or may be taken as $\frac{2(-1)^{n+1}}{n}$	1	
		$\therefore f(x)=\sum_{n=1}^{\infty}\frac{-2(-1)^n}{n}\cdot\sin n\pi x$ or $\sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\cdot\sin n\pi x$	1	8
	a) ii)	$f(x)=e^x$		
		Fourier expansion is $f(x)=\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$	1	
		Where $b_n=\frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$	1	
		Here, $l=1$.		
		$\therefore b_n=2\int_0^1 e^x\sin n\pi x\cdot dx$		
		$=2\left[\frac{e^x}{1+n^2\pi^2}\left(\sin n\pi x-n\pi\cos n\pi x\right)\right]_0^1$	2	
		$=2\left[\frac{e}{1+n^2\pi^2}\left(\sin n\pi-n\pi\cos n\pi\right)\right]-2\left[\frac{1}{1+n^2\pi^2}\left(\sin 0-n\pi\cos 0\right)\right]$	1	
		$=2\left[\frac{e}{1+n^2\pi^2}\left(0-n\pi(-1)^n\right)\right]-2\left[\frac{1}{1+n^2\pi^2}\left(0-n\pi\right)\right]$	1	
		$=\frac{2n\pi}{1+n^2\pi^2}\left[e(-1)^n+1\right]$	1	
		$\therefore f(x)=\sum_{n=1}^{\infty}\frac{2n\pi}{1+n^2\pi^2}\left[e(-1)^n+1\right]\sin n\pi x$	1	8

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	b) i)	$I=\int_0^{\pi/4}\log\left(1+\tan x\right)dx$		
		$=\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$	½	
		$=\int_0^{\pi/4}\log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$	1	
		$=\int_0^{\pi/4}\log\left[\frac{2}{1+\tan x}\right]dx$		
		$=\int_0^{\pi/4}\left[\log 2-\log\left(1+\tan x\right)\right]dx$	½	
		$=\int_0^{\pi/4}\log 2dx-\int_0^{\pi/4}\log\left(1+\tan x\right)dx$		
		$=\int_0^{\pi/4}\log 2dx-I$	½	
		$\therefore 2I=\int_0^{\pi/4}\log 2dx$		
		$=\log 2\int_0^{\pi/4}dx$		
		$=\log 2\left[x\right]_0^{\pi/4}$	½	
		$=\frac{\pi}{4}\log 2$	½	
		$\therefore I=\frac{\pi}{8}\log 2$	½	**4**
	ii)	$I=\int_4^5\frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$		
		$=\int_4^5\frac{\sqrt{5-(9-x)}}{\sqrt{(9-x)-4}+\sqrt{5-(9-x)}}dx$	½	
		$I=\int_4^5\frac{\sqrt{x-4}}{\sqrt{5-x}+\sqrt{x-4}}dx$	1	
		$\therefore 2I=\int_4^5\frac{\sqrt{x-4}+\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$	½	
		$=\int_4^5 1\cdot dx$	½	
		$=\left[x\right]_4^5$	½	
		$=5-4$		
		$=1$	½	
		$\therefore I=\frac{1}{2}$	½	**4**
		OR		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$I=\int_4^5 \frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}\,dx$ Replace $x \to 9-x$ $\therefore 5-x \to x-4$ & $x-4 \to 5-x$	½	
		$I=\int_4^5 \frac{\sqrt{x-4}}{\sqrt{5-x}+\sqrt{x-4}}\,dx$	1	
		$\therefore 2I=\int_4^5 \frac{\sqrt{x-4}+\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}\,dx$	½	
		$=\int_4^5 1\cdot dx$	½	
		$=[x]_4^5$	½	
		$=5-4$	½	
		$=1$		
		$\therefore I=\frac{1}{2}$	½	**4**
	iii)	Given $I=10\sin(100\pi t)$ Mean Value $=\frac{1}{b-a}\int_a^b I\cdot dt$		
		$=\frac{1}{\frac{1}{50}-0}\int_0^{1/50} 10\sin(100\pi t)\,dt$	1	
		$=500\int_0^{1/50}\sin(100\pi t)\,dt$		
		$=500\left[\frac{-\cos(100\pi t)}{100\pi}\right]_0^{1/50}$	1	
		$=\frac{-5}{\pi}\left[\cos(2\pi)-\cos 0\right]$	1	
		$=\frac{-5}{\pi}[1-1]$		
		$=0$	1	**4**
		Important Note **In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.**		