

17349

15116

3 Hours / 100 Marks

Seat No.

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**Instructions :** (1) All Questions are *compulsory*.

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

**Marks**

**1. Attempt any TEN of the following :**

**20**

(a) Find the gradient of the curve  $y = \sqrt{x^3}$  at  $x = 4$ .

(b) Evaluate  $\int (1 - x)^{10} dx$

(c) Evaluate  $\int \frac{dx}{2x + 1}$

(d) Evaluate  $\int \frac{x}{x + 1} dx$

(e) Evaluate  $\int_2^4 \frac{dx}{2x + 3}$

(f) Evaluate  $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

(g) Define order and degree of Differential Equation.

(h) Show that  $(3x^2 - y) dx - x dy = 0$  is an exact D.E.

(i) Find the order and degree of D.E.

$$\left(\frac{dy}{dx}\right)^3 - \frac{d^2y}{dx^2} - 2 = 0$$

**P.T.O.**

- (j) An unbiased die is thrown. Write down the sample space. Hence, write down the following event sets : (i) The number on the die is even (ii) The number on the die is odd.
- (k) Find the slope of tangent to the curve  $xy = 6$  at  $(1, 6)$ .
- (l) Two fair coins are tossed. Write down the corresponding sample space for the experiment. Hence, write down the following event sets :
- (i) Exactly two heads
- (ii) At least one head
- (m) Find  $E(X)$  for the following probability distribution :
- |        |   |               |               |               |               |                |
|--------|---|---------------|---------------|---------------|---------------|----------------|
| $X$    | : | 8             | 12            | 16            | 20            | 24             |
| $P(X)$ | : | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{12}$ |
- (n) A random variable  $X$  follows a Poisson distribution with mean value 3.4. Find  $P(X = 6)$ .

**2. Attempt any FOUR of the following :**

**16**

- (a) The slope of the curve  $2y^3 = ax^2 + b$  at  $(1, -1)$  is same as the slope of  $x + y = 0$ . Find  $a$  and  $b$ .
- (b) Evaluate  $\int \frac{e^{2x} dx}{\sqrt{3 - 2e^x - 3e^{2x}}}$
- (c) Evaluate  $\int \operatorname{cosec}^3 x dx$
- (d) Evaluate  $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$
- (e) Find area enclosed between the curve  $y = 3x - 2 - x^2$  and  $x$ -axis.
- (f) Verify that  $y = e^{m \sin^{-1} x}$  is the solution of the differential equation

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$

**3. Attempt any FOUR of the following :****16**

- (a) A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.
- (b) Evaluate  $\int \frac{1}{1 - \sin 2x - 2\cos 2x} dx$
- (c) Evaluate  $\int_1^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx$
- (d) Find the area between the parabola  $y = x^2 + 3$  and the line  $y = x + 3$ .
- (e) Solve  $(x^3 + y^3) \frac{dy}{dx} = x^2y$ , given  $y = 1$  when  $x = 0$ .
- (f) An unbiased coin is tossed 5 times. Find the probability of getting (i) three heads, (ii) at least 4 heads.

**4. Attempt any FOUR of the following :****16**

- (a) Find the radius of curvature of the curve  $\sqrt{x} + \sqrt{y} = 1$  at  $\left(\frac{1}{4}, \frac{1}{4}\right)$ .
- (b) Evaluate  $\int \frac{x}{(x+1)(x-2)} dx$
- (c) Find the area bounded by  $y = 4x - x^2$  meeting the  $x$ -axis and the ordinates  $x = 1, x = 3$ .
- (d) Solve :  $\frac{dy}{dx} = (4x + y + 1)^2$
- (e) A box contains 2 silver and 4 gold coins and a second box contains 4 silver and 3 gold coins. If a coin is selected at random from one box, what is the probability that it is a gold coin ?
- (f) The number of road accidents met with by taxi drivers follow Poisson distribution with mean 2 out of 5000 taxis in the city. Find the number of drivers :
- (i) who does not meet with an accident.
- (ii) who met with an accident more than 3 times.

**5. Attempt any FOUR of the following :****16**

- (a) A telegraph wire hangs in the form of a curve  $y = a \log \sec \left( \frac{x}{a} \right)$ , where 'a' is constant. Show that the curvature at any point is  $\frac{1}{a} \cdot \cos \left( \frac{x}{a} \right)$ .
- (b) Evaluate  $\int \frac{\log x}{(1 + \log x)^2} dx$
- (c) Evaluate  $\int_0^{\pi/4} \log (1 + \tan x) dx$
- (d) Solve D.E.  $(2xy + y - \tan y) dx + (x^2 - x \cdot \tan^2 y + \sec^2 y) dy = 0$
- (e) A husband and wife appeared in an interview for two vacancies in an office. The probability of husband's selection is  $\frac{1}{7}$  and that of wife's selection is  $\frac{1}{5}$ . Find the probability that
- both of them are selected.
  - only one of them is selected.
- (f) A firm produces articles of which 0.1 percent are defective, out of 500 articles. If wholesaler purchases 100 such cases, how many can be expected to have one defective ? Given :  $e^{-0.5} = 0.6065$ .

**6. Attempt any FOUR of the following :****16**

- (a) Find the maximum and minimum values of  $x^3 - 9x^2 + 24x$ .
- (b) Evaluate  $\int \frac{dx}{\sqrt{13 - 6x - x^2}}$
- (c) Evaluate  $\int_0^1 x \cdot \sin^{-1} x dx$
- (d) Evaluate  $\int_{-1}^1 \frac{x + x^2}{1 + x^2} dx$
- (e) Solve D.E. :  $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$
- (f) Solve D.E. :  $\left[ y \left( 1 + \frac{1}{x} \right) + \cos y \right] dx + [x + \log x - x \sin y] dy = 0$ .
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