Model Answers

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.

2) The model answer and the answer written by candidate may vary but the examiner may tryto assess the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).

4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.

6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.

7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		**Attempt any ten of the following:**		
	a)	**Evaluate:** $\int e^{\log x} dx$		
	Ans	$\int e^{\log x} dx$		
		$= \int x dx \qquad .....e^{\log f(x)} = f(x)$	1	
		$= \frac{x^2}{2} + c$	1	02
		In the above solution, each term of last step carries ½ marks.		
		Note: In the above solution of any integration problems, if the constant c is not added, ½ marks may be deducted.		
	b)	**Evaluate:** $\int \left[\frac{1}{1+x^2} - \frac{\cos x}{\sin^2 x} \right] dx$		
	Ans	$= \int \left[\frac{1}{1+x^2} - \frac{\cos x}{\sin x \sin x} \right] dx$		
		$= \int \left[\frac{1}{1+x^2} - \cot x \operatorname{cos} ecx \right] dx$	1	
		$= \tan^{-1} x + \cos ecx + c$	1	02
	c)	**Evaluate:** $\int e^{\tan x} \sec^2 x dx$		
	Ans	$\tan x = t$		
		$\therefore \sec^2 x dx = dt$		
		$= \int e^t dt$	½	
		$= e^t + c$	1	
		$= e^{\tan x} + c$	½	02
	d)	**Evaluate:** $\int \log x dx$		
	Ans	$= \int \log x .1 dx$		
		Integrating by part		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$= \log x \int 1dx - \int \left(\frac{d}{dx} \log x \int 1dx \right) dx$	½	
		$= \log x . x - \int \frac{1}{x} . x dx$	½	
		$= \log x . x - x + c$	1	02
		OR $= x(\log x - 1) + c$		
	e)	**Find order and degree of differential equation**		
		$\frac{d^3y}{dx^3} = \left[k + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}$		
	Ans	$\left(\frac{d^3y}{dx^3} \right)^2 = \left[k + \left(\frac{dy}{dx} \right)^2 \right]^3$	1	
		Order=3 Degree=2	1	02
	f)	Solve: $xdy - ydx = 0$		
	Ans	$xdy - ydx = 0$	½	
		$\therefore xdy = ydx$		
		$\therefore \frac{dy}{y} = \frac{dx}{x} dx$	½	
		$\therefore \int \frac{dy}{y} = \int \frac{dx}{x}$		
		$\therefore \log y = \log x + c$	1	02
	g)	Find the equation of the curve whose slope is $(x-3)$		
		And which passes through $(2,0)$		
	Ans	$\frac{dy}{dx} = x - 3$		
		$dy = (x-3)dx$	½	
		$\int dy = \int (x-3)dx$		
		$y = \frac{x^2}{2} - 3x + c$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$x=0, y=0$ $\therefore 0=\frac{2^2}{2}-3(2)+c$ $c=4$ $\therefore y=\frac{x^2}{2}-3x+4$	½	02
	h)	Find $L[\sin 2t\sin 3t]$		
	Ans	$L[\sin 2t\sin 3t]$ $=L\left\{\frac{1}{2}\left[\cos(2t-3t)-\cos(2t+3t)\right]\right\}$	½	
		$=L\left\{\frac{1}{2}\left[\cos(-t)-\cos 5t\right]\right\}$ $=\frac{1}{2}\left\{L[\cos t]-L[\cos 5t]\right\}$		
		$=\frac{1}{2}\left\{\frac{s}{(s^2+1)}-\frac{s}{s^2+25}\right\}$	1	
		$=\frac{12s}{(s^2+1)(s^2+25)}$	½	02
	i)	Find $L\left[te^t\right]$		
	Ans	$L[t]=\frac{1}{s^2}$	1	
		$\therefore L\left[te^t\right]=\frac{1}{(s-1)^2}$ ….by first shifting property	1	02
	j)	Find $L^{-1}\left[\frac{3s+12}{s^2+8}\right]$		
	Ans	$=L^{-1}\left[\frac{3s}{s^2+8}\right]+L^{-1}\left[\frac{12}{s^2+8}\right]$	½	
		$=3L^{-1}\left[\frac{s}{s^2+\left(\sqrt{8}\right)^2}\right]+L^{-1}\left[\frac{12}{s^2+\left(\sqrt{8}\right)^2}\right]$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$=3\cos\sqrt{8}t+\frac{12}{\sqrt{8}}\sin\sqrt{8}t$	½	02
	k)	**Evaluate:** $\int e^{-x}dx$		
		$=\left[\frac{e^{-x}}{-1}\right]_0^{\infty}$	1	
		$=\left[-\frac{1}{e^{x}}\right]_0^{\infty}$		
		$=-\left[\frac{1}{e^{\infty}}-\frac{1}{e^{0}}\right]_0^{\infty}$	½	
		$=-[0-1]$ $=1$	½	02
	l)	Form the Differential equation from $y^2=ax^2$		
	Ans	Let $y^2=ax^2$		
		$\therefore 2y\frac{dy}{dx}=2ax$	1	
		$\therefore y\frac{dy}{dx}=ax$		
		$\therefore y\frac{dy}{dx}=\frac{y^2}{x^2}x$	1	
		$\therefore x\frac{dy}{dx}-y=0$		02
2)		**Attempt any four of the following:**		
	a)	Verify that $y=\sin(\log x)$ solution of D.E. differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$		
	Ans	$y=\sin(\log x)$		
		$\therefore\frac{dy}{dx}=\frac{\cos(\log x)}{x}$	1	
		$\therefore x\frac{dy}{dx}=\cos(\log x)$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$\therefore x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=\dfrac{-\sin(\log x)}{x}$	1	
		$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$	½	
		$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$	1	04
	b)	Solve $\dfrac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$		
	Ans	$\dfrac{dy}{dx}=e^{3x}e^{-2y}+x^2e^{-2y}$	½	
		$\therefore \dfrac{dy}{dx}=e^{-2y}\left(e^{3x}+x^2\right)$	½	
		$\therefore \dfrac{dy}{e^{-2y}}=\left(e^{3x}+x^2\right)$	1	
		$\therefore \int e^{2y}dy=\int\left(e^{3x}+x^2\right)dx$	1	04
		$\therefore \dfrac{e^{2y}}{2}=\dfrac{e^{3x}}{3}+\dfrac{x^3}{3}+c$	1	
	c)	Solve $ydx=xdy+\sqrt{xy}dx$		
	Ans	$y=x\dfrac{dy}{dx}+\sqrt{xy}$		
		$\therefore x\dfrac{dy}{dx}-y=-\sqrt{x}\sqrt{y}$		
		$\therefore \dfrac{1}{\sqrt{y}}\dfrac{dy}{dx}-\dfrac{\sqrt{y}}{x}=-\dfrac{1}{\sqrt{x}}$	½	
		Put $\sqrt{y}=t$		
		$\therefore \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx}=\dfrac{dt}{dx}$		
		$\dfrac{1}{\sqrt{y}}\dfrac{dy}{dx}=\dfrac{2dt}{dx}$	1	
		$\therefore 2\dfrac{dt}{dx}-\dfrac{t}{x}=-\dfrac{1}{\sqrt{x}}$		
		$\dfrac{dt}{dx}+\left(\dfrac{-1}{2x}\right)t=-\dfrac{1}{2\sqrt{x}}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$I.F = e^{\int \frac{-1}{2x}dx}$ $= e^{-\frac{1}{2}\log x}$ $= e^{\frac{1}{2}\log\frac{1}{x}} = \frac{1}{\sqrt{x}}$	½	
		$\therefore t\frac{1}{\sqrt{x}} = \int -\frac{1}{2\sqrt{x}}\frac{1}{\sqrt{x}}dx + c$ $\therefore \frac{\sqrt{y}}{\sqrt{x}} = \frac{-1}{2}\int \frac{1}{x}dx + c$	1	04
		$\therefore \sqrt{\frac{y}{x}} = \frac{-1}{2}.\log x + c$		
	d)	A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battery of 20 volts. Find the current in the circuit at any instant, if the relation between L,R and E is $L\frac{di}{dt} + Ri = \text{E}$		
	Ans	$L\frac{di}{dt} + Ri = E$ $\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ $P = \frac{R}{L} \quad and \quad Q = \frac{E}{L}$ $\therefore IF = e^{\int pdt} = e^{\int \frac{R}{L}dt} = e^{\frac{R}{L}t}$	½	
		$i \cdot IF = \int Q \cdot IF \cdot dt + c$	½	
		$\therefore i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} \cdot e^{\frac{R}{L}t} \cdot dt + c$ $\therefore i \cdot e^{\frac{R}{L}t} = \frac{E}{L} \cdot \frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$	1	
		$\therefore i \cdot e^{\frac{R}{L}t} = \frac{E}{R} \cdot e^{\frac{R}{L}t} + c$	½	
		$At\ i = 0,\ t = 0,$ $\therefore 0 = \frac{E}{R} \cdot e^{0} + c$ $\therefore c = -\frac{E}{R}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	d)	$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}\cdot e^{\frac{R}{L}t}-\frac{E}{R}\quad or\quad i=e^{-\frac{R}{L}t}\left[e^{\frac{R}{L}t}-1\right]\frac{E}{R}$		
		$Given\ \ R=100,\ \ L=0.1,\ \ E=20.$		
		$\therefore i\cdot e^{1000t}=\frac{1}{5}\cdot e^{1000t}-\frac{1}{5}\quad or\quad i=e^{-1000t}\left[e^{1000t}-1\right]\frac{1}{5}$	1	04
		Note: In the above example, L, R, E are arbitrary constants whereas i and t are variables. Also the values of L, R, E are given in advance. Thus these values can be substituted directly in the given differential equation and then the equation can be solved as illustrated below.		
	e)	Solve: $\left(e^x+2xy^2+y^3\right)dx+\left(a^y+2x^2y+3xy^2\right)dy=0$		
	Ans	$M=e^x+2xy^2+y^3$		
		$N=a^y+2x^2y+3xy^2$		
		$\frac{\partial M}{\partial y}=4xy+3y^2$		
		$\frac{\partial N}{\partial x}=4xy+3y^2$		
		$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	2	
		$\therefore$ Given D.E.is exact.		
		$\therefore$ solution is,		
		$\int\limits_{y-\text{constant}} Mdx+\int\limits_{\substack{terms\ not\\ containing\ 'x'}} Ndy=c$		
		$\int_{y-const}\left(e^x+2xy^2+y^3\right)dx+\int a^y dy=c$	½	
		$e^x+x^2y^2+xy^3+\frac{a^y}{\log a}=c$	1½	04
	f)	Obtain Fourier coefficient a_0 for $f\left(x\right)=2x-x^2$ over $\left(0,3\right)$		
	Ans	$a_0=\frac{1}{l}\int_0^{2l} f\left(x\right)dx$		
		$=\frac{2}{3}\int_0^{3}\left(2x-x^2\right)dx$	1	

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2)	f)	$=\frac{2}{3}\left[x^2-\frac{x^3}{3}\right]_0^3$	1	
		$=\frac{2}{3}\left[(9-9)-(0)\right]$	1	
		$=0$	1	
3)		Attempt any four of the following:		
	a)	Find $L\left\{\cos(at+b)\right\}$		
	Ans	$L\left\{\cos(at+b)\right\}=L\{\cos at.\cos b-\sin at.\sin b\}$	2	
		$=\frac{s\cos b}{s^2+a^2}-\frac{a\sin b}{s^2+a^2}$	1	
		$=\frac{s\cos b-a\sin b}{s^2+a^2}$	1	04
	b)	Find $L[t\sin t]$		
	Ans	$\because L[\sin t]=\frac{1}{s^2+1}$	1	
		$L[t\sin t]=-\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$	1	
		$=-\left(\frac{-2s}{s^2+1}\right)$	1	
		$=\left(\frac{2s}{s^2+1}\right)$	1	04
	c)	Find $L^{-1}\left[\frac{1}{s^2-2s+17}\right]$		
	Ans	$L^{-1}\left[\frac{1}{s^2-2s+17}\right]$		
		$=L^{-1}\left[\frac{1}{s^2-2s+1+17-1}\right]$	1	
		$=L^{-1}\left[\frac{1}{(s-1)^2+16}\right]==L^{-1}\left[\frac{1}{(s-1)^2+4^2}\right]$	1+1	

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3)		$=\frac{1}{4}\sin 4t.e^{t}$	1	04
	d)	Apply convulation thm to evaluate $L^{-1}\left(\frac{1}{s^2(s+1)}\right)$		
	Ans	Let $L^{-1}\left[\frac{1}{s^2}\right]=t=f(t)$		
		$L^{-1}\left[\frac{1}{s+1}\right]=e^{-t}=g(t)$	1	
		$\therefore\ f(u)=u$ and $g(t-u)=e^{-(t-u)}$		
		$L^{-1}\left(\frac{1}{s^2(s+1)}\right)$		
		$=\int_0^t f(u).g(t-u)\,du$	½	
		$=\int_0^t ue^{(u-t)}du$		
		$=\int_0^t ue^{u}e^{-t}du$		
		$=e^{-t}\int_0^t ue^{u}du$		
		$=e^{-t}\left[u\int e^{u}du-\int\left(\frac{d}{du}u\int e^{u}du\right)du\right]_0^t$	½	
		$=e^{-t}\left[ue^{u}-\int e^{u}du\right]_0^t$		
	\	$=e^{-t}\left[ue^{u}-e^{u}\right]_0^t$	1	
		$=e^{-t}\left[e^{u}(u-1)\right]_0^t$		
		$=e^{-t}\left[e^{t}(t-1)-e^{0}(0-1)\right]$	½	
		$=e^{-t}\left[e^{t}(t-1)+1\right]$		
		$=t-1+e^{-t}$	½	04
	e)	Solve $\frac{dy}{dt}-y=3e^{-2t}$ if $y(0)=-1$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	Ans	$\frac{dy}{dt}-y=3e^{-2t}$		
		$\therefore sY(s)-Y(0)-Y(s)=\frac{3}{s+2}$	1	
		$\therefore sY(s)+1-Y(s)=\frac{3}{s+2}$		
		$\therefore Y(s)(s-1)=\frac{3}{s+2}-1$	1	
		$Y(s)=\frac{(1-s)}{(s+2)(s-1)}$		
		$=-\frac{1}{s+2}$	1	
		$\therefore y(t)=L^{-1}\left[\frac{-1}{s+2}\right]$		04
		$=-e^{-2t}$	1	
	f)	Find $L^{-1}\left[\frac{s^2+1}{s^3+3s^2+2s}\right]$		
	Ans	$\frac{s^2+1}{s^3+3s^2+2s}=\frac{s^2+1}{s(s+2)(s+1)}$	½	
		$\therefore \frac{s^2+1}{s(s+2)(s+1)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+1}$		
		$\therefore s^2+1=A(s+2)(s+1)+Bs(s+1)+Cs(s+2)$		
			½	
		$\therefore A=\frac{1}{2}$		
		$\therefore B=\frac{5}{2}$	½	
		$\therefore C=-2$	½	
		$L^{-1}\left[\frac{s^2+1}{s(s+2)(s+1)}\right]=L^{-1}\left[\frac{\frac{1}{2}}{s}+\frac{\frac{5}{2}}{s+2}-\frac{2}{s+1}\right]$	1	
		$=\frac{1}{2}+\frac{5}{2}e^{-2t}-2e^{-t}$	1	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		Attempt any four of the following:		
	a)	Evaluate $\int \frac{1}{x\cos^2(\log x)}dx$		
	Ans	Put $\log x = t$		
		$\therefore \frac{1}{x}dx = dt$	1	
		$= \int \frac{dt}{\cos^2 t}$	1	
		$= \int \sec^2 t dt$		
		$= \tan t + c$	1	
		$= \tan(\log x) + c$	1	04
	b)	Evaluate $\int x\tan^{-1}x dx$		
	ans	$\int x\tan^{-1}x dx$		
		$= \tan^{-1}x\int x dx - \int\left(\frac{d}{dx}\tan^{-1}x\int x dx\right)dx$	½	
		$= \tan^{-1}x.\frac{x^2}{2} - \int \frac{1}{1+x^2}\frac{x^2}{2}dx$	1	
		$= \tan^{-1}x.\frac{x^2}{2} - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}dx$	½	
		$= \tan^{-1}x.\frac{x^2}{2} - \frac{1}{2}\left[\int dx - \int \frac{1}{x^2+1}dx\right]$	1	
		$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left[x - \tan^{-1}x\right] + c$	1	04
	c)	Evaluate $\int \frac{dx}{5+4\cos x}$		
	Ans	Put $t = \tan\frac{x}{2}$		
		$\therefore dx = \frac{2dt}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$=\int \frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$	½	
		$=2\int \frac{dt}{5+5t^2+4-4t^2}$	½	
		$=2\int \frac{dt}{t^2+9}$	1	
		$=2\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1	
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	½	04
	d)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x+\sin x}dx$		
	Ans	Let $I=\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x+\sin x}dx$ ---------------------------(1)		
		$=\int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx$ ....by property	½	
		$I=\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}dx$ .....(2)	½	
		$\therefore (1)+(2)$		
		$\therefore 2I=\int_0^{\frac{\pi}{2}} \frac{\sin x+\cos x}{\sin x+\cos x}dx$	½	
		$\therefore 2I=\int_0^{\frac{\pi}{2}} dx=[x]_0^{\frac{\pi}{2}}$	1	
		$2I=\frac{\pi}{2}-0$	1	04
		$I=\frac{\pi}{4}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	e)	Find the area enclosed by the curve $y=4x-x^2$ and x axis.		
	Ans	$y=4x-x^2$		
		$\therefore 0=4x-x^2$		
		$\therefore 0=x(4-x)$		
		$\therefore x=0$ or $x=4$	1	
		$\therefore Area=\int_0^4 \left(4x-x^2\right)dx$		
		$=\left[2x^2-\dfrac{x^3}{3}\right]_0^4$	1	
		$=2(4)^2-\dfrac{4^3}{3}-0$	1	04
		$=\dfrac{32}{3}$ or 10.66	1	
	f)	Find the mean value of $I=10\sin 100\pi t$ over a complete peroid .		
	Ans	Let $f(t)=10\sin 100\pi t$		
		$T=\dfrac{2\pi}{100\pi}=\dfrac{1}{50}$		
		$\therefore a=0, b=\dfrac{1}{50}$	1	
		$\therefore$ mean value $\bar{y}=\dfrac{1}{b-a}\int_a^b f(t)dt$		
		$=\dfrac{1}{\frac{1}{50}-0}\int_0^{\frac{1}{50}} 10\sin 100\pi t dt$	½	
		$=500\left[\dfrac{-\cos 100\pi t}{100\pi}\right]_0^{\frac{1}{50}}$	1	
		$=\dfrac{-500}{100\pi}\left[\cos 2\pi-\cos 0\right]$	1	
		$=-\dfrac{5}{\pi}[1-1]=0$	½	04
5)		**Attempt any four of the following:**		
	a)	Find the positive root of the equqtion $x=e^{-x}$ using bisection method .		
	Ans	Let $f(x)=x-e^{-x}$		
		$\therefore f(0)=-1$		

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5)		$\therefore f(1) = 0.63$		
		Root lies in $(0,1)$	1	
		$\therefore a = 0$ b=1		
		$x_1 = \dfrac{a+b}{2}$	1	
		$= \dfrac{0+1}{2} = 0.5$		
		$f(0.5) = -0.10 < 0$		
		$\therefore$ root lies in $(0.5,1)$		
		$\therefore x_2 = \dfrac{0.5+1}{2} = 0.75$	1	
		$f(0.75) = 0.27 > 0$		
		$\therefore$ root lies in $(0.5,0.75)$		
		$x_3 = \dfrac{0.5+0.75}{2} = 0.625$	1	04
		OR		
		Let $f(x) = x - e^{-x}$ $\therefore f(0) = -1 < 0$		
		$\therefore f(1) = 0.63 > 0$		
		root lies in $(0,1)$	1	
		$\therefore a = 0$ b=1		
		<table><tr><td>a</td><td>b</td><td>$x = \dfrac{a+b}{2}$</td><td>$f(x)$</td></tr><tr><td>0</td><td>1</td><td>0.5</td><td>-0.10</td></tr><tr><td>0.5</td><td>1</td><td>0.75</td><td>0.27</td></tr><tr><td>0.5</td><td>0.75</td><td>0.625</td><td>---</td></tr></table>	1+1+1	
	b)	Obtain the root of the equation by Regula-Falsi method		04
		$x^3 - 2x - 5 = 0$ (three iterations only).		
	Ans	Let $f(x) = x^3 - 2x - 5$		
		$f(2) = -1$		
		$f(3) = 16$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\therefore$ root lies in (2,3)		04
		$x_1=\dfrac{af(b)-bf(a)}{f(b)-f(a)}=\dfrac{2(16)-3(-1)}{16-(-1)}=2.058$	1½	
		$f(2.058)=-0.399<0$		
		$\therefore$ root lies in (2.058,3)		
		$x_2=\dfrac{x_1f(b)-bf(x_1)}{f(b)-f(x_1)}=\dfrac{2.058(16)-3(-0.399)}{16-(-0.399)}=2.08$	1½	04
		OR		
		Let $f(x)=x^3-2x-5$		
		$f(2)=-1$ $f(3)=16$	1	
		(table below)	1½ +1½	04
	c)	Using Newton-Raphson method, find an approximate value of the equation $x^3-3x-5=0$ which lies near to $x=2$ (Carry out two iterations only)		
	Ans	$x^3-3x-5=0$		
		$f(x)=x^3-3x-5$		
		$f'(x)=3x^2-3$	1	
		$\therefore$ Initial root $x_0=2$		
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=2-\dfrac{(-3)}{9}=2.33$	1½	
		$x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}=2.33-\dfrac{0.659}{13.28}=2.28$	1½	04
	d)	Solve using Gauss elimination method:		
	Ans	$x+y+z=6, 3x-y+3z=10, 5x+5y-4z=3$		

a	b	$f(a)$	$f(b)$	$x=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$
2	3	-1	16	2.058	-0.399
2.058	3	-0.399	16	2.08	---

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	Ans	Given $x+y+z=6$ $3x-y+3z=10$ $5x+5y-4z=3$		
		$\begin{array}{r} x+y+z=6 \\ +\quad 3x-y+3z=10 \\ \hline 4x+4z=16 \end{array}$ and $\begin{array}{r} 15x-5y+15z=50 \\ +\quad 5x+5y-4z=3 \\ \hline 20x+11z=53 \end{array}$ $\therefore x+z=4$	1+1	
		$\begin{array}{r} 20x+20z=80 \\ 20x+11z=53 \\ -\quad -\quad - \\ \hline 9z=27 \end{array}$ $\therefore z=3$	1	
		$y=2$ $x=1$	1	04
	e)	Solve using Jacobi's method. (three iterations only) $20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$		
	Ans	$x=\frac{1}{20}(17-y+2z)$ $y=\frac{1}{20}(-18-3x+z)$ $z=\frac{1}{20}(25-2x+3y)$	1	
		Starting with $x_0=y_0=z_0=0$ $x_1=0.85$ $y_1=-0.9$ $z_1=1.25$	1	
		$x_2=1.02$ $y_2=-0.965$ $z_2=1.03$	1	
		$x_3=1.001$ $y_3=-1.001$ $z_3=1.003$	1	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	a)	Attempt any four of the following:		
	i)	Obtain Furier expansion for e^x in the interval $-\pi < x < \pi$		
	Ans	Let $f(x) = e^x$		
		$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x dx$		
		$= \frac{1}{\pi}\left[e^x\right]_{-\pi}^{\pi}$	1	
		$= \frac{1}{\pi}\left[e^{\pi} - e^{-\pi}\right]$	1	
		$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx dx$		
		$= \frac{1}{\pi}\left[\frac{e^x}{1+n^2}\left(\cos nx + n\sin nx\right)\right]_{-\pi}^{\pi}$	1	
		$= \frac{1}{\pi}\left[\frac{e^{\pi}}{1+n^2}\left(\cos n\pi + n\sin n\pi\right) - \frac{e^{-\pi}}{1+n^2}\left(\cos n\pi - n\sin n\pi\right)\right]$		
		$= \frac{1}{\pi}\left[\frac{e^{\pi}}{1+n^2}(-1)^n - \frac{e^{-\pi}}{1+n^2}(-1)^n\right]$	½	
		$= \frac{(-1)^n}{\pi(1+n^2)}\left[e^{\pi} - e^{-\pi}\right]$	1	
		$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx dx$		
		$= \frac{1}{\pi}\left[\frac{e^x}{1+n^2}\left(\sin nx - n\cos nx\right)\right]_{-\pi}^{\pi}$		
		$= \frac{1}{\pi}\left[\frac{e^{\pi}}{1+n^2}\left(\sin n\pi - n\cos n\pi\right) - \frac{e^{-\pi}}{1+n^2}\left(-\sin n\pi - n\cos n\pi\right)\right]$	1	
		$= \frac{1}{\pi}\left[-\frac{ne^{\pi}}{1+n^2}(-1)^n + \frac{ne^{-\pi}}{1+n^2}(-1)^n\right]$	½	
		$= \frac{n(-1)^n}{\pi(1+n^2)}\left[-e^{\pi} + e^{-\pi}\right]$	1	
		$\therefore f(x) = \frac{1}{2\pi}\left(e^{\pi} - e^{-\pi}\right) + \sum_{n=1}^{\infty}\left[\frac{(-1)^n}{\pi(1+n^2)}.\left(e^{\pi} - e^{-\pi}\right) - \frac{n(-1)^n}{\pi(1+n^2)}.\left(e^{\pi} - e^{-\pi}\right)\right]$	1	08

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	a) ii) Ans	Find F-series for the function 1 $f(x)=x-x^2, \quad 1<x<1$ Given $f(x)=x-x^2$		
		$a_0=\frac{1}{1}\int_{-1}^{1}(x-x^2)dx$	½	
		$=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{1}$	1	
		$=\left[\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)\right]$		
		$=\frac{-2}{3}$	½	
		$a_n=\frac{1}{1}\int_{-1}^{1}(x-x^2)\cos nx dx$	½	
		$=\left[(x-x^2)\frac{\sin nx}{n}-(1-2x)\frac{(-\cos nx)}{n^2}+(-2)\frac{(-\sin nx)}{n^3}\right]_{-1}^{1}$	1	
		$=\left[2\sin n\left(\frac{1}{n}+\frac{2}{n^3}\right)-\frac{4\sin n}{n^3}\right]$	1	
		$b_n=\frac{1}{1}\int_{-1}^{1}(x-x^2)\sin nx dx$	½	
		$=\left[(x-x^2)\frac{(-\cos nx)}{n}-(1-2x)\frac{(-\sin nx)}{n^2}+(-2)\frac{(\cos nx)}{n^3}\right]_{-1}^{1}$	1	
		$=\left[\sin n\left(\frac{-1}{n^2}+\frac{3}{n^3}\right)\right]$	1	
		$\therefore f(x)=-\frac{1}{3}+\sum_{n=1}^{\infty}\left[\left(\frac{1}{n}+\frac{2}{n^3}\right)2\sin n.\cos nx+\left(\frac{-1}{n^2}+\frac{3}{n^3}\right)\sin n.\sin nx\right]$	1	08
6)	b)	Attempt any two of the following:		
	i)	Evaluate: $\int_4^5\frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$		
	Ans	Let I= $\int_4^5\frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$ ---------------------------(1)		
		I= $\int_4^5\frac{\sqrt{5-(9-x)}}{\sqrt{(9-x)-4}+\sqrt{5-(9-x)}}dx$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$I=\int_4^5 \frac{\sqrt{x-4}}{\sqrt{5-x}+\sqrt{x-4}}dx$ --------------(2)	½	
		Add (1) and (2)		
		$I+I=\int_4^5 \frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx+\int_4^5 \frac{\sqrt{x-4}}{\sqrt{5-x}+\sqrt{x-4}}dx$	1	
		$2I=\int_4^5 \frac{\sqrt{5-x}+\sqrt{x-4}}{\sqrt{x-4}+\sqrt{5-x}}dx$		
		$2I=\int_4^5 dx$	½	
		$2I=\left[x\right]_4^5$	1	
		2I =5-4=1		
		$I=\frac{1}{2}$	½	04
	b) ii)	Evaluate: $\int \frac{dx}{\sin x+\sin 2x}$		
	Ans	$\int \frac{dx}{\sin x+\sin 2x}$		
		$=\int \frac{dx}{\sin x+2\sin x.\cos x}$		
		Put $\tan\frac{x}{2}=t$ $dx=\frac{2dt}{1+t^2}$		
		$\cos x=\frac{1-t^2}{1+t^2}$ $\sin x=\frac{2t}{1+t^2}$	½	
		$\int \frac{1}{\frac{2t}{1+t^2}+2.\frac{2t}{1+t^2}.\frac{1-t^2}{1+t^2}}.\frac{2dt}{1+t^2}$		
		$=2\int \frac{1+t^2}{2t\left(1+t^2\right)+4t-4t^3}.dt=2\int \frac{1+t^2}{2t-2t^3+4t}.dt$	½	
		$=\int \frac{1+t^2}{3t-t^3}.dt=\int \frac{1+t^2}{t\left(3-t^2\right)}dt$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$=\int \frac{1+t^2}{t\left(3-t^2\right)}dt$		
		$\frac{1+t^2}{t\left(\sqrt{3}-t\right)\left(\sqrt{3}+t\right)}=\frac{A}{t}+\frac{B}{\sqrt{3}-t}+\frac{c}{\sqrt{3}+t}$	½	
		$\therefore 1+t^2=\left(\sqrt{3}-t\right)\left(\sqrt{3}+t\right)A+t\left(\sqrt{3}+t\right)B+t\left(\sqrt{3}-t\right)C$		
		$A=\frac{1}{3}$	½	
		$B=\frac{2}{3}$	½	
		$C=\frac{-2}{3}$		
		$\int \frac{1+t^2}{t\left(\sqrt{3}-t\right)\left(\sqrt{3}+t\right)}dt=\int\left[\frac{1/3}{t}+\frac{2/3}{\sqrt{3}-t}+\frac{-2/3}{\sqrt{3}+t}\right]dt$		
		$=\frac{1}{3}\log t-\frac{2}{3}\log\left(\sqrt{3}-t\right)-\frac{2}{3}\log\left(\sqrt{3}+t\right)+c$	1	
		$=\frac{1}{3}\log\tan\frac{x}{2}-\frac{2}{3}\log\left(\sqrt{3}-\tan\frac{x}{2}\right)-\frac{2}{3}\log\left(\sqrt{3}-\tan\frac{x}{2}\right)+c$	½	04
	iii)	Find R.M.S.values of i where $i=I\sin pt$		
	Ans	Given $i=I\sin pt$		
		$R.M.S.value=\sqrt{\frac{1}{b-a}\int_a^b i^2dt}$		
		$a=0,b=2\pi$	½	
		$=\sqrt{\frac{1}{2\pi-0}\int_0^{2\pi} I^2\sin^2 ptdt}$		
		$=\sqrt{\frac{1}{2\pi}\int_0^{2\pi} I^2\left[\frac{1-\cos 2pt}{2}\right]dt}$	½	
		$=\frac{I}{2}\sqrt{\frac{1}{\pi}\left[t-\frac{\sin 2pt}{2p}\right]_0^{2\pi}}$	1	
		$=\frac{I}{2}\sqrt{\frac{1}{\pi}\left[2\pi-\frac{\sin 4p\pi}{2p}-0\right]}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$=\frac{I}{2}\sqrt{\frac{1}{\pi}[2\pi]}$	1	
		$=\frac{I\sqrt{2}}{2}$ or $\frac{I}{\sqrt{2}}$	½	04
		***<u>Important Note:</u>* In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.**		