Que. No.	Sub. Que.	Model answers	Marks	Total Marks

Important Instructions to the Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

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1)		**Attempt any TEN of the following:**		
	a)	If $f(x)=x-x^{-1}$ then show that $f\left(\frac{1}{x}\right)=-f(x)$.		
	Ans.	$f(x)=x-x^{-1}$		
		$\therefore f\left(\frac{1}{x}\right)=\frac{1}{x}-\left(\frac{1}{x}\right)^{-1}$	½	
		$=\frac{1}{x}-x$	½	
		$=-\left(x-\frac{1}{x}\right)$ *or* $-\left(x-x^{-1}\right)$	½	
		$=-f(x)$	½	**2**
		OR		
		$f(x)=x-x^{-1}$		
		$\therefore f\left(\frac{1}{x}\right)=\frac{1}{x}-\left(\frac{1}{x}\right)^{-1}$	½	
		$=\frac{1}{x}-x$	½	
		$-f(x)=-\left(x-x^{-1}\right)$		
		$=-\left(x-\frac{1}{x}\right)$		
		$=-x+\frac{1}{x}$	½	
		$\therefore f\left(\frac{1}{x}\right)=-f(x)$	½	**2**
		---		
	b)	If $f(x)=x^7-5x^3+3\sin x$ then find $f(x)+f(-x)$		
	Ans.	$f(x)=x^7-5x^3+3\sin x$		
		$\therefore f(-x)=(-x)^7-5(-x)^3+3\sin(-x)$	½	
		$=(-1)^7x^7-5(-1)^3(x)^3+3(-\sin x)$		
		$=-x^7+5x^3-3\sin x$	½	
		$\therefore f(x)+f(-x)=x^7-5x^3+3\sin x-x^7+5x^3-3\sin x$	½	
		$=0$	½	**2**
		OR		

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1)		$f(x)=x^7-5x^3+3\sin x$		
		$\therefore f(-x)=(-x)^7-5(-x)^3+3\sin(-x)$	½	
		$=(-1)^7x^7-5(-1)^3(x)^3+3(-\sin x)$		
		$=-x^7+5x^3-3\sin x$	½	
		$=-(x^7-5x^3+3\sin x)$		
		$=-f(x)$	½	
		$\therefore f(x)+f(-x)=0$	½	**2**
		OR		
		$f(x)=x^7-5x^3+3\sin x$		
		$\therefore f(x)+f(-x)=\left[x^7-5x^3+3\sin x\right]+\left[(-x)^7-5(-x)^3+3\sin(-x)\right]$	½	
		$=x^7-5x^3+3\sin x+(-1)^7x^7-5(-1)^3(x)^3+3(-\sin x)$		
		$=x^7-5x^3+3\sin x-x^7+5x^3-3\sin x$	1	**2**
		$=0$	½	
		---		
	c)	Evaluate $\lim\limits_{x\to 1}\dfrac{x^3+1}{x+1}$		
	Ans.	$\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}=\lim\limits_{x\to -1}\dfrac{(x+1)(x^2-x+1)}{x+1}$	½	
		$=\lim\limits_{x\to -1}(x^2-x+1)$	½	
		$=(-1)^2-(-1)+1$	½	
		$=3$	½	**2**
		OR		
		$\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}=\lim\limits_{x\to -1}\dfrac{x^3+1^3}{x+1}$	½	
		$=3(-1)^2$	1	
		$=3$	½	**2**
		Note: The above solution method is the direct application of the result $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$ *or* $\lim\limits_{x\to -a}\dfrac{x^n+a^n}{x+a}=n(-a)^{n-1}$		
		---		
	d)	Evaluate $\lim\limits_{x\to 0}\dfrac{\sin 7x}{\tan 3x}$		
	Ans.	$\lim\limits_{x\to 0}\dfrac{\sin 7x}{\tan 3x}=\lim\limits_{x\to 0}\dfrac{\frac{\sin 7x}{x}}{\frac{\tan 3x}{x}}$		

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1)		$=\lim\limits_{x\to 0}\dfrac{\dfrac{\sin 7x}{7x}\times 7}{\dfrac{\tan 3x}{3x}\times 3}$	½	
		$=\dfrac{1\times 7}{1\times 3}$	1	
		$=\dfrac{7}{3}$	½	**2**
		OR		
		$\lim\limits_{x\to 0}\dfrac{\sin 7x}{\tan 3x}=\lim\limits_{x\to 0}\dfrac{\sin 7x}{x}\times\dfrac{x}{\tan 3x}$		
		$=\lim\limits_{x\to 0}\dfrac{\sin 7x}{7x}\times 7\times\dfrac{3x}{\tan 3x}\times\dfrac{1}{3}$	½	
		$=1\times 7\times 1\times\dfrac{1}{3}$	1	
		$=\dfrac{7}{3}$	½	**2**
	e)	Evaluate $\lim\limits_{x\to 0}\dfrac{e^{5+2x}-e^{5}}{x}$		
	Ans.	$\lim\limits_{x\to 0}\dfrac{e^{5+2x}-e^{5}}{x}=\lim\limits_{x\to 0}\dfrac{e^{5}\cdot e^{2x}-e^{5}}{x}$		
		$=\lim\limits_{x\to 0}\dfrac{e^{5}\left(e^{2x}-1\right)}{x}$	½	
		$=\lim\limits_{x\to 0}e^{5}\left(\dfrac{e^{2x}-1}{x}\right)$		
		$=\lim\limits_{x\to 0}e^{5}\left(\dfrac{e^{2x}-1}{2x}\times 2\right)$	½	
		$=e^{5}\left(1\times 2\right)$	½	
		$=2e^{5}$	½	**2**
	f)	Evaluate $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{2x}\right)^{2x}$		
	Ans.	$\lim\limits_{x\to\infty}\left(1+\dfrac{3}{2x}\right)^{2x}$		
		$=\lim\limits_{x\to\infty}\left(1+\dfrac{3}{2x}\right)^{\frac{2x}{3}\times 3}$	1	
		$=e^{3}$	1	**2**

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1)	g)	If $y=\sec x+\tan x$, then show that $\frac{1}{y}\frac{dy}{dx}=\sec x$		
	Ans.	$y=\sec x+\tan x$		
		$\therefore \frac{dy}{dx}=\sec x\tan x+\sec^2 x$	1	
		$=\sec x(\sec x+\tan x)$		
		$=\sec x\cdot y$	½	
		$\therefore \frac{1}{y}\frac{dy}{dx}=\sec x$	½	**2**
		OR		
		$y=\sec x+\tan x$		
		$\therefore \frac{dy}{dx}=\sec x\tan x+\sec^2 x$	1	
		$=\sec x(\sec x+\tan x)$		
		$\therefore \frac{1}{y}\frac{dy}{dx}=\frac{1}{\sec x+\tan x}\times\sec x(\sec x+\tan x)$	½	
		$=\sec x$	½	**2**
		---		
	h)	Evaluate $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$		
	Ans.	$\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$		
		$=\lim\limits_{x\to 0}\frac{(1+x)-(1-x)}{x}\times\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$	½	
		$=\lim\limits_{x\to 0}\frac{1+x-1+x}{x}\times\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$		
		$=\lim\limits_{x\to 0}\frac{2x}{x}\times\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$	½	
		$=\lim\limits_{x\to 0}2\times\frac{1}{\sqrt{1+x}+\sqrt{1-x}}$		
		$=2\times\frac{1}{\sqrt{1}+\sqrt{1}}$	½	
		$=1$	½	**2**
		---		
	i)	Find $\frac{dy}{dx}$, if $y=x\sin x+\cos x$		
	Ans.	$\frac{dy}{dx}=x\cdot\cos x+\sin x\cdot 1-\sin x$	1	
		$=x\cos x$	1	**2**

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1)	j)	Find $\frac{dy}{dx}$, if $y=\cot(4x+5)$		
	Ans.	$\frac{dy}{dx}=-\operatorname{cosec}^2(4x+5)\cdot\frac{d}{dx}(4x+5)$	½	
		$=-\operatorname{cosec}^2(4x+5)\cdot(4+0)$	½	
		$=-4\operatorname{cosec}^2(4x+5)$	1	**2**
		OR		
		$Put\ u=4x+5$		
		$\therefore\frac{du}{dx}=4$	½	
		$\therefore y=\cot u$		
		$\therefore\frac{dy}{du}=-\operatorname{cosec}^2 u$	½	
		$\therefore\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$		
		$=-\operatorname{cosec}^2(4x+5)\cdot 4$		
		$=-4\operatorname{cosec}^2(4x+5)$	1	**2**
	k)	The mean of 20 observations is 25 and mean of 10 observations is 15. Find the mean of combined 30 observations.		
	Ans.	$\overline{x_1}=25 \quad N_1=20$		
		$\overline{x_2}=15 \quad N_2=10$		
		$\therefore\bar{x}=\frac{N_1\overline{x_1}+N_2\overline{x_2}}{N_1+N_2}$		
		$=\frac{20\times25+10\times15}{20+10}$	1	
		$=21.667$	1	**2**
	l)	Find the median and mode of the following data: 5, 8, 10, 9, 7, 6, 5, 8, 5.		
	Ans.	Rearrange the data as: 5, 5, 5, 6, 7, 8, 8, 9, 10		
		$Median=value\ \left(\frac{N+1}{2}\right)th\ item$		
		$=5th\ value$		
		$=7$	1	
		$Mode=item\ having\ \max.\ frequency$		
		$=5$	1	**2**

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2)		**Attempt any FOUR of the following:**		
	a)	Find $f(t)$, if $f(x)=\frac{x+2}{4x-3}$ and $t=\frac{2+3x}{4x-1}$.		
	Ans.	$\therefore f(t)=\frac{t+2}{4t-3}$		
		$=\frac{\frac{2+3x}{4x-1}+2}{4\left(\frac{2+3x}{4x-1}\right)-3}$	1	
		$=\frac{\frac{2+3x+2(4x-1)}{4x-1}}{\frac{4(2+3x)-3(4x-1)}{4x-1}}$		
		$=\frac{2+3x+2(4x-1)}{4(2+3x)-3(4x-1)}$	1	
		$=\frac{2+3x+8x-2}{8+12x-12x+3}$		
		$=\frac{11x}{11}$	1	
		$=x$	1	4
	b)	If $f(t)=4.5\left[\sin pt+\frac{1}{2}\sin 2pt\right]$, then show that $f\left(\frac{2\pi}{p}+t\right)=f(t)$.		
	Ans.	$f\left(\frac{2\pi}{p}+t\right)=4.5\left[\sin p\left(\frac{2\pi}{p}+t\right)+\frac{1}{2}\sin 2p\left(\frac{2\pi}{p}+t\right)\right]$	1	
		$=4.5\left[\sin(2\pi+pt)+\frac{1}{2}\sin(4\pi+2pt)\right]$	1	
		$=4.5\left[\sin pt+\frac{1}{2}\sin 2pt\right]$	1	
		$=f(t)$	1	4
	c)	Evaluate $\lim_{x\to3}\frac{x^2-x-6}{x^3-3x^2+x-3}$		
	Ans.	$\lim_{x\to3}\frac{x^2-x-6}{x^3-3x^2+x-3}=\lim_{x\to3}\frac{(x-3)(x+2)}{(x-3)(x^2+1)}$	1	
		$=\lim_{x\to3}\frac{x+2}{x^2+1}$	1	
		$=\frac{3+2}{3^2+1}$	1	
		$=\frac{1}{2}$	1	4

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2)	d)	Evaluate $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$		
	Ans.	$\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\times\dfrac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$		
		$=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\left(\sqrt{x}\right)^2-\left(\sqrt{a}\right)^2}\times\left(\sqrt{x}+\sqrt{a}\right)$		
		$=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}\times\left(\sqrt{x}+\sqrt{a}\right)$	½	
		$Put\ x-a=t \quad As\ \ x\to a,\ \ t\to 0$	½	
		$\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}=\lim\limits_{t\to 0}\dfrac{\sin(t+a)-\sin a}{t}\times\left(\sqrt{t+a}+\sqrt{a}\right)$	½	
		$=\lim\limits_{t\to 0}\dfrac{2\cos\left(\dfrac{t+2a}{2}\right)\sin\left(\dfrac{t}{2}\right)}{t}\times\left(\sqrt{t+a}+\sqrt{a}\right)$	1	
		$=\lim\limits_{t\to 0}2\cos\left(\dfrac{t+2a}{2}\right)\times\dfrac{\sin\left(\dfrac{t}{2}\right)}{\dfrac{t}{2}}\times\dfrac{1}{2}\times\left(\sqrt{t+a}+\sqrt{a}\right)$	½	
		$=2\cos(a)\times 1\times\dfrac{1}{2}\times\left(\sqrt{a}+\sqrt{a}\right)$	½	
		$=2\sqrt{a}\cos a$	½	**4**
		OR		
		$\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\times\dfrac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$		
		$=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}\times\left(\sqrt{x}+\sqrt{a}\right)$	½	
		$=\lim\limits_{x\to a}\dfrac{2\cos\left(\dfrac{x+a}{2}\right)\sin\left(\dfrac{x-a}{2}\right)}{x-a}\times\left(\sqrt{x}+\sqrt{a}\right)$	1	
		$=\lim\limits_{x\to a}2\cos\left(\dfrac{x+a}{2}\right)\times\dfrac{\sin\left(\dfrac{x-a}{2}\right)}{\dfrac{x-a}{2}}\times\dfrac{1}{2}\times\left(\sqrt{x}+\sqrt{a}\right)$	1	
		$=2\cos(a)\times 1\times\dfrac{1}{2}\times\left(\sqrt{a}+\sqrt{a}\right)$	1	
		$=2\sqrt{a}\cos a$	½	**4**

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2)	e)	Evaluate $\lim_{x\to 0}\frac{4^{2x}-2^{3x}}{x}$		
	Ans.	$\lim_{x\to 0}\frac{4^{2x}-2^{3x}}{x}=\lim_{x\to 0}\frac{4^{2x}-1-2^{3x}+1}{x}$	½	
		$=\lim_{x\to 0}\frac{\left(4^{2x}-1\right)-\left(2^{3x}-1\right)}{x}$		
		$=\lim_{x\to 0}\left(\frac{4^{2x}-1}{x}-\frac{2^{3x}-1}{x}\right)$	1	
		$=\lim_{x\to 0}\left(\frac{4^{2x}-1}{2x}\times 2-\frac{2^{3x}-1}{3x}\times 3\right)$	1	
		$=\log 4\times 2-\log 2\times 3$	1	
		$=2\log 4-3\log 2$	½	4
		OR		
		$\lim_{x\to 0}\frac{4^{2x}-2^{3x}}{x}=\lim_{x\to 0}\frac{16^{x}-8^{x}}{x}$	1	
		$=\lim_{x\to 0}\frac{16^{x}-1-8^{x}+1}{x}$	1	
		$=\lim_{x\to 0}\frac{\left(16^{x}-1\right)-\left(8^{x}-1\right)}{x}$		
		$=\lim_{x\to 0}\left(\frac{16^{x}-1}{x}-\frac{8^{x}-1}{x}\right)$	1	
		$=\log 16-\log 8$ OR $\log\left(\frac{16}{8}\right)=\log 2$	1	4
		OR $2\log 4-3\log 2$		
		---		
	f)	Evaluate $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x}$		
	Ans.	$\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x}=\lim_{x\to\infty}\left(\frac{1+\frac{2}{x}}{1+\frac{1}{x}}\right)^{x}$	1	
		$=\lim_{x\to\infty}\frac{\left(1+\frac{2}{x}\right)^{x}}{\left(1+\frac{1}{x}\right)^{x}}$		
		$=\lim_{x\to\infty}\frac{\left(1+\frac{2}{x}\right)^{\frac{x}{2}\times 2}}{\left(1+\frac{1}{x}\right)^{x}}$	1	
		$=\frac{e^{2}}{e}$	1	
		$=e$	1	4

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3)		**Attempt any FOUR of the following:**		
	a)	If $x=3\cos t-2\cos^3 t,\quad y=3\sin t-2\sin^3 t$ then find $\frac{dy}{dx}$.		
	Ans.	$x=3\cos t-2\cos^3 t$		
		$\frac{dx}{dt}=-3\sin t-2\times3\cos^2 t\left(-\sin t\right)$		
		$=-3\sin t+6\cos^2 t\sin t$		
		$=3\sin t\left(-1+2\cos^2 t\right)$	1	
		$y=3\sin t-2\sin^3 t$		
		$\frac{dy}{dt}=3\cos t-2\times3\sin^2 t\left(\cos t\right)$		
		$=3\cos t-6\sin^2 t\cos t$		
		$=3\cos t\left(1-2\sin^2 t\right)$	1	
		$\therefore\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3\cos t\left(1-2\sin^2 t\right)}{3\sin t\left(-1+2\cos^2 t\right)}$		
		$=\frac{\cos t\cos 2t}{\sin t\cos 2t}$	1	
		$=\cot t$	1	**4**
		---		
	b)	Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w. r. t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$		
	Ans.	$u=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$		
		$Put\ \ x=\cos\theta$		
		$u=\tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)=\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)=\tan^{-1}\left(\tan\theta\right)=\theta$		
		$\therefore u=\cos^{-1}x$	1	
		$\therefore\frac{du}{dx}=-\frac{1}{\sqrt{1-x^2}}$	½	
		$v=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$		
		$Put\ \ x=\sin\theta\qquad\left(or\ \ also\ \ x=\cos\theta\right)$		
		$\therefore v=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)=\sin^{-1}\left(2\sin\theta\cos\theta\right)=\sin^{-1}\left(\sin 2\theta\right)=2\theta$		
		$\therefore v=2\sin^{-1}x$	1	
		$\therefore\frac{dv}{dx}=\frac{2}{\sqrt{1-x^2}}$	½	

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3)		$\therefore \frac{du}{dv}=\frac{du/dx}{dv/dx}=\frac{-\frac{1}{\sqrt{1-x^2}}}{\frac{2}{\sqrt{1-x^2}}}=-\frac{1}{2}$	1	**4**
		--		
	c)	Find $\frac{dy}{dx}$, $\sin xy+\cos(x+y)=1$.		
	Ans.	$\sin xy+\cos(x+y)=1$		
		$\therefore \cos xy\frac{d}{dx}(xy)-\sin(x+y)\frac{d}{dx}(x+y)=0$	1	
		$\therefore \cos xy\left(x\frac{dy}{dx}+y\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)=0$	1	
		$\therefore x\cos xy\frac{dy}{dx}+y\cos xy-\sin(x+y)-\sin(x+y)\frac{dy}{dx}=0$		
		$\therefore [x\cos xy-\sin(x+y)]\frac{dy}{dx}=\sin(x+y)-y\cos xy$	1	
		$\therefore \frac{dy}{dx}=\frac{\sin(x+y)-y\cos xy}{x\cos xy-\sin(x+y)}$	1	**4**
		--		
	d)	Differentiate $(\sin x)^x$ w. r. t. x.		
	Ans.	$Let\ \ y=(\sin x)^x$		
		$\therefore \log y=x\log(\sin x)$	1	
		$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=x\cdot\frac{d}{dx}[\log(\sin x)]+\log(\sin x)\cdot\frac{d}{dx}(x)$		
		$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=x\left[\frac{1}{\sin x}\cdot\cos x\right]+\log(\sin x)\cdot[1]$	1	
		$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=x\cot x+\log(\sin x)$	1	
		$\therefore \frac{dy}{dx}=y[x\cot x+\log(\sin x)]$	1	**4**
		--		
	e)	If $x^y=e^{x-y}$, then show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$		
	Ans.	$x^y=e^{x-y}$		
		$\therefore y\log x=x-y$	½	
		$\therefore y\log x+y=x$		
		$\therefore y(\log x+1)=x$	½	

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3)		$\therefore y=\dfrac{x}{\log x+1}$	1	
		$\therefore \dfrac{dy}{dx}=\dfrac{(\log x+1)\cdot 1-x\left(\dfrac{1}{x}+0\right)}{(\log x+1)^2}$	1	
		$\therefore \dfrac{dy}{dx}=\dfrac{\log x+1-1}{(\log x+1)^2}$		
		$\therefore \dfrac{dy}{dx}=\dfrac{\log x}{(\log x+1)^2}$	1	4
		OR		
		$x^y=e^{x-y}$		
		$\therefore y\log x=x-y \quad ...(1)$	½	
		$\therefore y\log x+y=x \quad ...(2)$		
		$\therefore y\cdot\dfrac{1}{x}+\log x\dfrac{dy}{dx}+\dfrac{dy}{dx}=1$	1	
		$\therefore (\log x+1)\dfrac{dy}{dx}=1-\dfrac{y}{x}=\dfrac{x-y}{x}$	½	
		$\therefore \dfrac{dy}{dx}=\dfrac{x-y}{x(\log x+1)}$	1	
		$=\dfrac{y\log x}{y(\log x+1)(\log x+1)} \quad (\text{by } 1\ \&\ 2)$		
		$=\dfrac{\log x}{(\log x+1)^2}$	1	4
		--		
	f)	If $y=\sin\left(m\sin^{-1}x\right)$, the show that $\left(1-x^2\right)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+m^2y=0$		
	Ans.	$y=\sin\left(m\sin^{-1}x\right)$		
		$\therefore \dfrac{dy}{dx}=\cos\left(m\sin^{-1}x\right)\cdot m\cdot\dfrac{1}{\sqrt{1-x^2}}$	1	
		$\therefore \sqrt{1-x^2}\cdot\dfrac{dy}{dx}=m\cos\left(m\sin^{-1}x\right)$		
		$\therefore \dfrac{1}{2\sqrt{1-x^2}}(-2x)\cdot\dfrac{dy}{dx}+\sqrt{1-x^2}\dfrac{d^2y}{dx^2}\cdot=-m\cdot\sin\left(m\sin^{-1}x\right)\cdot m\cdot\dfrac{1}{\sqrt{1-x^2}}$	1	
		$\therefore -x\cdot\dfrac{dy}{dx}+\left(1-x^2\right)\dfrac{d^2y}{dx^2}\cdot=-m^2\cdot y$	1	
		$\therefore \left(1-x^2\right)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+m^2y=0$	1	4
		OR		

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3)		**OR**		
		$y=\sin\left(m\sin^{-1}x\right)$		
		$\therefore \frac{dy}{dx}=\cos\left(m\sin^{-1}x\right)\cdot m\cdot\frac{1}{\sqrt{1-x^2}}$	1	
		$\therefore \sqrt{1-x^2}\cdot\frac{dy}{dx}=m\cos\left(m\sin^{-1}x\right)$		
		$\therefore \left(1-x^2\right)\cdot\left(\frac{dy}{dx}\right)^2=m^2\cos^2\left(m\sin^{-1}x\right)$		
		$\therefore \left(1-x^2\right)\cdot 2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot(-2x)$		
		$=m^2\cdot 2\cos\left(m\sin^{-1}x\right)\left[-\sin\left(m\sin^{-1}x\right)\cdot m\cdot\frac{1}{\sqrt{1-x^2}}\right]$	1	
		$\therefore 2\cdot\frac{dy}{dx}\left[\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}\right]=-2m^2y\frac{dy}{dx}$	1	
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-m^2y$		
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$	1	4
		---		
4)		**Attempt any FOUR of the following:**		
	a)	In a potentiometer circuit, R is given by $R=\frac{1}{x}-\frac{1}{x-a}$ where a is constant. Find the value of x which makes R minimum. Also calculate this minimum value of R.		
	Ans.	$R=\frac{1}{x}-\frac{1}{x-a}$		
		$\therefore \frac{dR}{dx}=-\frac{1}{x^2}+\frac{1}{(x-a)^2}$	1	
		$\therefore \frac{d^2R}{dx^2}=\frac{2}{x^3}-\frac{2}{(x-a)^3}$	1	
		$For\ stationary\ values,\ \frac{dR}{dx}=0$		
		$\therefore -\frac{1}{x^2}+\frac{1}{(x-a)^2}=0 \quad or \quad \frac{1}{(x-a)^2}=\frac{1}{x^2} \quad or \quad x^2=(x-a)^2$		
		$\therefore 2ax=a^2$		
		$\therefore x=\frac{a}{2}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$At\ x=\frac{a}{2},\quad \frac{d^2R}{dx^2}=\frac{2}{\left(\frac{a}{2}\right)^3}-\frac{2}{\left(\left(\frac{a}{2}\right)-a\right)^3}=\frac{32}{a^3}>0$	½	
		$\therefore At\ x=\frac{a}{2},\ R\ has\ \min imum\ value\ \ and\ it\ is$ $R=\frac{1}{\frac{a}{2}}-\frac{1}{\frac{a}{2}-a}=\frac{4}{a}$	½	**4**
		--		
	b)	The rate of working of an engine is given by the expression $10v+\frac{4000}{v}$. Find the speed v at which the rate of working is the least.		
	Ans.	$Let\ R=10v+\frac{4000}{v}$		
		$\therefore \frac{dR}{dv}=10-\frac{4000}{v^2}$	1	
		$\therefore \frac{d^2R}{dv^2}=\frac{8000}{v^3}$	1	
		$For\ \ stationary\ values,\ \frac{dR}{dv}=0$		
		$\therefore 10-\frac{4000}{v^2}=0\quad or\ \ 10=\frac{4000}{v^2}\ \ or\ \ v^2=400$		
		$\therefore v=20,\ \ -20$	1	
		$At\ \ v=20,\quad \frac{d^2R}{dv^2}=\frac{8000}{20^3}>0$	½	
		$\therefore At\ \ v=20,\ R\ \ has\ \min imum\ value.$	½	**4**
		--		
	c)	Show that the radius of curvature for the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ at any point θ is $4a\cos\frac{\theta}{2}$.		
	Ans.	$x=a(\theta+\sin\theta)$		
		$\therefore \frac{dx}{d\theta}=a(1+\cos\theta)$	½	
		$y=a(1-\cos\theta)$		
		$\therefore \frac{dy}{d\theta}=a\sin\theta$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$\therefore \dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}$ $=\dfrac{a\sin\theta}{a(1+\cos\theta)}$ $=\dfrac{\sin\theta}{1+\cos\theta}$ $=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$ $=\tan\dfrac{\theta}{2}$	 1	
		$\therefore \dfrac{d^2y}{dx^2}=\sec^2\dfrac{\theta}{2}\cdot\dfrac{d\theta}{dx}\cdot\dfrac{1}{2}$ $=\dfrac{1}{2}\sec^2\dfrac{\theta}{2}\cdot\dfrac{1}{a(1+\cos\theta)}$ $=\dfrac{1}{2}\sec^2\dfrac{\theta}{2}\cdot\dfrac{1}{a\left(2\cos^2\frac{\theta}{2}\right)}$ $=\dfrac{1}{4a}\sec^4\left(\dfrac{\theta}{2}\right)$	 1	
		$\therefore \rho=\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ $=\dfrac{\left[1+\tan^2\left(\frac{\theta}{2}\right)\right]^{3/2}}{\frac{1}{4a}\sec^4\left(\frac{\theta}{2}\right)}$	 ½	
		$=\dfrac{\sec^3\left(\frac{\theta}{2}\right)}{\frac{1}{4a}\sec^4\left(\frac{\theta}{2}\right)}$ $=\dfrac{4a}{\sec\left(\frac{\theta}{2}\right)}$ $=4a\cos\left(\dfrac{\theta}{2}\right)$	 ½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	d)	If $y=4x-5$ touches the curve $y^2=ax^3+b$ at the point (2, 3). Find the value of a and b.		
	Ans.	$y^2=ax^3+b$		
		$\therefore 2y\dfrac{dy}{dx}=3ax^2$	½	
		$\therefore \dfrac{dy}{dx}=\dfrac{3ax^2}{2y}$	½	
		$\therefore at\left(2,\ 3\right),\ slope\ m=\dfrac{3a\left(2\right)^2}{2\left(3\right)}=2a$	½	
		But slope of given tangent is $m=4$		
		$\therefore 2a=4$	½	
		$\therefore \boxed{a=2}$	1	
		But the point (2, 3) is on the line.		
		$\therefore 3^2=a\left(2\right)^3+b$		
		$\therefore 9=8a+b=16+b$		
		$\therefore \boxed{b=-7}$	1	**4**
		--		
	e)	Find the median marks obtained by 49 students in the following distribution table graphically.		

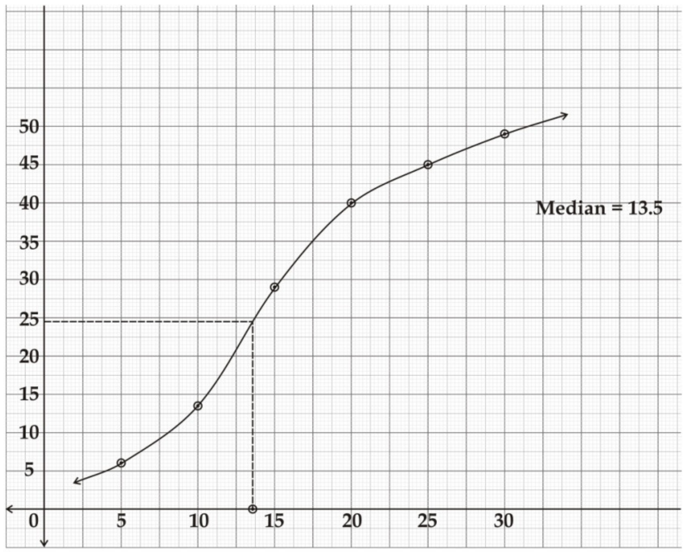
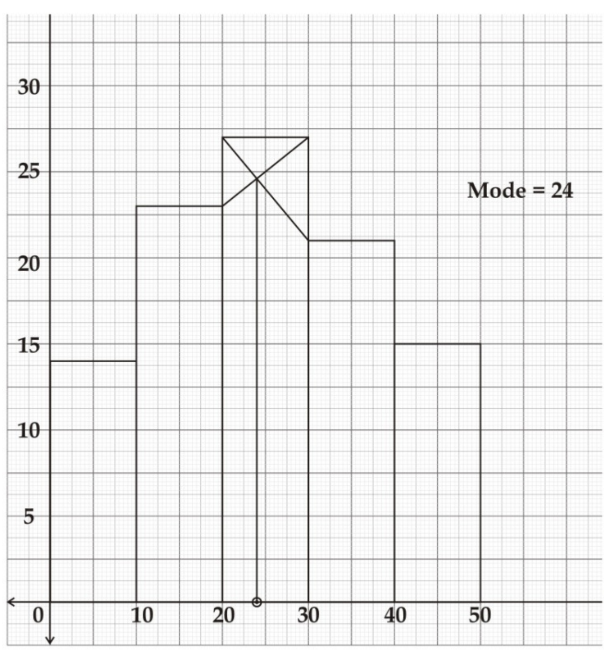
Marks	5-10	10-15	15-20	20-25	25-30	30-35
No. of Students	06	07	16	11	05	04

Ans.

Class	Fi	< c. f.
0-5	6	6
5-10	7	13
10-15	16	29
15-20	11	40
20-25	5	45
25-30	4	49

1

(Note: The median can also be calculated by drawing Greater than Ogive Curve and also by drawing both the ogives simultaneously. So marks to be given accordingly. If the graph is too small or not clear to understand, marks can be deducted. On x-axis, instead of writing points 0, 5, 10, 15, … etc., if class 0-5, 5-10, 10-15, … etc. are written, no marks to be given. The same is also applicable for histogram.)

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)			1+1+1	
		Marks Distribution: 1 mark for plotting points and drawing curve correctly. 1 mark for drawing line of median to x-axis. 1 mark for value of median. Note the value 16 is approximate value. Difference of **+0.5 or –0.5** is acceptable in case of graph.		**4**
	f)	Find the mode graphically for the following distribution.		
		C. I.: 0-10, 10-20, 20-30, 30-40, 40-50; Frequency: 14, 23, 27, 21, 15		
	Ans.		1+1+1+1	**4**

C. I.	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		**Marks Distribution**: 1 mark for plotting points and drawing histogram correctly. 1 mark for drawing the cross lines in the modal class. 1 mark for drawing line of mode to x-axis. 1 mark for value of mode. Note the value 24 is approximate value. Difference of **+0.5 or –0.5** is acceptable in case of graph.		
		(Note: **If the graph is too small or not clear to understand, marks can be deducted. On x-axis, instead of writing points 10, 20, 30, … etc., if class 0-10, 10-20, 20-30, … etc. are written, no marks to be given.)**		
5)		**Attempt any FOUR of the following:**		
	a)	The mean weight of 100 students is 55 kg. Two weights are wrongly recorded as 49 and 73 instead of 69 and 63 kg. Find the correct mean weight of the students.		
	Ans.	$Given\ \bar{x}=55,\quad n=100$		
		$\sum x_i = n\bar{x} = 100\times 55 = 5500$	1	
		$\therefore Incorrect \sum x_i = 5500$		
		$\therefore Correct \sum x_i = Incorrect \sum x_i - wrong + correct\ items$		
		$= 5500-49-73+69+63 = 5510$	2	
		$\therefore Correct\ mean = \dfrac{Correct \sum x_i}{n} = \dfrac{5510}{100} = 55.1$	1	4
	b)	Calculate the median of the following frequency distribution analytically:		

C.I.	45-60	60-75	75-90	90-105	105-120	120-135	135-150
fi	43	99	152	180	160	40	26

Ans.

Class	Fi	< c. f.
45-60	43	43
60-75	99	142
75-90	152	294
90-105	**180**	**474**
105-120	160	634
120-135	40	674
135-150	26	700

1

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$N=700 \qquad \therefore \frac{N}{2}=350$		
		$\therefore Median = L+\frac{\frac{N}{2}-C.F.}{f}\times h$		
		$=90+\frac{350-294}{180}\times 15$	2	
		$=94.66$	1	**4**
	c)	Find the mean deviation about mean of the following data:		

C. I.	0-10	10-20	20-30	30-40	40-50
Frequency	15	18	21	17	12

Class	xi	f_i	$f_i x_i$	$D_i=\left\|x_i-\bar{x}\right\|$	$f_i D_i$
0-10	5	15	75	19.157	287.355
10-20	15	18	270	9.157	164.826
20-30	25	21	525	0.843	17.703
30-40	35	17	595	10.843	184.331
40-50	45	12	540	20.843	250.116
		83	**2005**		**904.331**

1+1

$\bar{x}=\frac{\sum f_i x_i}{N}=\frac{2005}{83}=24.157$ 1

$M.D.=\frac{\sum f_i D_i}{N}$

$=\frac{904.331}{83}$

$=10.896$ 1 **4**

d) Find the mean deviation about median of the following distribution.

xi	3	4	5	6	7	8
fi	4	9	10	8	6	3

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	Ans.			

xi	f_i	C. F.	$D_i = \left\|x_i - \bar{x}\right\|$	$f_i D_i$
3	4	4	2	8
4	9	13	1	9
5	10	23	0	0
6	8	31	1	8
7	6	37	2	12
8	3	40	3	9
	40			**46**

Marks: 1+1

$N = 40 \qquad \therefore \frac{N}{2} = 20 \qquad \therefore Median = 5$ — 1

$$M.D. = \frac{\sum f_i D_i}{N}$$

$$= \frac{46}{40}$$

$$= 1.15$$

Marks: 1 — Total: **4**

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e) Find the standard deviation for the following frequency distribution:

C. I.	0-20	20-40	40-60	60-80	80-100
Frequency	20	130	220	70	60

Ans.

Class	xi	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
0-20	10	20	100	200	2000
20-40	30	130	900	3900	117000
40-60	50	220	2500	11000	550000
60-80	70	70	4900	4900	343000
80-100	90	60	8100	5400	486000
		500		25400	1498000

Marks: 2

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{1498000}{500} - \left(\frac{25400}{500}\right)^2}$$ — 1

$$= 20.38$$ — 1

Total: **4**

OR

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)				

Class	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-20	10	20	-2	-40	4	80
20-40	30	130	-1	-130	1	130
40-60	50	220	0	0	0	0
60-80	70	70	1	70	1	70
80-100	90	60	2	120	4	240
		500		**20**		**520**

(Marks: 2)

$A = 50 \quad h = 20, \quad d_i = \dfrac{x_i - A}{h}$

$$S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= 20 \times \sqrt{\frac{520}{500} - \left(\frac{20}{500}\right)^2}$$ (1)

$$= 20.38$$ (1) — Total: **4**

Note: Students may take any another value for A in the above/ below example. So the above table and corresponding values vary accordingly. But the final answer will be the same.

----

f) In two factories A and B, the average weekly wages in Rs. and the standard deviation are as follows:

Factory	Average wages	S. D.
A	34.5	5.0
B	28.5	4.5

Which factory A or B has greater variability in individual wages?

Ans.

$$C.V.(A) = \frac{\sigma}{\bar{x}} \times 100 = \frac{5}{34.5} \times 100 = 14.493$$ (1)

$$C.\,V.(B) = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.5}{28.5} \times 100 = 15.789$$ (1)

$$\therefore C.V.(A) < C.\,V.(B)$$ (1)

$\therefore$ B is more variable. (1) — Total: **4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		**For Civil, Electrical and Mechanical Groups**		
		Attempt any FOUR of the following:		
	a)	If $z_1 = 3+2i,\ z_2 = 3-5i$, find $\frac{z_1}{z_2} + \frac{\overline{z_1}}{\overline{z_2}}$		
	Ans.	$\therefore \overline{z_1} = 3-2i, \qquad \overline{z_2} = 3+5i$		
		$\frac{z_1}{z_2} + \frac{\overline{z_1}}{\overline{z_2}} = \frac{3+2i}{3-5i} + \frac{3-2i}{3+5i}$		
		$= \frac{3+2i}{3-5i} \times \frac{3+5i}{3+5i} + \frac{3-2i}{3+5i} \times \frac{3-5i}{3-5i}$	½	
		$= \frac{9+15i+6i-10}{9+25} + \frac{9-15i-6i-10}{9+25}$	1	
		$= \frac{-1+21i}{34} + \frac{-1-21i}{34}$	1	
		$= \frac{-1+21i-1-21i}{34}$	½	
		$= \frac{-2}{34}$	½	
		$= -\frac{1}{17}$	½	**4**
		OR		
		$\therefore \overline{z_1} = 3-2i, \qquad \overline{z_2} = 3+5i$		
		$\therefore \frac{z_1}{z_2} = \frac{3+2i}{3-5i}$		
		$= \frac{3+2i}{3-5i} \times \frac{3+5i}{3+5i}$		
		$= \frac{9+15i+6i-10}{9+25}$		
		$= \frac{-1+21i}{34}$	1	
		$\therefore \frac{\overline{z_1}}{\overline{z_2}} = \frac{3-2i}{3+5i}$	½	
		$= \frac{3-2i}{3+5i} \times \frac{3-5i}{3-5i}$		
		$= \frac{9-15i-6i-10}{9+25}$		
		$= \frac{-1-21i}{34}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$\therefore \frac{z_1}{z_2}+\overline{\frac{z_1}{z_2}}=\frac{-1+21i}{34}+\frac{-1-21i}{34}$		
		$=\frac{-1+21i-1-21i}{34}$	½	
		$=\frac{-2}{34}$	½	
		$=-\frac{1}{17}$	½	**4**
	b)	Express $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ in the polar form.		
	Ans.	$Let\ \ z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$		
		$\therefore r=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=1$	1½	
		$\theta=\tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)=\tan^{-1}\left(\sqrt{3}\right)=60^\circ\ \ or\ \ \frac{\pi}{3}$	1½	
		$\therefore z=r\left(\cos\theta+i\sin\theta\right)$		
		$=\cos 60^\circ+i\sin 60^\circ \quad or \quad \cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$	1	**4**
	c)	Prove that		
		$\left(1+\cos\theta+i\sin\theta\right)^n+\left(1+\cos\theta-i\sin\theta\right)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$		
	Ans.	$\left(1+\cos\theta+i\sin\theta\right)^n=\left[\left(1+\cos\theta\right)+i\left(\sin\theta\right)\right]^n$		
		$=\left[2\cos^2\frac{\theta}{2}+i\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\right]^n$	½	
		$=\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right]^n$		
		$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n$	½	
		$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)$	½	
		$\therefore\left(1+\cos\theta-i\sin\theta\right)^n=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$	1	

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<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>6)</td><td></td><td>
$\therefore \left(1+\cos\theta+i\sin\theta\right)^n+\left(1+\cos\theta-i\sin\theta\right)^n$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)+2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$
$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$
$=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$
</td><td>½
½
½</td><td>4</td></tr>
<tr><td></td><td></td><td>
OR
Let $z=1+\cos\theta+i\sin\theta=\left(1+\cos\theta\right)+i\left(\sin\theta\right)$
$\therefore r=\sqrt{\left(1+\cos\theta\right)^2+\sin^2\theta}$
$=\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}=\sqrt{1+2\cos\theta+1}$
$=\sqrt{2+2\cos\theta}$
$=\sqrt{2\left(1+\cos\theta\right)}=\sqrt{2\left(2\cos^2\frac{\theta}{2}\right)}$
$=\sqrt{4\cos^2\frac{\theta}{2}}$
$=2\cos\frac{\theta}{2}$
$\theta=\tan^{-1}\left[\frac{\sin\theta}{1+\cos\theta}\right]=\tan^{-1}\left[\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right]=\tan^{-1}\left[\tan\frac{\theta}{2}\right]=\frac{\theta}{2}$
$\therefore z=2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)$
$\left(1+\cos\theta+i\sin\theta\right)^n=\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right]^n$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)$
$\therefore \left(1+\cos\theta-i\sin\theta\right)^n=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$
$\therefore \left(1+\cos\theta+i\sin\theta\right)^n+\left(1+\cos\theta-i\sin\theta\right)^n$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)+2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$
$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$
</td><td>½
½
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1
½</td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$	½	
		$=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$	½	**4**
		OR		
		$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$		
		$=\left[2\cos^2\frac{\theta}{2}+i\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\right]^n+\left[2\cos^2\frac{\theta}{2}-i\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\right]^n$		
		$=\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right]^n+\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)\right]^n$	1	
		$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n+2^n\cos^n\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^n$	½	
		$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)+2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$	1	
		$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$	½	
		$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$	½	
		$=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$	½	**4**
		--		
	d)	Simplify using Demoiver's theorem $$\frac{(\cos3\theta+i\sin3\theta)^4(\cos5\theta-i\sin5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta+i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}}\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}}$$		
	Ans.	$\frac{(\cos3\theta+i\sin3\theta)^4(\cos5\theta-i\sin5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta+i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}}\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}}$		
		$=\frac{(\cos\theta+i\sin\theta)^{3\times4}(\cos\theta+i\sin\theta)^{-5\times\frac{4}{5}}}{(\cos\theta+i\sin\theta)^{\frac{9}{2}\times\frac{2}{3}}(\cos\theta+i\sin\theta)^{\frac{4}{5}\times10}}$	2	
		$=\frac{(\cos\theta+i\sin\theta)^{12}(\cos\theta+i\sin\theta)^{-4}}{(\cos\theta+i\sin\theta)^{3}(\cos\theta+i\sin\theta)^{8}}$	1	
		$=(\cos\theta+i\sin\theta)^{12-4-3-8}$		
		$=(\cos\theta+i\sin\theta)^{-3}$	½	
		$=\cos3\theta-i\sin3\theta$	½	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		**OR**		
		$\left(\cos 3\theta + i\sin 3\theta\right)^4 = \left(\cos\theta + i\sin\theta\right)^{3\times 4} = \left(\cos\theta + i\sin\theta\right)^{12}$	½	
		$\left(\cos 5\theta - i\sin 5\theta\right)^{\frac{4}{5}} = \left(\cos\theta + i\sin\theta\right)^{-5\times\frac{4}{5}} = \left(\cos\theta + i\sin\theta\right)^{-4}$	½	
		$\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}} = \left(\cos\theta + i\sin\theta\right)^{\frac{9}{2}\times\frac{2}{3}} = \left(\cos\theta + i\sin\theta\right)^{3}$	½	
		$\left(\cos\frac{4}{5}\theta + i\sin\frac{4}{5}\theta\right)^{10} = \left(\cos\theta + i\sin\theta\right)^{\frac{4}{5}\times 10} = \left(\cos\theta + i\sin\theta\right)^{8}$	½	
		$\therefore \dfrac{\left(\cos 3\theta + i\sin 3\theta\right)^4 \left(\cos 5\theta - i\sin 5\theta\right)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}} \left(\cos\frac{4}{5}\theta + i\sin\frac{4}{5}\theta\right)^{10}}$		
		$= \dfrac{\left(\cos\theta + i\sin\theta\right)^{12}\left(\cos\theta + i\sin\theta\right)^{-4}}{\left(\cos\theta + i\sin\theta\right)^{3}\left(\cos\theta + i\sin\theta\right)^{8}}$	1	
		$= \left(\cos\theta + i\sin\theta\right)^{12-4-3-8}$		
		$= \left(\cos\theta + i\sin\theta\right)^{-3}$	½	
		$= \cos 3\theta - i\sin 3\theta$	½	4
		OR		
		$\dfrac{\left(\cos 3\theta + i\sin 3\theta\right)^4 \left(\cos 5\theta - i\sin 5\theta\right)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}} \left(\cos\frac{4}{5}\theta + i\sin\frac{4}{5}\theta\right)^{10}}$		
		$= \dfrac{\left(e^{3i\theta}\right)^4 \left(e^{-5i\theta}\right)^{\frac{4}{5}}}{\left(e^{\frac{9}{2}i\theta}\right)^{\frac{2}{3}} \left(e^{\frac{4}{5}i\theta}\right)^{10}}$	2	
		$= \dfrac{\left(e^{12i\theta}\right)\left(e^{-4i\theta}\right)}{\left(e^{3i\theta}\right)\left(e^{8i\theta}\right)}$	1	
		$= e^{12i\theta - 4i\theta - 3i\theta - 8i\theta}$		
		$= e^{-3i\theta}$	½	
		$= \cos 3\theta - i\sin 3\theta$	½	4
		OR		
		$\left(\cos 3\theta + i\sin 3\theta\right)^4 = \left(e^{3i\theta}\right)^4 = e^{12i\theta}$	½	
		$\left(\cos 5\theta - i\sin 5\theta\right)^{\frac{4}{5}} = \left(e^{-5i\theta}\right)^{\frac{4}{5}} = e^{-4i\theta}$	½	
		$\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}} = \left(e^{\frac{9}{2}i\theta}\right)^{\frac{2}{3}} = e^{3i\theta}$	½	
		$\left(\cos\frac{4}{5}\theta + i\sin\frac{4}{5}\theta\right)^{10} = \left(e^{\frac{4}{5}i\theta}\right)^{10} = e^{8i\theta}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$\therefore \dfrac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 5\theta - i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{9}{2}\theta + i\sin\frac{9}{2}\theta\right)^{\frac{2}{3}} \left(\cos\frac{4}{5}\theta + i\sin\frac{4}{5}\theta\right)^{10}}$		
		$= \dfrac{(e^{12i\theta})(e^{-4i\theta})}{(e^{3i\theta})(e^{8i\theta})}$	1	
		$= e^{12i\theta - 4i\theta - 3i\theta - 8i\theta}$		
		$= e^{-3i\theta}$	½	
		$= \cos 3\theta - i\sin 3\theta$	½	**4**
		--		
	e)	Using Euler's formulae prove that $\cos^2\theta = 1 - \sin^2\theta$		
	Ans.	$\cos^2\theta = \left[\dfrac{e^{i\theta} + e^{-i\theta}}{2}\right]^2$		
		$= \dfrac{(e^{i\theta})^2 + 2e^{i\theta}e^{-i\theta} + (e^{-i\theta})^2}{4}$		
		$= \dfrac{e^{2i\theta} + 2 + e^{-2i\theta}}{4}$	1	
		$1 - \sin^2\theta = 1 - \left[\dfrac{e^{i\theta} - e^{-i\theta}}{2i}\right]^2$		
		$= 1 - \dfrac{e^{2i\theta} - 2e^{i\theta}e^{-i\theta} + e^{-2i\theta}}{-4}$		
		$= 1 + \dfrac{e^{2i\theta} - 2 + e^{-2i\theta}}{4}$	1	
		$= \dfrac{4 + e^{2i\theta} - 2 + e^{-2i\theta}}{4}$		
		$= \dfrac{e^{2i\theta} + 2 + e^{-2i\theta}}{4}$	1	
		$\therefore \cos^2\theta = 1 - \sin^2\theta$	1	**4**
		--		
	f)	Separate into real and imaginary parts $\cosh(x+iy)$.		
	Ans.	$\cosh(x+iy) = \cosh x \cosh iy + \sinh x \sinh iy$	2	
		$= \cosh x \cos y + i\sinh x \sin y$	2	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		**For Computer/Information Technology Group**		
		Attempt any FOUR of the following:		
	a)	Find the roots of the equation $x^3-4x-9=0$ using bisection method by taking three iterations.		
	Ans.	$f(x)=x^3-4x-9$		
		$\therefore f(2)=-9$	½	
		$f(3)=6$	½	
		$\therefore$ the root is in $(2,\ 3)$.	½	
		$\therefore x_1=\dfrac{2+3}{2}=2.5$	½	
		$\therefore f(2.5)=-3.375$	½	
		$\therefore$ the root is in $(2.5,\ 3)$.		
		$\therefore x_2=\dfrac{2.5+3}{2}=2.75$	½	
		$\therefore f(2.75)=0.797$	½	
		$\therefore$ the root is in $(2.5,\ 2.75)$.		
		$\therefore x_3=\dfrac{2.5+2.75}{2}=2.625$	½	4
		OR		
		$f(x)=x^3-4x-9$		
		$\therefore f(2)=-9$	½	
		$f(3)=6$	½	
		$\therefore$ the root is in $(2,\ 3)$.	½	

a	b	$f(a)$	$f(b)$	$x=\dfrac{a+b}{2}$	$f(x)$	Marks	Total Marks
2	3	-9	6	2.5	-3.375	1	
2.5	3	-3.375	6	2.75	0.797	1	
2.5	2.75	-3.375	0.797	2.625	---	½	4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
	b)	Solve the equation $e^x-4x=0$ using regula-falsi method by taking three iterations.		
	Ans.	$f(x)=e^x-4x$		
		$\therefore f(0)=1$	½	
		$f(1)=-1.282$	½	
		$\therefore$ the root is in $(0,\ 1)$.	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$\therefore x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = 0.438$	½	
		$\therefore f(0.438) = -0.202$	½	
		$\therefore$ the root is in $(0,\ 0.438)$.		
		$\therefore x_2 = 0.364$	½	
		$\therefore f(0.364) = -0.0169$	½	
		$\therefore$ the root is in $(0,\ 0.364)$.		
		$\therefore x_3 = 0.358$	½	4
		OR		
		$f(x) = e^x - 4x$		
		$\therefore f(0) = 1$	½	
		$f(1) = -1.282$	½	
		$\therefore$ the root is in $(0,\ 1)$.	½	

a	b	$f(a)$	$f(b)$	$x = \frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$	Marks
0	1	1	-1.282	0.438	-0.202	1
0	0.438	1	-0.202	0.364	-0.0169	1
0	0.364	1	-0.0169	0.358	---	½

Total Marks: 4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
	c)	Using Newton-Raphson method, find the approximate root of the equation $x^3 - 3x - 5 = 0$ by taking two iterations.		
	Ans.	$x^3 - 3x - 5 = 0$		
		$f(x) = x^3 - 3x - 5 \qquad \therefore f'(x) = 3x^2 - 3$	½	
		$\therefore f(2) = -3$	½	
		$f(3) = 13$	½	
		$\therefore$ the root is in (2, 3).		
		$\therefore x_i = \frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(3x^2-3) - [x^3 - 3x - 5]}{3x^2 - 3}$		
		$= \frac{3x^3 - 3x - x^3 + 3x + 5}{3x^2 - 3}$		
		$= \frac{2x^3 + 5}{3x^2 - 3} \qquad ....(*)$	1	
		$\therefore$ start with $x_0 = 2$		
		$\therefore x_1 = 2.333$	1	
		$x_2 = 2.281$	½	4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		**Note:** Once the formula (*) is formed, writing the direct values of $x_i's$ is permissible, as we allow it in case of table format for either bisection method or regula-falsi method.		
		OR		
		$x^3-3x-5=0$		
		$f(x)=x^3-3x-5$ $\therefore f'(x)=3x^2-3$	½	
		$\therefore f(2)=-3$	½	
		$f(3)=13$	½	
		$\therefore$ the root is in (2, 3).		
		$\therefore$ start with $x_0=2$		
		$\therefore x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}$		
		$=2-\dfrac{f(2)}{f'(2)}$		
		$=2-\dfrac{-3}{9}$	½	
		$=2.333$	1	
		$x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}$		
		$=2.333-\dfrac{f(2.333)}{f'(2.333)}$		
		$=2.333-\dfrac{0.699}{13.329}$		
		$=2.281$	1	**4**
		--		
	d)	Solve by Gauss elimination method: $x+y+z=6,\quad 2x+y+3z=13,\quad 3x+3y+3z=20$		
	Ans.	**Note for RAC** **No weightage should be given while assessing this bit.**		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	e)	Solve the system using Gauss-Seidel iterative method up to two iterations: $10x+y+z=12,\quad 2x+10y+z=13,\quad x+y+5z=7$		
	Ans.	$\therefore x=\frac{1}{10}(12-y-z)$ $y=\frac{1}{10}(13-2x-z)$ $z=\frac{1}{5}(7-x-y)$	1	
		Starting with $x_0=0=y_0=z_0$ $x_1=1.2$ $y_1=1.06$ $z_1=0.948$	1	
		$x_2=0.999$ $y_2=1.005$ $z_2=0.999$	1	
		$x_3=1$ $y_3=1$ $z_3=1$	1	**4**
		Note for Numerical Problems: For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal point. Due to the use of advance calculators, such as modern scientific non-programmable calculators, 1/3 is actually 0.333333333333 but can be taken as 0.333 or in case of 3/7 it is actually 0.428571428 but it is truncated as 0.429. Further it is preferred that in numerical methods the answers are to be in decimal forms, but still many times students keep answers in fractional form. In this case, no marks to be deducted. --		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	f)	Solve by Jacobi's method by taking three iterations: $5x+2y+z=12,\ x+4y+2z=15,\ x+2y+5z=20$		
	Ans.	$5x+2y+z=12$ $x+4y+2z=15$ $x+2y+5z=20$ $\therefore x=\frac{1}{5}(12-2y-z)$ $y=\frac{1}{4}(15-x-2z)$ $z=\frac{1}{5}(20-x-2y)$	1	
		Starting with $x_0=0=y_0=z_0$ $x_1=2.4$ $y_1=3.75$ $z_1=4$	1	
		$x_2=0.1$ $y_2=1.15$ $z_2=2.02$	1	
		$x_3=1.536$ $y_3=2.715$ $z_3=3.520$	1	**4**
		--		
		-- --		
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking. -- --		