Model Answers

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.

2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.

3)The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).

4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give
credit for any equivalent figure drawn.

5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.

6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.

7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		**Attempt any ten of the following:**		
	a) Ans	Evaluate: $\int \frac{dx}{4-x^2}$		
		$\int \frac{dx}{4-x^2}$		
		$=\int \frac{dx}{(2)^2-x^2}$	½	
		$=\frac{1}{4}\log\left\|\frac{2+x}{2-x}\right\|+c$	1+½	02
		***Note:** In the solution of any integration problems, if the constant c is not added, ½ mark may be deducted.*		
	b) Ans	Evaluate: $\int \sin^2 x \cos x dx$		
		$\int \sin^2 x \cos x dx$		
		Put $\sin x = t$		
		$\therefore \cos x dx = dt$	½	
		$=\int t^2 dt$		
		$=\frac{t^3}{3}+c$	1	
		$=\frac{\sin^3 x}{3}+c$	½	02
	c)	**Evaluate:** $\int xe^x dx$		
	Ans	$\int xe^x dx$		
		$=x\int e^x dx-\int\left[\frac{d}{dx}x.\int e^x dx\right]dx$	½	
		$=xe^x-\int 1.e^x dx$	1	
		$=xe^x-e^x+c$	½	02
	d)	If $\int_0^1 \left(3x^2+2x+k\right)dx=0$ .Find k		
	Ans	$\int_0^1 \left(3x^2+2x+k\right)dx=0$		
		$\therefore \left[x^3+x^2+kx\right]_0^1=0$	1	
		$1+1+k-0=0$	½	
		$2+k=0$		
		$\therefore k=-2$	½	02

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
	e)	Evaluate: $\int_{0}^{\frac{\pi}{2}} \sin x \cos x dx$		
	Ans	$\int_{0}^{\frac{\pi}{2}} \sin x \cos x dx$ Put $\sin x = t$ $\therefore \cos x dx = dt$	½	
		$\begin{array}{\|c\|c\|c\|}\hline x & 0 & \pi/2 \\ \hline \text{t} & 0 & 1 \\ \hline\end{array}$	½	
		$= \int_{0}^{1} t dt$		
		$= \left[\frac{t^2}{2}\right]_0^1$	½	
		$= \frac{1}{2}$	½	02
		OR		
		$= \int_{0}^{\frac{\pi}{2}} \sin x \cos x dx$		
		$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} 2\sin x \cos x dx$		
		$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} \sin 2x dx$	½	
		$= \frac{1}{2}\left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$	½	
		$= \frac{1}{4}\left\{\left[-\cos\pi - (-\cos 0)\right]\right\}$	½	
		$= \frac{1}{4}[1+1]$		
		$= \frac{1}{4}[2] = \frac{1}{2}$	½	02
	f)	Form a differential equation if $y = mx + c$		
	Ans	$y = mx + c$		
		$\therefore \frac{dy}{dx} = m$	1	
		$\therefore \frac{d^2y}{dx^2} = 0$	1	02

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1)	g)	Solve the D.E. $\frac{1}{y^2}dx=\frac{1}{x}dy$		
	Ans	$\therefore \frac{1}{y^2}dx=\frac{1}{x}dy$		
		$\therefore xdx=y^2dy$	½	
		Solution is,		
		$\int xdx=\int y^2dy$	½	
		$\frac{x^2}{2}=\frac{y^3}{3}+c$	1	02
	h)	Find the area bounded by $y=x$, $x=0$ and $x=4$		
	Ans	Area = $\int_a^b ydx$		
		$=\int_0^4 xdx$	½	
		$=\left[\frac{x^2}{2}\right]_0^4$	½	
		$=\frac{(4)^2}{2}-0$	½	
		$=8$	½	02
	i)	Find the probability that a card is drawn from a pack is a diamond.		
	Ans	n (S)= $^{52}C_1$=52	½	
		A=card is a diamond		
		n (A)= $^{13}C_1$=13	½	
		P(A)= $\frac{n(A)}{n(S)}=\frac{13}{52}=\frac{1}{4}=0.25$	1	02
	j)	Find order and degree of a D.E. $\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$		
	Ans	$\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$		
		$\therefore\left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$	1	
		Order=2, degree=2	1	02
	k)	When a die is thrown, find the probability of getting even numbers.		
	Ans	S= $\{1,2,3,4,5,6\}$	½	
		n (S)=6		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$A=\{2,4,6\}$		
		n (A)=3	½	
		P(A)=$\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}=0.5$	1	02
	l)	Two coins are tossed simultaneously, find the probability of getting atleast one head.		
		S=$\{HH,HT,TH,TT\}$ n (S)=4	½	
		A=$\{HH,HT,TH\}$		
		n (A)=3	½	
		P(A)=$\frac{n(A)}{n(S)}=\frac{3}{4}=0.75$	1	02
2)		**Attempt any four of the following:**		
	a)	Evaluate: $\int\frac{dx}{5+4\cos x}$		
	Ans	Put $\tan\frac{x}{2}=t$		
		$\cos x=\frac{1-t^2}{1+t^2}$, $dx=\frac{2dt}{1+t^2}$	1	
		$\int\frac{dx}{5+4\cos x}$		
		$=\int\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}.\frac{2dt}{1+t^2}$	½	
		$=\int\frac{1}{\frac{5(1+t^2)+4(1-t^2)}{1+t^2}}.\frac{2dt}{1+t^2}$		
		=2$\int\frac{dt}{5(1+t^2)+4(1-t^2)}$	½	
		=2$\int\frac{dt}{9+t^2}$		
		=2$\int\frac{dt}{(3)^2+t^2}$	½	
		=2.$\frac{1}{3}.\tan^{-1}\frac{t}{3}+c$	1	
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	½	04

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2)	**b)**	Evaluate: $\int \frac{x+1}{x^3-4x}dx$		
	Ans	$\int \frac{x+1}{x^3-4x}dx$		
		$=\int \frac{x+1}{x(x-2)(x+2)}dx$	½	
		Consider,		
		$\frac{x+1}{x(x-2)(x+2)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+2}$	½	
		$\therefore x+1=(x-2)(x+2)A+x(x+2)B+x(x-2)C$		
		Put $x=0$		
		1=(-2)(2)A+0+0		
		$\therefore$ 1=-4A $A=\frac{1}{-4}$	½	
		Put $x=2$		
		3=0+2(4)B+0		
		3=8B $B=\frac{3}{8}$	½	
		Put $x=-2$		
		-1=0+0+(-2)(-4)C		
		-1=8C $C=\frac{-1}{8}$	½	
		$\therefore \frac{x+1}{x(x-2)(x+2)}=\frac{\frac{1}{-4}}{x}+\frac{\frac{3}{8}}{x-2}+\frac{\frac{-1}{8}}{x+2}$		
		$\therefore \int \frac{x+1}{x(x-2)(x+2)}dx=\frac{1}{-4}\int \frac{1}{x}dx+\frac{3}{8}\int \frac{1}{x-2}dx-\frac{1}{8}\int \frac{1}{x+2}dx$	½	
		$=\frac{-1}{4}\log\lvert x\rvert+\frac{3}{8}\log\lvert x-2\rvert-\frac{1}{8}\log\lvert x+2\rvert+c$	1	04
	c)	Evaluate: $\int x\tan^{-1}xdx$		
	Ans	$\int x\tan^{-1}xdx$		
		$=\tan^{-1}x.\int xdx-\int\left[\frac{d}{dx}\tan^{-1}x.\int xdx\right]dx$	1	
		$=\tan^{-1}x.\frac{x^2}{2}-\int \frac{1}{1+x^2}.\frac{x^2}{2}.dx$	1	
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int \frac{x^2}{1+x^2}dx$		
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int \frac{1+x^2-1}{1+x^2}dx$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left[\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right]dx$		
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left[1-\frac{1}{1+x^2}\right]dx$		
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left[x-\tan^{-1}x\right]+c$	1	04
	d)	Evaluate: $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$		
	Ans	Let I= $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ --------------------(1)		
		I= $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	1	
		I= $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ----------------------------(2)	1	
		Add (1) and (2)		
		I+I= $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx+\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	½	
		2I= $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½	
		2I= $\int_0^{\frac{\pi}{2}}dx$		
		2I $=\left[x\right]_0^{\frac{\pi}{2}}$	½	
		2I $=\frac{\pi}{2}-0$		
		$\therefore$ I= $\frac{\pi}{4}$	½	04
	e)	Find the area of circle $x^2+y^2=36$ by using definite integral.		
	Ans	$x^2+y^2=36$		

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2)		$y^2=36-x^2$		
		$y=\sqrt{36-x^2}$	½	
		Area= $\int_a^b y dx$		
		A= $\int_0^6 \sqrt{36-x^2}dx=\int_0^6 \sqrt{(6)^2-x^2}dx$	½	
		$=\left[\frac{x}{2}\sqrt{(6)^2-x^2}+\frac{(6)^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$	1	
		$=\left[0+18\sin^{-1}(1)\right]-\left[0\right]$		
		$=18\frac{\pi}{2}=9\pi$	1	
		$\therefore$ Area of circle = $4(9\pi)$		
		$=36\pi$	1	04
	f)	Evaluate: $\int_4^5 \frac{\sqrt[3]{x-4}}{\sqrt[3]{x-4}+\sqrt[3]{5-x}}dx$		
	Ans	Let I= $\int_4^5 \frac{\sqrt[3]{x-4}}{\sqrt[3]{x-4}+\sqrt[3]{5-x}}dx$ ----------------------------(1)		
		I= $\int_4^5 \frac{\sqrt[3]{(5+4-x)-4}}{\sqrt[3]{(5+4-x)-4}+\sqrt[3]{5-(5+4-x)}}dx$	1	
		I= $\int_4^5 \frac{\sqrt[3]{5-x}}{\sqrt[3]{5-x}+\sqrt[3]{x-4}}dx$ --------------(2)	1	
		Add (1) and (2)		
		I+I= $\int_4^5 \frac{\sqrt[3]{x-4}}{\sqrt[3]{x-4}+\sqrt[3]{5-x}}dx+\int_4^5 \frac{\sqrt[3]{5-x}}{\sqrt[3]{5-x}+\sqrt[3]{x-4}}dx$	½	
		2I= $\int_4^5 \frac{\sqrt[3]{x-4}+\sqrt[3]{5-x}}{\sqrt[3]{x-4}+\sqrt[3]{5-x}}dx$		
		2I= $\int_4^5 dx$		
		2I $=\left[x\right]_4^5$	½	
		2I =5-4=1	½	
		I= $\frac{1}{2}$	½	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		**Attempt any four of the following**:		
	a)	Evaluate: $\int_0^{\pi} x\sin^3 x\cos^2 xdx$		
	Ans	Let I= $\int_0^{\pi} x\sin^3 x\cos^2 xdx$		
		$I=\int_0^{\pi}(\pi-x)\sin^3(\pi-x)\cos^2(\pi-x)dx$	½	
		$I=\int_0^{\pi}(\pi-x)\sin^3 x\cos^2 xdx$	½	
		$I=\int_0^{\pi}\pi\sin^3 x\cos^2 xdx-\int_0^{\pi}x\sin^3 x\cos^2 xdx$		
		$I=\int_0^{\pi}\pi\sin^3 x\cos^2 xdx-I$		
		$I+I=\pi\int_0^{\pi}\sin^3 x\cos^2 xdx$	½	
		$2I=\pi\int_0^{\pi}\sin^2 x\cos^2 x\sin xdx$		
		$2I=\pi\int_0^{\pi}\left(1-\cos^2 x\right)\cos^2 x\sin xdx$		
		Put $\cos x=t$ $-\sin xdx=dt$ $\sin xdx=-dt$		
		$\begin{array}{\|c\|c\|c\|}\hline x & 0 & \pi \\ \hline \text{t} & 1 & -1 \\ \hline\end{array}$	1	
		$\therefore 2I=\pi\int_1^{-1}\left(1-t^2\right)t^2\left(-dt\right)$		
		$\therefore 2I=-\pi\int_1^{-1}\left(t^2-t^4\right)dt$		
		$\therefore 2I=-\pi\left[\frac{t^3}{3}-\frac{t^5}{5}\right]_1^{-1}$	½	
		$\therefore 2I=-\pi\left[\left(\frac{-1}{3}+\frac{1}{5}\right)-\left(\frac{1}{3}-\frac{1}{5}\right)\right]$		
		$\therefore 2I=-\pi\left(\frac{-1}{3}+\frac{1}{5}-\frac{1}{3}+\frac{1}{5}\right)$	½	
		$\therefore 2I=-\pi\left(\frac{-2}{3}+\frac{2}{5}\right)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore 2I=-2\pi\left(\frac{-1}{3}+\frac{1}{5}\right)$		
		$\therefore I=\frac{2\pi}{15}$	½	04
	b)	Find the centre of gravity of an area enclosed by the curve $y=2x+3$ ordinates $x=1, x=2$ and $x-$axis.		
	Ans	$\bar{x}=\frac{\int_1^2 xydx}{\int_1^2 ydx}$ and $\bar{y}=\frac{\frac{1}{2}\int_1^2 y^2dx}{\int_1^2 ydx}$		
		Consider, $\int_1^2 xydx=\int_1^2\left(2x^2+3x\right)dx$		
		$=\left[\frac{2x^3}{3}+\frac{3x^2}{2}\right]_1^2$		
		$=\left[\left(\frac{16}{3}+\frac{12}{2}\right)-\left(\frac{2}{3}+\frac{3}{2}\right)\right]$		
		$=\frac{55}{6}=9.16$	1	
		$\int_1^2 ydx=\int_1^2\left(2x+3\right)dx$		
		$=\left[x^2+3x\right]_1^2=\left(4+6\right)-\left(1+3\right)$=6	1	
		$\int_1^2 y^2dx=\int_1^2\left(4x^2+12x+9\right)dx$		
		$=\left[\frac{4x^3}{3}+6x^2+9x\right]_1^2=\left[\left(\frac{32}{3}+24+18\right)-\left(\frac{4}{3}+6+9\right)\right]$		
		$=\frac{109}{3}$	1	
		$\bar{x}=\frac{\frac{55}{6}}{6}=\frac{55}{36}$ or 1.527		
		$\bar{y}=\frac{\frac{1}{2}\cdot\frac{109}{3}}{6}=\frac{109}{36}$ or 3.027	½	

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3)		$C.G.$ is $\left(\overline{x},\overline{y}\right)=\left(\frac{55}{36},\frac{109}{36}\right)$	½	04
		Note: *The above example can be solve by directly finding the values of* $\overline{x}$ *and* $\overline{y}$ *.*		
	c)	Find the volume obtained by revolving about the x-axis , the region bounded by x-axis, the curve $9x^2-4y^2=36$ and the line x=2,x=4.		
	Ans	Given, $9x^2-4y^2=36$		
		$\therefore y^2=\frac{1}{4}(9x^2-36)$	½	
		Volume V= $\pi\int_a^b y^2dx$		
		V= $\pi\int_2^4\frac{\left(9x^2-36\right)}{4}dx$	½	
		$=\frac{\pi}{4}\left[\frac{9x^3}{3}-36x\right]_2^4$	1	
		$=\frac{\pi}{4}\left[3x^3-36x\right]_2^4$	½	
		$=\frac{\pi}{4}[(192-144)-(24-72)]$	1	
		$=24\pi$	½	04
	d)	Evaluate: $\int\frac{1}{4\sin^2x+5\cos^2x}dx$		
	Ans	$\int\frac{1}{4\sin^2x+5\cos^2x}dx$		
		$=\int\frac{\frac{1}{\cos^2x}}{4\frac{\sin^2x}{\cos^2x}+5\frac{\cos^2x}{\cos^2x}}dx$	½	
		$=\int\frac{\sec^2x}{4\tan^2x+5}dx$	½	
		Put $\tan x=t$		
		$\therefore\ \sec^2xdx=dt$	½	
		$=\int\frac{1}{4t^2+5}dt$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$=\frac{1}{4}\int\frac{1}{t^2+\left(\frac{\sqrt{5}}{2}\right)^2}dt$	½	
		$=\frac{1}{4}.\frac{1}{\frac{\sqrt{5}}{2}}\tan^{-1}\left(\frac{t}{\frac{\sqrt{5}}{2}}\right)+c$	1	
		$=\frac{1}{2\sqrt{5}}.\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$	½	04
	e)	Evaluate: $\int\frac{e^x\sin\left(e^x\right)}{\cos^2\left(e^x\right)}dx$		
	Ans	$\int\frac{e^x\sin\left(e^x\right)}{\cos^2\left(e^x\right)}dx$		
		Put $e^x=t$		
		$\therefore\ e^x dx=dt$	1	
		$=\int\frac{\sin t}{\cos^2 t}dt$	½	
		$=\int\tan t\sec t dt$	1	
		=sect+c	1	
		$=\sec\left(e^x\right)+c$	½	04
	f)	Solve the D.E. $\frac{dy}{dx}=e^{x-y}+xe^{-y}$		
	Ans	$\frac{dy}{dx}=e^{x-y}+xe^{-y}$		
		$\frac{dy}{dx}=e^xe^{-y}+xe^{-y}$	½	
		$\frac{dy}{dx}=e^{-y}(e^x+x)$		
		$e^ydy=(e^x+x)dx$	1	
		$\int e^ydy=\int(e^x+x)dx$	½	

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3)				
		$e^y = e^x + \frac{x^2}{2} + c$	2	04
4)		**Attempt any four of the following:**		
	a)	Solve: $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$ given that y=2 when x=1.		
	Ans	$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$		
		Put $y = vx$	½	
		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	½	
		$\therefore v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{xvx}$		
		$v + x\frac{dv}{dx} = \frac{x^2(1+v^2)}{vx^2}$		
		$x\frac{dv}{dx} = \frac{1+v^2-v^2}{v}$	1	
		$\therefore vdv = \frac{dx}{x}$		
		$\int vdv = \int \frac{dx}{x}$		
		$\therefore \frac{v^2}{2} = \log\lvert x\rvert + c$	1	
		$\frac{\frac{y^2}{x^2}}{2} = \log\lvert x\rvert + c$		
		$x = 1, y = 2$		
		$\therefore \frac{4}{2} = \log(1) + c$	½	
		$\therefore c = 2$		
		$\frac{\frac{y^2}{x^2}}{2} = \log\lvert x\rvert + 2$	½	04

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4)	b)	Solve: $\left(e^x+2xy^2+y^3\right)dx+\left(a^y+2x^2y+3xy^2\right)dy=0$		
	Ans	$M=e^x+2xy^2+y^3$		
		$N=a^y+2x^2y+3xy^2$		
		$\frac{\partial M}{\partial y}=4xy+3y^2$		
		$\frac{\partial N}{\partial x}=4xy+3y^2$		
		$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	2	
		$\therefore$ Given D.E.is exact.		
		$\therefore$ solution is,		
		$\int\limits_{y-\text{constant}} Mdx+\int\limits_{\substack{\text{terms not}\\ \text{containing'x'}}} Ndy=c$	½	
		$\int_{y-const}\left(e^x+2xy^2+y^3\right)dx+\int a^y dy=c$	½	
		$e^x+x^2y^2+xy^3+\frac{a^y}{\log a}=c$	1	04
	c)	Solve: $\frac{dy}{dx}+xy=x^3y^3$		
	Ans	$\frac{dy}{dx}+xy=x^3y^3$		
		$\frac{1}{y^3}\frac{dy}{dx}+x.\frac{1}{y^2}=x^3$	½	
		Put $\frac{1}{y^2}=v$		
		$\frac{-2}{y^3}.\frac{dy}{dx}=\frac{dv}{dx}$	½	
		$\therefore \frac{1}{y^3}\frac{dy}{dx}=\frac{-1}{2}.\frac{dv}{dx}$		
		$\therefore \frac{-1}{2}\frac{dv}{dx}+xv=x^3$		
		$\therefore \frac{dv}{dx}-2xv=-2x^3$	½	

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4)		I.F.= $e^{-2\int xdx} = e^{-x^2}$	½	
		Its solution is,		
		$vI.F. = \int Q.I.F.dx + c$		
		$\frac{1}{y^2}.e^{-x^2} = \int -2x^3.e^{-x^2}dx + c$	½	
		put $x^2 = t$ in R.H.S $2xdx = dt$	½	
		$\therefore \frac{1}{y^2}e^{-x^2} = -\int te^{-t}dt + c$		
		$\therefore \frac{1}{y^2}e^{-x^2} = -\left[t\frac{e^{-t}}{-1} - \int 1.\frac{e^{-t}}{-1}dt\right] + c$	½	
		$\therefore \frac{1}{y^2}e^{-x^2} = te^{-t} + e^{-t} + c$		
		$\therefore \frac{1}{y^2}e^{-x^2} = x^2e^{-x^2} + e^{-x^2} + c$	½	04
	d)	Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3. Obtain the root by bisection method (three iterations only).		
		Let $f(x) = x^3 - 9x + 1$		
		$f(2) = -9 < 0$ $f(3) = 1 > 0$ $\therefore$ the root lies in (2,3)	1	
		$x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$	1	
		$f(2.5) = -5.875$ <0		
		$\therefore$ the root lies in (2.5,3)		
		$x_2 = \frac{x_1+b}{2} = \frac{2.5+3}{2} = 2.75$	1	
		$f(x_2) = -2.96 < 0$		
		$\therefore$ the root lies in (2.75,3)		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$x_3=\frac{x_2+b}{2}=\frac{2.75+3}{2}=2.875$	1	04
		OR		
		$f(2)=-9<0$		
		$f(3)=1>0$		
		$\therefore$ the root lies in (2,3)	1	

a	b	$x=\frac{a+b}{2}$	$f(x)$
2	3	2.5	-5.875
2.5	3	2.75	-2.96
2.75	3	2.875	---

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
			1+1+1	04
	e)	Obtain the root of the equation by Regula-Falsi method		
		$x^3-x-1=0$ (three iterations only).		
	Ans	Let $f(x)=x^3-x-1$		
		$f(1)=-1$		
		$f(2)=5$		
		$\therefore$ the root lies in (1,2)	1	
		$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{1(5)-2(-1)}{5-(-1)}=\frac{7}{6}=1.16$	1	
		$f(1.16)=-0.599$ <0		
		$\therefore$ the root lies in (1.16,2)	1	
		$x_2=\frac{x_1 f(b)-bf(x_1)}{f(b)-f(x_1)}=\frac{1.16(5)-2(-0.599)}{5-(-0.599)}=1.24$		
		$f(x_2)=-0.33<0$	1	
		$\therefore$ the root lies in (1.24,2)		
		$x_3=\frac{x_2 f(b)-bf(x_2)}{f(b)-f(x_2)}=\frac{1.24(5)-2(-0.33)}{5-(-0.33)}=1.28$	1	04

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>4)</td><td></td><td>OR
$f(x)=x^3-x-1$
$\therefore f(1)=-1<0$
$f(2)=5>0$
$\therefore$ the root is in $(1,\ 2)$.</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>
<table>
<tr><th>a</th><th>b</th><th>$f(a)$</th><th>$f(b)$</th><th>$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$</th><th>$f(x)$</th></tr>
<tr><td>1</td><td>2</td><td>-1</td><td>5</td><td>1.16</td><td>-0.599</td></tr>
<tr><td>1.16</td><td>2</td><td>-0.599</td><td>5</td><td>1.24</td><td>-0.33</td></tr>
<tr><td>1.24</td><td>2</td><td>-0.33</td><td>5</td><td>1.28</td><td>---</td></tr>
</table>
</td><td>1+1+1</td><td>04</td></tr>
<tr><td></td><td>f)</td><td>Using Newton-Raphson method, evaluate $\sqrt[3]{100}$. (Carry out three iterations only)</td><td></td><td></td></tr>
<tr><td></td><td>Ans</td><td>Let, $x=\sqrt[3]{100}$
$\therefore x^3=100$
$x^3-100=0$
$f(x)=x^3-100$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(4)=-36$
$f(5)=25$
$f'(x)=3x^2$
$\therefore f'(5)=75$
$\therefore$ Initial root $x_0=5$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x_1=x_0-\frac{f(x_0)}{f^{'}(x_0)}=5-\frac{25}{75}=4.66$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$f(4.66)=0.19$ and $f'(4.66)=65.14$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x_2=x_1-\frac{f(x_1)}{f^{'}(x_1)}=4.66-\frac{0.19}{65.14}=4.64$</td><td>1</td><td>04</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$f(4.64) = -0.102$ and $f'(4.64) = 64.58$		
		$x_3 = x_2 - \frac{f(x_2)}{f^{'}(x_2)} = 4.64 + \frac{0.102}{64.58} = 4.64$	1	04
5)		**Attempt any four of the following:**		
	a)	A particle starts from rest. Its acceleration at any time is (t+3) $m/\sec^2$. Find the distance travelled in 4 seconds.		
	Ans	From given $\frac{dv}{dt} = (t+3)$		
		$\therefore dv = (t+3)dt$		
		$\therefore \int dv = \int (t+3)dt$	½	
		$v = \frac{t^2}{2} + 3t + c$ --------------------------(1)	½	
		at t=0 , $v = 0$		
		$\therefore$ c=0	½	
		Equation (1) becomes,		
		$v = \frac{t^2}{2} + 3t$		
		$\frac{dx}{dt} = \frac{t^2}{2} + 3t$		
		$dx = \left(\frac{t^2}{2} + 3t\right)dt$		
		$\int dx = \int \left(\frac{t^2}{2} + 3t\right)dt$	½	
		$x = \frac{t^3}{6} + \frac{3t^2}{2} + c_1$	1	
		At $t = 0,\ x = 0$		
		$\therefore c_1 = 0$	½	
		$x = \frac{t^3}{6} + \frac{3t^2}{2}$		
		At t=4,		

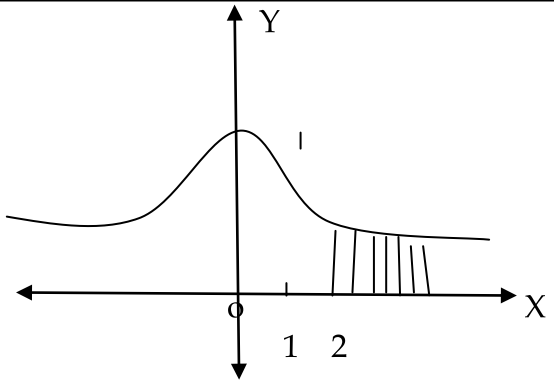
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$x=\frac{64}{6}+\frac{48}{2}=34.66$	½	04
	b)	Solve: $\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy$		
	Ans	$\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy$		
		$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ Linear in x	½	
		$\therefore$ I.F.$=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$	½	
		Its solution is ,		
		x I.F. $=\int Q.$I.F.$\,dy+c$		
		$xe^{\tan^{-1}y}=\int\frac{\tan^{-1}y}{1+y^2}.e^{\tan^{-1}y}dy+c$	½	
		Put $\tan^{-1}y=t$ in R.H.S.		
		$\frac{1}{1+y^2}dy=dt$		
		$xe^{\tan^{-1}y}=\int te^t dt+c$	½	
		$xe^{\tan^{-1}y}=te^t-\int e^t+c$	½	
		$\therefore\ xe^{\tan^{-1}y}=te^t-e^t+c$	1	
		$\therefore xe^{\tan^{-1}y}=e^t(\text{t}-1)+c$		
		$\therefore xe^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+c$	½	04
	c)	Solve $\frac{dy}{dx}=\frac{x+y}{x+y+2}$		
	Ans	$\frac{dy}{dx}=\frac{x+y}{x+y+2}$		
		Put $x+y=v$		
		$1+\frac{dy}{dx}=\frac{dv}{dx}$		
		$\frac{dy}{dx}=\frac{dv}{dx}-1$	½	
		$\therefore\ \frac{dv}{dx}-1=\frac{v}{v+2}$		
		$\frac{dv}{dx}=\frac{v}{v+2}+1$		
		$\frac{dv}{dx}=\frac{v+v+2}{v+2}=\frac{2\left(v+1\right)}{v+2}$	1	
		$\frac{1}{2}.\frac{v+2}{v+1}dv=dx$	½	
		$\therefore\frac{1}{2}\int\left[1+\frac{1}{1+v}\right]dv=\int dx$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\therefore \frac{1}{2}\left(v+\log\lvert 1+v\rvert\right)=x+c$	1	
		$\therefore \frac{1}{2}\left(x+y+\log\lvert 1+x+y\rvert\right)=x+c$	½	04
	d)	A particle executes S.H.M. according to the law $\frac{d^2x}{dt^2}=-4x$ .If x =2 and $\frac{dx}{dt}$ =3 at t=0. Find the displacement x at any time t.		
	Ans	Given, $\frac{d^2x}{dt^2}=-4x$		
		$v\frac{dv}{dx}=-4x$	½	
		$vdv=-4xdx$		
		$\int vdv=-4\int xdx$	½	
		$\frac{v^2}{2}=-2x^2+c$		
		At $x=2$, $v=3$ when t=0		
		$c=\frac{25}{2}$	½	
		$\frac{v^2}{2}=-2x^2+\frac{25}{2}$		
		$\therefore v^2=-4x^2+25$		
		$v=\sqrt{25-4x^2}$	½	
		i.e. $\frac{dx}{dt}=\sqrt{25-4x^2}$		
		$\frac{dx}{\sqrt{25-4x^2}}=dt$		
		$\int\frac{dx}{2\sqrt{\left(\frac{5}{2}\right)^2-x^2}}=\int dt$		
		$\frac{1}{2}.\sin^{-1}\left(\frac{x}{5/2}\right)=t+c$	½	
		At t=0, $x=2$		
		$\therefore \frac{1}{2}.\sin^{-1}\left(\frac{4}{5}\right)=c$	½	
		$\therefore \frac{1}{2}.\sin^{-1}\left(\frac{x}{5/2}\right)=t+\frac{1}{2}.\sin^{-1}\left(\frac{4}{5}\right)$	½	
		$\sin^{-1}\left(\frac{2x}{5}\right)=2t+\sin^{-1}\left(\frac{4}{5}\right)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\frac{2x}{5}=\sin(2t+\alpha)$ $x=\frac{5}{2}\sin\left(2t+\alpha\right)$ --------------$\left(\because \sin^{-1}\left(\frac{4}{5}\right)=\alpha\right)$	½	04
	e)	Solve using Gauss elimination method: $x+2y+3z=14, 3x+y+2z=11, 2x+3y+z=11$		
	Ans	Given, $x+2y+3z=14$ $3x+y+2z=11$ $2x+3y+z=11$		
		$\begin{array}{r} 3x+6y+9z=42 \\ 3x+y+2z=11 \\ \underline{-\quad -\quad -\qquad -} \\ 5y+7z=31 \end{array}$ and $\begin{array}{r} 2x+4y+6z=28 \\ 2x+3y+z=11 \\ \underline{-\quad -\quad -\quad -} \\ y+5z=17 \end{array}$	1+1	
		$5y+7z=31$ $\underline{5y+25z=85}$ $-18z=-54$	1	
		$\therefore z=3$ $y=2$ $x=1$	1	04
		Note: *In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking. Let us see, how the solution becomes by eliminating first y and then z to get the value of x, as illustrated below:* **OR**		
		$\begin{array}{r} x+2y+3z=14 \\ 6x+2y+4z=22 \\ \underline{-\quad -\quad -\quad -} \\ -5x-z=-8 \end{array}$ and $\begin{array}{r} 9x+3y+6z=33 \\ 2x+3y+z=11 \\ \underline{-\quad -\quad -\quad -} \\ 7x+5z=22 \end{array}$	1+1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$-25x-5z=-40$		
		$\underline{7x+5z=22}$		
		$-18x=-18$	1	
		$\therefore x=1$		
		$y=2$	1	04
		$z=3$		
	f)	Solve using Jacobi's method. (three iterations only) $10x+y+2z=13, 3x+10y+z=14, 2x+3y+10z=15$		
	Ans	$x=\frac{1}{10}(13-y-2z)$		
		$y=\frac{1}{10}(14-3x-z)$		
		$z=\frac{1}{10}(15-2x-3y)$	1	
		Starting with $x_0=y_0=z_0=0$		
		$x_1=1.3$		
		$y_1=1.4$		
		$z_1=1.5$	1	
		$x_2=0.86$		
		$y_2=0.86$		
		$z_2=0.82$	1	
		$x_3=1.05$		04
		$y_3=1.06$		
		$z_3=1.07$	1	
6)	a)	Solve using Gauss-Seidal method.(three iterations)		
		$5x+2y+z=12, x+4y+2z=15, x+2y+5z=20$		
	Ans	$x=\frac{1}{5}(12-2y-z)$		
		$y=\frac{1}{4}(15-x-2z)$		
		$z=\frac{1}{5}(20-x-2y)$		
		Starting with $y_0=z_0=0$		
		$I)\ x_1=2.4$	1	
		$y_1=3.15$		
		$z_1=2.26$		
			1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$II)\ x_2 = 0.688$ $y_2 = 2.44$ $z_2 = 2.88$	1	
		$III) x_3 = 0.848$ $y_3 = 2.098$ $z_3 = 2.99$	1	04
	b)	A box contains 10 radio valves of which 4 are defective find the probability that if two of them valves are taken from the box, they are both defective.		
	Ans	n (S) $= {}^{10}C_2 = 45$ A= two of them valves are defective	1	
		n (A) $= {}^{4}C_2 = 6$	1	
		P(A) = $\frac{n(A)}{n(S)} = \frac{6}{45} = 0.133$	2	04
	c)	If 3% of electric bulbs manufacture by a company is defective find the probability that in a sample 100 bulbs, exactly 5 bulbs are defective (Given e^{-3} = 0.04974)		
	Ans	P=3%=0.03, n=100	1	
		Mean , m=np =0.03(100) =3	1	
		$\therefore \text{P}(r) = \frac{e^{-m} m^r}{r!}$ $\text{P}(5) = \frac{e^{-3}(3)^5}{5!} = 0.1007$	2	04
	d)	The life times of certain kinds of electric devices have a mean of 300 hrs and S.D. of 25 hrs. Find the probability that any one of these electronic devices will have a life time of more than 350 hrs.(area between z=0 to z=2 is 0.4772).		
	Ans	Given, $\bar{x}$=300, σ=25, x=350 Standard normal variate $\bar{z}$ is, $\bar{z} = \frac{x-\bar{x}}{\sigma} = \frac{350-300}{25} = 2$	1	
		$\therefore$ Area =0.5-(area between z=0 to z=2) =0.5-0.4772 =0.0228	2	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)			1	04
	e)	A bag contains 6 white and 4 black balls.5 balls are drawn at random. What is the probability that 3 are white and 2 are black?		
	Ans	n (S)= $^{10}C_5 = 252$	1	
		A={2 balls are white and 2 balls are black} n (A)= $^{6}C_3 . ^{4}C_2 = 20(6) = 120$	1	04
		P(A)= $\dfrac{n(A)}{n(S)} = \dfrac{120}{252} = 0.4761$	2	
	f)	Two cards are drawn at random from well shuffled pack of 52 cards. Find the probability that i) Both the cards are spade ii) One king and other queen		
	Ans	$n(S) = {}^{52}C_2 = 1326$	1	
		i) A={both are spade} $n(A) = {}^{13}C_2 = 78$	½	
		P(A)= $\dfrac{n(A)}{n(S)} = \dfrac{78}{1326} = 0.0588$	1	
		ii) A={one king and other queen} $n(B) = {}^{4}C_1 . {}^{4}C_1 = 16$	½	
		P(B)= $\dfrac{n(A)}{n(S)} = \dfrac{16}{1326} = 0.0120$	**1**	04
		Important Note: ***In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.***		