

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

Subject Code: 17301 Winter-2015 Page No: 1/26 Model Answers **Important Instructions to examiners:** The model answer shall be the complete solution for each and every question on the question paper. Numerical shall be completely solved in a step by step manner along with step marking. All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert. In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors. In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors. In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment. In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator. Experts shall cross check the DTP of the final draft of the model answer prepared by them.



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Que.	Sub.	Model Answers	Mark	Total
No.	Que.	Attempt any FIVE of the following:	S	Marks 20
1)	a)	At what point on the curve $y = e^x$, slope is 1?		20
	Ans			
		$y = e^x$	1	
		$\therefore \frac{dy}{dx} = e^x$		
		dx $\therefore e^x = 1$	1	
			1	
		$\therefore x = 0$	1	
		$\therefore y = e^0 = 1$		4
		\therefore Point is $(0,1)$		4
	b)	Find the radius of curvature of $y = e^x$ at $(0,1)$		
	Ans	$y = e^x$	4/	
			1/2	
		$\therefore \frac{dy}{dx} = e^x$	1/2	
		d^2y	72	
		$\therefore \frac{d^2 y}{dx^2} = e^x$		
		at (0,1)	1/2	
		$\frac{dy}{dx} = e^0 = 1$		
			1/2	
		$\frac{d^2 y}{dx^2} = e^0 = 1$		
		Radius of curvature = $\frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$(1+(1)^2)^{\frac{7}{2}}$	1	
		$= \frac{\left(1 + \left(1\right)^{2}\right)^{\frac{3}{2}}}{1}$	4	
		=2.828	1	4
	c)	Evaluate: $\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}} dx$		
	Ans	$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}} dx$		
		Put $\sqrt{x} = t$	1/2	
		Put $\sqrt{x} = t$ $\therefore \frac{1}{2\sqrt{x}} dx = dt$		
		$\therefore \frac{1}{\sqrt{x}} dx = 2 dt$	1	
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Que.	Sub.	Model Answers	Marks	Total Marks
No. 1)	d) Ans	$= \int \sin t \ 2dt$ $= -2 \cos t + c$ $= -2 \cos \sqrt{x} + c$ Integrate w.r.t. $x = \frac{\sin x}{\cos^2 x}$ $\int \frac{\sin x}{\cos^2 x} dx$ $= \int \frac{\sin x}{\cos x \cos x} dx$ $= \int \tan x \sec x dx$	1 1 1/ ₂ 2 2	Marks 4
		OR $\int \frac{\sin x}{\cos^2 x} dx$ $put \cos x = t$ $\therefore -\sin x dx = dt$ $\therefore \sin x dx = -dt$ $\int \frac{-dt}{t^2}$ $= -\frac{t^{-1}}{-1} + c$ $= \frac{1}{t} + c$	1 1 1 1 ½	
	e) Ans	$= \frac{1}{\cos x} + c$ $= \sec x + c$ Evaluate: $\int xe^{x} dx$ $= x \int e^{x} dx - \int \left[\int e^{x} dx \frac{d}{dx} x \right] dx$ $= x e^{x} - \int 1 \cdot e^{x} dx$ $= x e^{x} - e^{x} + c$	1 1+1 1	4
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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	Evaluate: $\int \frac{1}{x(x+1)} dx$		14141183
	Ans			
		$\int \frac{1}{x(x+1)} dx$		
		consider $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$		
		$\therefore A = 1$	1 1	
		$B = -1$ $\therefore \int \frac{1}{x(x+1)} dx = \int \left(\frac{1}{x} + \frac{-1}{x+1}\right) dx$		
		$= \log x - \log (x+1) + c$	1+1	4
	g)	Evaluate: $\int_{0}^{1} \frac{1}{\sqrt{1-x^{2}}} dx$		
	Ans	$\int_{0}^{1} \frac{1}{\sqrt{1-x^2}} dx$	2	
		$= \left[\sin^{-1} x\right]_0^1$	1	
		$= \left[\sin^{-1}1\right] - \left[\sin^{-1}0\right]$		
		$=\frac{\pi}{2}$	1	4
	h)	Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with X-axis		
	Ans	$A = \int_{0}^{3} x^{2} dx$	1	
		$= \left[\frac{x^3}{3}\right]_0^3$	1	
		$ = \begin{bmatrix} 3^3 \\ \hline 3 \end{bmatrix} - \begin{bmatrix} 0^3 \\ \hline 3 \end{bmatrix} $	1	
		[3] [3] = 9 Sq.units	1	4
	.,			
	i) Ans	Find order and degree of the following differential equation. Order = 2		
		$\frac{d^2 y}{dx^2} = \sqrt{y + \left(\frac{dy}{dx}\right)^2} \therefore \left(\frac{d^2 y}{dx^2}\right)^2 = y + \left(\frac{dy}{dx}\right)^2$	2	
		Degree = 2	2	4



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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
1)	j)	Form the differential equation of the curve $y = ax^2$		
	Ans	$y = ax^2$	41/	
		$\frac{dy}{dx} = 2 ax$	1½	
		$\frac{dy}{dx} = 2 \frac{y}{x^2} x$	1½	
		$x\frac{dy}{dx} - 2y = 0$	1	4
	k)	Three cards are drawn from well shuffled pack of cards .Find the probability that all of them are king.		
	Ans	$n(S) = {}^{52}C_3$	1	
		$n(S) = {}^{52}C_3$ $n(A) = {}^4C_3$	1	
		$P(A) = \frac{n(A)}{n(S)} = \frac{4}{22100} = 0.00018$	2	4
	l) Ans	Two coins are tossed simultaneously, find the probability of getting atleast one head.		
		$S=\{HH, HT, TH, TT\}$ $n(S)=4$	1	
		$A = \{HH, HT, TH\}$ $n(A) = 3$	1	
		$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4} = 0.75$	2	4
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Que.	Sub.	Model Answers	Marks	Total
No.	Que.	Wiodel Thiswels	Widiks	Marks
2)		Attempt any FOUR of the following:		16
	a)	Find the equation of tangent and normal to the curve		
		$2x^2 - xy + 3y^2 = 18$ at (3,1)		
	Ans			
		$2x^2 - xy + 3y^2 = 18$		
		$4x - \left(x\frac{dy}{dx} + y\right) + 6y\frac{dy}{dx} = 0$	1/2	
		$4x - x\frac{dy}{dx} - y + 6y\frac{dy}{dx} = 0$		
		$\left(6y - x\right)\frac{dy}{dx} = y - 4x$		
		$\therefore \frac{dy}{dx} = \frac{y - 4x}{6y - x}$	1/2	
		at point (3,1)		
		slope of tangent = $\frac{-11}{3}$	1/2	
		slope of normal = $\frac{3}{11}$	1/2	
		Equation of tangent at (3,1) is	1/2	
		$y - 1 = \frac{-11}{3}(x - 3)$	1/2	
		$\therefore 11x + 3y - 36 = 0$	72	
		Equation of normal at (3,1) is	1/2	
		$y - 1 = \frac{3}{11}(x - 3)$	1/2	4
		$\therefore 3x - 11y + 2 = 0$		
	b)	Show that the radius of curvature at any point on the curve		
	Ans	$y = a \log \left(\sec \frac{x}{a} \right)$ where a is constant is $a \sec \left(\frac{x}{a} \right)$		
		$y = a \log \left(\sec \frac{x}{a} \right)$		
		$\therefore \frac{dy}{dx} = a \cdot \frac{1}{\sec \frac{x}{a}} \cdot \frac{d}{dx} \sec \frac{x}{a}$	1	
		$a = a \cdot \frac{1}{-1} \cdot \sec \frac{x}{-1} \tan \frac{x}{-1} \cdot \frac{1}{-1}$		
		$\frac{-a}{\sec \frac{x}{a}} = \frac{x}{a} = \frac{a}{a}$		
		$=\tan\frac{x}{a}$	1/2	



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Que.	Sub.	Model Answers	Marks	Total
No. 2)	Que.		1	Marks
		$\frac{d^2 y}{dx^2} = \sec^2\left(\frac{x}{a}\right) \frac{1}{a}$		
		Radius of curvature $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$= \frac{\left[1 + \tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\frac{1}{a}}$	1/2	
		$= \frac{\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\frac{1}{a}}$	1/2	
		$= a \sec\left(\frac{x}{a}\right)$	1/2	4
	c)	Find the maximum and minimum value of $x^3 - 9x^2 + 24x$		
	Ans	Let $y = x^3 - 9x^2 + 24x$	1/2	
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$		
		$\therefore \frac{d^2 y}{dx^2} = 6x - 18$	1/2	
		Consider $\frac{dy}{dx} = 0$		
		$3x^2 - 18x + 24 = 0$	1	
		$\therefore x = 2 \text{ or } x = 4$		
		at $x = 2$ $\frac{d^2 y}{dx^2} = 6(2) - 18 = -6 < 0$	1/2	
		$\therefore y \text{ is maximum at } x = 2$		
		$y_{\text{max}} = 2^3 - 9(2)^2 + 24(2)$	1/2	
		= 20		



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		TVICTING	Marks
2)		$at x = 4$ $\frac{d^2 y}{dx^2} = 6(4) - 18 = 6 > 0$ $\therefore y \text{ is minimum at } x = 4$	1/2	
		$y_{\min} = 4^3 - 9(4)^2 + 24(4)$ = 16	1/2	4
	d) Ans	Evaluate: $\int \cos^{-1} x dx$ $I = \int \cos^{-1} x . 1 dx$		
		$= \cos^{-1} x \int 1 dx - \int \left(\int 1 dx \frac{d}{dx} \cos^{-1} x \right) dx$	1	
		$= \left(\cos^{-1} x\right) x - \int \frac{-1}{\sqrt{1-x^2}} x dx$	1	
		$=x\cos^{-1}x+\int\frac{x}{\sqrt{1-x^2}}dx$		
		$= x \cos^{-1} x + \frac{1}{-2} \int \frac{-2x}{\sqrt{1-x^2}} dx$	1	
		$= x \cos^{-1} x - \frac{1}{2} \left(2\sqrt{1 - x^2} \right) + c$ $= x \cos^{-1} x - \sqrt{1 - x^2} + c$	1	4
	e) Ans	Evaluate: $\int \frac{\left(\tan^{-1} x\right)^3}{1+x^2} dx$ $\left(\tan^{-1} x\right)^3$		
		$I = \int \frac{\left(\tan^{-1} x\right)^3}{1+x^2} dx$ Put $\tan^{-1} x = t$	1/2	
		$\frac{1}{1+x^2}dx = dt$	1/2	
		$\therefore I = \int t^3 dt$ $= \frac{t^4}{4} + c$	1 1	
		$=\frac{\left(\tan^{-1}x\right)^4}{4}+c$	1	4
	f)	Evaluate: $\int \frac{e^x}{\left(e^x - 1\right)\left(e^x + 1\right)} dx$		



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		IVIAIKS	Marks
2)	Ans	$Put e^{x} = t$	1	
		$e^x dx = dt$	1	
		$I = \int \frac{dt}{(t-1)(t+1)}$		
			1	
		$=\int \frac{dt}{t^2-1}$	1	
		$= \frac{1}{2} \log \left(\frac{t-1}{t+1} \right) + c$	1½	
		$= \frac{1}{2} \log \left(\frac{e^x - 1}{e^x + 1} \right) + c$	1/2	4
		OR		
		Put $e^x = t$		
		$e^{x}dx = dt$	1	
		Let $\frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$		
		$1 = A \left(t+1\right) + B \left(t-1\right)$		
		Put t = -1		
		1 = B(-2)		
		$B = -\frac{1}{2}$	1/2	
		Put t = 1		
		1 = A(2)		
		$A = \frac{1}{2}$	1/2	
		$\frac{1}{2}$ $-\frac{1}{2}$		
		$\frac{1}{(t-1)(t+1)} = \frac{\frac{1}{2}}{t-1} + \frac{-\frac{1}{2}}{t+1}$		
		$\int \frac{dt}{(t-1)(t+1)} = \int \left(\frac{\frac{1}{2}}{t-1} + \frac{-\frac{1}{2}}{t+1} \right) dt$	1/2	
		$= \frac{1}{2} \log (t-1) - \frac{1}{2} \log (t+1) + c$	1	
		$= \frac{1}{2} \log \left(\frac{t-1}{t+1} \right) + c$		
		$=\frac{1}{2}\log\left(\frac{e^{x}-1}{e^{x}+1}\right)+c$	1/2	4



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Que.	Sub.	Model Answers	Marks	Total
No. 3)	Que.	Attempt any EOLID of the following		Marks 16
	2)	Attempt any <u>FOUR</u> of the following: $\frac{\pi}{2}$		
	a)	Evaluate $\int_{0}^{2} \frac{\cos x}{4 - \sin^{2} x} dx$		
	Ans	$Put \sin x = t$		
		$\cos x dx = dt$ when $x \to 0$ to $\frac{\pi}{2}$ $t \to 0$ to 1	1+1	
		$\therefore I = \int_0^1 \frac{1}{4-t^2} dt$		
		$I = \int_{0}^{1} \frac{1}{(2)^{2} - t^{2}} dt$		
		$I = \left[\frac{1}{2(2)} \log \left \frac{2+t}{2-t} \right \right]_0^1$	1	
		$I = \frac{1}{4} \left[\log \left \frac{3}{1} \right - \log \left \frac{2}{2} \right \right]$	1	
		$I = \frac{1}{4} \left[\log 3 - \log 1 \right]$		
		$I = \frac{1}{4} \log 3$		4
	b)	Evaluate $\int_{0}^{\frac{\pi}{4}} \log(1 + \tan x) dx$		
	Ans	$I = \int \log \left(1 + \tan x \right) dx$		
		$I = \int_{0}^{\frac{\pi}{4}} \log \left(1 + \tan \left(\frac{\pi}{4} - x \right) \right) dx$	1/2	
		$I = \int_{0}^{\frac{\pi}{4}} \log \left 1 + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right dx$		
		$I = \int_{0}^{\frac{\pi}{4}} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$	1	



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	2	$I = \int_{0}^{\frac{\pi}{4}} \log \left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x} \right) dx$		
		$I = \int_{0}^{\frac{\pi}{4}} \log \left(\frac{2}{1 + \tan x} \right) dx$	1/2	
		$I = \int_{0}^{\frac{\pi}{4}} \left[\log 2 - \log \left(1 + \tan x \right) \right] dx$	1/2	
		$I = \log 2 \int_{0}^{\frac{\pi}{4}} dx - \int_{0}^{\frac{\pi}{4}} \log (1 + \tan x) dx$		
		$I = \log 2 \left[x \right]_0^{\frac{\pi}{4}} - I$	1/2	
		$2I = \log 2 \left[\frac{\pi}{4} - 0 \right]$	1/2	
		$I = \frac{\pi}{8} \log 2$	1/2	4
	c)	Find the area of an ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ by integration		
	Ans	$\frac{x^2}{16} + \frac{y^2}{9} = 1$		
		$\therefore y^2 = \frac{9}{16} \left(16 - x^2 \right)$		
		$\therefore y = \frac{3}{4} \sqrt{16 - x^2}$		
		area, $A = 4 \int_{a}^{b} y dx$	1	
		$A = 4 \left[\frac{3}{4} \int_{0}^{4} \sqrt{(4)^{2} - x^{2}} dx \right]$		
		$A = 3 \left[\frac{x}{2} \sqrt{(4)^2 - x^2} + \frac{(4)^2}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$	1	
		$A = 3 \left[8 \sin^{-1} (1) - 0 \right]$	1	
		$A = 24 \frac{\pi}{2}$ $A = 12 \pi$	1	4



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.	O R		Marks
3)		$\frac{x^2}{16} + \frac{y^2}{9} = 1$		
		$\therefore y^2 = \frac{9}{16} \left(16 - x^2 \right)$		
		$\therefore y = \frac{3}{4} \sqrt{16 - x^2}$	1/4	
		area, $A = \int_{a} y dx$	1/2	
		$A = \left[\frac{3}{4} \int_{0}^{4} \sqrt{(4)^{2} - x^{2}} dx \right]$		
		$A = \frac{3}{4} \left[\frac{x}{2} \sqrt{(4)^2 - x^2} + \frac{(4)^2}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$	1	
		$A = \frac{3}{4} \left[8 \sin^{-1} (1) - 0 \right]$	1	
		$A = \frac{3}{4} \left[8 \frac{\pi}{2} \right]$	1	
		$A = 3\pi$		
		∴ area of ellipse is		
		$= 4 \times A$ $= 4 \times 3\pi$		
		$= 12\pi$	1/2	4
	d)	Solve $\frac{dy}{dx} = \cos(x + y)$		
	Ans	Put x + y = v		
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$	1	
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$		
		$\therefore \frac{dv}{dx} - 1 = \cos v$	1/2	
		$\frac{dv}{dx} = 1 + \cos v$		
		$\frac{1}{1+\cos v}dv=dx$	1/2	
		$\int \frac{1}{1 + \cos v} dv = \int dx$	72	



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\int \frac{1}{2\cos^2\left(\frac{v}{2}\right)} \ dv = x + c$	1/2	
		$\frac{1}{2} \int \sec^2 \left(\frac{v}{2}\right) dv = x + c$ $\tan \left(\frac{v}{2}\right)$		
		$\frac{1}{2} \frac{\tan\left(\frac{v}{2}\right)}{\frac{1}{2}} = x + c$	1	
		$\tan\left(\frac{v}{2}\right) = x + c$ $\tan\left(\frac{x+y}{2}\right) = x + c$	1/2	4
		OR $Solve \frac{dy}{dx} = \cos(x + y)$ Put $x + y = y$		
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$	1	
		$dx dx$ $dv dx - 1 = \cos v$ dv	1/2	
		$\frac{1}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$		
		$\int \frac{1}{1 + \cos v} dv = \int dx$ Put $\tan \frac{v}{2} = t$	1/2	
		$dv = \frac{2 dt}{1 + t^2}$ $\cos v = \frac{1 - t^2}{1 + t^2}$	1/2	
		$\therefore \int \frac{1}{1 + \frac{1 - t^2}{1 + t^2}} \frac{2 dt}{1 + t^2} = x + c$	1/2	
		$2\int \frac{1}{1+t^2+1-t^2} dt = x+c$		



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3) $2\int \frac{1}{2}ds = x + c$ $t = x + c$ $tan\left(\frac{y}{2}\right) = x + c$ $tan\left(\frac{x + y}{2}\right) = x + c$ OR $Solve \frac{dy}{dx} = cos(x + y)$ $Put x + y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = 1 + cos v$ $\frac{1}{1 + cos v} = 1 + cos v$ $\int \frac{1}{1 + cos v} dv = \int dx$ $\int \frac{1 - cos v}{1 - cos^2} v = \int dx$ $\int \frac{1 - cos v}{\sin^2 v} dv = x + c$ $\int (cos e^2 v - cot v cos e c v) dv = x + c$ $- cot v + cos e c v = x + c$ $- cot (x + y) + cos e c (x + y) = x + c$ $e) Solve the differential equation \frac{dy}{dx} = \frac{x^2 + y^2}{xy} Ans Put y = vx$	Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
$tan\left(\frac{v}{2}\right) = x + c$ $tan\left(\frac{v}{2}\right) = x + c$ $tan\left(\frac{x + y}{2}\right) = x + c$ OR $Solve \frac{dy}{dx} = cos(x + y)$ $Put x + y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} - 1$ $\frac{dv}{dx} - 1 = cos v$ $\frac{dv}{dx} - 1 = cos v$ $\frac{dv}{dx} = 1 + cos v$ $\frac{1}{1 + cos v} dv = dx$ $\int \frac{1}{1 + cos^2} dv = \int dx$ $\int \frac{1 - cos v}{\sin^2 v} dv = \int dx$ $\int \frac{1 - cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (cos ec^2 v - \cot v \cos ec v) dv = x + c$ $- \cot (x + y) + \cos ec (x + y) = x + c$ e e $Solve the differential equation \frac{dy}{dx} = \frac{x^2 + y^2}{xy}$			$2\int \frac{1}{-dt} = x + c$	1/2	
$\tan\left(\frac{v}{2}\right) = x + c$ $\tan\left(\frac{x + y}{2}\right) = x + c$ OR $Solve \frac{dy}{dx} = \cos(x + y)$ $Put x + y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\frac{dv}{dx} - 1 = \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = x + c$ $\int \left(\frac{\sin^2 v}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $= \cot(v + \cos ecv = x + c)$ $= \cot(x + y) + \csc(x + y) = x + c$ $= \cot(x + y) + \cot(x + y) $			2		
OR Solve $\frac{dy}{dx} = \cos(x + y)$ Put $x + y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\frac{dv}{dx} - 1 = \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int ((\cos ec^2 v - \cot v \cos ec v)) dv = x + c$ $-\cot(x + y) + \cos ec(x + y) = x + c$ Polytonia (1) 10 11 12 14 15 16 17 17 17 17 18 19 19 19 19 10 10 10 10 11 11				1/2	4
Solve $\frac{dy}{dx} = \cos(x + y)$ Put $x + y = y$ $1 + \frac{dy}{dx} = \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{dy}{dx} - 1$ $\frac{dy}{dx} = 1 + \cos y$ $\frac{dy}{dx} = 1 + \cos y$ $\frac{1}{1 + \cos y} dy = dx$ $\int \frac{1}{1 + \cos^2 y} dy = \int dx$ $\int \frac{1 - \cos y}{1 - \cos^2 y} dv = \int dx$ $\int \frac{1 - \cos y}{\sin^2 y} dv = x + c$ $\int \left(\frac{1}{\sin^2 y} - \frac{\cos y}{\sin^2 y}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ec v) dv = x + c$ $- \cot(x + y) + \cos ec(x + y) = x + c$ $= 0$ E) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$			$\tan\left(\frac{1}{2}\right) = x + c$		1
Put $x y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\frac{dv}{dx} - 1 = \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos^2 v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1 - \cos v}{\sin^2 v} dv = x + c\right)$ $\int (\cos ec^2 v - \cot v \cos ec v) dv = x + c$ $-\cot v + \cos ec v = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $e)$ Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$			OR		
Put $x + y = v$ $1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos e^2 v - \cot v \cos e c v) dv = x + c$ $- \cot v + \cos e c v = x + c$ $- \cot (x + y) + \cos e c (x + y) = x + c$ $e)$ Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$			Solve $\frac{dy}{dx} = \cos(x + y)$		
$\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\frac{dy}{dx} = 1 + \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos v} dv = \int dx$ $\int \frac{1}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1}{1 - \cos^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ecv) dv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $e)$ Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$					
$\frac{dx}{dx} = 1 + \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ecv) dv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos ec (x + y) = x + c$ $= \cos (x + y) + \cos (x + y) + \cos (x + y) = x + c$ $= \cos (x + y) + \cos (x + $			$1 + \frac{dy}{dx} = \frac{dv}{dx}$		
$\frac{dv}{dx} - 1 = \cos v$ $\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ec v) dv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $\frac{1}{\cos^2 v} + \cos^2 v + \cos^2 v + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$				1	
$\frac{dv}{dx} = 1 + \cos v$ $\frac{1}{1 + \cos v} dv = dx$ $\int \frac{1}{1 + \cos v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ec v) dv = x + c$ $-\cot v + \cos ec v = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $\frac{1}{2}$			$\therefore \frac{dv}{}-1=\cos v$		
$\frac{1}{1+\cos v}dv = dx$ $\int \frac{1}{1+\cos v}dv = \int dx$ $\int \frac{1-\cos v}{1-\cos^2 v}dv = \int dx$ $\int \frac{1-\cos v}{\sin^2 v}dv = x+c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right)dv = x+c$ $\int (\cos ec^2 v - \cot v \cos ecv)dv = x+c$ $-\cot v + \cos ecv = x+c$ $-\cot (x+y) + \cos ec (x+y) = x+c$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$			$\frac{dv}{dt} = 1 + \cos v$	1/2	
$\int \frac{1}{1 + \cos v} dv = \int dx$ $\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ecv) dv = x + c$ $-\cot v + \cos ecv = x + c$ $-\cot (x + y) + \csc (x + y) = x + c$ $\int (\cos ec^2 v - \cot v \cos ecv) dv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$					
$\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$ $\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ec v) dv = x + c$ $-\cot v + \cos ec v = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $\frac{1}{2}$			$1 + \cos v$		
e) Solve the differential equation $\frac{\int \frac{1-\cos^2 v}{1-\cos^2 v} dv = \int dx}{\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv} = x + c$ $\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int (\cos ec^2 v - \cot v \cos ecv) dv = x + c$ $-\cot v + \cos ecv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $\frac{1}{2}$			$\int \frac{1}{1 + \cos v} dv = \int dx$	1/2	
$\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$ $\int \left(\cos ec^2 v - \cot v \cos ec v\right) dv = x + c$ $-\cot v + \cos ec v = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $\frac{1}{2}$ $e) Solve the differential equation \frac{dy}{dx} = \frac{x^2 + y^2}{xy}$			$\int \frac{1 - \cos v}{1 - \cos^2 v} dv = \int dx$	1/2	
$\int (\cos ec^{2}v - \cot v \cos ecv) dv = x + c$ $-\cot v + \cos ecv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$			$\int \frac{1 - \cos v}{\sin^2 v} dv = x + c$		
$\int (\cos ec^{2}v - \cot v \cos ecv) dv = x + c$ $-\cot v + \cos ecv = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$ $-\cot (x + y) + \cos ec (x + y) = x + c$			$\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v} \right) dv = x + c$		
e) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$				1	
e) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$				1/2	4
			$-\cot(x+y)+\cos ec(x+y)=x+c$, -	•
			2 2		
		e)	Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$		
		Ans	Put y = vx		



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Que.	Sub.	Model Answers	Marks	Total
No. 3)	Que.		1	Marks
		$\frac{dy}{dx} = v + x \frac{dv}{dx}$	1	
		$dv = x^2 + (vx)^2$		
		$\therefore v + x \frac{dv}{dx} = \frac{x^2 + (vx)^2}{x(vx)}$		
		$v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{v x^2}$		
		$v + x \frac{dv}{dx} = \frac{1 + v^2}{v}$	1/2	
		$x\frac{dv}{dx} = \frac{1+v^2}{v} - v$	1/2	
		$x\frac{dv}{dx} = \frac{1+v^2-v^2}{v}$		
		$x\frac{dv}{dx} = \frac{1}{v}$	1/2	
		$v dv = \frac{1}{x} dx$		
		$\int v dv = \int \frac{1}{x} dx$	1/2	
		$\frac{v^2}{2} = \log x + c$	1/2	
		$\frac{y^2}{2x^2} = \log x + c$	1/2	4
	f)	Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$		
	Ans	$\frac{dy}{dx} - \frac{1}{x+1}y = e^x(x+1)$		
		$\therefore P = -\frac{1}{x+1} \text{ and } Q = e^{x} (x+1)$	1	
		$IF = e^{-\int \frac{1}{x+1} dx} = e^{-\log(x+1)} = \frac{1}{x+1}$	1	
		$\therefore yIF = \int QIF dx + c$	1	
		$y\frac{1}{x+1} = \int e^{x} (x+1) \frac{1}{x+1} dx + c$	1	
		$\frac{y}{x+1} = \int e^x dx + c$		4
		$\frac{y}{x+1} = e^x + c$	1	



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Que.	Sub.	Model Answers	Marks	Total
No. 4)	Que.	Attempt any FOUR of the following:		Marks 16
	a)	Evaluate $\int_{1}^{5} \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}} dx$		
	Ans	$I = \int_{1}^{5} \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}} dx \qquad(1)$		
		$I = \int_{1}^{5} \frac{\sqrt{9 - (1 + 5 - x)}}{\sqrt{9 - (1 + 5 - x)} + \sqrt{(1 + 5 - x) + 3}} dx$		
		$I = \int_{1}^{5} \frac{\sqrt{x+3}}{\sqrt{x+3} + \sqrt{9-x}} dx \qquad (2)$	1	
		add (1) and (2) $I + I = \int_{1}^{5} \frac{\sqrt{9 - x}}{\sqrt{9 - x} + \sqrt{x + 3}} dx + \int_{1}^{5} \frac{\sqrt{x + 3}}{\sqrt{x + 3} + \sqrt{9 - x}} dx$		
		$2I = \int_{1}^{5} \frac{\sqrt{9-x} + \sqrt{x+3}}{\sqrt{9-x} + \sqrt{x+3}} dx$	1	
		5	1/2	
		$2I = \int_{1} 1 dx$	1/2	
		$2I = \left[x\right]_{1}^{5}$	1/2	4
		2I = 5 - 1 $I = 2$	1/2	4
		I = 2		
	b)	Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{dx}{4 + 5\cos x}$		
	Ans	Put $\tan \frac{x}{2} = t$		
		$\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2 dt}{1+t^2}$ when $x \to 0$ to $\frac{\pi}{2}$ $t \to 0$ to 1	1	
		$\therefore I = \int_{0}^{1} \frac{1}{4+5\left(\frac{1-t^{2}}{1+t^{2}}\right)} \frac{2dt}{1+t^{2}}$	1/2	
		$I = 2\int_{0}^{1} \frac{1}{4(1+t^{2}) + 5(1-t^{2})} dt$		
		$I = 2\int_{0}^{1} \frac{1}{4+4t^{2}+5-5t^{2}} dt$		



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	C 1	17301 Willter-2013	Tage Ne	
Que.	Sub.	Model Answers	Marks	Total
No.	Que.			Marks
4)		$I = 2 \int_{0}^{1} \frac{1}{9 - t^{2}} dt$ $I = 2 \int_{0}^{1} \frac{1}{(3)^{2} - t^{2}} dt$	1/2	
		$I = 2 \left[\frac{1}{2(3)} \log \left \frac{3+t}{3-t} \right \right]_0^1$	1	
		$I = \frac{1}{3} \left[\log \left \frac{4}{2} \right - \log \left \frac{3}{3} \right \right]$	1/2	
		$I = \frac{1}{3} \left[\log \left 2 \right - \log \left 1 \right \right]$		
		$I = \frac{1}{3} \log 2 $	1/2	4
	c) Ans	Find the area between the parabola $y^2 = 4x$ and the line $y = 2x + 3$ As in the given problem Curves are Not intersecting thus finding the area between the given two curves is not possible. So while assessing this question Assessor NOT TO REWARD MARKS for this Question in either case whether candidate attempted the question or not.	Do Not Asse ss	Do Not Asse ss



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Subjec	(ISO/IEC - 27001 - 2005 Certified) Subject Code: 17301 Winter-2015 Page No: 18/26						
Que.	Sub.	Model Answers	Marks	Total			
No.	Que.	Model Allswers	Marks	Marks			
4)	d) Ans	Solve $\frac{dy}{dx} = e^{2x+y} + x^2 e^y$ $\frac{dy}{dx} = \left(e^{2x} + x^2\right) e^y$					
	1110	$\frac{dy}{dx} = \left(e^{2x} + x^2\right)e^{y}$	1				
		$e^{-y}dy = \left(e^{2x} + x^2\right)dx$					
		$\int e^{-y} dy = \int \left(e^{2x} + x^2\right) dx$	1				
		$\frac{e^{-y}}{-1} = \frac{e^{2x}}{2} + \frac{x^3}{3} + c$	2	4			
	e)	Solve $(2x + 3\cos y) dx + (2y - 3x\sin y) dy = 0$					
	Ans	$M = 2x + 3\cos y$, $N = 2y - 3x\sin y$					
		$\frac{\partial M}{\partial y} = -3\sin y ,$	1				
		$\frac{\partial N}{\partial y} = -3\sin y$	1				
		$\frac{1}{\partial x} = -3 \sin y$					
		$\therefore \frac{\partial M}{\partial M} = \frac{\partial N}{\partial M}$					
		$\partial y \qquad \partial x$					
		∴ equation is an exact D.E.					
		$\int_{\substack{y-cons \tan t}} M dx + \int_{\substack{terms free \\ from x}} N dy = c$					
		$\therefore \qquad \int \qquad (2x + 3\cos y) dx + \int 2 y dy = c$	1				
		$x^2 + 3x \cos y + y^2 = c$	1	4			
	f)	Show that $y = A \sin mx + B \cos mx$ is a solution of differential equation					
		$\frac{d^2y}{dx^2} + m^2y = 0$		ļ			
	Ans						
		$y = A \sin mx + B \cos mx$ $\frac{dy}{dx} = mA \cos mx - mB \sin mx$	1				
		dx					
		$\frac{d^2y}{dx^2} = -m^2 A \sin mx - m^2 B \cos mx$	1				
		$\frac{d^2y}{dx^2} = -m^2 \left(A \sin mx + B \cos mx \right)$	1				
		$\frac{d^2y}{dx^2} = -m^2y$					
		$\frac{d^2y}{dx^2} + m^2y = 0$		4			
		$\frac{1}{dx^2} + m y = 0$	1	4			
		OR					



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1	1/301 Winter-2015	Pager	
Sub. Que.	Model Answers	Marks	Total Marks
7	$y = A \sin mx + B \cos mx$ $\frac{dy}{dx} = mA \cos mx - mB \sin mx$	1	-
	$\frac{d^{2}y}{dx^{2}} = -m^{2}A\sin mx - m^{2}B\cos mx$ L.H.S. = $\frac{d^{2}y}{dx^{2}} + m^{2}y$	1	
	$= -m^{2} A \sin mx - m^{2} B \cos mx + m^{2} (A \sin mx + B \cos mx)$ $= -m^{2} A \sin mx - m^{2} B \cos mx + m^{2} A \sin mx + m^{2} B \cos mx$	1	
	= 0 = R.H.S.	1	4
	Attempt any <u>FOUR</u> of the following:		16
a)	A problem is given to three students X,Y,Z whose chances of solving are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ respectively. Find the probability that: i) The problem is solved by each of them.		
Ans	ii) The problem is not solved by any of them. $P(X) = \frac{1}{2} \qquad \therefore P(X') = 1 - \frac{1}{2} = \frac{1}{2}$ $P(Y) = \frac{1}{3} \qquad \therefore P(Y') = 1 - \frac{1}{3} = \frac{2}{3}$		
	i)Problem is solved by each of them is: $= P(X \cap Y \cap Z)$ $= P(X) \times P(Y) \times P(Z)$	1	
	$= \frac{-x - x}{2} = \frac{1}{3} = \frac{1}{4}$ $= \frac{1}{24} \text{or} 0.0417$ ii) Problem is not solved by any of them is:	1½	
	$P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z')$ $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ $= \frac{1}{2} or 0.25$	1½	
	Que.	Que. $y = A \sin mx + B \cos mx$ $\frac{dy}{dx} = mA \cos mx - mB \sin mx$ $\frac{d^2y}{dx^2} = -m^2 A \sin mx - m^2 B \cos mx$ $L.H.S. = \frac{d^2y}{dx^2} + m^2 y$ $= -m^2 A \sin mx - m^2 B \cos mx + m^2 (A \sin mx + B \cos mx)$ $= -m^2 A \sin mx - m^2 B \cos mx + m^2 A \sin mx + m^2 B \cos mx$ $= 0 = R.H.S.$ Attempt any FOUR of the following: a) A problem is given to three students X,Y,Z whose chances of solving are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Find the probability that: i) The problem is not solved by each of them. ii) The problem is not solved by any of them. $P(X) = \frac{1}{2} \qquad \therefore P(X') = 1 - \frac{1}{2} = \frac{1}{2}$ $P(Y) = \frac{1}{3} \qquad \therefore P(Y') = 1 - \frac{1}{4} = \frac{3}{4}$ i)Problem is solved by each of them is: $= P(X \cap Y \cap Z)$ $= P(X) \times P(Y) \times P(Z)$ $= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ $= \frac{1}{2} \text{ or } 0.0417$ ii)Problem is not solved by any of them is: $P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z')$ $= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ $= \frac{1}{24} \text{ or } 0.0417$ ii)Problem is not solved by any of them is: $P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z')$ $= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ $= \frac{1}{24} \text{ or } 0.0417$ ii)Problem is not solved by any of them is: $P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z')$ $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$	Que. Model Answers $ \frac{dy}{dx} = mA \cos mx + B \cos mx $ $ \frac{dy}{dx} = mA \cos mx - mB \sin mx $ $ \frac{d^2y}{dx^2} = -m^2 A \sin mx - m^2 B \cos mx $ $ 1 $ 1 S. $ \frac{d^2y}{dx^2} + m^2 y $ $ = -m^2 A \sin mx - m^2 B \cos mx + m^2 (A \sin mx + B \cos mx) $ $ = -m^2 A \sin mx - m^2 B \cos mx + m^2 A \sin mx + m^2 B \cos mx $ $ = 0 = R.H. S. $ 1 Attempt any FOUR of the following: a) A problem is given to three students X, Y, Z whose chances of solving are $ \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} $ $ \frac{1}{3} \cdot \frac{1}{4} $ i) The problem is not solved by any of them. $ P(X) = \frac{1}{2} \therefore P(X') = 1 - \frac{1}{2} = \frac{1}{2} $ $ P(Y) = \frac{1}{3} \therefore P(Y') = 1 - \frac{1}{3} = \frac{2}{3} $ $ P(Z) = \frac{1}{4} \therefore P(Z') = 1 - \frac{1}{4} = \frac{3}{4} $ i) Problem is solved by each of them is: $ = P(X \cap Y \cap Z) $ $ = P(X) \times P(Y) \times P(Z) $ $ = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} $ $ = \frac{1}{2} \text{or } 0.0417 $ ii) Problem is not solved by any of them is: $ P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z') $ $ = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} $ 11/2



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	T .	Willel-2013	Tage No	
Que	Sub.		\	Total
	Que	Model Answers	Mark	Mark
No.	~===		S	S
5)	b)	If 30% of the bulbs produced are defective, find the probability that out of		
		4 bulbs selected:		
		i) One is defective		
	Ans	ii) At the most two are defective		
		p = 30% = 0.3		
		q = 1 - p = 0.7		
		n = 4	1	
		Binomial Distribution is:		
		$p(r) = {}^{n}C_{r}p^{r}q^{n-r}$		
		i) One is defective, $r = 1$		
		$p(1) = {}^{4}C_{1}(0.3)^{1}(0.7)^{4-1}$		
			11/2	
		p(1) = 0.4116		
		ii) At the most two are defective		
		= p(0) + p(1) + p(2)		
		$= {}^{4}C_{0}(0.3)^{0}(0.7)^{4-0} + 0.4116 + {}^{4}C_{2}(0.3)^{2}(0.7)^{4-2}$		
		= 0.2401 + 0.4116 + 0.2646	41/	4
		= 0.9163	11/2	4
	c)	Using Poisson distribution, find the probability that the ace of spade will be		
	'	drawn from a pack of well shuffled cards at least once		
		in 104 consecutive trials.		
	Ans	Given $n = 104$		
		$p = \frac{1}{50}$		
		52	1	
		m = np		
		$= 104 \times \frac{1}{100} = 2$	1	
		52		
		r = atleast one		
		=1, 2, 3		
		$\therefore p(r) = \frac{e^{-m}m^r}{}$		
		r!		
		p(r) = 1 - p(0)		
		$=1-\frac{e^{-2}2^{0}}{}$	1	
		= 1 - 0!		
		= 0.8646	1	4
	<u> </u>		<u> </u>	



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		Widiks	Marks
5)	d)	Evaluate $\int \frac{dx}{2 + 3\cos x}$		
	A == 0	$\int 2 + 3 \cos x$		
	Ans	Put $\tan \frac{x}{-} = t$		
		2		
		$\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2 dt}{1+t^2}$		
			1	
		$\therefore I = \int \frac{1}{\left(1 + \frac{t^2}{2}\right)^2} \frac{2at}{1 + \frac{t^2}{2}}$	1/2	
		$\therefore I = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)} \frac{2dt}{1+t^2}$	72	
		$I = 2 \int \frac{1}{2(1+t^2) + 3(1-t^2)} dt$		
		$I = 2\int \frac{1}{2 + 2t^2 + 3 - 3t^2} dt$		
		$I = 2\int \frac{1}{5 - t^2} dt$	1/2	
		$I = 2\int \frac{1}{\left(\sqrt{5}\right)^2 - \left(t\right)^2} dt$	1/2	
		$I = \frac{2}{2\sqrt{5}}\log\left \frac{\sqrt{5}+t}{\sqrt{5}-t}\right + c$	1	
		$I = \frac{1}{\sqrt{5}} \log \left \frac{\sqrt{5} + \tan \frac{x}{2}}{\sqrt{5} - \tan \frac{x}{2}} \right + c$	1/2	4
	e)	Evaluate $\int_{0}^{1} x \tan^{-1} x dx$		
	Ans	$= \left[\tan^{-1} x \int x dx - \int \left(\int x dx \frac{d}{dx} \tan^{-1} x \right) dx \right]_0^1$	1/2	
		$= \left[\tan^{-1} x \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{1+x^2} dx \right]_0^1$	1	
		$= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{1+x^2-1}{1+x^2} dx\right]_0^1$		
		$= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{1 + x^2} \right) dx \right]_0^1$	1/2	
		$= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) \right]_0^1$	1	
		$= \left[\frac{1}{2} \tan^{-1} (1) - \frac{1}{2} (1 - \tan^{-1} 1)\right] - 0$	1/2	



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5)	f) Ans	$= \frac{1}{2} \frac{\pi}{4} - \frac{1}{2} + \frac{1}{2} \frac{\pi}{4}$ $= \frac{\pi}{4} - \frac{1}{2} \text{ or } \frac{1}{2} \left(\frac{\pi}{2} - 1 \right)$ $= \frac{3}{4} - \frac{1}{2} \text{ or } \frac{1}{2} \left(\frac{\pi}{2} - 1 \right)$ $= \frac{3}{4} - \frac{1}{2} + \frac{1}{2} \frac{\pi}{4}$ $= \frac{1}{4} - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \frac{\pi}{4}$ $= \frac{1}{4} - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \frac{\pi}{4}$ $= \frac{1}{4} - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \frac{\pi}{4}$ $= \frac{1}{4} - \frac{1}{2} + $	1/2	4
		dx dx	1/2	
		$\therefore v + x \frac{dv}{dx} = v + \sin v$	1/2	
		$x \frac{dv}{dx} = \sin v$		
		$\frac{1}{\sin v} dv = \frac{1}{x} dx$	1/2	
		$\int \cos e c v d v = \int \frac{1}{x} dx$ $\log \left \cos e c v - \cot v \right = \log \left x \right + c$	1/2+1/2	
		$\log \left \cos e c \frac{y}{x} - \cot \frac{y}{x} \right = \log \left x \right + c$	1/2	4
6)		Attempt any <u>FOUR</u> of the following:		16
	a)	A bag contains 20 tickets numbered from 1 to 20. One ticket is drawn at		
		random. Find the probability that it is numbered with multiple of 3 or 5.		
	Ans	$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$		
		$\therefore n(S) = 20$	1	
		number multiple of 3 or 5 $A = \{3, 5, 6, 9, 10, 12, 15, 18, 20\}$	1 ¹ / ₂	
		$\therefore n(A) = 9$		
		$p(A) = \frac{n(A)}{n(S)}$ $= \frac{9}{20} or 0.45$	1½	4
		OR		



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		VIII(E1-2013	1	J. 23/20
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$		
		$\therefore n(S) = 20$	1	
		number multiple of 3 or 5		
		$n(A \cup B) = n(A) + n(B) - n(A \cap B)$	1	
		$n(A \cup B) = 6 + 4 - 1$		
		$\therefore n(A \cup B) = 9$	1	
		$p(A \cup B) = \frac{n(A \cup B)}{n(S)}$		
		n(S)		
		$=\frac{9}{}$ or 0.45	1	4
		20		
	b)	A firm produces articles of which 0.1% are defective, out of 500		
		articles. If wholesaler purchases 100 such cases , how many		
		can be expected to have one defective? Given: $e^{-0.5} = 0.6065$		
	Ans	p = 0.1% = 0.001	1/2	
		n = 500	/2	
		$m ean m = np = 500 \times 0.001$		
		m = 0.5	1	
		Poisson Distribution is		
		$P(r) = \frac{e^{-m}m^r}{r!}$		
		One is defective		
		r = 1		
		$e^{-0.5} (0.5)^{1}$	1	
		$P(1) = \frac{e^{-0.5} (0.5)^{1}}{1!}$		
		P(1) = 0.30325	1/2	
		No. of cases = 100×0.30325		
		$= 30.325 \approx 30$	1	4
	c)	I.Q.'s are normally distributed with mean 100 and standard deviatio15.		
		Find the probability that a randomly		
		selected person has:		
		i) An I.Q.more than 130		
		ii) An I.Q.between 85 and 115		
		[z = 2, Area = 0.4772, z = 1, Area = 0.3413]		
	Ans	Given $x = 100$, $\sigma = 15$		
		$\frac{-}{x-x}$		
		Standard normal variate, $Z = \frac{x - x}{\sigma}$		



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		i) For $x = 130$, $Z = \frac{130 - 100}{15} = 2$	1/2	
		15		
		p = (area more than 2) = 0.5 - A(2)	1	
		= 0.5 - 0.4772	•	
		= 0.0228	1/2	
		ii) For $x = 85$, $Z = \frac{85 - 100}{15} = -1$		
		For $x = 115$, $Z = \frac{115 - 100}{15} = 1$	1/2	
		15		
		m (I O between 95 and 115) A (1) + A (1)	1	
		p(I.Q.between 85 and 115) = A(-1) + A(1)		
		= 0.3413 + 0.3413	1/2	4
		= 0.6826	72	
	d)	Divide 80 into two parts such that their product is maximum.		
	Ans	consider x and y be the two parts		
		$\therefore x + y = 80$		
		y = 80 - x		
		product is, $P = xy$		
		P = x(80 - x)		
		$P = 80 x - x^2$	_	
			1	
		$\frac{dP}{dx} = 80 - 2x$	1/2	
		d^2P		
		$\frac{d^2}{dx^2} = -2$	1/2	
		For maximum value $\frac{dP}{dt} = 0$		
		dx	1	
		$\therefore 80 - 2x = 0$		
		x = 40		
		At $x = 40$, $\frac{d^2 P}{dx^2} = -2$, <i>i.e.</i> Product is maximum	1/2	
		$\therefore x = 40, y = 40$	1/2	
				4
	6)			
	e)	The equation of the tangent at the point $(2,3)$ on the curve $y = ax^3 + b$		
		is $y = 4x - 5$. Find the values of a and b		
<u> </u>	1		I .	



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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
6)	Ans	The equation of the tan gent is,		
		y = 4x - 5		
		$\therefore slope = m = 4$	1/2	
		$y = ax^3 + b$		
			1	
		$\frac{dy}{dx} = 3ax^2$		
		at (2,3)		
		$\frac{dy}{dx} = 12a$	1/2	
		$\frac{}{dx} = 12a$	/2	
		m = 12a		
		$\therefore 4 = 12a$	1	
		$a = \frac{1}{a}$	1	
		3		
		$y = ax^3 + b$		
		$\therefore 3 = \left(\frac{1}{3}\right) (2)^3 + b$		
		(3)	1	
		$b=rac{1}{3}$	1	
				4
		$a = \frac{1}{3}, b = \frac{1}{3}$		4
	f) Ans	Find the area of circle $x^2 + y^2 = 16$ by integration		
	71115	$x^2 + y^2 = 16$		
		$\therefore y = \sqrt{16 - x^2}$		
		$area, A = 4 \int_{a}^{b} y dx$	1	
		$\begin{bmatrix} 4 & & \\ & & \\ & & \\ & & \end{bmatrix}$		
		$A = 4 \left[\int_0^4 \sqrt{\left(4\right)^2 - x^2} dx \right]$		
		$A = 4 \left[\frac{x}{2} \sqrt{(4)^2 - x^2} + \frac{(4)^2}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$	1	
		$A = 4 \left[8 \sin^{-1} (1) - 0 \right]$	1	
		$A = 4 \left\lceil 8 \frac{\pi}{2} \right\rceil$	_	
		$A = 16\pi$	1	4
			1	



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the		
		scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.		



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