



Winter - 2012 Examination

Subject & Code: Applied Maths (12013)

Model Answer

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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	a)	$f(x) = 3x^4 - 2x^2 + \cos x$ $\therefore f(-x) = 3(-x)^4 - 2(-x)^2 + \cos(-x)$ $= 3x^4 - 2x^2 + \cos x$ $= f(x)$ $\therefore f(x) \text{ is even.}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	b)	$f(x) = x^2 + 6x + 10$ $\therefore f(2) = 2^2 + 6(2) + 10 = 26$ $f(-2) = (-2)^2 + 6(-2) + 10 = 2$ $\therefore f(2) + f(-2) = 28$ <p>OR</p> $f(2) + f(-2) = [2^2 + 6(2) + 10] + [(-2)^2 + 6(-2) + 10]$ $= 28$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1 1	2
	c)	$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2}$ $= \lim_{x \rightarrow 2} (x + 2)$ $= 2 + 2$ $= 4$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	d)	$\lim_{x \rightarrow 0} \frac{\sin 4x}{3x} = \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot \frac{4}{3}$ $= 1 \times \frac{4}{3}$ $= \frac{4}{3}$	$\frac{1}{2}$ 1 $\frac{1}{2}$	2
	e)	$\lim_{x \rightarrow \infty} \frac{6x^3 + 5x^2 - 1}{3x^3 + 4x^2 + 7} = \lim_{x \rightarrow \infty} \frac{\frac{6x^3 + 5x^2 - 1}{x^3}}{\frac{3x^3 + 4x^2 + 7}{x^3}}$ $= \lim_{x \rightarrow \infty} \frac{6 + \frac{5}{x} - \frac{1}{x^3}}{3 + \frac{4}{x} + \frac{7}{x^3}}$ $= \frac{6 + 0 - 0}{3 + 0 + 0}$ $= 2$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$	2



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1)	f)	$\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{x}$ $= \lim_{x \rightarrow 0} \left[\frac{a^x - 1}{x} + \frac{b^x - 1}{x} + \frac{c^x - 1}{x} \right]$ $= \log a + \log b + \log c$ $= \log abc$	$\frac{1}{2}$ 1 $\frac{1}{2}$	2
	g)	$y = e^x \tan x$ $\frac{dy}{dx} = e^x \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(e^x)$ $= e^x \sec^2 x + \tan x \cdot e^x$ $= e^x (\sec^2 x + \tan x)$	$\frac{1}{2}$ 1 $\frac{1}{2}$	
	h)	$y = \tan(4 - 3x)$ $\frac{dy}{dx} = \sec^2(4 - 3x) \cdot \frac{d}{dx}(4 - 3x)$ $= \sec^2(4 - 3x) \cdot (-3)$ $= -3 \sec^2(4 - 3x)$	1 $\frac{1}{2}$ $\frac{1}{2}$	2
	i)	$y = \frac{e^x + 1}{e^x - 1}$ $\therefore \frac{dy}{dx} = \frac{(e^x - 1) \frac{d}{dx}(e^x + 1) - (e^x + 1) \frac{d}{dx}(e^x - 1)}{(e^x - 1)^2}$ $= \frac{(e^x - 1)e^x - (e^x + 1)e^x}{(e^x - 1)^2}$ $= \frac{e^x [e^x - 1 - e^x - 1]}{(e^x - 1)^2}$ $= \frac{-2e^x}{(e^x - 1)^2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	
		OR		
		$y = \frac{e^x + 1}{e^x - 1} = 1 + \frac{2}{e^x - 1}$ $\therefore \frac{dy}{dx} = 0 + \frac{-2}{(e^x - 1)^2} \cdot \frac{d}{dx}(e^x - 1)$ $= \frac{-2e^x}{(e^x - 1)^2}$	1 1	2



Que. No.	Sub. Que.	Model answers	Marks	Total Marks																					
1)	j)	$y = \log(x^2 + 2x + 5)$ $\frac{dy}{dx} = \frac{1}{x^2 + 2x + 5} \cdot \frac{d}{dx}(x^2 + 2x + 5)$ $= \frac{2x + 2}{x^2 + 2x + 5}$ <p style="text-align: center;">OR</p> $y = \log(x^2 + 2x + 5)$ $e^y = x^2 + 2x + 5$ $\therefore e^y \frac{dy}{dx} = 2x + 2$ $\therefore \frac{dy}{dx} = \frac{2x + 2}{e^y}$	1 1	2																					
	k)	Rearranging the terms: 3, 4, 5, 6, 6, 7, 8, 8, 9, 11 $\therefore \text{Median} = \frac{6 + 7}{2} = 6.5$ $\text{Mode} = 6 \quad \text{and} \quad 8$	1 1	2																					
	l)	<table border="1"><thead><tr><th>x_i</th><th>f_i</th><th>$x_i f_i$</th></tr></thead><tbody><tr><td>1</td><td>14</td><td>14</td></tr><tr><td>3</td><td>23</td><td>69</td></tr><tr><td>5</td><td>27</td><td>135</td></tr><tr><td>7</td><td>21</td><td>147</td></tr><tr><td>9</td><td>15</td><td>135</td></tr><tr><td></td><td>100</td><td>500</td></tr></tbody></table> $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{500}{100} = 5$	x_i	f_i	$x_i f_i$	1	14	14	3	23	69	5	27	135	7	21	147	9	15	135		100	500	1 1	2
	x_i	f_i	$x_i f_i$																						
	1	14	14																						
	3	23	69																						
5	27	135																							
7	21	147																							
9	15	135																							
	100	500																							
2)	a)	$f(t) = \frac{2t + 5}{3t - 4}$ $2\left(\frac{5 + 4x}{3x - 2}\right) + 5$ $= \frac{2(5 + 4x) + 5(3x - 2)}{3x - 2}$ $= \frac{3x - 2}{3(5 + 4x) - 4(3x - 2)}$ $3x - 2$	1/2 1																						



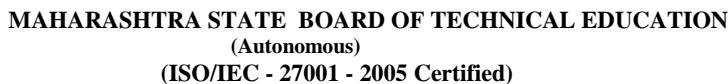
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$= \frac{10x+8x+15x-10}{15+12x-12x+8}$ $= \frac{23x}{23}$ $= x$	1	4
	b)	$f(x) = \log\left(\frac{1+x}{1-x}\right)$ $\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$ $= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$ $= \log\left[\frac{(1+x)^2}{(1-x)^2}\right]$ $= \log\left(\frac{1+x}{1-x}\right)^2$ $= 2\log\left(\frac{1+x}{1-x}\right)$ $= 2f(x)$	1 1/2	
	c)	$\lim_{x \rightarrow 4} \frac{x^4 - 64x}{\sqrt{x^2 + 9} - 5} = \lim_{x \rightarrow 4} \frac{x^4 - 64x}{\sqrt{x^2 + 9} - 5} \times \frac{\sqrt{x^2 + 9} + 5}{\sqrt{x^2 + 9} + 5}$ $= \lim_{x \rightarrow 4} \frac{(x^4 - 64x)}{x^2 + 9 - 25} (\sqrt{x^2 + 9} + 5)$ $= \lim_{x \rightarrow 4} \frac{x(x^3 - 64)}{x^2 - 16} (\sqrt{x^2 + 9} + 5)$ $= \lim_{x \rightarrow 4} \frac{x(x-4)(x^2 + 4x + 16)}{(x-4)(x+4)} (\sqrt{x^2 + 9} + 5)$ $= \lim_{x \rightarrow 4} \frac{x(x^2 + 4x + 16)}{(x+4)} (\sqrt{x^2 + 9} + 5)$ $= \frac{4(4^2 + 44 + 16)}{(4+4)} (\sqrt{4^2 + 9} + 5)$ $= 240$	1 1 1 1 1/2	4



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	d)	$\lim_{x \rightarrow 5} \left[\frac{\log x - \log 5}{x - 5} \right]$		
		$\text{Let } x = 5 + h$ $\text{as } x \rightarrow 5, h \rightarrow 0$		
		$= \lim_{h \rightarrow 0} \left[\frac{\log(5+h) - \log 5}{5+h-5} \right]$	1	
		$= \lim_{h \rightarrow 0} \frac{1}{h} \log \left(\frac{5+h}{5} \right)$		
		$= \lim_{h \rightarrow 0} \log \left(1 + \frac{h}{5} \right)^{1/h}$	1	
		$= \lim_{h \rightarrow 0} \log \left(1 + \frac{h}{5} \right)^{5/h \times 1/5}$		
		$= \lim_{h \rightarrow 0} \frac{1}{5} \log \left(1 + \frac{h}{5} \right)^{5/h}$		
		$= \frac{1}{5} \log e$	1	
		$= \frac{1}{5}$	1	4
	e)	$\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2} = \lim_{x \rightarrow 0} \frac{3^x \cdot 2^x - 3^x - 2^x + 1}{x^2}$	1	
		$= \lim_{x \rightarrow 0} \frac{3^x (2^x - 1) - (2^x - 1)}{x^2}$		
		$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(2^x - 1)}{x^2} \dots\dots\dots (*)$	1	
		$= \lim_{x \rightarrow 0} \frac{(3^x - 1)}{x} \times \frac{(2^x - 1)}{x}$	1	
	f)	$= \log 3 \times \log 2$	1	4
		<p>Note: Students may write the step (*) directly as we do in case of factors of polynomial. So this direct step may be considered for marks.</p>		
		$\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right)}{x^2}$		
		$= \lim_{x \rightarrow 0} \frac{-2 \sin 4x \sin x}{x^2}$	1	
		$= \lim_{x \rightarrow 0} -2 \times \frac{\sin 4x}{4x} \times \frac{\sin x}{x} \times 4$	1	
		$= -2 \times 1 \times 1 \times 4$	1	
		$= -8$	1	4



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	a)	$x = a(\cos \theta + \theta \sin \theta)$		
		$\therefore \frac{dx}{d\theta} = a(-\sin \theta + \theta \cos \theta + \sin \theta)$		
		$= a\theta \cos \theta$	1	
		$y = a(\sin \theta - \theta \cos \theta)$		
		$\therefore \frac{dy}{d\theta} = a(\cos \theta + \theta \sin \theta - \cos \theta)$		
		$= a\theta \sin \theta$	1	
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta \sin \theta}{a\theta \cos \theta}$	1	
		$= \tan \theta$	1	4
	b)	$y = (\sin x)\cos x$		
		$\therefore \frac{dy}{dx} = \sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x)$	1	
		$= \sin x(-\sin x) + \cos x(\cos x)$	1+1	
		$= \cos^2 x - \sin^2 x$	1	4
	c)	$x^3 + y^3 = 3axy$		
		$\therefore 3x^2 + 3y^2 \frac{dy}{dx} = 3a \left[x \frac{dy}{dx} + y \right]$		
		$\therefore 3y^2 \frac{dy}{dx} - 3ax \frac{dy}{dx} = 3ay - 3x^2$		
		$\therefore (3y^2 - 3ax) \frac{dy}{dx} = 3ay - 3x^2$		
		$\therefore \frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}$	1	
		$= \frac{3(ay - x^2)}{3(y^2 - ax)}$		
	d)	$= \frac{ay - x^2}{y^2 - ax}$	1	4
		$Let u = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$		
		$Put x = \tan \theta \therefore \theta = \tan^{-1} x$		
		$\therefore u = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) = \tan^{-1} (\tan 2\theta)$		

[illegible]

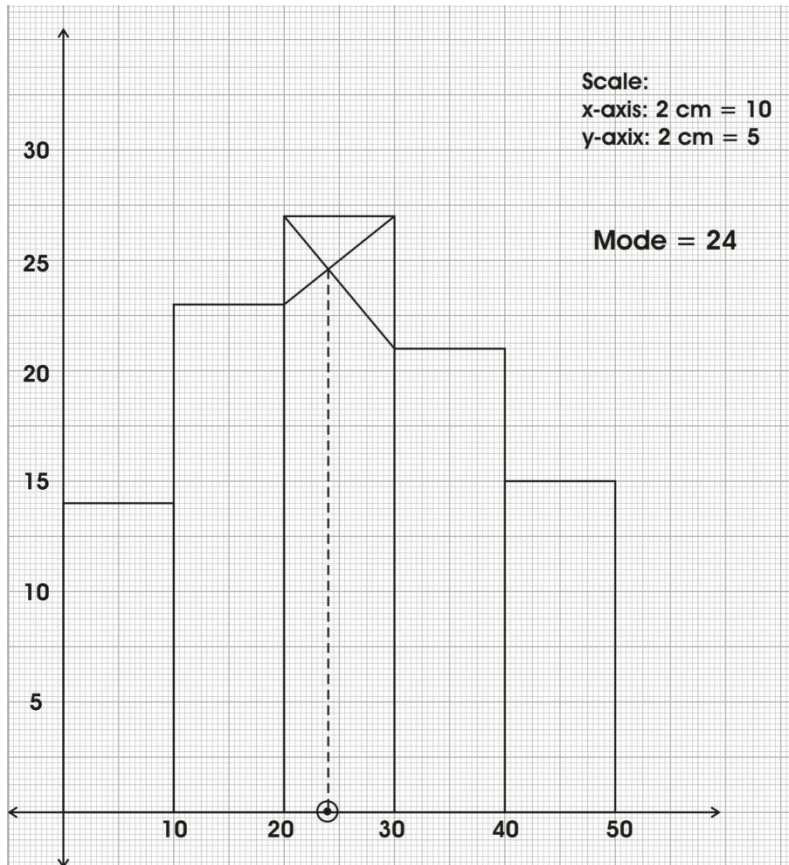


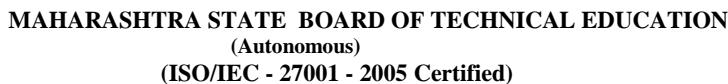
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	f)	$y = \tan^{-1} x$		
		$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$	1	
		$\frac{d^2 y}{dx^2} = \frac{-2x}{(1+x^2)^2}$	1	
		$\therefore (1+x^2) \frac{d^2 y}{dx^2} = -2x \cdot \frac{1}{1+x^2}$		
		$\therefore (1+x^2) \frac{d^2 y}{dx^2} = -2x \cdot \frac{dy}{dx}$	1	
		$\therefore (1+x^2) \frac{d^2 y}{dx^2} + 2x \cdot \frac{dy}{dx} = 0$	1	
		$\therefore (1+x^2) y'' + 2xy' = 0$		4
		$y = \sin^{-1} \left[\frac{\cos x + \sin x}{\sqrt{2}} \right]$		
		$= \sin^{-1} \left[\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right]$		
		$= \sin^{-1} \left[\sin \frac{\pi}{4} \cos x + \cos \frac{\pi}{4} \sin x \right]$	1	
4)	a)	$y = \sin^{-1} \left[\sin \left(\frac{\pi}{4} + x \right) \right]$	1	
		$= \frac{\pi}{4} + x$	1	
		$\therefore \frac{dy}{dx} = 1$	1	
			1	4
		Let $y = 2x^3 - 9x^2 + 12x + 5$		
		$\therefore \frac{dy}{dx} = 6x^2 - 18x + 12$	1/2	
		$\therefore \frac{d^2 y}{dx^2} = 12x - 18$	1/2	
		For extreme values, $\frac{dy}{dx} = 0$		
		$\therefore 6x^2 - 18x + 12 = 0$		
		$\therefore x = 1, 2$		
		At $x = 1$, $\frac{d^2 y}{dx^2} = -6 < 0$	1	
		$\therefore y$ is maximum at $x = 1$.	1/2	
		\therefore the maximum value is,		
		$y = 2(1)^3 - 9(1)^2 + 12(1) + 5 = 10$	1/2	



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		At $x = 1$, $\frac{d^2y}{dx^2} = 6 > 0$	$\frac{1}{2}$	4
		$\therefore y$ is minimum at $x = 2$.		
		\therefore the minimum value is,		
		$y = 2(2)^3 - 9(2)^2 + 12(2) + 5 = 9$	$\frac{1}{2}$	4
	b)	$y = x^3$		
		$\therefore \frac{dy}{dx} = 3x^2$	1	
		$\therefore \frac{d^2y}{dx^2} = 6x$	1	
		Radius of curvature, $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$= \frac{\left[1 + (3x^2)^2\right]^{\frac{3}{2}}}{6x}$	1	
		At point (1,1),		4
		$\rho = \frac{\left[1 + (3)^2\right]^{\frac{3}{2}}}{6} = \frac{(10)^{\frac{3}{2}}}{6} \quad \text{or} \quad 5.27$	1	
	c)	Let length = x , breadth = y		
		$\therefore 2x + 2y = 100 \quad \text{or} \quad x + y = 50$		
		$\therefore y = 50 - x$		
		Area, $A = xy$		
		$\therefore A = x(50 - x) = 50x - x^2$	1	
		$\therefore \frac{dA}{dx} = 50 - 2x$	$\frac{1}{2}$	
		$\therefore \frac{d^2A}{dx^2} = -2$	$\frac{1}{2}$	
		Now, $\frac{dA}{dx} = 0$ gives $x = 25$		
		At $x = 25$, $\frac{d^2A}{dx^2} = -2 < 0$		
		$\therefore A$ is maximum at $x = 25$	1	
		\therefore Length = 25, breadth = 25	1	4

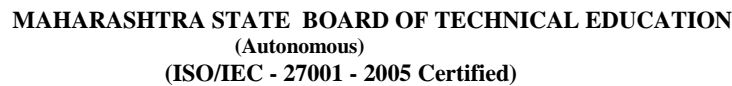


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4)	d)	$y = 3x - x^2$ $\therefore \frac{dy}{dx} = 3 - 2x$ From given slope = -5 $\therefore \frac{dy}{dx} = -5$ $\therefore 3 - 2x = -5$ $\therefore x = 4$ $y = -4$ \therefore point is (4, -4)	1 1 1 1	4
	e)	 <p>Scale: x-axis: 2 cm = 10 y-axis: 2 cm = 5</p> <p>Mode = 24</p> <p>Marks distribution: 1 mark for plotting points and drawing histogram correctly. 1 mark for drawing the cross lines in the modal class. 1 mark for drawing line of mode to x-axis. 1 mark for value of mode. Note the value 24 is approximate value. Difference of +0.5 or -0.5 is acceptable in case of graph. (Note: If the graph is too small or not clear to understand, marks can be deducted. On x-axis, instead of writing points 10, 20, 30, ... etc., if class 0-10, 10-20, 20-30, ... etc. are written, no marks to be given.)</p>	1+1+1+1	4

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5)	a)	<p>Given $\bar{x} = 40$, $n = 100$</p> <p>$\sum x_i = n\bar{x} = 100 \times 40 = 4000$</p> <p>$\therefore$ Incorrect $\sum x_i = 4000$</p> <p>\therefore Correct $\sum x_i = \text{Incorrect } \sum x_i - 83 + 53$</p> <p>$= 4000 - 83 + 53 = 3970$</p> <p>$\therefore$ Correct mean $= \frac{\text{Correct } \sum x_i}{n} = \frac{3970}{100} = 39.70$</p>	1	4																																																												
			1																																																													
			2																																																													
	b)	<p>Here $f_1 = 18$, $f_m = 25$, $f_2 = 15$</p> <p>Mode $= L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$</p> <p>$= 25 + \frac{25 - 18}{2(25) - 18 - 15} \times 5$</p> <p>$= 27.058$</p> <p>Note: The formula of mode is written in the various forms such as $\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$.</p>	1	4																																																												
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	c)	<table><tr><th>xi</th><th>f_i</th><th>$f_i x_i$</th><th>x_i^2</th><th>$f_i x_i^2$</th></tr><tr><td>1</td><td>7</td><td>7</td><td>1</td><td>7</td></tr><tr><td>4</td><td>46</td><td>184</td><td>16</td><td>736</td></tr><tr><td>7</td><td>165</td><td>1155</td><td>49</td><td>8085</td></tr><tr><td>10</td><td>195</td><td>1950</td><td>100</td><td>19500</td></tr><tr><td>13</td><td>189</td><td>2457</td><td>169</td><td>31941</td></tr><tr><td>16</td><td>89</td><td>1424</td><td>256</td><td>22784</td></tr><tr><td>19</td><td>28</td><td>532</td><td>361</td><td>10108</td></tr><tr><td>22</td><td>19</td><td>418</td><td>484</td><td>9196</td></tr><tr><td>25</td><td>09</td><td>225</td><td>625</td><td>5625</td></tr><tr><td>28</td><td>03</td><td>84</td><td>784</td><td>2352</td></tr><tr><td></td><td>750</td><td>8436</td><td></td><td>110334</td></tr></table> <p>$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$</p> <p>$= \sqrt{\frac{110334}{750} - \left(\frac{8436}{750}\right)^2}$</p> <p>$= 4.538$</p>	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$	1	7	7	1	7	4	46	184	16	736	7	165	1155	49	8085	10	195	1950	100	19500	13	189	2457	169	31941	16	89	1424	256	22784	19	28	532	361	10108	22	19	418	484	9196	25	09	225	625	5625	28	03	84	784	2352		750	8436		110334	1+1	4
	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$																																																											
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5)		<table><tr><th>xi</th><th>f_i</th><th>d_i</th><th>$f_i d_i$</th><th>d_i^2</th><th>$f_i d_i^2$</th></tr><tr><td>1</td><td>7</td><td>-4</td><td>-28</td><td>16</td><td>112</td></tr><tr><td>4</td><td>46</td><td>-3</td><td>-138</td><td>9</td><td>414</td></tr><tr><td>7</td><td>165</td><td>-2</td><td>330</td><td>4</td><td>660</td></tr><tr><td>10</td><td>195</td><td>-1</td><td>-195</td><td>1</td><td>195</td></tr><tr><td>13</td><td>189</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>16</td><td>89</td><td>1</td><td>89</td><td>1</td><td>89</td></tr><tr><td>19</td><td>28</td><td>2</td><td>56</td><td>4</td><td>112</td></tr><tr><td>22</td><td>19</td><td>3</td><td>57</td><td>9</td><td>171</td></tr><tr><td>25</td><td>09</td><td>4</td><td>36</td><td>16</td><td>144</td></tr><tr><td>28</td><td>03</td><td>5</td><td>15</td><td>25</td><td>75</td></tr><tr><td></td><td>750</td><td></td><td>-438</td><td></td><td>1972</td></tr></table> <p>$A = 13 \quad h = 3, \quad d_i = \frac{x_i - A}{h}$</p> <p>$S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2}$</p> <p>$= 3 \times \sqrt{\frac{1972}{750} - \left(\frac{-438}{750} \right)^2}$</p> <p>$= 4.538$</p> <p>Note: Students may take any another value for A in the above/ below example. So the above table and corresponding values vary accordingly. But the final answer will be the same.</p>	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$	1	7	-4	-28	16	112	4	46	-3	-138	9	414	7	165	-2	330	4	660	10	195	-1	-195	1	195	13	189	0	0	0	0	16	89	1	89	1	89	19	28	2	56	4	112	22	19	3	57	9	171	25	09	4	36	16	144	28	03	5	15	25	75		750		-438		1972	1+1	
xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$																																																																							
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10	195	-1	-195	1	195																																																																							
13	189	0	0	0	0																																																																							
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d)		<table><tr><th>Class</th><th>xi</th><th>f_i</th><th>d_i</th><th>$f_i d_i$</th></tr><tr><td>900-920</td><td>910</td><td>4</td><td>-3</td><td>-12</td></tr><tr><td>920-940</td><td>930</td><td>11</td><td>-2</td><td>-22</td></tr><tr><td>940-960</td><td>950</td><td>63</td><td>-1</td><td>-63</td></tr><tr><td>960-980</td><td>970</td><td>90</td><td>0</td><td>0</td></tr><tr><td>980-1000</td><td>990</td><td>73</td><td>1</td><td>73</td></tr><tr><td>1000-1020</td><td>1010</td><td>38</td><td>2</td><td>76</td></tr><tr><td>1020-1040</td><td>1030</td><td>16</td><td>3</td><td>48</td></tr><tr><td>1040-1060</td><td>1050</td><td>5</td><td>4</td><td>20</td></tr><tr><td></td><td></td><td>300</td><td></td><td>120</td></tr></table> <p>$A = 970, \quad h = 20$</p> <p>$\therefore \bar{x} = A + \frac{\sum f_i d_i}{N} \times h$</p> <p>$= 970 + \left(\frac{120}{300} \right) \times 20$</p> <p>$= 978$</p>	Class	xi	f_i	d_i	$f_i d_i$	900-920	910	4	-3	-12	920-940	930	11	-2	-22	940-960	950	63	-1	-63	960-980	970	90	0	0	980-1000	990	73	1	73	1000-1020	1010	38	2	76	1020-1040	1030	16	3	48	1040-1060	1050	5	4	20			300		120	2																							
Class	xi	f_i	d_i	$f_i d_i$																																																																								
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940-960	950	63	-1	-63																																																																								
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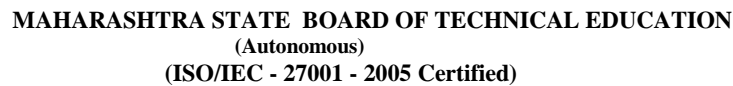
Que. No.	Sub. Que.	Model answers	Marks	Total Marks																																																						
5)	e)	<table border="1"><thead><tr><th>Class</th><th>xi</th><th>f_i</th><th>$f_i x_i$</th><th>$D_i = x_i - \bar{x}$</th><th>$f_i D_i$</th></tr></thead><tbody><tr><td>10-15</td><td>12.5</td><td>7</td><td>87.5</td><td>15.3</td><td>107.1</td></tr><tr><td>15-20</td><td>17.5</td><td>12</td><td>210</td><td>10.3</td><td>123.6</td></tr><tr><td>20-25</td><td>22.5</td><td>16</td><td>360</td><td>5.3</td><td>84.8</td></tr><tr><td>25-30</td><td>27.5</td><td>25</td><td>687.5</td><td>0.3</td><td>7.5</td></tr><tr><td>30-35</td><td>32.5</td><td>19</td><td>617.5</td><td>4.7</td><td>89.3</td></tr><tr><td>35-40</td><td>37.5</td><td>15</td><td>562.5</td><td>9.7</td><td>145.5</td></tr><tr><td>40-45</td><td>42.5</td><td>6</td><td>255</td><td>14.7</td><td>88.2</td></tr><tr><td></td><td></td><td>100</td><td>2780</td><td></td><td>646</td></tr></tbody></table>	Class	xi	f_i	$f_i x_i$	$D_i = x_i - \bar{x} $	$f_i D_i$	10-15	12.5	7	87.5	15.3	107.1	15-20	17.5	12	210	10.3	123.6	20-25	22.5	16	360	5.3	84.8	25-30	27.5	25	687.5	0.3	7.5	30-35	32.5	19	617.5	4.7	89.3	35-40	37.5	15	562.5	9.7	145.5	40-45	42.5	6	255	14.7	88.2			100	2780		646	1+1	
		Class	xi	f_i	$f_i x_i$	$D_i = x_i - \bar{x} $	$f_i D_i$																																																			
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$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2780}{100} = 27.8$	1																																																									
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$= 6.46$																																																										
6)	f)	$C.V.(A) = \frac{\sigma}{x} \times 100 = \frac{6}{80} \times 100 = 7.5$	1																																																							
		$C.V.(B) = \frac{\sigma}{x} \times 100 = \frac{7.2}{60} \times 100 = 12$	1																																																							
		$\therefore C.V.(A) < C.V.(B)$	1																																																							
		$\therefore \text{Group } B \text{ is more variable.}$	1																																																							
	a)	$z_1 = -3 + 4i, z_2 = 5 - 3i$																																																								
		$\frac{z_1}{z_2} = \frac{-3 + 4i}{5 - 3i}$																																																								
		$= \frac{-3 + 4i}{5 - 3i} \times \frac{5 + 3i}{5 + 3i}$	1																																																							
		$= \frac{-15 - 9i + 20i + 12i^2}{25 - 9i^2}$																																																								
		$= \frac{-15 + 11i - 12}{25 + 9}$	1																																																							
		$= \frac{-27 + 11i}{34}$	1																																																							
		$= \frac{-27}{34} + \frac{11}{34}i$	1																																																							



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	b) i)	$LHS = \sin 2\theta$		
		$= \frac{e^{2i\theta} - e^{-2i\theta}}{2i}$	1/2	
		$RHS = 2 \sin \theta \cos \theta$		
		$= 2 \cdot \frac{e^{i\theta} - e^{-i\theta}}{2i} \cdot \frac{e^{i\theta} + e^{-i\theta}}{2i}$	1/2	
		$= \frac{(e^{i\theta})^2 - (e^{-i\theta})^2}{2i}$		
		$= \frac{e^{2i\theta} - e^{-2i\theta}}{2i}$	1/2	
		$\therefore LHS = RHS$	1/2	



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	d)	$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$ $= \sin x \cosh y + \cos x i \sinh y$ $= \sin x \cosh y + i \cos x \sinh y$ $\therefore \text{Real part} = \sin x \cosh y$ $\text{Imaginary part} = \cos x \sinh y$	1	4
	e) i)	$z_1 z_2 = (4-5i)(3+7i)$ $= 12 + 28i - 15i - 35i^2$ $= 47 + 13i$	1	
		$\therefore z_1 z_2 = \sqrt{47^2 + 13^2}$ $= \sqrt{2378} \text{ or } 48.765$	1	
			1	
	e) ii)	$\frac{z_1}{z_2} = \frac{4-5i}{3+7i}$ $= \frac{4-5i}{3+7i} \times \frac{3-7i}{3-7i}$ $= \frac{12-28i-15i+35i^2}{9-49i^2}$ $= \frac{-23-43i}{58}$ $= \frac{-23}{58} - \frac{43}{58}i$	1/2	4
		$\therefore \left \frac{z_1}{z_2} \right = \sqrt{\left(\frac{-23}{58} \right)^2 + \left(\frac{-43}{58} \right)^2}$ $= \sqrt{\frac{2378}{3364}} \text{ or } 0.84$	1	
	f)	$\text{Let } z = 1 - \cos 2 + i \sin 2$ $\therefore z = 2 \sin^2 1 + i \sin 1 \cos 1$ $= 2 \sin 1 (\sin 1 + i \cos 1)$ $= 2 \sin 1 \left[\cos \left(\frac{\pi}{2} - 1 \right) + i \sin \left(\frac{\pi}{2} - 1 \right) \right]$	1	
		$\therefore \text{modulus } r = 2 \sin 1$ $\text{amplitude } \theta = \frac{\pi}{2} - 1$	1	
			1	
			1	



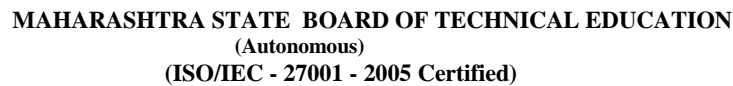
Que. No.	Sub. Que.	Model answers	Marks	Total Marks															
6)		For Computer/Information Technology Group																	
	a)	$f(x) = x^3 - 5x - 11$ $\therefore f(2) = -13, \quad f(3) = 1$ \therefore the root is in $(2, 3)$. $\therefore x_1 = \frac{2+3}{2} = 2.5$ $\therefore f(2.5) = -7.875$ \therefore the root is in $(2.5, 3)$. $\therefore x_2 = \frac{2.5+3}{2} = 2.75$ $\therefore f(2.75) = -3.95$ \therefore the root is in $(2.75, 3)$. $\therefore x_3 = \frac{2.75+3}{2} = 2.875$ OR $f(x) = x^3 - 5x - 11$ $\therefore f(2) = -13, \quad f(3) = 1$ \therefore the root is in $(2, 3)$. <table border="1"><tr><td>a</td><td>b</td><td>$x = \frac{a+b}{2}$</td><td>$f(x)$</td></tr><tr><td>2</td><td>3</td><td>2.5</td><td>-7.875</td></tr><tr><td>2.5</td><td>3</td><td>2.75</td><td>-3.95</td></tr><tr><td>2.75</td><td>3</td><td>2.875</td><td>---</td></tr></table>	a	b	$x = \frac{a+b}{2}$	$f(x)$	2	3	2.5	-7.875	2.5	3	2.75	-3.95	2.75	3	2.875	---	1 <
a	b	$x = \frac{a+b}{2}$	$f(x)$																
2	3	2.5	-7.875																
2.5	3	2.75	-3.95																
2.75	3	2.875	---																



Que. No.	Sub. Que.	Model answers	Marks	Total Marks																								
6)		<p style="text-align: center;">OR</p> $f(x) = x^3 - 2x - 5$ $\therefore f(2) = -1$ $f(3) = 16$ $\therefore \text{ the root is in } (2, 3).$ <table border="1"><thead><tr><th>a</th><th>b</th><th>$f(a)$</th><th>$f(b)$</th><th>$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$</th><th>$f(x)$</th></tr></thead><tbody><tr><td>2</td><td>3</td><td>-1</td><td>16</td><td>2.0588</td><td>-0.39</td></tr><tr><td>2.0588</td><td>3</td><td>-0.39</td><td>16</td><td>2.08</td><td>-0.166</td></tr><tr><td>2.08</td><td>3</td><td>-0.16</td><td>16</td><td>2.089</td><td>---</td></tr></tbody></table>	a	b	$f(a)$	$f(b)$	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	$f(x)$	2	3	-1	16	2.0588	-0.39	2.0588	3	-0.39	16	2.08	-0.166	2.08	3	-0.16	16	2.089	---	1	4
a	b	$f(a)$	$f(b)$	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	$f(x)$																							
2	3	-1	16	2.0588	-0.39																							
2.0588	3	-0.39	16	2.08	-0.166																							
2.08	3	-0.16	16	2.089	---																							
c)	$f(x) = x^3 - 20$	1																										
	$f'(x) = 3x^2$	1/2																										
	$f(2) = -12$	1/2																										
	$f(3) = 7$	1																										
	$x - \frac{f(x)}{f'(x)} = x - \frac{x^3 - 20}{3x^2}$ $= \frac{2x^3 + 20}{3x^2}$ <p>Start with $x_0 = 3$,</p> $\therefore x_1 = 2.741$ $x_2 = 2.715$ $x_3 = 2.714$ <p style="text-align: center;">OR</p> $f(x) = x^3 - 20$ $f'(x) = 3x^2$ $f(2) = -12$ $f(3) = 7$ <p>Start with $x_0 = 3$,</p> $\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 3 - \frac{f(3)}{f'(3)}$ $= 3 - \frac{-11}{27}$ $= 2.741$	1	4																									
			1/2																									
			1/2																									
			1																									



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.741 - \frac{f(2.741)}{f'(2.741)}$ $= 2.741 - \frac{0.593}{22.539}$ $= 2.715$	1	4
		$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.714 - \frac{f(2.714)}{f'(2.714)}$ $= 2.714 - \frac{0.593}{22.539}$ $= 2.714$	1	
	d)	$\begin{aligned} 2x + 3y + 2z &= 2 \\ 10x + 3y + 4z &= 16 \\ 3x + 6y + z &= -6 \end{aligned}$ $\begin{array}{rcl} 2x + 3y + 2z &= & 2 \\ 10x + 3y + 4z &= & 16 \\ \hline -8x - 2z &= & -14 \end{array} \quad \text{and} \quad \begin{array}{rcl} 20x + 6y + 8z &= & 32 \\ 3x + 6y + z &= & -6 \\ \hline 17x + 7z &= & 38 \end{array}$ $\begin{aligned} 28x + 7z &= 49 \\ \underline{17x + 7z} &= \underline{38} \\ 11x &= 11 \\ \therefore x &= 1 \\ z &= 3 \\ y &= -2 \end{aligned}$	1 1 1	4
	e)	$\begin{aligned} 15x + 2y + z &= 18 \\ 2x + 20y - 3z &= 19 \\ 3x - 6y + 25z &= 22 \end{aligned}$		
		<p>Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.</p>		



Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	e)	$\therefore x = \frac{18-2y-z}{15}$ $y = \frac{19-2x+3z}{20}$ $z = \frac{22-3x+6y}{25}$ Starting with $x_0 = 0 = y_0 = z_0$ $x_1 = 1.2$ $y_1 = 0.95$ $z_1 = 0.88$ $x_2 = 1.0146$ $y_2 = 0.962$ $z_2 = 0.9864$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	4
f)	$27x + 6y - z = 85$ $6x + 15y + 2z = 72$ $x + y + 54z = 110$ $\therefore x = \frac{85-6y+z}{27}$ $y = \frac{72-6x-2z}{15}$ $z = \frac{110-x-y}{54}$ Starting with $x_0 = 0 = y_0 = z_0$ $x_1 = 3.148$ $y_1 = 3.54$ $z_1 = 1.91$ $x_2 = 2.43$ $y_2 = 3.57$ $z_2 = 1.925$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	4	
<p style="text-align: center;">Important Note</p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</p>				