MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

SUMMER - 2017 Examination Model Answer

Subject Code:

17105

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling error should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 1 | | Attempt any <u>TEN</u> of the following: | 20 |
| | a) | $\begin{vmatrix} Solve & \begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$ | 02 |
| | Ans | $\begin{vmatrix} \cdot \cdot \begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$ | |
| | | $\therefore 6 - (-12) = x^2 - (-2)$ $18 = x^2 + 2$ | 1 |
| | | $16 = x^2$ $\therefore x = \pm 4$ | 1 |
| | b) | Find 'x' if $\begin{vmatrix} 0 & 7 & -2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0$ | 02 |
| | Ans | $\begin{vmatrix} 0 & 7 & -2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0$ | |
| | | 0(x-80)-7(11-40)+(-2)(88-4x)=0 203-176+8x=0 | 1 |
| | | $\therefore 8x = -27 \therefore x = \frac{-27}{8} \text{ or } -3.375$ | 1 |
| | | | |
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| Model Answe | r |
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| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 1 | c) | $\begin{vmatrix} Solve & 2 & 3 & x \\ 1 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 8 \\ 2 & 1 \end{vmatrix}$ | 02 |
| | Ans | $\begin{vmatrix} 2 & 3 & x \\ 1 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 8 \\ 2 & 1 \end{vmatrix}$ | |
| | | $\therefore 2(0-(-3))-3(0-(-6))+x(-1-0)=-1-16$ | 1 |
| | | $6-18-x=-17$ $\therefore x=5$ | 1 |
| | d) | Define singular and non-singular matrix. | 02 |
| | Ans | For every square matrix A, If $ A = 0$ then A is singular matrix & If $ A \neq 0$ then A is non-singular matrix. | 02 |
| | e) | Define orthogonal matrix. | 02 |
| | Ans | If $AA^T = I$ then A is orthogonal matrix. | 02 |
| | f) | If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ find $ AB $ | 02 |
| | Ans | $AB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ | |
| | | $= \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix}$ | |
| | | $= \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$ | 1 |
| | | $\begin{vmatrix} AB = \begin{vmatrix} 5 & 2 \\ 9 & -6 \end{vmatrix} \\ = -30 - 18 $ | |
| | | = -48 | 1 |
| | | | |
| 1 | ı | | |



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|-----------|--------------|--|-------------------|
| Q. No. | Sub Q. N. | Answer | Marking Scheme |
| 1 | g) | Resolve into partial fractions $\frac{1}{x^2 - x}$ | 02 |
| | Ans | $\frac{1}{x^2 - x} = \frac{1}{x(x - 1)} = \frac{A}{x} + \frac{B}{x - 1}$ | 1/2 |
| | | $\therefore 1 = A(x-1) + Bx$ put $x = 0$, $\therefore A = -1$ | 1/2 |
| | | $put x = 1, \therefore B = 1$ | 1/2 |
| | | $\therefore \frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$ | 1/2 |
| | h) | Without using calculator, find the value of sin75 ⁰ | 02 |
| | Ans | $\sin 75^{\circ} = \sin \left(30^{\circ} + 45^{\circ}\right)$ | 1/2 |
| | 7113 | $= \sin 30^{\circ} \cos 45^{\circ} + \cos 30^{\circ} \sin 45^{\circ}$ | |
| | | $= \frac{1}{2} \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}}$ | 1/2 |
| | | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 1/2 |
| | | $=\frac{\sqrt{3}+1}{2\sqrt{2}} \ or \ 0.9659$ | 1/2 |
| | i) | Prove that $\frac{\sin \theta - \sin 3\theta}{\sin^2 \theta - \cos^2 \theta} = 2\sin \theta$ | 02 |
| | Ans | $LHS = \frac{\sin \theta - \sin 3\theta}{\sin^2 \theta - \cos^2 \theta}$ | |
| | | $= \frac{\sin \theta - 3\sin \theta + 4\sin^3 \theta}{\sin^2 \theta - (1 - \sin^2 \theta)}$ | 1 |
| | | $=\frac{-2\sin\theta+4\sin^3\theta}{2\sin^2\theta-1}$ | |
| | | $=\frac{2\sin\theta\left(2\sin^2\theta-1\right)}{2\sin^2\theta-1}$ | 1/2 |
| | | $= 2\sin\theta$ $= RHS$ | 1/2 |
| | | | |
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|-----------|--------------|--|-------------------|
| Q. No. | Sub Q. N. | Answer | Marking Scheme |
| 1 | j) | Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$ | 02 |
| | Ans | $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right)$ | 4 |
| | | 2 3) | 1 |
| | | $= \tan^{-1} \left(1 \right)$ | 1/2 |
| | | $=\frac{\pi}{4}$ | 1/2 |
| | k) | Proceed that $a^{\frac{1}{2}}a^{\frac{1}{2}}$ and $a^{\frac{1}{2}}a^{\frac{1}{2}}$ | 02 |
| | | Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ | 02 |
| | Ans | Let $\sin^{-1} x = \theta$ | 1/2 |
| | | $\therefore x = \sin \theta$ $\therefore x = \cos \left(\frac{\pi}{2} - \theta\right)$ | 1/2 |
| | | $\therefore \cos^{-1} x = \frac{\pi}{2} - \theta$ | |
| | | $\therefore \theta + \cos^{-1} x = \frac{\pi}{2}$ | 1/2 |
| | | $\therefore \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ | 1/2 |
| | l) | Find the value of k , if the lines $kx - 6y - 9 = 0$ and $6x + 5y - 13 = 0$ | 02 |
| | | are perpendicular to each other. | |
| | Ans | Let L_1 : $kx - 6y - 9 = 0$ and | |
| | | $L_2: 6x + 5y - 13 = 0$ | |
| | | slope of L_1 , $m_1 = -\frac{k}{-6} = \frac{k}{6}$ and | 1/2 |
| | | slope of L_2 , $m_2 = \frac{-6}{5}$ | 1/2 |
| | | ∴ Lines are perpendicular | |
| | | $\therefore m_1 m_2 = -1$ | |
| | | $\therefore \left(\frac{k}{6}\right) \left(\frac{-6}{5}\right) = -1$ | 1/2 |
| | | $\therefore k = 5$ | 1/2 |
| | | | |
| | | | |
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|-----------|--------------|---|-------------------|
| 2 | | Attempt any FOUR of the following | 16 |
| | a) | Solve by Cramer's rule | 04 |
| | Ans | x + y = 5, $y + z = 8$, $z + x = 7$ | |
| | AllS | | |
| | | $D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0)-1(0-1)-0(0-1) = 2$ | 1 |
| | | | |
| | | $D_{x} = \begin{vmatrix} 5 & 1 & 0 \\ 8 & 1 & 1 \\ 7 & 0 & 1 \end{vmatrix} = 5(1-0)-1(8-7)-0(0-7) = 4$ | 1/2 |
| | | $D_x = \begin{bmatrix} 8 & 1 & 1 \\ 7 & 0 & 1 \end{bmatrix} = 5(1-0)-1(8-7)-0(0-7) = 4$ | |
| | | | |
| | | $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 8 & 1 \end{bmatrix} = 1(8-7) = 5(0-1) = 0(0-8) = 6$ | |
| | | $D_{y} = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 8 & 1 \\ 1 & 7 & 1 \end{vmatrix} = 1(8-7)-5(0-1)-0(0-8) = 6$ | 1/2 |
| | | | |
| | | $D_z = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 8 \\ 1 & 0 & 7 \end{vmatrix} = 1(7-0)-1(0-8)+5(0-1)=10$ | 1/2 |
| | | 1 0 7 | |
| | | D 4 D 6 D 10 | 1/ . 1/ . 1/ |
| | | $\therefore x = \frac{D_x}{D} = \frac{4}{2} = 2 \therefore y = \frac{D_y}{D} = \frac{6}{2} = 3 \therefore z = \frac{D_z}{D} = \frac{10}{2} = 5$ | 1/2+1/2+1/2 |
| | | | |
| | | ([2 4] [4 4]) [2 60] | |
| | b) | Solve $2 \left\{ \begin{vmatrix} 3x & -1 \\ 8 & 5 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ -2 & -y \end{vmatrix} \right\} = \begin{vmatrix} 260 \\ 128 \end{vmatrix}$ | 04 |
| | | $\begin{bmatrix} \begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} -2 & -y \end{bmatrix} \end{bmatrix}$ | |
| | Ans | $\therefore 2\left\{ \begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix} \right\} = \begin{bmatrix} 260 \\ 128 \end{bmatrix}$ | |
| | | | |
| | | $\therefore 2 \begin{vmatrix} 3x+4 & -1+1 \\ 8-2 & 5-y \end{vmatrix} = \begin{vmatrix} 260 \\ 128 \end{vmatrix}$ | 1 |
| | | | |
| | | $\begin{bmatrix} \therefore 2 \begin{bmatrix} 3x+4 & 0 \\ 6 & 5-y \end{bmatrix} = \begin{bmatrix} 260 \\ 128 \end{bmatrix}$ | 1 |
| | | | 1 |
| | | $\left \begin{array}{ccc} \cdot \cdot \begin{bmatrix} 6x + 8 & 0 \\ 12 & 10 - 2y \end{array} \right = \begin{bmatrix} 260 \\ 128 \end{bmatrix}$ | |
| | | ∴ order of LHS ≠ order of RHS | 1 |
| | | Note: If student attempted to solve the problem and concluded above result's | |
| | | then reward full credit to students OR student attempted to solve and carried | |
| | | out steps then reward appropriate marks. | |
| | 1 | Page No. | |



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| Q. No. | Sub | Answer | Marking Scheme |
|-----------|-------|--|-------------------|
| INO. | Q. N. | | Scheme |
| 2 | c) | If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$, show that $A^2 - 8A$ is a scalar matrix | 04 |
| | Ans | $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ $A^{2} = AA = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ $= \begin{bmatrix} 4+16+16 & 8+8+16 & 8+16+8 \\ 8+8+16 & 16+4+16 & 16+8+8 \\ 8+16+8 & 16+8+8 & 16+16+4 \end{bmatrix}$ | 1/2 |
| | | $= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix}$ | 1 |
| | | $ 8A = 8 \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix} $ | 1 |
| | | $\begin{vmatrix} A^2 - 8A = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ | 1 |
| | | $\therefore A^2 - 8A$ is scalar matrix. | 1/2 |
| | d) | Express the matrix $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ as the sum of symmetric and skew-symmetric matrices. | 04 |
| | Ans | Consider $\frac{1}{2}(A+A^T)$ | |



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| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| 2 | Ans | $= \frac{1}{2} \left(\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 5 \\ 7 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} -2 & 9 & 6 \\ 9 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix} \right)$ | 1 |
| | | $= \begin{bmatrix} -1 & \frac{9}{2} & 3 \\ \frac{9}{2} & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix}$ | 1∕2 |
| | | Consider $\frac{1}{2}(A-A^T)$ | |
| | | $= \frac{1}{2} \left(\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 5 \\ 7 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix} \right) = \frac{1}{2} \left(\begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 4 \\ 4 & -4 & 0 \end{bmatrix} \right)$ | 1 |
| | | $= \begin{bmatrix} 0 & \frac{5}{2} & -2 \\ -\frac{5}{2} & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ | 1/2 |
| | | Consider $A = \frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T)$ | |
| | | $= \begin{bmatrix} -1 & \frac{9}{2} & 3 \\ \frac{9}{2} & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & -2 \\ -\frac{5}{2} & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ | 1 |
| | | = symmetric matrix +skew- symmetric matrix | |
| | e) | If $A = \begin{bmatrix} 1 & 2 & -1 \\ 6 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ verify that $(AB)^T = B^T A^T$ | 04 |
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Model Answer

| Q. | Sub | | Marking |
|-----|-------|--|---------|
| No. | Q. N. | Answer | Scheme |
| 2 | e) | | |
| _ | | Consider $AB = \begin{bmatrix} 1 & 2 & -1 \\ 6 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ | |
| | Ans | | |
| | | $\begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \end{bmatrix}$ | |
| | | $= \begin{vmatrix} 1+4+6 & 0+2-1 & 0+6-3 \\ 6+0+0 & 0+0+2 & 0+0+6 \end{vmatrix}$ | |
| | | $\begin{vmatrix} 4+10+0 & 0+5+0 & 0+0+0 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{vmatrix}$ | |
| | | | |
| | | $= \begin{bmatrix} 5 & 1 & -3 \\ 6 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ | 1 |
| | | 14 5 0 | 1 |
| | | | |
| | | $\therefore (AB)^T = \begin{bmatrix} 5 & 6 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix}$ | 1/2 |
| | | _3 6 0] | |
| | | | 1 |
| | | $\mathbf{B}^{T} \mathbf{A}^{T} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}$ | 1 |
| | | | |
| | | $ = \begin{bmatrix} 1+4+0 & 6+0+0 & 4+10+0 \\ 0+2-1 & 0+0+2 & 0+5+0 \\ 0+0-3 & 0+0+6 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix} $ | 1 |
| | | $= \begin{vmatrix} 0+2-1 & 0+0+2 & 0+5+0 \\ \end{vmatrix} = \begin{vmatrix} 1 & 2 & 5 \\ \end{vmatrix}$ | _ |
| | | | |
| | | $\therefore (AB)^T = B^T A^T$ | 1/2 |
| | | $(\tan \theta + 1)$ | |
| | f) | Resolve into partial fractions $\frac{(\tan \theta + 1)}{(\tan \theta - 1)(\tan \theta + 2)}$ | 04 |
| | Ans | Let $\tan \theta = t$ | 1/ |
| | | (t+1) $A B$ | 1/2 |
| | | $\frac{(t+1)}{(t-1)(t+2)} = \frac{A}{t-1} + \frac{B}{t+2}$ | 1/2 |
| | | $\therefore t+1 = A(t+2) + B(t-1)$ | |
| | | put $t=1$, $\therefore A=\frac{2}{3}$ and | 1 |
| | | 3 | 1 |
| | | $put t = -2, \therefore B = \frac{1}{3}$ | 1 |
| | | $(t+1)$ $\frac{2}{3}$ $\frac{1}{3}$ | |
| | | $\therefore \frac{(t+1)}{(t-1)(t+2)} = \frac{\frac{2}{3}}{t-1} + \frac{\frac{1}{3}}{t+2}$ $\therefore \frac{(\tan\theta + 1)}{(\tan\theta - 1)(\tan\theta + 2)} = \frac{\frac{2}{3}}{\tan\theta - 1} + \frac{\frac{1}{3}}{\tan\theta + 2}$ | 1/2 |
| | | 2 1 | |
| | | $\frac{(\tan\theta+1)}{1+\cos\theta+1} = \frac{3}{3} + \frac{3}{3}$ | 1/2 |
| | | $\frac{(\tan \theta - 1)(\tan \theta + 2)}{(\tan \theta - 1)(\tan \theta + 2)} = \tan \theta - 1$ | |
| | | Page No. C | |



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Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| 3 | | Attempt any FOUR of the following : | 16 |
| | a) | Find the adjoint of $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ | 04 |
| | Ans | $Let A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ | |
| | | Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 2 & 3 & 1 & 3 & 1 & 2 \\ 1 & 2 & 3 & 2 & 3 & 1 \\ 3 & 1 & 2 & 1 & 2 & 3 \\ 1 & 2 & 3 & 2 & 3 & 1 \\ 3 & 1 & 2 & 1 & 2 & 3 \\ 2 & 3 & 1 & 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -7 & -5 \\ 5 & 1 & -7 \\ 7 & 5 & 1 \end{bmatrix}$ | 2 |
| | | Matrix of cofactors = $\begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}$ | 1 |
| | | OR | |
| | | $\begin{vmatrix} C_{11} = + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1, \ C_{12} = - \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = -(2 - 9) = 7$ | |
| | | $\begin{vmatrix} C_{13} = + \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = -5, \ C_{21} = - \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -(6 - 1) = -5$ | |
| | | $\begin{vmatrix} C_{22} = + \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1, \ C_{23} = - \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -(2 - 9) = 7$ | |
| | | $\begin{vmatrix} C_{31} = + \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 9 - 2 = 7, \ C_{32} = - \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -(6 - 1) = -5$ | 2 |
| | | $C_{33} = + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1,$ | |
| | | Matrix of cofactors = $\begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}$ | 1 |
| | | $Adj.A = \begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{bmatrix}$ | 1 |
| | | | |



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| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 3 | b) | Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ | 04 |
| | Ans | $\begin{vmatrix} \det A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} $ $\therefore A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1(24 - 25) - 2(12 - 15) + 3(10 - 12)$ | |
| | | $= -1 + 6 - 6$ $ A = -1 \neq 0$ $\therefore A^{-1} \text{ exists}$ | 1/2 |
| | | Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 4 & 5 & 2 & 5 & 2 & 4 \\ 5 & 6 & 3 & 6 & 3 & 5 \\ 2 & 3 & 1 & 3 & 1 & 2 \\ 5 & 6 & 3 & 6 & 3 & 5 \\ 2 & 3 & 1 & 3 & 1 & 2 \\ 4 & 5 & 2 & 5 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ -3 & -3 & -1 \\ -2 & -1 & 0 \end{bmatrix}$ | 1 |
| | | Matrix of cofactors = $\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ OR | 1 |
| | | $C_{11} = + \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1, \ C_{12} = - \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -(12 - 15) = 3$ $C_{13} = + \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 10 - 12 = -2, \ C_{21} = - \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -(12 - 15) = 3$ $C_{22} = + \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} = 6 - 9 = -3, C_{23} = - \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = -(5 - 6) = 1$ $C_{31} = + \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2, \ C_{32} = - \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = -(5 - 6) = 1$ $C_{33} = + \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0,$ | 1 |



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Model Answer

| | | THOUGH ANSWEL | |
|-----------|--------------|--|-------------------------|
| Q. No. | Sub Q. N. | Answer | Marking Scheme |
| 3 | b) | Matrix of cofactors = $\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 & 3 & -2 \end{bmatrix}$ | 1 |
| | | $Adj.A = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ $A^{-1} = \frac{1}{ A }Adj.A$ | 1 / ₂ |
| | | $A^{-1} = \frac{1}{-1} \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$ | 1 |
| | c) Ans | Using matrix inversion method, solve $2x + y = 3$, $2y + 3z = 4$, $2z + 2x = 8$ $\begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$ | 04 |
| | | Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix}$ $ A = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 2 & 0 & 2 \end{vmatrix} = 2(4-0)-1(0-6)+0(0-4)$ | |
| | | $= 8 + 6$ $\therefore A = 14 \neq 0$ | 1/ |
| | | $\therefore A^{-1} \text{ exists}$ $\begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 0 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} \end{bmatrix}$ | 1/2 |
| | | Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 2 & 3 & 0 & 3 & 0 & 2 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 1 & 0 & 2 & 0 & 2 & 1 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 1 & 0 & 2 & 0 & 2 & 1 \\ 2 & 3 & 0 & 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -6 & -4 \\ 2 & 4 & -2 \\ 3 & 6 & 4 \end{bmatrix}$ | 1 |
| | | Matrix of cofactors = $\begin{bmatrix} 4 & 6 & -4 \\ -2 & 4 & 2 \\ 3 & -6 & 4 \end{bmatrix}$ | 1/2 |
| | | OR | |



SUMMER – 17 EXAMINATION

Model Answer

| | | <u>Model Aliswei</u> Subject Code. | |
|-----------|--------------|--|-------------------|
| Q. No. | Sub Q. N. | Answer | Marking Scheme |
| 3 | | $C_{11} = + \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4, C_{12} = - \begin{vmatrix} 0 & 3 \\ 2 & 2 \end{vmatrix} = -(0 - 6) = 6$ $C_{13} = + \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = 0 - 4 = -4, C_{21} = - \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = -(2 - 0) = -2$ $C_{22} = + \begin{vmatrix} 2 & 0 \\ 2 & 2 \end{vmatrix} = 4 - 0 = 4, C_{23} = - \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = -(0 - 2) = 2$ $C_{31} = + \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 3 - 0 = 3, C_{32} = - \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = -(6 - 0) = -6$ | 1 |
| | | $C_{33} = + \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4$ Matrix of cofactors = $ \begin{bmatrix} 4 & 6 & -4 \\ -2 & 4 & 2 \\ 3 & -6 & 4 \end{bmatrix} $ | 1/2 |
| | | $Adj.A = \begin{bmatrix} 4 & -2 & 3 \\ 6 & 4 & -6 \\ -4 & 2 & 4 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 \\ Adj.A \end{bmatrix}$ | 1/2 |
| | | $A^{-1} = \frac{1}{ A } A dj.A$ $= \frac{1}{14} \begin{bmatrix} 4 & -2 & 3 \\ 6 & 4 & -6 \\ -4 & 2 & 4 \end{bmatrix}$ $\therefore X = A^{-1}B = \frac{1}{14} \begin{bmatrix} 4 & -2 & 3 \\ 6 & 4 & -6 \\ -4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$ | 1/2 |
| | | $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 12 - 8 + 24 \\ 18 + 16 - 48 \\ -12 + 8 + 32 \end{bmatrix}$ $= \frac{1}{14} \begin{bmatrix} 28 \\ -14 \\ 28 \end{bmatrix}$ | 1/2 |
| | | $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ $\therefore x = 2, y = -1, z = 2.$ | ½ |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| 3 | d) | Resolve into partial fractions $\frac{x^2 - x + 3}{(x-2)(x^2+1)}$ | 04 |
| | Ans | Let $\frac{x^2 - x + 3}{(x - 2)(x^2 + 1)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1}$ | |
| | | $\therefore x^2 - x + 3 = (x^2 + 1)A + (x - 2)(Bx + C)$ | 1/2 |
| | | Put $x = 2$ | |
| | | $(2)^2 - 2 + 3 = ((2)^2 + 1)A$ | |
| | | 5=5A | |
| | | $\therefore A = 1$ | 1 |
| | | Put $x = 0$, A = 1 | |
| | | 3 = (0+1)(1)+(0-2)(0+C) | |
| | | 3=1-2C:. C=-1 | 1 |
| | | Put $x = 1$, $A = 1$, $C = -1$ | |
| | | $(1)^{2}-1+3=((1)^{2}+1)(1)+(1-2)(B(1)+(-1))$ | |
| | | 3 = 2 - B + 1 | |
| | | $\therefore B = 0$ | 1 |
| | | $\therefore \frac{x^2 - x + 3}{(x - 2)(x^2 + 1)} = \frac{1}{x - 2} + \frac{-1}{x^2 + 1}$ | 1/2 |
| | e) | Resolve into partial fractions $\frac{2x-3}{(x+1)(x^2-1)}$ | 04 |
| | Ans | Let $\frac{2x-3}{(x+1)(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ | |
| | | $\therefore 2x-3 = (x+1)^2 A + (x+1)(x-1)B + (x-1)C$ | 1/2 |
| | | Put $x = 1$ | |
| | | $\therefore 2(1) - 3 = (1+1)^2 A$ | |
| | | $\therefore A = -\frac{1}{4}$ | 1 |
| | | Put $x = -1$ | |
| | | $\therefore 2(-1)-3=(-1-1)C$ | |
| | | $-5 = -2C \qquad \therefore C = \frac{5}{2}$ | 1 |
| | | | |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| 3 | | Put $x = 0$, $A = -\frac{1}{4}$, $C = \frac{5}{2}$ $2(0) - 3 = (0+1)^{2} \left(-\frac{1}{4}\right) + (0+1)(0-1)B + (0-1)\left(\frac{5}{2}\right)$ $-3 = -\frac{1}{4} - B - \frac{5}{2} : B = \frac{1}{4}$ $\frac{2x - 3}{(x+1)(x-1)(x+1)} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{5}{2}}{(x+1)^{2}}$ | 1 1/2 |
| | f) | Resolve into partial fractions $\frac{x^4}{x^2-1}$ | 04 |
| | Ans | $x^{2}-1) \overline{\smash{\big)}\ x^{4}}$ $x^{4}-x^{2}$ | |
| | | - + | |
| | | x^2 $x^2 - 1$ $- +$ | 1 |
| | | 1 | |
| | | $\frac{x^4}{x^2 - 1} = (x^2 + 1) + \frac{1}{x^2 - 1}$ | 1/2 |
| | | $\frac{1}{x^2 - 1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ $\therefore 1 = (x-1)A + (x+1)B$ | 1/2 |
| | | Put $x = -1$: $A = -\frac{1}{2}$ | 1/2 |
| | | Put $x = 1$: $B = \frac{1}{2}$ | 1/2 |
| | | $\frac{1}{x^2 - 1} = \frac{1}{(x+1)(x-1)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$ | 1/2 |
| | | $\frac{x^4}{x^2 - 1} = \left(x^2 + 1\right) + \frac{-\frac{1}{2}}{x + 1} + \frac{\frac{1}{2}}{x - 1} = \left(x^2 + 1\right) + \frac{1}{2}\left(\frac{-1}{x + 1} + \frac{1}{x - 1}\right)$ | 1/2 |



SUMMER – 17 EXAMINATION

Model Answer

Subject Code:

17105

| Q. | Sub | _ | Marking |
|-----|-------|--|---------|
| No. | Q. N. | Answer | Scheme |
| 4 | | Attempt any FOUR of the following : | 16 |
| | a) | Prove that $sin(A + B).sin(A - B) = sin^2 A - sin^2 B$ | 04 |
| | Ans | $LHS = \sin(A+B).\sin(A-B)$ | |
| | | $= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$ | 1 |
| | | $= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$ | 1 |
| | | $= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$ | 1 |
| | | $= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$ | |
| | | $= \sin^2 A - \sin^2 B = RHS$ | 1 |
| | | | |
| | b) | Prove that $\tan 40^{\circ} + 2 \tan 10^{\circ} = \tan 50^{\circ}$ | 04 |
| | Ans | $\tan 50^\circ = \tan \left(40^\circ + 10^\circ\right)$ | |
| | | $= \frac{\tan 40^{\circ} + \tan 10^{\circ}}{1 - \tan 40^{\circ} \tan 10^{\circ}}$ | 1 |
| | | $\therefore \tan 50^{\circ} \left(1 - \tan 40^{\circ} \tan 10^{\circ} \right) = \tan 40^{\circ} + \tan 10^{\circ}$ | |
| | | $\tan 50^{\circ} - \tan 50^{\circ} \tan 40^{\circ} \tan 10^{\circ} = \tan 40^{\circ} + \tan 10^{\circ}$ | 1/2 |
| | | $\tan 50^{\circ} - \cot 40^{\circ} \tan 40^{\circ} \tan 10^{\circ} = \tan 40^{\circ} + \tan 10^{\circ} (\because \tan 50^{\circ} = \cot (90^{\circ} - 50) = \cot 40^{\circ})$ | 1 |
| | | $\tan 50^{\circ} - (1) \tan 10^{\circ} = \tan 40^{\circ} + \tan 10^{\circ}$ | 1 |
| | | $\therefore \tan 40^{\circ} + 2 \tan 10^{\circ} = \tan 50^{\circ}$ | 1 1/2 |
| | | | |
| | c) | Prove that $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos 2A + \sin A \tan 3A}$ | 04 |
| | Ans | $\cos A + 2\cos 3A + \cos 5A$ $L.H.S. = \frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos 2A + \cos 6A}$ | |
| | AllS | $\cos A + 2\cos 3A + \cos 5A$ | |
| | | $=\frac{2\cos 4A + (\cos 2A + \cos 6A)}{(\cos 2A + \cos 6A)}$ | |
| | | $\frac{-2\cos 3A + (\cos A + \cos 5A)}{2\cos 3A + (\cos A + \cos 5A)}$ | |
| | | $2\cos 4A + 2\cos\left(\frac{2A+6A}{2}\right).\cos\left(\frac{2A-6A}{2}\right)$ | |
| | | $= \frac{2\cos 3A + 2\cos\left(\frac{A+5A}{2}\right).\cos\left(\frac{A-5A}{2}\right)}{2\cos 3A + 2\cos\left(\frac{A-5A}{2}\right)}$ | 1 |
| | | $= \frac{2\cos 4A + 2\cos 4A \cdot \cos(-2A)}{2\cos 3A + 2\cos 3A \cdot \cos(-2A)}$ | 1/2 |
| | | | |
| | | $=\frac{2\cos 4A(1+\cos(-2A))}{2\cos 3A(1+\cos(-2A))}$ | |
| | | $2\cos 3A(1+\cos(-2A))$ | |
| | I | Page No 1 | E /2E |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 4 | c) | $= \frac{\cos 4A}{\cos 3A}$ $= \frac{\cos (3A+A)}{\cos 3A}$ $= \frac{\cos 3A \cos A - \sin 3A \sin A}{\cos 3A}$ $= \cos A - \sin A \tan 3A = \text{R.H.S.}$ | 1 1 1/2 |
| | d) Ans | Show that : $\sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ} = \frac{3}{16}$ $\sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ}$ | 04 |
| | | $= \sin 20^{0} \sin 40^{0} \frac{\sqrt{3}}{2} \sin 80^{0}$ $= \frac{\sqrt{3}}{4} \left[2 \sin 20^{0} \sin 40^{0} \right] \sin 80^{0}$ | 1/2 |
| | | $= \frac{\sqrt{3}}{4} \left[\cos(-20^{\circ}) - \cos 60^{\circ} \right] \sin 80^{\circ}$ $= \frac{\sqrt{3}}{4} \left[\cos 20^{\circ} - \frac{1}{2} \right] \sin 80^{\circ}$ | 1 |
| | | $= \frac{\sqrt{3}}{4} \left[\cos 20^{0} \sin 80^{0} - \frac{1}{2} \sin 80^{0} \right]$ $= \frac{\sqrt{3}}{4} \left[\frac{1}{2} 2 \cos 20^{0} \sin 80^{0} - \frac{1}{2} \sin 80^{0} \right]$ $= \frac{\sqrt{3}}{4} \left[\cos 20^{0} \sin 80^{0} - \frac{1}{2} \sin 80^{0} \right]$ | 1/2 |
| | | $= \frac{\sqrt{3}}{8} \left[\sin 100^{0} - \sin (-60) - \sin 80^{0} \right]$ $= \frac{\sqrt{3}}{8} \left[\sin (2 \times 90^{0} - 80) + \frac{\sqrt{3}}{2} - \sin 80^{0} \right]$ | 1/2 |
| | e) | $= \frac{\sqrt{3}}{8} \left[\sin 80^{0} + \frac{\sqrt{3}}{2} - \sin 80^{0} \right]$ $= \frac{3}{16}$ Prove that $\cos 11^{\circ} + \sin 11^{\circ} - \cos 56^{\circ}$ | 1/2 |
| | Ans | Prove that $\frac{\cos 11^{\circ} + \sin 11^{\circ}}{\cos 11^{\circ} - \sin 11^{\circ}} = \tan 56^{\circ}$ $LHS = \frac{\cos 11^{\circ} + \sin 11^{\circ}}{\cos 11^{\circ} - \sin 11^{\circ}}$ | 04 |
| | | Page No. | |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 4 | e) | $= \frac{\cos 11^{\circ} + \cos 79^{\circ}}{\cos 11^{\circ} - \cos 79^{\circ}} \qquad (\because \sin 11^{\circ} = \cos (90^{\circ} - 11^{\circ}) = \cos 79^{\circ})$ $= \frac{2 \cos \left(\frac{11^{\circ} + 79^{\circ}}{2}\right) \cdot \cos \left(\frac{11^{\circ} - 79^{\circ}}{2}\right)}{-2 \sin \left(\frac{11^{\circ} + 79^{\circ}}{2}\right) \cdot \sin \left(\frac{11^{\circ} - 79^{\circ}}{2}\right)}$ | 1 |
| | | $= \frac{2\cos 45^{\circ}.\cos(-34^{\circ})}{-2\sin 45^{\circ}.\sin(-34^{\circ})} = \frac{2\left(\frac{1}{\sqrt{2}}\right).\cos 34^{\circ}}{-2\left(\frac{1}{\sqrt{2}}\right).(-\sin 34^{\circ})}$ | 1 |
| | | $= \frac{\cos 34^{\circ}}{\sin 34^{\circ}} = \cot 34^{\circ} = \tan (90^{\circ} - 34^{\circ})$ | 1/2 |
| | | $= \tan 56^{\circ} = RHS$ | 1/2 |
| | f) | Prove that $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \cot^{-1}\left(\frac{9}{2}\right)$ | 04 |
| | Ans | $\therefore \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$ | |
| | | $= \tan^{-1} \left[\frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{7} \times \frac{1}{13}} \right]$ | 1 |
| | | $= \tan^{-1} \left[\frac{\frac{20}{91}}{1 - \frac{1}{91}} \right]$ | 1 |
| | | $= \tan^{-1} \left[\frac{\frac{20}{91}}{\frac{90}{91}} \right]$ | |
| | | $= \tan^{-1}\left(\frac{2}{9}\right)$ $= \cot^{-1}\left(\frac{9}{2}\right)$ | 1 |
| | | $=\cot^{-1}\left(\frac{1}{2}\right)$ | 1 |
| | | | |
| <u> </u> | <u> </u> | Page No. 1 | |



SUMMER – 17 EXAMINATION

Model Answer

| Q. | Sub | | Marking |
|-----|-------|--|---------|
| No. | Q. N. | Answer | Scheme |
| 5 | | Attempt any FOUR of the following : | 16 |
| | a) | Prove that $\sqrt{2 + \sqrt{2 + 2\cos 8\theta}} = 2\cos \theta$ | 04 |
| | Ans | $LHS = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}$ | |
| | | $=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$ | |
| | | | 1 |
| | | $=\sqrt{2+\sqrt{2+\sqrt{2\left(2\cos^2 4\theta\right)}}}$ | |
| | | $=\sqrt{2+\sqrt{2+2\cos 4\theta}}$ | 1/2 |
| | | $=\sqrt{2+\sqrt{2\left(1+\cos 4\theta\right)}}$ | 1 |
| | | $=\sqrt{2+\sqrt{2\left(2\cos^2 2\theta\right)}}$ | |
| | | $=\sqrt{2+2\cos 2\theta}$ | 1/2 |
| | | $= \sqrt{2(1+\cos 2\theta)} = \sqrt{2(2\cos^2 \theta)} = 2\cos \theta = RHS$ | 1 |
| | | | |
| | b) | $\mathbf{P}_{\text{rove that}} \sec 8\theta - 1 = \tan 8\theta$ | 04 |
| | | Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$ | |
| | Ans | $LHS = \frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{\cos \theta} - 1}$ | 1/2 |
| | | $\sec 4\theta - 1 \frac{1}{\cos 4\theta} - 1$ | |
| | | $\frac{1-\cos 8\theta}{\cos \theta}$ | |
| | | $=\frac{\cos 8\theta}{1-\cos 4\theta}$ | |
| | | $\cos 4\theta \\ \cos 4\theta (2\sin^2 4\theta)$ | |
| | | $=\frac{\cos 4\theta \left(2\sin^2 4\theta\right)}{\cos 8\theta \left(2\sin^2 2\theta\right)}$ | 1 |
| | | $= \frac{(2\sin 4\theta \cos 4\theta)\sin 4\theta}{(2\sin 4\theta \cos 4\theta)\sin 4\theta}$ | |
| | | $\cos 8	heta \left(2\sin^2 2	heta ight)$ | |
| | | $=\frac{\sin 8\theta (2\sin 2\theta \cos 2\theta)}{\cos (2\cos^2 2\theta)}$ | 1 |
| | | $-\cos 8\theta (2\sin^2 2\theta)$ $\sin 8\theta \cos 2\theta$ | 1/ |
| | | $=\frac{1}{\cos 8\theta \sin 2\theta}$ | 1/2 |
| | | $= \tan 8\theta \cot 2\theta = \frac{\tan 8\theta}{\tan 2\theta} = RHS$ | 1 |
| | | tan 20 | 0/25 |



SUMMER – 17 EXAMINATION

Model Answer

| Q. | Sub | _ | Marking |
|-----|-----------|---|---------|
| No. | Q. N. | Answer | Scheme |
| 5 | c) | In any triangle ABC, prove that | 04 |
| | Ans | $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ | |
| | 7 | In any triangle ABC, $\angle A + \angle B + \angle C = 180^{\circ}$ | 1/2 |
| | | $\therefore A + B = 180^{\circ} - C$ | |
| | | $\therefore \tan(A+B) = \tan(180^{\circ} - C)$ | 1/2 |
| | | $\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$ | 1+1 |
| | | | 1/ |
| | | $\therefore \tan A + \tan B = -\tan C \left(1 - \tan A \tan B \right)$ | 1/2 |
| | | $\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C$ | |
| | | $\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$ | 1/2 |
| | d) Ans | Prove that $\frac{\sin 2A + 2\sin 4A + \sin 6A}{\sin A + 2\sin 3A + \sin 5A} = \cos A + \cot 3A \sin A$ L.H.S. = $\frac{\sin 2A + 2\sin 4A + \sin 6A}{\sin A + 2\sin 3A + \sin 5A}$ $2\sin 4A + (\sin 2A + \sin 6A)$ | 04 |
| | | $ 2\sin 3A + (\sin A + \sin 5A) = \frac{2\sin 4A + 2\sin\left(\frac{2A + 6A}{2}\right) \cdot \cos\left(\frac{2A - 6A}{2}\right)}{2\sin 3A + 2\sin\left(\frac{A + 5A}{2}\right) \cdot \cos\left(\frac{A - 5A}{2}\right)} = \frac{2\sin 4A + 2\sin 4A \cdot \cos\left(-2A\right)}{2\sin 3A + 2\sin 3A \cdot \cos\left(-2A\right)} $ | 1 |
| | | $2\sin 3A + 2\sin 3A.\cos(-2A)$ $= \frac{2\sin 4A(1+\cos(-2A))}{2\sin 3A(1+\cos(-2A))}$ | 1/2 |
| | | $=\frac{\sin 4A}{\cos x}$ | 1/2 |
| | | $\sin 3A$ | |
| | | $=\frac{\sin(3A+A)}{\cos(ab)}$ | 1/2 |
| | | $-\sin 3A$ | |
| | | $= \frac{\sin 3A \cos A + \cos 3A \sin A}{\sin A}$ | 1 |
| | | sin 3A | |
| | | $= \cos A + \cot 3A \sin A$ $= D \cup C$ | 1/2 |
| | | = R.H.S. | ½ |

SUMMER – 17 EXAMINATION

Model Answer

| | Cls | | N 4 a vilvi a a |
|-----------|--------------|---|-------------------|
| Q. No. | Sub Q. N. | Answer | Marking Scheme |
| 5 | e) | Prove that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ | 04 |
| | Ans | Let $A = \sin^{-1}\left(\frac{3}{5}\right)$ $B = \sin^{-1}\left(\frac{8}{17}\right)$ | |
| | | $\therefore \sin A = \frac{3}{5} \qquad \qquad \sin B = \frac{8}{17}$ | |
| | | 3 8 17 | |
| | | 4 15 . 4 | 1 |
| | | $\therefore \cos A = \frac{4}{5} \qquad \cos B = \frac{15}{17}$ OR | |
| | | Let $A = \sin^{-1}\left(\frac{3}{5}\right)$ and $B = \sin^{-1}\left(\frac{8}{17}\right)$ | |
| | | $\therefore \sin A = \frac{3}{5} \qquad , \qquad \sin B = \frac{8}{17}$ | |
| | | $\cos A = \sqrt{1 - \sin^2 A} \text{and} \cos B = \sqrt{1 - \sin^2 B}$ | |
| | | $=\sqrt{1-\left(\frac{3}{5}\right)^2} \qquad =\sqrt{1-\left(\frac{8}{17}\right)^2}$ | |
| | | $\therefore \cos A = \frac{4}{5} \qquad \qquad \cos B = \frac{15}{17}$ | 1 |
| | | $\therefore \sin(A-B) = \sin A \cos B - \cos A \sin B$ $= \left(\frac{3}{5}\right) \left(\frac{15}{17}\right) - \left(\frac{4}{5}\right) \left(\frac{8}{17}\right)$ | |
| | | | 4 |
| | | $=\frac{13}{85}$ | 1 |
| | | $A - B = \sin^{-1}\left(\frac{13}{85}\right) \therefore \sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{13}{85}\right)$ | 1/2 |
| | | $Let C = \sin^{-1}\left(\frac{13}{85}\right) \therefore \sin C = \frac{13}{85}$ | |
| | | $\cos C = \frac{84}{85} \therefore C = \cos^{-1}\left(\frac{84}{85}\right)$ | 1/2 |
| | | $\therefore \sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ 84 C | |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 5 | f) | Prove that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$ | 04 |
| | Ans | Let $A = \cos^{-1}\left(\frac{12}{13}\right)$ and $B = \sin^{-1}\left(\frac{3}{5}\right)$ | |
| | | $\therefore \cos A = \frac{12}{13} \qquad , \qquad \sin B = \frac{3}{5}$ $5 \qquad \qquad$ | 1 |
| | | $\sin A = \sqrt{1 - \cos^2 A} \text{and} \cos B = \sqrt{1 - \sin^2 B}$ $= \sqrt{1 - \left(\frac{12}{12}\right)^2}$ $= \sqrt{1 - \left(\frac{3}{12}\right)^2}$ | |
| | | $=\sqrt{1-\left(\frac{12}{13}\right)^2}$ $=\sqrt{1-\left(\frac{3}{5}\right)^2}$ $\therefore \sin A = \frac{5}{13}$ $\cos B = \frac{4}{5}$ $\therefore \sin (A+B) = \sin A \cos B + \cos A \sin B$ $=\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$ | 1 |
| | | $=\frac{56}{65}$ | 1 |
| | | $A + B = \sin^{-1}\left(\frac{56}{65}\right)$ $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$ | 1 |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | 17105 g |
|-----------|--------------|--|---------|
| 6 | | Attempt any FOUR of the following : | 16 |
| | a) | Find the equation of the line passing through the point of intersection of lines | 04 |
| | Ans | 2x+3y=13; $5x-y=7$ and perpendicular to $3x-y+7=0$. | |
| | 7113 | 2x + 3y = 13 | |
| | | 5x - y = 7 | |
| | | 2x+3y=13 | |
| | | $\therefore \frac{15x - 3y = 21}{15x - 3y = 21}$ | |
| | | 17x = 34 | 1 |
| | | x = 2 | |
| | | $\therefore 5(2) - y = 7$ | 1 |
| | | $\therefore -y = -3 \therefore y = 3$ | |
| | | $\therefore \text{ Point of intersection} = (2, 3)$ Stand of the line $2x + x + 7 = 0$ is | |
| | | Slope of the line $3x - y + 7 = 0$. is, | |
| | | $m_0 = -\frac{a}{b} = -\frac{3}{-1} = 3$ | |
| | | ∴ Slope of the required line is, | |
| | | $m - \frac{1}{2} - \frac{1}{2}$ | 1 |
| | | $m = -\frac{1}{m_0} = -\frac{1}{3}$ | |
| | | :. equation is, | |
| | | $y - y_1 = m(x - x_1)$ | |
| | | $\therefore y-3=-\frac{1}{2}(x-2)$ | |
| | | $\therefore x + 3y - 11 = 0$ | 1 |
| | | | |
| | b) | Find the counties of the line manifes the second the se | 04 |
| | 0) | Find the equation of the line passing through the point of intersection of lines | 04 |
| | | x + y = 0 and $2x - y = 9$ and through the point $(2,5)$. | |
| | Ans | x + y = 0 $2x - y = 0$ | |
| | | 2x - y = 9 $3x = 9$ | |
| | | $\therefore x = 3$ | 1 |
| | | y = -3 | 1 |
| | | $\therefore \text{ Point of intersection} = (3, -3)$ | 1 |
| | | (-, -) | |
| | | Page No. | 22/25 |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| INO. | Q. IV. | | JUILETTIE |
| 6 | b) | $\therefore equation is,$ $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ | |
| | | $\therefore \frac{y-5}{-3-5} = \frac{x-2}{3-2}$ | 1 |
| | | $\therefore 8x + y - 21 = 0$ | 1 |
| | | OR ∴ Point of intersection = $(3, -3)$ | |
| | | $\therefore Slope m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{3 - 2} = -8$ | 1 |
| | | $\therefore equation is, y - y_1 = m(x - x_1)$ | |
| | | $\therefore y - 5 = -8(x - 2) \qquad \text{OR} y + 3 = -8(x - 3)$ $\therefore 8x + y - 21 = 0$ | 1 |
| | c) Ans | If m_1 and m_2 are the slopes of two lines then prove that the acute angle between two lines is $\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $ | 04 |
| | | θ_2 θ_1 | 1 |
| | | Let θ_1 = Angle of inclination of L_1 θ_2 = Angle of inclination of L_2 $\therefore \text{ slope of } L_1 \text{ is } m_1 = \tan \theta_1$ $\text{ slope of } L_2 \text{ is } m_2 = \tan \theta_2$ $\therefore \text{ from figure,}$ | |
| | | $\theta_1 = \theta + \theta_2$ $\therefore \theta = \theta_1 - \theta_2$ | 1 |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|--|-------------------|
| 6 | c) | $\therefore \tan \theta = \tan (\theta_1 - \theta_2)$ | |
| | | $= \frac{\tan(\theta_1) - \tan(\theta_2)}{1 + \tan(\theta_1)\tan(\theta_2)}$ $= \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$ $\therefore \theta = \tan^{-1} \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$ | 1 |
| | | For angle to be acute, | |
| | | $\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $ | 1 |
| | d) | Find the length of perpendicular from $(-3, -4)$ on the line $4(x+2)=3(y-4)$. | 04 |
| | Ans | point $(x_1, y_1) = (-3, -4)$ and $L: 4x - 3y + 20 = 0$ $p = \left \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right $ | |
| | | $= \left \frac{4(-3) - 3(-4) + 20}{\sqrt{(4)^2 + (-3)^2}} \right $ $= \left \frac{-12 + 12 + 20}{\sqrt{16 + 9}} \right $ $= \left \frac{20}{\sqrt{25}} \right $ | 2 |
| | | $= \frac{20}{5}$ $p = 4 \text{ units}$ | 2 |
| | e) | Prove the perpendicular distance between two parallel lines $ax + by + c_1 = 0$ and $c_2 - c_1$ | 04 |
| | Ans | $ax + by + c_2 = 0 \text{ is } \left \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $ $L_1 \qquad L_2$ | |
| | | Page No 24/2 | 1 |



SUMMER – 17 EXAMINATION

Model Answer

| Q. No. | Sub Q. N. | Answer | Marking Scheme |
|-----------|--------------|---|-------------------|
| 6 | e) | $L_1: ax + by + c_1 = 0$ | |
| | <i>C)</i> | $L_1 : ax + by + c_1 = 0$ $L_2 : ax + by + c_2 = 0$ | |
| | | Let $P(x_1, y_1)$ be any point on the line L_2 | |
| | | $\therefore ax_1 + by_1 + c_2 = 0$ | |
| | | $\therefore ax_1 + by_1 + c_2 = 0$ $\therefore ax_1 + by_1 = -c_2$ | 1 |
| | | length of perpendicular on the line L_1 | 1 |
| | | | |
| | | $\therefore \text{ perpendicular length} = \left \frac{ax_1 + by_1 + c_1}{\sqrt{a^2 + b^2}} \right $ | 1 |
| | | $\therefore d = \left \frac{-c_2 + c_1}{\sqrt{a^2 + b^2}} \right \text{or } d = \left \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $ | 1 |
| | | | |
| | f) | Find the acute angle between the lines $3x - 2y + 4 = 0$ and | 04 |
| | Ans | 2x-3y-7=0 | |
| | | For $L_1: 3x - 2y + 4 = 0$ | |
| | | $slope \ m_1 = -\frac{a}{b} = \frac{3}{2}$ | 1 |
| | | | |
| | | $For L_2: 2x - 3y - 7 = 0,$ | 1 |
| | | slope $m_2 = -\frac{a}{b} = -\frac{2}{-3} = \frac{2}{3}$ | 1 |
| | | $\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $ | |
| | | $\left \begin{array}{c c} \frac{3}{2} - \frac{2}{3} \end{array}\right $ 5 | 1 |
| | | $= \frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\right) \cdot \left(\frac{2}{3}\right)} = \frac{5}{12} or 0.417$ | |
| | | $\therefore \theta = \tan^{-1} \left(\frac{5}{12} \right) or \tan^{-1} \left(0.417 \right)$ | 1 |
| | | <u>Important Note</u> | |
| | | In the solution of the question paper, wherever possible all the possible alternative | |
| | | methods of solution are given for the sake of convenience. Still student may follow a | |
| | | method other than the given herein. In such case, first see whether the method falls | |
| | | within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking. | |
| | | | |
| | | Page No 2 | |