

#### MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

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#### Winter 2014 Examination

Subject & Code: Applied Maths (17301) Model Answer Page No: 1/28

Que. No.	Sub. Que.	Model Answers	Marks	Total Mark
		Important Instructions to the Examiners:		
		1) The Answers should be examined by key words and not as		
		word-to-word as given in the model answer scheme.		
		2) The model answer and the answer written by candidate may		
		vary but the examiner may try to assess the understanding level of the candidate.		
		3) The language errors such as grammatical, spelling errors		
		should not be given more importance. (Not applicable for		
		subject English and Communication Skills.)		
		4) While assessing figures, examiner may give credit for		
		principal components indicated in the figure. The figures		
		drawn by the candidate and those in the model answer may		
		vary. The examiner may give credit for any equivalent		
		figure drawn.		
		5) Credits may be given step wise for numerical problems. In		
		some cases, the assumed constant values may vary and there		
		may be some difference in the candidate's Answers and the model answer.		
		6) In case of some questions credit may be given by judgment		
		on part of examiner of relevant answer based on candidate's understanding.		
		7) For programming language papers, credit may be given to		
		any other program based on equivalent concept.		



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	Que.	Attempt any TEN of the following:		Warks
	a)	Find the gradient of the tangent of the curve $y = \sqrt{x^3}$ at $x = 4$ .		
	Ans.	$y = \sqrt{x^3} = x^{3/2}$		
		$y = \sqrt{x} = x$ $\therefore \frac{dy}{dx} = \frac{3}{2}x^{1/2}$	1	
		$\therefore \frac{dx}{dx} - \frac{dx}{2}$ $\therefore \text{ the gradient at } x = 4 \text{ is,}$		
		$\frac{dy}{dx} = \frac{3}{2}(4)^{1/2} = 3$	1	
		$\frac{1}{dx} - \frac{1}{2} (4) = 3$ OR	1	
		$y = \sqrt{x^3}$		2
		$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x^3}} \cdot \frac{d}{dx} (x^3)$		
		$= \frac{1}{2\sqrt{x^3}} \cdot 3x^2$	1	
		$2\sqrt{x^3}$ ∴ the gradient at $x = 4$ is,		
		$\frac{dy}{dx} = \frac{1}{2\sqrt{4^3}} \cdot 3(4)^2 = 3$	1	2
		$dx  2\sqrt{4^3}  (1)  3$		_
	b)	Find the radius of the curvature of the curve $y^2 = 4ax$ at the point $(a, 2a)$ .		
	Ans.	$y^2 = 4ax$		
		$\therefore 2y \frac{dy}{dx} = 4a$	1/2	
		$\therefore \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$		
		$\therefore \frac{d^2 y}{dx^2} = -\frac{2a}{y^2} \cdot \frac{dy}{dx}$	1/2	
		$\therefore at (a, 2a),$		
		$\frac{dy}{dx} = \frac{2a}{2a} = 1$ and $\frac{d^2y}{dx^2} = -\frac{2a}{(2a)^2} \cdot 1 = -\frac{1}{2a}$	1/2	
		$\frac{dy}{dx} = \frac{2u}{2a} = 1  and  \frac{dy}{dx^2} = -\frac{2u}{(2a)^2} \cdot 1 = -\frac{1}{2a}$ $\therefore \kappa = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} = \frac{-\frac{1}{2a}}{\left[1 + \left(1\right)^2\right]^{\frac{3}{2}}} = -\frac{1}{2a \cdot 2^{3/2}} = -\frac{1}{2^{5/2}a}$		

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Que.	Sub.	Model Answers	Marks	Total
No. <b>1)</b>	Que.			Marks
-/		$\therefore \rho = \frac{1}{\kappa} = -2^{5/2}a$	1/2	
		OR	OR	
		$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^{2}\right]^{\frac{3}{2}}}{\frac{d^{2}y}{dx^{2}}} = \frac{\left[1 + (1)^{2}\right]^{\frac{3}{2}}}{-\frac{1}{2}a} = -2a \cdot 2^{3/2} = -2^{5/2}a$	1/2	2
		OR		
		$y^2 = 4ax$		
		$y^2 = 4ax$ $\therefore 2y \frac{dy}{dx} = 4a$	1/2	
		$\therefore 2y\frac{d^2y}{dx^2} + 2\frac{dy}{dx} \cdot \frac{dy}{dx} = 0  or  2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$		
		$\therefore \frac{d^2 y}{dx^2} = -\frac{1}{y} \left(\frac{dy}{dx}\right)^2$	1/2	
		$\therefore at (a, 2a),$ $\frac{dy}{dx} = \frac{4a}{2 \cdot 2a} = 1  and  \frac{d^2y}{dx^2} = -\frac{1}{2a} \cdot 1 = -\frac{1}{2a}$	1/2	
		$\therefore \kappa = -\frac{1}{2^{5/2}a}$ $\therefore \rho = -2^{5/2}a$	1/2	
		p = -2 u	/2	2
	c)	Evaluate $\int (\tan x + \cot x)^2 dx$		
	Ans.	$\int (\tan x + \cot x)^2 dx$		
		$= \int (\tan^2 x + 2 \tan x \cot x + \cot^2 x) \cdot dx$		
		$= \int \left(\tan^2 x + 2 + \cot^2 x\right) \cdot dx$	1/2	
		$= \int \left(\sec^2 x - 1 + 2 + \cos ec^2 x - 1\right) \cdot dx$		
		$= \int \left(\sec^2 x + \cos ec^2 x\right) \cdot dx$	1/2	
		$= \tan x - \cot x + c$	1	2
		Note: In the solution of any INDEFINITE integration		
		<b>problems</b> , if the constant c is not added, ½ mark may be deducted.		



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	d)	Evaluate $\int \sec^2(\log x) \frac{1}{x} dx$		
	Ans.	$\int \sec^2 (\log x) \frac{1}{x} dx = \int \sec^2 t dt$ $Put  \log x = t$ $\therefore \frac{1}{x} dx = dt$	1/2 + 1/2	
		$= \tan t + c$ $= \tan (\log x) + c$	1/ <sub>2</sub> 1/ <sub>2</sub>	2
		OR  Put log v = t		
		$Put \log x = t$ $\therefore \frac{1}{x} dx = dt$	1/2	
		$\therefore \int \sec^2(\log x) \frac{1}{x} dx = \int \sec^2 t dt$	1/2	
		$= \tan t + c$	1/2	
		$= \tan(\log x) + c$	1/2	2
	e)	Evaluate $\int xe^x dx$ .		
	Ans.	$\int xe^x dx = x \int e^x dx - \int \left[ \int e^x dx \right] \frac{d}{dx}(x) \cdot dx$	1/2	
		$= xe^x - \int e^x dx$	1/2	
		$= xe^x - e^x + c$	1/2+1/2	2
	f)	Evaluate $\int \frac{1}{x^2 + 3x + 2} dx$		
	Ans.	$x^{2} + 3x + 2 = x^{2} + 3x + \frac{9}{4} - \frac{9}{4} + 2 = \left(x + \frac{3}{2}\right)^{2} - \left(\frac{1}{2}\right)^{2}$	1/2+1/2	
		$\int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$		
		$= \frac{1}{2\left(\frac{1}{2}\right)} \log \left(\frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}}\right) + c$	1/2	
		$=\log\left(\frac{x+1}{x+2}\right)+c$	1/2	2

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Que.	Sub.	Model Answers	Marks	Total
No.	Que.		IVIUINS	Marks
1)		OR $\int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2} dx$	1/2	
		$=\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$	1/2	
		$= \frac{1}{2\left(\frac{1}{2}\right)} \log \left(\frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}}\right) + c$	1/2	
		$=\log\left(\frac{x+1}{x+2}\right)+c$	1/2	2
		OR		
		$I = \int \frac{1}{x^2 + 3x + 2} dx = \int \frac{1}{(x+1)(x+2)} dx$		
		$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$		
		$\therefore A = 1$	1/2	
		$B = -1 \qquad(*)$	1/2	
		$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$ $\therefore I = \int \left[ \frac{1}{x+1} + \frac{-1}{x+2} \right] dx$ $= \log(x+1) - \log(x+2) + c$	1	2
		Note (*): There are various methods to find the values of A and B to partially factorize the given expression including direct method. Students may apply any one of the methods. Take in count all such methods.		
	g)	Evaluate $\int_{1}^{2} \frac{dx}{3x-2}$		
	Ans.	$\int_{1}^{2} \frac{dx}{3x - 2} = \left[ \frac{\log(3x - 2)}{3} \right]_{1}^{2}$	1	
		$= \frac{\log(6-2)}{3} - \frac{\log(3-2)}{3}$ $\log 4 - \log 1$	1/2	
		$= \frac{\log 4}{3} - \frac{\log 1}{3}$ $= \frac{\log 4}{3}$	1/2	2



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.	Wiodel Aliswers	Iviaiks	Marks
1)	h)	Find the area above the x-axis bounded by $y = \sin x$ and the ordinates $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ .		
	Ans.	$A = \int_{\pi/6}^{\pi/3} \sin x dx$ = $\left[ -\cos x \right]_{\pi/6}^{\pi/3}$	1/2	
		$= \left[-\cos\frac{\pi}{3}\right] - \left[-\cos\frac{\pi}{6}\right] (*)$	1/2	
		$=\frac{-1+\sqrt{3}}{2}  or  0.366$ OR	1	2
		$A = \int_{\pi/3}^{\pi/6} \sin x dx$		
		$= \left[ -\cos x \right]_{\pi/3}^{\pi/6}$	1/2	
		$= \left[-\cos\frac{\pi}{6}\right] - \left[-\cos\frac{\pi}{3}\right] (*)$	1/2	
		$=\frac{-\sqrt{3}+1}{2}$ or $-0.366$	1/2	
		$\therefore Area \ A = \frac{-1 + \sqrt{3}}{2}  or  0.366$	1/2	2
		Note: Due to the use of advance non-programmable scientific calculators, writing directly the value of the step (*) as 0.366 or -0.366 is permissible. No marks to be deducted for calculating directly the value.		
	i)	Find the order and degree of the equation $2\frac{d^2y}{dx^2} + \left[ 3\sqrt{1 - \left(\frac{dy}{dx}\right)^2} - y \right] = 0$		
	Ans.	Order = 2	1	
		$\sqrt{1 - \left(\frac{dy}{dx}\right)^2} = \frac{1}{3} \left(y - 2\frac{d^2y}{dx^2}\right)$		
		$\therefore 1 - \left(\frac{dy}{dx}\right)^2 = \frac{1}{9} \left(y - 2\frac{d^2y}{dx^2}\right)^2$		
		Degree = 2	1	2



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	C 1		1	Tr ( 1
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	j) Ans.	Verify that $y = Ae^x + Be^{-x}$ is a solution of $\frac{d^2y}{dx^2} - y = 0$		
		$y = Ae^{x} + Be^{-x}$ $\therefore \frac{dy}{dx} = Ae^{x} + Be^{-x} \cdot (-1) = Ae^{x} - Be^{-x}$ $\therefore \frac{d^{2}y}{dx^{2}} = Ae^{x} - Be^{-x} \cdot (-1)$	1/2	
		$=Ae^{x}+Be^{-x}$	1/2	
		$\therefore \frac{d^2y}{dx^2} = y$	1/2	
		$\therefore \frac{d^2 y}{dx^2} - y = 0$	1/2	2
	k)	A bag contains 7 white balls, 5 black balls, and 4 red balls. If two balls are drawn at random from the bag, find the probability that both the balls are white.		
	Ans.	Total = $7 + 5 + 4 = 16$ $n = {}^{16}C_2 = 120$	1/2	
		$\therefore m = {}^{7}C_{2} = 21$	1/2	
		$\therefore p = \frac{m}{n} = \frac{21}{120} = \frac{7}{40}  or  0.175$	1	2
		OR		
		Total = 7 + 5 + 4 = 16		
		$\therefore p = \frac{{}^{7}C_2}{{}^{16}C_2}$	1	
		$=\frac{21}{120} = \frac{7}{40}  or  0.175$	1	2
		<b>Note:</b> Due to the use of advance non-programmable scientific calculators which is permissible in the board examination, writing directly the values of ${}^{n}C_{r}$ or ${}^{n}C_{r}p^{r}q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.		



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Page No: 8/28 Que. Sub. Total **Model Answers** Marks No. Que. Marks 1) What is the probability of getting more than 4 in a single throw 1) of a die? Ans. n = 6 $1/_{2}$  $A = \{5, 6\}$  $1/_{2}$  $\therefore m = 2$  $\therefore p = \frac{m}{n} = \frac{2}{6} = \frac{1}{3} \quad or \quad 0.333$ 2 1 2) Attempt any four. Find the equations of the tangent and normal to the curve a)  $4x^2 + 9y^2 = 40$  at the point (1, 2). Ans.  $4x^2 + 9y^2 = 40$  $\therefore 4 \cdot 2x + 9 \cdot 2y \frac{dy}{dx} = 0 \quad or \quad 8x + 18y \frac{dy}{dx} = 0$  $1/_{2}$  $\therefore 18y \frac{dy}{dx} = -8x$  $\therefore \frac{dy}{dx} = \frac{-8x}{18y} = -\frac{4x}{9y}$  $\frac{1}{2}$  $\therefore$  at (1, 2), the slope of tangent is  $m = \frac{dy}{dx} = -\frac{4 \cdot 1}{9 \cdot 2} = -\frac{2}{9}$  $1/_{2}$ : the equation of tangent is  $y-2=-\frac{2}{9}(x-1)$  $1/_{2}$  $\therefore 9y - 18 = -2x + 2$  $\therefore 2x + 9y - 20 = 0$  or -2x - 9y + 20 = 0 $1/_{2}$  $\therefore$  at (1, 2), the slope of normal is  $m=\frac{9}{3}$  $1/_{2}$ : the equation of tangent is  $y-2=\frac{9}{2}(x-1)$  $1/_{2}$  $\therefore 2y - 4 = 9x - 9$ 4  $1/_{2}$  $\therefore 9x - 2y - 5 = 0$  or -9x + 2y + 5 = 0



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Que.	Sub.	26.114	3.6.1	Total
No.	Que.	Model Answers	Marks	Marks
2)	b)	A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$ , find the radius of curvature of the beam at this point $x = \frac{\pi}{2}$ .		
	Ans.	$y = 2\sin x - \sin 2x$		
		$\therefore \frac{dy}{dx} = 2\cos x - 2\cos 2x$	1	
		$\& \frac{d^2y}{dx^2} = -2\sin x + 4\sin 2x$	1	
		$\therefore at \ x = \frac{\pi}{2},$		
		$\frac{dy}{dx} = 2\cos\left(\frac{\pi}{2}\right) - 2\cos 2\left(\frac{\pi}{2}\right) = 2$	1/2	
		and $\frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$	1/2	
		$\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2} = -5.590$	1	
		$OR$ $d^2y$	OR	
		$\kappa = \frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} = \frac{-2}{\left[1 + (2)^2\right]^{\frac{3}{2}}} = -0.1789$	1/2	
		$\therefore \rho = \frac{1}{\kappa} = -5.590$	1/2	4
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.		
	Ans.	Let x and y be the sides of rectangle. $\therefore 2x+2y=36$ or $x+y=18$		
		$\therefore y = 18 - x$	1/2	
		But area $A = xy = x(18-x) = 18x - x^2$	1	
		$\therefore \frac{dA}{dx} = 18 - 2x$	1/2	



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Outo	Cub		1	Total
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	2	$\therefore \frac{d^2 A}{dx^2} = -2$ For stationary values, $\frac{dA}{dx} = 0$	1/2	
		$\therefore 18 - 2x = 0$ $\therefore x = 9$	1/2	
		$At \ x = 9, \ \frac{d^2A}{dx^2} = -2 < 0$	1/2	
		∴ At $x = 9$ , A has max imum value and the other side is y = 18 - x = 9	1/2	4
	d)	Evaluate $\int \frac{x-3}{x^3 - 3x^2 - 16x + 48} dx$		
	Ans.	$\int \frac{x-3}{x^3 - 3x^2 - 16x + 48} dx = \int \frac{x-3}{(x-3)(x^2 - 16)} dx$		
		$=\int \frac{1}{x^2 - 16} dx$	1	
		$=\int \frac{1}{x^2 - 4^2} dx$	1	
		$= \frac{1}{8} \log \left( \frac{x-4}{x+4} \right) + c$	2	4
		OR		
		$\int \frac{x-3}{x^3 - 3x^2 - 16x + 48} dx = \int \frac{x-3}{(x-3)(x-4)(x+4)} dx$		
		$=\int \frac{1}{(x-4)(x+4)} dx$	1	
		$=\int \left[\frac{1/8}{x-4} + \frac{-1/8}{x+4}\right] dx$	1	
		$= \frac{1}{8} \log(x-4) - \frac{1}{8} \log(x+4) + c$	2	4
	e)	Evaluate $\int \frac{1}{x \left[9 + (\log x)^2\right]} dx$		
	Ans.	Put $\log x = t$	1	
		$\therefore \frac{1}{x} dx = dt$	1	



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Que.	Sub.	25.11.	3.5.1	Total
No.	Que.	Model Answers	Marks	Marks
2)		$\therefore \int \frac{1}{x \left[9 + \left(\log x\right)^2\right]} dx = \int \frac{1}{9 + t^2} dt$ $= \int \frac{1}{3^2 + t^2} dt$	1/2	
		$=\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1	
		$= \frac{1}{3} \tan^{-1} \left( \frac{\log x}{3} \right) + c$	1	4
		OR		
		$\therefore \int \frac{1}{x \left[9 + \left(\log x\right)^2\right]} dx$ $\left[\begin{array}{c} Put & \log x = t \\ \therefore \frac{1}{x} dx = dt \end{array}\right]$	1	
		$=\int \frac{1}{9+t^2} dt$	1/2	
		$=\int \frac{1}{3^2+t^2}dt$	1/2	
		$= \frac{1}{3} \tan^{-1} \left( \frac{t}{3} \right) + c$	1	
		$= \frac{1}{3} \tan^{-1} \left( \frac{\log x}{3} \right) + c$	1	4
	f)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$		
		$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$ $Put \tan x = t$ $\therefore \sec^2 x dx = dt$	1	
		$=\int \frac{1}{(1+t)(3+t)}dt$		
		$\therefore \frac{1}{(1+t)(3+t)} = \frac{A}{1+t} + \frac{B}{3+t}$		
		$\therefore A = \frac{1}{2}$	1	
		$B = -\frac{1}{2}$	1	
		[Please refer the note written in the question 1 (f).]		

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3)		$ \therefore \frac{1}{(1+t)(3+t)} = \frac{\frac{1}{2}}{1+t} + \frac{-\frac{1}{2}}{3+t} $ $ \int \frac{1}{(1+t)(3+t)} dt = \int \left[ \frac{\frac{1}{2}}{1+t} + \frac{-\frac{1}{2}}{3+t} \right] dx $ $ = \frac{1}{2} \log[1+t] - \frac{1}{2} \log[3+t] + c $ $ = \frac{1}{2} \log[1+\tan x] - \frac{1}{2} \log[3+\tan x] + c $ Attempt any four.	1/2 1/2	4
	a)	Evaluate $\int_0^{\pi/4} x \sec^2 x dx$		
	Ans.	$\int_0^{\pi/4} x \sec^2 x dx = \left[ x \int \sec^2 x dx - \int \left( \int \sec^2 x dx \right) \frac{d}{dx} (x) dx \right]_0^{\pi/4}$	1	
		$= \left[ x \tan x - \int \tan x dx \right]_0^{\pi/4}$	1	
		$= \left[x \tan x - \log(\sec x)\right]_0^{\pi/4}$	1	
		$= \left[ \frac{\pi}{4} \tan \frac{\pi}{4} - \log \left( \sec \frac{\pi}{4} \right) \right] - \left[ 0 - \log \left( \sec 0 \right) \right]$	1/2	
		$=\frac{\pi}{4}-\log\sqrt{2}  or  0.439$	1/2	4
		<b>Note:</b> In case of definite integrations, the problem may be solved by without limits and then the limits would be applied, as illustrated below:	2	
		$\int x \sec^2 x dx = x \int \sec^2 x dx - \int \left( \int \sec^2 x dx \right) \frac{d}{dx} (x) dx$	1	
		$= x \tan x - \int \tan x dx$	1	
		$= x \tan x - \log(\sec x)$	1	
		$\int_0^{\pi/4} x \sec^2 x dx = \left[ x \tan x - \log (\sec x) \right]_0^{\pi/4}$		
		$= \left[\frac{\pi}{4} \tan \frac{\pi}{4} - \log \left(\sec \frac{\pi}{4}\right)\right] - \left[0 - \log \left(\sec 0\right)\right]$	1/2	

 $=\frac{\pi}{4} - \log\sqrt{2} \quad or \quad 0.439$ 



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No. Que. Model Answers Marks No. Que. Evaluate $\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \left[x\right]_{1}^{2}$ $\therefore 2I = 2 - 1$ $\therefore I = \frac{1}{2}$ OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $= \int_{1}^{2} \frac{\sqrt{x} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $= \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = \left[x\right]_{1}^{2}$	Subject & C	Code: Applied Maths (17301)	<b>Page No:</b> 13/	28
b) Evaluate $\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \left[x\right]_{1}^{2}$ $\therefore 2I = 2 - 1$ $\therefore I = \frac{1}{2}$ OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $= \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x} + \sqrt{3} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x} + \sqrt{x} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} \frac{1}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} 1 dx$ $\therefore 2I = \left[x\right]_{1}^{2}$	No. Que	Model Answers	Marks	Total Marks
$I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $\therefore 2I = 2 - 1$ $\therefore I = \frac{1}{2}$ OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3} - x} dx$ $= \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{3}} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{3} - x}{\sqrt{3} - x + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3} - x}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = \left[ x \right]_{1}^{2}$	*	Evaluate $\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$		
		$I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $Re \ place \ x \to 3 - x$ $\therefore 3 - x \to x$	1	
$\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $\therefore 2I = 2 - 1$ $\therefore I = \frac{1}{2}$ OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $= \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{3} - (3 - x)} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3 - x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{1}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $1/2$		$\therefore I = \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{x} + \sqrt{2}} dx$	1	
$\therefore 2I = 2 - 1$ $\therefore I = \frac{1}{2}$ OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $= \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{3 - (3 - x)}} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3 - x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $1/2$		$\therefore 2I = \int_{1}^{2} 1 \cdot dx$	1/2	
OR $I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $= \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{3 - (3 - x)}} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3 - x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \left[x\right]_{1}^{2}$ $\therefore 2I = \left[x\right]_{1}^{2}$ $1/2$		-	1	
$I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $= \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{3 - (3 - x)}} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3 - x}}{\sqrt{3 - x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3 - x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $1/2$		$\therefore I = \frac{1}{2}$	1/2	4
$= \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{3} - (3-x)} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{x} + \sqrt{3} - x} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $1/2$		OR		
$= \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{3} - (3-x)} dx$ $\therefore I = \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$ $1/2$		$I = \int_{1}^{2} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}} dx$		
$\therefore 2I = \int_{1}^{2} \frac{\sqrt{x} + \sqrt{3 - x}}{\sqrt{x} + \sqrt{3 - x}} dx$ $\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$			1/2	
$\therefore 2I = \int_{1}^{2} 1 \cdot dx$ $\therefore 2I = [x]_{1}^{2}$		1 '	1/2	
$\therefore 2I = \left[x\right]_1^2$		•	1	
		1	1/2	
$  \cdot \cdot \cdot \angle I - \angle - 1  $		$\therefore 2I = [x]_1$ $\therefore 2I = 2 - 1$	1	

 $\frac{1}{2}$ 



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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
3)	c)	Find the area bounded by the curve $y = x^2$ and the line $y = x$ .		
	Ans.	Given $y = x^2$ , $y = x$		
		$\therefore x^2 = x$		
		$\therefore x^2 - x = 0$		
		$\therefore x = 0, 1$	1	
		$\therefore A = \int_a^b (y_2 - y_1) dx$		
		$=\int_0^1 \left(x-x^2\right) dx$	1/2	
		$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$	1	
		$=\left[\frac{1}{2} - \frac{1}{3}\right] - [0 - 0]$	1/2	
		= 0.167	1	4
		OR		
		Given $y = x^2$ , $y = x$		
		$\therefore x^2 = x$		
		$\therefore x^2 - x = 0$	1	
		$\therefore x = 0, 1$	1	
		$\therefore A = \int_a^b (y_2 - y_1) dx$		
		$=\int_0^1 \left(x^2 - x\right) dx$	1/2	
		$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$	1	
		$= \left[\frac{1}{3} - \frac{1}{2}\right] - \left[0 - 0\right]$	1/2	
		$=-\frac{1}{6}$ or $-0.167$	1/2	
		$\therefore Area \ A = \frac{1}{6}  or  0.167$	1/2	4



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.	Wiodel Aliswers	IVIAIKS	Marks
3)	d)	Solve $1-2\frac{dy}{dx} = \cos^2(x-2y)$		
	Ans.	ax		
		$Put  x-2y=v$ ∴ 1-2\frac{dy}{dx} = \frac{dv}{dx}	1	
		$\therefore \frac{dv}{dx} = \cos^2 v$	1	
		$\therefore \frac{dv}{\cos^2 v} = dx$		
		$\therefore \int \sec^2 v dv = \int dx$	1/2	
		$\therefore \tan v = x + c$ $\therefore \tan (x - 2y) = x + c$	1	
		(x 2y) = x + c	1/2	4
		dv. v. v		
	e)	Evaluate $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$		
	Ans.	$\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$		
		$Put \frac{y}{x} = v  or  y = vx$		
		$\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$	1	
		$\therefore v + x \frac{dv}{dx} = v + \sin v$	1/2	
		$\therefore x \frac{dv}{dx} = \sin v$	1/2	
		$\therefore \cos ecv  dv = \frac{dx}{x}$		
		$\therefore \int \cos ecv  dv = \int \frac{dx}{x}$	1/2	
		$\therefore \log(\cos ec  v - \cot v) = \log x + c$	1/2+1/2	
		$\therefore \log \left( \cos ec  \frac{y}{x} - \cot \frac{y}{x} \right) = \log x + c$	1/2	4

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(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

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Que. No.	Sub.	Model Answers	Marks	Total Marks
3)	Que.	Evaluate $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$		Widtks
	Ans.	$(x+1)\frac{dy}{dx} - y = e^x (x+1)^2$ $\therefore \frac{dy}{dx} - \frac{1}{x+1} \cdot y = e^x (x+1)$ $P = -\frac{1}{x+1}  \text{and}  Q = e^x (x+1)$ $\therefore IF = e^{\int pdx} = e^{\int -\frac{1}{x+1}dx} = e^{-\log(x+1)} = \frac{1}{x+1}$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$	1	
		$\therefore y \cdot \frac{1}{x+1} = \int \frac{1}{x+1} \cdot e^x (x+1) \cdot dx$	1	
		$\therefore y \cdot \frac{1}{x+1} = \int e^x \cdot dx$	1	
		$\therefore y \cdot \frac{1}{x+1} = e^x + c$	1	4
4)		Attempt any four.		
	a)	Evaluate $\int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$		
	Ans.	$I = \int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$	1	
		$\therefore 2I = \int_{0}^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$	1	
		$\therefore 2I = \int_{0}^{\pi/2} 1 \cdot dx$ $\therefore 2I = \left[x\right]_{0}^{\pi/2}$	1/2	
		$=\frac{\pi}{2}-0$	1	
		$ \begin{aligned} &: I = [x]_0 \\ &= \frac{\pi}{2} - 0 \\ &: I = \frac{\pi}{4} \end{aligned} $	1/2	4



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\frac{\pi}{2}$ $\sqrt{\cos x}$		
		$I = \int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$		
		$= \int_{0}^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$	1/2	
		$\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$	1/2	
		$\therefore 2I = \int_{0}^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$	1	
		$\therefore 2I = \int_{0}^{\pi/2} 1 \cdot dx$ $\therefore 2I = \left[x\right]_{0}^{\pi/2}$	1/2	
		$=\frac{\pi}{2}-0$	1	
		$\therefore I = \frac{\pi}{4}$	1/2	4
	b)	Evaluate $\int_0^1 x^2 \sqrt{1-x} \cdot dx$		
	Ans.	$I = \int_0^1 x^2 \sqrt{1 - x} \cdot dx$		
		$\therefore I = \int_0^1 (1-x)^2 \sqrt{1-(1-x)} \cdot dx$ $= \int_0^1 (1-2x+x^2) \sqrt{x} \cdot dx$	1/2	
		$= \int_0^1 \left( \sqrt{x} - 2x^{\frac{3}{2}} + x^{\frac{5}{2}} \right) \cdot dx$	1	
		$= \left[\frac{2}{3}x^{\frac{3}{2}} - 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_{0}^{1}$	1	
		$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}}\right]_0^1$		
		$= \left[\frac{2}{3}1^{\frac{3}{2}} - \frac{4}{5}1^{\frac{5}{2}} + \frac{2}{7}1^{\frac{7}{2}}\right] - [0 - 0 + 0]$	1	4
		$=\frac{16}{105}$ or 0.152	1/2	*



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Que.	Sub.	Model Agences	Marilia	Total
No.	Que.	Model Answers	Marks	Marks
4)	c)	Find by integration the area of the circle $x^2 + y^2 = a^2$ .		
	Ans.	$x^2 + y^2 = a^2$		
		$\therefore y^2 = a^2 - x^2$		
		$\therefore y = \sqrt{a^2 - x^2}$		
		$At \ y = 0,  a^2 - x^2 = 0$		
		$\therefore x = -a,  a$	1	
		$\therefore A = 4 \int_a^b y dx$		
		$=4\int_0^a \sqrt{a^2-x^2} dx$	1/2	
		$= 4 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_0^a$	1	
		$=4\left[0+\frac{a^2}{2}\sin^{-1}(1)\right]-\left[0+\frac{a^2}{2}\sin^{-1}(0)\right]$	1/2	
		$=4\left\lceil \frac{a^2}{2} \cdot \frac{\pi}{2} \right\rceil$	1/2	_
		$=\pi a^2$	1/2	4
		OR		
		x = -a, $a$	1	
		$\therefore A = 2\int_a^b y dx$		
		$=2\int_{-a}^{a}\sqrt{a^2-x^2}dx$	1/2	
		$= 2 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_{-a}^{a}$	1	
		$=2\left[0+\frac{a^2}{2}\sin^{-1}(1)\right]-\left[0+\frac{a^2}{2}\sin^{-1}(-1)\right]$	1/2	
		$=2\left[\frac{a^2}{2}\cdot\frac{\pi}{2}+\frac{a^2}{2}\cdot\frac{\pi}{2}\right]$	1/2	
		$=\pi a^2$	1/2	4
	d)	Evaluate $\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$		
	Ans.	$\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$		
	,	$dx = e^{2x} \cdot e^{-3y} + 4x^2 e^{-3y}$		
		$= (e^{2x} + 4x^2)e^{-3y}$	1	



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	2	$\therefore \frac{dy}{e^{-3y}} = \left(e^{2x} + 4x^2\right) dx$	1	
		$\therefore \int e^{3y} dy = \int \left(e^{2x} + 4x^2\right) dx$	1	
		$\therefore \frac{e^{3y}}{3} = \frac{e^{2x}}{2} + \frac{4}{3}x^3 + c$		4
		3 2 3	1	4
	e)	Evaluate $(2xy + y^2)dx + (x^2 + 2xy + \sin y)dy = 0$		
	Ans.	$(2xy + y^2)dx + (x^2 + 2xy + \sin y)dy = 0$		
		$M = 2xy + y^2$		
		$\therefore \frac{\partial M}{\partial y} = 2x + 2y$	1	
		$N = x^2 + 2xy + \sin y$		
		$\therefore \frac{\partial N}{\partial x} = 2x + 2y$	1/2	
		∴the equation is exact.	1/2	
		$\int_{y \ constant} Mdx + \int_{terms \ free \ from \ x} Ndy = c$		
		$\int (2xy + y^2) dx + \int \sin y dy = c$	1	
		$\therefore 2y \cdot \frac{x^2}{2} + y^2x - \cos y = c$		
		$or  x^2y + xy^2 - \cos y = c$	1	4
	f)	Show that $y^2 = ax^2$ is a solution of $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + ax = 0$ .		
	Ans.	$y^2 = ax^2$		
		$\therefore 2y \frac{dy}{dx} = 2ax$	1	
		$\therefore \frac{dy}{dx} = \frac{2ax}{2y} = \frac{ax}{y}$		
		$\therefore x \left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + ax = x\left(\frac{ax}{y}\right)^2 - 2y\cdot\frac{ax}{y} + ax$	1	
		$=\frac{a^2x^3}{v^2}-2ax+ax$		
		$=\frac{a^2x^3}{ax^2}-2ax+ax$	1	
		$= \frac{1}{ax^2} - 2ax + ax$ $= 0$	1	4



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Que.	Sub.	Mo Jol American	Maulia	Total
No.	Que.	Model Answers	Marks	Marks
5)		Attempt any four.		
	a)	A husband and wife appear in an interview for two vacancies		
		in the same post. The probability of husband's selection is $\frac{1}{7}$		
		and that of wife selection is $\frac{1}{5}$ . What is probability that:		
		<ol> <li>Both of them will be selected,</li> <li>None of them will be selected.</li> </ol>		
	Ans.	$P(H) = \frac{1}{7} \qquad P(H') = 1 - \frac{1}{7} = \frac{6}{7}$ $P(W) = \frac{1}{5} \qquad P(W') = 1 - \frac{1}{5} = \frac{4}{5}$	1/2	
			1/2	
		P(Both selected) = P(H & W)		
		$= P(H) \cdot P(W)$		
		$=\frac{1}{7}\cdot\frac{1}{5}$	1	
		$=\frac{1}{35}$ or 0.0286	1/2	
		$P(None \ is \ selected) = p(H'\&W')$		
		$=P(H')\cdot P(W')$		
		$=\frac{6}{7}\cdot\frac{4}{5}$	1	
		$=\frac{24}{35}$ or 0.686	1/2	4
	b)	The overall percentage of failures in a certain examination is 20. If six candidates appear in an examination, what is the probability that at least five pass the examination?		
	Ans.	% of failure = 20% ∴ % of passing = 80%		
		$\therefore p = \frac{80}{100} = 0.8 \qquad q = 1 - 0.8 = 0.2$	1	
		$n = 6$ $\therefore n(at least 5) - n(5) + n(6)$	1	
		$\therefore p(at least 5) = p(5) + p(6)$ $= {}^{6}C_{5}(0.8)^{5}(0.2)^{1} + {}^{6}C_{6}(0.8)^{6}(0.2)^{0}$	1	
		$= C_5(0.8) (0.2) + C_6(0.8) (0.2)$ $= 0.6553$		4
			1	

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Que.	Sub.	Model Answers	Marks	Total
No. <b>6)</b>	Que.	1120 1101 1210 11 020	1/10/11/0	Marks
		<b>Note:</b> Due to the use of advance non-programmable scientific calculators, writing directly the values of ${}^{n}C_{r}$ or ${}^{n}C_{r}p^{r}q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.		
	c)	A skilled typist, on routine work, kept a record of mistakes per day during 300 working days. Fit a Poisson distribution to the set of observations.		
		x     0     1     2     3     4     5     6       y     143     90     42     12     9     3     1		
	Ans.	$\begin{array}{c cccc} x & y & xy \\ \hline 0 & 143 & 0 \\ \hline 1 & 90 & 90 \\ \hline 2 & 42 & 84 \\ \hline 3 & 12 & 36 \\ \hline 4 & 9 & 36 \\ \hline 5 & 3 & 15 \\ \hline 6 & 1 & 6 \\ \hline & 300 & 267 \\ \\ & & \\ & & \\ e^{-m}m^r & e^{-0.89}(0.89)^r \end{array}$ $\therefore mean \ m = \frac{267}{300} = 0.89$	1 2 1	4
		$\therefore p = \frac{c \cdot m}{r!} = \frac{c \cdot (s \circ s)}{r!}$		<b>T</b>
	d)	Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$		
	Ans.	$Put \tan \frac{x}{2} = t$ $2dt \qquad 2t \qquad 1 - t^2$	1	
		$\therefore dx = \frac{2dt}{1+t^2},  \sin x = \frac{2t}{1+t^2},  \cos x = \frac{1-t^2}{1+t^2}$	1	
		$\int \frac{dx}{\sin x + \cos x + 1} = \int \frac{\frac{2dt}{1 + t^2}}{1 + \frac{2t}{1 + t^2} + \frac{1 - t^2}{1 + t^2}}$ $\frac{2dt}{1 + \frac{2t}{1 + t^2}}$	1/2	
		$= \int \frac{\frac{2dt}{1+t^2}}{\frac{1+t^2+2t+1-t^2}{1+t^2}}$		



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Que.	Sub.	Model Answers	Marks	Total
No. <b>5)</b>	Que.	1120 110 110 110 110	1,141118	Marks
3)		$= 2\int \frac{dt}{2+2t}$ $= \int \frac{dt}{1+t}$	1/2	
		$-J_{1+t}$ $= \log[1+t] + c$	1	
		$= \log \left[ 1 + \tan \frac{x}{2} \right] + c$	1	4
	e)	Evaluate $\int_{0}^{1} x (1-x)^{3/2} dx$		
	Ans.	$\int_{0}^{1} x (1-x)^{3/2} dx = \int_{0}^{1} (1-x) \left[ 1 - (1-x) \right]^{3/2} dx$	1/2	
		$= \int_{0}^{1} (1-x) x^{3/2} dx$	1/2	
		$= \int_{0}^{1} (x^{3/2} - x \cdot x^{3/2}) dx$ $= \int_{0}^{1} (x^{3/2} - x^{5/2}) dx$		
		$= \int_{0}^{1} \left( x^{3/2} - x^{5/2} \right) dx$	1/2	
		$= \left[\frac{x^{5/2}}{5/2} - \frac{x^{7/2}}{7/2}\right]_0^1$	1/2+1/2	
		$= \left[ \frac{1}{5/2} - \frac{1}{7/2} \right] - 0$	1	
		$=\frac{4}{35}$ or 0.114	1/2	4
	f)	Solve $\frac{dy}{dx} = -\frac{y\cos x + \sin y + y}{\sin x + x\cos y + x}$		
	Ans.	$\frac{dy}{dx} = -\frac{y\cos x + \sin y + y}{\sin x + x\cos y + x}$		
		$\therefore (y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$ $M = y\cos x + \sin y + y$		
		$\therefore \frac{\partial M}{\partial y} = \cos x + \cos y + 1$	1	

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Que. No.	Sub.	Model Answers	Marks	Total
6)	Que.			Marks
,		$N = \sin x + x \cos y + x$ $\therefore \frac{\partial N}{\partial x} = \cos x + \cos y + 1$ $\therefore \text{the equation is exact.}$	1/2 1/2	
6)		$\int_{y \text{ cons tan }t} Mdx + \int_{terms \text{ free from }x} Ndy = c$ $\int_{y \text{ cons }x + \sin y + y} dx + \int_{y \text{ odd }y = c} 0dy = c$ $\therefore_{y \text{ sin }x + x \text{ sin }y + xy = c}$	1 1	4
0)		Attempt any four.		
	a)	A coin is tossed and a die is rolled. Show that the events head and six are independent and mutually exclusive.		
	Ans.	Case I) consider the experiment of two events "A coin is tossed and a die is rolled" are taken together. $\therefore S = \begin{cases} (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), \\ (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \end{cases}$ $\therefore n = 12.$ Let A = event of occurring head, $\therefore A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6) \}$ $\therefore m = 6$		
		$\therefore P(A) = \frac{m}{n} = \frac{6}{12} = \frac{1}{2}$ Let B = event of occurring six $\therefore B = \{(H, 6), (T, 6)\}$	1/2	
		$\therefore m = 2$ $\therefore P(B) = \frac{m}{n} = \frac{2}{12} = \frac{1}{6}$ Now $A \cap B = \{(H, 6)\}$	1/2	
		∴ $m = 1$ The probability of happening head and six is		
		$P(A \cap B) = \frac{m}{n} = \frac{1}{12}$ But $P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ .		
		But $P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ . ∴ $P(A \cap B) = P(A)P(B)$ ∴ the events are independent.	1/2	

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks		
6)	2.00	But $P(A \cap B) \neq 0$ [or also $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ] $\therefore$ the events are not mutually exclusive.	1/2			
		Case II) Consider the experiment of two events "A coin is tossed and a die is rolled" are not taken together and done exclusively.  i. the set of tossing coin is { H, T}. Consequently n = 2. Now let A = event of occurring head, then m = 1 and hence $P(A) = \frac{m}{n} = \frac{1}{2}$ .  ii. the set of rolling of die is {1, 2, 3, 4, 5, 6}.  Consequently n = 6. Now let B = event of occurring six, then m = 1 in this case and hence $P(B) = \frac{m}{n} = \frac{1}{6}$ .  Now here in this case $A \cap B = \Phi$ and hence $P(A \cap B) = 0$ shows that the events are mutually exclusive but the events are not independent as: $P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \neq P(A \cap B).$				
	b)	If A and B are two events such that $P(A) = \frac{1}{2}$ , $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{7}{12}$ , find $P(A' \cap B')$				
	Ans.	$P(A' \cap B') = P(A \cup B)'$ $= 1 - P(A \cup B)$ $= 1 - [P(A) + P(B) - P(A \cap B)]$	1			
		$=1 - \left[ \frac{1}{2} + \frac{1}{3} - \frac{7}{12} \right]$ $= \frac{3}{4}  or  0.75$	1	4		
		OR				



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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	Que.			Widiks
		OR		
		$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	1	
		$=\frac{1}{2}+\frac{1}{3}-\frac{7}{12}$	1	
		$=\frac{1}{4} \ or \ 0.25$	1/2	
		$P(A' \cap B') = P(A \cup B)'$		
		$=1-P(A\cup B)$	1	
		$=1-\frac{1}{4}$ or $1-0.25$		
		$=\frac{3}{4}  or  0.75$	1/2	4
	c)	In a sample of 1000 cases the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find 1) how many students score between 12 and 15? and 2) how many students score above 18?		
	Ans.	Given $\bar{x} = 14$ $\sigma = 2.5$ $N = 1000$		
		1) $z = \frac{12-14}{2.5} = -0.8$ $z = \frac{15-14}{2.5} = 0.4$	1/2	
		$\therefore P(12 \le x \le 15) = P(-0.8 \le z \le 0.4)$		
		$= P(-0.8 \le z \le 0) + P(0 \le z \le 0.4)$		
		$= P(0 \le z \le 0.8) + P(0 \le z \le 0.4)$	1/2	
		Note: To get the further solution of the problem, the students are required the set of the values of the area under standard normal curves. And this set of values is not provided with this question. These values can only be obtained from the Table of Area Under Standard Normal Curve. This table is provided at the end of the solution for the sake of your convenience. Please go through the same. The further solution is formed using the values obtained from this table only.		
		$P(12 \le x \le 15) = 0.2881 + 0.1554$ $= 0.4435$	1/2	
		:. no. of students = $N \cdot P = 1000 \times 0.4435 = 443.5$ i.e., 444	1/2	



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Que.	Sub.			Total
No.	Que.	Model Answers	Marks	Marks
6)		$2) \ z = \frac{18 - 14}{2.5} = 1.6$	1/2	
		$\therefore P(18 \le x) = P(1.6 \le z)$	1/2	
		$= 0.5 - P(0 \le z \le 1.6)$	/2	
		=0.5-0.4452	1/	
		= 0.0548	1/ <sub>2</sub> 1/ <sub>2</sub>	4
		:. no. of students = $N \cdot P = 1000 \times 0.0548 = 54.8$ i.e., 55	, _	
		Note: If the students have adopted any other assumptions, due		
		credit may be given.		
	d)	Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2$		
		at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$ .		
	Ans.	$\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2$		
		$\therefore m \left(\frac{x}{a}\right)^{m-1} \cdot \frac{1}{a} + m \left(\frac{y}{b}\right)^{m-1} \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$	1	
		$\therefore$ at $(a, b)$ ,		
		$m\left(\frac{a}{a}\right)^{m-1} \cdot \frac{1}{a} + m\left(\frac{b}{b}\right)^{m-1} \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$	1/2	
		$\therefore m \cdot \frac{1}{a} + m \cdot \frac{1}{b} \cdot \frac{dy}{dx} = 0$	1/2	
		∴ slope of tangent at $(a, b) = \frac{dy}{dx} = -\frac{b}{a}$	1	
		∴ the equation of tangent is		
		$y - b = -\frac{b}{a}(x - a)$	1/2	
		$\therefore ay - ab = -bx + ab$		
		$\therefore bx + ay = 2ab$		
		$\therefore \frac{x}{a} + \frac{y}{b} = 2$	1/2	4
		a b		



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Que.	Sub.	Model Answers	Marks	Total
No.	Que.	WIOGGI / HISWCIS	IVIGINS	Marks
6)	e)	Divide 80 into two parts such that their product is maximum.		
	Ans.	Let x, y be the numbers. But $x + y = 80$ i. e., $y = 80 - x$		
		To maximize, $p = xy = x(80-x)$		
		$\therefore p = 80x - x^2$	1	
		$\therefore \frac{dp}{dx} = 80 - 2x$	1/2	
		$\therefore \frac{d^2 p}{dx^2} = -2$	1/2	
		For stationary values, $\frac{dp}{dx} = 0$		
		$\therefore 80 - 2x = 0  or  80 = 2x$	1/2	
		$\therefore x = 40$	1/2	
		$At \ x = 40, \ \frac{d^2p}{dx^2} = -2 < 0$	1/2	
		$\therefore At \ x = 40, \ p \ has \ max \ imum \ value.$	1/2	4
	f)	Two points $A(1, 4)$ and $B(9, 12)$ are on the parabola $y^2 = 16x$ . Show that the area enclosed between the chord AB and the parabola is $\frac{16}{3}$ .		
	Ans.	$\frac{x-1}{9-1} = \frac{y-4}{12-4}$ $\therefore x-1 = y-4$ $\therefore y = x+3$	1	
		Note: Students may use another form to find the equation of this line, such as slope point form.		
		$\therefore A = \int_a^b (y_2 - y_1) dx$		
		$=\int_{1}^{9} \left(4\sqrt{x}-x-3\right) dx$	1	
		$= \left[4 \cdot \frac{2}{3} x^{3/2} - \frac{x^2}{2} - 3x\right]_1^9$	1	
		$= \left[\frac{8}{3}(9)^{3/2} - \frac{9^2}{2} - 3(9)\right] - \left[\frac{8}{3} - \frac{1}{2} - 3\right]$	1/2	
		$=\frac{16}{3}$ or 5.333	1/2	4



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Que.	Sub.	25.114	3.6.1	Total
No.	Que.	Model Answers	Marks	Marks
No. <b>6)</b>	Que.	OR $\frac{x-1}{9-1} = \frac{y-4}{12-4}$ $\therefore x-1 = y-4$ $\therefore y = x+3$ $\therefore A = \int_{a}^{b} (y_2 - y_1) dx$ $= \left[\frac{x^2}{2} + 3x - 4 \cdot \frac{2}{3} x^{3/2}\right]_{1}^{9}$ $= \left[\frac{9^2}{2} + 3(9) - \frac{8}{3}(9)^{3/2}\right] - \left[\frac{1}{2} + 3 - \frac{8}{3}\right]$ $= -\frac{16}{3}  or  -5.333$ $\therefore Area  A = \frac{16}{3}  or  5.333$	1 1 1 ½	Marks 4
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.		

#### MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

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### Area Under Standard Normal Curve

0											
0.1         0.0398         0.0438         0.0478         0.0517         0.0557         0.0596         0.0636         0.0675         0.0714         0.0753           0.2         0.0793         0.0832         0.0971         0.091         0.0948         0.0987         0.1026         0.1064         0.1103         0.1141           0.3         0.1179         0.1217         0.1255         0.1293         0.1331         0.1368         0.1406         0.1443         0.148         0.1517           0.4         8.1554         9.1591         0.1628         8.1664         8.17         0.1736         0.1772         0.1808         0.1844         0.1879           0.5         0.1915         0.195         0.1985         0.2019         0.2054         0.2088         0.2123         0.2157         0.219         0.2224           0.6         0.2257         0.2291         0.2324         0.2367         0.2389         0.2422         0.2454         0.2486         0.2517         0.2549           0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2852           0.8         0.2881         0.291         0		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.2         0.0793         0.0832         0.0871         0.091         0.0948         0.0987         0.1026         0.1064         0.1103         0.1141           0.3         0.1179         0.1217         0.1255         0.1293         0.1331         0.1368         0.1406         0.1443         0.148         0.1517           0.4         8.1654         8.1691         0.1628         8.1664         9.17         0.1736         0.1772         0.1808         0.1844         0.1879           0.5         0.1915         0.195         0.1985         0.2019         0.2054         0.2088         0.2123         0.2157         0.219         0.2224           0.6         0.2257         0.2291         0.2324         0.2357         0.2389         0.2422         0.2454         0.2486         0.2517         0.2549           0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2852           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3079         0.3106         0.3313           0.9         0.3133         0.3433         0	0	0	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.3         0.1179         0.1217         0.1265         0.1293         0.1331         0.1368         0.1406         0.1443         0.1484         0.1517           0.4         8.1664         9.1664         9.1736         0.1772         0.1808         0.1844         0.1879           0.5         0.1915         0.1995         0.2019         0.2054         0.2088         0.2123         0.2157         0.219         0.2244           0.6         0.2257         0.2291         0.2324         0.2367         0.2389         0.2422         0.2464         0.2794         0.2764         0.2794         0.2794         0.2794         0.2794         0.2794         0.2794         0.2794         0.2794         0.2794         0.2823         0.2852           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3378         0.3106         0.3133           0.9         0.3159         0.3186         0.3212         0.3238         0.3508         0.3531         0.3544         0.3577         0.3599         0.3621           1.1         0.34343         0.3666         0.3686         0.3708         0.3729         0.3749         0.377         0.379	0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.4         8.1654         9.1694         9.1694         9.1736         0.1772         0.1808         0.1844         0.1879           0.5         0.1915         0.1965         0.1985         0.2019         0.2054         0.2088         0.2123         0.2157         0.219         0.2224           0.6         0.2257         0.2291         0.2324         0.2357         0.2389         0.2422         0.2454         0.2496         0.2517         0.2549           0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2852           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3348         0.3106         0.3238         0.3264         0.3289         0.3315         0.334         0.3365         0.3389           1         0.3443         0.3461         0.3485         0.3508         0.3531         0.3554         0.3577         0.3599         0.3811           1.1         0.3843         0.3686         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383	0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.5         0.1915         0.195         0.1985         0.2019         0.2054         0.2088         0.2123         0.2157         0.219         0.2224           0.6         0.2257         0.2291         0.2324         0.2367         0.2389         0.2422         0.2454         0.2486         0.2517         0.2549           0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2652           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3078         0.3106         0.3133           0.9         0.3186         0.3212         0.3238         0.3624         0.3289         0.3315         0.334         0.3365         0.3389           1         0.3413         0.3438         0.3461         0.3486         0.3508         0.3503         0.3571         0.3579         0.3599         0.3621           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.3777         0.3599         0.3811           1.2         0.3849         0.3889         0.3888         0.3907         0	0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.6         0.2257         0.2291         0.2324         0.2367         0.2389         0.2422         0.2454         0.2486         0.2517         0.2549           0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2862           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3078         0.3106         0.3133           0.9         0.3159         0.3186         0.3212         0.3238         0.3264         0.3289         0.3315         0.334         0.3365         0.3389           1         0.3413         0.3488         0.3485         0.3508         0.3531         0.3554         0.3577         0.3599         0.3621           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3898         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4046         0.4066         0.40	0.4	0.1554	8.1591	0.1628	8.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.7         0.258         0.2611         0.2642         0.2673         0.2704         0.2734         0.2764         0.2794         0.2823         0.2823         0.2852           0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3078         0.3106         0.3133           0.9         0.3159         0.3186         0.3212         0.3288         0.3508         0.3531         0.3554         0.3577         0.3599         0.3661           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4261         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0	0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.8         0.2881         0.291         0.2939         0.2967         0.2995         0.3023         0.3051         0.3078         0.3106         0.3133           0.9         0.3159         0.3186         0.3212         0.3238         0.3264         0.3289         0.3315         0.334         0.3365         0.3389           1         0.3413         0.3438         0.3461         0.3485         0.3508         0.3531         0.3577         0.3599         0.3621           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4343         0.4463         0.4474         0.4484         0.4	0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.9         0.3159         0.3186         0.3212         0.3238         0.3264         0.3289         0.3315         0.334         0.3365         0.3389           1         0.3413         0.3438         0.3461         0.3485         0.3508         0.3531         0.3554         0.3577         0.3599         0.3621           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4463         0.4474         0.4	0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
1         0.3413         0.3438         0.3461         0.3485         0.3508         0.3531         0.3554         0.3577         0.3599         0.3621           1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4564         0.4573         0.	0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
1.1         0.3643         0.3665         0.3686         0.3708         0.3729         0.3749         0.377         0.379         0.381         0.383           1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4566         0.4664         0.4671         0.4678         0.4686         0.4693         0.4699         0.4706           1.9         0.4772         0.4778         0.4783	0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1.2         0.3849         0.3869         0.3888         0.3907         0.3925         0.3944         0.3962         0.398         0.3997         0.4015           1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4633           1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4693         0.4761         0.4767           2         0.4772         0.4778 <t< td=""><td>1</td><td>0.3413</td><td>0.3438</td><td>0.3461</td><td>0.3485</td><td>0.3508</td><td>0.3531</td><td>0.3554</td><td>0.3577</td><td>0.3599</td><td>0.3621</td></t<>	1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.3         0.4032         0.4049         0.4066         0.4082         0.4099         0.4115         0.4131         0.4147         0.4162         0.4177           1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4633           1.8         0.4641         0.4649         0.4666         0.4664         0.4671         0.4678         0.4686         0.4699         0.4761         0.4767           1.9         0.4713         0.4779         0.4726         0.4732         0.4738         0.4744         0.475         0.4761         0.4767           2.1         0.4821         0.4826         0.4833	1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.4         0.4192         0.4207         0.4222         0.4236         0.4251         0.4265         0.4279         0.4292         0.4306         0.4319           1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4533           1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4693         0.4699         0.4706           1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4788         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483 <td< td=""><td>1.2</td><td>0.3849</td><td>0.3869</td><td>0.3888</td><td>0.3907</td><td>0.3925</td><td>0.3944</td><td>0.3962</td><td>0.398</td><td>0.3997</td><td>0.4015</td></td<>	1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.5         0.4332         0.4345         0.4357         0.437         0.4382         0.4394         0.4406         0.4418         0.4429         0.4441           1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4633           1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4693         0.4699         0.4706           1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4756         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4871         0.4875         0.4878         0.4884         0.4884         0.4887         0.4896           2.2         0.4861         0.4868         0.4871 <td< td=""><td>1.3</td><td>0.4032</td><td>0.4049</td><td>0.4066</td><td>0.4082</td><td>0.4099</td><td>0.4115</td><td>0.4131</td><td>0.4147</td><td>0.4162</td><td>0.4177</td></td<>	1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.6         0.4452         0.4463         0.4474         0.4484         0.4495         0.4505         0.4515         0.4525         0.4535         0.4545           1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4633           1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4699         0.4706           1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4756         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.4854         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4897         0.489           2.3         0.4893         0.4896         0.4898         0.4901         0.4904 <td< td=""><td>1.4</td><td>0.4192</td><td>0.4207</td><td>0.4222</td><td>0.4236</td><td>0.4251</td><td>0.4265</td><td>0.4279</td><td>0.4292</td><td>0.4306</td><td>0.4319</td></td<>	1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.7         0.4554         0.4564         0.4573         0.4582         0.4591         0.4599         0.4608         0.4616         0.4625         0.4633           1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4693         0.4699         0.4706           1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4756         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4783         0.4793         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.4854         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4884         0.4887         0.489           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.4922         0.4925         0.4927         0.4929 <td< td=""><td>1.5</td><td>0.4332</td><td>0.4345</td><td>0.4357</td><td>0.437</td><td>0.4382</td><td>0.4394</td><td>0.4406</td><td>0.4418</td><td>0.4429</td><td>0.4441</td></td<>	1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.8         0.4641         0.4649         0.4656         0.4664         0.4671         0.4678         0.4686         0.4693         0.4699         0.4706           1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4756         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.485         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4897           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.4922         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4935           2.5         0.4938         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949 <td< td=""><td>1.6</td><td>0.4452</td><td>0.4463</td><td>0.4474</td><td>0.4484</td><td>0.4495</td><td>0.4505</td><td>0.4515</td><td>0.4525</td><td>0.4535</td><td>0.4545</td></td<>	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.9         0.4713         0.4719         0.4726         0.4732         0.4738         0.4744         0.475         0.4756         0.4761         0.4767           2         0.4772         0.4778         0.4783         0.4788         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.485         0.4854         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4887         0.489           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.4922         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
2         0.4772         0.4778         0.4783         0.4788         0.4793         0.4798         0.4803         0.4808         0.4812         0.4817           2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.485         0.4854         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4887         0.489           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4913         0.4916           2.4         0.4918         0.4922         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.4966         0.4962         0.4963         0.4964           2.7         0.4965         0.4966         0.4967         0.4968 <td< td=""><td>1.8</td><td>0.4641</td><td>0.4649</td><td>0.4656</td><td>0.4664</td><td>0.4671</td><td>0.4678</td><td>0.4686</td><td>0.4693</td><td>0.4699</td><td>0.4706</td></td<>	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
2.1         0.4821         0.4826         0.483         0.4834         0.4838         0.4842         0.4846         0.485         0.4854         0.4857           2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4887         0.489           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.492         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.4941         0.4943         0.4945         0.4946         0.4948         0.49491         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.496         0.4961         0.4962         0.4963         0.4964           2.7         0.4965         0.4967         0.4968         0.4969         0.497         0.4971         0.4972         0.4973         0.4973         0.4973         0.4974           2.8         0.4974         0.4975         0.4976         0.4977 <td< td=""><td>1.9</td><td>0.4713</td><td>0.4719</td><td>0.4726</td><td>0.4732</td><td>0.4738</td><td>0.4744</td><td>0.475</td><td>0.4756</td><td>0.4761</td><td>0.4767</td></td<>	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2.2         0.4861         0.4864         0.4868         0.4871         0.4875         0.4878         0.4881         0.4884         0.4887         0.4898           2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.492         0.4922         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.496         0.4961         0.4962         0.4963         0.4964           2.7         0.4965         0.4967         0.4968         0.4969         0.497         0.4971         0.4972         0.4973         0.4974           2.8         0.4974         0.4975         0.4986         0.4977         0.4978         0.4979         0.4979         0.4986         0.4986           2.9         0.4981         0.4982         0.4983         0.4984         0.4984         <	2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.3         0.4893         0.4896         0.4898         0.4901         0.4904         0.4906         0.4909         0.4911         0.4913         0.4916           2.4         0.4918         0.492         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.494         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.496         0.4961         0.4962         0.4963         0.4964           2.7         0.4965         0.4967         0.4968         0.4969         0.497         0.4971         0.4972         0.4973         0.4974           2.8         0.4974         0.4975         0.4976         0.4977         0.4977         0.4978         0.4979         0.4979         0.4986         0.4986           2.9         0.4981         0.4982         0.4983         0.4984         0.4984         0.4985         0.4985         0.4986         0.4986         0.4986	2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.4         0.4918         0.492         0.4922         0.4925         0.4927         0.4929         0.4931         0.4932         0.4934         0.4936           2.5         0.4938         0.494         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.496         0.4961         0.4962         0.4963         0.4964           2.7         0.4965         0.4967         0.4968         0.4969         0.497         0.4971         0.4972         0.4973         0.4974           2.8         0.4974         0.4975         0.4976         0.4977         0.4977         0.4978         0.4979         0.4979         0.4980         0.4981           2.9         0.4981         0.4982         0.4983         0.4984         0.4984         0.4985         0.4985         0.4986         0.4986         0.4986	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.5         0.4938         0.494         0.4941         0.4943         0.4945         0.4946         0.4948         0.4949         0.4951         0.4952           2.6         0.4953         0.4955         0.4956         0.4957         0.4959         0.496         0.4961         0.4962         0.4963         0.4964           2.7         0.4965         0.4966         0.4967         0.4968         0.4969         0.497         0.4971         0.4972         0.4973         0.4974           2.8         0.4974         0.4975         0.4976         0.4977         0.4977         0.4978         0.4979         0.4979         0.4980         0.4986           2.9         0.4981         0.4982         0.4983         0.4984         0.4984         0.4985         0.4985         0.4986         0.4986	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.6     0.4953     0.4955     0.4956     0.4957     0.4959     0.496     0.4961     0.4962     0.4963     0.4964       2.7     0.4965     0.4966     0.4967     0.4968     0.4969     0.497     0.4971     0.4972     0.4973     0.4974       2.8     0.4974     0.4975     0.4976     0.4977     0.4977     0.4978     0.4979     0.4979     0.498     0.4981       2.9     0.4981     0.4982     0.4983     0.4984     0.4984     0.4985     0.4985     0.4986     0.4986	2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.7     0.4965     0.4966     0.4967     0.4968     0.4969     0.497     0.4971     0.4972     0.4973     0.4974       2.8     0.4974     0.4975     0.4976     0.4977     0.4977     0.4978     0.4979     0.4979     0.498     0.4981       2.9     0.4981     0.4982     0.4983     0.4984     0.4984     0.4985     0.4985     0.4986     0.4986     0.4986	2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.8     0.4974     0.4975     0.4976     0.4977     0.4977     0.4978     0.4979     0.4979     0.4981       2.9     0.4981     0.4982     0.4983     0.4984     0.4984     0.4985     0.4985     0.4986     0.4986	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.9 0.4981 0.4982 0.4982 0.4983 0.4984 0.4984 0.4985 0.4985 0.4986 0.4986	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
3 0.4987 0.4987 0.4988 0.4988 0.4989 0.4989 0.4989 0.4989 0.499	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
	3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

e. g., The value of  $P(0 \le z \le 0.45)$  is 0.1736.