MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

WINTER- 16 EXAMINATION Model Answer

Subject Code:

17216

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q.	Sub	Answer	Marking
No.	Q. N.		Scheme
Q. 1		Attempt any <u>TEN</u> of the following:	20
	a)	If $(a-2bi)+(b-3ai)=5+2i$ find a and b	02
	Ans	$\left(a-2bi\right)+\left(b-3ai\right)=5+2i$	
		$\therefore a - 2bi + b - 3ai = 5 + 2i$	1/2
		$\therefore (a+b)+(-3a-2b)i=5+2i$	/2
		$\therefore a+b=5$	
		-3a-2b=2	
		$\therefore 2a + 2b = 10$	1/2
		-3a-2b=2	/2
		-a=12	1/2
		$\therefore a = -12$	
		b = 17	1/2
	b)	Express in the form $x + iy$, $\frac{\left(2 + i\right)^2}{2 + 3i}$, where $x, y \in R$ and $i = \sqrt{-1}$	02
	A a	$\left(2+i\right)^2$	
	Ans	$\frac{1}{2+3i}$	
		$=\frac{4+4i+i^2}{}$	1/2
		2 + 3i	/2



WINTER – 16 EXAMINATION

Model Answer

Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
1	b)	$= \frac{4+4i-1}{2+3i} = \frac{3+4i}{2+3i}$	
		$= \frac{3+4i}{2+3i} \times \frac{2-3i}{2-3i}$	1/2
		$=\frac{6-9i+8i-12i^2}{4-9i^2}$	1/
		$=\frac{6-9i+8i+12}{4+9}$	1/2
		4 + 9	
		$=\frac{18-i}{}$	1/2
		13	
		$= \frac{18}{13} - \frac{1}{13}i$	
		13 13	
	c)	If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$	02
	Ans	f(0) = 7	1/2
	Alls	f(2) = 16 - 4 + 7 = 19	1/2
		$f(2) = 10^{-4} + 7 = 15$ $f(0) + f(2) = 7 + 19$	
		= 26	1/2
		= 26	1/2
	d)	If $f(x) = 16^x + \log_2 x$, find the value of $f\left(\frac{1}{4}\right)$	02
	Ans	, · ·	
		$f(x) = 16^x + \log_2 x ,$	1
		$\therefore f\left(\frac{1}{4}\right) = \left(16\right)^{\frac{1}{4}} + \log_2\left(\frac{1}{4}\right)$	_
		$= 2 - \log_2 4$	
		$= 2 - \log_2 4$ $= 2 - \log_2 \left(2\right)^2$	
		$= 2 - 2 \log_2 2$	
		=2-2(1)	
		= 0	1
	e)	Evaluate $\lim_{x\to 1} \left(\frac{x^3-1}{x-1} \right)$	02
	Ans	$\lim_{x \to \infty} \frac{x^3 - 1}{x^3 - 1}$	
		$\lim_{x \to 1} {x-1}$	
	<u> </u>	Page No O	



WINTER - 16 EXAMINATION

Model Answer

Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
1	e)	$= \lim_{x \to \infty} \frac{(x-1)(x^2+x+1)}{x^2+x+1}$	1/2
		$x \to 1$ $x = 1$	1/2
		$= \lim_{x \to 1} \left(x^2 + x + 1 \right)$	1/2
		$=(1)^2+1+1$	1/2
		= 3	
	C)	Evaluate $\lim_{x\to 0} \frac{3\sin x + 4x}{7x - 2\tan x}$	02
	f)	$3 \sin x + 4x$	
	Ans	$\lim_{x \to 0} \frac{3 \sin x + 1x}{7 x - 2 \tan x}$	
		$\frac{3\sin x + 4x}{}$	1/2
		$= \lim_{x \to 0} \frac{x}{7x - 2 \tan x}$, -
		\overline{x}	1/
		$3\left(\lim_{x\to 0}\frac{\sin x}{x}\right) + \frac{4x}{x}$	1/2
		$= \frac{7x}{x} - 2\left(\lim_{x \to 0} \frac{\tan x}{x}\right)$	
			1/2
		$=\frac{3(1)+4}{7-2(1)}$	
		7 - 2 (1)	1/2
		= - 5	
	g)	Evaluate $\lim \frac{3^x - 2^x}{}$	02
	Ans	$x \to 0 \sin \pi x$	
		$\lim_{x \to 0} \frac{3^x - 2^x}{\sin \pi x}$	
		$(3^x - 1 - 2^x + 1)$	1/2
		$= \lim_{x \to 0} \frac{x}{\sin \pi x}$	/2
		$\lim_{x \to 0} \left(\frac{\left(3^x - 1\right) - \left(2^x - 1\right)}{x} \right)$	
			1/2
		$\left[\lim_{x\to 0}\frac{\sin \pi x}{\pi x}\right]\pi$	
		Page No.0	2 (2-



WINTER – 16 EXAMINATION

Model Answer

Q.	Sub	<u> </u>	Marking
No.	Q. N.	Answer	Scheme
1	g)	$= \frac{\lim_{x \to 0} \left(\frac{3^{x} - 1}{x} - \frac{2^{x} - 1}{x} \right)}{\left(\lim_{x \to 0} \frac{\sin \pi x}{\pi x} \right) \pi}$	
		$= \frac{\left(\lim_{x \to 0} \frac{3^{x} - 1}{x}\right) - \left(\lim_{x \to 0} \frac{2^{x} - 1}{x}\right)}{\left(\lim_{x \to 0} \frac{\sin \pi x}{\pi x}\right) \pi}$	1/2
		$= \frac{\log 3 - \log 2}{(1)\pi}$ $= \frac{1}{\pi} (\log 3 - \log 2) = \frac{1}{\pi} \log \left(\frac{3}{2}\right)$	1/2
	h)	If $y = e^{7x} \cos 7x$, find $\frac{dy}{dx}$	02
	Ans	$y = e^{7x} \cos 7x, \text{ find } dx$ $y = e^{7x} \cos 7x$	
		$\frac{dy}{dx} = e^{7x} \frac{d}{dx} \cos 7x + \cos 7x \frac{d}{dx} e^{7x}$	1
		$\therefore \frac{dy}{dx} = e^{7x} \left(-\sin 7x \right).7 + \cos 7x \ e^{7x}.7$ $\therefore \frac{dy}{dx} = 7e^{7x} \left(-\sin 7x + \cos 7x \right)$	1
	i)	If $y = \log(x \sin 2x)$, find $\frac{dy}{dx}$	02
	Ans	$y = \log(x\sin 2x)$	1/
		$\frac{dy}{dx} = \frac{1}{x \sin 2x} \frac{d}{dx} (x \sin 2x)$	1/2
		$\frac{dy}{dx} = \frac{1}{x \sin 2x} \left(x \cos 2x \cdot 2 + \sin 2x \cdot 1 \right)$	1
		$\frac{dy}{dx} = \frac{\left(2x\cos 2x + \sin 2x\right)}{x\sin 2x}$	1/2
	j)	Find $\frac{dy}{dx}$, if $x = 3 \sin 4\theta$, $y = 4 \cos 3\theta$	02
	Ans		
	<u> </u>	I .	



WINTER - 16 EXAMINATION

Model Answer

		Model Allower Subject code.	
Q. No.	Sub Q. N.	Answer	Marking Scheme
1	j)	$\frac{dx}{d\theta} = 12\cos 4\theta \text{and} \frac{dy}{d\theta} = -12\sin 3\theta$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-12\sin 3\theta}{12\cos 4\theta}$ $\frac{dy}{dx} = \frac{-\sin 3\theta}{\cos 4\theta}$	½+½ ½ ½ ½
	k)	Show that their exist the root of the equation $x^3 - 5x - 11 = 0$ between 2 and 3	02
	Ans	Let $f(x) = x^3 - 5x - 11$	1/2
		f(2) = -13 < 0 f(3) = 1 > 0	1/2
		∴ root lies between 2 and 3	1/2
			1/2
	I)	Solve the following equations by using Jacobi's method (only first iteration)	02
	A	4x - y + z = 4, x + 6y + 2z = 9, -x - 2y + 5z = 2	
	Ans	Initial approximations: $x_0 = y_0 = z_0 = 0$ $x = \frac{4 + y - z}{4}$ $y = \frac{9 - x - 2z}{6}$	
		$z = \frac{2+x+2y}{5}$	1/2
		x = 1 , $y = 1.5$, $z = 0.4$	1½
2		Attempt any <u>FOUR</u> of the following:	16
	a)	Express $(1+i)$ in polar form	04
	Ans	Let $z = 1 + i$ Re $(z) = 1$, Im $(z) = 1$	
		$r = z = \sqrt{1+1} = \sqrt{2}$	
		$\theta = \tan^{-1} \left(\frac{1}{1} \right) = \frac{\pi}{4}$	1½
			11/2
		Polar form, $z = r(\cos \theta + i \sin \theta)$	
		Page No. (



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	a)	$\therefore 1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$	1
	b)	Simplify using De-Moiver's theorem $\frac{\left(\cos 2\theta + j\sin 2\theta\right)^{\frac{3}{2}} \left(\cos \theta - j\sin \theta\right)^{\frac{3}{2}}}{\left(\cos 3\theta - j\sin 3\theta\right)^{2} \left(\cos 5\theta - j\sin 5\theta\right)^{\frac{2}{5}}}$	04
	Ans	$\frac{\left(\cos 2\theta + j\sin 2\theta\right)^{\frac{3}{2}} \left(\cos \theta - j\sin \theta\right)^{\frac{3}{2}}}{\left(\cos 3\theta - j\sin 3\theta\right)^{2} \left(\cos 5\theta - j\sin 5\theta\right)^{\frac{2}{5}}}$	
		$= \frac{\left(\cos\theta + j\sin\theta\right)^{3} \left(\cos\theta + j\sin\theta\right)^{-3}}{\left(\cos\theta + j\sin\theta\right)^{-6} \left(\cos\theta + j\sin\theta\right)^{-2}}$	2
			1
		$= (\cos \theta + j \sin \theta)^{3-3+6+2}$ $= (\cos \theta + j \sin \theta)^{8}$	1/2
		$= (\cos \theta + j \sin \theta)$ $= \cos 8\theta + j \sin 8\theta$	1/2
	c)	Use De-Moiver's theorem to solve $x^3 - 1 = 0$	04
	Ans	$x^3 - 1 = 0$	
		$\therefore x^3 = 1$ $Put x^3 = z$	
		$\therefore x = z^{\frac{1}{3}}$	
		$\therefore \ z = z$ $\therefore \ z = 1 + 0i$	
		Re(z) = 1, Im(z) = 0	
		$r = \left z \right = \sqrt{1+0} = 1$	
		$\theta = \tan^{-1} \left(\frac{0}{1} \right) = 0$	1/2
		$z = r(\cos\theta + i\sin\theta)$	1/2
		$z = 1(\cos 0 + i \sin 0)$	1/2
		In general polar form, $z = r(\cos(2\pi k + \theta) + i\sin(2\pi k + \theta))$ $z = 1(\cos 2\pi k + i\sin 2\pi k)$	
		$\frac{1}{z^{3}} = (\cos 2\pi k + i \sin 2\pi k)^{\frac{1}{3}}$	1/2
		$z = \left(\cos 2\pi k + i \sin 2\pi k\right)^{3}$ $z = \cos\left(\frac{2\pi k}{3}\right) + i \sin\left(\frac{2\pi k}{3}\right) ; k = 0, 1, 2$	1/2
		$\left(\frac{z-\cos\left(\frac{\pi}{3}\right)+t\sin\left(\frac{\pi}{3}\right)}{3}\right) = \frac{1}{3} + t\sin\left(\frac{\pi}{3}\right)$	



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c)	when $k = 0$ $z_1 = \cos 0 + i \sin 0 = 1$ when $k = 1$	1/2
		$z_2 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$	1/2
		when $k = 2$ $z_3 = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$	1/2
	d)	Separate into real and imaginary parts of $\cosh\left(\alpha+ieta ight)$	04
	Ans	$\cosh(\alpha + i\beta) = \cos(\alpha + i\beta)$	1/2
		$= \cos \alpha \cos i \beta - \sin \alpha \sin i \beta$ $= \cos \alpha \cosh \beta - i \sin \alpha \sinh \beta$	1
		$\therefore \text{ Real part} = \cos \alpha \cosh \beta \text{ and}$	2
		Imaginary part = $-\sin \alpha \sinh \beta$	1/2
			,,,
	e) Ans	If $f(x) = \frac{1}{1-x}$ show that $f[f(x)] = x$	04
		$f\left[f\left\{f\left(x\right)\right\}\right]$	
		$= f \left[f \left\{ \frac{1}{1-x} \right\} \right]$	1/2
		$= f \left[\frac{1}{1 - \frac{1}{1 - x}} \right]$	1/2
		$= f \left[\frac{1-x}{1-x-1} \right]$ $f \left[1-x \right]$	1
		$= f \left[\frac{1-x}{-x} \right]$	
		$=\frac{1}{1-\left(\frac{1-x}{-x}\right)}$	1/2
		$=\frac{1}{1+\frac{1-x}{x}}$	1/2



WINTER - 16 EXAMINATION

Model Answer

		L_	NAs di
Q. No.	Sub Q. N.	Answer	Marking Scheme
	- 1		
2	e)	$=\frac{x}{1}$	
		$\begin{vmatrix} 1 \\ = x \end{vmatrix}$	1
	f)	If $f(x) = x^2 - 3x + 4$, find x if $f(1-x) = f(2x+1)$	04
	Ans	$f(1-x) = (1-x)^2 - 3(1-x) + 4$	
		$= 1 - 2x + x^2 - 3 + 3x + 4$	
		$=x^2+x+2$	1
		f(2x+1)	
		$= (2x+1)^2 - 3(2x+1) + 4$	
		$= 4x^2 + 4x + 1 - 6x - 3 + 4$	
		$=4x^2-2x+2$	1
		Given $f(1-x) = f(2x+1)$	
		$\therefore x^2 + x + 2 = 4x^2 - 2x + 2$	1/2
		$\therefore -3x^2 + 3x = 0$	
		$\therefore 3x^2 - 3x = 0$	1/2
		3x(x-1)=0	
		$\therefore x = 0,1$	1/2+1/2
3		Attempt any <u>FOUR</u> of the following:	16
	a)	If $y = f(x) = \frac{2x-3}{3x-2}$ show that $x = f(y)$	04
	Ans	$f(y) = \frac{2y-3}{3y-2}$	1/2
		$2\left(\frac{x}{3x-2}\right)-3$	1
		$= \frac{2\left(\frac{2x-3}{3x-2}\right) - 3}{3\left(\frac{2x-3}{3x-2}\right) - 2}$	
		$\frac{2(2x-3)-3(3x-2)}{3x-2}$	
		$=\frac{3x-2}{3(2x-3)-2(3x-2)}$	1
		3x-2	
		$= \frac{4x - 6 - 9x + 6}{6x - 9 - 6x + 4}$	1/2
		6x - 9 - 6x + 4	
	1	Page No C	



WINTER – 16 EXAMINATION

Model Answer

Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
3	a)	-5x	1/2
		$=\frac{-5x}{-5}$	
		= x	1/2
	b)	Evaluate $\lim_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$	04
	Ans	$\lim_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$	
		$= \lim_{x \to 3} \frac{(x-3)(x+2)}{x^2(x-3)+1(x-3)}$	
		$= \lim_{x \to 3} \frac{(x-3)(x+2)}{(x^2+1)(x-3)}$	1
		$=\lim_{x\to 3}\frac{\left(x+2\right)}{\left(x^2+1\right)}$	1
		$=\frac{3+2}{(3)^2+1}$	1
		$=\frac{5}{}$	
		$=\frac{1}{10}$	
		$=\frac{1}{-}$	1
			1
	c)	Evaluate $\lim_{x \to 2} \left(\frac{x}{x-2} - \frac{4}{x^2 - 2x} \right)$	04
	Ans	$\lim_{x \to 2} \left(\frac{x}{x-2} - \frac{4}{x^2 - 2x} \right)$	
		$= \lim_{x \to 2} \left(\frac{x}{x-2} - \frac{4}{x(x-2)} \right)$	1/2
		$= \lim_{x \to 2} \left(\frac{x^2}{x(x-2)} - \frac{4}{x(x-2)} \right)$	1
		$= \lim_{x \to 2} \frac{x^2 - 4}{x(x - 2)}$	
		$= \lim_{x \to 2} \frac{\left(x-2\right)\left(x+2\right)}{x\left(x-2\right)}$	1/2
		$=\lim_{x\to 2}\frac{\left(x+2\right)}{x}$	1/2
	1	Page No.0	00/27



WINTER - 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	$=\frac{2+2}{2}$	1/2
		2 4	
		$=\frac{-}{2}$	
		= 2	1
	d)	Evaluate $\lim_{\theta \to 0} \frac{2 \sin \theta - \sin 2\theta}{\theta^3}$	04
	Ans		
		$\lim_{\theta \to 0} \frac{2\sin\theta - \sin 2\theta}{\theta^3}$	
		$=\lim_{\theta\to 0} \frac{2\sin\theta - 2\sin\theta\cos\theta}{\theta^3}$	1/2
		$=\lim_{\theta\to 0}\frac{2\sin\theta\left(1-\cos\theta\right)}{\theta^{3}}$	
		$= \lim_{\theta \to 0} \frac{2 \sin \theta 2 \sin^2 \left(\frac{\theta}{2}\right)}{\theta^3}$	1
		$=4\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\frac{\sin^2\left(\frac{\theta}{2}\right)}{\theta^2}$	1/2
		$= 4 \left(\lim_{\theta \to 0} \frac{\sin \theta}{\theta} \right) \left(\lim_{\theta \to 0} \frac{\sin \left(\frac{\theta}{2} \right)}{\frac{\theta}{2}} \cdot \frac{1}{2} \right)^{2}$	1
		$=4\left(1\right)\left(1.\frac{1}{2}\right)^{2}$	1/2
		$=\frac{4}{-}$	
		4 = 1	1/
			1/2
	e)	Evaluate $\lim_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$	04
	Ans		
	,	$= \lim_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$	1
		$= \lim_{x \to 0} \frac{3^{x} 2^{x} - 3^{x} - 2^{x} + 1}{x^{2}}$	
		$= \lim_{x \to 0} \frac{3^{x} (2^{x} - 1) - (2^{x} - 1)}{x^{2}}$	
		$x \to 0$ x^2	



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	e)	$= \lim_{x \to 0} \frac{(3^{x} - 1)(2^{x} - 1)}{x^{2}}$	1
		$= \left(\lim_{x \to 0} \frac{3^x - 1}{x}\right) \left(\lim_{x \to 0} \frac{2^x - 1}{x}\right)$	1
		= (log 3)(log 2)	1
	f)	Evaluate $f(x) = \log\left(\frac{1+x}{1-x}\right)$, prove that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$	04
	Ans	$f(a) + f(b) = \log\left(\frac{1+a}{1-a}\right) + \log\left(\frac{1+b}{1-b}\right)$	½+½ ½
		$= \log\left(\frac{1+a}{1-a} \cdot \frac{1+b}{1-b}\right)$ $\left(1+a+b+ab\right)$	1/2
		$= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$ $(a+b)$	
		$f\left(\frac{a+b}{1+ab}\right) = \log\left(\frac{1+\frac{a+b}{1+ab}}{1-\frac{a+b}{1+ab}}\right)$	1/2
		$= \log \left(\frac{\frac{1 + ab + a + b}{1 + ab}}{\frac{1 + ab - (a + b)}{1 + ab}} \right)$	1/2
		$= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$	1/2
		$\therefore f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$ OR	1/2
		$f(a) + f(b) = \log\left(\frac{1+a}{1-a}\right) + \log\left(\frac{1+b}{1-b}\right)$	1/2+1/2
		$= \log\left(\frac{1+a}{1-a} \cdot \frac{1+b}{1-b}\right)$	1/2
		$= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$	1/2
		$= \log \left(\frac{1 + ab + a + b}{1 + ab - (a + b)} \right)$	1/2



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	f)	$= \log \left \frac{1 + \left(\frac{a+b}{1+ab}\right)}{1 - \left(\frac{a+b}{1+ab}\right)} \right $ $= f\left(\frac{a+b}{1+ab}\right)$	1 1/2
4		Attempt any <u>FOUR</u> of the following:	16
	a)	Using first principle of derivative find derivative of $f(x) = \cos x$	04
	Ans	$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
			1
		$\frac{dy}{dx} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$	
		$\frac{dy}{dx} = \lim_{h \to 0} \frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$	1
		$\frac{dy}{dx} = -2\lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$ $\frac{dy}{dx} = -2\left(\lim_{h \to 0}\sin\left(\frac{2x+h}{2}\right)\right) \left \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{h} \cdot \frac{1}{2}\right $	1
		$\frac{dy}{dx} = -2\left(\sin x\right)\frac{1}{2}$	1/2
		$\frac{dy}{dy} = -\sin x$	1/2
		dx	
	b) Ans	If u and v are differentiable functions of x then prove that $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{dv}{dx}$ Given $y = uv$	04
		Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding	
		to increment δx in x .	1/2
		$y + \delta y = (u + \delta u)(v + \delta v)$ $y + \delta y = uv + u\delta v + v\delta u + \delta u\delta v$	1/2
		$\delta y = uv + u\delta v + v\delta u + \delta u\delta v - y$	
		Page No 12	



WINTER – 16 EXAMINATION

Model Answer

		Model Allswer	
Q. No.	Sub Q. N.	Answer	Marking Scheme
4	b)	$\delta y = uv + u\delta v + v\delta u + \delta u\delta v - uv$	
		$\delta y = u \delta v + v \delta u + \delta u \delta v$	1/2
		$\delta u, \delta v$ are very small.	
		$\therefore \delta u \delta v$ is negligible.	
		$\therefore \delta y = u \delta v + v \delta u$	
		$\therefore \frac{\delta y}{\delta x} = \frac{u \delta v + v \delta u}{\delta x}$	1/2
		$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = u \lim_{\delta x \to 0} \frac{\delta v}{\delta x} + v \lim_{\delta x \to 0} \frac{\delta u}{\delta x}$	1
		$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$	1
		$\int dx dx dx$	
		(oos x) du	
	c)	If $y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$ find $\frac{dy}{dx}$	04
	Ans	$y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$	
			1/
		$\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \frac{d}{dx} \left(\frac{\cos x}{1 + \sin x}\right)$	1/2
		$\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{\left(1 + \sin x\right)\left(-\sin x\right) - \cos x\left(0 + \cos x\right)}{\left(1 + \sin x\right)^2}\right)$	1
		$\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{-\sin x - \sin^2 x - \cos^2 x}{\left(1 + \sin x\right)^2}\right)$	1/2
		$\therefore \frac{dy}{dx} = \frac{-1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{\sin x + \sin^2 x + \cos^2 x}{\left(1 + \sin x\right)^2}\right)$	
		$\therefore \frac{dy}{dx} = \frac{-1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{\sin x + 1}{\left(1 + \sin x\right)^2}\right)$	
		$\therefore \frac{dy}{dx} = \frac{-1}{\frac{\left(1+\sin x\right)^2 + \cos^2 x}{\left(1+\sin x\right)^2}} \left(\frac{1}{1+\sin x}\right)$	1/2



WINTER – 16 EXAMINATION

Model Answer

Q.	Sub	Answer	Marking
No.	Q. N.	Allswei	Scheme
4	c)	$\therefore \frac{dy}{dx} = \frac{-1}{\frac{1+2\sin x + \sin^2 x + \cos^2 x}{1+\sin x}}$ $\therefore \frac{dy}{dx} = \frac{-(1+\sin x)}{1+2\sin x + 1}$ $\therefore \frac{dy}{dx} = \frac{-(1+\sin x)}{2+2\sin x}$	1/2
		$\therefore \frac{dy}{dx} = \frac{-\left(1 + \sin x\right)}{2\left(1 + \sin x\right)}$	1/2
		$dx = 2(1 + \sin x)$ $dy = \frac{-1}{2}$	1/2
		OR	
		$y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$	
		$y = \tan^{-1} \left(\frac{\sin \left(\frac{\pi}{2} - x \right)}{1 + \cos \left(\frac{\pi}{2} - x \right)} \right)$	1
		$y = \tan^{-1} \left(\frac{2 \sin \left(\frac{\pi}{4} - \frac{x}{2} \right) \cos \left(\frac{\pi}{4} - \frac{x}{2} \right)}{2 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2} \right)} \right)$	1
		$\therefore y = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right)$	1
		$\therefore y = \frac{\pi}{4} - \frac{x}{2}$	1/2
		$\therefore \frac{dy}{dx} = \frac{-1}{2}$ OR	1/2
		$y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$	
		$\therefore \tan y = \frac{\cos x}{1 + \sin x}$	1/2
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{\left(1 + \sin x\right)\left(-\sin x\right) - \cos x\left(0 + \cos x\right)}{\left(1 + \sin x\right)^2}$	1½
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{-\sin x - \sin^2 x - \cos^2 x}{\left(1 + \sin x\right)^2}$	1/2
	1	Page No.1	4/27



WINTER – 16 EXAMINATION

Model Answer

Subject Code:

17216

Q. No.	Sub Q. N.	Answer	Marking Scheme
4		$\therefore \sec^2 y \frac{dy}{dx} = \frac{-\left(\sin x + \sin^2 x + \cos^2 x\right)}{\left(1 + \sin x\right)^2}$ $\therefore \sec^2 y \frac{dy}{dx} = \frac{-\left(\sin x + 1\right)}{\left(1 + \sin x\right)^2}$ $\therefore \sec^2 y \frac{dy}{dx} = \frac{-1}{\left(1 + \sin x\right)^2}$	½ ½
		$\therefore \frac{dy}{dx} = \frac{-1}{\sec^2 y (1 + \sin x)}$	1/2
	d) Ans	If $4x + 3y = \log(4x - 3y)$ find $\frac{dy}{dx}$ $4x + 3y = \log(4x - 3y)$	04
		$\therefore 4 + 3 \frac{dy}{dx} = \frac{1}{4x - 3y} \frac{d}{dx} (4x - 3y)$	1/2
		$\therefore 4 + 3\frac{dy}{dx} = \frac{1}{4x - 3y} \left(4 - 3\frac{dy}{dx} \right)$	1
		$\therefore 4 + 3 \frac{dy}{dx} = \frac{4}{4x - 3y} - \frac{3}{4x - 3y} \frac{dy}{dx}$ $\therefore 3 \frac{dy}{dx} + \frac{3}{4x - 3y} \frac{dy}{dx} = \frac{4}{4x - 3y} - 4$	1/2
		$\therefore 3\left(1+\frac{1}{4x-3y}\right)\frac{dy}{dx} = 4\left(\frac{1}{4x-3y}-1\right)$	1/2
		$\therefore 3\left(\frac{4x-3y+1}{4x-3y}\right)\frac{dy}{dx} = 4\left(\frac{1-4x+3y}{4x-3y}\right)$	1/2
		$\therefore 3 \frac{dy}{dx} = 4 \left(\frac{1 - 4x + 3y}{4x - 3y} \right) \left(\frac{4x - 3y}{4x - 3y + 1} \right)$	1/2
		$\therefore \frac{dy}{dx} = \frac{4}{3} \left(\frac{1 - 4x + 3y}{4x - 3y + 1} \right)$	1/2
	e)	If $x^3 \cdot y^2 = (x + y)^5$ show that $\frac{dy}{dx} = \frac{y}{x}$	04
	Ans	$\log (x^3 y^2) = \log (x + y)^5$	04 ½
		$\log x^3 + \log y^2 = \log (x + y)^5$	



WINTER – 16 EXAMINATION

Model Answer

			ı
Q. No.	Sub Q. N.	Answer	Marking Scheme
4	e)	$3\log x + 2\log y = 5\log (x + y)$	1/2
		$3\frac{1}{x} + 2\frac{1}{y}\frac{dy}{dx} = 5\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$	1
		$\frac{3}{x} + \frac{2}{y} \frac{dy}{dx} = \frac{5}{x+y} + \left(\frac{5}{x+y}\right) \frac{dy}{dx}$	
		$\left(\frac{2}{y} - \frac{5}{x+y}\right) \frac{dy}{dx} = \frac{5}{x+y} - \frac{3}{x}$	1
		$\left(\frac{2x+2y-5y}{y(x+y)}\right)\frac{dy}{dx} = \frac{5x-3x-3y}{x(x+y)}$	
		$\left(\frac{2x-3y}{y}\right)\frac{dy}{dx} = \frac{2x-3y}{x}$	1/2
		$\therefore \frac{dy}{dx} = \frac{y}{x}$	1/2
	f)	If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	04
	Ans	$x = a(2\theta - \sin 2\theta), y = a(1 - \cos 2\theta)$	
		$\frac{dx}{d\theta} = a\left(2 - 2\cos 2\theta\right)$	
		$\therefore \frac{dx}{d\theta} = 2a\left(1 - \cos 2\theta\right)$	1
		$\therefore \frac{dy}{d\theta} = 2a\sin 2\theta$	1
		$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{dx}}$	
		$\therefore \frac{dy}{dx} = \frac{2a\sin 2\theta}{2a(1-\cos 2\theta)}$	1/2
		$\frac{dx}{dx} = \frac{2a(1-\cos 2\theta)}{\sin 2\theta}$	
		$dx = 1 - \cos 2\theta$	
		at $\theta = \frac{\pi}{4}$	
		$\therefore \frac{dy}{dx} = \frac{\sin 2\left(\frac{\pi}{4}\right)}{1 + \cos 2\left(\frac{\pi}{4}\right)}$	1/2
		$1-\cos 2\left(\frac{\pi}{4}\right)$	/2
<u> </u>	1		<u> </u>



WINTER – 16 EXAMINATION

Model Answer

		<u>Model Aliswei</u> Subject Code.	1/210
Q. No.	Sub Q. N.	Answer	Marking Scheme
4	f)	$\frac{dy}{dx} = \frac{\sin\left(\frac{\pi}{2}\right)}{1 - \cos\left(\frac{\pi}{2}\right)}$ $\frac{dy}{dx} = \frac{1}{1 - 0}$ $\frac{dy}{dx} = 1$ OR $x = a(2\theta - \sin 2\theta), y = a(1 - \cos 2\theta)$ dx	1
		$\frac{dx}{d\theta} = a \left(2 - 2 \cos 2\theta \right)$ $\therefore \frac{dx}{d\theta} = 2a \left(1 - \cos 2\theta \right)$ $\therefore \frac{dy}{d\theta} = 2a \sin 2\theta$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	1
		$\therefore \frac{dy}{dx} = \frac{2a\sin 2\theta}{2a(1-\cos 2\theta)}$ $\therefore \frac{dy}{dx} = \frac{\sin 2\theta}{1-\cos 2\theta}$ $\therefore \frac{dy}{dx} = \frac{2\sin \theta \cos \theta}{2\sin^2 \theta}$ $\therefore \frac{dy}{dx} = \frac{\cos \theta}{\sin \theta}$	<i>Y</i> ₂
		$\therefore \frac{dy}{dx} = \cot \theta$ at $\theta = \frac{\pi}{4}$	1/2
		$\therefore \frac{dy}{dx} = \cot \frac{\pi}{4}$ $\therefore \frac{dy}{dx} = 1$	½ ½
			,-



WINTER – 16 EXAMINATION

Model Answer

0	Sub		Marking
Q. No.	Q. N.	Answer	Marking Scheme
5		Attempt any FOUR of the following:	16
	a)	Evaluate $\lim_{x \to 0} \frac{\left(5^x - 1\right) \tan x}{\sqrt{x^2 + 16} - 4}$	04
	Ans	$\int_{x \to 0}^{1} \frac{(5^x - 1) \tan x}{\sqrt{x^2 + 16} - 4}$	
		$= \lim_{x \to 0} \frac{\left(5^{x} - 1\right) \tan x}{\sqrt{x^{2} + 16} - 4} \times \frac{\sqrt{x^{2} + 16} + 4}{\sqrt{x^{2} + 16} + 4}$	1
		$= \lim_{x \to 0} \frac{\left(5^x - 1\right) \tan x \left(\sqrt{x^2 + 16} + 4\right)}{x^2 + 16 - 16}$	
		$= \lim_{x \to 0} \frac{x^2 + 16 - 16}{(5^x - 1)\tan x \left(\sqrt{x^2 + 16} + 4\right)}$ $= \lim_{x \to 0} \frac{t}{x^2}$	1
		$= \left(lt \atop x \to 0 \atop x \to 0 \right) \left(lt \atop x \to 0 \atop x \to 0 \right) \left(lt \atop x \to 0 \atop x \to 0 \right) \left(lt \atop x \to 0 \atop x \to 0 \right) \left(lt \to 0 \right) \left(lt \atop x \to 0 \right) \left(lt \to 0$	
		$= (\log 5)(1)(\sqrt{16} + 4)$	1
		$= (\log 5)(1)(4+4)$	
		$= 8 \log 5$	1
	b)	Evaluate: $\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$	04
	Ans	$\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$	
		$Put x = 3 + h \text{ as } x \to 3, h \to 0$	
		$= \lim_{h \to 0} \frac{\log(3+h) - \log 3}{3+h-3}$	1
		$= lt \frac{\log\left(\frac{3+h}{3}\right)}{l}$	
		$= \lim_{h \to 0} \frac{1}{h} \log \left(1 + \frac{h}{3} \right)$	1
		$= \lim_{h \to 0} \log \left(1 + \frac{h}{3} \right)^{\frac{1}{h}}$	1/2
		$= \log \left[lt \left(1 + \frac{h}{3} \right)^{\frac{3}{h}} \right]^{\frac{1}{3}}$	1/2
		Page No. 1	



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer						
5	b)	$= \log e^{\frac{1}{3}}$ $= \frac{1}{3} \log e$ $= \frac{1}{-}$	1/2					
		3	1/2					
	c)	Using Bisection method find the approximate root of the equation $x^3 - 5x + 1 = 0$	04					
	Ans	(three iterations only)						
	7	$\operatorname{Let} f(x) = x^3 - 5x + 1$	1/2					
		f(2) = -1 < 0 f(3) = 13 > 0	1/2					
		$\therefore \text{ root lies in } (2,3)$	1/2					
		$x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$	1/2					
			1/2					
		$f(x_1) = 4.125 > 0$ the root lies in $(2, 2.5)$	/2					
		$x_2 = \frac{a + x_1}{2} = \frac{2 + 2.5}{2} = 2.25$	1/2					
		$f\left(x_{2}\right) = 1.14 > 0$	1/2					
		the root lies in $(2,2.25)$						
		$x_3 = \frac{a + x_2}{2} = \frac{2 + 2.25}{2} = 2.125$	1/2					
		O R						
		$Let f(x) = x^3 - 5x + 1$	1/					
		$f\left(2\right) = -1 < 0$	1/2					
		$f\left(3\right) = 13 > 0$	1/2					
		\therefore root lies in $(2,3)$	1/2					
		Iterations a b $x = \frac{a+b}{2}$ $f(x)$						
		I 2 3 2.5 4.125	1					
		II 2 2.5 2.25 1.14	1					
		III 2 2.25 2.125	1/2					
		Page No.1	0/27					



WINTER – 16 EXAMINATION

Model Answer

5 d) Find the root of $x^2 - x - 1 = 0$ by using Regula Falsi method up to third approximation. Ans $ \begin{aligned} &\text{Let } f(x) = x^2 - x - 1 \\ &f(1) = -1 < 0 \end{aligned} &f(2) = 1 > 0 \end{aligned} &\text{the root lies in } (1,2)$ $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5 \end{aligned} &f(x_1) = -0.25 < 0 \text{ the root lies in } (1.5, 2)$ $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6 \end{aligned} &f(x_2) = -0.04 < 0 \end{aligned} &\text{the root lies in } (1.6.2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615 $ OR $ \begin{aligned} &\text{Let } f(x) = x^2 - x - 1 \\ &f(1) = -1 < 0 \end{aligned} &f(2) = 1 > 0 \end{aligned} &\text{the root lies in } (1.2)$ $ \end{aligned} $ Iterations $ \begin{aligned} &\text{a} & \text{b} & f(a) & f(b) & x = \frac{af(b) - bf(a)}{f(b) - f(a)} & f(x) \\ &f(b) - f(a) & f(b) - f(a) \end{aligned} $ Iterations $ \end{aligned} \end{aligned} e) \end{aligned} Find the positive root of x^3 + x - 1 = 0 by Newton-Raphson method up to three iterations only \end{aligned} \end{aligned} \end{aligned} \end{aligned} \end{aligned} 1. Expressions only \end{aligned} \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 2. Pind the positive root of x^3 + x - 1 = 0 by Newton-Raphson method up to three iterations only \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 2. Pind the positive root of x^2 + x - 1 = 0 by Newton-Raphson method up to three iterations only \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 1. Let f(x) = x^2 + x - 1 \end{aligned} 2. The different continuation of the continuation of th$	Q.	Sub				Δ				Marking	
Ans Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ ∴ the root lies in (1.2) $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$ $f(x_1) = -0.25 < 0$ the root lies in (1.5, 2) $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_2) = -0.04 < 0$ the root lies is (1.6, 2) $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ ∴ the root lies in (1, 2) Iterations a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$ Iterations a b $f(a)$ $f(b)$ $f(b$	No.	Q. N.				А	nswer			Scheme	
$f(1) = -1 < 0$ $f(2) = 1 > 0$ $\therefore \text{ the root lies in } (1.2)$ $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$ $f(x_1) = -0.25 < 0$ $\text{the root lies in } (1.5, 2)$ $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_2) = -0.04 < 0$ $\text{the root lies in } (1.6, 2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR $\text{Let } f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ $\therefore \text{ the root lies in } (1.2)$ Iterations a b f(a) f(b) x = \frac{af(b) - bf(a)}{f(b) - f(a)} = f(x) $f(x)$ $f(x) = -0.04 < 0$ $f(x) = -0.04$	5	d)	Find the root of	$f x^2 - x -$	1 = 0	by using R	egula Fals	si method up to third ap	proximation.	04	
$f(2) = 1 > 0$ ∴ the root lies in $(1,2)$ $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$ $f(x_1) = -0.25 < 0$ the root lies in $(1.5, 2)$ $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_2) = -0.04 < 0$ the root lies in $(1.6.2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ 0 R $Let f(x) = x^2 - x - 1 f(1) = -1 < 0 f(2) = 1 > 0 ∴ the root lies in (1.2) Iterations a b f(a) f(b) x = \frac{af(b) - bf(a)}{f(b) - f(a)} f(x) f(b) = -1 < 0 f(b) = -1 $		Ans	$\operatorname{Let} f(x) = x^2 -$	$\operatorname{Let} f(x) = x^2 - x - 1$							
$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$ $f(x_1) = -0.25 < 0$ the root lies in $(1.5, 2)$ $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_2) = -0.04 < 0$ the root lies in $(1.6, 2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ \therefore the root lies in (1.2) $\frac{1}{11} = \frac{1}{1.5} = 1$			$f\left(1\right) = -1 < 0$							1/2	
$x_{1} = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$ $f(x_{1}) = -0.25 < 0$ the root lies in $(1.5, 2)$ $x_{2} = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_{2}) = -0.04 < 0$ the root lies in $(1.6, 2)$ $x_{3} = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR $Let f(x) = x^{2} - x - 1 f(1) = -1 < 0 f(2) = 1 > 0 \therefore \text{ the root lies in } (1, 2) \frac{1}{1} = \frac{1}{1} = \frac{1}{2} = \frac{1}{1} $			$f\left(2\right)=1>0$							1/2	
$x_{1} = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} - 1.5$ $f(x_{1}) = -0.25 < 0$ the root lies in (1.5, 2) $x_{2} = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_{2}) = -0.04 < 0$ the root lies in (1.6, 2) $x_{3} = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR $Let f(x) = x^{2} - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ $\therefore \text{ the root lies in (1.2)}$ Iterations a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$ $1 $: the root lies i	n (1, 2)						1/4	
the root lies in $(1.5, 2)$ $x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_3) = -0.04 < 0$ the root lies in $(1.6, 2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ \therefore the root lies in $(1, 2)$ Iterations a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$ I			$x_{1} = \frac{af(b) - bf}{f(b) - f}$	$\frac{\left(a\right)}{\left(a\right)} = \frac{1}{1}$	1) - 2	$\frac{(-1)}{1} = 1.5$					
$x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$ $f(x_2) = -0.04 < 0$ the root lies in $(1.6, 2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR $Let f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ $\therefore \text{ the root lies in } (1.2)$ Iterations a b f(a) f(b) $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ f(x) In the root lies in (1.2) Iterations a b f(a) f(b) f(b) f(c) f(c) In the root lies in (1.2) In the root lies in (1.5) In the root lies in (1.6) In the root lie			$f\left(x_{1}\right) = -0.25 <$	< 0						1/2	
$f\left(x_{2}\right)=-0.04<0$ the root lies in $(1.6,2)$ $x_{3}=\frac{1.6(1)-2(-0.04)}{1+0.04}=1.615$ OR $Let f\left(x\right)=x^{2}-x-1 f\left(1\right)=-1<0 f\left(2\right)=1>0 \therefore \text{ the root lies in } (1.2) Iterations a b f\left(a\right) f\left(b\right) x=\frac{af\left(b\right)-bf\left(a\right)}{f\left(b\right)-f\left(a\right)} f\left(x\right) I $			the root lies in	(1.5,2)							
the root lies in $(1.6, 2)$ $x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ \therefore the root lies in (1.2) Iterations a b f(a) f(b) $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ f(x) II 1.5 2 -0.25 1 1.6 -0.04 III 1.6 2 -0.04 1 1.615 Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			$x_2 = \frac{1.5(1) - 2(1)}{1 + 0.2}$	$\frac{-0.25}{25}$ =	: 1.6					1/2	
$x_{s} = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$ OR Let $f(x) = x^{2} - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ \therefore the root lies in $(1, 2)$ The root lies in $(1, 2)$ Iterations a b f(a) f(b) $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ f(x) I I I I I I I I I I I I I			$f\left(x_{2}\right) = -0.04$	< 0						1/2	
Proof of the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ Iterations a b f(a) f(b) $f(b)$			the root lies in	(1.6,2)							
Let $f(x) = x^2 - x - 1$ $f(1) = -1 < 0$ $f(2) = 1 > 0$ \therefore the root lies in $(1, 2)$ Iterations a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$ I 1 2 -1 1 1.5 -0.25 II 1.5 2 -0.25 1 1.6 -0.04 III 1.6 2 -0.04 1 1.615 e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			$x_3 = \frac{1.6(1) - 2(1)}{1 + 0.0}$	$\frac{-0.04}{04} =$	1.61	5				1/2	
$f(1) = -1 < 0$ $f(2) = 1 > 0$ $\therefore \text{ the root lies in } (1,2)$ $Iterations a b f(a) f(b) x = \frac{af(b) - bf(a)}{f(b) - f(a)} f(x)$ $I 1 2 -1 1 1.5 -0.25$ $II 1.5 2 -0.25 1 1.6 -0.04$ $III 1.6 2 -0.04 1 1.615 $ $Find the positive root of x^3 + x - 1 = 0 by Newton-Raphson method up to three iterations only Let f(x) = x^3 + x - 1 f(0) = -1 < 0 f(1) = 1 > 0 2x = x^3 + x - 1 2x = x^4 + x - 1 2x = x = x + x - 1 2x = x + x + x - 1 2x = x + x + x + x + x + x + x + x + x + $			O R								
## Iterations a b $f(a)$ $f(b)$ $f(a)$ $f(b)$ $f(a)$ $f(a$				<i>x</i> – 1						1/2	
## Comparison of the root lies in (1,2) Iterations a b f(a) f(b) $x = \frac{af(b) - bf(a)}{f(b) - f(a)} f(x)$ I 1 2 -1 1 1.5 -0.25 II 1.5 2 -0.25 1 1.6 -0.04 III 1.6 2 -0.04 1 1.615 Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$ $f(1$										1/2	
Iterations a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$ I 1 2 -1 1 1.5 -0.25 II 1.5 2 -0.25 1 1.6 -0.04 III 1.6 2 -0.04 1 1.615 Prind the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$ Ye Ans Let $f(x) = x^3 + x - 1$											
e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			∴ the root lies i	n (1, 2)						/2	
e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			Iterations	a	b	f(a)	f(b)	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(x)		
e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			I	1	2	- 1	1	1.5	- 0.25	1	
e) Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$			II	1.5	2	- 0.25	1	1.6	- 0.04	1	
Ans iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$ 22			III	1.6	2	- 0.04	1	1.615		1/2	
Ans iterations only Let $f(x) = x^3 + x - 1$ $f(0) = -1 < 0$ $f(1) = 1 > 0$ 22		0)	Find the positive	ve root o	f x ³ -	+ x - 1 = 0 b	v Newton		o three		
f(0) = -1 < 0 $f(1) = 1 > 0$ 2			iterations only				,			04	
f(0) = -1 < 0 $f(1) = 1 > 0$ $2 = 2$				x-1						1/2	
f'(x) = 3x + 1											
/2			$f'(x) = 3x^2 + 1$							1/2	



WINTER – 16 EXAMINATION

Model Answer

	- ·		
Q. No.	Sub Q. N.	Answer	Marking Scheme
INU.	Q. N.		Scheine
5	e)	Initial root $x_0 = 1$	
		$\therefore f'(1) = 4$	
		$x_1 = x_0 - \frac{f(x_0)}{f(x_0)} = 1 - \frac{f(1)}{f(1)} = 0.75$	1
		$x_2 = 0.75 - \frac{f(0.75)}{f(0.75)} = 0.686$	1
		$x_2 = 0.686 - \frac{f(0.686)}{f(0.686)} = 0.682$	1/2
		OR	
		$Let f(x) = x^3 + x - 1$	
		$f\left(0\right) = -1 < 0$	1/2
		$f\left(1\right)=1>0$	1/2
		$f'(x) = 3x^2 + 1$	1/2
		Initial root $x_0 = 1$	'-
		$x_{i} = x - \frac{f(x)}{f(x)}$	
		$x_i = x - \frac{x^3 + x - 1}{3x^2 + 1}$	
		$x_{i} = \frac{3x^{3} + x - x^{3} - x + 1}{3x^{2} + 1}$	
		$x_{i} = \frac{2x^{3} + 1}{3x^{2} + 1}$	1
		$\therefore f'(1) = 4$	
		$x_1 = 0.75$	1/2
		$x_2 = 0.686$	1/2
		$x_3 = 0.682$	1/2
	f)	Use Newton-Raphson method to find $\sqrt[3]{20}$ correct to three decimal places.	04
	٨٥٥	(third iteration)	
	Ans	Let $x = \sqrt[3]{20}$	
		$\therefore x^3 = 20$	
		$\therefore x^3 - 20 = 0$ $\therefore f(x) = x^3 - 20$ $f(2) = -12 < 0$	
		$\therefore f(x) = x - 20$	
		f(2) = -12 < 0	1/2
L	1	Dago No. 1	_1



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	f)	f(3) = 7 > 0	1/2
		$f'(x) = 3x^2$	1/2
		Initial root $x_0 = 3$	
		$\therefore f'(3) = 27$	
		$x_1 = x_0 - \frac{f(x_0)}{f(x_0)} = 3 - \frac{f(3)}{f(3)} = 2.741$	1
		$x_2 = 2.74 - \frac{f(2.741)}{f(2.741)} = 2.715$	1
		$x_2 = 2.71 - \frac{f(2.715)}{f(2.715)} = 2.714$	1/2
		O R	
		$Let x = \sqrt[3]{20}$	
		$\therefore x^3 = 20$	
		$\therefore x^3 - 20 = 0$	
		$\therefore f(x) = x^3 - 20$	1/2
		f(2) = -12 < 0 f(3) = 7 > 0	1/2
		$f'(x) = 3x^2$	1/2
		Initial root $x_0 = 3$	/2
		$\therefore f'(3) = 27$	
		$x_i = x - \frac{f(x)}{f(x)} = x - \frac{x^3 - 20}{3x^2}$	
		$=\frac{3x^3-x^3+20}{3x^2}$	
		$=\frac{2x^{3}+20}{3x^{2}}$	1
		$3x^2$ $x_1 = 2.741$	1/2
		$x_1 = 2.741$ $x_2 = 2.715$	
		$x_3 = 2.714$	1/2
			1/2
6		Attempt any <u>FOUR</u> of the following:	16
	a)	If $y = (x + \sqrt{x^2 + 1})^m$ show that $(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$	04



WINTER – 16 EXAMINATION

Model Answer

		L_	
Q. No.	Sub Q. N.	Answer	Marking Scheme
6	Ans	$y = \left(x + \sqrt{x^2 + 1}\right)^m$	
		$\therefore \frac{dy}{dx} = m\left(x + \sqrt{x^2 + 1}\right)^{m-1} \frac{d}{dx}\left(x + \sqrt{x^2 + 1}\right)$	1/2
		$\therefore \frac{dy}{dx} = m\left(x + \sqrt{x^2 + 1}\right)^{m-1} \left(1 + \frac{1}{2\sqrt{x^2 + 1}} 2x\right)$	1/2
		$\therefore \frac{dy}{dx} = m\left(x + \sqrt{x^2 + 1}\right)^{m-1} \left(1 + \frac{x}{\sqrt{x^2 + 1}}\right)$	
		$\therefore \frac{dy}{dx} = m\left(x + \sqrt{x^2 + 1}\right)^{m-1} \left(\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}\right)$	1
		$\therefore \frac{dy}{dx} = m\left(x + \sqrt{x^2 + 1}\right)^m \frac{1}{\sqrt{x^2 + 1}}$	1
		$\therefore \frac{dy}{dx} = my \frac{1}{\sqrt{x^2 + 1}}$	
		$\therefore \sqrt{x^2 + 1} \frac{dy}{dx} = my$	
		$\therefore \left(x^2 + 1\right) \left(\frac{dy}{dx}\right)^2 = m^2 y^2$	1/2
		$\therefore \left(x^2 + 1\right) 2 \frac{dy}{dx} \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 2x = 2m^2y \frac{dy}{dx}$	1
		$\therefore \left(x^2 + 1\right) \frac{d^2 y}{dx^2} + \frac{dy}{dx} x = m^2 y$	1/2
		$\therefore \left(x^2 + 1\right) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$	
	b)	If $x = 2\cos t - \cos 2t$, $y = 2\sin t - \sin 2t$ find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$	04
	Ans	$x = 2\cos t - \cos 2t, y = 2\sin t - \sin 2t$	
		$\frac{dx}{dt} = -2\sin t + 2\sin 2t$	1/2
		$\frac{dy}{dt} = 2\cos t - 2\cos 2t$	1/2
		$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dt}{dx}}$	
		$\frac{dx}{dt}$	
	1	Page No 2	



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	b)	$\therefore \frac{dy}{dx} = \frac{2\cos t - 2\cos 2t}{-2\sin t + 2\sin 2t}$ $\therefore \frac{dy}{dx} = \frac{\cos t - \cos 2t}{-\sin t + \sin 2t}$	1/2
		$\therefore \frac{d^2 y}{dx^2} = \frac{\left(-\sin t + \sin 2t\right)\left(-\sin t + 2\sin 2t\right) - \left(\cos t - \cos 2t\right)\left(-\cos t + 2\cos 2t\right)}{\left(-\sin t + \sin 2t\right)^2} \frac{dt}{dx}$	1
		$ \frac{d^2 y}{dx^2} = \frac{\left(-\sin t + \sin 2t\right)\left(-\sin t + 2\sin 2t\right) - \left(\cos t - \cos 2t\right)\left(-\cos t + 2\cos 2t\right)}{\left(-\sin t + \sin 2t\right)^2} \frac{1}{\left(-2\sin t + 2\sin 2t\right)} $ $ \frac{d^2 y}{dx^2} = \frac{\left(-\sin t + \sin 2t\right)\left(-\sin t + 2\sin 2t\right) - \left(\cos t - \cos 2t\right)\left(-\cos t + 2\cos 2t\right)}{\left(-\cos t + 2\cos 2t\right)} $	
		$\therefore \frac{d^2 y}{dx^2} = \frac{\left(-\sin t + \sin 2t\right)\left(-\sin t + 2\sin 2t\right) - \left(\cos t - \cos 2t\right)\left(-\cos t + 2\cos 2t\right)}{2\left(-\sin t + \sin 2t\right)^3}$ $\therefore \frac{d^2 y}{dx^2} = \frac{\sin^2 t + 2\sin^2 2t - 3\sin t \sin 2t + \cos^2 t + 2\cos^2 2t - 3\cos t \cos 2t}{2\left(-\sin t + \sin 2t\right)^3}$	1/2
		$at t = \frac{\pi}{2}$	
		$\therefore \frac{d^2 y}{dx^2} = \frac{\sin^2\left(\frac{\pi}{2}\right) + 2\sin^22\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)\sin2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right) + 2\cos^22\left(\frac{\pi}{2}\right) - 3\cos\left(\frac{\pi}{2}\right)\cos2\left(\frac{\pi}{2}\right)}{2\left(-\sin\left(\frac{\pi}{2}\right) + \sin2\left(\frac{\pi}{2}\right)\right)^3}$	
		$\therefore \frac{d^2 y}{dx^2} = \frac{\left(1\right)^2 + 0 - 0 + 0 + 2\left(-1\right)^2 - 0}{2\left(-1 + 0\right)^3}$ $\therefore \frac{d^2 y}{dx^2} = \frac{3}{2\left(-1\right)}$	
		$\therefore \frac{d^2 y}{dx^2} = \frac{3}{-2}$ $\therefore \frac{d^2 y}{dx^2} = \frac{3}{-2}$	1
	c)	Solve the following equations by Gauss elimination method $x + 2y + 3z = 14$, $3x + 3y + 5z = 24$, $4x + 5y + 7z = 35$	04
	Ans	x + 2y + 3z = 14 $3x + 3y + 5z = 24$ $4x + 5y + 7z = 35$	
		3x + 6y + 9z = 42 $4x + 8y + 12z = 563x + 3y + 5z = 24$ and $4x + 5y + 7z = 35$	
		3y + 4z = 18 $3y + 5z = 21$	1/2 +1/2



WINTER – 16 EXAMINATION

Model Answer

Г			T
Q. No.	Sub Q. N.	Answer	Marking Scheme
6	c)	3y + 4z = 18	
		3y + 5z = 21	
		-z=-3	
		$\therefore z = 3$	1
		$\therefore x = 1$	1
		y = 2	±
		z = 3	1
		Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.	
	d)	Solve the following equations by Jacobi's method (take three iterations)	04
		5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20	
	Ans	$x = \frac{1}{5} (12 - 2y - z)$	
		$x = \frac{1}{5}(12 - 2y - z)$ $y = \frac{1}{4}(15 - x - 2z)$ $z = \frac{1}{5}(20 - x - 2y)$	
		$z = \frac{1}{2} (20 - x - 2y)$	1
		Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 2.4$	
		$y_1 = 3.75$	
		$z_1 = 4$	1
		$x_2 = 0.1$	
		$y_2 = 1.15$	
		$z_2 = 2.02$	1
		$x_3 = 1.536$	
		$y_3 = 2.715$	
		$z_3 = 3.52$	1
	<u></u> _		
		Page No. 3	



WINTER – 16 EXAMINATION

Model Answer

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	e)	Solve the equations by Gauss-Seidel method up to two iterations	04
		10x + 2y + z = 9, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$	
	Ans	$x = \frac{1}{10} (9 - 2 y - z)$	
		$y = \frac{1}{10} (-22 - x + z)$	
		$z = \frac{1}{6} (22 + 2x - 3y)$	1
		Starting with $y_0 = z_0 = 0$	
		$x_1 = 0.9$	
		$y_1 = -2.29$	1½
		$z_1 = 3.067$	
		$x_2 = 1.051$	
		$y_2 = -1.998$ $z_2 = 3.009$	1½
	f)	Solve the following equations by Jacobi's method (take three iterations)	04
	,	10x + y + z = 12, x + 10y + z = 12, x + y + 10z = 12	
	Ans	$x = \frac{1}{10} (12 - y - z)$	
	7 1113		
		$y = \frac{1}{10} (12 - x - z)$	
		$z = \frac{1}{10} (12 - x - y)$	1
		Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 1.2$	
		$y_1 = 1.2$	1
		$z_1 = 1.2$	
		$x_2 = 0.96$	1
		$y_2 = 0.96$	
		$z_2 = 0.96$	
		$x_3 = 1.008$	
		$y_3 = 1.008$ $z_3 = 1.008$	1
			<u> </u>



WINTER - 16 EXAMINATION

Q.	Sub	Answer	Marking
No.	Q. N.		Scheme
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	