

Winter - 2012 Examination

Model Answer Subject & Code: Applied Maths (12013) **Page No:** 1/20

Que. No.	Sub. Que.	Model answers	Marks	Total Marks	
1)	a)	$f(x) = 3x^4 - 2x^2 + \cos x$		IVICIINS	
	,	$f(-x) = 3(-x)^4 - 2(-x)^2 + \cos(-x)$			
		$=3x^4-2x^2+\cos x$	1/2 1/2		
		= f(x)	1/2		
		$\therefore f(x) \text{ is even.}$	1/2	2	
	b)	$f(x) = x^2 + 6x + 10$			
		$\therefore f(2) = 2^2 + 6(2) + 10 = 26$	1/2		
		$f(-2) = (-2)^2 + 6(-2) + 10 = 2$	1/2		
		$\therefore f(2) + f(-2) = 28$	1	2	
		OR			
		$f(2) + f(-2) = \left[2^2 + 6(2) + 10\right] + \left[(-2)^2 + 6(-2) + 10\right]$	1		
		= 28	1	2	
	c)	$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2}$	1/2		
			/2		
		$=\lim_{x\to 2}(x+2)$	1/2		
		=2+2	1/2	2	
		= 4	1/2		
	d)	$\lim_{x \to 0} \frac{\sin 4x}{3x} = \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{4}{3}$	1/2		
	,	$x \to 0$ $3x$ $x \to 0$ $4x$ 3			
		$=1\times\frac{4}{3}$	1		
		$=\frac{4}{3}$	1/2	2	
	e)	$\lim_{x \to \infty} \frac{6x^3 + 5x^2 - 1}{3x^3 + 4x^2 + 7} = \lim_{x \to \infty} \frac{\frac{6x^3 + 5x^2 - 1}{x^3}}{3x^3 + 4x^2 + 7}$			
		${x^3}$			
		$6 + \frac{5}{1} - \frac{1}{3}$			
		$= \lim_{x \to \infty} \frac{6 + \frac{5}{x} - \frac{1}{x^3}}{3 + \frac{4}{x^3} + \frac{7}{x^3}}$	1/2		
		$= \frac{6+0-0}{3+0+0}$	1		
		$-\frac{3+0+0}{3+0+0}$ $= 2$	1/2	2	



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	f)	$\lim_{x \to 0} \frac{a^x + b^x + c^x - 3}{a^2}$		
		$X \rightarrow 0$ X		
		$= \lim_{x \to 0} \left[\frac{a^x - 1}{x} + \frac{b^x - 1}{x} + \frac{c^x - 1}{x} \right]$	1/2	
		$= \log a + \log b + \log c$	1	
		$=\log abc$	1/2	2
	g)	$y = e^x \tan x$		
		$\frac{dy}{dx} = e^x \frac{d}{dx} (\tan x) + \tan x \frac{d}{dx} (e^x)$	1/2	
		$= e^x \sec^2 x + \tan x \cdot e^x$	1	2
		$=e^x\left(\sec^2x+\tan x\right)$	1/2	2
	h)	$y = \tan\left(4 - 3x\right)$		
	,	$\frac{dy}{dx} = \sec^2(4-3x) \cdot \frac{d}{dx}(4-3x)$	1	
		$\begin{vmatrix} ax & ax \\ = \sec^2(4-3x)\cdot(-3) \end{vmatrix}$	1/2	
		$=-3\sec^2(4-3x)$	1/2	2
	i)	$e^x + 1$		
	1)	$y = \frac{e^x + 1}{e^x - 1}$		
		$\int_{a}^{b} dy \left(e^{x}-1\right) \frac{d}{dx} \left(e^{x}+1\right) - \left(e^{x}+1\right) \frac{d}{dx} \left(e^{x}-1\right)$		
		$\therefore \frac{dx}{dx} = \frac{dx}{\left(e^x - 1\right)^2}$	1/2	
		$(e^{x}-1)e^{x}-(e^{x}+1)e^{x}$	1/2	
		$= \frac{(e^{x}-1)e^{x}-(e^{x}+1)e^{x}}{(e^{x}-1)^{2}}$		
		$=\frac{e^x\left[e^x-1-e^x-1\right]}{\left(e^x-1\right)^2}$	1/	
		$\left(e^{x}-1\right)^{2}$	1/2	
		$=\frac{-2e^x}{\left(e^x-1\right)^2}$	1/2	2
		$(e^{x}-1)$ OR	/2	
		$y = \frac{e^x + 1}{e^x - 1} = 1 + \frac{2}{e^x - 1}$		
			1	
		$\therefore \frac{dy}{dx} = 0 + \frac{-2}{\left(e^x - 1\right)^2} \cdot \frac{d}{dx} \left(e^x - 1\right)$	1	
		$=\frac{-2e^x}{\left(e^x-1\right)^2}$	1	2
		$\left(e^x-1\right)^2$		



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	j)	$y = \log(x^2 + 2x + 5)$		IVIdIKS
		$\frac{dy}{dx} = \frac{1}{x^2 + 2x + 5} \cdot \frac{d}{dx} \left(x^2 + 2x + 5 \right)$	1	
		$=\frac{2x+2}{x^2+2x+5}$	1	2
		$ OR $ $ y = \log(x^2 + 2x + 5) $		
		$e^{y} = x^{2} + 2x + 5$		
		$\therefore e^{y} \frac{dy}{dx} = 2x + 2$	1	
		$\therefore \frac{dy}{dx} = \frac{2x+2}{e^y}$	1	2
	k)	Rearranging the terms:		
		3, 4, 5, 6, 6, 7, 8, 8, 9, 11		
		$\therefore Median = \frac{6+7}{2} = 6.5$	1	
		Mode = 6 and 8	1	2
	1)		1	
		$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{500}{100} = 5$	1	2
2)	a)	$f(t) = \frac{2t+5}{3t-4}$	1/2	
		$= \frac{2\left(\frac{5+4x}{3x-2}\right)+5}{3\left(\frac{5+4x}{3x-2}\right)-4}$ $2(5+4x)+5(3x-2)$	1	
		$=\frac{3x-2}{3(5+4x)-4(3x-2)}$ $3x-2$		



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Que.	Sub.	Model answers	Marks	Total
No.	Que.		IVIAINS	Marks
2)		$=\frac{10x+8x+15x-10}{15+12x-12x+8}$	1	
		$=\frac{23x}{23}$	1 1/2	4
		=x	72	*
	b)	$f(x) = \log\left(\frac{1+x}{1-x}\right)$		
		$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$	1	
		$=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$	1	
		$=\log\left[\frac{\left(1+x\right)^2}{\left(1-x\right)^2}\right]$		
		$=\log\left(\frac{1+x}{1-x}\right)^2$	1/2	
		$=2\log\left(\frac{1+x}{1-x}\right)$	1	
		=2f(x)	1/2	4
	c)	$\lim_{x \to 4} \frac{x^4 - 64x}{\sqrt{x^2 + 9} - 5} = \lim_{x \to 4} \frac{x^4 - 64x}{\sqrt{x^2 + 9} - 5} \times \frac{\sqrt{x^2 + 9} + 5}{\sqrt{x^2 + 9} + 5}$		
		$= \lim_{x \to 4} \frac{\left(x^4 - 64x\right)}{x^2 + 9 - 25} \left(\sqrt{x^2 + 9} + 5\right)$	1	
		$= \lim_{x \to 4} \frac{x(x^3 - 64)}{x^2 - 16} \left(\sqrt{x^2 + 9} + 5\right)$		
		$= \lim_{x \to 4} \frac{x(x-4)(x^2+4x+16)}{(x-4)(x+4)} \left(\sqrt{x^2+9}+5\right)$	1	
		$= \lim_{x \to 4} \frac{x(x^2 + 4x + 16)}{(x+4)} (\sqrt{x^2 + 9} + 5)$	1	
		$= \frac{4(4^2 + 44 + 16)}{(4+4)} (\sqrt{4^2 + 9} + 5)$ $= 240$	1	4



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Que.	Sub.	Model answers	Marks	Total
No.	Que.			Marks
2)	d)	$\lim_{x \to 5} \left[\frac{\log x - \log 5}{x - 5} \right] \qquad Let \ x = 5 + h$ $as \ x \to 5, \ h \to 0$		
		$= \lim_{h \to 0} \left[\frac{\log(5+h) - \log 5}{5+h-5} \right]$	1	
		$=\lim_{h\to 0}\frac{1}{h}\log\left(\frac{5+h}{5}\right)$		
		$=\lim_{h\to 0}\log\left(1+\frac{h}{5}\right)^{1/h}$	1	
		$= \lim_{h \to 0} \log \left(1 + \frac{h}{5} \right)^{\frac{5}{h} \times \frac{1}{5}}$		
		$=\lim_{h\to 0}\frac{1}{5}\log\left(1+\frac{h}{5}\right)^{5/h}$		
		$=\frac{1}{5}\log e$	1	
		$=\frac{1}{5}$	1	4
	e)	$\lim_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2} = \lim_{x \to 0} \frac{3^x \cdot 2^x - 3^x - 2^x + 1}{x^2}$	1	
		$= \lim_{x \to 0} \frac{3^x (2^x - 1) - (2^x - 1)}{x^2}$		
		$= \lim_{x \to 0} \frac{(3^x - 1)(2^x - 1)}{x^2} \dots (*)$	1	
		$= \lim_{x \to 0} \frac{\left(3^x - 1\right)}{x} \times \frac{\left(2^x - 1\right)}{x}$	1	
		$= \log 3 \times \log 2$	1	4
		Note: Students may write the step (*) directly as we do in case of factors of polynomial. So this direct step may be considered for marks.		
	f)	$\lim_{x \to 0} \frac{\cos 5x - \cos 3x}{x^2} = \lim_{x \to 0} \frac{-2\sin\left(\frac{5x + 3x}{2}\right)\sin\left(\frac{5x - 3x}{2}\right)}{x^2}$		
	,	$=\lim_{x\to 0}\frac{-2\sin 4x\sin x}{x^2}$	1	
		$= \lim_{x \to 0} -2 \times \frac{\sin 4x}{4x} \times \frac{\sin x}{x} \times 4$	1	
		$= -2 \times 1 \times 1 \times 4$ $= -8$	1 1	4



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Que.	Sub.			Total
No.	Que.	Model answers	Marks	Marks
3)	a)	$x = a(\cos\theta + \theta\sin\theta)$		
		$\therefore \frac{dx}{d\theta} = a\left(-\sin\theta + \theta\cos\theta + \sin\theta\right)$		
		$=a\theta\cos\theta$	1	
		$y = a(\sin\theta - \theta\cos\theta)$		
		$\therefore \frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta)$	1	
		$= a\theta \sin \theta$ $dy /$	1	
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta \sin \theta}{a\theta \cos \theta}$	1	
		$= \tan \theta$	1	4
	b)	$y = (\sin x)\cos x$	1	
		$\therefore \frac{dy}{dx} = \sin x \frac{d}{dx} (\cos x) + \cos x \frac{d}{dx} (\sin x)$	1	
		$= \sin x (-\sin x) + \cos x (\cos x)$	1+1	
		$=\cos^2 x - \sin^2 x$	1	4
	,	$x^3 + y^3 = 3axy$		
		$\therefore 3x^2 + 3y^2 \frac{dy}{dx} = 3a \left[x \frac{dy}{dx} + y \right]$	1+1	
		$\therefore 3y^2 \frac{dy}{dx} - 3ax \frac{dy}{dx} = 3ay - 3x^2$		
		$\therefore (3y^2 - 3ax)\frac{dy}{dx} = 3ay - 3x^2$		
		$\therefore \frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}$	1	
		$=\frac{3(ay-x^2)}{3(y^2-ax)}$		
		$=\frac{ay-x^2}{y^2-ax}$	1	4
	d)	$Let \ u = \tan^{-1} \left(\frac{2x}{1 - x^2} \right)$		
		$Put x = \tan \theta \therefore \theta = \tan^{-1} x$		
		$\therefore u = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) = \tan^{-1} \left(\tan 2\theta \right)$		



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Que.	Sub.	Model answers	Marks	Total
No.	Que.		IVICIA	Marks
3)		$= 2\theta$ $= 2 \tan^{-1} x \qquad \dots \dots$	1	
		$\therefore \frac{du}{dx} = \frac{2}{1+x^2}$	1/2	
			/2	
		$Let \ v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$		
		$Put x = \tan \theta \therefore \theta = \tan^{-1} x$		
		$\therefore v = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}\left(\cos 2\theta\right)$		
		$=2\theta$		
		$= 2 \tan^{-1} x \qquad \dots \dots \dots \dots \dots (ii)$	1	
		$\therefore \frac{dv}{dx} = \frac{2}{1+x^2}$	1/2	
		$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}} = 1$	1	
		$\int_{OR} dx = 1 + x^2$		
		By(i) and (ii) , $u = v$	1	
		$\therefore \frac{du}{dv} = 1$	1	4
		$y = \tan^{-1} x$		
	e)	$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$	1	
		$\int_{0}^{\infty} dx = 1 + x^{2}$		
		$\therefore \left(1+x^2\right) \frac{dy}{dx} = 1$	1	
		$\therefore \left(1+x^2\right) \frac{d^2y}{dx^2} + \frac{dy}{dx} \left(2x\right) = 0$	1	
		$\therefore (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$	1	4
		$\therefore (1+x^2)y'' + 2xy' = 0$		_
		OR	1	
		$\therefore y' = \frac{1}{1+x^2}$	1	
		$y'' = \frac{-2x}{(1+x^2)^2}$	1	
		$(1+x^2)y'' + 2xy' = (1+x^2)\frac{-2x}{(1+x^2)^2} + 2x\frac{1}{1+x^2}$	1	
			1	4
		_ U	1	



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Que. No.	Sub. Que.	Model answers		Total Marks	
3)	Que.	$y = \tan^{-1} x$		11101110	
,					
		$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$	1		
		$d^2y -2x$			
		$\frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2}$	1		
		$\therefore (1+x^2) \frac{d^2 y}{dx^2} = -2x \cdot \frac{1}{1+x^2}$			
		$\therefore (1+x^2)\frac{d^2y}{dx^2} = -2x \cdot \frac{dy}{dx}$	1		
		$\therefore (1+x^2)\frac{d^2y}{dx^2} + 2x \cdot \frac{dy}{dx} = 0$			
		un un	1		
		$\therefore (1+x^2)y'' + 2xy' = 0$		4	
	f)	$y = \sin^{-1} \left[\frac{\cos x + \sin x}{\sqrt{2}} \right]$			
	1)				
		$=\sin^{-1}\left[\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right]$			
		$= \sin^{-1} \left[\sin \frac{\pi}{4} \cos x + \cos \frac{\pi}{4} \sin x \right]$			
			1		
		$=\sin^{-1}\left[\sin\left(\frac{\pi}{4}+x\right)\right]$			
			1		
		$=\frac{\pi}{4}+x$	1		
		$\frac{dy}{dy}$	1		
		$\therefore \frac{dy}{dx} = 1$	1	4	
4)	a)	Let $y = 2x^3 - 9x^2 + 12x + 5$		_	
")	(a)	$\therefore \frac{dy}{dx} = 6x^2 - 18x + 12$			
			1/2		
		$\therefore \frac{d^2y}{dx^2} = 12x - 18$	1/2		
		For extreme values, $\frac{dy}{dx} = 0$			
		$\therefore 6x^2 - 18x + 12 = 0$			
		$\therefore x = 1, 2$	1		
		At $x = 1$, $\frac{d^2y}{dx^2} = -6 < 0$	1		
		$\therefore \text{ y is maximum at } x = 1.$	1/2		
		∴ the maximum value is,	/2		
		$y = 2(1)^3 - 9(1)^2 + 12(1) + 5 = 10$	1/2		



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Que. Su No. Qu	Model answers	Marks	Total Marks	
)	At $x=1$, $\frac{d^2y}{dx^2} = 6 > 0$	1/2		
	dx^2 $\therefore y \text{ is minimum at } x = 2.$			
	∴ the minimum value is,			
	$y = 2(2)^{3} - 9(2)^{2} + 12(2) + 5 = 9$	1/2	4	
b)	$y = x^3$			
	$\int_{0}^{y-x} dy dy = 3x^{2}$	1		
	$\frac{1}{dx} = 3x$			
	$y = x^{3}$ $\therefore \frac{dy}{dx} = 3x^{2}$ $\therefore \frac{d^{2}y}{dx^{2}} = 6x$	1		
	Radius of curvature, $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$			
	$= \frac{\left[1 + \left(3x^2\right)^2\right]^{\frac{3}{2}}}{6x}$ At point (1,1),	1		
	$\rho = \frac{\left[1 + (3)^2\right]^{\frac{3}{2}}}{6} = \frac{(10)^{\frac{3}{2}}}{6} or 5.27$	1	4	
c)	Let $length = x$, $breadth = y$			
	$\therefore 2x + 2y = 100 \text{ or } x + y = 50$			
	$\therefore y = 50 - x$			
	Area, $A = xy$	1		
	$\therefore A = x(50 - x) = 50x - x^2$	1		
	$\therefore \frac{dA}{dx} = 50 - 2x$	1/2		
	$\therefore \frac{d^2A}{dx^2} = -2$	1/2		
	Now, $\frac{dA}{dx} = 0$ gives $x = 25$			
	$At \ x = 25, \frac{d^2A}{dx^2} = -2 < 0$			
	\therefore A is maximum at $x = 25$	1	4	
	\therefore Length = 25, breadth = 25	1	-	



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Que.	Sub.	Model answers	Marks	Total
No.	Que.		Iviains	Marks
4)	d)	$y = 3x - x^{2}$ $\therefore \frac{dy}{dx} = 3 - 2x$ From given slope = -5 $\therefore \frac{dy}{dx} = -5$ $\therefore 3 - 2x = -5$ $\therefore x = 4$ $y = -4$	1 1 1	
		$\therefore \text{ point is } (4,-4)$	1	4
	e)	Scale: x-oxis: 2 cm = 10 y-oxix: 2 cm = 5 Mode = 24 Mode = 24 Mode = 24 Marks distribution: 1 mark for plotting points and drawing histogram correctly. 1 mark for drawing the cross lines in the modal class. 1 mark for drawing line of mode to x-axis. 1 mark for value of mode. Note the value 24 is approximate value. Difference of +0.5 or -0.5 is acceptable in case of graph. (Note: If the graph is too small or not clear to understand, marks can be deducted. On x-axis, instead of writing points 10, 20, 30, etc., if class 0-10, 10-20, 20-30, etc. are written, no marks to be given.)	1+1+1+1	4



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Que.	Sub.	26.11	3.5.1	Total
No.	Que.	Model answers	Marks	Marks
4)	f)	Class Fi < c. f. 0-5 10 10 5-10 15 25 10-15 17 42 15-20 21 63 20-25 8 71 25-30 16 87	1	
		80 70 Scale:	1+1+1	4
		Marks distribution: 1 mark for plotting points and drawing curve correctly. 1 mark for drawing line of median to x-axis. 1 mark for value of median. Note the value 16 is approximate value. Difference of +0.5 or -0.5 is acceptable in case of graph. (Note: The median can also be calculated by drawing Greater than Ogive Curve and also by drawing both the ogives simultaneously. So marks to be given accordingly. If the graph is too small or not clear to understand, marks can be deducted. On x-axis, instead of writing points 0, 5, 10, 15, etc., if class 0-5, 5-10, 10-15, etc. are written, no marks to be given. The same is also applicable for histogram.)		

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Que.	Sub.			N	Model aı	nswers			Marks	Total
No. 5)	Que.	Given $\overline{x} = 40$,	n – 1	00						Marks
- /		$\sum x_i = n\overline{x} = 10$)					
					,				1	
	$\therefore Incorrect \sum_{i} x_i = 4000$									
		\therefore Correct $\sum x$	$x_i = In$	correct	$\sum x_i - 8$	3 + 53				
			=40	00 - 83	+53 = 39	970			1	
		∴ Correct med	_ C	orrect]	$\sum x_i = 3$	970 _ 2	0.70			_
		Correct med	ın = —	n	=	$\frac{1}{100} = 3$	9.70		2	4
	b)	Here $f_1 = 18$,	$f_m =$	25, f_2	=15				1	
		$Mode = L + \frac{1}{2j}$	f_m – .	$f_1 \sim I$	1				1	
		$\frac{1000e-L+}{2j}$	$f_m - \overline{f_1}$	$-f_2$	ı				1	
		$=25+\frac{1}{2}$	25 - 2(25) -	-18 -18-15	×5				1	
		= 27.058	, ,	10 10					1	
		- 27.030								4
		Note: The formula of mode is written in the various forms such								
		as <i>Mod</i>	e = L +	$+\frac{f_1}{2}$	$\frac{-f_0}{2} \times f_0$	h.				
		as $Mode = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$.								
	c)		xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$]		
			1	7	7	1	7	_		
			4	46	184	16	736	<u>-</u>		
			7	165	1155	49	8085			
			10	195	1950	100	19500			
			13	189	2457	169	31941			
			16	89	1424	256	22784		1+1	
			19	28	532	361	10108			
			22	19	418	484	9196	_		
			25 28	09	225 84	625 784	5625 2352	-		
			20	750	8436	704	110334			
				750	0430		110334			
		$\sum f_{ij}$	r ² ($\frac{1}{\sum f(x)}$	$\sqrt{2}$					
		$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N}} - \left(\frac{\sum f_i x_i}{N}\right)^2$								
								1		
	$=\sqrt{\frac{110334}{750} - \left(\frac{8436}{750}\right)^2}$								1	
	=4.538								1	4



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Que.	Sub.	Model answers							Marks	Total		
No.	Que.				1,100						1/10/11/0	Marks
5)		Г		C	1	<i>C</i> 1		12	C 12	\neg		
		_	xi	f_i	d_i	$f_i d_i$		d_i^2	$f_i d_i^2$			
		_	1	7	-4	-28		16	112			
		-	4	46	-3	-138		9	414			
			7	165	-2	330		4	660			
			10 13	195 189	-1 0	-195 0		1 0	195 0			
			16	89	1	89		1	89			
		_	19	28	2	56		$\frac{1}{4}$	112		1+1	
		_	22	19	3	57		9	171			
			25	09	4	36		16	144			
			28	03	5	15		25	75			
				750		-438			1972			
			1 0	, X:-	- A		-		<u> </u>			
		A=13	h=3,	$d_i = \frac{1}{h}$!							
				c 12 (\(\nabla\)	7 (1)	2						
		$S.D. = h \times$	<u> </u>	$\frac{f_i a_i^2}{2} - \frac{2}{3}$	$\int_{a}^{b} f_{i}d_{i}$							
			V .	N	N							
			107	2 (_138	$\sqrt{2}$							
		$=3\times$	$\sqrt{\frac{1772}{750}}$	$\frac{2}{1} - \frac{-436}{750}$							1	
			$A = 13 h = 3, d_i = \frac{x_i - A}{h}$ $S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$ $= 3 \times \sqrt{\frac{1972}{750} - \left(\frac{-438}{750}\right)^2}$								4	
		= 4.5	=4.538							1		
		N. C.	1 .	. 1		.1	1	C A	1 1	/1 1		
			Note: Students may take any another value for A in the above/ below example. So the above table and corresponding values vary									
				gly. But th						ics vary		
				<i>O</i>)								
	d)			Class	xi	f	c i	d_{i}	$f_i d_i$			
	(d)			900-920	91			-3	-12			
				920-940	930			-2	-22			
				940-960	950			<u>-</u> 1	-63			
				960-980	97			0	0			
				980-1000	99	0 73	3	1	73		2	
			1	1000-1020	101	.0 3	8	2	76			
			1	1020-1040	103	0 1	6	3	48			
			1	1040-1060	105	50 5	5	4	20			
						30	00		120			
		A = 970,							_ _			
			$\sum f_i$	d_i ,								
		$\therefore \overline{x} = A + \frac{\sum f_i d_i}{N} \times h$										
		$= 970 + \left(\frac{120}{300}\right) \times 20$							1			
		= 970	$+ \sqrt{\frac{1}{30}}$	$\frac{1}{0}$ $\times 20$							1	
		= 978		<i>-)</i>							1	4

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Que. No.	Sub.				Mo	del answ	vers		Marks	Total
5)	Que.									Marks
٥,			Class	xi	f_{i}	$f_i x_i$	$D_i = x_i - \overline{x} $	f_iD_i		
							' '			
		l —	10-15 15-20	12.5 17.5	7 12	87.5 210	15.3 10.3	107.1 123.6		
		l —	20-25	22.5	16	360	5.3	84.8		
		I —	25-30	27.5	25	687.5	0.3	7.5	1.1	
			30-35	32.5	19	617.5	4.7	89.3	1+1	
			35-40	37.5	15	562.5	9.7	145.5		
			40-45	42.5	6	255	14.7	88.2		
					100	2780		646		
		$\bar{x} = \frac{\sum_{N} M.D.}{M.D.}$	$\frac{f_i x_i}{f_i} = \frac{2}{f_i}$	$\frac{780}{} = 27$	7.8				1	
		l N	V 1	.00					1	
		M.D. =	$\sum f_i D_i$	_						
			N							
		=	646							
									1	4
		=	6.46							
	f)		σ	100 6	100	7.5			1	
	1)	f) $C.V.(A) = \frac{\sigma}{x} \times 100 = \frac{6}{80} \times 100 = 7.5$ $C.V.(B) = \frac{\sigma}{x} \times 100 = \frac{7.2}{60} \times 100 = 12$						1		
		CV(I	$\sigma_{\rm p}$	100 - 7	.2	1 – 12			1	
		C. V.(1	$\frac{x}{x}$	6	$\frac{-}{0}$	1-12				
		∴ C.V.((A) < C	V.(B)					1	
		∴ Grou	p B is n	nore vari	able.				1	
			_							4
6)		For C	Civil,	Electr	ical a	and M	echanical (Groups		
٠,								-		
	a)	$z_1 = -3$	$+4i, z_2$	= 5 - 3i						
		$z_1 = -3$	3+4i							
		$\frac{z_1}{z_2} = \frac{-3}{5}$	$\overline{5-3i}$							
		-3	3+4i 5	5+3i					1	
		$=$ $\frac{1}{5}$	$\frac{3+4i}{-3i} \times \frac{5}{5}$	$\overline{5+3i}$					1	
		-1	15 - 9i +	20i + 12i	\dot{t}^2					
			25 -	$\frac{20i+12i}{-9i^2}$	_					
		$=$ $\frac{-1}{}$	15+11 <i>i</i> -	-12					1	
			25 + 9							
		$=\frac{-2}{-2}$	27 + 11i						1	
			34						1	
		$=\frac{-2}{}$	$\frac{27}{1} + \frac{11}{21}i$	į					1	4
		34	4 34						_	



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Que.	Sub.	Model answers	Marks	Total
No.	Que.			Marks
6)	b) i)	$LHS = \sin 2\theta$		
		$=\frac{e^{2i\theta}-e^{-2i\theta}}{2i}$	1/2	
		$RHS = 2\sin\theta\cos\theta$,-	
		$=2\cdotrac{e^{i heta}-e^{-i heta}}{2i}\cdotrac{e^{i heta}+e^{-i heta}}{2i}$	1/2	
		$=\frac{\left(e^{i\theta}\right)^2-\left(e^{-i\theta}\right)^2}{2i}$		
		2.		
		$=\frac{e^{2i\theta}-e^{-2i\theta}}{2i}$	1/2	
			1/2	
		$\therefore LHS = RHS$ OR		
		$2\sin\theta\cos\theta = 2 \cdot \frac{e^{i\theta} - e^{-i\theta}}{2i} \cdot \frac{e^{i\theta} + e^{-i\theta}}{2i}$		
		$2\sin\theta\cos\theta = 2\cdot{2i}\cdot{2i}$	1/2	
		$=\frac{\left(e^{i\theta}\right)^2-\left(e^{-i\theta}\right)^2}{2!}$		
		$=\frac{\langle \cdot \rangle \langle \cdot \rangle}{2i}$	1/2	
		$e^{2i\theta}-e^{-2i\theta}$	1/2	
		$=\frac{e^{2i\theta}-e^{-2i\theta}}{2i}$,-	
		$=\sin 2\theta$	1/2	
		$(a - a)^2 (a - a)^2$		
	b) ii)	$\cosh^2 \theta - \sinh^2 \theta = \left(\frac{e^{\theta} + e^{-\theta}}{2}\right)^2 - \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)^2$	1/2	
		$= \frac{e^{2\theta} + 2e^{\theta}e^{-\theta} + e^{-2\theta}}{1 - e^{-\theta}e^{-\theta} + e^{-\theta}e^{-\theta}} - \frac{e^{2\theta} - 2e^{\theta}e^{-\theta} + e^{-2\theta}e^{-\theta}e^{-\theta}}{1 - e^{-\theta}e$	1/2	
		4 4	1/2	
		$=\frac{4e^{\theta}e^{-\theta}}{4}$	72	
		=1	1/2	4
		_ I		
	-)	$\left[\cos 3\theta + i\sin 3\theta\right]^4 \left[\cos 4\theta - i\sin 4\theta\right]^5$		
	c)	$\frac{\left[\cos 4\theta + i \sin 4\theta\right]^{3} \left[\cos 5\theta + i \sin 5\theta\right]^{-4}}{\left[\cos 4\theta + i \sin 4\theta\right]^{3} \left[\cos 5\theta + i \sin 5\theta\right]^{-4}}$		
		$= \frac{\left[\cos\theta + i\sin\theta\right]^{3\square 4} \left[\cos\theta + i\sin\theta\right]^{-4\square 5}}{\left[\cos\theta + i\sin\theta\right]^{4\square 3} \left[\cos\theta + i\sin\theta\right]^{5\square (-4)}} \qquad \dots (*)$	1/2+1/2+	
			1/2+1/2	
		$= \frac{\left[\cos\theta + i\sin\theta\right]^{12} \left[\cos\theta + i\sin\theta\right]^{-20}}{\left[\cos\theta + i\sin\theta\right]^{12} \left[\cos\theta + i\sin\theta\right]^{-20}}$	1	
		$-\frac{1}{\left[\cos\theta+i\sin\theta\right]^{12}\left[\cos\theta+i\sin\theta\right]^{-20}}$	1	
		=1	1	4
		Note: Every term in the step (*) carries ½ marks.		



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Que.	Sub.			Total
No.	Que.	Model answers	Marks	Marks
6)	d)	$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$	1	
		$= \sin x \cosh y + \cos x i \sinh y$		
		$= \sin x \cosh y + i \cos x \sinh y$	1	
		$\therefore \text{Real part} = \sin x \cosh y$	1	
		Imaginary part = $\cos x \sinh y$	1	4
	e) i)	$z_1 z_2 = (4 - 5i)(3 + 7i)$		
		$= 12 + 28i - 15i - 35i^2$		
		=47+13i	1	
		$=\sqrt{2378} \ or \ 48.765$	1	
		$=\sqrt{2378} \text{ or } 48.703$		
	e) ii)	$\frac{z_1}{z_2} = \frac{4 - 5i}{3 + 7i}$		
			1/2	
		$= \frac{4-5i}{3+7i} \times \frac{3-7i}{3-7i}$		
		$12 - 28i - 15i + 35i^2$		
		$=\frac{12-28i-15i+35i^2}{9-49i^2}$		
		$=\frac{-23-43i}{58}$	1/2	
			, –	
		$=\frac{-23}{58}-\frac{43}{58}i$		
		$\left \therefore \left \frac{z_1}{z_2} \right = \sqrt{\left(\frac{-23}{58} \right)^2 + \left(\frac{-43}{58} \right)^2} \right $		
		$=\sqrt{\frac{2378}{3364}}$ or 0.84	1	4
		V 3364		
	f)	$Let z = 1 - \cos 2 + i \sin 2$		
	1)	$\therefore z = 2\sin^2 1 + i\sin 1\cos 1$		
		$= 2\sin 1(\sin 1 + i\cos 1)$	1	
		$= 2\sin 1 \left[\cos\left(\frac{\pi}{2} - 1\right) + i\sin\left(\frac{\pi}{2} - 1\right)\right]$	1	
		$\therefore \bmod u l u s \ r = 2 \sin 1$	1	
			1	
		amplitude $\theta = \frac{\pi}{2} - 1$		4
			1	T



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Que.	Sub.	Model answers	Marks	Total
No. 6)	Que.	For Computer/Information Technology Group		Marks
ĺ				
	a)	$f(x) = x^3 - 5x - 11$		
		f(2) = -13, f(3) = 1	1	
		$\therefore \text{ the root is in } (2, 3).$		
		$\therefore x_1 = \frac{2+3}{2} = 2.5$	1	
		$\therefore f(2.5) = -7.875$		
		\therefore the root is in $(2.5, 3)$.		
		$\therefore x_2 = \frac{2.5 + 3}{2} = 2.75$	1	
		f(2.75) = -3.95		
		\therefore the root is in $(2.75, 3)$.		
		$\therefore x_3 = \frac{2.75 + 3}{2} = 2.875$	1	4
		OR		
		$f(x) = x^3 - 5x - 11$		
		f(2) = -13, f(3) = 1	1	
		\therefore the root is in $(2, 3)$.		
		a b $x = \frac{a+b}{2}$ $f(x)$		
		2 3 2.5 -7.875		
		2.5 3 2.75 -3.95	1 1	
		2.75 3 2.875	1	4
	b)	$f(x) = x^3 - 2x - 5$		
		$\therefore f(2) = -1$		
		f(3) = 16	1	
		\therefore the root is in $(2, 3)$.	1	
		$\therefore x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = 2.0588$	1	
		$\therefore f(2.0588) = -0.39$		
		$\therefore \text{ the root is in } (2.0588, 3).$		
		$\therefore x_2 = 2.08$	1	
		$\therefore f(2.08) = -0.16$		
		\therefore the root is in $(2.08, 3)$.		
		$\therefore x_3 = 2.089$	1	4



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	Que.	$f(x) = x^3 - 2x - 5$		Warks
		f(2) = -1 $f(3) = 16$	1	
		$\therefore \text{ the root is in } (2, 3).$	1	
		a b $f(a)$ $f(b)$ $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ $f(x)$		
		2 3 -1 16 2.0588 -0.39	1	
		2.0588 3 -0.39 16 2.08 -0.166 2.08 3 -0.16 16 2.089	1 1	4
	(م)	$f\left(x\right) = x^3 - 20$		
	c)	$f'(x) = 3x^2$	1/2	
		f(2) = -12	1/2	
		$f(3) = 7$ $f(x) x^3 - 20$	/2	
		$x - \frac{f(x)}{f'(x)} = x - \frac{x^3 - 20}{3x^2}$		
		$=\frac{2x^3 + 20}{3x^2}$	1	
		Start with $x_0 = 3$, $\therefore x_1 = 2.741$		
		$x_1 = 2.741$ $x_2 = 2.715$	1 1	4
		$x_3 = 2.714$ OR		
		$f\left(x\right) = x^3 - 20$		
		$f'(x) = 3x^2$ $f(2) = -12$	1/2	
		f(3) = 7	1/2	
		Start with $x_0 = 3$,		
		$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$		
		$=3-\frac{f(3)}{f'(3)}$		
		$=3-\frac{-11}{27}$		
		= 2.741	1	



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	Que.	$x_{2} = x_{1} - \frac{f(x_{1})}{f'(x_{1})} = 2.741 - \frac{f(2.741)}{f'(2.741)}$ $= 2.741 - \frac{0.593}{22.539}$ $= 2.715$ $x_{3} = x_{2} - \frac{f(x_{2})}{f'(x_{2})} = 2.714 - \frac{f(2.714)}{f'(2.714)}$ $= 2.714 - \frac{0.593}{22.539}$	1	4
	d)	= 2.714 $2x+3y+2z=2$ $10x+3y+4z=16$ $3x+6y+z=-6$ $2x+3y+2z=2$ $10x+3y+4z=16$ $$ and $3x+6y+z=-6$ $$	1	
		-8x-2z=-14 $28x+7z=49$ $17x+7z=38$ $11x=11$ ∴ $x=1$ $z=3$ $y=-2$ Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per	1 1 1	4
	e)	above scheme of marking. 15x + 2y + z = 18 2x + 20y - 3z = 19 3x - 6y + 25z = 22		



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Que.	Sub.	Model answers	Marks	Total
No.	Que.	Wiodel aliswers	iviains	Marks
6)	e)	$\therefore x = \frac{18 - 2y - z}{15}$		
			1	
		$y = \frac{19 - 2x + 3z}{20}$	1	
		$z = \frac{22 - 3x + 6y}{25}$		
		Starting with $x_0 = 0 = y_0 = z_0$		
		$x_1 = 1.2$	1/2	
		$y_1 = 0.95$	1/2	
		$z_1 = 0.88$	1/2	
		$x_2 = 1.0146$	1/	
		$y_2 = 0.962$	1/ ₂ 1/ ₂	4
		$z_2 = 0.9864$	1/2	
		$\lambda_2 = 0.9804$	/2	
	f)	27x + 6y - z = 85		
	f)	6x + 15y + 2z = 72		
		x + y + 54z = 110		
		$\therefore x = \frac{85 - 6y + z}{27}$		
		$y = \frac{72 - 6x - 2z}{15}$	1	
		$z = \frac{110 - x - y}{54}$		
		Starting with $x_0 = 0 = y_0 = z_0$		
		$x_1 = 3.148$	1/2	
		$y_1 = 3.54$	1/2	
		$z_1 = 1.91$	1/2	
			1/2	
		$x_2 = 2.43$	1/2	
		$y_2 = 3.57$	1/2	4
		$z_2 = 1.925$		
		Important Note		
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then		
		only give appropriate marks in accordance with the scheme of		
		marking.		