Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	a)	$\int (x+1)^2\, dx$		
		$= \int (x^2+2x+1)\, dx$	1	
		$= \frac{x^3}{3} + x^2 + x + c$	1	**2**
		OR		
		$\int (x+1)^2\, dx = \frac{(x+1)^3}{3} + c$	2	**2**
		Note: In solution of integration problems, if the constant 'c' is not added, ½ mark **may be** deducted.		
	b)	$\int \frac{x}{x^2+3x-4}\, dx$		
		$= \int \frac{x}{(x-1)(x+4)}\, dx$		
		$= \int \left[\frac{1/5}{x-1} + \frac{4/5}{x+4}\right] dx$	1	
		$= \frac{1}{5}\log(x-1) + \frac{4}{5}\log(x+4) + c$	1	**2**
		Note: To find the partial fractions of LHS, traditional partial fraction method is generally used. But apart from this direct method of partial fraction is also allowed here.		
	c)	$\int \sin^2 x \cos x dx$ $Put\ \sin x = t$, $\therefore \cos x dx = dt$	½	
		$= \int t^2 dt$	½	
		$= \frac{t^3}{3} + c$	½	
		$= \frac{\sin^3 x}{3} + c$	½	**2**
	d)	$\int_4^9 \frac{dx}{x^{3/2}} = \left[\frac{x^{-1/2}}{-1/2}\right]_4^9$	½	
		$= \left[-\frac{2}{\sqrt{x}}\right]_4^9$		
		$= \left[-\frac{2}{\sqrt{9}}\right] - \left[-\frac{2}{\sqrt{4}}\right]$	1	
		$= \frac{1}{3}$ or 0.333	½	**2**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		**OR**		
		$\int_4^9 \frac{dx}{x^{3/2}} = \left[\frac{x^{-1/2}}{-1/2}\right]_4^9$	½	
		$= \left[\frac{9^{-1/2}}{-1/2}\right] - \left[\frac{4^{-1/2}}{-1/2}\right]$	1	
		$= \frac{1}{3} \quad or \quad 0.333$	½	**2**
	e)	$\int_0^{\log 2} e^{2x} dx = \left[\frac{e^{2x}}{2}\right]_0^{\log 2}$	½	
		$= \frac{e^{2\log 2}}{2} - \frac{e^0}{2}$	½	
		$= \frac{4}{2} - \frac{1}{2}$		
		$= \frac{3}{2} \quad or \quad 1.5$	1	**2**
	f)	$\sqrt[3]{\frac{dy}{dx} + y} = \sqrt[4]{\frac{d^2y}{dx^2}}$		
		$Order = 2$	1	
		$\left(\frac{dy}{dx} + y\right)^4 = \left(\frac{d^2y}{dx^2}\right)^3$		
		$Degree = 3$	1	**2**
	g)	$xdy - ydx = 0$		
		$\therefore \frac{dy}{y} - \frac{dx}{x} = 0$		
		$\therefore \int \frac{dy}{y} - \int \frac{dx}{x} = c$	1	
		$\therefore \log y - \log x = c$	1	**2**
		Note: The above may also be further reduced into the form of $\frac{y}{x} = k \quad or \quad y = kx$ but it is desirable.		
	h)	$n = n(S) = 52$		
		$m = n(A) = 13$	1	
		$\therefore p = \frac{m}{n} = \frac{13}{52}$	½	
		$= \frac{1}{4} \quad or \quad 0.25$	½	**2**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	i)	$n=n(S)=6$	1	
		$m=n(A)=2$		
		$\therefore p=\frac{m}{n}=\frac{2}{6}$	½	
		$=\frac{1}{3}\ \ or\ \ 0.333$	½	**2**
	j)	$n=n(S)={}^{52}C_2=1326$		
		$m=n(King\ and\ Queen)=4\times4=16$	1	
		$\therefore p=\frac{m}{n}=\frac{16}{1326}$	½	
		$=\frac{8}{663}\ \ or\ \ 0.012$	½	**2**
	k)	$A=\int_a^b y\,dx$		
		$=\int_0^3 x^2dx$	½	
		$=\left[\frac{x^3}{3}\right]_0^3$	½	
		$=\frac{3^3}{3}-0$	½	
		$=9$	½	**2**
	l)	$n=n(S)=2^2=4$		
		$m=n(A)=3$	1	
		$\therefore p=\frac{m}{n}=\frac{3}{4}\ \ or\ \ 0.75$	1	**2**
		Note for Numerical Problems: For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal point. Thus 8/663 is actually 0.01206636500754147812971342383107 but can be taken as 0.012. Due to the use of advance calculators, such as modern scientific non-programmable calculators, the step 8/663 may not be written by the students and then directly the answer 0.012 is written. In this case, no marks to be deducted.		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	a)	Put $\tan\frac{x}{2}=t \quad \therefore dx=\frac{2dt}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$	1	
		$\therefore \int\frac{dx}{5+4\cos x}=\int\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$		
		$=2\int\frac{1}{t^2+9}dt$	1	
		$=2\int\frac{1}{t^2+3^2}dt$		
		$=2\times\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1	
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	1	**4**
	b)	$\int\frac{\log x}{x(2+\log x)(3+\log x)}dx$ Put $\log x=t$, $\therefore \frac{1}{x}dx=dt$	1	
		$=\int\frac{t}{(2+t)(3+t)}dt$	½	
		$=\int\left[\frac{-2}{2+t}+\frac{3}{3+t}\right]dt$	1	
		$=-2\log(2+t)+3\log(3+t)+c$	1	
		$=-2\log(2+\log x)+3\log(3+\log x)+c$	½	**4**
		Note: Direct method of partial fraction is allowed.		
	c)	$\int x^2\tan^{-1}x\,dx$		
		$=\tan^{-1}x\int x^2dx-\int\left(\int x^2dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx+c \;-----(*)$	1	
		$=\tan^{-1}x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{1+x^2}dx+c$	1	
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\frac{x^3}{1+x^2}dx+c$		
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\left[x-\frac{x}{1+x^2}\right]dx+c$	1	
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\left[\frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)\right]+c$	1	**4**
		(*) Note: The constant 'c' may not be added by the students in the step (*). If done so, it must have been added at least in the last step.		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	d)	$Put\ \sin x = t$		
		$\therefore \cos x dx = dt$	½	
		x: 0, $\pi/2$; t: 0, 1	½	
		$\int_0^{\pi/2} \frac{\cos x}{4-\sin^2 x} dx = \int_0^1 \frac{1}{4-t^2} dt \quad ------(*)$	1	
		$= \int_0^1 \frac{1}{2^2 - t^2} dt$		
		$= \left[\frac{1}{2\times 2} \log\left(\frac{2+t}{2-t}\right) \right]_0^1$	1	
		$= \frac{1}{4}\log\left(\frac{2+1}{2-1}\right) - \frac{1}{4}\log\left(\frac{2+0}{2-0}\right)$		
		$= \frac{1}{4}\log 3$	1	**4**
		Note: In the step (*) if the limits are kept unchanged i. e., the step is written as $\int_0^{\pi/2} \frac{\cos x}{4-\sin^2 x} dx = \int_0^{\pi/2} \frac{1}{4-t^2} dt$, no further marks are to be given. Further many times the problem is first solved by without limits and then limiting values are applied. This method is also permissible. Give appropriate marks.		
	e)	$x^2 + y^2 = 36$		
		$y = \sqrt{36 - x^2}$		
		$y = 0\ gives\ x^2 = 36$		
		$\therefore x = 6,\ -6$	1	
		$A = 4\int_0^a y dx$	1	
		$= 4\int_0^6 \sqrt{36 - x^2} dx$		
		$= 4\left[\frac{x}{2}\sqrt{36 - x^2} + \frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right) \right]_0^6$	1	
		$= 4\left[\frac{6}{2}\sqrt{36 - 6^2} + \frac{6^2}{2}\sin^{-1} 1 \right] - 4\left[0 + \frac{6^2}{2}\sin^{-1} 0 \right]$		
		$= 4\left[0 + \frac{6^2}{2}\cdot\frac{\pi}{2} \right] - 4\left[0 + \frac{6^2}{2}\sin^{-1} 0 \right]$		
		$= 36\pi$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	f)	$y^2 = 9x$ $y = 0$ $gives$ $x = 0.$ $\therefore x = 0$ & $x = 3$ are the limits. $\int_0^3 ydx = \int_0^3 3\sqrt{x}dx$ $= \left[3 \cdot \frac{2}{3} x^{3/2}\right]_0^3$ $= \left[2x^{3/2}\right]_0^3$ $= \left[2 \cdot 3^{3/2}\right] - 0$ $= 2 \cdot 3^{3/2}$ or 10.392	 ½ ½	
		$\int_0^3 xydx = \int_0^3 3x\sqrt{x}dx$ $= 3\int_0^3 x^{3/2}dx$ $= \left[3 \cdot \frac{2}{5} x^{5/2}\right]_0^3$ $= \left[\frac{6}{5} \cdot 3^{5/2}\right] - 0$ $= \frac{6}{5} \cdot 3^{5/2}$ or 18.706	 ½ ½	
		$\int_0^3 y^2dx = \int_0^3 9xdx$ $= \left[\frac{9}{2} x^2\right]_0^3$ $= \left[\frac{9}{2} \cdot 3^2\right] - 0$ $= \frac{81}{2}$ or 40.5	½ ½	
		$\therefore \bar{x} = \frac{\int_0^3 xydx}{\int_0^3 ydx} = \frac{\frac{6}{5} \cdot 3^{5/2}}{2 \cdot 3^{3/2}} = \frac{18}{10}$ or $\frac{18.706}{10.392} = 1.8$	½	
		$\therefore \bar{y} = \frac{\frac{1}{2}\int_0^3 y^2dx}{\int_0^3 ydx} = \frac{\frac{81}{2}}{2 \cdot 3^{3/2}} = \frac{1}{4} \cdot 3^{5/2}$ or $\frac{40.5}{10.392} = 3.897$	½	
		$\therefore C.G. = \left(\frac{18}{10}, \frac{1}{4} \cdot 3^{5/2}\right)$ or $(1.8, 3.897)$ **Note:** The above example can be solve by directly finding the values of $\bar{x}$ and $\bar{y}$.		4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	a)	$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$		
		$=\int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	½	
		$I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	1	
		$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½	
		$=\int_0^{\pi/2}1\cdot dx$	½	
		$=[x]_0^{\pi/2}$	½	
		$=\frac{\pi}{2}$	½	
		$\therefore I=\frac{\pi}{4}$	½	**4**
		OR		
		$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ Replace $x\to\frac{\pi}{2}-x$ $\therefore \sin x\to\cos x$ $\&\cos x\to\sin x$	½	
		$\therefore I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	1	
		$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½	
		$=\int_0^{\pi/2}1\cdot dx$	½	
		$=[x]_0^{\pi/2}$	½	
		$=\frac{\pi}{2}$	½	
		$\therefore I=\frac{\pi}{4}$	½	**4**
	b)	$I=\int_1^5\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$		
		$=\int_1^5\frac{\sqrt[3]{9-(6-x)}}{\sqrt[3]{9-(6-x)}+\sqrt[3]{(6-x)+3}}dx$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$I=\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$	1	
		$\therefore 2I=\int_1^5 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+3}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$	½	
		$=\int_1^5 1\cdot dx$	½	
		$=[x]_1^5$	½	
		$=5-1$		
		$=4$	½	
		$\therefore I=2$	½	**4**
		OR		
		$I=\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$ Replace $x\to 6-x$, $\therefore 9-x\to x+3$ & $x+3\to 9-x$	½	
		$I=\int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3}+\sqrt[3]{9-x}}dx$	1	
		$\therefore 2I=\int_1^5 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+3}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$	½	
		$=\int_1^5 1\cdot dx$	½	
		$=[x]_1^5$	½	
		$=5-1$		
		$=4$	½	
		$\therefore I=2$	½	**4**
	c)	$I=\int_0^\pi x\sin^3 x\cos^2 xdx$		
		$=\int_0^\pi (\pi-x)\sin^3(\pi-x)\cos^2(\pi-x)dx$	½	
		$=\int_0^\pi (\pi-x)\sin^3 x\cos^2 xdx$	½	
		$=\int_0^\pi \pi\sin^3 x\cos^2 xdx-\int_0^\pi x\sin^3 x\cos^2 xdx$		
		$=\pi\int_0^\pi \sin^3 x\cos^2 xdx-I$		
		$\therefore 2I=\pi\int_0^\pi \sin^3 x\cos^2 xdx$	½	
		$=\pi\int_0^\pi \sin x\sin^2 x\cos^2 xdx$		
		$=\pi\int_0^\pi \sin x\left(1-\cos^2 x\right)\cos^2 xdx$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$Put\ \cos x = t \quad \therefore -\sin x dx = dt$	½	
		<table><tr><td>x</td><td>t</td></tr><tr><td>0</td><td>1</td></tr><tr><td>π</td><td>-1</td></tr></table>	½	
		$\therefore 2I = -\pi\int_{1}^{-1}\left(1-t^2\right)t^2 dt$		
		$= \pi\int_{-1}^{1}\left(t^2 - t^4\right)dt$		
		$= \pi\left[\frac{t^3}{3} - \frac{t^5}{5}\right]_{-1}^{1}$	½	
		$= \pi\left[\frac{1}{3} - \frac{1}{5}\right] - \pi\left[\frac{-1}{3} - \frac{-1}{5}\right]$		
		$= \frac{4}{15}\pi$	½	
		$\therefore I = \frac{2}{15}\pi$	½	**4**
	d)	Given $y^2 = 9x$ and $x^2 = 9y$		
		$\therefore \left(\frac{x^2}{9}\right)^2 = 9x$		
		$\therefore x = 0, \quad x = 9$	1	
		$A = \int_a^b \left(y_2 - y_1\right)dx$		
		$= \int_0^9 \left[3\sqrt{x} - \frac{x^2}{9}\right]dx$	1	
		$= \left[3\cdot\frac{2}{3}x^{3/2} - \frac{1}{9}\cdot\frac{x^3}{3}\right]_0^9$	1	
		$= \left[2\cdot 9^{3/2} - \frac{1}{27}\cdot 9^3\right] - 0$		
		$= 27$	1	**4**
	e)	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		
		$\therefore y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$		
		$\therefore y^2 = \frac{b^2}{a^2}\left(a^2 - x^2\right)$		
		$\therefore y = \frac{b}{a}\sqrt{a^2 - x^2}$		

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>3)</td><td></td><td>$Now\ y=0\ gives\ a^2-x^2=0\ \ i.e.,\ x=a,\ -a$
$\therefore V=\pi\int_{-a}^{a}y^2dx$
$=\pi\int_{-a}^{a}\frac{b^2}{a^2}\left(a^2-x^2\right)dx$
$=\frac{\pi b^2}{a^2}\left[a^2x-\frac{x^3}{3}\right]_{-a}^{a}$
$=\frac{\pi b^2}{a^2}\left[a^3-\frac{a^3}{3}\right]-\frac{\pi b^2}{a^2}\left[-a^3+\frac{a^3}{3}\right]$
$=\frac{4\pi ab^2}{3}$</td><td>1

1
1
1</td><td>4</td></tr>
<tr><td></td><td>f)</td><td>$\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$
$\therefore \frac{\sec^2 x}{\tan x}dx+\frac{\sec^2 y}{\tan y}dy=0$
$\therefore \int\frac{\sec^2 x}{\tan x}dx+\int\frac{\sec^2 y}{\tan y}dy=c$
$\therefore \log\tan x+\log\tan y=c$ ....(i)
$At\ \ y=\frac{\pi}{4},\ \ x=\frac{\pi}{4},\quad c=0$
$\therefore \log\tan x+\log\tan y=0$
$\left[OR\ \tan x=\cot y\quad OR\ \ x=\frac{\pi}{2}-y\right]$
Note: In the above solution, the step (i) is also formed by 'substitution method'. But direct method is also permissible as shown above.</td><td>1
1
1
½
½</td><td>4</td></tr>
<tr><td>4)</td><td>a)</td><td>$x^2ydx-\left(x^3+y^3\right)dy=0$
$\therefore \frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$
Put $y=vx\qquad \therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$
$\therefore v+x\frac{dv}{dx}=\frac{x^2vx}{x^3+\left(vx\right)^3}=\frac{v}{1+v^3}$
$\therefore x\frac{dv}{dx}=\frac{v}{1+v^3}-v$
$\therefore x\frac{dv}{dx}=-\frac{v^4}{1+v^3}$
$\therefore \frac{1+v^3}{v^4}dv=-\frac{1}{x}dx$</td><td>1

1</td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$\therefore \int\left(\frac{1}{v^4}+\frac{1}{v}\right)dv=-\int\frac{1}{x}dx$		
		$\therefore \frac{v^{-3}}{-3}+\log v=-\log x+c$	1	
		$\therefore \frac{1}{-3v^3}+\log v=-\log x+c$		
		$\therefore \frac{x^3}{-3y^3}+\log\left(\frac{y}{x}\right)=-\log x+c$	1	**4**
	b)	$\cos^2 x\frac{dy}{dx}+y=\tan x$		
		$\therefore \frac{dy}{dx}+\sec^2 x\cdot y=\tan x\cdot\sec^2 x$		
		$\therefore P=\sec^2 x$ and $Q=\tan x\cdot\sec^2 x$		
		$\therefore IF=e^{\int pdx}=e^{\int\sec^2 xdx}=e^{\tan x}$	1	
		$\therefore y\cdot IF=\int Q\cdot IF\cdot dx+c$		
		$\therefore y\cdot e^{\tan x}=\int\tan x\cdot\sec^2 x\cdot e^{\tan x}\cdot dx+c$	1	
		Put $\tan x=t$ $\therefore \sec^2 x\cdot dx=dt$	½	
		$\therefore y\cdot e^{\tan x}=\int t\,e^{t}\cdot dt+c$		
		$\therefore y\cdot e^{\tan x}=te^t-e^t+c$	½	
		$\therefore y\cdot e^{\tan x}=\tan xe^{\tan x}-e^{\tan x}+c$	1	
		$\therefore y\cdot e^{\tan x}=e^{\tan x}\left(\tan x-1\right)+c$		
	c)	$\frac{dy}{dx}=x^3y^3-xy$		
		$\therefore \frac{dy}{dx}+xy=x^3y^3$		
		$\therefore \frac{1}{y^3}\frac{dy}{dx}+\frac{x}{y^2}=x^3$		
		$Put\ \frac{1}{y^2}=t$		
		$\therefore \frac{-2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$	1	
		$\therefore \frac{1}{-2}\frac{dt}{dx}+x\cdot t=x^3$		
		$\therefore \frac{dt}{dx}-2x\cdot t=-2x^3$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$P=-2x,\quad Q=-2x^3$		
		$\therefore IF=e^{\int pdx}=e^{\int -2xdx}=e^{-x^2}$		
		$\therefore t\cdot e^{-x^2}=\int -2x^3\cdot e^{-x^2}\cdot dx+c$	1	
		$\therefore t\cdot e^{-x^2}=-\int 2x\cdot x^2\cdot e^{-x^2}\cdot dx+c$		
		$Put\ x^2=u\quad \therefore 2xdx=du$		
		$\therefore t\cdot e^{-x^2}=-\int u\cdot e^{-u}\cdot du+c$		
		$\therefore t\cdot e^{-x^2}=-\left(-ue^{-u}+e^{-u}\right)+c$	1	
		$\therefore \frac{1}{y^2}\cdot e^{-x^2}=-\left(-x^2e^{-x^2}+e^{-x^2}\right)+c$	1	**4**
		$\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^2\right)dy=0$		
	d)	$M=3x^2+6xy^2$		
		$\therefore \frac{\partial M}{\partial y}=12xy$	1	
		$N=6x^2y+4y^2$		
		$\therefore \frac{\partial N}{\partial x}=12xy$	1	
		$\therefore$ *the equation is exact.*		
		$\therefore$ *the solution is,*		
		$\int\limits_{y\ constant} Mdx+\int\limits_{terms\ free\ from\ x} Ndy=c$		
		$\int\left(3x^2+6xy^2\right)dx+\int 4y^2dy=c$	1	
		$\therefore 3.\frac{x^3}{3}+6y^2\frac{x^2}{2}+4\frac{y^3}{3}=c$		
		$\therefore x^3+3x^2y^2+\frac{4}{3}y^3=c$	1	**4**
	e)	$x+2y+3z=14$		
		$3x+y+2z=11$		
		$2x+3y+z=11$		
		$\begin{array}{r}3x+6y+9z=42\\3x+y+2z=11\\-\quad-\quad-\quad-\\\hline 5y+7z=31\end{array}$ and $\begin{array}{r}2x+4y+6z=28\\2x+3y+z=11\\-\quad-\quad-\quad-\\\hline y+5z=17\end{array}$	1	

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>4)</td><td></td><td>$5y+7z=31$
$\underline{5y+25z=85}$
$-18z=-54$
$\therefore z=3$
$y=2$
$x=1$

Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking. Let us see, how the solution becomes by eliminating first y and then z to get the value of x, as illustrated below:

$x+2y+3z=14$
$6x+2y+4z=22$
$\underline{-\quad -\quad -\quad -}$
$-5x-z=-8$
and
$9x+3y+6z=33$
$2x+3y+z=11$
$\underline{-\quad -\quad -\quad -}$
$7x+5z=22$

$-25x-5z=-40$
$\underline{7x+5z=22}$
$-18x=-18$
$\therefore x=1$
$y=2$
$z=3$</td><td>1
1
1

1

1
1
1</td><td>4

4</td></tr>
<tr><td></td><td>f)</td><td>$5x-y=9$
$x-5y+z=-4$
$y-5z=6$
$\therefore x=\dfrac{9+y}{5}$
$y=\dfrac{-4-x-z}{-5}$
$z=\dfrac{6-y}{-5}$
Starting with $x_0=0=y_0=z_0$
$x_1=1.8$
$y_1=1.16$
$z_1=-0.968$</td><td>1

1</td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$x_2 = 2.032$ $y_2 = 1.0128$ $z_2 = -0.997$	1	
		$x_3 = 2.003$ $y_3 = 1.001$ $z_3 = -0.9998$	1	**4**
5)	a)	$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ Put $x+y=t$ $\therefore 1+\frac{dy}{dx} = \frac{dt}{dx}$	1	
		$\therefore \frac{dt}{dx} - 1 = \frac{t+1}{t-1}$ $\therefore \frac{dt}{dx} = \frac{t+1}{t-1} + 1 = \frac{2t}{t-1}$ $\therefore \frac{t-1}{2t} dt = dx$ $\therefore \int \frac{t-1}{2t} dt = \int dx$ $\therefore \frac{1}{2} \int \left(1 - \frac{1}{t}\right) dt = \int dx$	1	
		$\therefore \frac{1}{2}\left(t - \log t\right) = x + c$	1	
		$\therefore \frac{1}{2}\left[x+y-\log\left(x+y\right)\right] = x + c$ $OR \quad x+y-\log\left(x+y\right) = 2x+k$	1	**4**
	b)	$\frac{dv}{dt} = 5 - 2t$ $\therefore dv = \left(5-2t\right) dt$ $\therefore \int dv = \int \left(5-2t\right) dt$ $\therefore v = 5t - t^2 + c$	1	
		$At\ t = 0,\ v = 4.$ $\therefore c = 4$ $\therefore v = 5t - t^2 + 4$	1	

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5)</td><td></td><td>$\therefore \frac{ds}{dt} = 5t - t^2 + 4$
$\therefore \int ds = \int (5t - t^2 + 4)\,dt$
$\therefore s = \frac{5}{2}t^2 - \frac{t^3}{3} + 4t + c$
$At\ t = 0,\ s = 0.$
$\therefore c = 0$
$\therefore s = \frac{5}{2}t^2 - \frac{t^3}{3} + 4t$
$\therefore at\ t = 3\sec,$
$s = \frac{5}{2}\cdot 3^2 - \frac{3^3}{3} + 4\cdot 3 = \frac{51}{2}\ \ or\ \ 25.5$</td><td>½
½
1</td><td>4</td></tr>
<tr><td></td><td>c)</td><td>$\frac{d^2x}{dt^2} = 3t^2$
$\therefore \frac{dv}{dt} = 3t^2$
$\therefore dv = 3t^2 dt$
$\therefore \int dv = \int 3t^2 dt$
$\therefore v = t^3 + c$
$At\ t = 1,\ v = 2.$
$\therefore c = 1$
$\therefore v = t^3 + 1$</td><td>1
1
1
1</td><td>4</td></tr>
<tr><td></td><td>d)</td><td>$f(x) = x^3 - x - 4$
$\therefore\ f(1) = -4$
$f(2) = 2$
$\therefore$ the root is in $(1,\ 2)$.
$\therefore x_1 = \frac{1+2}{2} = 1.5$
$\therefore\ f(1.5) = -2.125$
$\therefore$ the root is in $(1.5,\ 2)$.
$\therefore x_2 = \frac{1.5+2}{2} = 1.75$
$\therefore\ f(1.75) = -0.391$
$\therefore$ the root is in $(1.75,\ 2)$.
$\therefore x_3 = \frac{1.75+2}{2} = 1.875$</td><td>1
1
1
1</td><td>4</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		**OR**		
		$f(x)=x^3-x-4$		
		$\therefore f(1)=-4$		
		$f(2)=2$	1	
		$\therefore$ the root is in $(1,\ 2)$.		

a	b	$x=\frac{a+b}{2}$	$f(x)$	Marks
1	2	1.5	-2.125	1
1.5	2	1.75	-0.391	1
1.75	2	1.875	---	1

Total Marks: **4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
	e)	$f(x)=x^3-x-1$		
		$\therefore f(1)=-1$		
		$f(2)=5$	1	
		$\therefore$ the root is in $(1,\ 2)$.		
		$\therefore x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=1.167$	1	
		$\therefore f(1.167)=-0.579$		
		$\therefore$ the root is in $(1.167,\ 2)$.		
		$\therefore x_2=1.253$	1	
		$\therefore f(1.253)=-0.286$		
		$\therefore$ the root is in $(1.253,\ 2)$.		
		$\therefore x_3=1.293$	1	**4**
		OR		
		$f(x)=x^3-x-1$		
		$\therefore f(1)=-1$		
		$f(2)=5$	1	
		$\therefore$ the root is in $(1,\ 2)$.		

a	b	$f(a)$	$f(b)$	$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$	Marks
1	2	-1	5	1.167	-0.0579	1
1.167	2	-0.0579	5	1.253	-0.286	1
1.253	2	-0.286	5	1.293	---	1

Total Marks: **4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	f)	$5x+2y+z=12$ $x+4y+2z=15$ $x+2y+5z=20$		
		$\therefore x=\dfrac{12-2y-z}{5}$ $y=\dfrac{15-x-2z}{4}$ $z=\dfrac{20-x-2y}{5}$	1	
		Starting with $x_0=0=y_0=z_0$ $x_1=2.4$ $y_1=3.25$ $z_1=4$	1	
		$x_2=0.1$ $y_2=1.15$ $z_2=2.02$	1	
		$x_3=1.536$ $y_3=2.172$ $z_3=3.52$	1	**4**
6)	a)	$S=\{1,\ 2,---,20\}$ $n=n(S)=20$	1	
		$A=\{3,5,6,9,10,12,15,18,20\}$ $m=n(A)=9$	1	
		$\therefore p=\dfrac{m}{n}=\dfrac{9}{20}$ or 0.45	2	**4**
		OR		
		$S=\{1,\ 2,---,20\}$ $n(S)=20$ $A=\{3,6,9,12,15,18\}$ $n(A)=6$		
		$p(A)=\dfrac{6}{20}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$B=\{5,10,15,20\}$		
		$n(B)=4$		
		$p(B)=\frac{4}{20}$	1	
		$A=\{15\}$		
		$n(A\cap B)=1$		
		$p(A\cap B)=\frac{1}{20}$	1	
		$\therefore p=p(A)+p(B)-p(A\cap B)=\frac{9}{20}\quad or\quad 0.45$	1	**4**
	b)	$Total\ Balls=10+5+5=20$		
		$n=n(S)={}^{20}C_2=190$	1	
		$m=n(\text{not of same colour})$		
		$=n(\text{1R1W or 1W1B or 1R1B})$		
		$=10\times5+5\times5+10\times5$	1	
		$=125$	1	
		$\therefore p=\frac{m}{n}=\frac{125}{190}\quad or\quad 0.659$	1	**4**
	c)	$p=0.2$		
		$\therefore q=0.8$		
		$Here\ \ n=4$		
		$i)\ p={}^nC_r\,p^r q^{n-r}$		
		$={}^4C_1(0.2)^1(0.8)^3$		
		$=0.4096$	1	
		$ii)\ p=p(0)+p(1)+p(2)$		
		$={}^4C_0(0.2)^0(0.8)^4+{}^4C_1(0.2)^1(0.8)^3+{}^4C_2(0.2)^2(0.8)^2$	1	
		$=0.4096+0.4096+0.1536$	1	
		$=0.8192$	1	**4**
	d)	$n=100,\quad p=3\%=\frac{3}{100}=0.03$	1	
		$m=np=3$	1	
		$\therefore p=\frac{e^{-m}m^r}{r!}$		
		$=\frac{e^{-3}3^5}{5!}$	1	
		$=0.1008$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	e)	$\bar{x}=14, \quad \sigma=2.5$		
		$Here \quad x=18$		
		$\therefore z=\frac{x-\bar{x}}{\sigma}=\frac{18-14}{2.5}=1.6$	1	
		p = Area more than z=1.6 under the normal curve		
		$=0.5-A(z=0 \ to \ z=1.6)$		
		$=0.5-0.4452$		
		$=0.0548$	1	
		$\therefore$ N $= p\times 1000=54.8$	1	
		$\therefore$ the number of students $=55$	1	**4**
	f)	$P(A')=\frac{2}{3}$		
		$P(B')=\frac{1}{5}$		
		$P(C')=\frac{5}{7}$		
		$\therefore p=p(\text{the problem is solved})$		
		$=1-p(\text{the problem is not solved by all A, B, C})$	1	
		$=1-p(A'\cap B'\cap C')$	1	
		$=1-\frac{2}{3}\cdot\frac{1}{5}\cdot\frac{5}{7}$	1	
		$=\frac{95}{105} \ or \ \frac{19}{21} \quad or \ 0.905$	1	**4**
		Important Note		
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.		