



Summer 2015 Examination

Subject & Code: Basic Maths (17105)

Model Answer

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		<p><b>Important Instructions to the Examiners:</b></p> <ol style="list-style-type: none"><li>1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.</li><li>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.</li><li>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)</li><li>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.</li><li>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.</li><li>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.</li><li>7) For programming language papers, credit may be given to any other program based on equivalent concept.</li></ol> <p>-----</p> <p>-----</p> <p><b>Important Note</b></p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. <b>In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY</b> give appropriate marks in accordance with the scheme of marking.</p> <p>-----</p> <p>-----</p>		



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1)		<b>Attempt any TEN of the following:</b>		
	a)	Solve $\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$		
	Ans.	$\therefore 6+12 = x^2 + 2$ $\therefore x^2 = 16$ or $x^2 - 16 = 0$ or $-x^2 + 16 = 0$ $\therefore x = 4, -4$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	2
		<b>OR</b>		
		$\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = 6+12 = 18$ $\begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix} = x^2 + 2$ $\therefore 18 = x^2 + 2$ $\therefore x^2 = 16$ or $x^2 - 16 = 0$ or $-x^2 + 16 = 0$ $\therefore x = 4, -4$	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	2
	b)	Find x, if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$ .		
	Ans.	$\therefore 4(-2x-28) - 3(3x-77) + 9(12+22) = 0$ $\therefore -8x - 112 - 9x + 231 + 306 = 0$ $\therefore -17x + 425 = 0$ $\therefore x = \frac{425}{17}$ $\therefore \boxed{x = 25}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	c)	If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ , find $3A - 2B$ .		
	Ans.	$\therefore 3A - 2B = 3 \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix} - 2 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ $= \begin{bmatrix} 15 & 9 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & 4 \end{bmatrix}$ $= \begin{bmatrix} 11 & 11 \\ -9 & -1 \end{bmatrix}$	$\frac{1}{2} + \frac{1}{2}$  1	2



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1)		<p style="text-align: center;"><b>OR</b></p> $3A = 3 \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & 3 \end{bmatrix}$ $2B = 2 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 6 & 4 \end{bmatrix}$ $\therefore 3A - 2B = \begin{bmatrix} 15 & 9 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & 4 \end{bmatrix}$ $= \begin{bmatrix} 11 & 11 \\ -9 & -1 \end{bmatrix}$ <hr/> <p>d) If <math>A = \begin{bmatrix} 3 &amp; 9 \\ -1 &amp; -3 \end{bmatrix}</math>, then show that <math>A^2</math> is a null matrix.</p> <p>Ans. <math>A^2 = \begin{bmatrix} 3 &amp; 9 \\ -1 &amp; -3 \end{bmatrix} \cdot \begin{bmatrix} 3 &amp; 9 \\ -1 &amp; -3 \end{bmatrix}</math></p> $= \begin{bmatrix} 9-9 & 27-27 \\ -3+3 & -9+9 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ <p><math>\therefore A^2</math> is a null matrix.</p> <hr/> <p>e) Define singular and non-singular matrix.</p> <p>Ans. <b>Singular Matix:</b> Let A be a square matrix. Then A is singular matrix, if <math> A  = 0</math></p> <p><b>Non-Singular Matix:</b> Let A be a square matrix. Then A is singular matrix, if <math> A  \neq 0</math></p> <p><b>Note:</b> The above definition is a sample format. Students may express the same into other words also. Please give due credit to the students.</p> <hr/>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>	<p>2</p> <p>2</p> <p>2</p>



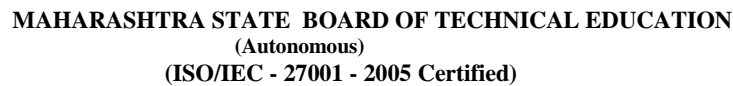
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	<p>Resolve into partial fractions: <math>\frac{1}{x^2 + 3x + 2}</math></p> <p>Ans. <math>\frac{1}{x^2 + 3x + 2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}</math></p> <p><math>\therefore 1 = (x+2)A + (x+1)B</math></p> <p>Put <math>x = -1</math></p> <p><math>\therefore 1 = (-1+2)A + 0</math></p> <p><math>\therefore \boxed{A=1}</math></p> <p>Put <math>x = -2</math></p> <p><math>\therefore 1 = 0 + (-2+1)B</math></p> <p><math>\therefore \boxed{B=-1}</math></p> <p><math>\therefore \boxed{\frac{1}{x^2 + 3x + 2} = \frac{1}{x+1} + \frac{-1}{x+2}}</math></p> <p><b>Note for partial fraction problems:</b> The problems of partial fractions could also be solved by the method of "equating equal power coefficients". This method is also applicable. Give appropriate marks in accordance with the scheme of marking in the later problems as the solution by this method is not discussed. For the sake of convenience, the solution of the above problem with the help of this method is illustrated hereunder.</p> <p><math>\frac{1}{x^2 + 3x + 2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}</math></p> <p><math>\therefore 1 = (x+2)A + (x+1)B</math></p> <p><math>\therefore 0x + 1 = (A+B)x + (2A+B)</math></p> <p>By equating equal power coefficients,</p> <p><math>A+B=0</math> and <math>2A+B=1</math></p> <p><math>\therefore \boxed{A=1}</math></p> <p><math>\boxed{B=-1}</math></p> <p><math>\therefore \boxed{\frac{1}{x^2 + 3x + 2} = \frac{1}{x+1} + \frac{-1}{x+2}}</math></p> <p>-----</p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	<p>2</p> <p>2</p>



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1)	g)	If $\angle A = 30^\circ$ , verify that $\sin 3A = 3\sin A - 4\sin^3 A$		
	Ans.	$LHS = \sin 3A = \sin 3(30^\circ) = 1$	$\frac{1}{2}$	
		$RHS = 3\sin A - 4\sin^3 A$		
		$= 3\sin 30^\circ - 4\sin^3 30^\circ$	$\frac{1}{2}$	
		$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 \quad \dots (*)$	$\frac{1}{2}$	
		$= 1 \quad \dots (**)$	$\frac{1}{2}$	
		<b>Note (*):</b> Due to the use of advance scientific calculator, writing directly the step (**) after (*) is allowed. No marks to be deducted.		2
	h)	Without using calculator prove that $\frac{\cos 21^\circ + \sin 21^\circ}{\cos 21^\circ - \sin 21^\circ} = \cot 24^\circ$		
	Ans.	$\frac{\cos 21^\circ + \sin 21^\circ}{\cos 21^\circ - \sin 21^\circ} = \frac{\sin 69^\circ + \sin 21^\circ}{\sin 69^\circ - \sin 21^\circ}$	$\frac{1}{2}$	
		$= \frac{2\sin\left(\frac{69^\circ + 21^\circ}{2}\right)\cos\left(\frac{69^\circ - 21^\circ}{2}\right)}{2\cos\left(\frac{69^\circ + 21^\circ}{2}\right)\sin\left(\frac{69^\circ - 21^\circ}{2}\right)}$		
		$= \frac{\sin 45^\circ \cos 24^\circ}{\cos 45^\circ \sin 24^\circ}$	$\frac{1}{2}$	
		$= \frac{1}{\sqrt{2}} \cos 24^\circ$	$\frac{1}{2}$	
		$= \frac{\frac{1}{\sqrt{2}} \cos 24^\circ}{\frac{1}{\sqrt{2}} \sin 24^\circ}$	$\frac{1}{2}$	
		$= \cot 24^\circ$		2
		OR		
		$\frac{\cos 21^\circ + \sin 21^\circ}{\cos 21^\circ - \sin 21^\circ} = \frac{1 + \tan 21^\circ}{1 - \tan 21^\circ}$	$\frac{1}{2}$	
		$= \tan(45^\circ + 21^\circ)$	$\frac{1}{2}$	
		$= \tan(66^\circ)$		
		$= \tan(90^\circ - 24^\circ)$	$\frac{1}{2}$	
		$= \cot 24^\circ$	$\frac{1}{2}$	2



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	i)	If $2 \sin 50^\circ \cos 70^\circ = \sin A - \sin B$ , find A and B.		
	Ans.	$2 \sin 50^\circ \cos 70^\circ = \sin A - \sin B$ $\therefore \sin(50^\circ + 70^\circ) + \sin(50^\circ - 70^\circ) = \sin A - \sin B$ $\therefore \sin(120^\circ) + \sin(-20^\circ) = \sin A - \sin B$ $\therefore \sin(120^\circ) - \sin(20^\circ) = \sin A - \sin B$ $\therefore A = 120^\circ$ $B = 20^\circ$	1/2	2
			1/2	
			1/2	
			1/2	
		<b>OR</b>		
		$2 \sin 50^\circ \cos 70^\circ = \sin A - \sin B$ $\therefore 2 \sin 50^\circ \cos 70^\circ = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$ $\therefore \frac{A+B}{2} = 70^\circ \text{ and } \frac{A-B}{2} = 50^\circ$ $\therefore A+B = 140^\circ$ $A-B = 100^\circ$ $\therefore A = 120^\circ$ $B = 20^\circ$	1/2	2
			1/2	
			1/2	
			1/2	
	j)	Find the principal value of $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right]$		
	Ans.	$\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right] = \cos\left[\frac{\pi}{2} - \frac{\pi}{6}\right]$ $= \cos\left[\frac{\pi}{3}\right]$ $= \frac{1}{2} \text{ or } 0.5$	1/2	2
			1/2	
			1	
			1	
		<b>OR</b>		
		$\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right] = \cos[90^\circ - 30^\circ]$ $= \cos[60^\circ]$ $= \frac{1}{2} \text{ or } 0.5$	1/2	2
			1/2	
			1	
			1	
		<b>OR</b>		

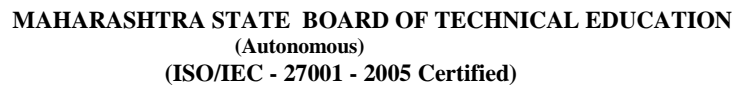


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1)		$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$ $\therefore \cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$ $=\cos\left[\frac{\pi}{3}\right]$ $=\frac{1}{2} \text{ or } 0.5$ <hr/>	½   ½  1	2
k)	Prove that $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\tan^{-1}(\infty)$			
Ans.	$\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{2}$ $\tan^{-1}(\infty)=\frac{\pi}{2}$ $\therefore \sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\tan^{-1}(\infty)$ <p style="text-align: center;"><b>OR</b></p> $\sin^{-1}\left(-\frac{1}{2}\right)=-\sin^{-1}\left(\frac{1}{2}\right)=-\frac{\pi}{6}$ $\cos^{-1}\left(-\frac{1}{2}\right)=\pi-\cos^{-1}\left(\frac{1}{2}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ $\therefore \sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}+\frac{2\pi}{3}=\frac{\pi}{2}$ $\text{and } \tan^{-1}(\infty)=\frac{\pi}{2}$ $\therefore \sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\tan^{-1}(\infty)$ <hr/>	1  ½  ½  ½  ½  ½	2	
				2



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1)	l)	Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$ .		
	Ans.	<p>For <math>3x - 2y + 4 = 0</math>,</p> $\text{slope } m_1 = -\frac{a}{b} = -\frac{3}{-2} = \frac{3}{2}$ <p>For <math>2x - 3y - 7 = 0</math>,</p> $\text{slope } m_2 = -\frac{a}{b} = -\frac{2}{-3} = \frac{2}{3}$ $\therefore \tan \theta = \left  \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $ $= \left  \frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\right) \cdot \left(\frac{2}{3}\right)} \right $ $= \frac{5}{12} \quad \text{or} \quad 0.417$ $\therefore \theta = \tan^{-1}\left(\frac{5}{12}\right) \quad \text{or} \quad \tan^{-1}(0.417) \quad \text{or} \quad 22.636^\circ \quad \text{or} \quad 0.395^c$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	2
2)	a)	<p><b>Attempt any four of the following:</b></p> <p>Solve by Cramer's rule</p> $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1, \quad \frac{3}{x} + \frac{1}{y} + \frac{2}{z} = 4, \quad \frac{9}{x} + \frac{1}{y} + \frac{4}{z} = 16$		
	Ans.	$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 9 & 1 & 4 \end{vmatrix} = 1(4-2) - 1(12-18) + 1(3-9)$ $= 2$ $D_{\frac{1}{x}} = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & 2 \\ 16 & 1 & 4 \end{vmatrix} = 1(4-2) - 1(16-32) + 1(4-16)$ $= 6$ $D_{\frac{1}{y}} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 2 \\ 9 & 16 & 4 \end{vmatrix} = 1(16-32) - 1(12-18) + 1(48-36)$ $= 2$	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	





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2)		$D_{\frac{1}{z}} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 4 \\ 9 & 1 & 16 \end{vmatrix} = 1(16-4) - 1(48-36) + 1(3-9)$ $= -6$ $\therefore x = \frac{D}{D_x} = \frac{2}{6} = \frac{1}{3}$ $y = \frac{D}{D_y} = \frac{2}{2} = 1$ $z = \frac{D}{D_z} = \frac{2}{-6} = -\frac{1}{3}$ <p style="text-align: center;"><b>OR</b></p> <p>Put <math>\frac{1}{x} = a, \quad \frac{1}{y} = b, \quad \frac{1}{z} = c</math></p> $\therefore a + b + c = 1$ $3a + b + 2c = 4$ $9a + b + 4c = 16$ $D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 9 & 1 & 4 \end{vmatrix} = 1(4-2) - 1(12-18) + 1(3-9)$ $= 2$ $D_a = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & 2 \\ 16 & 1 & 4 \end{vmatrix} = 1(4-2) - 1(16-32) + 1(4-16)$ $= 6$ $D_b = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 2 \\ 9 & 16 & 4 \end{vmatrix} = 1(16-32) - 1(12-18) + 1(48-36)$ $= 2$ $D_c = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 4 \\ 9 & 1 & 16 \end{vmatrix} = 1(16-4) - 1(48-36) + 1(3-9)$ $= -6$ $\therefore x = \frac{1}{a} = \frac{D}{D_a} = \frac{2}{6} = \frac{1}{3}$ $y = \frac{1}{b} = \frac{D}{D_b} = \frac{2}{2} = 1$ $z = \frac{1}{c} = \frac{D}{D_c} = \frac{2}{-6} = -\frac{1}{3}$	<div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1</div> <div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1/2</div>	<div>4</div> <div>4</div>



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2)	b)	<p>If <math>\left\{ \begin{bmatrix} 3 &amp; 1 \\ 3 &amp; 4 &amp; 0 \\ 3 &amp; -3 \end{bmatrix} - 2 \begin{bmatrix} 0 &amp; 2 \\ -2 &amp; 3 \\ -5 &amp; 4 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math>, find <math>x, y, z</math>.</p> <p>Ans. <math>\left\{ \begin{bmatrix} 3 &amp; 1 \\ 3 &amp; 4 &amp; 0 \\ 3 &amp; -3 \end{bmatrix} - 2 \begin{bmatrix} 0 &amp; 2 \\ -2 &amp; 3 \\ -5 &amp; 4 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \left\{ \begin{bmatrix} 9 &amp; 3 \\ 12 &amp; 0 \\ 9 &amp; -9 \end{bmatrix} - \begin{bmatrix} 0 &amp; 4 \\ -4 &amp; 6 \\ -10 &amp; 8 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} 9 &amp; -1 \\ 16 &amp; -6 \\ 19 &amp; -17 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} -9-2 \\ -16-12 \\ -19-34 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} -11 \\ -28 \\ -53 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore x = -11, y = -28, z = -53</math></p> <p style="text-align: center;"><b>OR</b></p> <p><math>3 \begin{bmatrix} 3 &amp; 1 \\ 4 &amp; 0 \\ 3 &amp; -3 \end{bmatrix} - 2 \begin{bmatrix} 0 &amp; 2 \\ -2 &amp; 3 \\ -5 &amp; 4 \end{bmatrix} = \begin{bmatrix} 9 &amp; 3 \\ 12 &amp; 0 \\ 9 &amp; -9 \end{bmatrix} - \begin{bmatrix} 0 &amp; 4 \\ -4 &amp; 6 \\ -10 &amp; 8 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 9 &amp; -1 \\ 16 &amp; -6 \\ 19 &amp; -17 \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} 9 &amp; -1 \\ 16 &amp; -6 \\ 19 &amp; -17 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} -9-2 \\ -16-12 \\ -19-34 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore \begin{bmatrix} -11 \\ -28 \\ -53 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}</math></p> <p><math>\therefore x = -11, y = -28, z = -53</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>4</p> <p>4</p>



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2)		<p><b>Note:</b> In this way the problem could be solved in different ways. Further note that, due to simple mistake, if one of the values of unknowns becomes wrong and others are correct, give appropriate marks. For example, consider</p> $3 \begin{bmatrix} 3 & 1 \\ 4 & 0 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 12 & 0 \\ 9 & -3 \end{bmatrix}.$ <p>This mistake will lead to the value of z to be wrong.</p> <hr/> <p>c) Find the inverse of <math>\begin{bmatrix} 1 &amp; 1 &amp; 1 \\ 1 &amp; 2 &amp; 3 \\ 1 &amp; 4 &amp; 9 \end{bmatrix}</math> by adjoint method.</p> <p>Ans. Let <math>A = \begin{bmatrix} 1 &amp; 1 &amp; 1 \\ 1 &amp; 2 &amp; 3 \\ 1 &amp; 4 &amp; 9 \end{bmatrix}</math></p> <p><math>\therefore  A  = 1(18-12) - 1(9-3) + 1(4-2) = 2</math></p> <p><math>\therefore A^{-1}</math> exists.</p> <p>Matrix of Cofactor of A is,</p> $C(A) = \begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix} \quad \text{---(*)}$ $\text{adj}(A) = \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$ <p><math>\therefore A^{-1} = \frac{1}{ A } \text{adj}(A)</math></p> $= \frac{1}{2} \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p>	<p>4</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		<p>(*) <b>Note:</b> In the matrix <math>C(A)</math>, if 1 to 3 elements are wrong (either in sign or value), deduct <math>\frac{1}{2}</math> mark, if 4 to 6 elements are wrong, deduct <math>1\frac{1}{2}</math> marks, if 7 to 9 are wrong, deduct all the 2 marks. Further, if all the elements in the last i.e., <math>adj(A)</math> are correct, then only give <math>\frac{1}{2}</math> mark.</p> <p style="text-align: center;"><b>OR</b></p> <p>Matrix of minors of A is,</p> $M(A) = \begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} 6 & 6 & 2 \\ 5 & 8 & 3 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{---(*)}$ $C(A) = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$ <p style="text-align: center;"><b>OR</b></p> $A_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6 \quad A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6 \quad A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2$ $A_{21} = -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -5 \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8 \quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3$ $A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1 \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2 \quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$ <p><b>Note:</b> In the above, if 1 to 3 elements are wrong, deduct <math>\frac{1}{2}</math> mark, if 4 to 6 elements are wrong, deduct 1 marks, and if 7 to 9 are wrong, deduct all the marks. Further, if all the elements in the following matrices <math>C(A)</math> and <math>adj(A)</math> are correct, then only give the marks.</p>	<p style="text-align: center;">OR</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p style="text-align: center;">OR</p> <p><math>1\frac{1}{2}</math></p>	<p>4</p> <p>4</p>



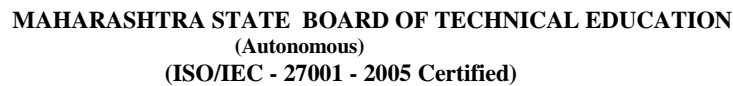
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	d)	<p>Resolve into partial fractions: <math>\frac{x-5}{x^3+x^2-6x}</math></p> <p>Ans. <math>\frac{x-5}{x^3+x^2-6x} = \frac{x-5}{x(x+3)(x-2)}</math></p> $= \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$ <p><math>\therefore x-5 = (x+3)(x-2)A + x(x-2)B + x(x+3)C</math></p> <p>Put <math>x = 0</math></p> <p><math>\therefore 0-5 = (0+3)(0-2)A + 0+0</math></p> <p><math>\therefore -5 = -6A</math></p> <p><math>\therefore \boxed{\frac{5}{6} = A}</math></p> <p>Put <math>x+3=0 \therefore x = -3</math></p> <p><math>\therefore -3-5 = 0-3(-3-2)B + 0</math></p> <p><math>\therefore -8 = 15B</math></p> <p><math>\therefore \boxed{-\frac{8}{15} = B}</math></p> <p>Put <math>x-2=0 \therefore x = 2</math></p> <p><math>\therefore 2-5 = 0+0+2(2+3)C</math></p> <p><math>\therefore -3 = 10C</math></p> <p><math>\therefore \boxed{-\frac{3}{10} = C}</math></p> <p><math>\therefore \frac{x-5}{x^3+x^2-6x} = \frac{5}{6x} - \frac{8}{15(x+3)} - \frac{3}{10(x-2)}</math></p>	<p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p>	4
<p><b>Note for Partial Fraction Methods:</b> The above problems of partial fractions could be solved by the method of "equating equal power coefficients" also. This method is also applicable, which is illustrated in the problem Q. 1 (f). If such method is applied, give appropriate marks in accordance with the scheme of marking. As this method is very tedious and complicated, hardly someone use this method in such cases. So such solution methods for partial fraction problems are not illustrated herein.</p>				



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	e)	<p>If <math>A = \begin{bmatrix} 2 &amp; -3 \\ 1 &amp; 5 \end{bmatrix}</math>, <math>B = \begin{bmatrix} 3 &amp; -1 &amp; 2 \\ 1 &amp; 0 &amp; 1 \end{bmatrix}</math>, verify that <math>(AB)' = B' \cdot A'</math>.</p>		
	Ans.	$\therefore AB = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 6-3 & -2-0 & 4-3 \\ 3+5 & -1+0 & 2+5 \end{bmatrix}$ $= \begin{bmatrix} 3 & -2 & 1 \\ 8 & -1 & 7 \end{bmatrix}$ $(AB)' = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$ $B' \cdot A' = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ $= \begin{bmatrix} 6-3 & 3+5 \\ -2+0 & -1+0 \\ 4-3 & 2+5 \end{bmatrix}$ $= \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$ $\therefore (AB)' = B' \cdot A'$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	4
	f)	<p>If <math>A = \begin{bmatrix} 2 &amp; 4 \\ 1 &amp; 1 \end{bmatrix}</math>, then show that <math>A^2 - 3A = 2I</math> where I is unit matrix of order 2</p>		
	Ans.	$A^2 = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ $= \begin{bmatrix} 4+4 & 8+4 \\ 2+1 & 4+1 \end{bmatrix}$ $= \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix}$ $3A = 3 \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$A^2 - 3A = \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$ $= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $= 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= 2I$ <p style="text-align: center;"><b>OR</b></p> $A^2 - 3A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ $= \begin{bmatrix} 4+4 & 8+4 \\ 2+1 & 4+1 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$ $= \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$ $= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $= 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= 2I$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2+1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>	<p>4</p> <p>4</p>
3)	a)	<p><b>Attempt any four of the following:</b></p> <p>Find AB if <math>A = \begin{bmatrix} 3 &amp; 2 &amp; 1 \\ -4 &amp; 0 &amp; 2 \end{bmatrix}</math>, <math>B = \begin{bmatrix} 3 &amp; -1 &amp; 2 \\ 4 &amp; 2 &amp; 0 \\ 5 &amp; -7 &amp; 6 \end{bmatrix}</math></p>		
	Ans.	$AB = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 0 \\ 5 & -7 & 6 \end{bmatrix}$ $= \begin{bmatrix} 9+8+5 & -3+4-7 & 6+0+6 \\ -12+0+10 & 4+0-14 & -8+0+12 \end{bmatrix}$ $= \begin{bmatrix} 22 & -6 & 12 \\ -2 & -10 & 4 \end{bmatrix}$	<p>2</p> <p>2</p>	<p>4</p>
<p><b>Note:</b> If one or two elements in the final answer are wrong, give appropriate marks. Don't deduct full 2 marks.</p>				

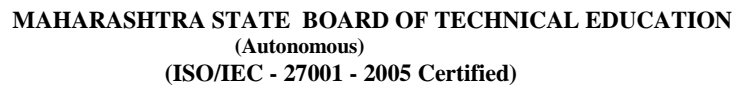


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	b)	<p>If <math>A = \begin{bmatrix} 1 &amp; -2 \\ -3 &amp; 1 \end{bmatrix}</math>, <math>B = \begin{bmatrix} 4 &amp; 2 &amp; -5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math>, <math>C = \begin{bmatrix} 6 &amp; -7 &amp; 0 \\ -1 &amp; 2 &amp; 5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math>, prove that</p> <p><math>(AB)C = A(BC)</math></p>		
	Ans.	<p><math>A = \begin{bmatrix} 1 &amp; -2 \\ -3 &amp; 1 \end{bmatrix}</math>, <math>B = \begin{bmatrix} 4 &amp; 2 &amp; -5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math>, <math>C = \begin{bmatrix} 6 &amp; -7 &amp; 0 \\ -1 &amp; 2 &amp; 5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math></p> <p><math>AB = \begin{bmatrix} 1 &amp; -2 \\ -3 &amp; 1 \end{bmatrix} \begin{bmatrix} 4 &amp; 2 &amp; -5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 4-2 &amp; 2-0 &amp; -5-6 \\ -12+1 &amp; -6-0 &amp; 15+3 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 2 &amp; 2 &amp; -11 \\ -11 &amp; -6 &amp; 18 \end{bmatrix}</math></p> <p><math>(AB)C = \begin{bmatrix} 2 &amp; 2 &amp; -11 \\ -11 &amp; -6 &amp; 18 \end{bmatrix} \begin{bmatrix} 6 &amp; -7 &amp; 0 \\ -1 &amp; 2 &amp; 5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 12-2-11 &amp; -14+4-0 &amp; 0+10-33 \\ -66+6+18 &amp; 77-12+0 &amp; 0-30+54 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} -1 &amp; -10 &amp; -23 \\ -42 &amp; 65 &amp; 24 \end{bmatrix}</math></p> <p><math>BC = \begin{bmatrix} 4 &amp; 2 &amp; -5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix} \begin{bmatrix} 6 &amp; -7 &amp; 0 \\ -1 &amp; 2 &amp; 5 \\ 1 &amp; 0 &amp; 3 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 24-2-5 &amp; -28+4-0 &amp; 0+10-15 \\ 6-0+3 &amp; -7+0+0 &amp; 0+0+9 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 17 &amp; -24 &amp; -5 \\ 9 &amp; -7 &amp; 9 \end{bmatrix}</math></p> <p><math>A(BC) = \begin{bmatrix} 1 &amp; -2 \\ -3 &amp; 1 \end{bmatrix} \begin{bmatrix} 17 &amp; -24 &amp; -5 \\ 9 &amp; -7 &amp; 9 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 17-18 &amp; -24+14 &amp; -5-18 \\ -51+9 &amp; 72-7 &amp; 15+9 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} -1 &amp; -10 &amp; -23 \\ -42 &amp; 65 &amp; 24 \end{bmatrix}</math></p> <p><math>\therefore (AB)C = A(BC)</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	
		OR		
			$\frac{1}{2}$	4



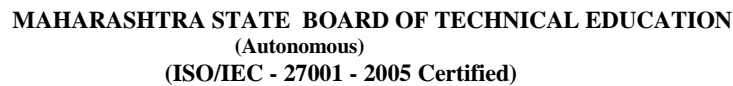


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$(AB)C = \left\{ \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix} \right\} \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$ $= \left\{ \begin{bmatrix} 4-2 & 2-0 & -5-6 \\ -12+1 & -6-0 & 15+3 \end{bmatrix} \right\} \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$ $= \begin{bmatrix} 2 & 2 & -11 \\ -11 & -6 & 18 \end{bmatrix} \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$ $= \begin{bmatrix} 12-2-11 & -14+4-0 & 0+10-33 \\ -66+6+18 & 77-12+0 & 0-30+54 \end{bmatrix}$ $= \begin{bmatrix} -1 & -10 & -23 \\ -42 & 65 & 24 \end{bmatrix}$ $A(BC) = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix} \right\}$ $= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 24-2-5 & -28+4-0 & 0+10-15 \\ 6-0+3 & -7+0+0 & 0+0+9 \end{bmatrix} \right\}$ $= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 17 & -24 & -5 \\ 9 & -7 & 9 \end{bmatrix}$ $= \begin{bmatrix} 17-18 & -24+14 & -5-18 \\ -51+9 & 72-7 & 15+9 \end{bmatrix}$ $= \begin{bmatrix} -1 & -10 & -23 \\ -42 & 65 & 24 \end{bmatrix}$ $\therefore (AB)C = A(BC)$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	
	c)	Solve the equations by matrix method: $x+y+z=3, 2x-y+3z=4, 3x+4y+z=8$		
	Ans.	$\therefore A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 4 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$ $\therefore  A  = 1(-1-12) - 1(2-9) + 1(8+3)$ $= 5$	$\frac{1}{2}$	4

[illegible]



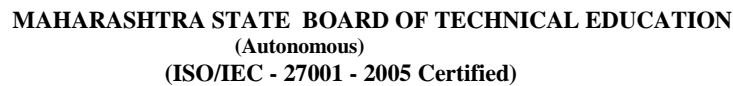
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		<p>Put <math>x = 0</math></p> $\therefore 0^2 + 23(0) = (0^2 + 1)A + (0 + 3)(0 + C)$ $\therefore 0 = A + 3C$ $\therefore 0 = -6 + 3C$ $\therefore 6 = 3C$ $\therefore \boxed{2 = C}$ <p>Put <math>x = 1</math></p> $\therefore 1^2 + 23(1) = (1^2 + 1)A + (1 + 3)(B + C)$ $\therefore 24 = 2A + 4B + 4C$ $\therefore 24 = 2(-6) + 4B + 4(2)$ $\therefore 28 = 4B$ $\therefore \boxed{7 = B}$ $\therefore \frac{x^2 + 23x}{(x+3)(x^2+1)} = \frac{-6}{x+3} + \frac{7x+2}{x^2+1}$ <p>-----</p> <p>e) Resolve into partial fractions: <math>\frac{x^4}{x^3+1}</math></p> <p>Ans. <math>\frac{x^4}{x^3+1} = x - \frac{x}{x^3+1}</math></p> $\therefore \frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)}$ $= \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ $\therefore \boxed{x = (x^2-x+1)A + (x+1)(Bx+C)}$ <p>Put <math>x = -1</math></p> $\therefore -1 = [(-1)^2 - (-1) + 1]A + 0$ $\therefore -1 = 3A$ $\therefore \boxed{-\frac{1}{3} = A}$ <p>Put <math>x = 0</math></p> $\therefore 0 = (0^2 - 0 + 1)A + (0 + 1)(0 + C)$ $\therefore 0 = A + C$	<p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\therefore 0 = -\frac{1}{3} + C$ $\therefore \boxed{\frac{1}{3} = C}$ <p>Put <math>x = 1</math></p> $\therefore 1 = (1^2 - 1 + 1)A + (1 + 1)(B + C)$ $\therefore 1 = A + 2B + 2C$ $\therefore 1 = -\frac{1}{3} + 2B + 2\left(\frac{1}{3}\right)$ $\therefore 1 + \frac{1}{3} - 2\left(\frac{1}{3}\right) = 2B$ $\therefore \frac{2}{3} = 2B$ $\therefore \boxed{\frac{1}{3} = B}$ $\therefore \frac{x}{x^3+1} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$ $\therefore \boxed{\frac{x^4}{x^3+1} = x + \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}}$ <hr style="border-top: 1px dashed black;"/> <p>f) Resolve into partial fractions <math>\frac{\sin \theta + 1}{(\sin \theta - 1)(\sin \theta + 2)}</math></p> <p>Ans. Put <math>\sin \theta = x</math></p> $\frac{\sin \theta + 1}{(\sin \theta - 1)(\sin \theta + 2)} = \frac{x + 1}{(x - 1)(x + 2)} = \frac{A}{x - 1} + \frac{B}{x + 2}$ $\therefore \boxed{x + 1 = (x + 2)A + (x - 1)B}$ <p>Put <math>x = 1</math></p> $\therefore 1 + 1 = (1 + 2)A + 0$ $\therefore 2 = 3A$ $\therefore \boxed{\frac{2}{3} = A}$	<div>½</div> <div>½</div> <div>½</div> <div>4</div> <div>½</div> <div>1</div>	



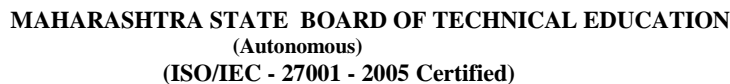
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		<p>Put <math>x = -2</math></p> $\therefore -2 + 1 = 0 + (-2 - 1)B$ $\therefore -1 = -3B$ $\therefore \boxed{\frac{1}{3} = B}$ $\therefore \frac{x+1}{(x-1)(x+2)} = \frac{\frac{2}{3}}{x-1} + \frac{\frac{1}{3}}{x+2}$ $\therefore \boxed{\frac{\sin \theta + 1}{(\sin \theta - 1)(\sin \theta + 2)} = \frac{\frac{2}{3}}{\sin \theta - 1} + \frac{\frac{1}{3}}{\sin \theta + 2}}$ <p style="text-align: center;"><b>OR</b></p> $\frac{\sin \theta + 1}{(\sin \theta - 1)(\sin \theta + 2)} = \frac{A}{\sin \theta - 1} + \frac{B}{\sin \theta + 2}$ $\therefore \boxed{\sin \theta + 1 = (\sin \theta + 2)A + (\sin \theta - 1)B}$ <p>Put <math>\sin \theta = 1</math></p> $\therefore 1 + 1 = (1 + 2)A + 0$ $\therefore 2 = 3A$ $\therefore \boxed{\frac{2}{3} = A}$ <p>Put <math>\sin \theta = -2</math></p> $\therefore -2 + 1 = 0 + (-2 - 1)B$ $\therefore -1 = -3B$ $\therefore \boxed{\frac{1}{3} = B}$ $\therefore \boxed{\frac{\sin \theta + 1}{(\sin \theta - 1)(\sin \theta + 2)} = \frac{\frac{2}{3}}{\sin \theta - 1} + \frac{\frac{1}{3}}{\sin \theta + 2}}$ <hr style="border-top: 1px dashed black;"/>	<p>1</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p>	<p>4</p> <p>4</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		<b>Attempt any four of the following:</b>		
	a)	Prove that $\sin A \sin(60^\circ - A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$		
	Ans.	$\begin{aligned}\sin A \sin(60^\circ - A) \sin(60^\circ + A) &= \sin A (\sin^2 60^\circ - \sin^2 A) \\&= \sin A \left(\frac{3}{4} - \sin^2 A\right) \\&= \frac{1}{4} \sin A [3 - 4 \sin^2 A] \\&= \frac{1}{4} [3 \sin A - 4 \sin^3 A] \\&= \frac{1}{4} \sin 3A\end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned}\sin A \sin(60^\circ - A) \sin(60^\circ + A) &= \sin A \cdot \frac{1}{-2} (\cos 120^\circ - \cos 2A) \\&= -\frac{1}{2} \sin A [\cos(90^\circ + 30^\circ) - \cos 2A] \\&= -\frac{1}{2} \sin A [-\sin 30^\circ - \cos 2A] \\&= \frac{1}{2} \sin A \cdot \left[\frac{1}{2} + 1 - 2 \sin^2 A\right] \\&= \frac{1}{2} \sin A \cdot \left(\frac{3}{2} - 2 \sin^2 A\right) \\&= \frac{1}{4} \sin A [3 - 4 \sin^2 A] \\&= \frac{1}{4} [3 \sin A - 4 \sin^3 A] \\&= \frac{1}{4} \sin 3A\end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned}&\sin A \sin(60^\circ - A) \sin(60^\circ + A) \\&= \sin A (\sin 60^\circ \cos A - \cos 60^\circ \sin A)(\sin 60^\circ \cos A + \cos 60^\circ \sin A) \\&= \sin A (\sin^2 60^\circ \cos^2 A - \cos^2 60^\circ \sin^2 A) \\&= \sin A \left(\frac{3}{4} \cos^2 A - \frac{1}{4} \sin^2 A\right)\end{aligned}$	<div>1</div> <div>1</div> <div>1</div> <div>1</div> <div>1</div> <div>1</div> <div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1/2</div> <div>1</div> <div>1</div>	<div>4</div> <div>4</div>

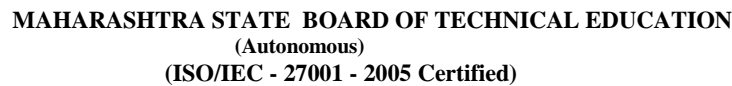


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$= \frac{1}{4} \sin A (3 \cos^2 A - \sin^2 A)$ $= \frac{1}{4} \sin A [3(1 - \sin^2 A) - \sin^2 A]$ $= \frac{1}{4} \sin A [3 - 4 \sin^2 A]$ $= \frac{1}{4} [3 \sin A - 4 \sin^3 A]$ $= \frac{1}{4} \sin 3A$	$\frac{1}{2}$     1	4
	b)	Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$		
	Ans.	$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$ $= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$ $= \tan^{-1}(1)$ $= \frac{\pi}{4}$	1     1   1	
	c)	Prove that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$		4
	Ans.	$\sin 3\theta = \sin(\theta + 2\theta)$ $= \sin \theta \cos 2\theta + \cos \theta \sin 2\theta$ $= \sin \theta (1 - 2 \sin^2 \theta) + \cos \theta (2 \sin \theta \cos \theta)$ $= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta \cos^2 \theta$ $= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta (1 - \sin^2 \theta)$ $= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$	$\frac{1}{2}$ 1 $\frac{1}{2} + \frac{1}{2}$  $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	d)	Prove that $\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A} = \tan\left(\frac{5A}{2}\right)$		
	Ans.	$\begin{aligned}& \frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A}\\&= \frac{(\sin A + \sin 4A) + (\sin 2A + \sin 3A)}{(\cos A + \cos 4A) + (\cos 2A + \cos 3A)}\\&= \frac{2 \sin\left(\frac{5A}{2}\right) \cos\left(\frac{-3A}{2}\right) + 2 \sin\left(\frac{5A}{2}\right) \cos\left(\frac{-A}{2}\right)}{2 \cos\left(\frac{5A}{2}\right) \cos\left(\frac{-3A}{2}\right) + 2 \cos\left(\frac{5A}{2}\right) \cos\left(\frac{-A}{2}\right)}\\&= \frac{2 \sin\left(\frac{5A}{2}\right) \left[ \cos\left(\frac{-3A}{2}\right) + \cos\left(\frac{-A}{2}\right) \right]}{2 \cos\left(\frac{5A}{2}\right) \left[ \cos\left(\frac{-3A}{2}\right) + \cos\left(\frac{-A}{2}\right) \right]}\\&= \frac{\sin\left(\frac{5A}{2}\right)}{\cos\left(\frac{5A}{2}\right)}\\&= \tan\left(\frac{5A}{2}\right)\end{aligned}$ <hr style="border-top: 1px dashed black;"/>	1  1  1  1	4
	e)	Prove that $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$		
	Ans.	$\begin{aligned}\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ &= \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ \\&= \frac{\sqrt{3}}{2} \cdot \frac{-1}{2} (-2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ \\&= -\frac{\sqrt{3}}{4} (\cos 60^\circ - \cos 20^\circ) \sin 80^\circ \\&= -\frac{\sqrt{3}}{4} \left( \frac{1}{2} - \cos 20^\circ \right) \sin 80^\circ \\&= -\frac{\sqrt{3}}{4} \left( \frac{1}{2} \sin 80^\circ - \sin 80^\circ \cos 20^\circ \right) \\&= -\frac{\sqrt{3}}{4} \left( \frac{1}{2} \sin 80^\circ - \frac{1}{2} \cdot 2 \sin 80^\circ \cos 20^\circ \right)\end{aligned}$	$\frac{1}{2}$   $\frac{1}{2}$ $\frac{1}{2}$	

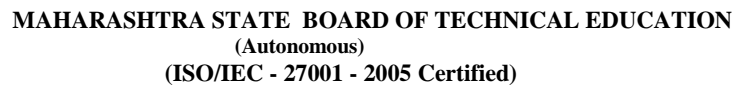


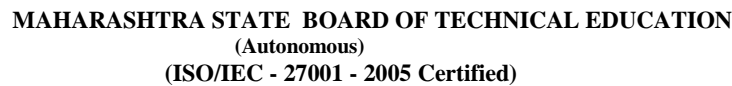


Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\begin{aligned} &= -\frac{\sqrt{3}}{4} \cdot \frac{1}{2} [\sin 80^\circ - (\sin 100^\circ + \sin 60^\circ)] \\ &= -\frac{\sqrt{3}}{8} \left[ \sin 80^\circ - \sin 100^\circ - \frac{\sqrt{3}}{2} \right] \\ &= -\frac{\sqrt{3}}{8} \left[ 2 \cos 90^\circ \sin 20^\circ - \frac{\sqrt{3}}{2} \right] \\ &= -\frac{\sqrt{3}}{8} \left[ 0 - \frac{\sqrt{3}}{2} \right] \\ &= \frac{3}{16} \end{aligned}$ <p><b>Note: 1)</b> If the above problem is proved, using the values of <math>\sin 20^\circ</math>, <math>\sin 40^\circ</math>, <math>\sin 80^\circ</math> with the help of calculator, no marks to be given because under the constraint of the MSBTE Curriculum, it is expected that such problems are to be solved without using calculator.</p> <p><b>Note 2)</b> The above problem may also be solved by making various combinations of sine ratios. Consequently the solutions vary in accordance with the combinations. Please give the appropriate marks in accordance with the scheme of marking. For the sake of convenience one of the solutions is illustrated hereunder.</p> $\begin{aligned} \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ &= \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{-1}{2} (-2 \sin 40^\circ \sin 80^\circ) \sin 20^\circ \\ &= -\frac{\sqrt{3}}{4} (\cos 120^\circ - \cos 40^\circ) \sin 20^\circ \\ &= -\frac{\sqrt{3}}{4} (\cos(90^\circ + 30^\circ) - \cos 40^\circ) \sin 20^\circ \\ &= -\frac{\sqrt{3}}{4} (-\sin 30^\circ - \cos 40^\circ) \sin 20^\circ \\ &= -\frac{\sqrt{3}}{4} \left( -\frac{1}{2} - \cos 40^\circ \right) \sin 20^\circ \\ &= -\frac{\sqrt{3}}{4} \left( -\frac{1}{2} \sin 20^\circ - \sin 20^\circ \cos 40^\circ \right) \end{aligned}$	<div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div> <div style="text-align: center;">1/2</div>	<b>4</b>



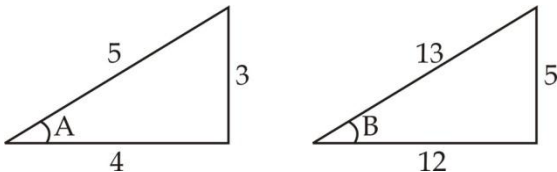
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$= -\frac{\sqrt{3}}{4} \left( -\frac{1}{2} \sin 20^\circ - \frac{1}{2} \cdot 2 \sin 20^\circ \cos 40^\circ \right)$ $= \frac{\sqrt{3}}{4} \cdot \frac{1}{2} [\sin 20^\circ + \sin 60^\circ + \sin (-20^\circ)]$ $= \frac{\sqrt{3}}{8} \left[ \sin 20^\circ + \frac{\sqrt{3}}{2} - \sin 20^\circ \right]$ $= \frac{\sqrt{3}}{8} \left[ \frac{\sqrt{3}}{2} \right]$ $= \frac{3}{16}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	4
	f)	Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$		
	Ans.	$\sin(A+B)\sin(A-B) = -\frac{1}{2} [-2\sin(A+B)\sin(A-B)]$ $= -\frac{1}{2} [\cos[(A+B)+(A-B)] - \cos[(A+B)-(A-B)]]$ $= -\frac{1}{2} [\cos 2A - \cos 2B]$ $= -\frac{1}{2} [1 - 2\sin^2 A - 1 + 2\sin^2 B]$ $= \sin^2 A - \sin^2 B$ <p style="text-align: center;"><b>OR</b></p> $\sin(A+B)\sin(A-B)$ $= [\sin A \cos B + \cos A \sin B][\sin A \cos B - \cos A \sin B]$ $= (\sin A \cos B)^2 - (\cos A \sin B)^2$ $= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$ $= \sin^2 A [1 - \sin^2 B] - [1 - \sin^2 A] \sin^2 B$ $= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$ $= \sin^2 A - \sin^2 B$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	4

[illegible]



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	b)	Prove that $\tan 15^\circ + \tan 75^\circ = 4$		
	Ans.	$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$ $= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}$ $= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$	½	
		$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$ $= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$ $= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$	½	
		$\therefore \tan 15^\circ + \tan 75^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ $= \frac{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2}$ $= 4$	½	4
		<b>OR</b>		
		$\tan 15^\circ + \tan 75^\circ = \tan(45^\circ - 30^\circ) + \tan(45^\circ + 30^\circ)$ $= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} + \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$ $= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} + \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$ $= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ $= \frac{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2}$ $= 4$	½+½	
			½+½	
			½+½	
			1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	c)	<p>Prove that <math>\cos^{-1}\left(\frac{4}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{63}{65}\right)</math></p> <p>Ans.</p> <p>Let <math>A = \cos^{-1}\left(\frac{4}{5}\right)</math>      <math>B = \cos^{-1}\left(\frac{12}{13}\right)</math></p> <p><math>\therefore \cos A = \frac{4}{5}</math>      <math>\cos B = \frac{12}{13}</math></p>  <p><math>\cos(A - B) = \cos A \cos B + \sin A \sin B</math></p> <p><math>= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13} \quad \text{--- (*)}</math></p> <p><math>= \frac{48}{65} + \frac{15}{65}</math></p> <p><math>= \frac{48+15}{65}</math></p> <p><math>= \frac{63}{65} \quad \text{--- (**)}</math></p> <p><math>\therefore A - B = \cos^{-1}\left(\frac{63}{65}\right)</math></p> <p><math>\therefore \cos^{-1}\left(\frac{4}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{63}{65}\right)</math></p> <p><b>Note:</b> Due to the use of advance scientific calculators which is permissible in the exam, students may write the step (**) directly after step (*). Writing such step are to be considered.</p> <p><b>Note:</b> To evaluate value of sin A and sin B, many times the relation between sine ratio and cosine ratio is used, instead of using Triangle Method as illustrated in the above solution. As the main object is to find the values, please consider these methods also. This is illustrated hereunder:</p> <p><math>\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}</math></p> <p><math>\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}</math></p>	1 1 1 1/2 1/2	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	d)	<p>Prove that <math>\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A \tan 3A</math></p>		
	Ans.	$\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \frac{(\cos 2A + \cos 6A) + 2\cos 4A}{(\cos A + \cos 5A) + 2\cos 3A}$ $= \frac{2\cos 4A \cos(-2A) + 2\cos 4A}{2\cos 3A \cos(-2A) + 2\cos 3A}$ $= \frac{2\cos 4A [\cos(-2A) + 1]}{2\cos 3A [\cos(-2A) + 1]}$ $= \frac{\cos 4A}{\cos 3A}$ $= \frac{\cos(A + 3A)}{\cos 3A}$ $= \frac{\cos A \cos 3A - \sin A \sin 3A}{\cos 3A}$ $= \cos A - \sin A \tan 3A$ <p style="text-align: center;"><b>OR</b></p> $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \frac{(\cos 6A + \cos 2A) + 2\cos 4A}{(\cos 5A + \cos A) + 2\cos 3A}$ $= \frac{2\cos 4A \cos 2A + 2\cos 4A}{2\cos 3A \cos 2A + 2\cos 3A}$ $= \frac{2\cos 4A [\cos 2A + 1]}{2\cos 3A [\cos 2A + 1]}$ $= \frac{\cos 4A}{\cos 3A}$ $= \frac{\cos(A + 3A)}{\cos 3A}$ $= \frac{\cos A \cos 3A - \sin A \sin 3A}{\cos 3A}$ $= \cos A - \sin A \tan 3A$ <p style="text-align: center;"><b>OR</b></p> $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \frac{\cos 2A + \cos 4A + \cos 4A + \cos 6A}{\cos A + \cos 3A + \cos 3A + \cos 5A}$ $= \frac{2\cos 3A \cos A + 2\cos 5A \cos A}{2\cos 2A \cos A + 2\cos 4A \cos A}$	<p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p>	<p>4</p> <p>4</p>

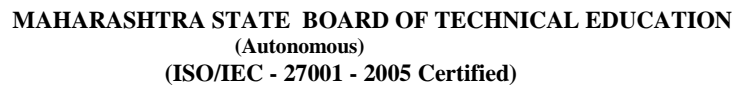


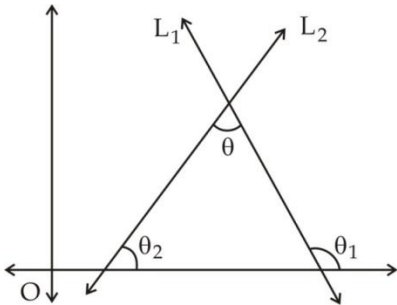
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$  \begin{aligned}  &= \frac{2 \cos A (\cos 3A + \cos 5A)}{2 \cos A (\cos 2A + \cos 4A)} \\  &= \frac{\cos 3A + \cos 5A}{\cos 2A + \cos 4A} \\  &= \frac{2 \cos 4A \cos A}{2 \cos 3A \cos A} \\  &= \frac{\cos 4A}{\cos 3A} \\  &= \frac{\cos (A + 3A)}{\cos 3A} \\  &= \frac{\cos A \cos 3A - \sin A \sin 3A}{\cos 3A} \\  &= \cos A - \sin A \tan 3A  \end{aligned}  $ <p style="text-align: center;"><b>OR</b></p> $  \begin{aligned}  LHS &= \frac{\cos 2A + 2 \cos 4A + \cos 6A}{\cos A + 2 \cos 3A + \cos 5A} \\  &= \frac{(\cos 2A + \cos 6A) + 2 \cos 4A}{(\cos A + \cos 5A) + 2 \cos 3A} \\  &= \frac{2 \cos 4A \cos (-2A) + 2 \cos 4A}{2 \cos 3A \cos (-2A) + 2 \cos 3A} \\  &= \frac{2 \cos 4A [\cos (-2A) + 1]}{2 \cos 3A [\cos (-2A) + 1]} \\  &= \frac{\cos 4A}{\cos 3A} \\  RHS &= \cos A - \sin A \tan 3A \\  &= \cos A - \sin A \cdot \frac{\sin 3A}{\cos 3A} \\  &= \frac{\cos A \cos 3A - \sin A \sin 3A}{\cos 3A} \\  &= \frac{\cos (A + 3A)}{\cos 3A} \\  &= \frac{\cos 4A}{\cos 3A} \\  \therefore LHS &= RHS  \end{aligned}  $ <p>-----</p>	<p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p>1</p> <p>1</p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p>	<p>4</p> <p>4</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	e)	<p>If x and y are positive, then prove that</p> $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right), \quad 1-xy \geq 0$		
	Ans.	<p>Put <math>\tan^{-1} x = A</math> and <math>\tan^{-1} y = B</math>  <math>\therefore x = \tan A</math> and <math>y = \tan B</math>  <math>\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}</math>  <math display="block">= \frac{x+y}{1-xy}</math>  <math>\therefore A+B = \tan^{-1} \left( \frac{x+y}{1-xy} \right)</math>  <math>\therefore \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)</math></p> <p style="text-align: center;"><b>OR</b></p> $\therefore \tan(\tan^{-1} x + \tan^{-1} y) = \frac{\tan(\tan^{-1} x) + \tan(\tan^{-1} y)}{1 - \tan(\tan^{-1} x) \tan(\tan^{-1} y)}$ $= \frac{x+y}{1-xy}$ $\therefore \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$ <hr/>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1 ½</p> <p>1 ½</p> <p>1</p>	<p><b>4</b></p> <p><b>4</b></p>
	f)	<p>Prove that <math>\cos C + \cos D = 2 \cos \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right)</math></p>		
	Ans.	<p>We know that,</p> $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ <p>Put <math>A+B = C</math>  <math>A-B = D</math>  <math>\therefore A = \frac{C+D}{2}</math> and <math>B = \frac{C-D}{2}</math>  <math>\therefore \cos C + \cos D = 2 \cos \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right)</math></p> <hr/>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p><b>4</b></p>





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<b>Attempt any four of the following:</b>		
	a)	Find the equation of the line passing through the point of intersection of the lines $x + y = 0$ , $2x - y = 9$ and parallel to the line $3x + 2y - 1 = 0$ .		
	Ans.	$x + y = 0$ $\underline{2x - y = 9}$ $\therefore 3x = 9$ $\therefore x = 3$ $y = -3$ $\therefore \text{Point of intersection} = (3, -3)$ <p>Slope of the line <math>3x + 2y - 1 = 0</math> is,</p> $m_0 = -\frac{a}{b} = -\frac{3}{2}$ $\therefore \text{Slope of the required line is,}$ $m = m_0 = -\frac{3}{2}$ $\therefore \text{equation is,}$ $y - y_1 = m(x - x_1)$ $\therefore y + 3 = -\frac{3}{2}(x - 3)$ $\therefore 2y + 6 = -3x + 9$ $\therefore 3x + 2y - 3 = 0$ <hr style="border-top: 1px dashed black;"/>	<div style="text-align: right;">1/2</div> <div style="text-align: right;">1/2</div> <div style="text-align: right;">1</div> <div style="text-align: right;">1</div> <div style="text-align: right;">1/2</div> <div style="text-align: right;">1/2</div>	
	b)	If $m_1$ and $m_2$ are the slopes of two lines, prove that the acute angle between the lines is $\theta = \tan^{-1} \left  \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right $		
	Ans.		1	
				<b>4</b>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<p>Let <math>\theta_1</math> = Angle of inclination of <math>L_1</math>  <math>\theta_2</math> = Angle of inclination of <math>L_2</math>  <math>\therefore</math> Slope of <math>L_1</math> is <math>m_1 = \tan \theta_1</math>  Slope of <math>L_2</math> is <math>m_2 = \tan \theta_2</math></p> <p><math>\therefore</math> from figure,  <math>\theta = \theta_1 - \theta_2</math>  <math>\therefore \tan \theta = \tan (\theta_1 - \theta_2)</math>  <math display="block">= \frac{\tan (\theta_1) - \tan (\theta_2)}{1 + \tan (\theta_1) \tan (\theta_2)}</math>  <math display="block">= \frac{m_1 - m_2}{1 + m_1 \cdot m_2}</math>  <math display="block">\therefore \theta = \tan^{-1} \left( \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right)</math>  For angle to be acute,  <math display="block">\theta = \tan^{-1} \left  \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right </math></p> <hr/>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	4
	c)	Find the length of the perpendicular from (3, 2) on the line $4x - 6y = 5$ .		
	Ans.	<p>Given <math>4x - 6y - 5 = 0</math>  <math>\therefore A = 4, B = -6, C = -5</math>  <math>\therefore</math> the length of the perpendicular is,</p> $p = \left  \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right $ $= \left  \frac{4(3) - 6(2) - 5}{\sqrt{4^2 + (-6)^2}} \right $ $= \frac{5}{\sqrt{52}} \quad \text{or} \quad 0.693$	<p>2</p> <p>1+1</p>	4
		<p><b>Note:</b> If -ve sign is left with the answer, 1 mark is to be deducted.</p> <hr/>		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	d)	Find the equation of a line passing through the point of intersection of $x + y = 0$ , $2x - y = 9$ and through the point (2, 5)		
	Ans.	$\begin{aligned} x + y &= 0 \\ 2x - y &= 9 \\ \hline \therefore 3x &= 9 \\ \therefore x &= 3 \\ y &= -3 \\ \therefore \text{Point of intersection} &= (3, -3) \\ \therefore \text{equation is,} \\ \frac{y - y_1}{y_2 - y_1} &= \frac{x - x_1}{x_2 - x_1} \\ \therefore \frac{y - 5}{-3 - 5} &= \frac{x - 2}{3 - 2} \\ \therefore 8x + y - 21 &= 0 \end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned} \therefore \text{Point of intersection} &= (3, -3) \\ \therefore \text{Slope } m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{3 - 2} = -8 \\ \therefore \text{equation is,} \\ y - y_1 &= m(x - x_1) \\ \therefore y - 5 &= -8(x - 2) \\ \therefore 8x + y - 21 &= 0 \end{aligned}$ <hr style="border-top: 1px dashed black;"/>	<p>1 1</p> <p>1 1</p> <p>OR</p> <p>1/2</p> <p>1/2</p> <p>1</p>	<p>4</p> <p>4</p>
	e)	Find the length of perpendicular from $(-3, -4)$ on the line $4(x + 2) = 3(y - 4)$ .		
	Ans.	$\begin{aligned} \text{Given } 4(x + 2) &= 3(y - 4) \\ \therefore 4x - 3y + 20 &= 0 \\ \therefore \text{the length of perpendicular is,} \\ P &= \left  \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right  \\ &= \left  \frac{4(-3) - 3(-4) + 20}{\sqrt{4^2 + (-3)^2}} \right  \\ &= 4 \end{aligned}$	<p>1</p> <p>2</p> <p>1</p>	<p>4</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	f)	Find the distance between the lines $5x - 12y + 1 = 0$ and $10x - 24y = 1$ . Also prove that these lines are parallel to each other.		
	Ans.	<p>Given <math>5x - 12y + 1 = 0</math> and <math>10x - 24y = 1</math>  <math>\therefore 10x - 24y + 2 = 0</math> and <math>10x - 24y - 1 = 0</math>  <math>\therefore A = 10, B = -24, C_1 = 2</math> and <math>C_2 = -1</math></p> $\therefore p = \frac{ C_1 - C_2 }{\sqrt{A^2 + B^2}}$ $= \frac{ 2 + 1 }{\sqrt{10^2 + (-24)^2}}$ $= \frac{3}{26} \quad \text{or} \quad 0.115$ <p style="text-align: center;"><b>OR</b></p> <p>Given <math>5x - 12y + 1 = 0</math> and <math>10x - 24y = 1</math>  <math>\therefore 5x - 12y + 1 = 0</math> and <math>5x - 12y - \frac{1}{2} = 0</math>  <math>\therefore A = 5, B = -12, C_1 = 1</math> and <math>C_2 = -\frac{1}{2}</math></p> $\therefore p = \frac{ C_1 - C_2 }{\sqrt{A^2 + B^2}} = \frac{ 1 + \frac{1}{2} }{\sqrt{5^2 + (-12)^2}}$ $= \frac{3}{26} \quad \text{or} \quad 0.115$ <p><b>Note:</b> If the -ve value is written by the student (i.e., <math>-\frac{3}{26}</math> or <math>-0.115</math>, deduct <math>\frac{1}{2}</math> mark.</p> <p>For <math>5x - 12y + 1 = 0</math>,  <math>m_1 = -\frac{a}{b} = -\frac{5}{-12} = \frac{5}{12}</math>  For <math>10x - 24y = 1</math>,  <math>m_2 = -\frac{a}{b} = -\frac{10}{-24} = \frac{5}{12}</math>  <math>\therefore m_1 = m_2</math>  <math>\therefore</math> the lines are parallel.</p>	<p>1</p> <p>1</p> <p>OR</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	4