17217

21314

3 Hours/100 Marks

Seat No. | | | | | | | | |

Instructions : *(1)* ***All*** *questions are* ***compulsory****.*
(2) Answer ***each*** *next* ***main*** *question on a* ***new*** *page.*
(3) Illustrate your answers with neat sketches ***wherever necessary****.*
(4) Figures to the ***right*** *indicate* ***full*** *marks.*
(5) Assume ***suitable data****, if necessary.*
(6) ***Use*** *of non-programmable Electronic Pocket Calculator is permissible.*

MARKS

1. Attempt **any ten** of the following : **20**

a) If f(x) = tan x, prove that $f(2x) = \frac{2f(x)}{1-f^2(x)}$.

b) Evaluate $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$.

c) If $y = e^{2\log x} + e^{x\log 2}$ find $\frac{dy}{dx}$.

d) If $y = \log_{10} x + \log_e x + \log_5 5$ find $\frac{dy}{dx}$.

e) If $y = \tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$ find $\frac{dy}{dx}$.

f) If $y = \frac{e^x - 1}{e^x + 1}$ find $\frac{dy}{dx}$.

g) Evaluate $\int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$.

P.T.O.

MARKS

h) Evaluate $\int_0^{\infty} e^{-x} dx$.

i) The crushing strength of some concrete blocks are given as follows :

$4.8 \times 10^4, 4.2 \times 10^4, 5.1 \times 10^4, 3.8 \times 10^4, 4.4 \times 10^4, 4.7 \times 10^4$. Find the mean crushing strength.

j) Find the median of following data :

x_i :	4	7	10	13	16	19
f_i :	7	10	25	20	25	30

k) Find the standard deviation for following data :
1, 2, 3, 4, 5, 6, 7, 8, 9.

l) Evaluate $\int \sin^2 x \, dx$.

2. Attempt **any four** of the following : **16**

a) If f(x) = log (1 + tanx) prove that $f\left(\frac{\pi}{4} - x\right) = \log_2 - f(x)$.

b) Evaluate $\lim_{x \to 0} \frac{\log(x+a) - \log a}{x}$.

c) Evaluate $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

d) Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

e) Differentiate w.r.t. x. $x^{\sin x} + (\sin x)^x$.

f) If $x = a\cos^3\theta, y = a\sin^3\theta$ find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ at $\theta = \pi/4$.

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3. Attempt **any four** of following : **16**

a) Evaluate $\int \frac{\log\left(\tan \frac{x}{2}\right)}{\sin x} dx$.

b) Evaluate $\int \frac{\sec^2 x}{3\tan^2 x - 2\tan x - 5} dx$.

c) Evaluate $\int \frac{dx}{2 - 3\cos^2 x}$.

d) Evaluate $\int_0^1 x \tan^{-1} x \, dx$.

e) Evaluate $\int_0^{\pi/2} \frac{dx}{1 + \sqrt[3]{\cot x}}$.

f) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x \, dx}{\sin^2 x + 3\sin x + 2}$.

4. Attempt **any four** of the following : **16**

a) Show that $\frac{x}{a} + \frac{y}{b} = 2$ is the tangent to the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^n = 2$.

b) A metal wire 36 cm long is bent to form a rectangle. Find its dimensions, when its area is maximum.

c) The mean marks recorded by 27 candidates happened to be 64%. During the time of verification it is found that one candidates actually scores 74% marks instead of 47% as recorded. Find the correct mean.

d) The mean weight of one group of student is 55.5 kg and that of another group is 63.6 kg. If the combined mean is 58.2 kg. Find ratio between students of two groups.

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e) Find the mode of the following data :

Wages	45 – 59	60 – 74	75 – 89	90 – 104	105 – 119	120 – 134	135 – 149
No. of workers	43	99	152	175	160	40	25

f) Find the median graphically by plotting the ogive curve of the following distribution :

Marks	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
No. of student	4	6	10	5	7	3	9	6

5. Attempt **any four** of the following : **16**

a) Find the mean deviation from mean of the following data :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of student	5	8	15	16	6

b) Find the variance of the following :

Expenditure below	5	10	15	20	25
No. of student	6	16	28	38	46

c) Following are marks obtained by two students x and y :

Marks obtained by x	44	80	76	48	52	72	68	56
Marks obtained by y	48	75	54	60	63	69	72	51

Which of two students is more consistent ?

MARKS

d) Find the combined standard deviation of groups A and B taken together given that :

Group	Size	Arithmetic mean	Standard deviation
A	100	66	6
B	200	63	4

e) Find the quartile deviation of the following data :

C.I.	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
f_i	3	5	9	15	20	16	10	2

f) Find the mean deviation from median :

x_i	10	11	12	13	14
f_i	3	12	18	12	3

6. Attempt **any two** of the following : **16**

a) Following are the values of import of raw material and export of finished product in suitable units :

Export	10	11	14	14	20	22	16	12	15	13
Import	12	14	15	16	21	26	21	15	16	14

Calculate the Karl Pearson's coefficient of correlation between the import and export values.

MARKS

b) Calculate Spearman's rank correlation coefficient between the following marks given by two judges in series of eight one-act plays in a drama competition :

One-act play No.	1	2	3	4	5	6	7	8
Marks by Judge A	81	72	60	33	29	11	56	42
Marks by Judge B	75	56	42	15	30	20	60	80

c) Obtain the regression lines for following data :

x :	2	3	5	7	8	10	12	15
y :	2	5	8	10	12	14	15	16

Find estimate of :

i) y when x = 6

ii) x when y = 20.
