MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC - 27001 - 2005 Certified)

Winter – 2012 Examination

Subject & Code: Basic Maths (12003) **Model Answer**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	a)	$\log_3 243 = \log_3\left(3^5\right)$	½	
		$= 5\log_3(3)$	½	**2**
		$= 5$	1	
		OR		
		$Put\ \log_3 243 = x$		
		$\therefore 3^x = 243$	½	
		$\therefore 3^x = 3^5$	½	
		$\therefore x = 5$	½	
		$\therefore \log_3 243 = 5$	½	**2**
	b)	$\dfrac{x+4}{x(x+1)} = \dfrac{A}{x} + \dfrac{B}{x+1}$	½	
		$\therefore x+4 = (x+1)A + xB$		
		$Put\ x = 0$		
		$\therefore 0+4 = (1)A$		
		$\therefore \boxed{A = 4}$	½	
		$Put\ x = -1$		
		$\therefore -1+4 = 0 + (-1)B$		
		$\therefore \boxed{B = -3}$	½	
		$\therefore \boxed{\dfrac{x+4}{x(x+1)} = \dfrac{4}{x} + \dfrac{-3}{x+1}}$	½	**2**
	c)	$\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix} = 0$		
		$\therefore 2(-2x-2x) - 3(-12-8) + 1(6x-4x) = 0$	1	
		$\therefore -8x + 60 + 2x = 0$		
		$\therefore -6x + 60 = 0$		
		$\therefore \boxed{x = 10}$	1	**2**
	d)	$A - 3B = \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 3 & -2 \end{bmatrix}$		
		$= \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 9 & -6 \end{bmatrix}$	1	
		$= \begin{bmatrix} 6 & 6 \\ -7 & 7 \end{bmatrix}$	1	**2**

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1)		**OR**		
		$3B=3\begin{bmatrix}0&-1\\3&-2\end{bmatrix}=\begin{bmatrix}0&-3\\9&-6\end{bmatrix}$	1	
		$\therefore A-3B=\begin{bmatrix}6&3\\2&1\end{bmatrix}-\begin{bmatrix}0&-3\\9&-6\end{bmatrix}$		
		$=\begin{bmatrix}6&6\\-7&7\end{bmatrix}$	1	**2**
	e)	$\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$		
		$=\dfrac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$ *or* $\dfrac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}$		
		$=\dfrac{2}{\sin^2\theta}$	1	
		$=2\cos ec^2\theta$	1	**2**
	f)	$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$	½	
		$=\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}$	½	
		$=1$	1	**2**
	g)	$\sin 3A=3\sin A-4\sin^3 A$	½	
		$=3(0.6)-4(0.6)^3$	½	
		$=0.936$	1	**2**
	h)	$\cos\left[\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{1}{2}\right)\right]=\cos\left[\dfrac{\pi}{2}-\dfrac{\pi}{6}\right]$	½	
		$=\cos\left[\dfrac{\pi}{3}\right]$	½	
		$=\dfrac{1}{2}$ *or* 0.5	1	**2**
		OR		
		$\cos\left[\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{1}{2}\right)\right]=\cos[90^\circ-30^\circ]$	½	
		$=\cos[60^\circ]$	½	
		$=\dfrac{1}{2}$ *or* 0.5	1	**2**

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1)		**OR**		
		$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	½	
		$\therefore \cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$		
		$=\cos\left[\frac{\pi}{3}\right]$	½	
		$=\frac{1}{2}$ *or* 0.5	1	**2**
	i)	*Let* $A=(4,3),\quad B=(-1,1)$		
		$\therefore d(AB)$ *or* $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$		
		$=\sqrt{(-1-4)^2+(1-3)^2}$	1	
		$=\sqrt{29}$ *or* 5.385	1	**2**
		OR		
		The distance between $(4,3),\ (-1,1)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$		
		$=\sqrt{(-1-4)^2+(1-3)^2}$	1	
		$=\sqrt{29}$ *or* 5.385	1	**2**
	j)	$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$		
		$\therefore \frac{y+3}{6+3}=\frac{x-2}{-4-2}$	½	
		$\therefore \frac{y+3}{9}=\frac{x-2}{-6}$		
		$\therefore -6(y+3)=9(x-2)$	½	
		$\therefore 9x+6y=0$ *or* $3x+2y=0$	1	**2**
		OR		
		$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6+3}{-4-2}$		
		$\therefore m=\frac{9}{-6}=-\frac{3}{2}$	½	
		$\therefore$ *the equation is,*		
		$y-y_1=m(x-x_1)$		
		$\therefore y+3=-\frac{3}{2}(x-2)$	½	
		$\therefore 3x+2y=0$	1	**2**

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1)	k)	$Here\ a=5,\ b=-2\sqrt{6},\ \ c_1=1,\ \ c_2=-10$ $p=\left\|\dfrac{c_2-c_1}{\sqrt{a^2+b^2}}\right\|$ $=\left\|\dfrac{-10-1}{\sqrt{5^2+\left(-2\sqrt{6}\right)^2}}\right\|$ $=\dfrac{11}{7}\quad or\quad 1.571$	 1 1	 **2**
	l)	$2g=0,\ \ 2f=-12,\ \ c=5$ $\therefore g=0,\ \ f=-6,\ \ c=5$ $\therefore Center=\left(-g,\ -f\right)=\left(0,\ 6\right)$ $Radius=\sqrt{g^2+f^2-c}$ $=\sqrt{0^2+6^2-5}$ $=\sqrt{31}\quad or\quad 5.568$	 1 1	 **2**
2)	a)	$\dfrac{1}{\log_{ab} abc}+\dfrac{1}{\log_{bc} abc}+\dfrac{1}{\log_{ca} abc}$ $=\log_{abc} ab+\log_{abc} bc+\log_{abc} ca$ $=\log_{abc}\left(ab\times bc\times ca\right)$ $=\log_{abc}\left(abc\right)^2$ $=2\log_{abc}\left(abc\right)$ $=2$ **OR** $\dfrac{1}{\log_{ab} abc}+\dfrac{1}{\log_{bc} abc}+\dfrac{1}{\log_{ca} abc}$ $=\dfrac{\log ab}{\log abc}+\dfrac{\log bc}{\log abc}+\dfrac{\log ca}{\log abc}$ $=\log_{abc} ab+\log_{abc} bc+\log_{abc} ca$ $=\log_{abc}\left(ab\times bc\times ca\right)$ $=\log_{abc}\left(abc\right)^2$ $=2\log_{abc}\left(abc\right)$ $=2$ **OR**	 1 1 1 1 1 1 1 1	 **4** **4**

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2)		$\frac{1}{\log_{ab} abc}+\frac{1}{\log_{bc} abc}+\frac{1}{\log_{ca} abc}$		
		$=\frac{1}{\frac{\log abc}{\log ab}}+\frac{1}{\frac{\log abc}{\log bc}}+\frac{1}{\frac{\log abc}{\log ca}}$		
		$=\frac{\log ab}{\log abc}+\frac{\log bc}{\log abc}+\frac{\log ca}{\log abc}$	1	
		$=\frac{\log ab+\log bc+\log ca}{\log abc}$		
		$=\frac{\log\left(ab\times bc\times ca\right)}{\log abc}$	1	
		$=\frac{\log\left(abc\right)^2}{\log abc}$		
		$=\frac{2\log\left(abc\right)}{\log abc}$	1	
		$=2$	1	**4**
	b)	$\frac{4x^2+x-1}{x^3-x}=\frac{4x^2+x-1}{x\left(x+1\right)\left(x-1\right)}$	½	
		$=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$		
		$\therefore 4x^2+x-1=\left(x+1\right)\left(x-1\right)A+x\left(x-1\right)B+x\left(x+1\right)C$		
		$Put\ \ x=0$		
		$\therefore 0+0-1=\left(0+1\right)\left(0-1\right)A+0+0$		
		$\therefore -1=-A$	1	
		$\therefore \boxed{A=1}$		
		$Put\ \ x=-1$		
		$\therefore 4\left(-1\right)^2-1-1=0-1\left(-1-1\right)B+0$		
		$\therefore 2=2B$	1	
		$\therefore \boxed{B=1}$		
		$Put\ \ x=1$		
		$\therefore 4\left(1\right)^2+1-1=0+0+1\left(1+1\right)C$		
		$\therefore 4=2C$		
		$\therefore \boxed{C=2}$	1	
		$\therefore \boxed{\frac{4x^2+x-1}{x^3-x}=\frac{1}{x}+\frac{1}{x+1}+\frac{2}{x-1}}$	½	**4**

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2)	c)	$\frac{x-2}{x^3+1}=\frac{x-2}{(x+1)(x^2-x+1)}$ $=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$ $\therefore x-2=(x^2-x+1)A+(x+1)(Bx+C)$ $Put\ x=-1$ $\therefore -1-2=((-1)^2+1+1)A+0$ $\therefore -3=3A$ $\therefore \boxed{A=-1}$ $Put\ x=0$ $\therefore 0-2=(0^2-0+1)A+(0+1)(0+C)$ $\therefore -2=A+C$ $\therefore -2=-1+C$ $\therefore \boxed{C=-1}$ $Put\ x=1$ $\therefore 1-2=(1^2-1+1)A+(1+1)(B+C)$ $\therefore -1=A+2B+2C$ $\therefore -1=-1+2B-2$ $\therefore \boxed{B=1}$ $\therefore \boxed{\frac{x-2}{x^3+1}=\frac{-1}{x+1}+\frac{x-1}{x^2-x+1}}$ **Note for Partial Fraction Methods:** The above problems of partial fractions could be solved by the method of "equating equal power coefficients" also. This method is also applicable. Give appropriate marks in accordance with the scheme of marking. As this method is very tedious and complicated, hardly someone use this method in such cases. So such solution methods for partial fraction problems are not illustrated herein.	½ 1 1 1 ½	 **4**
	d)	$D=\begin{vmatrix}2&3&1\\1&1&1\\4&4&1\end{vmatrix}=2(1-4)-3(1-4)+1(4-4)$ $=3$	1	

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2)		$D_x = \begin{vmatrix} 4 & 3 & 1 \\ 1 & 1 & 1 \\ 16 & 4 & 1 \end{vmatrix} = 4(1-4)-3(1-16)+1(4-16)$ $= 21$	½	
		$D_y = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 1 & 1 \\ 4 & 16 & 1 \end{vmatrix} = 2(1-16)-4(1-4)+1(16-4)$ $= -6$	½	
		$D_z = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 4 & 4 & 16 \end{vmatrix} = 2(16-4)-3(16-4)+4(4-4)$ $= -12$	½	
		$\therefore x = \frac{D_x}{D} = \frac{21}{3} = 7$	½	
		$y = \frac{D_y}{D} = \frac{-6}{3} = -2$	½	
		$z = \frac{D_z}{D} = \frac{-12}{3} = -4$	½	
	e)	$\frac{1}{1-x} = (1-x)^{-1}$		
		$= 1-(-1)x + \frac{(-1)(-2)}{2!}x^2 - \frac{(-1)(-2)(-3)}{3!}x^3 + \ldots$	2	
		$= 1 + x + x^2 + x^3 + \ldots$	2	**4**
	f)	$T_{r+1} = {}^nC_r a^r b^{n-r}$		
		$Here\ n = 10,\ a = \frac{x}{y},\quad b = -\frac{y}{x},\quad r = 6$		
		$\therefore T_7 = T_{6+1}$		
		$= {}^{10}C_6 \left(\frac{x}{y}\right)^6 \left(-\frac{y}{x}\right)^{10-6}$	1	
		$= 210 \cdot \frac{x^6}{y^6} \cdot \frac{y^4}{x^4}$	1+1	
		$= 210 \cdot \frac{x^2}{y^2}$	1	**4**
		Note: As the use of non-programmable scientific calculator is allowed, the value of ${}^nC_r$ can be calculated directly using calculator, so students are not supposed to use the corresponding formula.		

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3)	a)	$A^2-9A+14I=\begin{bmatrix}4&3\\2&5\end{bmatrix}\begin{bmatrix}4&3\\2&5\end{bmatrix}-9\begin{bmatrix}4&3\\2&5\end{bmatrix}+14\begin{bmatrix}1&0\\0&1\end{bmatrix}$		
		$=\begin{bmatrix}22&27\\18&31\end{bmatrix}-\begin{bmatrix}36&27\\18&45\end{bmatrix}+\begin{bmatrix}14&0\\0&14\end{bmatrix}$	1+1+1	
		$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$	1	
		OR		
		$A^2=\begin{bmatrix}4&3\\2&5\end{bmatrix}\begin{bmatrix}4&3\\2&5\end{bmatrix}$		
		$=\begin{bmatrix}22&27\\18&31\end{bmatrix}$	1	
		$9A=9\begin{bmatrix}4&3\\2&5\end{bmatrix}$		
		$=\begin{bmatrix}36&27\\18&45\end{bmatrix}$	1	
		$14I=14\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}14&0\\0&14\end{bmatrix}$	1	
		$\therefore A^2-9A+14I=\begin{bmatrix}22&27\\18&31\end{bmatrix}-\begin{bmatrix}36&27\\18&45\end{bmatrix}+\begin{bmatrix}14&0\\0&14\end{bmatrix}$		
		$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$	1	**4**
	b)	The cofactor matrix of A is,		
		$C(A)=\begin{bmatrix}\begin{vmatrix}-3&0\\0&-1\end{vmatrix}&-\begin{vmatrix}2&0\\6&-1\end{vmatrix}&\begin{vmatrix}2&-3\\6&0\end{vmatrix}\\-\begin{vmatrix}4&1\\0&-1\end{vmatrix}&\begin{vmatrix}1&1\\6&-1\end{vmatrix}&-\begin{vmatrix}1&4\\6&0\end{vmatrix}\\\begin{vmatrix}4&1\\-3&0\end{vmatrix}&-\begin{vmatrix}1&1\\2&0\end{vmatrix}&\begin{vmatrix}1&4\\2&-3\end{vmatrix}\end{bmatrix}$	1	
		$=\begin{bmatrix}3&2&18\\4&-7&24\\3&2&-11\end{bmatrix}$ ----------(*)	2*	
		$\therefore adj(A)=\begin{bmatrix}3&4&3\\2&-7&2\\18&24&-11\end{bmatrix}$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		**(*) Note:** In the matrix *C(A)*, if 1 to 3 elements are wrong (either in sign or value), deduct ½ mark, if 4 to 6 elements are wrong, deduct 1½ marks, if 7 to 9 are wrong, deduct all the 2 marks. Further, if all the elements in the last i.e., *adj (A)* are correct, then only give 1 mark.		
		OR		
		The matrix of minors is, $M(A)=\begin{bmatrix} \begin{vmatrix}-3&0\\0&-1\end{vmatrix} & \begin{vmatrix}2&0\\6&-1\end{vmatrix} & \begin{vmatrix}2&-3\\6&0\end{vmatrix} \\ \begin{vmatrix}4&1\\0&-1\end{vmatrix} & \begin{vmatrix}1&1\\6&-1\end{vmatrix} & \begin{vmatrix}1&4\\6&0\end{vmatrix} \\ \begin{vmatrix}4&1\\-3&0\end{vmatrix} & \begin{vmatrix}1&1\\2&0\end{vmatrix} & \begin{vmatrix}1&4\\2&-3\end{vmatrix}\end{bmatrix}$	1	
		$=\begin{bmatrix}3&-2&18\\-4&-7&-24\\3&-2&-11\end{bmatrix}$	1	
		$\therefore$ the matrix of cofactors is, $C(A)=\begin{bmatrix}3&2&18\\4&-7&24\\3&2&-11\end{bmatrix}$	1	
		$\therefore adj(A)=\begin{bmatrix}3&4&3\\2&-7&2\\18&24&-11\end{bmatrix}$	1	**4**
		OR		
		$A_{11}=\begin{vmatrix}-3&0\\0&-1\end{vmatrix}$ $A_{12}=-\begin{vmatrix}2&0\\6&-1\end{vmatrix}$ $A_{13}=\begin{vmatrix}2&-3\\6&0\end{vmatrix}$ $A_{21}=-\begin{vmatrix}4&1\\0&-1\end{vmatrix}$ $A_{22}=\begin{vmatrix}1&1\\6&-1\end{vmatrix}$ $A_{23}=-\begin{vmatrix}1&4\\6&0\end{vmatrix}$ $A_{31}=\begin{vmatrix}4&1\\-3&0\end{vmatrix}$ $A_{32}=-\begin{vmatrix}1&1\\2&0\end{vmatrix}$ $A_{33}=\begin{vmatrix}1&4\\2&-3\end{vmatrix}$	2	
		Note: In the above, if 1 to 3 elements are wrong, deduct ½ mark, if 4 to 6 elements are wrong, deduct 1½ marks, and if 7 to 9 are wrong, deduct all the marks. Further, if all the elements in the following matrices C(A) and adj (A) are correct, then only give the marks.		

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3)		$\therefore$ the matrix of cofactors is, $$C(A)=\begin{bmatrix}3 & 2 & 18\\4 & -7 & 24\\3 & 2 & -11\end{bmatrix}$$	1	
		$$\therefore adj(A)=\begin{bmatrix}3 & 4 & 3\\2 & -7 & 2\\18 & 24 & -11\end{bmatrix}$$	1	**4**
	c)	$$\lvert A\rvert=\begin{vmatrix}2 & -1 & 0\\1 & 0 & 4\\1 & -1 & 1\end{vmatrix}=2(0+4)+1(1-4)+0$$ $$=5$$	1	
		Note: To find the adj (A), students may follow any of methods as shown in the question 3 (b). Please give appropriate marks, as per scheme of marking discussed in the question 3 (b).		
		$$adj(A)=\begin{bmatrix}4 & 1 & -4\\3 & 2 & -8\\-1 & 1 & 1\end{bmatrix}$$	2	
		$$\therefore A^{-1}=\frac{1}{\lvert A\rvert}adj(A)$$ $$=\frac{1}{5}\begin{bmatrix}4 & 1 & -4\\3 & 2 & -8\\-1 & 1 & 1\end{bmatrix}$$	1	**4**
	d)	$x+y+z=6$ $x-y+2z=5$ $2x+y-z=1$ $$\therefore A=\begin{bmatrix}1 & 1 & 1\\1 & -1 & 2\\2 & 1 & -1\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}6\\5\\1\end{bmatrix}$$		
		$$\therefore \lvert A\rvert=\begin{vmatrix}1 & 1 & 1\\1 & -1 & 2\\2 & 1 & -1\end{vmatrix}=1(1-2)-1(-1-4)+1(1+2)$$ $$=7$$	½	
		Note: To find the adj (A), students may follow any of methods as shown in the question 3 (b). Please give appropriate marks, as per scheme of marking discussed in the question 3 (b).		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore adj(A)=\begin{bmatrix}-1 & 2 & 3\\ 5 & -3 & -1\\ 3 & 1 & -2\end{bmatrix}$	1	
		$\therefore A^{-1}=\frac{1}{\lvert A\rvert}adj(A)$		
		$=\frac{1}{7}\begin{bmatrix}-1 & 2 & 3\\ 5 & -3 & -1\\ 3 & 1 & -2\end{bmatrix}$	1	
		$\therefore$ the solution is,		
		$X=A^{-1}B$		
		$=\frac{1}{7}\begin{bmatrix}-1 & 2 & 3\\ 5 & -3 & -1\\ 3 & 1 & -2\end{bmatrix}\begin{bmatrix}6\\ 5\\ 1\end{bmatrix}$		
		$=\frac{1}{7}\begin{bmatrix}7\\ 14\\ 21\end{bmatrix}$		
		$=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$	1	
		$\therefore x=1,\quad y=2,\quad z=3$	½	**4**
	e)	$\sin(75^o+A)\sin(75^o-A)-\cos(15^o+A)\cos(15^o-A)$		
		$=-\frac{1}{2}\left[\cos(150^o)-\cos(2A)\right]-\frac{1}{2}\left[\cos(30^o)+\cos(2A)\right]$	1	
		$=\frac{1}{2}\left[-\cos(150^o)-\cos(30^o)\right]$	1	
		$=\frac{1}{2}\left[-\cos(180^o-30^o)-\cos(30^o)\right]$ *or* $=\frac{1}{2}\left[-\cos(90^o+60^o)-\cos(30^o)\right]$		
		$=\frac{1}{2}\left[+\cos(30^o)-\cos(30^o)\right]$ $\quad=\frac{1}{2}\left[+\sin(60^o)-\cos(30^o)\right]$	1	
		$=0$ $\quad=\frac{1}{2}\left[\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right]=0$	1	**4**
		OR		
		$\sin(75^o+A)\sin(75^o-A)-\cos(15^o+A)\cos(15^o-A)$		
		$=\sin(75^o+A)\sin(75^o-A)-\cos\left[90^o-(75^o-A)\right]\cos\left[90^o-(75^o+A)\right]$	1	
		$=\sin(75^o+A)\sin(75^o-A)-\sin(75^o-A)\sin(75^o+A)$	1+1	
		$=0$	1	**4**

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3)	f)	$\frac{\sin 2A+2\sin 4A+\sin 6A}{\sin A+2\sin 3A+\sin 5A}=\frac{\sin 2A+\sin 6A+2\sin 4A}{\sin A+\sin 5A+2\sin 3A}$		
		$=\frac{2\sin 4A\cos(-2A)+2\sin 4A}{2\sin 3A\cos(-2A)+2\sin 3A}$	1	
		$=\frac{\sin 4A[2\cos(-2A)+2]}{\sin 3A[2\cos(-2A)+2]}$		
		$=\frac{\sin 4A}{\sin 3A}$	1	
		$=\frac{\sin(A+3A)}{\sin 3A}$		
		$=\frac{\sin A\cos 3A+\cos A\sin 3A}{\sin 3A}$	1	
		$=\frac{\sin A\cos 3A}{\sin 3A}+\frac{\cos A\sin 3A}{\sin 3A}$		**4**
		$=\sin A\cot 3A+\cos A$	1	
4)	a)	$\frac{\sec A}{\sec A-1}+\frac{\sec A}{\sec A+1}=\frac{\sec A(\sec A+1)+\sec A(\sec A-1)}{(\sec A-1)(\sec A+1)}$		
		$=\frac{\sec^2 A+\sec A+\sec^2 A-\sec A}{\sec^2 A-1}$		
		$=\frac{2\sec^2 A}{\sec^2 A-1}$	1	
		$=\frac{2\sec^2 A}{\tan^2 A}$ *or* $=\frac{\frac{2}{\cos^2 A}}{\frac{1-\cos^2 A}{\cos^2 A}}$	1	
		$=2\cdot\frac{1}{\cos^2 A}\cdot\frac{\cos^2 A}{\sin^2 A}$ $\quad=\frac{2}{1-\cos^2 A}$	1	
		$=2\cdot\frac{1}{\sin^2 A}$ $\quad=\frac{2}{\sin^2 A}$		
		$=2\cdot\cos ec^2 A$ $\quad=2\cdot\cos ec^2 A$	1	**4**
	b)	*Given* $\sin A=\frac{5}{13},\ \cos A=-\frac{4}{5}$		
		Note that, both ratios are of A only.		
		and we have to evaluate $\cos(A+B)$.		
		$\therefore \cos(A+B)=\cos A\cos B-\sin A\sin B$	2	
		$=-\frac{4}{5}\cos B-\frac{5}{13}\sin B$	2	**4**

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4)	c)	$\sin A\sin(60-A)\sin(60+A)$		
		$=\sin A(\sin 60\cos A-\cos 60\sin A)(\sin 60\cos A+\cos 60\sin A)$	1	
		$=\sin A(\sin^2 60\cos^2 A-\cos^2 60\sin^2 A)$		
		$=\sin A\left(\frac{3}{4}\cos^2 A-\frac{1}{4}\sin^2 A\right)$	1	
		$=\frac{1}{4}\sin A(3\cos^2 A-\sin^2 A)$		
		$=\frac{1}{4}\sin A\left[3(1-\sin^2 A)-\sin^2 A\right]$		
		$=\frac{1}{4}\sin A\left[3-4\sin^2 A\right]$	1	
		$=\frac{1}{4}\left[3\sin A-4\sin^3 A\right]$		
		$=\frac{1}{4}\sin 3A$	1	**4**
		OR		
		$\sin A\sin(60-A)\sin(60+A)=\sin A(\sin^2 60-\sin^2 A)$	1	
		$=\sin A\left(\frac{3}{4}-\sin^2 A\right)$	1	
		$=\frac{1}{4}\sin A\left[3-4\sin^2 A\right]$		
		$=\frac{1}{4}\left[3\sin A-4\sin^3 A\right]$	1	
		$=\frac{1}{4}\sin 3A$	1	**4**
		OR		
		$\sin A\sin(60-A)\sin(60+A)=\sin A\cdot\frac{1}{-2}(\cos 120-\cos 2A)$	1	
		$=-\frac{1}{2}\sin A\cdot\left[\cos(90+30)-\cos 2A\right]$		
		$=-\frac{1}{2}\sin A\cdot\left[-\sin 30-\cos 2A\right]$	1	
		$=\frac{1}{2}\sin A\cdot\left[\frac{1}{2}+1-2\sin^2 A\right]$		
		$=\frac{1}{2}\sin A\cdot\left(\frac{3}{2}-2\sin^2 A\right)$		
		$=\frac{1}{4}\sin A\left[3-4\sin^2 A\right]$		
		$=\frac{1}{4}\left[3\sin A-4\sin^3 A\right]$	1	
		$=\frac{1}{4}\sin 3A$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	d)	$\frac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta}=\frac{\sin 2(2\theta)+\sin 2\theta}{1+\cos 2(2\theta)+\cos 2\theta}$		
		$=\frac{2\sin 2\theta\cos 2\theta+\sin 2\theta}{2\cos^2 2\theta+\cos 2\theta}$	1	
		$=\frac{\sin 2\theta(2\cos 2\theta+1)}{\cos 2\theta(2\cos 2\theta+1)}$	1	
		$=\frac{\sin 2\theta}{\cos 2\theta}$	1	
		$=\tan 2\theta$	1	**4**
	e)	$\frac{1+\sin x-\cos x}{1+\sin x+\cos x}=\frac{1-\cos x+\sin x}{1+\cos x+\sin x}$		
		$=\frac{2\sin^2\left(\frac{x}{2}\right)+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)+2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$	1	
		$=\frac{2\sin\left(\frac{x}{2}\right)\left[1+\cos\left(\frac{x}{2}\right)\right]}{2\cos\left(\frac{x}{2}\right)\left[1+\cos\left(\frac{x}{2}\right)\right]}$	1	
		$=\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$	1	
		$=\tan\left(\frac{x}{2}\right)$	1	**4**
	f)	$\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\cdot\frac{1}{13}}\right)$	1	
		$=\tan^{-1}\left(\frac{20}{90}\right)$	1	
		$=\tan^{-1}\left(\frac{2}{9}\right)$	1	
		$=\cot^{-1}\left(\frac{9}{2}\right)$	1	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	a)	$Let\ A=(-1,-4),\ B=(4,6),\ C=(-4,10)$		
		$\therefore AB=\sqrt{(4+1)^2+(6+4)^2}=\sqrt{125}$	1	
		$BC=\sqrt{(-4-4)^2+(10-6)^2}=\sqrt{80}$	1	
		$CA=\sqrt{(-4+1)^2+(10+4)^2}=\sqrt{205}$	1	
		$\therefore (\sqrt{125})^2+(\sqrt{80})^2=(\sqrt{205})^2$		
		$\therefore AB^2+BC^2=CA^2$	1	
		$\therefore$ the triangle is right angled triangle.		**4**
	b)	$Area\ is,\ \Delta=\frac{1}{2}\begin{vmatrix}-8 & -2 & 1\\ -4 & -6 & 1\\ -1 & 5 & 1\end{vmatrix}$	1 ½	
		$=\frac{1}{2}[-8(-6-5)+2(-4+1)+1(-20-6)]$	1½	
		$=28$	1	**4**
	c)	$Given\ P=(1,2),\ Q=(3,4),\ R=(1,0)$		
		$M=$ Midpoint of QR		
		$=\left(\frac{3+1}{2},\frac{4+0}{2}\right)$		
		$=(2,2)$	1+1	
		$Slope\ of\ PM=\frac{2-2}{2-1}=0$	1	
		$\therefore PM$ is parallel to x-axis.		
		$\therefore$ *equation of PM is*		
		$y=$ y-coordinate of P or M *or* $y-2=0(x-1)$		
		$\therefore y=2$	1	**4**
	d)	$2x+3y=1$		
		$3x-4y=4$		
		$\therefore 8x+12y=4$		
		$9x-12y=12$		
		$\therefore 17x=16$		
		$\therefore x=\frac{16}{17}$	½	
		$y=-\frac{5}{17}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\therefore$ Point of intersection $=\left(\frac{16}{17}, -\frac{5}{17}\right)$	1	
		$\therefore$ *equation is,*		
		$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$		
		$\therefore \frac{y-2}{-\frac{5}{17}-2}=\frac{x-3}{\frac{16}{17}-3}$	1	
		$\therefore 39x-35y-47=0$	1	**4**
		OR		
		$\therefore$ Point of intersection $=\left(\frac{16}{17}, -\frac{5}{17}\right)$	1	
		$\therefore$ *Slope* $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-\frac{5}{17}-2}{\frac{16}{17}-3}=\frac{39}{35}$	1	
		$\therefore$ *equation is,*		
		$y-y_1=m(x-x_1)$		
		$\therefore y-2=\frac{39}{35}(x-3)$	1	
		$\therefore 39x-35y-47=0$	1	**4**
	e)	*Given* $4(x+2)=3(y-4)$		
		$\therefore 4x-3y+20=0$	1	
		$\therefore$ the length of perpendicular is,		
		$P=\left\|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\|$		
		$=\left\|\frac{4(-3)-3(-4)+20}{\sqrt{4^2+(-3)^2}}\right\|$	2	
		$=4$	1	**4**
	f)	*For* $2x+x=1$ *i.e.* $3x=1$, *the slope is*		
		$m_1=\infty$	1	
		OR the line is parallel to y-axis.		
		$\therefore$ its angle is $\theta_1=90^\circ$		
		For line $x+3y=6$, *the slope is*		
		$m_2=\tan\theta_2=-\frac{1}{3}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\therefore \theta_2 = \tan^{-1}\left(-\frac{1}{3}\right)$ *or* -18.435^o	1	
		$\therefore$ the acute angle is,		
		$\theta = \theta_1 + \theta_2 = 90^o + \tan^{-1}\left(-\frac{1}{3}\right) = 90^o - 18.435^o = 71.565^o$	1	**4**
6)	a)	*Let* $A = (5, 4),\ B = (2, 3),\ C = (1, 0),\ D = (6, 1)$		
		$AB = \sqrt{(2-5)^2 + (3-4)^2} = \sqrt{10}$	½	
		$BC = \sqrt{(1-2)^2 + (0-3)^2} = \sqrt{10}$	½	
		$CD = \sqrt{(4-1)^2 + (1-0)^2} = \sqrt{10}$	½	
		$AD = \sqrt{(4-5)^2 + (1-4)^2} = \sqrt{10}$	½	
		$\therefore AB = BC = CD = AD$	1	
		$\therefore ABCD$ is a rhombus.	1	**4**
	b)	*Let center* $C = (4, 5)$, *and* $P = (-2, -3)$		
		$\therefore$ *radius is* $r = CP = \sqrt{(-2-4)^2 + (-3-5)^2} = 10$	1	
		$\therefore$ *the equation is,* $(x-4)^2 + (y-5)^2 = 10^2$	2	
		$\therefore x^2 + y^2 - 8x - 10y - 59 = 0$	1	**4**
	c)	The equation of concentric circle is,		
		$x^2 + y^2 + 6x - 4y + c = 0$	1	
		But the circle is passing through $(2, 3)$		
		$\therefore 2^2 + 3^2 + 6(2) - 4(3) + c = 0$	1	
		$\therefore c = -13$	1	
		$\therefore x^2 + y^2 + 6x - 4y - 13 = 0$	1	**4**
		OR		
		$x^2 + y^2 + 6x - 4y + c = 0$		
		$\therefore 2g = 6,\ 2f = -4,\ c = -12$		
		$\therefore g = 3,\ f = -2,\ c = -12$		
		$\therefore$ *centre* $C = (-g, -f) = (-3, 2)$	1	
		But the circle is passing through $P = (2, 3)$		
		$\therefore$ *radius* $r = CP = \sqrt{(2+3)^2 + (3-2)^2} = \sqrt{26}$	1	
		$\therefore$ the equation is $(x+3)^2 + (y-2)^2 = \left(\sqrt{26}\right)^2$	1	
		$\therefore x^2 + y^2 + 6x - 4y - 13 = 0$	1	**4**

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>6)</td><td>d)</td><td>$\bar{a}\times\bar{b}=\begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$
$=3i+2j-2k$
$\therefore \left|\bar{a}\times\bar{b}\right|=\sqrt{3^2+2^2+(-2)^2}=\sqrt{17}$
$Unit\ Vector=\frac{\bar{a}\times\bar{b}}{\left|\bar{a}\times\bar{b}\right|}=\frac{3i+2j-2k}{\sqrt{17}}$</td><td>1

1
1
1</td><td>4</td></tr>
<tr><td></td><td>e)</td><td>The resultant force is,
$\bar{F}=(3\bar{i}+\bar{j}+7\bar{k})+(-2\bar{i}-2\bar{j}+\bar{k})+(4\bar{i}+\bar{j}-5\bar{k})$
$=5\bar{i}+3\bar{k}$
$Let\ A=(1,-1,3),\quad B=(4,2,-2)$
$\therefore \overline{AB}=\bar{b}-\bar{a}$
$=(4-1)\bar{i}+(2+1)\bar{j}+(-2-3)\bar{k}$
$=3\bar{i}+3\bar{j}-5\bar{k}$
$\therefore$ Work done $W=\bar{F}\cdot\overline{AB}$
$=(5\bar{i}+3\bar{k})\cdot(3\bar{i}+3\bar{j}-5\bar{k})$
$=0$</td><td>1

1
1
1</td><td>4</td></tr>
<tr><td></td><td>f)</td><td>$\overline{AP}=\bar{p}-\bar{a}$
$=(1-1)\bar{i}+(-1-2)\bar{j}+(2-3)\bar{k}$
$=-3\bar{j}-\bar{k}$
$\therefore Moment\ of\ force=\overline{AP}\times\bar{F}$
$=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -3 & -1 \\ 2 & 3 & 1 \end{vmatrix}$
$=(-3+3)\bar{i}-(0+2)\bar{j}+(0+6)\bar{k}$
$=-2\bar{j}+6\bar{k}$
$\therefore Moment\ of\ force\ about\ the\ line$
$=\text{Re}solved\ part\ of\ the\ moment\ of\ \bar{F}\ about\ A\ along\ the\ line$
$=(-2\bar{j}+6\bar{k})\cdot\left(\frac{\bar{i}+3\bar{j}+\bar{k}}{\sqrt{1^2+3^2+1^2}}\right)$
$=\frac{1}{\sqrt{11}}(0\times1+(-2)3+6\times1)$
$=0$</td><td>1
1
1
1</td><td>4</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		**Important Note** **In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.**		