Winter – 2012 Examination

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	a)	$\int \sin^2 x dx$		
		$= \int \frac{1-\cos 2x}{2} dx$	1	
		$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$	1	**2**
		Note: In solution of integration problems, if the constant 'c' is not added, ½ mark **may be** deducted.		
	b)	$\int \frac{1}{x \log x} dx$ Put $\log x = t$, $\therefore \frac{1}{x} dx = dt$	½	
		$= \int \frac{1}{t} dt$	½	
		$= \log t + c$	½	
		$= \log(\log x) + c$	½	**2**
	c)	$\int \frac{1}{\sqrt{4x^2+25}} dx$		
		$= \frac{1}{2}\int \frac{1}{\sqrt{x^2 + \left(\frac{5}{2}\right)^2}} dx$	1	
		$= \frac{1}{2}\log\left[x + \sqrt{x^2 + \left(\frac{5}{2}\right)^2}\right] + c$	1	**2**
		OR		
		$\int \frac{1}{\sqrt{4x^2+25}} dx$		
		$= \int \frac{1}{\sqrt{(2x)^2 + 5^2}} dx$	1	
		$= \frac{\log\left[2x + \sqrt{(2x)^2 + 5^2}\right]}{2} + c$	1	**2**
	d)	$\frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$		
		$\therefore A = \frac{1}{2}$ & $B = \frac{-1}{2}$	½ +½	

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1)		$\therefore \frac{1}{(x+1)(x+3)} = \frac{1/2}{x+1} - \frac{1/2}{x+3}$		
		$\therefore \int \frac{1}{(x+1)(x+3)} dx = \int \left[\frac{1/2}{x+1} - \frac{1/2}{x+3}\right] dx$		
		$= \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3) + c$	½ + ½	**2**
		Note: In the solution of the problem of partial fractions, if one of the values A or B in the partial fraction is wrong but other values are correct and all the further solution is correct, it is advised to give appropriate marks.		
		OR		
		$\int \frac{1}{(x+1)(x+3)} dx = \int \frac{1}{x^2+4x+3} dx$		
		$= \int \frac{1}{(x+2)^2 - 1} dx$	1	
		$= \frac{1}{2}\log\left(\frac{x+1}{x+3}\right) + c$	1	**2**
	e)	Put $\sin x = t$		
		$\therefore \cos x dx = dt$	½	
		<table><tr><td>x</td><td>t</td></tr><tr><td>0</td><td>0</td></tr><tr><td>$\pi/2$</td><td>1</td></tr></table>	½	
		$\int_0^{\pi/2} \sin x \cos x dx = \int_0^1 t dt$		
		$= \left[\frac{t^2}{2}\right]_0^1$	½	
		$= \frac{1}{2}$	½	**2**
		OR		
		$\int_0^{\pi/2} \sin x \cos x dx = \int_0^{\pi/2} \frac{\sin 2x}{2} dx$	½	
		$= \frac{1}{2}\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2}$	½	
		$= \frac{1}{2}\left[\frac{-\cos \pi}{2}\right] - \frac{1}{2}\left[\frac{-\cos 0}{2}\right]$	½	
		$= \frac{1}{2}$	½	**2**

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1)	f)	$\int \tan^{-1}\left(\frac{\sin x}{1-\cos x}\right)dx$		
		$= \int \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$		
		$= \int \tan^{-1}\left(\cot\frac{x}{2}\right)dx$	½	
		$= \int \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]dx$	½	
		$= \int\left(\frac{\pi}{2}-\frac{x}{2}\right)dx$	½	
		$= \frac{\pi}{2}x - \frac{x^2}{4} + c$	½	**2**
	g)	$\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^3}$		
		$Order = 2$	1	
		$\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3$		
		$Degree = 2$	1	**2**
	h)	$\frac{dy}{dx} + y\tan x = \cos^2 x$		
		$\therefore P = \tan x \quad and \quad Q = \cos^2 x$		
		$IF = e^{\int pdx}$		
		$= e^{\int \tan x dx}$	½	
		$= e^{\log\sec x}$	½	
		$= \sec x$	1	**2**
	i)	$\left(\frac{\Delta^2}{E}\right)x^3 = \frac{(E-1)^2}{E}x^3$	½	
		$= \frac{E^2-2E+1}{E}x^3$		
		$= \left(E-2+\frac{1}{E}\right)x^3$	½	

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1)		$= E\left(x^3\right) - 2\left(x^3\right) + \frac{1}{E}\left(x^3\right)$		
		$= (x+1)^3 - 2x^3 + (x-1)^3$	½	
		$= 6x$	½	**2**
	j)	(table below)	½	

x	0	0.5	1	1.5	2
$f(x)$	1	1.125	2	4.375	9
	y_0	y_1	y_2	y_3	y_4

Note: For marks, only the first two rows are to be considered. The third row of y_i's is just written for the sake of convenience.

$$\int_0^2 \left(1+x^3\right)dx = \frac{h}{2}\left[y_0 + y_4 + 2\left(y_1 + y_2 + y_3\right)\right]$$

$$= \frac{0.5}{2}\left[1+9+2\left(1.125+2+4.375\right)\right]$$ 1

$$= 6.25$$ ½ **2**

k)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	9	3	0	$a-18$	$96-4a$
1	12	3	$a-18$	$78-3a$	
2	15	$a-15$	$60-2a$		
3	a	$45-a$			
4	45				

1

$But\ \Delta^4 y = 0$

$\therefore 96 - 4a = 0$ ½

$\therefore 96 = 4a$

$\therefore a = 24$ ½ **2**

Note: In the above problem, backward difference table can also be used to find the value of the unknown.

l)

$A = \{1, 2, 3, 4\}$ *and* $B = \{4, 5, 7\}$

$\therefore A - B = \{1, 2, 3\}$ ½

and $B - A = \{5, 7\}$ ½

$\therefore (A-B) \cup (B-A) = \{1, 2, 3, 5, 7\}$ 1 **2**

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2)	a)	$y=e^{m\tan^{-1}x}$		
		$\therefore \frac{dy}{dx}=e^{m\tan^{-1}x}\cdot m\cdot\frac{1}{1+x^2}$	2	
		$\therefore \left(1+x^2\right)\frac{dy}{dx}=my$	1	
		$\therefore \left(1+x^2\right)\frac{dy}{dx}-my=0$	1	**4**
		OR		
		$y=e^{m\tan^{-1}x}$		
		$\therefore \log y=m\tan^{-1}x$		
		$\therefore \frac{1}{y}\frac{dy}{dx}=m\cdot\frac{1}{1+x^2}$	2	
		$\therefore \left(1+x^2\right)\frac{dy}{dx}=my$	1	
		$\therefore \left(1+x^2\right)\frac{dy}{dx}-my=0$	1	**4**
	b)	$y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$		
		$\therefore y^2=xy\frac{dy}{dx}-x^2\frac{dy}{dx}$		
		$\therefore y^2=\left(xy-x^2\right)\frac{dy}{dx}$		
		$\therefore \frac{dy}{dx}=\frac{y^2}{xy-x^2}$		
		$Put\ \frac{y}{x}=v\ \ or\ \ y=vx$		
		$\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$	1	
		$\therefore v+x\frac{dv}{dx}=\frac{(vx)^2}{x.vx-x^2}$		
		$\therefore v+x\frac{dv}{dx}=\frac{v^2}{v-1}$		
		$\therefore x\frac{dv}{dx}=\frac{v^2}{v-1}-v$		
		$\therefore x\frac{dv}{dx}=\frac{v}{v-1}$	1	
		$\therefore \frac{v-1}{v}dv=\frac{dx}{x}$		

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2)		$\therefore \int \frac{v-1}{v} dv = \int \frac{dx}{x}$	½	
		$\therefore \int \left(1-\frac{1}{v}\right) dv = \int \frac{dx}{x}$		
		$\therefore v - \log v = \log x + c$	1	
		$\therefore \frac{y}{x} - \log\left(\frac{y}{x}\right) = \log x + c$	½	**4**
	c)	$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$		
		$M = 2xy + y - \tan y$		
		$\therefore \frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$	1	
		$N = x^2 - x\tan^2 y + \sec^2 y$		
		$\therefore \frac{\partial N}{\partial x} = 2x - \tan^2 y$		
		$= 2x + 1 - \sec^2 y$	1	
		$\therefore$ *the equation is exact.*		
		$\therefore$ *the solution is,*		
		$\int\limits_{y\ constant} Mdx + \int\limits_{terms\ free\ from\ x} Ndy = c$		
		$\int (2xy + y - \tan y)dx + \int \sec^2 y dy = c$	1	
		$\therefore 2y.\frac{x^2}{2} + yx - \tan y \cdot x + \tan y = c$		
		$\therefore x^2 y + xy - x\tan y + \tan y = c$	1	**4**
	d)	$\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$		
		$\therefore \frac{1}{\cos^2 y}\frac{dy}{dx} + \frac{2x\sin y\cos y}{\cos^2 y} = x^3$		
		$\therefore \sec^2 y \frac{dy}{dx} + 2x\tan y = x^3$	½	
		Put $\tan y = t$		
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$	½	
		$\therefore \frac{dt}{dx} + 2xt = x^3$		
		$P = 2x$ *and* $Q = x^3$		
		$\therefore IF = e^{\int pdx} = e^{\int 2xdx} = e^{x^2}$	½	

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2)		$\therefore$ *the solution is,*		
		$t \cdot IF = \int Q \cdot IF \cdot dx + c$		
		$\therefore t \cdot e^{x^2} = \int x^3 \cdot e^{x^2} \cdot dx + c$	½	
		$Put\ x^2 = u$	½	
		$\therefore 2xdx = du$		
		$\therefore t \cdot e^{x^2} = \frac{1}{2}\int u \cdot e^u \cdot du + c$		
		$\therefore t \cdot e^{x^2} = \frac{1}{2}\left[u \cdot e^u - e^u\right] + c$	½	
		$\therefore t \cdot e^{x^2} = \frac{1}{2}[u-1]e^u + c$		
		$\therefore \tan y \cdot e^{x^2} = \frac{1}{2}\left[x^2-1\right]e^{x^2} + c$	1	**4**
	e)	$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$		
		Put $x+y=t$		
		$\therefore 1 + \frac{dy}{dx} = \frac{dt}{dx}$	1	
		$\therefore \frac{dt}{dx} - 1 = \frac{t+1}{2t+3}$		
		$\therefore \frac{dt}{dx} = 1 + \frac{t+1}{2t+3} = \frac{3t+4}{2t+3}$	1	
		$\therefore \frac{2t+3}{3t+4}dt = dx$		
		$\therefore \int \frac{2t+3}{3t+4}dt = \int dx$		
		$\therefore \int \left(\frac{2}{3} + \frac{1/3}{3t+4}\right)dt = \int dx$		
		$\therefore \frac{2}{3}t + \frac{1}{3} \cdot \frac{\log(3t+4)}{3} = x + c$	1	
		$\therefore \frac{2}{3}(x+y) + \frac{1}{9}\log(3x+3y+4) = x + c$	1	**4**
	f)	$\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$		
		$P = \frac{1}{RC}$ *and* $Q = \frac{E}{R}$		
		$\therefore IF = e^{\int pdt} = e^{\int \frac{1}{RC}dt} = e^{\frac{1}{RC}t}$	1	

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2)		$\therefore$ *the solution is,* $q\cdot IF=\int Q\cdot IF\cdot dx+c$ $\therefore q\cdot e^{\frac{1}{RC}t}=\int\frac{E}{R}\cdot e^{\frac{1}{RC}t}\cdot dt+c$ $\therefore q\cdot e^{\frac{1}{RC}t}=\frac{E}{R}\cdot\frac{e^{\frac{1}{RC}t}}{\frac{1}{RC}}+c$	1	
		$\therefore q\cdot e^{\frac{1}{RC}t}=EC\cdot e^{\frac{1}{RC}t}+c$ $At\ q=0,\ t=0,$ $\therefore 0=EC\cdot e^{0}+c$ $\therefore c=-EC$	1	
		$\therefore q\cdot e^{\frac{1}{RC}t}=EC\cdot e^{\frac{1}{RC}t}-EC$ OR $\therefore q\cdot e^{\frac{1}{RC}t}=EC\cdot\left(e^{\frac{1}{RC}t}-1\right)$	1	**4**
3)	a)	$t(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\times y_0+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\times y_1+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\times y_2$ $=\frac{(x-2)(x-3)}{(1-2)(1-3)}\times 7+\frac{(x-1)(x-3)}{(2-1)(2-3)}\times 18+\frac{(x-1)(x-2)}{(3-1)(3-2)}\times 35$	1	
		$=3x^2+2x+2$	1	
		$t(2.5)=3(2.5)^2+2(2.5)+2$ ------------- (i)	1	
		$=25.75$ ------------- (ii)	1 (2)	**4**
		Note for Numerical Method Problems: 1) Due to the use of advance calculators, such as modern scientific non-programmable calculators, the step number (i) may not calculated/written by the students and then directly the step number (ii) is written. In this case, no marks to be deducted. 2) Generally for function $f(x)$ symbol is used. But here $t(x)$ is given. If $f(x)$ is used by students, you may consider it for marks. 3) For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus in this paper solution, the calculation is taken up to 3 decimal points only. If this is not followed, the final answer varies accordingly. If the answer is truncated up to two decimal points, deduct 1 mark. If answer is truncated up to one decimal points, deduct 1½ marks. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal point. By either way, if final answer varies only for the last decimal points by small amount, no mark may be deducted. By keeping aside opinion differences, difference of ± 0.005 is generally acceptable.		

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3)	b)	(table below)	1	

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1921	46				
1931	66	20			
1941	81	15	-5		
1951	93	12	-3	2	
1961	101	8	-4	-1	-3

Note: In the theory of numerical methods, the forward difference table is generally written in upward direction (i.e., the last number is coming exactly opposite to the first number) as shown above, the backward difference table is written in the downward direction as shown in the next example and the central difference table is written in the central direction (i.e., the last number is coming exactly opposite to the central value or center of the table).

$$m=\frac{x-x_n}{h}=\frac{1955-1961}{10}=-0.6$$ ½

$$f(x)=y_n+m\nabla y_n+\frac{m(m+1)}{2!}\nabla^2 y_n+\frac{m(m+1)(m+2)}{3!}\nabla^3 y_n+\frac{m(m+1)(m+2)(m+3)}{4!}\nabla^4 y_n$$

$$=101+(-0.6)(8)+\frac{-0.6(-0.6+1)}{2!}(-4)+\frac{-0.6(-0.6+1)(-0.6+2)}{3!}(-1)+\frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(-3)$$ 1 ½

$$=101-4.8+0.48+0.056+0.1008$$

$$=96.8368$$ 1

c)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	5	4	2	0
1	9	6	2	0
2	15	8	2	
3	23	10		
4	33			

1

$$m=\frac{x-x_0}{h}=\frac{x-0}{1}=x$$ 1

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3)

$$f(x)=y_0+m\Delta y_0+\frac{m(m-1)}{2!}\Delta^2 y_0+\frac{m(m-1)(m-2)}{3!}\Delta^3 y_0$$

$$=5+x\cdot 4+\frac{x(x-1)}{2!}\times 2+0$$ 1

$$=5+4x+x^2-x$$

$$\therefore f(x)=x^2+3x+5$$ 1 — Total: **4**

d)

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
2	8			
5	125	117		
8	512	387	270	
11	1331	819	432	162

1

$$m=\frac{x-x_n}{h}=\frac{12-11}{3}=0.333$$ ½

$$f(x)=y_n+m\nabla y_n+\frac{m(m+1)}{2!}\nabla^2 y_n+\frac{m(m+1)(m+2)}{3!}\nabla^3 y_n$$

$$=1331+0.333\times 819+\frac{0.333(0.333+1)}{2!}\times 432$$

$$+\frac{0.333(0.333+1)(0.333+2)}{3!}\times 162$$ 1½

$$=1331+272.727+95.88+27.961$$

$$=1727.568$$ 1

e)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
140	3.685	1.169	0.279	0.047	0.002
150	4.854	1.448	0.326	0.049	
160	6.302	1.774	0.375		
170	8.076	2.149			
180	10.225				

1½

$$m=\frac{x-x_0}{h}=\frac{142-140}{10}=0.2$$ ½

$$f(x)=y_0+m\Delta y_0+\frac{m(m-1)}{2!}\Delta^2 y_0+\frac{m(m-1)(m-2)}{3!}\Delta^3 y_0+\frac{m(m-1)(m-2)(m-3)}{4!}\Delta^4 y_0$$

$$=3.685+0.2(1.169)+\frac{0.2(0.2-1)}{2!}\times 0.279+\frac{0.2(0.2-1)(0.2-2)}{3!}\times 0.047$$

$$+\frac{0.2(0.2-1)(0.2-2)(0.2-3)}{4!}\times 0.002$$ 1

$$=3.685+0.234-0.0223+0.002-0.000$$

$$=3.898$$ 1 — Total: **4**

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3)	f)	$L\frac{di}{dt}+Ri=E$ $\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$ $P=\frac{R}{L}$ and $Q=\frac{E}{L}$ $\therefore IF=e^{\int pdt}=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1	
		$\therefore$ the solution is, $i\cdot IF=\int Q\cdot IF\cdot dt+c$ $\therefore i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\cdot e^{\frac{R}{L}t}\cdot dt+c$ $\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\cdot\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$ $\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}\cdot e^{\frac{R}{L}t}+c$	1	
		At $i=0,\ t=0,$ $\therefore 0=\frac{E}{R}\cdot e^{0}+c$ $\therefore c=-\frac{E}{R}$ $\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}\cdot e^{\frac{R}{L}t}-\frac{E}{R}$ or $i=e^{-\frac{R}{L}t}\left[e^{\frac{R}{L}t}-1\right]\frac{E}{R}$	1	
		Given $R=100,\ L=0.1,\ E=20.$ $\therefore i\cdot e^{1000t}=\frac{1}{5}\cdot e^{1000t}-\frac{1}{5}$ or $i=e^{-1000t}\left[e^{1000t}-1\right]\frac{1}{5}$	1	
		Note: In the above example, L, R, E are arbitrary constants whereas i and t are variables. Also the values of L, R, E are given in advance. Thus these values can be substituted directly in the given differential equation and then the equation can be solved as illustrated below.		
		$0.1\frac{di}{dt}+100i=20$ $\therefore \frac{di}{dt}+1000i=200$	1	
		$P=1000$ and $Q=200$ $\therefore IF=e^{\int pdt}=e^{\int 1000dt}=e^{1000t}$	1	

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3)		$\therefore$ *the solution is,* $i\cdot IF=\int Q\cdot IF\cdot dt+c$ $\therefore i\cdot e^{1000t}=\int 200\cdot e^{1000t}\cdot dt+c$ $\therefore i\cdot e^{1000t}=200\cdot\frac{e^{1000t}}{1000}+c$ $\therefore i\cdot e^{1000t}=0.2\cdot e^{1000t}+c$	1	
		At $i=0,\ t=0,$ $\therefore 0=0.2\cdot e^{0}+c$ $\therefore c=-0.2$ $\therefore i\cdot e^{1000t}=0.2\cdot e^{1000t}-0.2\quad or\quad i=0.2e^{-1000t}\left[e^{1000t}-1\right]$	1	**4**
4)	a)	*Given example is* $\int\frac{\log(\tan x/2)}{\sin x}dx$ *Taking as* $\int\frac{\log\left(\tan\frac{x}{2}\right)}{\sin x}dx$ *Put* $\log\left(\tan\frac{x}{2}\right)=t$ $\therefore\frac{1}{\tan\frac{x}{2}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}dx=dt$	1	
		$\therefore\frac{1}{\sin x}dx=dt$ $\therefore\int\frac{\log\left(\tan\frac{x}{2}\right)}{\sin x}dx=\int tdt$	1	
		$=\frac{t^2}{2}+c$	1	
		$=\frac{\left[\log\left(\tan\frac{x}{2}\right)\right]^2}{2}+c$	1	**4**
	b)	*Put* $\tan\frac{x}{2}=t$ $\therefore dx=\frac{2dt}{1+t^2}\quad and\quad \sin x=\frac{2t}{1+t^2}$	1	
		$\therefore\int\frac{1}{3+2\sin x}dx=\int\frac{1}{3+2\left(\frac{2t}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$		

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4)		$\therefore \int \frac{1}{3+2\sin x}dx = 2\int \frac{1}{3\left(1+t^2\right)+2\left(2t\right)}\cdot dt$		
		$= 2\int \frac{1}{3t^2+4t+3}\cdot dt$		
		$= \frac{2}{3}\int \frac{1}{\left(t+\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}\cdot dt$	1	
		$= \frac{2}{3}\cdot\frac{1}{\frac{\sqrt{5}}{3}}\tan^{-1}\left[\frac{t+\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right]+c$	1	
		$= \frac{2}{\sqrt{5}}\tan^{-1}\left[\frac{\tan\frac{x}{2}+\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right]+c$	1	**4**
		$OR \quad \frac{2}{\sqrt{5}}\tan^{-1}\left[\frac{3\tan\frac{x}{2}+2}{\sqrt{5}}\right]+c$		
	c)	$I = \int_0^{\pi/2}\frac{1}{1+\sqrt[3]{\cot x}}dx$		
		$= \int_0^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$	½	
		$= \int_0^{\pi/2}\frac{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}}dx$	1	
		$I = \int_0^{\pi/2}\frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$		
		$\therefore 2I = \int_0^{\pi/2}\frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$	½	
		$= \int_0^{\pi/2}1\cdot dx$	½	
		$= \left[x\right]_0^{\pi/2}$	½	
		$= \frac{\pi}{2}$	½	**4**
		$\therefore I = \frac{\pi}{4}$	½	
		OR		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$I=\int_0^{\pi/2}\frac{1}{1+\sqrt[3]{\cot x}}dx$		
		$=\int_0^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ Replace $x\to\frac{\pi}{2}-x$ $\therefore \sin x\to\cos x$ *and* $\cos x\to\sin x$	½	
		$I=\int_0^{\pi/2}\frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$	1	
		$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$	½	
		$=\int_0^{\pi/2}1\cdot dx$	½	
		$=[x]_0^{\pi/2}$	½	
		$=\frac{\pi}{2}$	½	
		$\therefore I=\frac{\pi}{4}$	½	**4**
	d)	$\int_0^1 x\sin^{-1}x dx$		
		$=\left[\sin^{-1}x\int xdx-\int\left(\int xdx\right)\frac{d}{dx}\left(\sin^{-1}x\right)dx\right]_0^1$	½	
		$=\left[\sin^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\frac{1}{\sqrt{1-x^2}}dx\right]_0^1$	½	
		$=\left[\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx\right]_0^1$		
		$=\left[\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\int\left(\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)dx\right]_0^1$	1	
		$=\left[\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right)\right]_0^1$	1	
		$=\left[\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}x\right)\right]_0^1$		
		$=\left[\frac{1.\sin^{-1}1}{2}+\frac{1}{2}\left(\frac{1}{2}\sqrt{1-1^2}-\frac{1}{2}\sin^{-1}1\right)\right]-\left[0+\frac{1}{2}\left(0-\frac{1}{2}\sin^{-1}0\right)\right]$		
		$=\left[\frac{\pi/2}{2}+\frac{1}{2}\left(0-\frac{1}{2}\cdot\frac{\pi}{2}\right)\right]-0$	½	
		$=\frac{\pi}{8}$	½	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	e)	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		
		$\therefore y^2=b^2\left(1-\frac{x^2}{a^2}\right)$		
		$\therefore y^2=\frac{b^2}{a^2}\left(a^2-x^2\right)$		
		$\therefore y=\frac{b}{a}\sqrt{a^2-x^2}$		
		$Now\ y=0\ gives\ a^2-x^2=0\ \ i.e.,\ x=a,\ -a$	1	
		$\therefore A=4\int_0^a ydx$		
		$=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$	1	
		$=4\cdot\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$	1	
		$=\frac{4b}{a}\left[0+\frac{a^2}{2}\sin^{-1}(1)\right]-0$		
		$=\frac{4b}{a}\left[\frac{a^2}{2}\cdot\frac{\pi}{2}\right]$		
		$=\pi ab$	1	**4**
	f)	$Given\ \ y^2=4x\ \ and\ \ \ 2x-y=4$		
		$\therefore (2x-4)^2=4x$		
		$\therefore x=1,\ 4$	1	
		$\therefore A=\int_1^4 (y_2-y_1)dx$		
		$=\int_1^4\left(2\sqrt{x}-2x+4\right)dx$	1	
		$=\left[2\cdot\frac{2}{3}x^{3/2}-x^2+4x\right]_1^4$		
		$=\left[\frac{4}{3}\cdot 4^{3/2}-16+16\right]-\left[\frac{4}{3}-1+4\right]$	1	
		$=\frac{19}{3}\ \ or\ \ 6.333$	1	**4**
		Note: The above example could be solved by taking, $\therefore A=\int_1^4 (y_2-y_1)dx=\int_1^4\left(2x-4-2\sqrt{x}\right)dx$ In this case, we get $A=-\frac{19}{3}\ or\ -6.333$ and thus the final answer would become $A=\frac{19}{3}\ or\ 6.333$.		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	a)			

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	0	0.1736	-0.0052	-0.0052	0.0004
10	0.1736	0.1684	-0.0104	-0.0048	
20	0.3420	0.1580	-0.0152		
30	0.5	0.1428			
40	0.6428				

2

$$f'(x)=\frac{1}{h}\left[\Delta y_1-\frac{1}{2}\Delta^2 y_1+\frac{1}{3}\Delta^3 y_1-\frac{1}{4}\Delta^4 y_1+.....\right]$$

Common difference of x is h $=10.$

$$\cos 10^\circ=\frac{1}{10}\left[0.1684-\frac{1}{2}(-0.0104)+\frac{1}{3}(-0.0048)\right]$$ 1

$$=0.0172$$ 1

4

b)

$$h=\frac{b-a}{n}=\frac{7-2}{5}=1$$ ½

x	2	3	4	5	6	7
$f(x)$	0.5	0.333	0.25	0.2	0.167	0.143
	y_0	y_1	y_2	y_3	y_4	y_5

1 ½

$$\int_a^b f(x)\,dx=\frac{h}{2}\left[y_0+y_n+2(y_1+y_2+.....+y_{n-1})\right]$$

$$\therefore \int_2^7 \frac{1}{x}\,dx=\frac{1}{2}\left[y_0+y_5+2(y_1+y_2+y_3+y_4)\right]$$

$$=\frac{1}{2}\left[0.5+0.143+2(0.333+0.25+0.2+0.167)\right]$$ 1

$$=1.2715$$ 1

4

c)

Here $h=1$

x	0	1/6	2/6	3/6	4/6	5/6	6/6
	0	1/6	1/3	1 /2	2/3	5/6	1
$f(x)$	1	0.973	0.9	0.8	0.692	0.59	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

1

$$\int_a^b f(x)\,dx=\frac{h}{3}\left[y_0+y_n+4(y_1+y_3+.....)+2(y_2+y_4+.....)\right]$$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$\therefore \int_0^1 \frac{1}{1+x^2}dx = \frac{h}{3}\left[y_0+y_6+4\left(y_1+y_3+y_5\right)+2\left(y_2+y_4\right)\right]$		
		$= \frac{1/6}{3}\left[1+0.5+4\left(0.973+0.8+0.59\right)+2\left(0.9+0.692\right)\right]$	½	
		$= 0.7853$	½	
		$\therefore \int_0^1 \frac{1}{1+x^2}dx = 0.7853$		
		$\therefore \left[\tan^{-1}x\right]_0^1 = 0.7853$	½	
		$\therefore \tan^{-1}1 = 0.7853$	½	
		$\therefore \frac{\pi}{4} = 0.7853$	½	
		$\therefore \pi = 3.141$	½	**4**
	d)	*Here* $h=1$		
		(table below)	1 ½	
		$\int_a^b f(x)dx = \frac{h}{3}\left[y_0+y_n+4\left(y_1+y_3+.....\right)+2\left(y_2+y_4+.....\right)\right]$		
		$\therefore \int_0^6 \frac{1}{1+x}dx = \frac{h}{3}\left[y_0+y_6+4\left(y_1+y_3+y_5\right)+2\left(y_2+y_4\right)\right]$		
		$= \frac{1}{3}\left[1+0.143+4\left(0.5+0.25+0.167\right)+2\left(0.333+0.2\right)\right]$	1 ½	
		$= 1.959$	1	**4**
	e)	*Given* $\frac{dy}{dx} = y' = x^2+xy$		
		$\therefore f(x,\ y) = x^2+xy$		
		Here $x_0 = 0, \quad y_0 = 1$		
		$x_1 = 0.2, \quad y_1 = ?$		
		$\therefore h = x_1 - x_0 = 0.2$		
		$K_1 = h\cdot f(x_0,\ y_0)$		
		$= 0.2\cdot f(0,1)$		
		$= 0.2\left[0^2+0(1)\right]$		
		$= 0$	1	

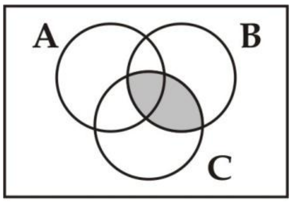
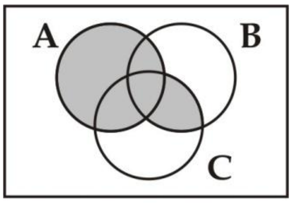
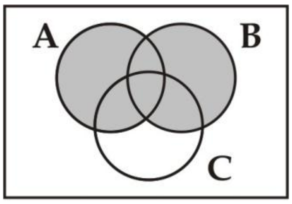
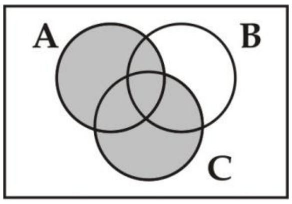
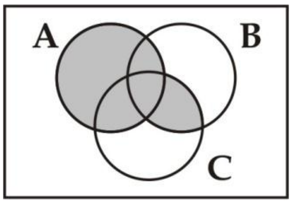
Table for 5) d):

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.333	0.25	0.2	0.167	0.143
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$K_2 = h \cdot f\left(x_0 + h,\ y_0 + K_1\right)$		
		$= 0.2 \cdot f\left(0 + 0.2,\ 1 + 0\right)$		
		$= 0.2 \cdot f\left(0.2,\ 1\right)$		
		$= 0.2\left[\left(0.2\right)^2 + 0.2\left(1\right)\right]$		
		$= 0.048$	1	
		$\therefore K = \dfrac{K_1 + K_2}{2} = \dfrac{0 + 0.048}{2} = 0.024$	1	
		$\therefore y = y_0 + K = 1 + 0.024 = 1.024$	1	
		OR		
		$\therefore y = y_0 + \dfrac{1}{2}\left(K_1 + K_2\right)$		
		$= 1 + \dfrac{1}{2}\left(0 + 0.048\right)$		
		$= 1.024$	2	**4**
	f)	$Given\ \dfrac{dy}{dx} = y' = x + y$		
		$\therefore f\left(x,\ y\right) = x + y$		
		$Here\quad x_0 = 0, \qquad y_0 = 1$		
		$x_1 = 0.2, \quad y_1 = ?$		
		$\therefore h = x_1 - x_0 = 0.2$		
		$K_1 = h \cdot f\left(x_0,\ y_0\right)$		
		$= 0.2 \cdot f\left(0,\ 1\right)$		
		$= 0.2\left[0 + 1\right]$		
		$= 0.2$	½	
		$K_2 = h \cdot f\left(x_0 + \dfrac{h}{2},\ y_0 + \dfrac{K_1}{2}\right)$		
		$= 0.2 \cdot f\left(0 + \dfrac{0.2}{2},\ 1 + \dfrac{0.2}{2}\right)$		
		$= 0.2 \cdot f\left(0.1,\ 1.1\right)$		
		$= 0.2\left[0.1 + 1.1\right]$		
		$= 0.24$	1	
		$K_3 = h \cdot f\left(x_0 + \dfrac{h}{2},\ y_0 + \dfrac{K_2}{2}\right)$		
		$= 0.2 \cdot f\left(0 + \dfrac{0.2}{2},\ 1 + \dfrac{0.24}{2}\right)$		
		$= 0.2 \cdot f\left(0.1,\ 1.12\right)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$=0.2[0.1+1.12]$ $=0.244$ $K_4=h\cdot f(x_0+h,\ y_0+K_3)$ $=0.2\cdot f(0,1)$ $=0.2[0+1]$ $=0.2$ $\therefore K=\dfrac{K_1+K_2}{2}=\dfrac{0+0.048}{2}=0.024$ $\therefore y=y_0+K=1+0.024=1.024$ *OR* $\therefore y=y_0+\dfrac{1}{2}(K_1+K_2)$ $=1+\dfrac{1}{2}(0+0.048)$ $=1.024$	1 ½ ½ ½ 1	**4**
6)	a)	*Given* $\dfrac{dy}{dx}=y'=x-y^2$ $\therefore f(x,\ y)=x-y^2$ *Given* $h=0.1$ *Stage I) Here* $x_0=0,\quad y_0=1$ $x_1=0.1,\quad y_1=?$ $K_1=h\cdot f(x_0,\ y_0)$ $=0.1\cdot f(0,1)$ $=0.1\left[0-(1)^2\right]$ $=-0.1$ $K_2=h\cdot f(x_0+h,\ y_0+K_1)$ $=0.1\cdot f(0+0.1,\ 1-0.1)$ $=0.1\cdot f(0.1,\ 0.9)$ $=0.1\left[0.1-(0.9)^2\right]$ $=-0.071$ $\therefore y_1=y_0+\dfrac{1}{2}(K_1+K_2)$ $=1+\dfrac{1}{2}(-0.1-0.071)$ $=0.9145$	 ½ ½ 1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		*Stage II*) *Now* $x_1 = 0.1, \quad y_1 = 0.9145$ $x_2 = 0.2, \quad y_2 = ?$ $K_1 = h \cdot f(x_1, y_1)$ $= 0.1 \cdot f(0.1, 0.9145)$ $= 0.1\left[0.1 - (0.9145)^2\right]$ $= -0.0736$	½	
		$K_2 = h \cdot f(x_1 + h, y_1 + K_1)$ $= 0.1 \cdot f(0.1 + 0.1, 0.9145 - 0.0736)$ $= 0.1 \cdot f(0.2, 0.8409)$ $= 0.1\left[0.2 - (0.8409)^2\right]$ $= -0.0507$	½	
		$\therefore y_2 = y_1 + \frac{1}{2}(K_1 + K_2)$ $= 0.9145 + \frac{1}{2}(-0.0736 - 0.0507)$ $= 0.8523$	1	4
	b)	$n(X) = 200$ $A =$ numbers divisible by 4. $\therefore n(A) = \frac{200}{4} = 50$	½	
		$B =$ numbers divisible by 5. $\therefore n(B) = \frac{200}{5} = 40$	½	
		$A \cap B =$ numbers divisible by 4 and 5. $\therefore n(A \cap B) = \frac{200}{4 \times 5} = 10$	1	
		$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $= 50 + 40 - 10$ $= 80$	1	
		$(A \cup B)' =$ number not divisible by 4 nor by 5. $\therefore n\left[(A \cup B)'\right] = n(X) - n(A \cup B)$ $= 200 - 80$ $= 120$	1	4

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	c)	$X=\{0,\ 1,\ 2,\ 3,\ 4,\ 5\},\quad A=\{0,\ 1,\ 2,\ 3,\ 5\},\quad B=\{0,\ 1,\ 2,\ 3\}$		
		$i)\ A\cup B=\{0,\ 1,\ 2,\ 3,\ 5\}$	1	
		$\therefore\ (A\cup B)'=\{4\}$	1	
		$ii)\ A-B=\{5\}$	½	
		$B-A=\{\ \}$	½	
		$\therefore A\oplus B=(A-B)\cup(B-A)=\{5\}$	½	
		$\therefore (A\oplus B)'=\{0,\ 1,\ 2,\ 3,\ 4\}$	½	**4**
	d)	 B∩C	1	
		 A∪(B∩C)	1	
		 A∪B	½	
		 A∪C	½	
		 (A∪B)∩(A∪C)	1	**4**
		$\therefore A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	e)	$I=\int_4^5 \frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$ Replace $x \to 9-x$ $\therefore 5-x \to x-4$ & $x-4 \to 5-x$	½	
		$I=\int_4^5 \frac{\sqrt{x-4}}{\sqrt{5-x}+\sqrt{x-4}}dx$	1	
		$\therefore 2I=\int_4^5 \frac{\sqrt{x-4}+\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}}dx$	½	
		$=\int_4^5 1\cdot dx$	½	
		$=[x]_4^5$	½	
		$=5-4$		
		$=1$	½	
		$\therefore I=\frac{1}{2}$	½	**4**
	f)	$\int \frac{\log x}{x(1+\log x)(2+\log x)}dx$ Put $\log x=t$ $\therefore \frac{1}{x}dx=dt$	1	
		$=\int \frac{t}{(1+t)(2+t)}dt$	½	
		$=\int\left[\frac{-1}{1+t}+\frac{2}{2+t}\right]dt$	1	
		$=-\log(1+t)+2\log(2+t)+c$	1	
		$=-\log(1+\log x)+2\log(2+\log x)+c$	½	**4**
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.		