MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
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SUMMER- 13 EXAMINATION

Subject code : 17204 **Model Answer :Engineering Mechanics**

Important instruction to examiners:

1) The answers should be examined by key words and not as word to word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given stepwise for numerical problems. In some cases the assumed constant values may be vary and there may be some difference in the candidates answer and model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q 1	Attempt any TEN	20 Marks
(a)	Simple Machine :- It is a device by which heavy load is lifted by using less effort.	1 M
	Compound Machine :- It is a device which consists of number of Simple machines to do heavy work with less effort at greater speed.	1 M
(b)	Input of Machine :- The amount of workdone by effort on the machine is called Input of Machine $\therefore$ Input $= P \times y$ $\therefore P =$ effort $\therefore y =$ dist. moved by P	1M
	Output of Machine :- The amount of workdone by load by the Machine is called output of Machine $\therefore$ output $= W \times x$ $\therefore W =$ Load $\therefore x =$ dist. moved by W	1M
(c)	Ideal Machine :- It is a machine in which friction is zero and its efficiency is 100 %.	1 M
	Ideal effort :- It is the effort required to lift the load when there is no friction in Machine. $\therefore P_i = \frac{W}{V.R.}$	1 M
(d)	Principle of transmissibility of forces. :- "If a force acts at a point on a rigid body, it is assumed to act at any other point on the line of action of a force within the same body."	2 M.

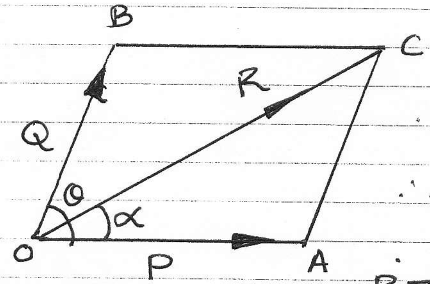
Q 1

(e) characteristics of force :— $\frac{1}{2}$ Mark for each

(i) Magnitude
(ii) sense or nature
(iii) Direction
(iv) Point of application

(f) Law of Parallelogram of forces.

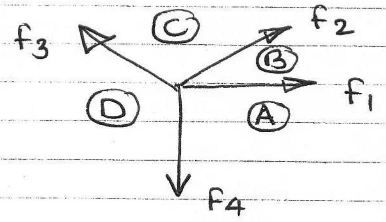
ie If two forces acting at and away from the point be represented in Magnitude and direction by the two adjacent sides of a Parallelogram, then the diagonal of the Parallelogram Passing thraugh the Point of intersection of the two forces, represents the resultant in Magnitude and direction 2 M



where
∴ P & Q are forces
∴ R is resultant

$$R=\sqrt{P^2+Q^2+2PQ\cos\theta}$$

(g) spacediagram :- The diagram in which number of forces acting on body are drawn in space to a suitable scale and naming the spaces in order by Bow's notation is called space diagram. 1 M

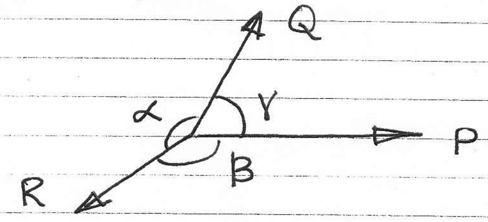


space diagram

Vector diagram :- After drawing the space diagram, all forces are drawn to suitable scale and Parallel to their lines of action in space diagram, in Proper order then the resulting diagram is called vector diagram. 1m

Q 1.

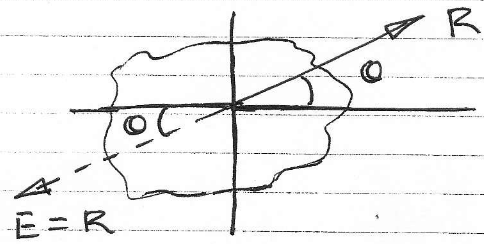
(h) Lami's theorem :- If three forces acting at a point on a body keep it at rest, then each force is proportional to the sine of the angle between the other two forces. 1M



$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ 1M

(i) Relation between resultant and equilibrant.

Resultant and equilibrant are equal in magnitude, colinear but acting in opposite direction. 1M



1M

(j) Laws of static friction :-

(i) frictional force always acts tangential to the plane of contact and opposite to the motion of body.

(ii) Force of friction is a self adjusting force upto limiting friction

(iii) The coefficient of friction depends upon the nature of surface in contact. and is independent of the surface area in contact

(iv) The static friction is more than dynamic friction

(v) when body is in limiting equilibrium, then ratio of limiting friction and normal reaction is constant, called as coefficient of friction (μ)

2 Marks (Any four)

Q 1(K) Define coefficient of friction (μ) :-

The ratio of limiting friction to normal reaction at the surfaces of contact is constant, which is called as coefficient of Friction. 1 M

Angle of Repose :- It is the angle made by the inclined plane with the horizontal plane, when body placed on it, is just on the point of moving down the plane with its own weight is called as angle of Repose. 1 M

(l) conditions of equilibrium for coplanar forces.

$\therefore \Sigma fx = 0$ where Σfx = sum of all horizontal forces 2 M (Any Two)

$\therefore \Sigma fy = 0$ $\therefore fy$ = sum of all vertical forces

$\therefore \Sigma M = 0$ $\therefore \Sigma M$ = sum of moment of all forces @ any point

Q 2(a) $\therefore$ Find M.A., V.R. & Law of M/c

Data

$\therefore W_1 = 10\,KN = 10000\,N$

$\therefore P_1 = 300\,N$

$\eta\% = 75$

$W_2 = 20\,KN$

$P_2 = 550\,N$

$$\therefore M.A = \frac{W_1}{P_1} = \frac{10000}{300} = 33.333$$ 1 M

$$\therefore \eta = \frac{M.A}{V.R.} \times 100$$

$$\therefore V.R. = \frac{M.A.}{\eta} \times 100$$

$$\therefore V.R. = \frac{33.333}{75} \times 100$$

$$\therefore V.R = 44.444$$ 1 M

finding law of Machine

$$300 = 10000m + c \quad \text{---} (1)$$

$$550 = 20000m + c \quad \text{---} (2)$$

$$-250 = -10000m$$

	$\therefore 250 = 10000m$	
	$\therefore m = 0.025$	1M
	put value of m in eq^n ①	
	$300 = 10000 \times 0.025 + C$	
	$\therefore C = 50N$	1M
	Law of Machine	
	$P = 0.025W + 50N$	
Q2(b)	find M.A., V.R & η%. Data: $W = 30KN$, $P = 400N$, $L = 600mm$, Pitch $= 15mm$	
	M.A.	
	(i) $M.A. = \frac{W}{P}$	
	$M.A. = \frac{30000}{400}$	
	$M.A. = 75$	1M
	(ii) $V.R. = \frac{2\pi L}{\text{pitch}}$	
	$V.R. = \frac{2\pi \times 600}{15}$	
	$V.R. = 251.327$	2M
	(iii) $\eta = \frac{M.A.}{V.R.} \times 100$	
	$\therefore \eta = \frac{75}{251.327} \times 100$	
	$\therefore \eta = 29.842\%$	1M
Q2(C)	Data: $D = 360mm$, $d_1 = 90mm$, $d_2 = 60mm$, $\eta\% = 80$, $W = ?$, $P = 100N$	
	$M.A. = \frac{W}{P}$	
	$= \frac{W}{100}$	

Q2 (c) cont.^{d}	$\therefore V.R. = \frac{2D}{d_1 - d_2}$	
	$\therefore V.R. = \frac{2 \times 360}{90-60}$	
	$\therefore V.R. = 24$	
	$\therefore \eta\% = \frac{M.A}{V.R} \times 100$	
	$\therefore 80 = \frac{W}{100 \times 24} \times 100$	
	$W = 1920\ N$	
(d)	Data: $\therefore D = 250\ mm$, $d = 100\ mm$, $P = 2$, $W = 2kN = 2000\ N$, $\eta\% = 80\%$	
	$\therefore M.A. = \frac{W}{P}$	
	$= \frac{2000}{P}$	1M
	$\therefore V.R. = \frac{2D}{D-d}$	
	$= \frac{2 \times 250}{250-100}$	
	$= 3.333$	1M
	$\therefore \eta\% = \frac{M.A.}{V.R.} \times 100$	1M
	$\therefore 80 = \frac{2000}{P \times 3.333} \times 100$	
	$\therefore P = \frac{2000 \times 100}{80 \times 3.333}$	
	$\therefore P = 750.075\ N$	1M.
(e)	$\therefore$ find M.A., $\eta\%$ & Pf for 1000 N load. Data: $\therefore V.R. = 25$, $\therefore P = 0.01W + 5N$	
	find effort for 1000N load	
	$\therefore P = 0.01 \times 1000 + 5$	
	$\therefore P = 15\ N$	1M

Q 2(e) cont.^{d}	$M.A. = \frac{W}{P}$	
	$= \frac{1000}{15}$	
	$= 66.666$	1M
	$\eta\% = \frac{M.A.}{V.R.} \times 100$	
	$= \frac{66.666}{25} \times 100$	
	$= 266.664$	1M
	Note :- Data for V.R. in given Problem is not correct therefore $\eta\% > 100$ which is not true for actual Machine	
	As $\eta\% > 50\%$, the machine is Reversible Machine	1 M.
(F)	Data: $D = 400$ mm, $d = 500$ mm, $T = 60$, $W = ?$, $P = 100$ N, $P_i = ?$, $\eta\% = 45\%$	
	$M.A. = \frac{W}{P}$	
	$= \frac{W}{100}$	1M
	$V.R. = \frac{DT}{nd}$	
	$= \frac{400 \times 60}{1 \times 500}$	
	$= 48$	1M
	$\eta\% = \frac{M.A.}{V.R.} \times 100$	
	$\eta\% = \frac{W}{100 \times V.R} \times 100$	
	$45 = \frac{W}{100 \times 48} \times 100$	
	$W = 2160$ N.	1M
	$P_i = \frac{W}{V.R.} = \frac{2160}{48} = 45$	1M

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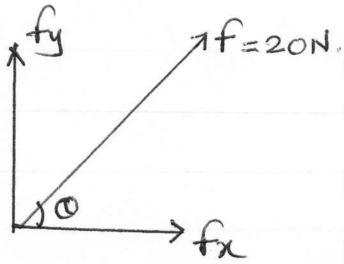
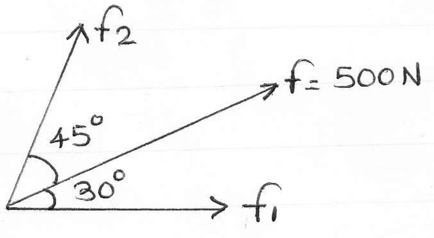
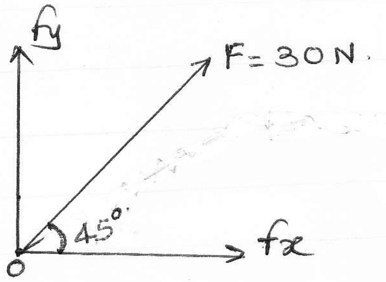
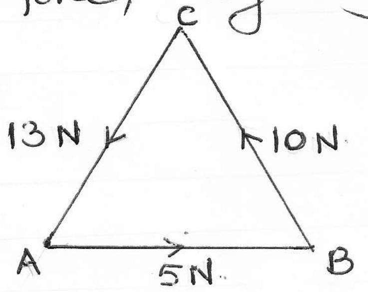
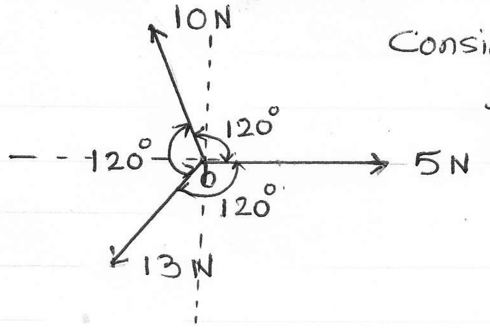
17204 Engg. Mechanics.

Q.No.	Answer	Marks
3 a)	Resolve a force of 20 N into two components that are right angle & in ratio 3:4	
	Given : $F = 20\,N$. $\frac{F_x}{F_y} = \frac{3}{4}$	
		
	But $f_x = f\cos\theta$ $f_y = f\sin\theta$.	
	$\therefore \frac{f\cos\theta}{f\sin\theta} = \frac{3}{4}$	1
	$\therefore \cot\theta = \frac{3}{4}$, $\tan\theta = \frac{4}{3} = 1.33$	
	$\therefore \theta = \tan^{-1}(1.33) = 53.13^\circ$	1
	$\therefore$	
	$f_x = f\cos\theta = 20\cos 53.13^\circ = 12\,N$	1
	$f_y = f\sin\theta = 20\sin 53.13^\circ = 16\,N$.	1
3 b)	Resolve a force of 500N into two directions @ 30° & 45° on either side of it.	
	Given: $F = 500N$, $\alpha = 30^\circ$ and $\beta = 45^\circ$	
		
	$f_1 = \frac{f\sin\beta}{\sin(\alpha+\beta)}$	1
	$f_1 = \frac{500\sin 45}{\sin(30+45)}$	
	$f_1 = 366.03\,N$.	1

Q.No.	Answer	Marks
	$f_2 = \dfrac{500 \sin \alpha}{\sin(\alpha+\beta)}$	1
	$f_2 = \dfrac{500 \sin 30}{\sin(30+45)} = 258.82\ N.$	1
3c)	Resolve a force of 30 N acting North-East away from the point.	
	Given :	
		
	$\therefore f_x = f\cos\theta$.	2
	$\therefore f_x = 30 \cos 45^\circ = 21.21\ N.$	
	$f_y = f\sin\theta$.	2
	$f_y = 30 \sin 45^\circ = 21.21\ N.$	
3d)	A particle is acted upon by three force of 5 N, 10 N & 13 N along three sides of an equilateral triangle, taken in order. find magnitude of the Resultant force, analytically.	
	Given :	
		

Q.No.	Answer	Marks



Considering forces acting away from the particle. — 1

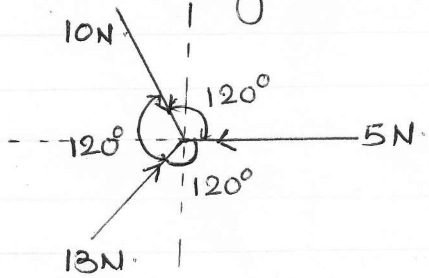
$\Sigma fx = +5.0 - 10\cos 60 - 13\cos 60 = -6.50\,N.$ — 1

$\Sigma fy = +10\sin 60 - 13\sin 60 = -2.60\,N.$ — 1

$R = \sqrt{(\Sigma fx)^2 + (\Sigma fy)^2} = \sqrt{(-6.50)^2 + (-2.60)^2} = 7.00\,N.$ — 1.

$\boxed{\text{OR.}}$

Considering forces acting towards the particle.



1

$\Sigma fx = -5 + 10\cos 60 + 13\cos 60 = +6.50\,N.$ — 1

$\Sigma fy = -10\sin 60 + 13\sin 60 = +2.60\,N.$ — 1

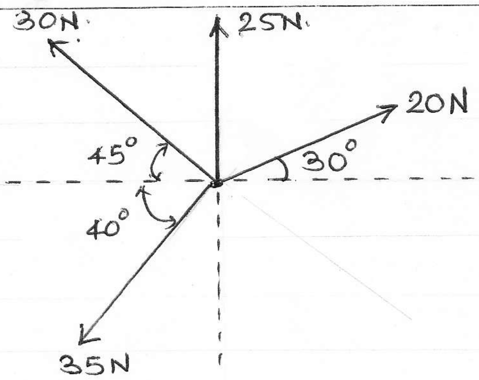
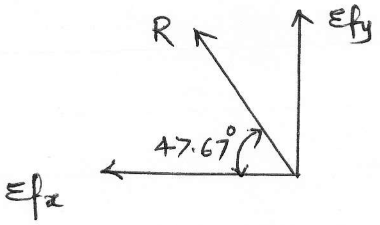
$\therefore R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$

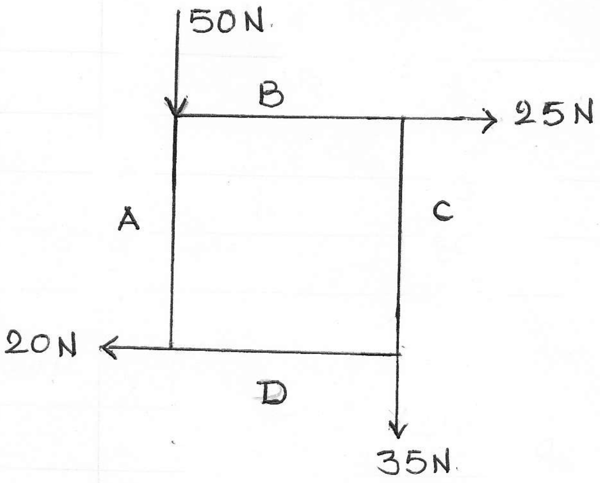
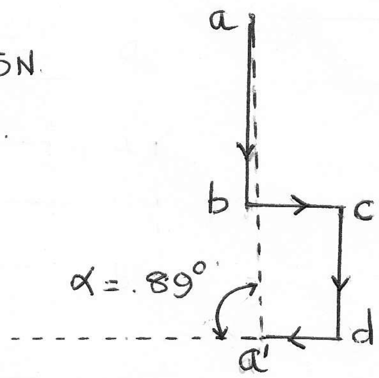
$R = \sqrt{(+6.50)^2 + (2.60)^2}$ — 1

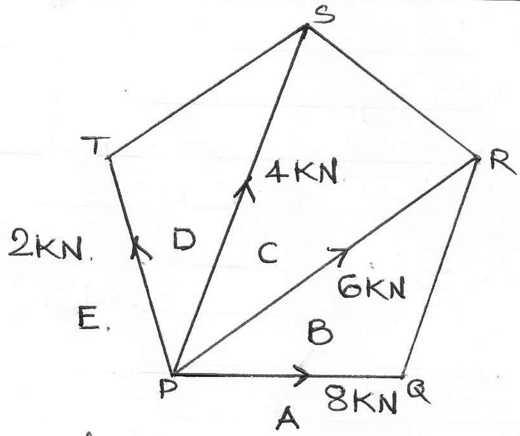
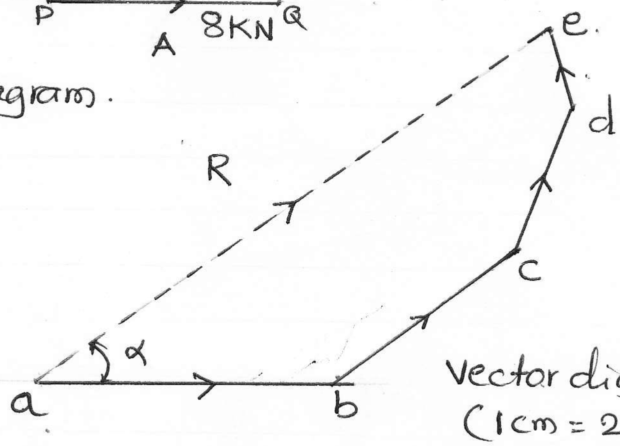
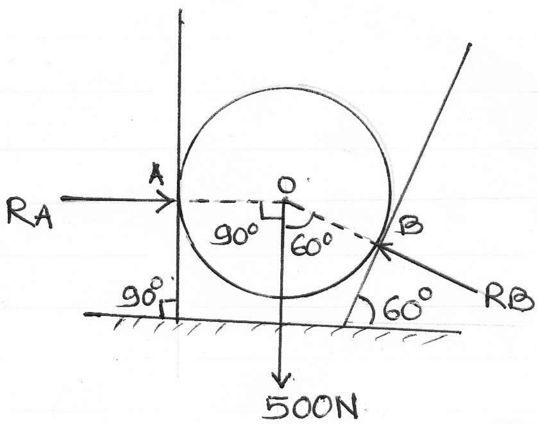
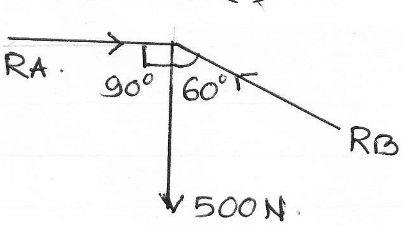
$R = 7.00\,N.$

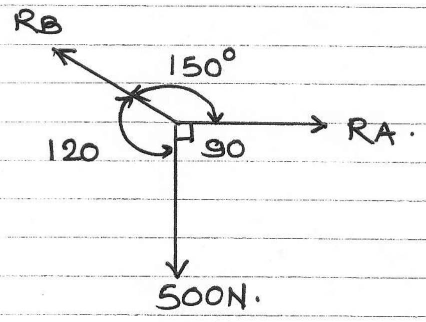
(Note: As in the question it is not mentioned clearly whether forces are acting towards the point or away from the point hence marks should be given to one of cases as mentioned above).

Q.No.	Answer	Marks
3e).	Data: Two forces P & Q.	
	when $\theta = 90^\circ$ $R = \sqrt{10}$ KN. and.	
	When $\theta = 60^\circ$ $R = \sqrt{13}$ KN.	
	To find. P & Q. when $\theta = 90^\circ$	
	$R^2 = P^2 + Q^2 + 2PQ\cos\theta$.	
	$(\sqrt{10})^2 = P^2 + Q^2 + 2PQ\cos 90$.	
	$\therefore\ 10 = P^2 + Q^2$ .....(i)	1
	when $\theta = 60^\circ$	
	$(\sqrt{13})^2 = P^2 + Q^2 + 2PQ\cos 60$.	
	$\therefore\ 13 = (P^2 + Q^2) + PQ$.	
	$\therefore\ PQ = 13 - 10 = 3$ ...(ii)	1
	Taking. $(P+Q)^2 = P^2 + Q^2 + 2PQ$.	
	putting the values from (i) & (ii)	
	$(P+Q)^2 = 10 + 2(3) = 16$.	
	$\therefore\ P+Q = 4$ ....(iii)	1
	Taking $(P-Q)^2 = P^2 + Q^2 - 2PQ$.	
	Putting the values from (i) & (ii)	
	$(P-Q)^2 = 10 - (2\times 3) = 4$.	
	$\therefore\ P-Q = 2$ ...(iv).	
	Adding (iii) & (iv) $\begin{array}{r} P+Q = 4 \\ P-Q = 2 \\ \hline 2P = 6 \end{array}$	1
	$\therefore$ P = 3 N and Q = 1 N. (Ans).	

Q.No.	Answer	Marks
3f.	Data: 	
	$\Sigma fx = +20\cos 30 - 30\cos 45 - 35\cos 40$	
	$= -30.70$ N	1
	$\Sigma fy = +20\sin 30 + 25 + 30\sin 45 - 35\sin 40$	
	$= 33.71$ N	1
	$R = \sqrt{(-30.70)^2 + (33.71)^2}$	
	$= 45.59$ N	1
	$\alpha = \tan^{-1}\left(\frac{\Sigma fy}{\Sigma fx}\right)$	
	$= \tan^{-1}\left(\frac{33.71}{-30.70}\right)$	1
	$\alpha = -47.67°$ 	

Q.No.	Answer	Marks
4) a).	Data: $P = Q = 100N$, $\theta = ?$ $R = 100N$ (resultant). Using law of parallelogram of forces. $R^2 = P^2 + Q^2 + 2PQ\cos\theta$	1
	$(100)^2 = (100)^2 + (100)^2 + 2\times100\times100\cos\theta$	1
	Divide all by $(100)^2$ $1 = 2(1+\cos\theta)$.	1
	$\therefore 1+\cos\theta = \frac{1}{2}$ $\cos\theta = -\frac{1}{2}$. $\therefore \theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$ (Ans)	1
4 b).	 Space Diagram	1
	 Vector diagram (1mm = 2N).	2
	$R = 42.5\text{ mm} = 85N$ & $\alpha = 89^\circ$	1

Q.No.	Answer	Marks
4c		1
	Space diagram.	
	 Vector diagram (1cm = 2KN).	2
	$R = 8.30\,cm\,(ae) = 16.6\,KN$	1
	$\alpha = 35^\circ$	
4d.		
	Method (I) 	1
	Using Lami's theorem equation: $\frac{P}{\sin\theta_1} = \frac{Q}{\sin\theta_2} = \frac{R}{\sin\theta_3}$.	
	$\frac{500}{\sin 150} = \frac{RA}{\sin 60^\circ} = \frac{RB}{\sin 90}$	1
	$RA = \frac{500}{\sin 150} \times \sin 60 = 866.03\ N$	1
	$RB = \frac{500}{\sin 150} \times \sin 90 = 1000.00\ N$	1

4 d.	Method II: 	1
	Using Lamis theorem.	
	$\frac{500}{\sin 150} = \frac{RA}{\sin 120} = \frac{RB}{\sin 90}$	1
	$\therefore RA = \frac{500}{\sin 150} \times \sin 120 = 866.03\ N.$	1
	$RB = \frac{500}{\sin 150} \times \sin 90 = 1000.00\ N$	1
	(Note: Student can solve the problem by any one method as mentioned above)	

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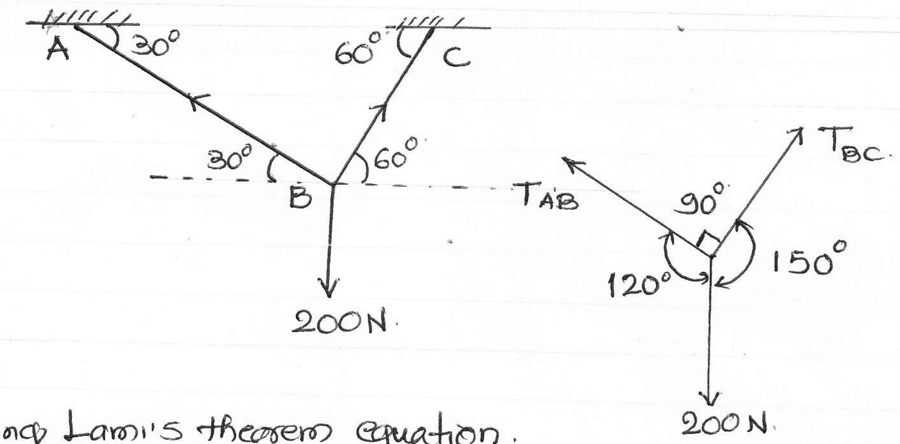
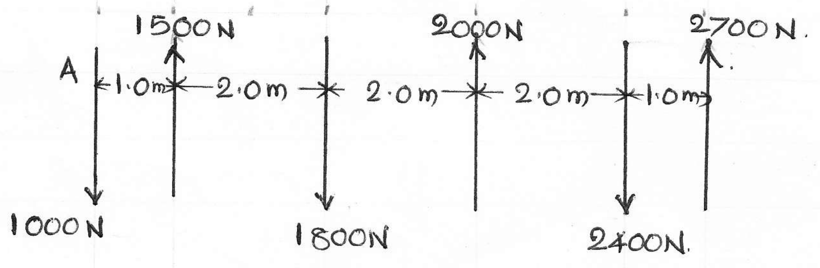
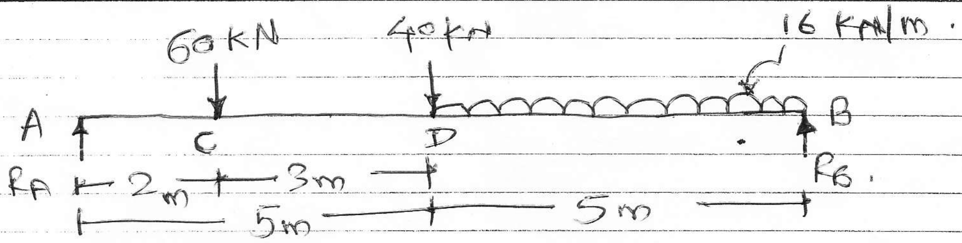
4 e).		1
	Using Lami's theorem equation. $\dfrac{f_1}{\sin\theta_1} = \dfrac{f_2}{\sin\theta_2} = \dfrac{f_3}{\sin\theta_3}$.	
	$\dfrac{200}{\sin 90} = \dfrac{T_{AB}}{\sin 150} = \dfrac{T_{BC}}{\sin 120}$	1
	$\therefore\ T_{AB} = \dfrac{200}{\sin 90} \times \sin 150 = 100\ N.$	1
	$T_{BC} = \dfrac{200}{\sin 90} \times \sin 120 = 173.21\ N.$	1
4 f)		1
	$R = \Sigma fy = -1000 + 1500 - 1800 + 2000 - 2400 + 2700$ $= +1.000\ N.\ (\uparrow).$	1
	To find position of resultant from 'A'. Using Varignon's theorem of moment. $f_1x_1 + f_2x_2 + \cdots f_nx_n = R \times x$.	
	$-(1500 \times 1) + (1800 \times 3) - (2000 \times 5) + (2400 \times 7) - (2700 \times 8) = 1000 \times x$.	1
	$\therefore\ x = \dfrac{10,900}{1000} = 10.9\ m$ from 'A'.	1

Q.5 (a)



Consider conditions of Equillibrium

$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M = 0.$

$\Sigma F_x = 0$ (Not reqd. since No Hz force).

$\Sigma F_y = 0 = -60 - 40 - (16 \times 5) + R_A + R_B$ (01)

$\therefore \quad R_A + R_B = 180$ N. Eq^n — (1).

Taking moment about 'A' and Considering equillibrium.

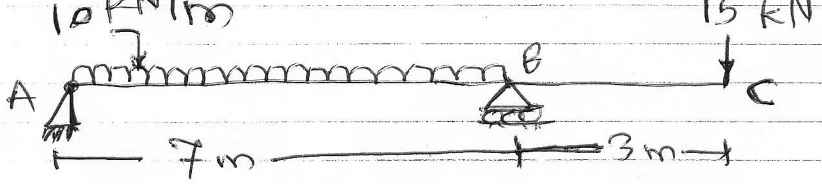
$\therefore \Sigma M_A = 0 = 60 \times 2 + 40 \times 5 + (16 \times 5) \times 7.5 - 10 R_B.$ (02)

$\therefore \quad \boxed{R_B = 92 \text{ kN} \uparrow}$ — Eq^n (II).

Put value of R_B in eq^n (1).

$\therefore \quad \boxed{R_A = 180 - 92 = 88 \quad \text{kN} \uparrow}$ (01)

(b)



Assume R_A = Reaction at 'A'
R_B = Reaction at 'B'

Consider equillibrium conditions.

$\Sigma F_x = 0$ $\quad \Sigma F_y = 0$ $\quad \Sigma M = 0$.

$\Sigma F_x = 0$ (Not reqd.).

$\Sigma F_y = 0 = -(10 \times 7) - 15 + R_A + R_B$

$\therefore R_A + R_B = 70 + 15$

$R_A + R_B = 85$ kN. ——— Eq^n ① (01)

Taking moments about 'A'.

$\therefore \Sigma M_A = 0 = 70 \times 3.5 + 15 \times 10 - (7 R_B)$

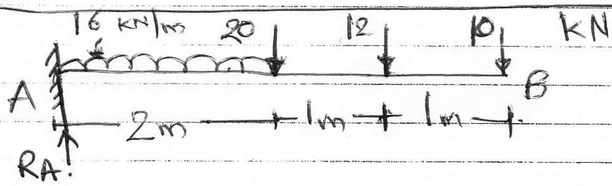
$\therefore 7 R_B = 245 + 150$ (02)

$$\boxed{\begin{array}{l} \therefore R_B = 56.42 \text{ kN} \uparrow \\ R_A = 28.58 \text{ kN} \uparrow \end{array}}$$

——— Eq^n ⑪

(from eq^n ① & ⑪) (01)

(c)

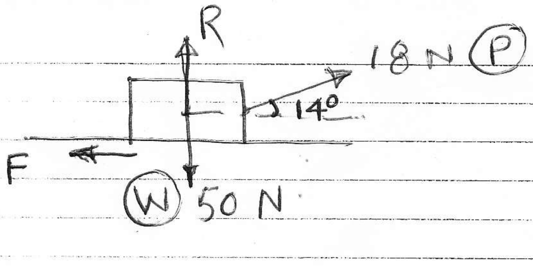


Consider Equillibrium condition. (02)

$\Sigma F_y = 0 = (16 \times 2) + 20 + 12 + 10 + (R_A)$

$$\boxed{\therefore R_A = 74 \text{ kN} \uparrow}$$ (02)

(d)



R = Reaction.

W = Weight of block.

P = Pull.

F = frictional force = μR.

$\Sigma f_x = 0 = 18\cos 14 - F$

$\therefore F = 18\cos 14$

$\therefore F = 17.465 \text{ N}$ (02)

$\Sigma f_y = 0 = R + 18\sin 14 - 50$

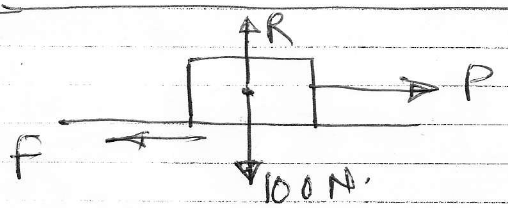
$0 = R + (-45.65)$

$\therefore R = \underline{45.65} \text{ N}$ (01)

$\therefore \mu = \frac{F}{R} = \frac{17.465}{45.65}$ (01)

$\boxed{\therefore \mu = 0.39}$

Q5 e



Given

Angle of Repose $= 15^0$.

Coefficient of friction $= \mu = \tan\phi$

$\therefore \mu = \underline{0.27}$ (01)

Use Equillibrium condition.

$\therefore \Sigma f_x = 0 = F - P = 0$

$\therefore \mu R = P$ —— (1) (01)

20/30

$\Sigma f_y = 0 = R - 100 = 0$.

$\therefore R = 100\ N$. — (II) (01)

Put μ & R in eqn (I).

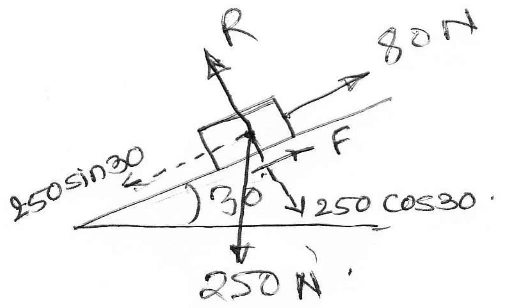
$\therefore 0.27 \times 100 = P$.

$\boxed{\therefore P = 27\ N}$ (01)

$\therefore$ Horizontal force Reqd $= \boxed{27\ N}$.

Q (5)

(f).



$\therefore$ Resolving forces parallel to the plane and Perpendicular to the plane.

(I) Parallel to the plane $= \Sigma f_x$

$\Sigma f_x = 80 + F - 250 \sin 30^\circ = 0$.

$F = 250 \sin 30 - 80 = 45\ N$. (02)

(II) Perpendicular to the plane $= \Sigma f_y$.

$\Sigma f_y = R - 250 \cos 30 = 0$. (01)

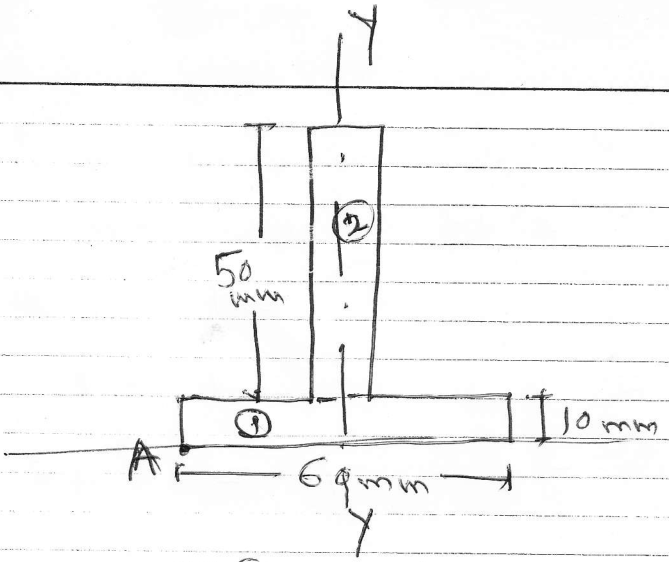
$\therefore R = 216.50\ N$.

$F = \mu R$. $\therefore 45 = \mu \times R$. (01)

$\therefore$ $\boxed{\therefore \mu = \frac{45}{216.50} = 0.21}$

Q 6

(a)



∴ If a line parallel to Y axis is drawn through centre of 'T' section, it will be seen that it is symmetrical about 'Y' axis. Hence the distance of centroid from left edge (i.e. point A) will be = 30 mm.

i.e. $\bar{x} = 30$ mm. (02)

(OR)

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{(a_1 + a_2)}$$

a_1 = Area of flange

x_1 = Centroid of flange from left edge.

a_2 = Area of web.

x_2 = Centroid of web from left edge.

$a_1 = 60 \times 10 = 600 \text{ mm}^2$.

$x_1 = \frac{60}{2} = 30$ mm.

$a_2 = 50 \times 10 = 500 \text{ mm}^2$.

$x_2 = \frac{60}{2} = \underline{30 \text{ mm}}$.

$\therefore \bar{x} = \frac{600 \times 30 + 500 \times 30}{600 + 500} = \underline{30 \text{ mm}}$

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2}$$

Y_1 = Distance of centroid from bottom of flange of fig. 1

Y_2 = Distance of centroid from bottom of flange of figure 2

$\therefore Y_1 = 05 \text{ mm}$.

$Y_2 = 10 + 25 = 35 \text{ mm}$.

$\therefore \bar{Y} = \frac{600 \times 5 + 500 \times 35}{600 + 500} = \underline{18.64} \text{ mm}$.

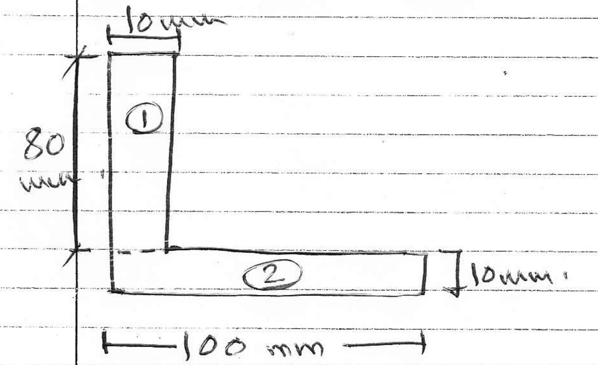
$\therefore \bar{Y} = 18.64$ mm from Extreme bottom. 02.

$\boxed{\begin{array}{l}\bar{x} = 30 \text{ mm. from left edge.} \\ \bar{Y} = 18.64 \text{ mm from extreme bottom.}\end{array}}$

Q. 6 (b) Since diagram of 'L' section is not given in the question paper, students may draw the 'L' section in different ways. [Since these are I year students, may get confused with web dimenssions and flange dimenssions.]

Considering the above probability, three alternatives for solution are given herewith. Examiners shall consider it and full marks may be given accordingly. Examiners shall see that, if the student is capable of calculating centroid correctly of the figure he has drawn for this problem.

Alternative (I).



Consider Reference axis at extreme left edge (vertical) and at extreme bottom. The centroid is to be calculated from these reference axis.

$$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$a_1 = 80 \times 10 = 800\ mm^2$	$x_1 = \frac{10}{2} = 5\ mm.$
$a_2 = 100 \times 10 = 1000\ mm^2$	$x_2 = \frac{100}{2} = 50\ mm.$

(01)

$$\therefore \bar{x} = \frac{800 \times 5 + 1000 \times 50}{800 + 1000} = 30\ mm.$$

(01)

$\boxed{\therefore \bar{x} = 30\ mm.}$ from left edge.

$\therefore \bar{Y} = \dfrac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2}$

$Y_1 = (10 + \frac{80}{2}) = 50$ mm. (01)

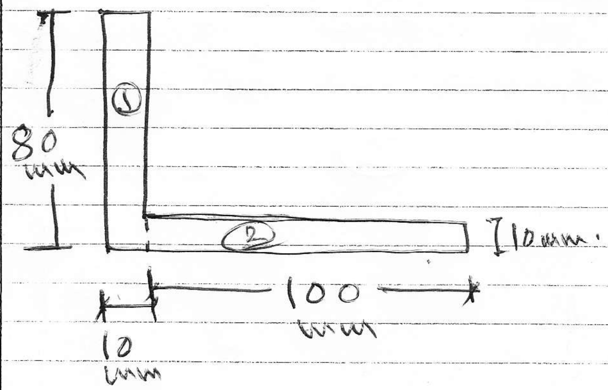
$Y_2 = \frac{10}{2} = 5$ mm.

$\therefore \bar{Y} = \dfrac{800 \times 50 + 1000 \times 5}{800 + 1000} = 25$ mm. ~~from~~ bottom

$\boxed{\therefore \bar{Y} = 25 \text{ mm from bottom.}}$ (01)

Q 6 b

Alternative II



$\bar{x} = \dfrac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$

$a_1 = 80 \times 10 = 800 \text{ mm}^2$

$a_2 = 100 \times 10 = 1000 \text{ mm}^2$

$x_1 = \frac{10}{2} = 5$ mm. (01)

$x_2 = (10 + \frac{100}{2})$

$x_2 = 60$ mm.

$\therefore \bar{x} = \dfrac{800 \times 5 + 1000 \times 60}{800 + 1000} = \boxed{35.55 \text{ mm}}$ (01)

from left edge

$\therefore \bar{Y} = \dfrac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2}$

$Y_1 = \frac{80}{2} = 40$ mm.

$Y_2 = \frac{10}{2} = 5$ mm. (01)

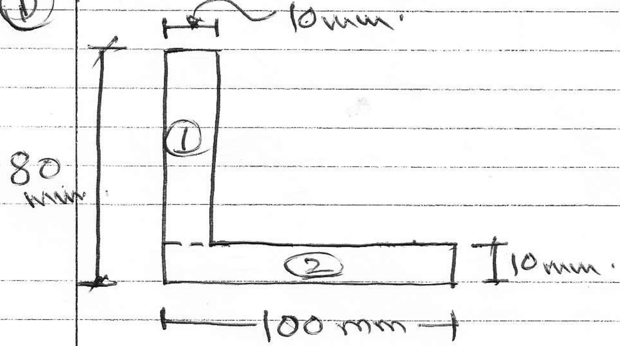
$\therefore \bar{Y} = \dfrac{800 \times 40 + 1000 \times 5}{800 + 1000} = \boxed{20.55 \text{ mm.}}$ (01)

from bottom.

$\therefore \bar{x} = 35.55$ mm.	$\bar{Y} = 20.55$ mm.

Q 6 (b)

Alternative III



$\bar{x} = \dfrac{a_1x_1 + a_2x_2}{a_1 + a_2}$

$a_1 = 70 \times 10 = 700 \text{ mm}^2$

$a_2 = 100 \times 10 = 1000 \text{ mm}^2$

$x_1 = \dfrac{10}{2} = 5 \text{ mm}$ 01

$x_2 = \dfrac{100}{2} = 50 \text{ mm}$

$\therefore \bar{x} = \dfrac{700 \times 5 + 1000 \times 50}{700 + 1000} = 31.47 \text{ mm}$ 01

$\boxed{\therefore \bar{x} = 31.47 \text{ mm}}$

$\bar{y} = \dfrac{a_1y_1 + a_2y_2}{a_1 + a_2}$

$y_1 = (10 + \dfrac{70}{2})$

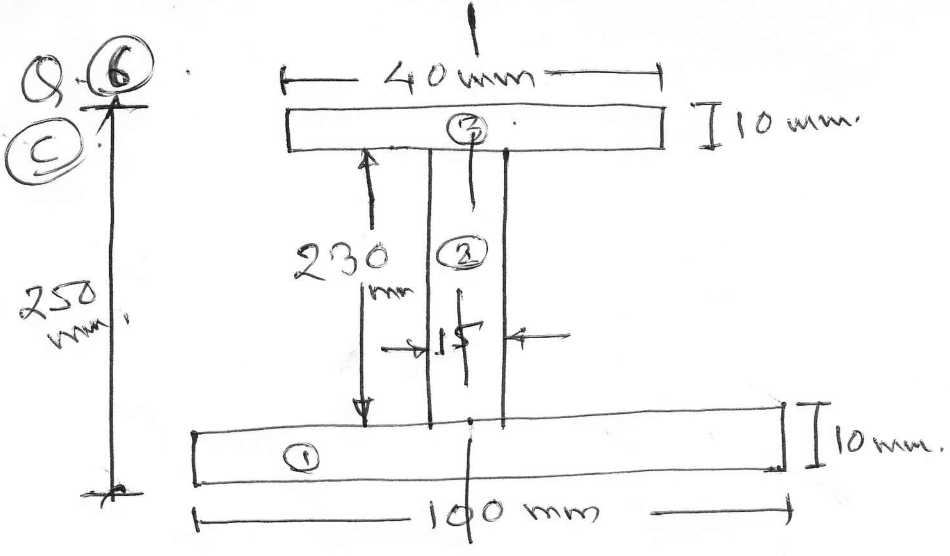
$= 10 + 35 = 45 \text{ mm}$ 01

$y_2 = \dfrac{10}{2} = 5 \text{ mm}$

$\therefore \bar{y} = \dfrac{700 \times 45 + 1000 \times 5}{700 + 1000} = 21.47 \text{ mm}$ 01

$\boxed{\bar{y} = 21.47 \text{ mm}}$

Q 6 (c)



$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

Since it is symmetrical about Y axis drawn through centre (as shown) the $\bar{x} = 50\,mm$ from left edge

$a_1 = 100 \times 10 = 1000\ mm^2$

$x_1 = \frac{100}{2} = 50\ mm$

$a_2 = 230 \times 15$

$= 3450\ mm^2$

$x_2 = 50\ mm$

$a_3 = 40 \times 10 = 400\ mm^2$

$x_3 = 50\ mm$

$$\bar{x} = \frac{1000 \times 50 + 3450 \times 50 + 400 \times 50}{1000 + 3450 + 400}$$

02

$$\boxed{\therefore \bar{x} = 50\ mm}$$

$y_1 = \frac{10}{2} = 5.0\ mm$

$y_2 = 10 + \frac{230}{2}$

$= 125\ mm$

$y_3 = 240 + \frac{10}{2}$

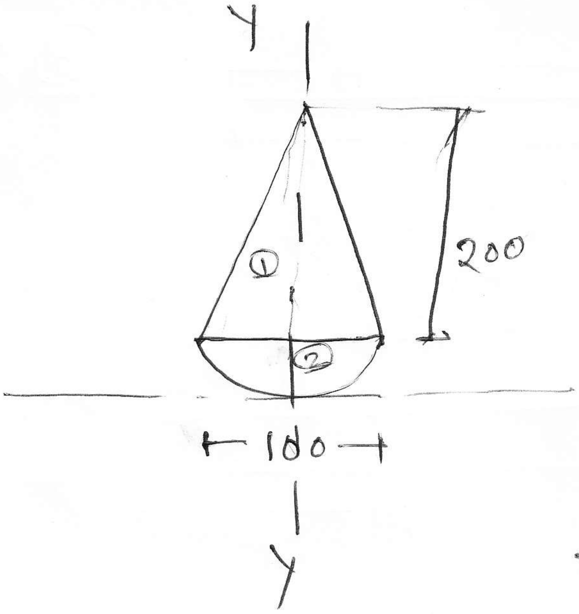
$= 245\ mm$

$$\bar{Y} = \frac{1000 \times 5 + 3450 \times 125 + 400 \times 245}{1000 + 3450 + 400}$$

02

$\boxed{\bar{Y} = 110.15\ mm}$ from bottom.

Q:6 d.



The vertical line YY is Passing through centre and the figure is Symetrical about YY Hence.

$\bar{x}$ can be directly written as

$$\boxed{\therefore \bar{x} = \frac{100}{2} = \underline{50\,mm} \text{ from extreme Left edge}}$$

(02)

$$\therefore \bar{Y} = \frac{V_1 Y_1 + V_2 Y_2}{V_1 + V_2}$$

V_1 = volume of Cone.

$= \frac{1}{3}\pi (50)^2 \times 200$

$= 523.59 \times 10^3\ mm^3$.

$Y_1 = 50 + \frac{200}{4} = 100\,mm$.

V_2 = volume of hemisphere

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi (50)^3$

$= 261.80 \times 10^3\ mm^3$.

$Y_2 = 50 - \frac{3r}{8} = 50 - \frac{3 \times 50}{8} = \underline{31.25\,mm}$.

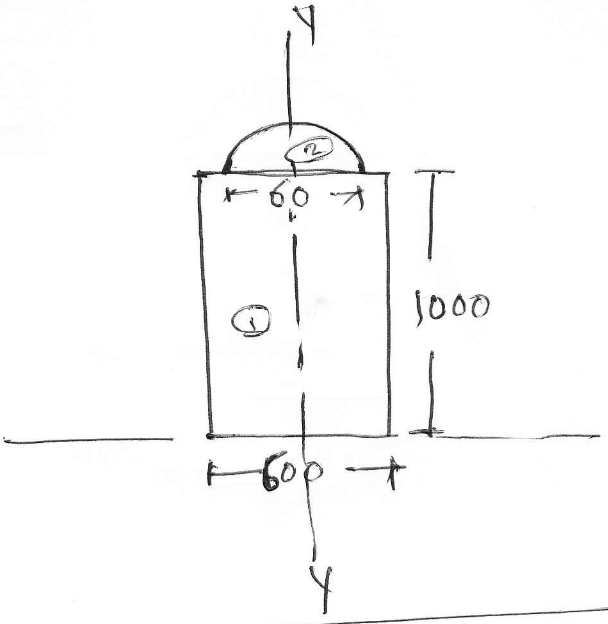
(02)

$$\therefore \bar{Y} = \frac{523.59 \times 10^3 \times 100 + 261.80 \times 10^3 \times 31.25}{523.59 \times 10^3 + 261.80 \times 10^3}$$

$\boxed{\bar{Y} = 77.08\ mm}$ From Extreme bottom.

28/30 marks.

Q. 6
(e)



The figure is symetrical about Y-Y axis (as shown)

Hence $\bar{x}$ will be at 300 mm. from extreme left

$\boxed{\therefore \bar{x} = 300 \text{ mm} \quad \text{from extreme left}}$ → 02.

V_1 = Volume of cylinder

$= \pi r^2 h$.

$= \pi \times (300)^2 \times 1000$

$V_1 = 282.74 \times 10^6 \text{ mm}^3$.

V_2 = vol. of hemisphere

$= \frac{2}{3} \pi r^3$.

$= \frac{2}{3} \times \pi \times (30)^3$

$= 56.55 \times 10^3 \text{ mm}^3$

$Y_1 = \frac{1000}{2} = 500 \text{ mm}$. (2).

$Y_2 = 1000 + \left(\frac{3 \times 30}{8}\right)$

$Y_2 = 1011.25 \text{ mm}$.

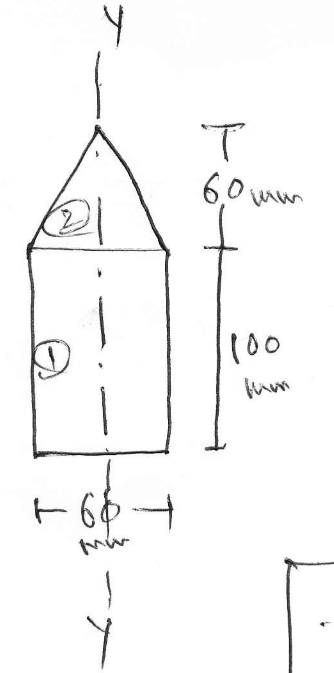
$$\bar{Y} = \frac{V_1 Y_1 + V_2 Y_2}{V_1 + V_2}$$

$$\bar{Y} = \frac{282.74 \times 10^6 \times 500 + 56.55 \times 10^3 \times 1011.25}{(282.74 \times 10^6 + 56.55 \times 10^3)}$$

$\boxed{\bar{Y} = 500.102 \text{ mm}}$

Q 6

(f)



Since it is symmetrical about Y-Y axis the $\bar{x}$ will lie on it.

$\therefore \bar{x} = 30$ mm from left edge (2)

$\therefore \bar{Y} = \frac{V_1 Y_1 + V_2 Y_2}{V_1 + V_2}$

$V_1 = \pi r^2 h$

$= \pi \times (30)^2 \times 100$

$= 282.74 \times 10^3 \text{ mm}^3$

$Y_1 = 50$ mm.

$V_2 = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi \times (30)^2 \times 60$

$= 56.55 \times 10^3 \text{ mm}^3$

$Y_2 = 100 + \frac{60}{4}$

$= 115$ mm.

$\therefore \bar{Y} = \frac{282.74 \times 10^3 \times 50 + 56.55 \times 10^3 \times 115}{(282.74 \times 10^3 + 56.55 \times 10^3)}$ (2)

$\therefore \bar{Y} = 60.83$ mm from Bottom.

$\bar{x} = 30$ mm from left edge

Note for Examiner: In the above answers if students are writing some additional points or information which may be correct but not included in the model answer sheet. Examiners are requested to go through each answer carefully.