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#### MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

#### WINTER-2017 EXAMINATION

Subject Code:

17104

#### **Model Answer**

#### **Important Instructions to examiners:**

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any <u>TEN</u> of the following:	20
	a)	Find the value of 'P' if $\begin{vmatrix} P & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$	02
	Ans	P  4  -4	
		$\begin{vmatrix} P & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$	
		P(-2+4)-4(3+2)-4(-12-4)=0	1
		2P - 20 + 64 = 0	1
		$\therefore P = -22$	<b>-</b>
		If $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$ find matrix X such that $A + 2X = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix}$	02
	Ans	$A + 2X = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix}$	
		$\therefore 2X = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix} - A$	
		$\therefore 2X = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$	1



### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$\therefore 2X = \begin{bmatrix} 2 & 8 \\ -4 & -2 \end{bmatrix}$ $\therefore X = \frac{1}{2} \begin{bmatrix} 2 & 8 \\ -4 & -2 \end{bmatrix}$ $\therefore X = \begin{bmatrix} 1 & 4 \\ -2 & -1 \end{bmatrix}$	1
		If $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$ find $(AB)^T$	02
	Ans	$AB = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$	
		$AB = \begin{bmatrix} 12+0+30 & 2+20+42 \\ 0+0+10 & 0+4+14 \end{bmatrix} = \begin{bmatrix} 42 & 64 \\ 10 & 18 \end{bmatrix}$	1
		$\therefore (AB)^T = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix}$	1
	d) Ans	If $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ show that $A^2$ is a null matrix. $A^2 = A \cdot A$	02
		$\begin{vmatrix} A - A \cdot A \\ = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ $= \begin{bmatrix} 4 - 4 & 8 - 8 \\ -2 + 2 & -4 + 4 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	1
		$\therefore A^2$ is a null matrix.	1
	e)	Resolve into partial fraction $\frac{1}{x^2 + x}$	02
	Ans	Let $\therefore \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$	1/2



### WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub	Answer	Marking
No.	Q. N.	Allswei	Scheme
1.	e)	1 = (x+1)A + xB	
		Put $x = 0$	
		$\therefore A = 1$ Put $x = -1$	1/2
		$\therefore B = -1$	1/2
		$\therefore \frac{1}{x(x+1)} = \frac{1}{x} + \frac{-1}{x+1}$	1/2
		x(x+1) $x$ $x+1$	, -
		Prove that $\sin 2\theta \cos 2\theta$	02
	f)	Prove that $\frac{\sin 2\theta}{\sin \theta} - \frac{\cos 2\theta}{\cos \theta} = \sec \theta$	02
	Ans	Consider $\frac{\sin 2\theta}{\sin \theta} - \frac{\cos 2\theta}{\cos \theta}$	
	Alls	$\sin \theta  \cos \theta$ $\sin 2\theta \cdot \cos \theta - \cos 2\theta \cdot \sin \theta$	
		$= \frac{\sin 2\theta \cdot \cos \theta - \cos 2\theta \cdot \sin \theta}{\sin \theta \cdot \cos \theta}$	1/2
		$\sin(2\theta-\theta)$	17
		$=\frac{1}{\sin\theta.\cos\theta}$	1/2
		$=\frac{\sin(\theta)}{\cos(\theta)}$	1/2
		$\sin \theta . \cos \theta$	
		$=\frac{1}{\cos\theta}$	
		$= \sec \theta$	1/2
	g)	Evaluate: $2\cos 75^{\circ}.\cos 15^{\circ}$	02
	Ans	$2\cos 75^{\circ}.\cos 15^{\circ}$	
		$= \cos(75^{\circ} + 15^{\circ}) + \cos(75^{\circ} - 15^{\circ})$	1/2
		$=\cos 90^{\circ} + \cos 60^{\circ}$	1/2
		$=0+\frac{1}{2}$	
		$=\frac{1}{2}$ or 0.5	1
	h)	Find principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$	02
	Ans	$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$	1
			1



#### WINTER – 2017 EXAMINATION

### **Model Answer**

			Model Answer Subject Code.	17104
	Q. No.	Sub Q. N.	Answer	Marking Scheme
	1.	h)	$=\pi - \frac{\pi}{3} \text{ or } 180^{0} - 60^{0}$	1/2
			$=\frac{2\pi}{3}$ or $120^{\circ}$	1/2
			<u>OR</u>	
			Let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$	
			$\therefore -\frac{1}{2} = \cos \theta$	1/2
			But $-\frac{1}{2} = -\cos 60$	1/2
			$\because \cos(180-60) = -\cos 60$	1/2
			$\therefore -\frac{1}{2} = \cos\left(180 - 60\right)$	
			$-\frac{1}{2} = \cos(120) = \cos\theta$	
			$\begin{array}{l} 2 \\ \therefore \theta = 120 \end{array}$	1/2
		i)	Without using calculator find the value of $\sin\left(\frac{\pi}{12}\right)^c$	02
		Ans	$\sin\left(\frac{\pi}{12}\right) = \sin\left(15^{\circ}\right) = \sin\left(45^{\circ} - 30^{\circ}\right) \qquad OR  \sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$	1/2
			$= \sin 45^{\circ} \cdot \cos 30^{\circ} - \cos 45^{\circ} \cdot \sin 30^{\circ} \qquad OR  \sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4} - \cos \frac{\pi}{4} \cdot \sin \frac{\pi}{4}$	1/2
			$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$	
			$=\frac{\sqrt{3}-1}{2\sqrt{2}}$	1
			$2\sqrt{2}$	
			(A) 1	
		j)	If $\tan\left(\frac{A}{2}\right) = \frac{1}{\sqrt{3}}$ find $\sin A$	02
		Ans	$\tan\left(\frac{A}{2}\right) = \frac{1}{\sqrt{3}}$	
			$\therefore \frac{A}{2} = \tan^{-1} \left( \frac{1}{\sqrt{3}} \right)$	1/2
			$2 \qquad (\sqrt{3})$	
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#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub	Answer	Marking
No.	Q. N.	Miswei	Scheme
1.	j)	$\frac{A}{2} = 30^{\circ}$	1/2
		$\therefore A = 60^{\circ}$	1/2
		$\therefore \sin A = \sin \left(60^{\circ}\right) = \frac{\sqrt{3}}{2}$	1/2
	k)	Find the slope and X-intercept of the line $\frac{x}{2} - \frac{y}{3} = \frac{1}{4}$	02
	Ans	$\frac{x}{2} - \frac{y}{3} - \frac{1}{4} = 0$	
		Slope = $-\frac{a}{b} = -\frac{\frac{1}{2}}{-\frac{1}{3}} = \frac{3}{2}$	1
		X-intercept = $-\frac{c}{a} = -\frac{-\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$ OR put $y = 0$ : $x = \frac{1}{2}$	1
		OR	
		$\begin{vmatrix} \frac{x}{2} - \frac{y}{3} - \frac{1}{4} = 0 \\ 6x - 4y - 3 = 0 \end{vmatrix}$	
		Slope = $-\frac{a}{b} = -\frac{6}{-4} = \frac{3}{2}$	1
		$X-intercept = -\frac{c}{a} = -\frac{-3}{6} = \frac{1}{2}$	1
	l) Ans	Find the range and coefficient of range of the data :5,25,65,55,35,45,15  Range = Largest value – Smallest value = $L - S$	02
		= 65 - 5 $= 60$	1
		Coefficient of range = $\frac{L-S}{L+S} = \frac{60}{70} = 0.857$	1
2.		Attempt any FOUR of the following:	16
	a)	Solve the following equations by using Cramer's rule:	04
		3x+3y-z=11 $2x-y+2z=9$ $4x+3y+2z=25$	
		Page No	05/20



#### WINTER – 2017 EXAMINATION

### **Model Answer**

		Model Miswel Subject Code.	17104
Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	3 3 -1	
	Ans	$D = \begin{vmatrix} 2 & -1 & 2 \end{vmatrix}$	
	Alls	$D = \begin{vmatrix} 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$	
		=3(-2-6)-3(4-8)-1(6+4)	
		$\begin{vmatrix} -22 \end{vmatrix} = -22$	1
			1
		$D = \begin{bmatrix} 1 & 3 & 1 \\ 9 & -1 & 2 \end{bmatrix}$	
		$D_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$	
		=11(-2-6)-3(18-50)-1(27+25)	
		=-44	
		$\therefore x = \frac{D_x}{D} = \frac{-44}{-22} = 2$	1
		3 11 -1	
		$D_{y} = \begin{vmatrix} 2 & 9 & 2 \end{vmatrix}$	
		$D_{y} = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$	
		=3(18-50)-11(4-8)-1(50-36)	
		= -66	
		$\therefore y = \frac{D_y}{D} = \frac{-66}{-22} = 3$	
		$\therefore y = \frac{1}{D} = \frac{1}{-22} = 3$	1
		3 3 11	
		$D_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$	
		=3(-25-27)-3(50-36)+11(6+4)	
		= -88	
		$\therefore z = \frac{D_z}{D} = \frac{-88}{-22} = 4$	1
		D $-22$	
		If $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	
	b)	$  \text{ If } A =   3 \ 0 \ 1 \  , B =   1 \ 4 \ 5 \  , C =   2 \  , X =   y \  $	04
		such that $(A+2B)C = X$ find $x, y, z$	
	Ans	$\begin{bmatrix} \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} & \begin{bmatrix} 3 & 0 & 2 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} x \end{bmatrix}$	
		$ \begin{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} $	
		$\left  \begin{array}{cccccccccccccccccccccccccccccccccccc$	
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WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)		1
		$\begin{bmatrix} 7 & 3 & 6 \\ 5 & 8 & 11 \\ 7 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1
		$\begin{bmatrix} 7+6+18\\ 5+16+33\\ 7+6+6 \end{bmatrix} = \begin{bmatrix} 31\\ 54\\ 19 \end{bmatrix} = \begin{bmatrix} x\\ y\\ z \end{bmatrix}$ by equating	2
		x = 31, y = 54, z = 19	
	c)	If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find $A^{-1}$ by adjoint method.	04
	Ans	$ A  = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3+0) - 2(-1-0) - 2(2-0) = 1 \neq 0 : A^{-1} \text{ exists}$	1
		Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 3 & -1 & 2 \\ -2 & 1 & -2 \\ 6 & -2 & 5 \end{bmatrix}$	1
		Matrix of cofactors = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$	1
		$\begin{vmatrix} OR \\ c_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3,  c_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = 1,  c_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = 2,$	
		$\begin{vmatrix} c_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = 2,  c_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1,  c_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = 2,$	2
		$\begin{vmatrix} c_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 6,  c_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = 2,  c_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 5$	



### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	$Adj.A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	1/2
		$A^{-1} = \frac{1}{ A }.adj.A = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	1/2
	d)	If $A = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ , $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & -1 \end{bmatrix}$ , $C = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ Verify that $A(BC) = (AB)C$	04
	Ans	L.H.S = $A(BC) = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$	
		$A(BC) = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1+3 \\ 2+0 \\ 3-3 \end{bmatrix}$	
		$A(BC) = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$	1
		$A(BC) = \begin{bmatrix} 12+2-0\\12+2+0 \end{bmatrix}$ $A(BC) = \begin{bmatrix} 14\\14 \end{bmatrix}$	1
		R.H.S = $(AB)C = \begin{cases} \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$	
		$(AB)C = \begin{bmatrix} 3+2-3 & 3+0+1 \\ 3+2+6 & 3+0-2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$	
		$(AB)C = \begin{bmatrix} 2 & 4 \\ 11 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $(AB)C = \begin{bmatrix} 2+12 \\ 11+3 \end{bmatrix}$	1
		$\begin{bmatrix} 11+3 \end{bmatrix}$	



### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$(AB)C = \begin{bmatrix} 14\\14 \end{bmatrix}$	1
		$\therefore A(BC) = (AB)C$	
	e)	Resolve into partial fraction: $\frac{x+3}{(x^2-1)(x+5)}$	04
	Ans	Let $\frac{x+3}{(x-1)(x+1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+5}$	1/2
		x+3 = A(x+1)(x+5) + B(x-1)(x+5) + C(x-1)(x+1)	
		Put $x = 1$	
		1+3=A(1+1)(1+5)	
		4 = A(12)	
		$\therefore A = \frac{1}{3}$	1
		Put $x = -1$	
		-1+3=B(-1-1)(-1+5)	
		2 = B(-8)	
		$\therefore B = -\frac{1}{4}$	1
		Put $x = -5$	
		-5+3=C(-5-1)(-5+1)	
		-2 = C(24)	
		$-2 = C(24)$ $\therefore C = -\frac{1}{12}$	1
		$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{4}$ $-\frac{1}{42}$	
		$\therefore \frac{x+3}{(x-1)(x+1)(x+5)} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{12}}{x+5}$	1/2
	f)	Resolve into partial fraction $\frac{e^x}{e^{2x} + 4e^x + 3}$	04
	Ans	$e^{-x} + 4e^{x} + 3$ Put $e^{x} = m$	1/2
		$\therefore \frac{m}{m^2 + 4m + 3} = \frac{m}{(m+1)(m+3)}$	
		$\therefore \frac{m}{(m+1)(m+3)} = \frac{A}{m+1} + \frac{B}{m+3}$	1/2
		(m+1)(m+3) $m+1$ $m+3$	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	f)	$\therefore m = (m+3)A + (m+1)B$	
		Put $m = -1$	
		$\therefore -1 = A(-1+3)$	
		$\therefore A = -\frac{1}{2}$	1
		Put $m = -3$	
		$\therefore -3 = B\left(-3+1\right)$	
		$\therefore B = \frac{3}{2}$	1
		$-\frac{1}{2}$	
		$\therefore \frac{m}{(m+1)(m+3)} = \frac{-\frac{1}{2}}{m+1} + \frac{\frac{3}{2}}{m+3}$	1/2
		$\frac{1}{2}$ $\frac{3}{2}$	
		$\therefore \frac{e^{x}}{(e^{x}+1)(e^{x}+3)} = \frac{-\frac{1}{2}}{e^{x}+1} + \frac{\frac{3}{2}}{e^{x}+3}$	1/2
3.		Attempt any FOUR of the following:	16
	a)	Solve the simultaneous equations by using matrix inversion	04
		method: $2x + 3y - z = -3$ , $5x + y + 3z = 10$ , $4x + 3y - 2z = -3$	
	Ans	Let $A = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 1 & 3 \\ 4 & 3 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} -3 \\ 10 \\ -3 \end{bmatrix}$	
		$\begin{bmatrix} 4 & 3 & -2 \end{bmatrix}$ $\begin{bmatrix} z \end{bmatrix}$ $\begin{bmatrix} -3 \end{bmatrix}$	
		$\begin{bmatrix} 2 & 3 & -1 \\ \text{Consider} &  A  = 5 & 1 & 3 \end{bmatrix}$	
		Consider, $ A  = \begin{vmatrix} 5 & 1 & 3 \\ 4 & 3 & -2 \end{vmatrix}$	
		=2(-2-9)-3(-10-12)-1(15-4)	
		$=33 \neq 0$ : $A^{-1}$ exists	1/2
		$egin{bmatrix}  1 & 3  &  5 & 3  &  5 & 1  \  3 & -2  &  4 & -2  &  4 & 3  \end{bmatrix}$	
		Matrix of minors = $\begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -3 & 0 & -6 \end{vmatrix}$	1
		Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -11 & -22 & 11 \\ -3 & 0 & -6 \\ 10 & 11 & -13 \end{bmatrix}$	1
		$\begin{bmatrix} \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 5 & 1 \end{vmatrix} \end{bmatrix}$	
		Page No 1	0.420



#### WINTER -2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	a)	Matrix of cofactors = $\begin{bmatrix} -11 & 22 & 11 \\ 3 & 0 & 6 \\ 10 & -11 & -13 \end{bmatrix}$ <i>OR</i>	1/2
		$\begin{vmatrix} c_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = -11, \ c_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 3 \\ 4 & -2 \end{vmatrix} = 22, \ c_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} = 11,$ $c_{21} = (-1)^{2+1} \begin{vmatrix} 3 & -1 \\ 3 & -2 \end{vmatrix} = 3,  c_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = 0,  c_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} = 6,$ $c_{31} = (-1)^{3+1} \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 10,  c_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ 5 & 3 \end{vmatrix} = -11, \ c_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} = -13$	1½
		$AdjA = \begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix}$ $A^{-1} = \frac{1}{ A }.adj.A = \frac{1}{33} \begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix}$ $X = A^{-1}B$	1
		$= \frac{1}{33} \begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix} \begin{bmatrix} -3 \\ 10 \\ -3 \end{bmatrix}$ $= \frac{1}{33} \begin{bmatrix} 33+30-30 \\ -66+0+33 \\ -33+60+39 \end{bmatrix} = \frac{1}{33} \begin{bmatrix} 33 \\ -33 \\ 66 \end{bmatrix}$	
		$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$	1
	b)	Resolve into partial fractions: $\frac{x^3 + x}{x^2 - 9}$	04
	Ans	$x^{2}-9)\overline{x^{3}+x}$ $x^{3}-9x$ $- +$ $10x$	



#### WINTER – 2017 EXAMINATION

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Q. Sub No. Q. N. Answer	Marking Scheme
3. b) $ \therefore \frac{x^3 + x}{x^2 - 9} = x + \frac{10x}{x^2 - 9} $	1
10x = (x-3)A + (x+3)B Put $x = -3$ $-30 = -6A$	
$\therefore A = 5$ Put $x = 3$ $\therefore B = 5$	1
$\therefore \frac{10x}{(x+3)(x-3)} = \frac{5}{x+3} + \frac{5}{x-3}$	1/2
From (1) $\Rightarrow \frac{x^3 + x}{x^2 - 9} = x + \frac{5}{x + 3} + \frac{5}{x - 3}$	1/2
Resolve into partial fractions $\frac{x^2 - 2x + 3}{x^3 + x}$	04
Ans $\frac{x^2 - 2x + 3}{x^3 + x} = \frac{x^2 - 2x + 3}{x(x^2 + 1)}$ $\therefore \frac{x^2 - 2x + 3}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$	1/2
$\therefore x^2 - 2x + 3 = (x^2 + 1)A + (x)(Bx + C)$ Put $x = 0$	
$\therefore 3 = (1)A$ $\therefore A = 3$ Put $x = 1$	1
2 = (2)A + (1)(B+C) $2 = 6+B+C$ $B+C = -4$ (1)	
Put $x = -1$ 6 = (2)A + (-1)(-B + C) 6 = 6 + B - C	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	B-C=0	1+1
	d) Ans	Prove that $\frac{\cos 3A \sin 9A - \sin A \cos 5A}{\cos A \cos 5A - \sin 3A \sin 9A} = \tan 8A$ $L.H.S. = \frac{\cos 3A \sin 9A - \sin A \cos 5A}{\cos A \cos 5A - \sin 3A \sin 9A}$ $= \frac{2\cos 3A \sin 9A - 2\sin A \cos 5A}{2\cos A \cos 5A - 2\sin 3A \sin 9A}$ $\left[\sin (3A + 9A) - \sin (3A - 9A)\right] - \left[\sin (A + 5A) + \sin (A - 5A)\right]$	04
		$= \frac{\left[\sin(3A+9A) - \sin(3A-9A)\right] - \left[\sin(A+5A) + \sin(A-5A)\right]}{\left[\cos(A+5A) + \cos(A-5A)\right] - \left[\cos(3A-9A) - \cos(3A+9A)\right]}$ $= \frac{\left[\sin(12A) - \sin(-6A)\right] - \left[\sin(6A) + \sin(-4A)\right]}{\left[\cos(6A) + \cos(-4A)\right] - \left[\cos(-6A) - \cos(12A)\right]}$ $\left[\sin(12A) + \sin(6A)\right] - \left[\sin(6A) - \sin(4A)\right]$	1 1/2
		$= \frac{\left[\sin(12A) + \sin(6A)\right] - \left[\sin(6A) - \sin(4A)\right]}{\left[\cos(6A) + \cos(4A)\right] - \left[\cos(6A) - \cos(12A)\right]}$ $= \frac{\sin(12A) + \sin(6A) - \sin(6A) + \sin(4A)}{\cos(6A) + \cos(4A) - \cos(6A) + \cos(12A)}$ $= \frac{\sin(12A) + \sin(4A)}{\cos(12A) + \sin(4A)}$	1/2
		$= \frac{2\sin\left(\frac{12A+4A}{2}\right).\cos\left(\frac{12A-4A}{2}\right)}{2\cos\left(\frac{4A+12A}{2}\right).\cos\left(\frac{4A-12A}{2}\right)}$ $= \frac{2\sin(8A).\cos(4A)}{2\cos(8A).\cos(-4A)}$	1
		$= \frac{\sin(8A).\cos(-4A)}{\cos(8A).\cos(4A)}$ $= \tan 8A = R.H.S.$	1/2
	e)	Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ Page No.1	04



#### WINTER -2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.		A	nswer	Marking Scheme
3.	e) Ans	A+B R	A		1
		Right Angled Triangle	Acute Angle	Trigonometric Ratios	
		Δ OMP	∠MOP = A	$\sin A = \frac{PM}{OP},  \cos A = \frac{OM}{OP}$	
		Δ OPQ	∠POQ = B	$\sin B = \frac{PQ}{OQ},  \cos B = \frac{OP}{OQ}$	1
		Δ PRQ	∠PQR = A	$\sin A = \frac{PR}{PQ},  \cos A = \frac{QR}{PQ}$	
		Δ ONQ	∠NOQ = A+B	$\sin(A+B) = \frac{QN}{OQ},  \cos(A+B) = \frac{ON}{OQ}$	
		$\therefore \sin(A+B) = \frac{QN}{OQ}$	l		
		$=\frac{QR+RN}{OQ}$			1/2
		$=\frac{QR+PM}{OQ}$			
		$=\frac{QR}{OQ}+\frac{PM}{OQ}$	nu on		1/2
		$=\frac{QR}{PQ}\times\frac{PQ}{OQ}+$	$\frac{PM}{OP} \times \frac{OP}{OQ}$		
		$= \cos A \sin B +$ $= \sin A \cos B +$			1
		511111000 D	23011011112		



WINTER – 2017 EXAMINATION

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	e)	OR Q	
		Consider a standard unit circle Let P,Q,R,S be points such that	1
		$\angle XOP = A$ , $\angle XOQ = B$ , $\angle XOR = A - B$	
		From fig. $A = A + B = A + B$	
		$\angle POQ = A - B$	
		$\therefore \angle POQ = \angle XOR$	
		$\therefore \text{Chord } PQ = \text{Chord RS}$	
		$P(\cos A, \sin A)$ , $Q(\cos B, \sin B)$	
		$R \left[\cos(A-B),\sin(A-B)\right]$ , S(1,0)	1/2
		PQ = RS	
		$\sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} = \sqrt{[\cos(A - B) - 1]^2 + [\sin(A - B) - 0]^2}$	
		$\left[ (\cos A - \cos B)^{2} + (\sin A - \sin B)^{2} = \left[ \cos (A - B) - 1 \right]^{2} + \left[ \sin (A - B) - 0 \right]^{2}$	1
		$\therefore \cos^2 A + \cos^2 B - 2\cos A\cos B + \sin^2 A + \sin^2 B - 2\sin A\sin B =$	
		$\cos^2(A-B)+1-2\cos(A-B)+\sin^2(A-B)$	
		$\therefore 1 + 1 - 2(\cos A \cos B + \sin A \sin B) = 1 + 1 - 2\cos(A - B)$	
		$\therefore \cos A \cos B + \sin A \sin B = \cos (A - B)$	1/2
		Consider $\sin(A+B) = \cos\left(\frac{\pi}{2} - (A+B)\right)$	1/2
		$=\cos\left(\left(\frac{\pi}{2}-A\right)-B\right)$	
		$=\cos\left(\frac{\pi}{2}-A\right)\cos B+\sin\left(\frac{\pi}{2}-A\right)\sin B$	
		$= \sin A \cos B + \cos A \sin B$	1/2
	f)	Prove that $\cot^{-1}\left(\frac{6}{5}\right) + \tan^{-1}\left(\frac{1}{11}\right) = \sec^{-1}\left(\sqrt{2}\right)$	04
	Ans	Let $\cot^{-1}\left(\frac{6}{5}\right) = \tan^{-1}\left(\frac{5}{6}\right)$	
		$L.H.S. = \cot^{-1}\left(\frac{6}{5}\right) + \tan^{-1}\left(\frac{1}{11}\right)$	
		Page No.1	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
3.	f)	$= \tan^{-1}\left(\frac{5}{6}\right) + \tan^{-1}\left(\frac{1}{11}\right)$	1/2
		$= \tan^{-1} \left( \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{611}} \right)$ $= \tan^{-1} \left( \frac{\frac{55 + 6}{66}}{\frac{66 - 5}{66}} \right)$	1
		$= \tan^{-1}(1)$	1
		$=\frac{\pi}{4}=R.H.S.$	1/2
		$\therefore R.H.S = \sec^{-1}\left(\sqrt{2}\right) = \frac{\pi}{4}$	1
4.		Attempt any FOUR of the following:	16
	a)	Without using calculator find the value of $\sin(150^{\circ}) + \cos(300^{\circ}) - \tan(315^{\circ}) + \sec^2(3660^{\circ})$	04
	Ans	$\sin(150^{\circ}) = \sin(2 \times 90 - 30) = \sin 30 = \frac{1}{2}$	1/2
		$\cos(300^{\circ}) = \cos(4 \times 90 - 60) = \cos 60 = \frac{1}{2}$	1/2
		$\tan(315^{\circ}) = \tan(4 \times 90 - 45) = -\tan 45 = -1$	1/2
		$\sec^{2}(3660^{0}) = \left[\sec(3660^{0})\right]^{2} = \left[\sec(40 \times 90 + 60)\right]^{2} = \left[\sec(60)\right]^{2} = \left[2\right]^{2} = 4$	1
		$ :: \sin(150^{\circ}) + \cos(300^{\circ}) - \tan(315^{\circ}) + \sec^{2}(3660^{\circ}) $ $ 1 : 1 $	1
		$= \frac{1}{2} + \frac{1}{2} - (-1) + 4$	1
		= 6	1/2
	b) Ans	In any $\triangle ABC$ , $A+B+C=\pi$ prove that $\sin 2A + \sin 2B - \sin 2C = 4\cos A\cos B\sin C$ $A+B+C=\pi$	04
		$A + B = \pi - C$ $\therefore \sin(A + B) = \sin(\pi - C) = \sin C$	1/2
		$ \sin(A+B) = \sin(\pi - C) = \sin C $	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$\therefore \cos(A+B) = \cos(\pi-C) = -\cos C$	1/2
	Ans	$L.H.S. = \sin 2A + \sin 2B - \sin 2C$	
		$=2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)-\sin 2C$	
		$= 2\sin(A+B)\cos(A-B) - \sin 2C$	1
		$= 2\sin C\cos(A-B) - \sin 2C$	
		$= 2\sin C\cos(A-B) - 2\sin C\cos C$	1/2
		$= 2\sin C \Big[\cos (A-B) - \cos C\Big]$	
		$= 2\sin C \Big[\cos (A-B) + \cos (A+B)\Big]$	
		$= 2\sin C \left[ 2\cos\left(\frac{A-B+A+B}{2}\right)\cos\left(\frac{A-B-A-B}{2}\right) \right]$	1
		$= 2\sin C 2\cos(A)\cos(-B)$	
		$= 4\sin C\cos A\cos B = R.H.S.$	1/2
	c)	Show that $\sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ} = \frac{3}{16}$	04
	Ans	$L.H.S. = \sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ}$	
		-	1/
		$=\sin 20^{\circ}\sin 40^{\circ}\frac{\sqrt{3}}{2}\sin 80^{\circ}$	1/2
		$=\frac{\sqrt{3}}{4}[2\sin 20^{\circ}\sin 40^{\circ}]\sin 80^{\circ}$	
		$=\frac{\sqrt{3}}{4}\left(\cos\left(-20^{0}\right)-\cos 60^{0}\right)\sin 80^{0}$	1
		$=\frac{\sqrt{3}}{4}\left(\cos 20^{\circ}\sin 80^{\circ}-\cos 60^{\circ}\sin 80^{\circ}\right)$	
		$=\frac{\sqrt{3}}{8}\left(2\cos 20^{0}\sin 80^{0}-2\frac{1}{2}\sin 80^{0}\right)$	1/2
		$=\frac{\sqrt{3}}{8}\left(\sin 100^{0}+\sin 60^{0}-\sin 80^{0}\right)$	1
		$= \frac{\sqrt{3}}{8} \left( \sin \left( 2 \times 90 - 80 \right) + \frac{\sqrt{3}}{2} - \sin 80^{\circ} \right)$	1/2
		$=\frac{\sqrt{3}}{8}\left(\sin 80^{0}+\frac{\sqrt{3}}{2}-\sin 80^{0}\right)$	
		$=\frac{3}{16}=R.H.S.$	1/2
		Page No.1	17/20



#### WINTER – 2017 EXAMINATION

### **Model Answer**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	If $x > 0$ , $y > 0$ and $xy < 1$ then prove that	04
		$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$	
	Ans	Let $\tan^{-1} x = A$ and $\tan^{-1} y = B$ : $x = \tan A$ and $y = \tan B$	1
		$\tan\left(A+B\right) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	1
		$A + B = \tan^{-1} \left( \frac{\tan A + \tan B}{1 - \tan A \tan B} \right)$	1
		$\therefore \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$	1
		OR	
		Let $\tan^{-1} x = A$ and $\tan^{-1} y = B$ : $x = \tan A$ and $y = \tan B$	1
		$R.H.S. = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	
		$= \tan^{-1} \left( \frac{\tan A + \tan B}{1 - \tan A \tan B} \right)$	1
		$= \tan^{-1} \left( \tan \left( A + B \right) \right)$	1
		$= A + B = \tan^{-1} x + \tan^{-1} y$	1
		= L.H.S.	1
	e)	Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\cot 2A}{\cot 8A}$	04
	Ans	$L.H.S. = \frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$	1
		$=\frac{(1-\cos 8A)\cos 4A}{(1-\cos 4A)\cos 8A}$	1
		$= \frac{2\sin^2 4A\cos 4A}{2\sin^2 2A\cos 8A}$	
		$\sin 4A \sin 4A \cos 4A$	
		$= \frac{1}{\sin 2A \sin 2A \cos 8A}$	
		$= \frac{2\sin 2A\cos 2A\sin 4A\cos 4A}{\sin 2A\sin 2A\cos 8A}$	
			1
		$= \frac{\sin 2A \cos 8A}{\sin 2A \cos 8A}$	
		$= \cot 2A \times \tan 8A$	
		Page No.1	18/20



### WINTER – 2017 EXAMINATION

### **Model Answer**

		Subject Code.	1/104
Q. No.	Sub Q. N.	Answer	Marking Scheme
4	e)	$=\frac{\cot 2A}{\cot 8A}$	1
	f)	Prove that: $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$	04
	Ans.	Put $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ $\therefore \sin \theta = \frac{3}{5}$	
		$\therefore \tan \theta = \frac{3}{4} \qquad \qquad \therefore \theta = \tan^{-1} \left(\frac{3}{4}\right)$	
		$\therefore \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$	1
		Put $\sin^{-1}\left(\frac{8}{17}\right) = \alpha$ $\therefore \sin \alpha = \frac{8}{17}$	
		$\therefore \tan \alpha = \frac{8}{15} \qquad \qquad \therefore \alpha = \tan^{-1} \left( \frac{8}{15} \right)$	
		$\therefore \sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}\left(\frac{8}{15}\right)$ 15 $\alpha$	1
		Put $\cos^{-1}\left(\frac{84}{85}\right) = \beta$ $\therefore \cos \beta = \frac{84}{85}$	
		$\therefore \tan \beta = \frac{13}{84} \qquad \qquad \therefore \beta = \tan^{-1} \left( \frac{13}{84} \right)$	
		$\therefore \cos^{-1}\left(\frac{84}{85}\right) = \tan^{-1}\left(\frac{13}{84}\right)$	1
		$\therefore L.H.S. = \sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right)$	
		$\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{8}{15}\right)$	
		$= \tan^{-1} \left( \frac{\frac{3}{4} - \frac{8}{15}}{1 + \frac{3}{4} \frac{8}{15}} \right)$	1/2
		$= \tan^{-1}\left(\frac{13}{84}\right)$	1/2
		$=\cos^{-1}\left(\frac{84}{85}\right) = R.H.S$	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub	Amorrom	Marking
No.	Q. N.	Answer	Scheme
5.		Attempt any FOUR of the following:	16
	a)	$\sin \alpha = \frac{12}{13}$ , $\cos \beta = \frac{3}{5}$ , then $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$ find $\cos(\alpha + \beta)$ .	04
	Ans	$\therefore \cos^2 \alpha = 1 - \sin^2 \alpha$	
		$=1-\frac{144}{169}=\frac{25}{169}$	
		$\therefore \cos \alpha = \pm \frac{5}{13}$	
		$\therefore \cos \alpha = -\frac{5}{13}  \text{as } \frac{\pi}{2} < \alpha < \pi$	1
		$\cos \beta = \frac{3}{5}$	
		$\therefore \sin^2 \beta = 1 - \cos^2 \beta$	
		$=1-\frac{9}{25}=\frac{16}{25}$	
		$\therefore \sin \beta = \pm \frac{4}{5}$	
		$\therefore \sin \beta = \frac{4}{5}  \text{as } 0 < \beta < \frac{\pi}{2}$	1
		$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(-\frac{5}{13} \cdot \frac{3}{5}\right) - \left(\frac{12}{13} \cdot \frac{4}{5}\right)$	1
		$=\frac{-63}{65}$	1
			04
	b) Ans	Show that $\cos 59^{0} + \sin 59^{0} = \sqrt{2} \cos 14^{0}$ L.H.S. = $\cos 59^{0} + \sin 59^{0}$	04
		$= \cos 59^{0} + \cos 31^{0} \qquad \left[ \because \sin 59^{0} = \cos \left( 90^{0} - 59^{0} \right) = \cos 31^{0} \right]$	1
		$=2\cos\left(\frac{59^{0}+31^{0}}{2}\right)\cdot\cos\left(\frac{59^{0}-31^{0}}{2}\right)$	1
		$=2\cos 45^{\circ}\cdot\cos 14^{\circ}$	1/2
		$=2\left(\frac{1}{\sqrt{2}}\right)\cdot\cos 14^{0}$	1
		$=\sqrt{2}\cos 14^{0}$	1/2
		= R.H.S.	
	c)	Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right)$	04
		Page No.	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

			1
Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	Let $\cos^{-1}\left(\frac{4}{5}\right) = \theta$	
	Ans	$\therefore \cos \theta = \frac{4}{5}$ $\therefore \tan \theta = \frac{3}{4}$	
		$\therefore \tan \theta = \frac{3}{4}$	
		$\therefore \theta = \tan^{-1} \left( \frac{3}{4} \right)$	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$	1
		$\therefore L.H.S. = \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right)$	
		$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$	
		$= \tan^{-1} \left( \frac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{3}{4} \frac{3}{5}} \right)$	1
		$= \tan^{-1} \left( \frac{\frac{15+12}{20}}{1-\frac{9}{20}} \right) = \tan^{-1} \left( \frac{27}{11} \right) = R.H.S.$	2
	d)	If p is the length of perpendicular from a point $p(x_1, y_1)$ to the	
	,	line $ax + by + c = 0$ then prove that $P = \left  \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right $	04
	Ans	Y $R\left(0,-\frac{c}{b}\right)$ $P(x_{1},y_{1})$ $Q\left(-\frac{c}{a},0\right)$ $A$ Let $Q\left(\frac{-c}{a},0\right)$ and $R\left(0,\frac{-c}{b}\right)$	1
	<u> </u>	Page No.	21/20



#### WINTER – 2017 EXAMINATION

### **Model Answer**

	ı		1
Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	d)	$A(\Delta PQR) = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ -c & 0 & 1 \\ 0 & \frac{-c}{b} & 1 \end{vmatrix} = \frac{1}{2} \left[ x_1 \left( 0 + \frac{c}{b} \right) - y_1 \left( \frac{-c}{a} - 0 \right) + 1 \left( \frac{c^2}{ab} \right) \right]$ $= \frac{1}{2} \left[ \frac{x_1 c}{b} + \frac{y_1 c}{a} + \frac{c^2}{ab} \right] = \frac{1}{2} \left[ \frac{c}{ab} \left( ax_1 + by_1 + c \right) \right]$ $\therefore d(QR) = \sqrt{\left( \frac{-c}{a} - 0 \right)^2 + \left( 0 - \frac{-c}{b} \right)^2}$ $= \sqrt{\left( \frac{c^2}{a^2} \right) + \left( \frac{c^2}{b^2} \right)}$ $= \frac{c}{ab} \sqrt{a^2 + b^2}$	1
		$A(\Delta PQR) = \frac{1}{2} \times d(QR) \times PM$ $= \frac{1}{2} \times \frac{c}{ab} \sqrt{a^2 + b^2} \times PM$	1
		$\therefore \frac{1}{2} \frac{c}{ab} \left( ax_1 + by_1 + c \right) = \frac{1}{2} \frac{c}{ab} \sqrt{a^2 + b^2} \times PM$ $\therefore PM = \left  \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right $	1
	e) Ans	Find the equation of line passing through $(-1,1)$ and making an angle $\frac{\pi}{4}$ with the line $2x + 3y = 6$	04
	Alls	The slope of the given line $2x + 3y = 6$ is $m_1 = \frac{-2}{3}$ Let the slope of the required line be 'm' $\therefore \tan \theta = \left  \frac{m_1 - m_2}{1 + m_1 m_2} \right $	1/2
		putting $\theta = 45^{\circ}$ , $m_1 = \frac{-2}{3}$ , $m_2 = m$	



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
5.	e)	$\therefore \tan 45^{\circ} = \left  \frac{\frac{-2}{3} - m}{1 + \left(\frac{-2}{3}\right)m} \right $ $\therefore 1 = \left  \frac{-2 - 3m}{3 - 2m} \right $	1/2
		$\begin{vmatrix} &   &   &   &   &   &   &   &   &  $	
		$3-2m  ∴ -2-3m = 3-2m  ∴ m = -5 or m = \frac{1}{5}$ $3-2m  or -2-3m = -3+2m$	1
		Hence the equation of lines, in slope-point form is  (i) for $m = -5$ $y - y_1 = m(x - x_1)$	
		$\therefore y - 1 = -5(x + 1)$ $\therefore 5x + y + 4 = 0$ $(ii) \text{ for } m = \frac{1}{5}$	1
		$y - y_1 = m(x - x_1)$ $\therefore y - 1 = \frac{1}{5}(x + 1)$ $\therefore x - 5y + 6 = 0$	1
		x-3y+0-0	1
	f)	Find the co-ordinate of the foot of perpendicular drawn from $(3,4)$ to the straight line $4x-2y+9=0$ .	04
	Ans	The slope of the given line $4x - 2y + 9 = 0$ is $m_1 = \frac{-4}{-2} = 2$	1/2
		For perpendicular lines, $m_1 m_2 = -1$ $\therefore m_2 = \frac{-1}{2}$	1/2
		Equation of perpendicular is	
		$(y-y_1) = m(x-x_1)$ $\therefore y-4 = \frac{-1}{2}(x-3)$	
	1	Page No.	23/20



#### WINTER – 2017 EXAMINATION

### **Model Answer**

		<u>Model Miswel</u> Subject Code.	17104
Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	f)	$\therefore x + 2y - 11 = 0$	1
		Foot of the perpendicular = Point of intersection of two lines	
		Solving the equations	
		4x - 2y = -9	
		x + 2y = 11	
		$\therefore 4x - 2y = -9$	
		+ x + 2y = 11	
		5x = 2	
		$\therefore x = \frac{2}{5} \text{ and } y = \frac{53}{10}$	1+1
		$\begin{array}{c} 10 \\ 10 \end{array}$	
		$(x_1, y_1) = (\frac{2}{5}, \frac{53}{10})$	
		(5 10)	
6.		Attempt any FOUR of the following:	16
	a)	Show that the points $(6,1),(-1,8),(3,-2)$ are the vertices of right angled triangle	10
		by using slopes.	04
	Ans	Let A(6,1), B(-1,8), C(3,-2) are the vertices of the $\triangle$ ABC,	
			1
		Slope of side AB = $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 1}{-1 - 6} = -1$	1
		Slope of side BC = $m_2 = \frac{-2 - 8}{3 + 1} = \frac{-5}{2}$	1
			1
		Slope of side AC = $m_3 = \frac{-2-1}{3-6} = 1$	1
		We observe that $m_1 \times m_3 = -1$	1/2
		$\therefore$ side $AB \perp$ side AC	72
		∴ ΔABC is right angled triangle at vertex A.	1/2
		The second standard at the second standard second s	/2
		Show that distance between two parallel lines $ax + by + 4 = 0$ and	0.4
	b)	$\begin{vmatrix} c_2 - c_1 \end{vmatrix}$	04
		$ax + by + c_2 = 0$ is $d = \left  \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $	
	Ans	Considering equations as $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$	
		$L_1: ax + by + c_1 = 0$	
		$L_2: ax + by + c_2 = 0$	
		Let $P(x_1, y_1)$ be any point on the line $L_1$	
		$\therefore ax_1 + by_1 + c_1 = 0$	
		Page No 24/2	

WINTER – 2017 EXAMINATION

### **Model Answer**

Q.	Sub	Answer	Marking								
No.	Q. N.	Allswei									
		<u> </u>	Scheme								
6.	b)	$P(x_1, y_1)$ $ax + by + c = 0$									
		$ax + by + c_1 = 0$ $ax + by + c_2 = 0$									
		$\therefore ax_1 + by_1 = -c_1$									
		$PM$ is perpendicular on the line $L_2$									
		$\therefore PM = \left  \frac{ax_1 + by_1 + c_2}{\sqrt{a^2 + b^2}} \right $	1								
		$\therefore PM = \left  \frac{-c_1 + c_2}{\sqrt{a^2 + b^2}} \right  = \left  \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $	1								
		Note: If student has attempted to solve this question reward appropriate marks.									
	c)	Following are the marks obtained by two students A and B	04								
		Marks obtained by A       44       80       76       48       52       72       68       56       60       64         Marks obtained by B       48       75       54       60       63       69       72       51       57       56									
	Ans	which of the two students is more consistent?									
	Alls	For Student A:									
		$\begin{array}{c cccc} x_i & x_i^2 & \\ \hline 44 & 1936 & \\ \end{array}$									
		80 6400									
		76 5776									
		48 2304									
		52         2704           72         5184									
		68 4624									
		56 3136									
		60 3600									
		64 4096									
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$									
		Mean $\bar{x} = \frac{\sum x_i}{N} = \frac{620}{10} = 62$	1/2								
		S.D.= $\sigma = \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2} = \sqrt{\frac{39760}{10} - (62)^2} = 11.49$	1								
		Coefficient of Variance = $\frac{S.D.}{x} \times 100 = \frac{11.49}{62} \times 100 = 18.53$	1/2								
			25/20								



WINTER – 2017 EXAMINATION

Q. No.	Sub Q. N.	Answer	Marking Scheme								
6.	c)	For Student B:									
	Ans.	$ \begin{array}{c cccc} x_i & x_i^2 \\ 48 & 2304 \\ 75 & 5625 \\ 54 & 2916 \\ 60 & 3600 \\ 63 & 3969 \\ 69 & 4761 \\ 72 & 5184 \\ 51 & 2601 \\ 57 & 3249 \\ 56 & 3136 \\ \hline 605 & 37345 \end{array} $									
		Mean $\bar{x} = \frac{\sum x_i}{N} = \frac{605}{10}$ $\bar{x} = 60.5$ S.D.= $\sigma = \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2}$ $= \sqrt{\frac{37345}{10} - (60.5)^2}$									
		$\sigma = 8.62$ Coefficient of Variance = $\frac{S.D.}{\overline{x}} \times 100$ $= \frac{8.62}{x} \times 100$									
		60.5 = 14.25	1/2								
		$\therefore CV(B) < CV(A)$	17								
	d)	∴ Student B is more consistent.	1/2								
		Class Interval         0-10         10-20         20-30         30-40         40-50									
		Frequency 14 23 27 21 15	04								



#### WINTER – 2017 EXAMINATION

#### **Model Answer**

Subject Code:

17104

Q. No.	Sub Q. N.	Answer									Marking Scheme	
6.	d)											
	Ang	CI		C		<i>C</i>	2		c 2			
	Ans	Class	<i>x</i> <sub>i</sub> 5	$f_i$		$\frac{f_i x_i}{70}$	$x_i^2$		$\frac{c_i x_i^2}{c_i x_i^2}$			
		0-10		14		70	25		175			
		10-20	15	23		345	225		175			
		20-30 30-40	25 35	27 21		675 735	625 1225		5875 5725			2
		40-50	45	15		675	2025		)375			
		40-30	73	100	,	2500	2023		3500			
				100	<u> </u>	2500		, , c	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,			
		Mean $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2500}{100} = 25$ S.D.= $\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$ = $\sqrt{\frac{78500}{100} - (25)^2}$									1	
		O = 12	$\sigma = 12.65$									
		<u>OR</u>	<u>OR</u>									
		Class	$x_i$ $f_i$ 5 14	$d_i$	$f_i d_i$	$d_{i}^{2}$	$f_i d_i^2$					
		0-10 10-20	5 14 15 23	-2	-28 -23	4	56 23					
		20-30	25 27	0	0	0	0					2
		30-40	35 21	1	21	1	21					
		40-50	45 15	2	30	4	60					
			100		00		160					
		$S.D. = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$										
		$= \sqrt{\frac{160}{100} - \left(\frac{0}{100}\right)^2} \times 10$ $= 12.65$									2	
	e)	Find the me	ean deviatio	n from	the me	edian of t	he follow	wing data	a.			04
			of wood log	gs	10-20	20-30	30-40	40-50	50-60	60-70		
		Num	ber of logs		4	6	10	18	9	3		



WINTER – 2017 EXAMINATION

### **Model Answer**

	C1-										Marking
Q. No.	Sub Q. N.	Answer									
INO.	Q. IV.										Scheme
6.	e)										
	Ans	Class	$x_i$		$f_{i}$		c.f.	$ x_i $	-M	$f_i  x_i - M $	
	AllS	10-20	) 15		4		4	27	7.78	111.12	
		20-30			6		10	17	7.78	106.68	
		30-40			10		20	7	.78	77.8	2
		40-50	1		18		38		.22	39.96	2
		50-60			9		47		2.22	109.98	
		60-70	) 65		3		50	22	2.22	66.66	
					50					512.2	
		$M = Median = L + \left(\frac{N}{2} - cf\right) \times h$ $= 40 + \left(\frac{25 - 20}{18}\right) \times 10 \qquad \therefore \text{ Median class } 40 - 50$ $= 42.78$ $M.D. = \frac{\sum f_i  x_i - M }{N}$ $= \frac{512.2}{50}$ $= 10.24$									
	f)	Find the coeff	icent of va		the 5	follov	ving 15	data:	 7		04
			No.of st	tudents	6		28	38 46			
	Ans.		X <sub>i</sub>	$f_i$		$f_i x_i$		$x_{i}^{2}$	$f_i x_i^2$		
			5	6		30		25	150		2
			10	16		160		100	1600		
		_	15	28		420		225	6300		
			20	38		760		400	15200		
			25	46		1150		625	28750		
		L		134		2520			52000		
											7- 20/20



#### WINTER – 2017 EXAMINATION

### **Model Answer**

Subject Code: 17104

Q.	Sub	Answer	Marking
No.	Q. N.		Scheme
6	f)	Mean $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2520}{134} = 18.81$	1/2
		S.D. $\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\overline{x}\right)^2}$	
		$=\sqrt{\frac{52000}{134}-\left(18.81\right)^2}$	
		$\sigma = 5.85$	1
		Coefficient of Variance = $\frac{S.D.}{\overline{x}} \times 100$	
		$=\frac{5.85}{18.81} \times 100$	1/2
		= 31.10	/2
		<u>Important Note</u>	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls	
		within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
L	1	Paga No.	20/20

Page No.29/29