

Important Instruction to Examiners:-

- 1) The answers should be examined by key words & not as word to word as given in the model answers scheme.**
- 2) The model answers & answers written by the candidate may vary but the examiner may try to access the understanding level of the candidate.**
- 3) The language errors such as grammatical, spelling errors should not be given more importance.
- 4) While assessing figures, examiners, may give credit for principle components indicated in the figure.
- 5) The figures drawn by candidate & model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credit may be given step wise for numerical problems. In some cases, the assumed contact values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding.
- 7) For programming language papers, credit may be given to any other programme based on equivalent concept.

Important notes to examiner

A-1	A> Attempt any six of the following	
a)i)	Define elasticity & plasticity the property by virtue of which a material regains its shape & size on removal of external load is called as plasticity. all metals possesses the property of elasticity.	1 M
ii)	plasticity - it is the property of a material due to which material undergo permanent deformation without failure or rupture on application of load in short, lack of elasticity is called plasticity.	1 M
b) i)	principal planes. A planes across which only the normal stresses acts with no shear stress is called as principal planes	1 M
ii)	principal stresses. The normal stresses or normal stresses acting on the principal plane is called as principal stresses	1 M
c)	Find polar moment of inertia of circle of 50mm diameter.	
→	$d = 50\text{mm}$, $I_p = \text{Polar MI}$	1 M
	$I_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 50^4$	Formula
	$I_p = 613.542 \times 10^3 \text{ mm}^4$	1 M calculation

SUMMER - 16 EXAMINATIONS

Subject Code: 17304

Model Answer- Strength of material

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- d) State the expression for power transmitted by a shaft giving meaning of each term used.

$$\text{power} = \left(\frac{2\pi NT}{60} \right) \text{Watts}$$

01 M

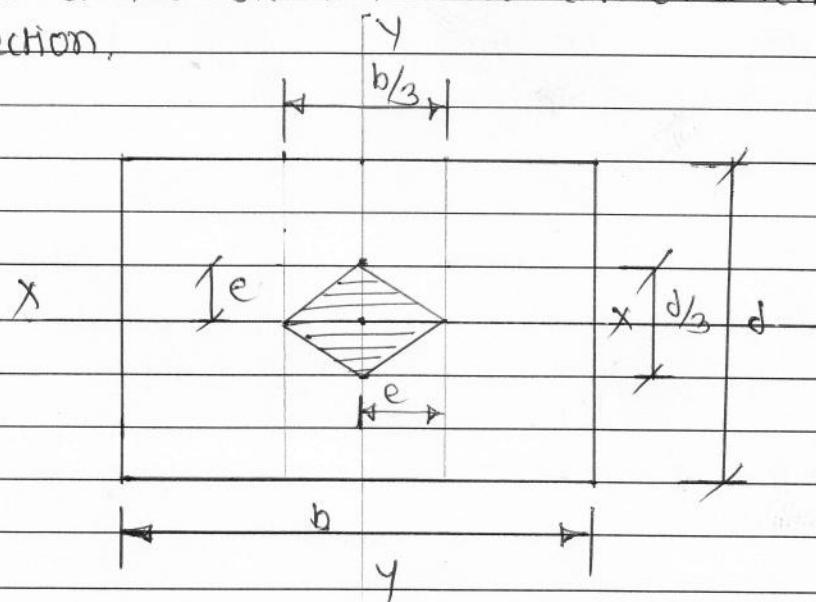
Where P - power in Watts

01 M

T = Avg. torque or mean torque in N-m

N = Number of revolution of shaft per minute

- e) Draw a neat sketch to show core of a rectangular section.



02 M

- f) State Hooke's law

It states that when a elastic material is loaded within its elastic limit, then stress produced is directly proportional to strain

01 M

mathematically $\sigma \propto e$ σ - stress

e = strain or linear strain 01 M

g) write the equation of circumferential stress in thin cylinder and explain each term

$$\sigma_c = \frac{P \cdot d}{2t} \quad \text{where}$$

01M

P = intensity of internal pressure

d = Diameter of the shell

01M

t = thickness of the shell

h) State the condition for no tension at the base of a column.

Direct stress (σ_d) = max. bending stress (σ_b)

$\sigma_d = \sigma_b$ (No tension condition)

02M

g) Attempt any two of the following

a) A mild steel flat 150mm wide by 20mm thick, 6m long, carries an axial pull of 300 kN. If the modulus of elasticity of steel is 200 GPa & poisson's ratio = 0.25 calculate the change in length width, thickness volume of the flat.

i) To find change in length (δL)

$$\sigma = \frac{P}{A} = \frac{300}{150 \times 20} = 0.1 \text{ kN/mm}^2$$

1/2M

$$\epsilon = \frac{\sigma}{E} = \frac{0.1}{200} = 5 \times 10^{-4}$$

1/2M

$$\text{But } \epsilon = \frac{\delta L}{L} \therefore \delta L = \epsilon \times L \therefore \delta L = 5 \times 10^{-4} \times 6000$$

$\boxed{\delta L = 3 \text{ mm (increase)}}$

01M

To find change in width (δb) & thickness (δt)

$$\text{Lateral strain } (\epsilon_l) = \mu \cdot e = 0.25 \times 5 \times 10^{-4} = 1.25 \times 10^{-4}$$

$$\text{Buck lateral strain} = \frac{\delta b}{b} - \frac{\delta t}{t} = 1.25 \times 10^{-4}$$

$$\therefore \delta b = 1.25 \times 10^{-4} \times 150 = 0.01875 \text{ (decrease)} \quad \frac{1}{2} M$$

$$\delta t = 1.25 \times 10^{-4} \times 20 = 2.5 \times 10^{-3} \text{ (decrease)} \quad \frac{1}{2} M$$

To find change in volume (δV)

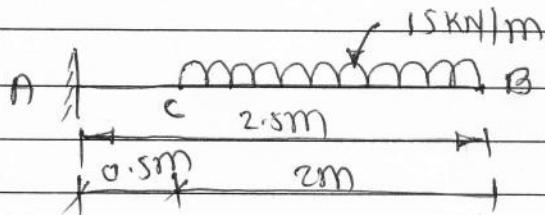
$$\text{we know } \frac{\delta V}{V} = e(1-2\mu)$$

$$\therefore \delta V = e(1-2\mu)V$$

$$= 5 \times 10^{-4} (1-2 \times 0.25) (6000 \times 150 \times 20)$$

$$\boxed{\delta V = 4500 \text{ mm}^3} \quad \frac{1}{2} M$$

- b) A cantilever beam 2.5m long carries a load of 15 KN/m over a length of 2m from the free end
Draw Shear force and Bending moment diagram.



A) S.F. calculation

$$\text{i) } F_B = 0$$

$$\text{ii) } F_{CR} = 15 \times 2 = 30 \text{ KN}$$

$$\text{iii) } F_{CL} = 30 \text{ KN}$$

1M

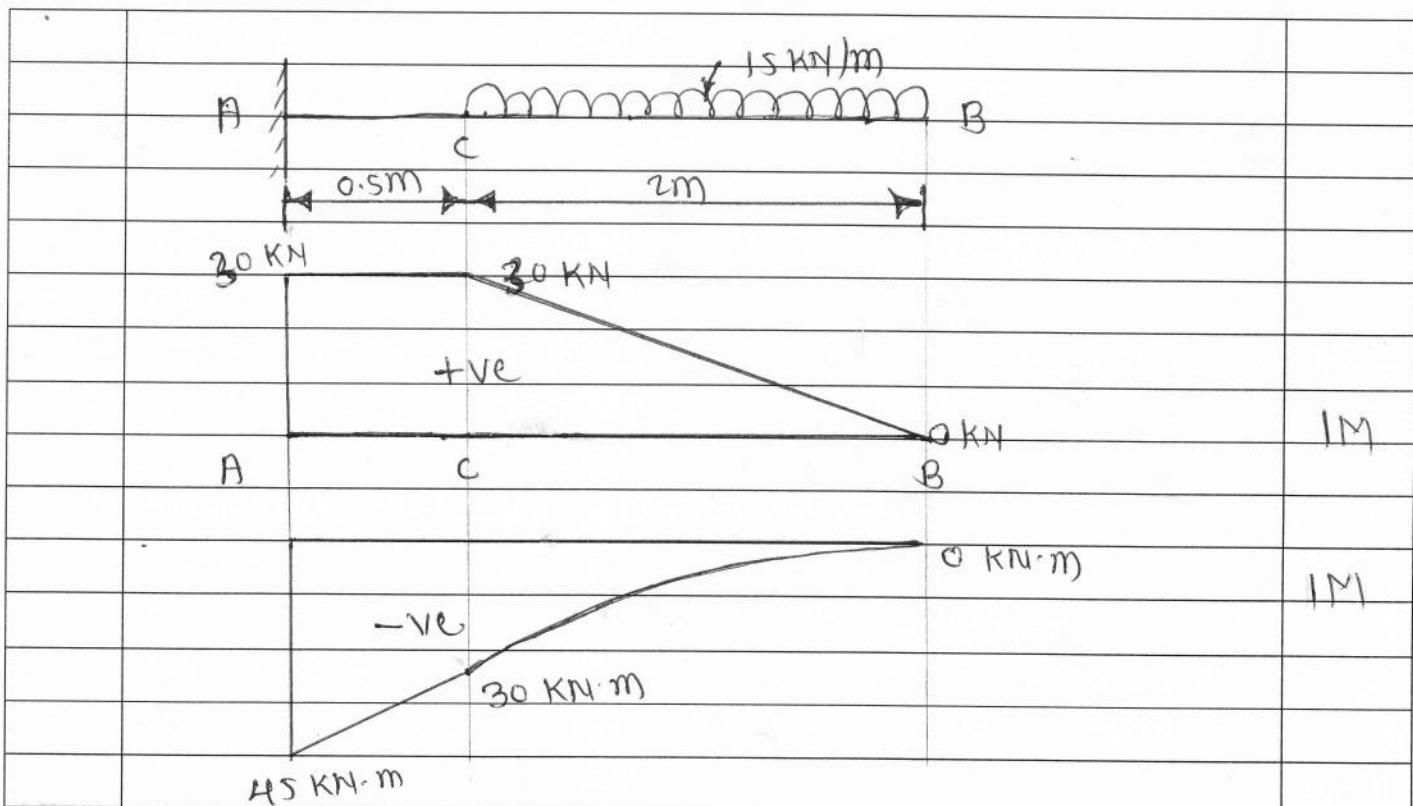
B) B.M. calculation

$$\text{i) } M_B = 0 \text{ KN-m}$$

$$\text{ii) } M_C = -15 \times 2 \times 1 = -30 \text{ KN-m}$$

$$\text{iii) } M_A = -15 \times 2 \times \left(\frac{2}{2} + 0.5\right) = -45 \text{ KN-m}$$

1M



- c) A simply supported beam of 5m span, carries a udl of 3 kN/m over the entire span. If the bending stress is not exceed 165 N/mm², find the value of section modulus for the beam and diameter when beam is circular.

i) Bending moment for simply supported beam

$$\text{Beam} = \frac{WL^2}{8} = \frac{3 \times 5^2}{8} = 9.375 \text{ kN}\cdot\text{m}$$

1M

$$\text{ii) } G_b(\text{permissible}) = 165 \text{ N/mm} \quad (\text{given})$$

$$\text{iii) } z = \frac{M}{G_b} = \frac{9.375 \times 10^3}{165} = 56.82 \text{ m}^3$$

1M

$$\text{iv) } z = \frac{\pi d^3}{32} \quad \therefore 56.82 = \frac{\pi \times d^3}{32}$$

1M

$$\therefore 56.82 \times 32 = \frac{\pi}{32} d^3$$

$$\therefore \frac{\pi}{32} \times 578.75 = d^3$$

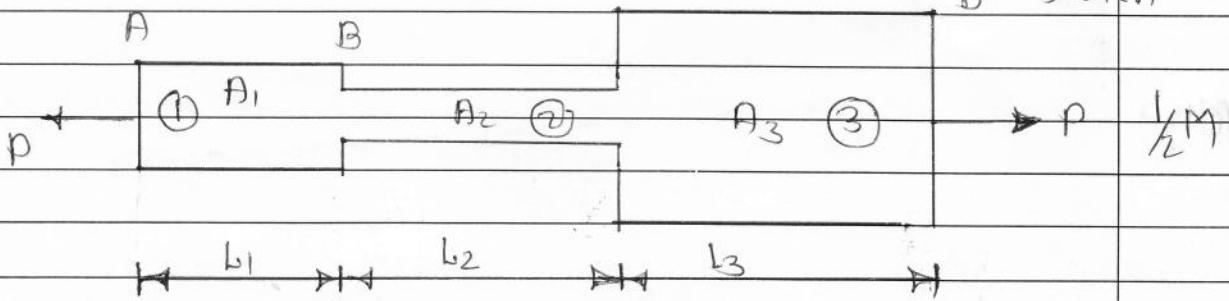
1M

$$\therefore d = 8.33 \text{ m}$$

Q-2 (a) i) Draw the sketch of stepped section showing axial load and state expression for change in length of it

a metal bar of varying cross-section $A_1, A_2 \text{ and } A_3$ $\frac{1}{2} \text{ m}$
and it is subjected to an axial pull P

$\delta L = \text{change of length}$, L_1, L_2, L_3 be the length of (1), (2) & (3)



$$\delta L = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] - \{ E = \text{constant} \} \text{ OIM}$$

or

$$\delta L = \frac{P}{A_1 E_1 + A_2 E_2 + A_3 E_3} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] - \{ E = \text{different} \} \text{ OIM}$$

GR

ii) State the effective length for both ends hinged column and for one end fixed and other end free

a) Both ends hinged.

Effective length (L_{eff}) = l .

OIM

b) One end fixed and other end free

$L_{eff} = 2l$

OIM

- b) A steel rod 4m long 30mm diameter is used as column with one end fixed and other hinged. Determine the crippling load by Euler's formula
 $E = 2 \times 10^5 \text{ MPa}$

$$\rightarrow L_{\text{eff}} = \frac{L}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828 \text{ m}$$

$$I = \frac{\pi}{64} \times 30^4 = 39.76 \times 10^3 \text{ mm}^4$$

Euler's crippling load

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 2 \times 10^5 \times 39.76 \times 10^3}{(2828)^2}$$

$$P_E = 9813.54 \text{ N}$$

c)

- A rod 300 mm long and 20 mm in diameter is heated through 100°C and at the same time pulled by a force P . If the total extension is 0.4 mm what is the magnitude of P ? $E = 2 \times 10^5 \text{ N/mm}^2$ and $\alpha = 12 \times 10^{-6} / {}^\circ\text{C}$

\rightarrow Total extension of the rod = free expansion +

Extension due to 'P'

$$0.4 = (L\alpha t) + \delta_L \text{ due to force } P$$

$$0.4 = (300 \times 12 \times 10^{-6} \times 100) + \delta_L$$

$$0.4 = 0.36 + \delta_L$$

$$\delta_L = 0.4 - 0.36 = 0.04 \text{ mm extension due to } P$$

$$\text{But } \delta_L = \frac{P L}{AE} \quad \delta_L = \frac{P \times 300}{\left(\frac{\pi \times 20^2}{4}\right) \times 2 \times 10^5}$$

$$[P = 8.377 \times 10^3 \text{ N}]$$

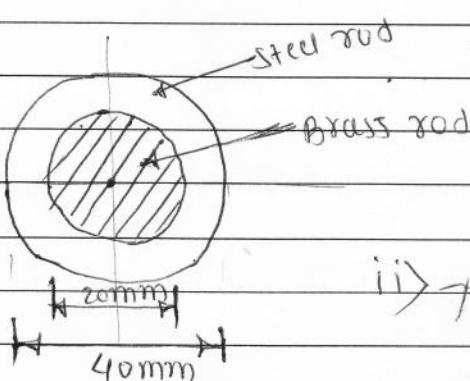
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Model Answer- Strength of material

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- d) A brass rod of 250mm length and 20 mm diameter is fixed inside a steel tube of 40 mm external diameter and 20 mm internal diameter and of same length. The composite bar is subjected to an axial pull of 140 kN. Find the stress in each metal.
Take 200 GPa & $E_{brass} = 110 \text{ GPa}$



$$i) A_{\text{steel}} = \frac{\pi}{4} \times (40^2 - 20^2)$$

$$A_{\text{steel}} = 942.45 \text{ mm}^2$$

1/2M

$$ii) A_{\text{brass}} = \frac{\pi}{4} \times 20^2$$

$$A_{\text{brass}} = 314.159 \text{ mm}^2$$

1/2M

iii) To find stress in each material

$$\sigma_s = \text{stress in steel}$$

$$\sigma_b = \text{stress in brass}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

1/2M

$$\sigma_s = \frac{E_s}{E_b} \sigma_b$$

$$\sigma_s = \frac{200}{110} \sigma_b$$

$$\boxed{\sigma_s = 1.818 \sigma_b}$$

1M

$$iv) P = P_s + P_b$$

$$P = 140 \text{ kN} \quad \text{given}$$

$$\therefore P = \sigma_s \cdot A_s + \sigma_b \cdot A_b$$

1/2M

$$140 \times 10^3 = 1.818 \sigma_b \times 942.45 + \sigma_b \times 314.159$$

$$140 \times 10^3 = 1713.54 \times \sigma_b + 314.159 \sigma_b$$

$$140 \times 10^3 = 2027.70 \sigma_b$$

$$\boxed{\sigma_b = 69.0435 \text{ N/mm}^2}$$

1/2M

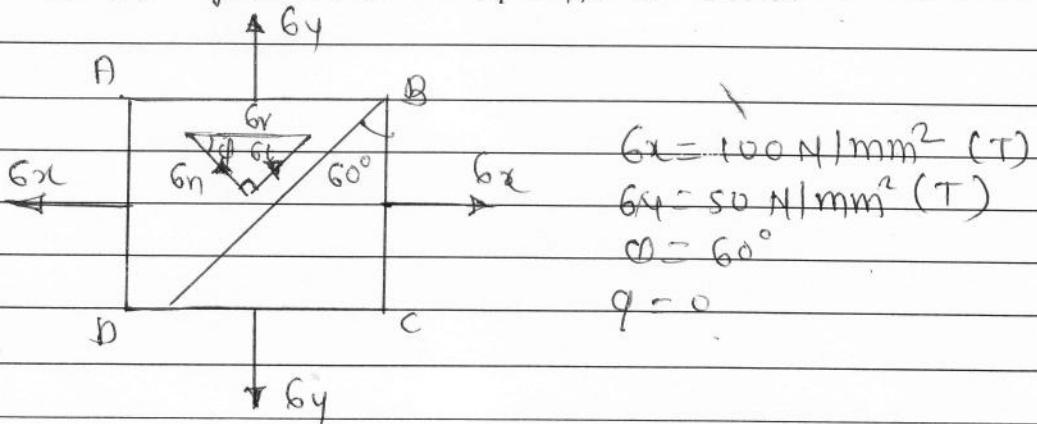
$$\sigma_s = 1.818 \sigma_b$$

$$\sigma_s = 1.818 \times 69.0435$$

$$\boxed{\sigma_s = 125.52 \text{ N/mm}^2}$$

1/2M

- e) The principle stresses at a point in the section of a member are 100 N/mm^2 and 50 N/mm^2 both tensile find the normal and tangential stresses across a plane passing through that point inclined at 60° to the plane have 100 N/mm^2 shear



i) To find normal stress ($6n$)

$$6n = \left(\frac{6x + 6y}{2} \right) + \left(\frac{6x - 6y}{2} \right) \cos 2\theta + q \sin 2\theta \quad 1 \text{ M}$$

$$6n = \left(\frac{100 + 50}{2} \right) + \left(\frac{100 - 50}{2} \right) \cos 2 \times 60 + 0$$

$$6n = 75 + 25 \times \cos 120^\circ$$

$$6n = 62.5 \text{ N/mm}^2 (\text{T}) \quad 1 \text{ M}$$

ii) To find tangential stress ($6t$)

$$6t = \left(\frac{6x - 6y}{2} \right) \sin 2\theta - q \cos 2\theta \quad 1 \text{ M}$$

$$6t = \left(\frac{100 - 50}{2} \right) \sin 2 \times 60 + 0$$

$$6t = 25 \times \sin 120^\circ$$

$$\boxed{6t = 21.65 \text{ N/mm}^2} \quad 1 \text{ M}$$

f) find hoop stress and longitudinal stress induced in a cylindrical boiler 1.6 m internal diameter subjected to an internal pressure of 2.5 MPa. thickness of wall is 30mm

$$d = 1.6 \text{ m}$$

$$P = 2.5 \text{ MPa}$$

$$t = 30 \text{ mm}$$

$$\text{i)} \text{ longitudinal stress } (\sigma_L) = \frac{Pd}{4t}$$

$$= \frac{2.5 \times 1600}{4 \times 30}$$

$$\sigma_L = 33.33 \text{ N/mm}^2$$

$$\text{ii)} \sigma_c = \text{hoop stress} = \frac{Pd}{2t}$$

$$= \frac{2.5 \times 1600}{2 \times 30}$$

$$\sigma_c = 66.67 \text{ N/mm}^2$$

OR

OR

$$\text{Relation betn } \sigma_c = 2\sigma_L$$

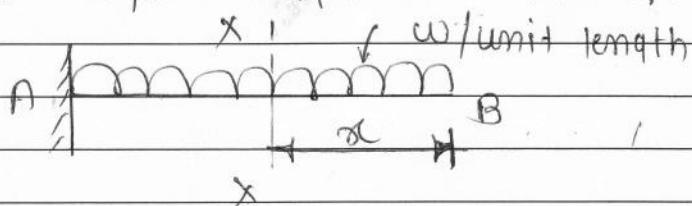
1M

$$\sigma_c = 2 \times 33.33$$

$$\sigma_c = 66.67 \text{ N/mm}^2$$

1M

Q-3 (a) A cantilever beam of span L is subjected to load of $w/\text{unit length}$ over entire span. Draw S.F & B.M dia.



i) S.F calculation

$$F_B = \text{shear force at } B = 0 \text{ kN}$$

1/2 M

Let Shear Force at any section ($x-x$) at distance x from free end

$$F_x = \text{Rate of loading} \times \text{length} \\ = -w \cdot x$$

1/2 M

Hence, we find that S.F is zero at B where $x=0$ and increase linearly with increase in length x

$$F_A = w \cdot L$$

1/2 M

ii) B.M calculation

load over length or from free end of cantilever

$$= w \cdot x \text{ it acts at } \frac{x}{2} \text{ from section } x-x$$

$$M_x = -w \cdot x \cdot \frac{x}{2} = -\frac{wx^2}{2}$$

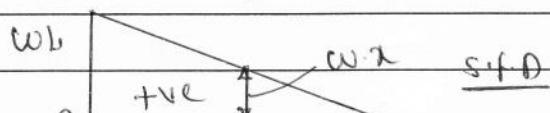
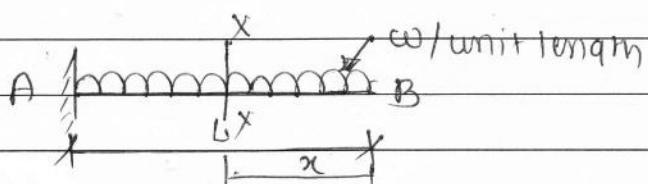
1/2 M

$$\text{at } x=0 \therefore M_B = B.M \text{ at } B=0$$

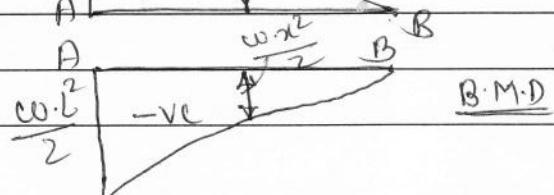
1/2 M

$$\text{at } x=L \therefore M_A = B.M \text{ at } A = -\frac{wL^2}{2}$$

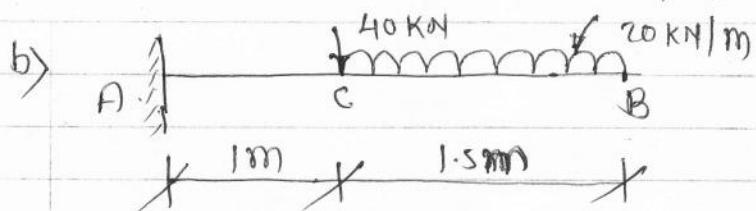
1/2 M



1/2 M



1/2 M



A) S.F. calculation

i) $F_B = S.F \text{ at } B = 0$

ii) $F_{CQ} = 20 \times 1.5 = 30 \text{ kN}$

iii) $F_{CL} = 20 \times 1.5 + 40 = 70 \text{ kN}$

iv) $F_A = 70 \text{ kN}$

IM

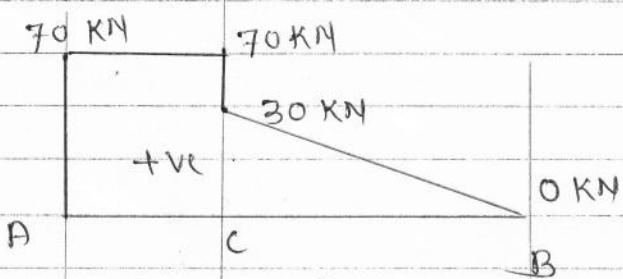
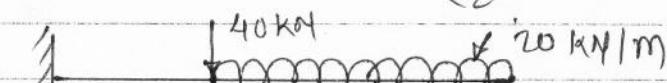
B) B.M. calculation

i) $M_B = 0 \text{ kN-m}$

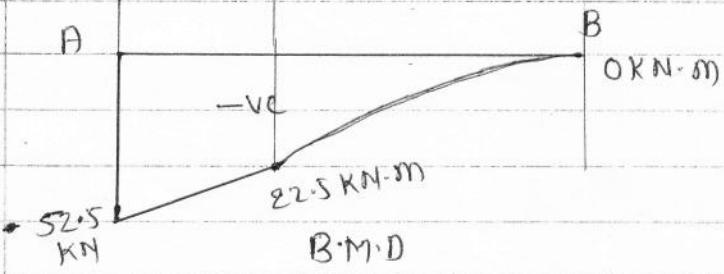
ii) $M_C = -20 \times 1.5 \times \frac{1.5}{2} = -22.5 \text{ kN-m}$

iii) $M_A = -20 \times 1.5 \times \left(\frac{1.5}{2} + 1\right) = -52.5 \text{ kN-m}$

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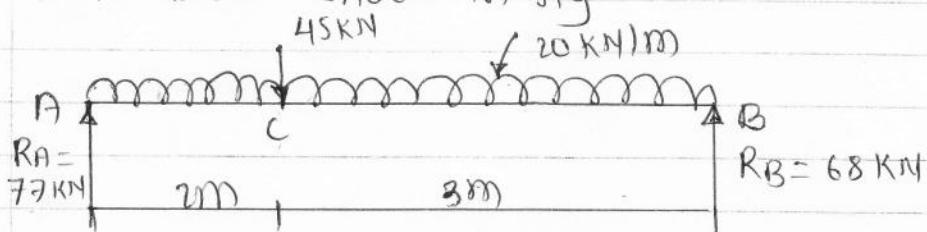


IM



IM

Q-3 (c) Draw S.F & B.M. diagram for the simply supported beam shown in fig



i) To find R_A & R_B

$$\sum F_y = 0$$

$$R_A + R_B = 45 + 20 \times 5$$

$$R_A + R_B = 145 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$- R_B \times 5 + 20 \times 5 \times \frac{5}{2} + 45 \times 2 = 0$$

$$- R_B \times 5 + 250 + 90 = 0$$

$$- R_B \times 5 + 340 = 0$$

$$- R_B \times 5 = -340$$

$$\boxed{R_B = 68 \text{ kN}}$$

$$\therefore \boxed{R_A = 77 \text{ kN}}$$

ii) S.F. calculation

$$F_B = -68 \text{ kN}$$

$$F_{CR} = -68 + 20 \times 3 = -8 \text{ kN}$$

$$F_{CL} = -8 + 45 = +37 \text{ kN}$$

$$F_A = 37 + 20 \times 2 = +77 \text{ kN}$$

1M

iii) B.M. calculation

$$M_B = 0 \text{ KN-m}$$

$$M_C = R_B \times 3 - 20 \times 3 \times \frac{3}{2}$$

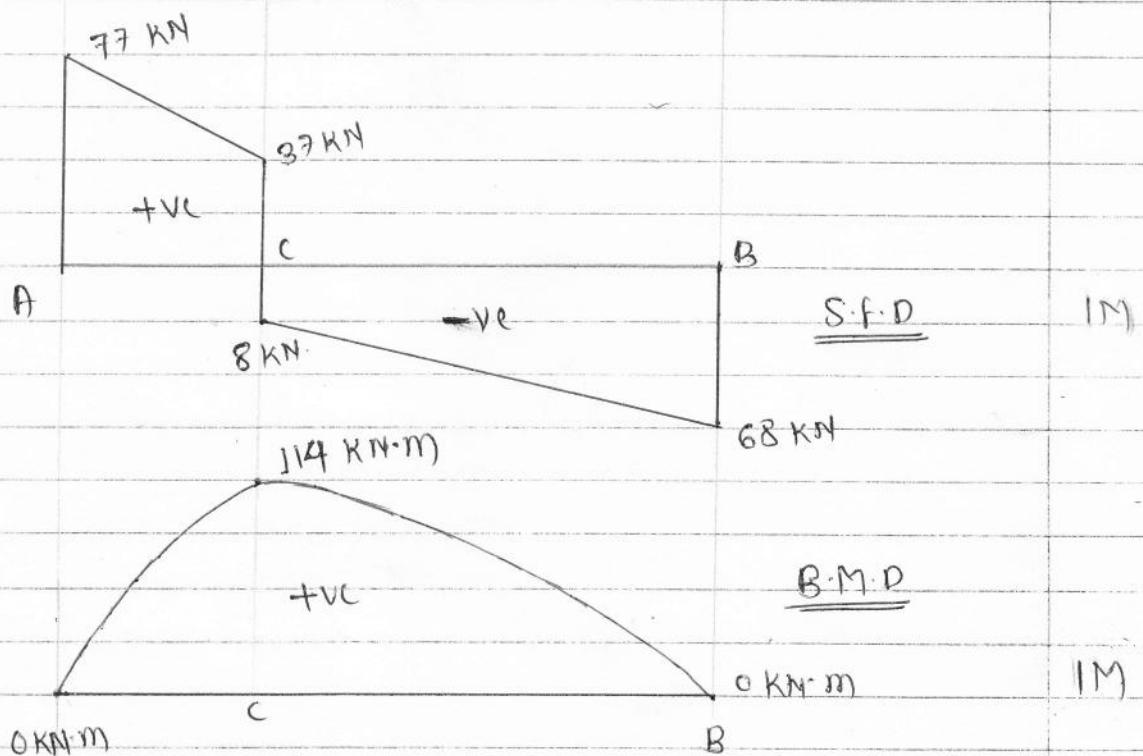
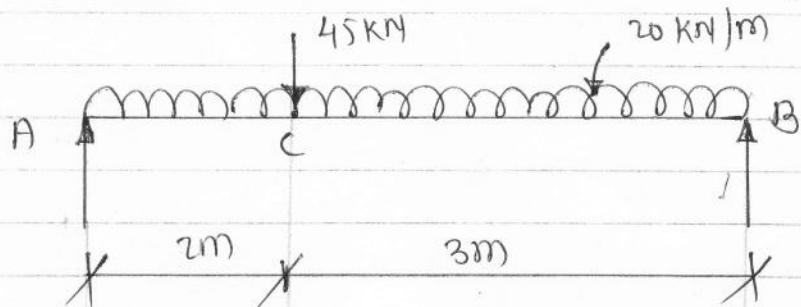
$$M_C = 68 \times 3 - 20 \times 3 \times \frac{3}{2}$$

$$M_C = +204 - 90$$

$$M_A = 0 \text{ KN-m}$$

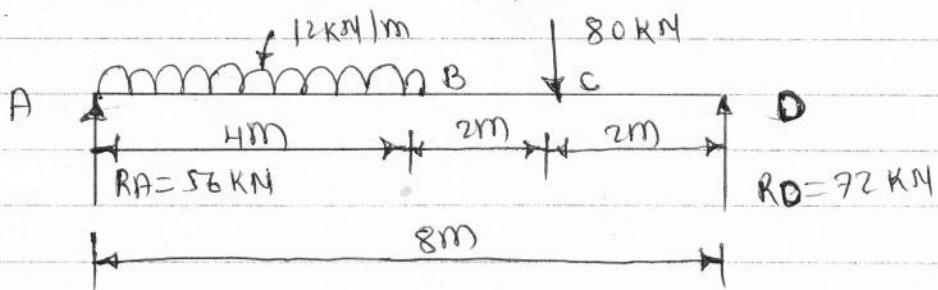
$$\boxed{M_C = 114 \text{ KN-m}}$$

1M



- d) A simply supported beam of span 8m carries a load of 12 kN/m over 4m length from L.H.S and a point load of 80 kN at 2m from the right support.

DRAW S.F & B.M. diagram



i) Reaction calculation

$$\sum F_y = 0$$

$$R_A + R_B = 12 \times 4 + 80$$

$$R_A + R_B = 128 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$-R_B \times 8 + 80 \times 6 + 12 \times 4 \times 2 = 0$$

$$-R_B \times 8 + 480 + 96 = 0$$

$$R_B = 72 \text{ KN}$$

$$\therefore R_A = 56 \text{ KN}$$

ii) S.F.D. calculation

$$F_A = 56 \text{ KN}$$

$$F_B = 56 - 12 \times 4$$

$$F_B = 8 \text{ KN}$$

$$F_{C1} = 8 \text{ KN}$$

$$F_{C2} = 8 - 80 = -72 \text{ KN}$$

$$F_D = -72 \text{ KN}$$

1M

iii) B.M.D. calculation

$$\sum M_A = 0 \text{ KN} \cdot \text{m}$$

$$M_C = R_B \times 2 = 72 \times 2 = 144 \text{ KN} \cdot \text{m}$$

$$M_B = R_B \times 4 - 80 \times 2$$

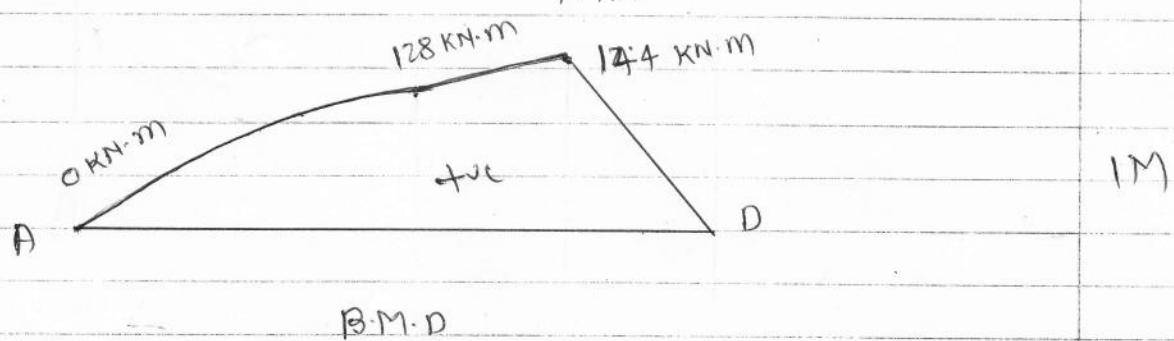
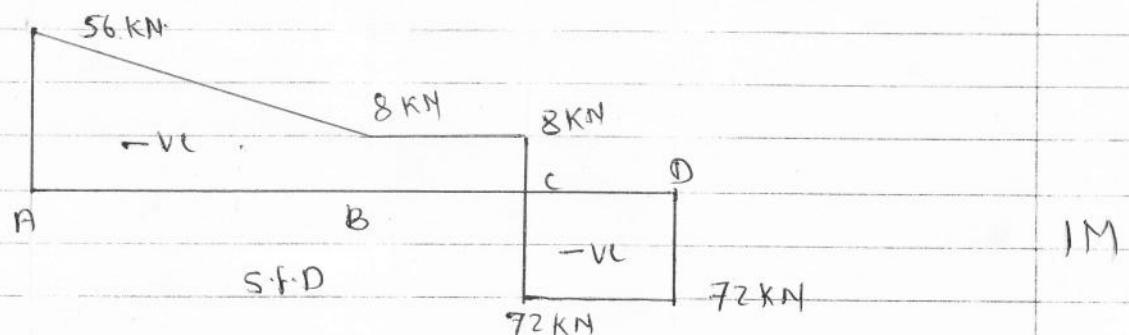
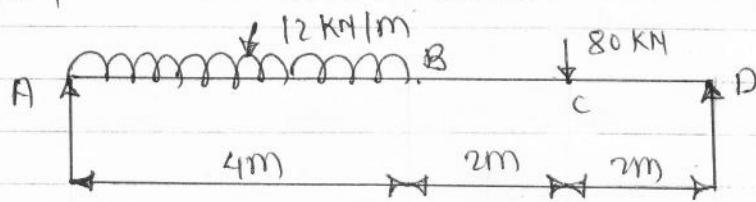
$$= 72 \times 4 - 80 \times 2$$

$$= 288 - 160$$

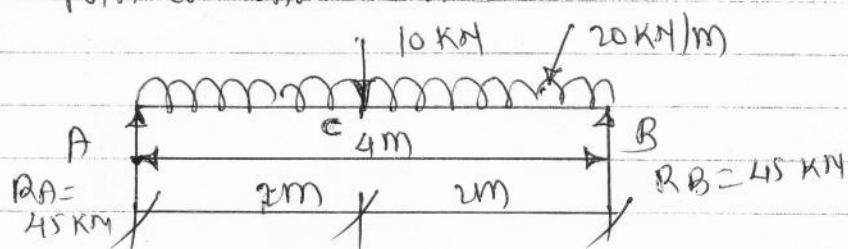
$$= 128 \text{ KN} \cdot \text{m}$$

$$M_D = 0 \text{ KN} \cdot \text{m}$$

1M



- e) A beam ABC 6m long is supported at A & B. 2 m apart, BC = 2m. The beam carries a UDL of 20 KN/m over entire length along with a downward load of 10 KN at point 'C'. Plot S.F.D & B.M.D for Beam & locate point of contraflexure.



i) Reaction calculation

$$\Sigma H = 0$$

$$R_A + R_B = 10 + 20 \times 4$$

$$R_A + R_B = 10 + 80$$

$$R_A + R_B = 90 \quad \text{--- (1)}$$

$$\Sigma M_A = 0$$

$$-RB \times 4 + 20 \times 4 \times 2 + 10 \times 2 = 0$$

$$-RB \times 4 + 160 + 20 = 0$$

$$-RB \times 4 + 180 = 0$$

$$-RB \times 4 = -180$$

$$RB = 45 \text{ KN}$$

$$\therefore RA = 45 \text{ KN}$$

ii) S.F. calculation

$$i) F_B = -45 \text{ KN}$$

$$ii) F_{CR} = -45 + 20 \times 2 = -5 \text{ KN}$$

$$iii) F_{CL} = -45 + 20 \times 2 + 10 = 5 \text{ KN}$$

$$iv) F_A = -45 + 20 \times 4 + 10 = 45 \text{ KN}$$

1M

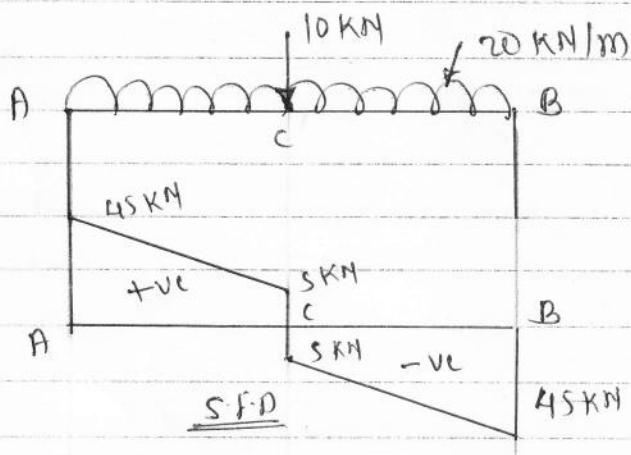
iii) B.M. calculation

$$i) M_B = 0 \text{ KN.m}$$

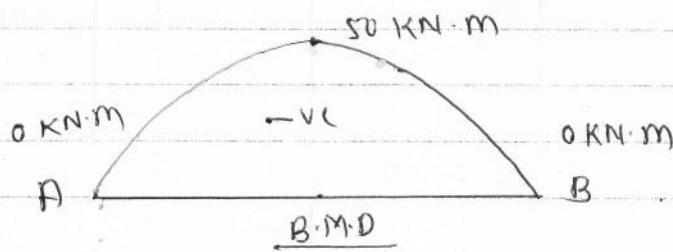
$$ii) M_C = 45 \times 2 - 20 \times 2 \times 1 = 90 - 40 = 50 \text{ KN.m}$$

$$iii) M_A = 45 \times 4 - 20 \times 4 \times 2 - 10 \times 2 = 0 \text{ KN.m}$$

1M

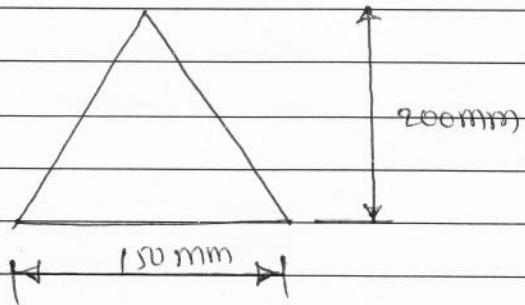


1M



1M

- f) calculate the M.I of triangle having base 150 mm & height 200 mm about its base and centroidal axis parallel to base



i) M.I about base

$$M.I_{\text{base}} = \frac{bh^3}{12} = \frac{150 \times 200^3}{12} = 100 \times 10^6 \text{ mm}^4 \quad 02 \text{ M}$$

ii) M.I about centroidal axis parallel to base

$$M.I = \frac{bh^3}{36}$$

$$M.I = \frac{150 \times 200^3}{36} = 33.33 \times 10^6 \text{ mm}^4 \quad 02 \text{ M}$$

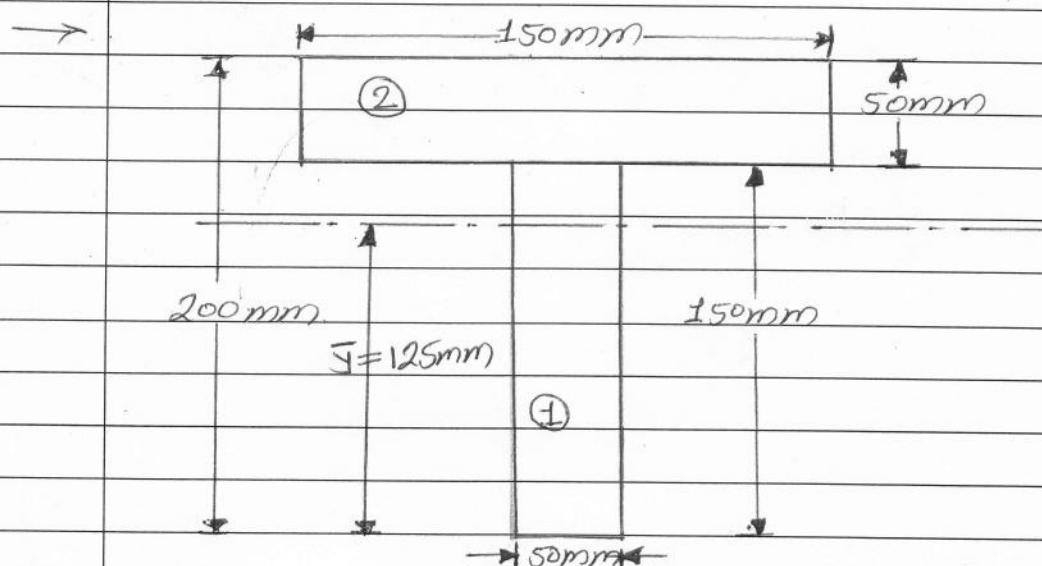
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Q4a)



1) Position of Centroidal axis

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2 \quad a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

$$x_1 = 50 + 50 = 75 \text{ mm} \quad x_2 = \frac{150}{2} = 75 \text{ mm}$$

$$y_1 = \frac{150}{2} = 75 \text{ mm} \quad y_2 = 150 + \frac{50}{2} = 175 \text{ mm}$$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 75) + (7500 \times 175)}{7500 + 7500}$$

$\bar{y} = 125 \text{ mm}$ from bottom.

1M

2) Moment of Inertia about x-x axis

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$I_{xx1} = I_{G_1} + A_1 h_1^2 = \frac{b d^3}{12} + a_1 (\bar{y} - y_1)^2 \quad \frac{1}{2} \text{ M}$$

$$I_{xx1} = \frac{50 \times 150^3}{12} + 7500 (125 - 75)^2 = 32.81 \times 10^6 \text{ mm}^4 \quad \frac{1}{2} \text{ M}$$

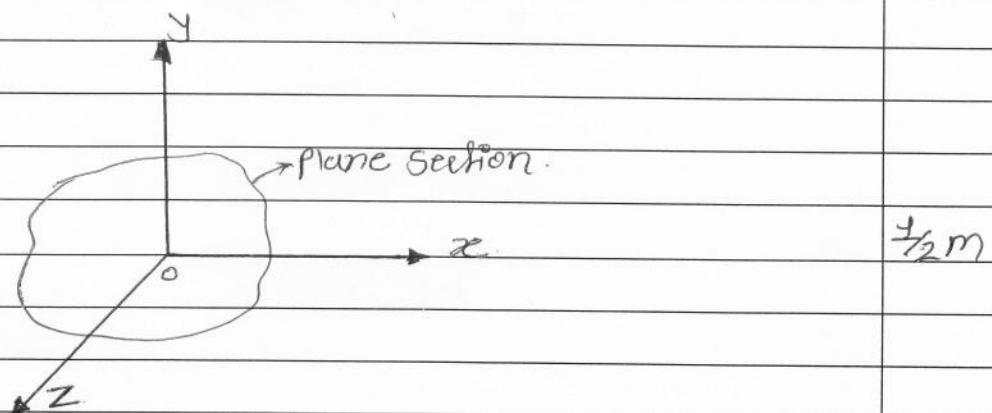
$$I_{xx2} = I_{G_2} + A_2 h_2^2 = \frac{b d^3}{12} + a_2 (\bar{y} - y_2)^2 \quad \frac{1}{2} \text{ M}$$

$$I_{xx2} = \frac{150 \times 50^3}{12} + 7500 (125 - 175)^2 = 20.31 \times 10^6 \text{ mm}^4 \quad \frac{1}{2} \text{ M}$$

$$\therefore I_{xx} = 32.81 \times 10^6 + 20.31 \times 10^6 = 53.12 \times 10^6 \text{ mm}^4 \quad 1 \text{ M}$$

Q4b

→ i) Perpendicular axis theorem:-

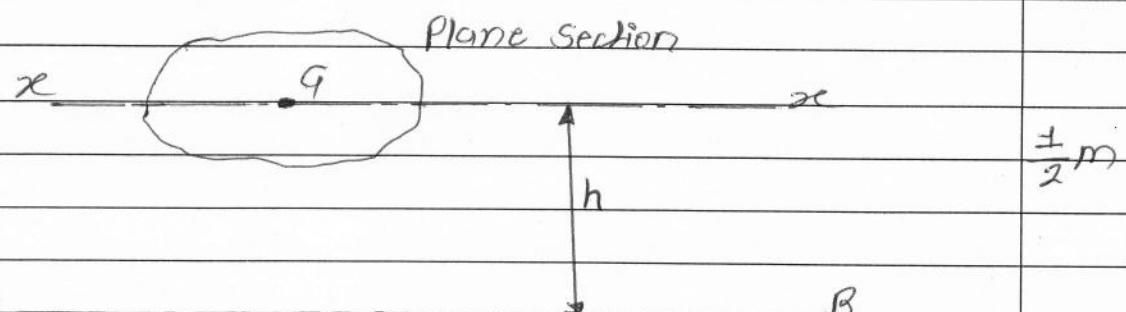


If I_{xx} & I_{yy} are the moment of inertia of a plane section about the two mutually perpendicular axes meeting at 'O', then the moment of inertia I_{zz} about the third axis Z-Z perpendicular to the plane and passing through the intersection of x-x and y-y is given by.

$$I_{zz} = I_{xx} + I_{yy}$$

 $\frac{1}{2}m$

ii) Parallel axis theorem:-

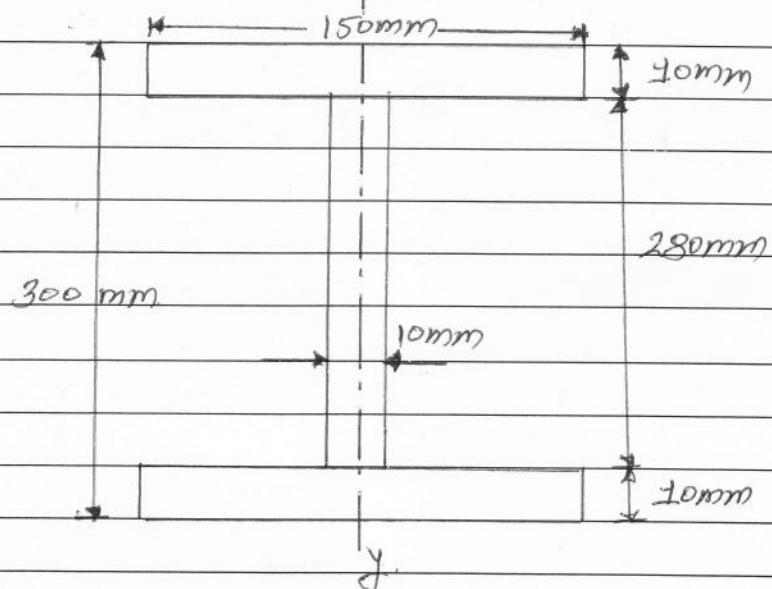


The moment of inertia of a plane section about any axis parallel to Centroidal axis is equal to the moment of inertia of a section about the Centroidal axis plus the product of the area of section and the square of the distance between the two axes.

$$I_{AB} = I_G + Ah^2$$

 $\frac{1}{2}m$

Q4c)



1) M.I about axis perpendicular to flanges (Y-Y axis)

Flanges and web are symmetrical about Y-Y axis,
hence no need to apply parallel axis theorem.

$$I_{yy} = 2 \times \text{M.I of flange} + \text{M.I of web}$$

$$= 2 \times \left[\frac{cb^3}{12} \right] + \left[\frac{cb^3}{12} \right]_w$$

$$= 2 \times \frac{10 \times 150^3}{12} + \frac{280 \times 10^3}{12}$$

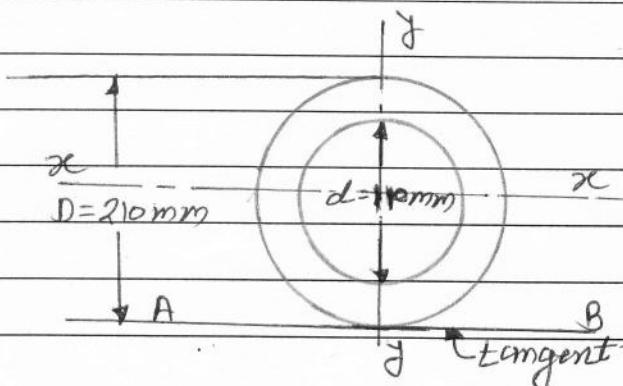
$$= 5.625 \times 10^6 + 23.33 \times 10^3$$

$$I_{yy} = 5.648 \times 10^6 \text{ mm}^4$$

Q4d)

$$\rightarrow D = 210\text{mm}$$

$$d = 110\text{mm}$$



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Q4d

1) M.I. of hollow circular section

cont...

$$I_{xx} = I_{yy} = \frac{\pi}{64} (D^4 - d^4)$$

$\frac{1}{2}m$

$$= \frac{\pi}{64} (210^4 - 110^4)$$

$$I_{xx} = I_{yy} = 88.278 \times 10^6 \text{ mm}^4$$

$\frac{1}{2}m$

2) Polar moment of Inertia (I_{zz})

$$I_p = I_{zz} = I_{xx} + I_{yy}$$

$\frac{1}{2}m$

$$= 88.278 \times 10^6 + 88.278 \times 10^6$$

$$I_p = I_{zz} = 176.556 \times 10^6 \text{ mm}^4$$

$\frac{1}{2}m$

3) Moment of Inertia about tangent AB

By parallel axis theorem.

$$I_{AB} = I_G + Ah^2$$

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (210^2 - 110^2) = 25.13 \times 10^3 \text{ mm}^2$$

$$h = \frac{D}{2} = \frac{210}{2} = 105 \text{ mm}$$

$\frac{1}{2}m$

$\frac{1}{2}m$

$$\therefore I_{AB} = 88.278 \times 10^6 + 25.13 \times 10^3 (105)^2$$

$$I_{AB} = 365.33 \times 10^6 \text{ mm}^4$$

1m

Q4e)

→ Assumptions of theory of simple bending.

1) Transverse section of beam which is plane before bending will remain plane after the bending.

2) The material of the beam is homogeneous & isotropic.

1m

3) The value of Young's modulus (E) is same in tension and compression.

each
for
any

4) the elastic limit is not exceeded.

Four

5) The beam is initially straight and unstressed.

6) The deformation of the section due to shear force is neglected.

- 7) The beam is stressed well up to proportional limit such that it must obeys Hooke's law.
- 8) Each longitudinal fiber is free to expand or contract independently from every other layer
- 9) The resultant force across transverse section of the beam is zero.

Q4f)

$$\rightarrow b = 100 \text{ mm}$$

$$S = 50 \text{ kN}$$

$$T_{\max} = 4 \text{ N/mm}^2 \text{ or } q_{\max} = 4 \text{ N/mm}^2$$

$$i) q_{\max} = 1.5 \cdot q_{avg}. \quad -(i) \quad 1m$$

$$q_{avg} = \frac{\text{Shear force}}{\text{c/s area}} = \frac{50 \times 10^3}{100 \times d}$$

$$q_{avg} = \frac{500}{d} \quad \text{--- substitute in eqn (i)}$$

$$q_{\max} = 1.5 \times \frac{500}{d}$$

$$4 = 1.5 \times \frac{500}{d}$$

$$\therefore d = \frac{1.5 \times 500}{4}$$

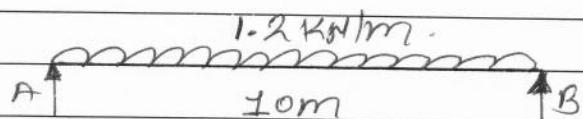
$$d = 187.5 \text{ mm.} \quad 1m$$

Q5a)

$$\rightarrow D = 120 \text{ mm}$$

$$L = 10 \text{ m}$$

$$w = 1200 \text{ N/m} = 1.2 \text{ KN/m.}$$



i) maximum Bending moment

$$\text{Max. B.M} = \frac{wL^2}{8} = \frac{1.2 \times 10^2}{8} = 15 \text{ KN.m.} \quad \pm 1\text{m}$$

ii) Using flexural formula

$$\frac{M}{I} = \frac{\sigma}{y} \quad \frac{1}{2} \text{ m}$$

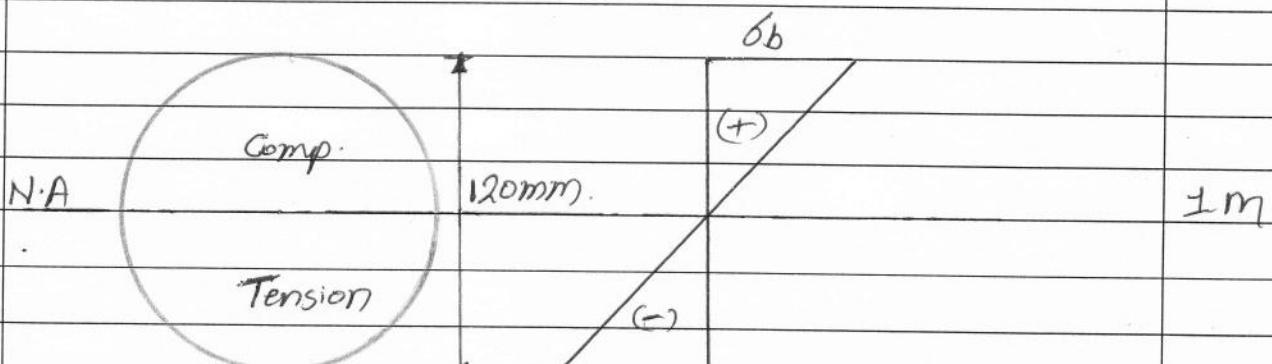
$$M = 15 \times 10^6 \text{ N-mm}$$

$$I = \frac{\pi D^4}{64} = \frac{\pi (120)^4}{64} = 10.178 \times 10^6 \text{ mm}^4 \quad \frac{1}{2} \text{ m}$$

$$y = \frac{D}{2} = \frac{120}{2} = 60 \text{ mm.}$$

$$\frac{15 \times 10^6}{10.178 \times 10^6} = \frac{6b}{60}$$

$$\therefore 6b = 88.42 \text{ N/mm}^2 \quad \pm 1\text{m}$$



q/s of beam

Bending stress distribution diagram.

(Q5b)

→ $b = d = 210 \text{ mm}$ - square column.

$$P = 105 \text{ kN}$$

$$e = 65 \text{ mm}$$

1) Geometrical properties of section.

i) C/S area $A = b \times d = 210 \times 210 = 44100 \text{ mm}^2$

1M

ii) Section modulus $Z = \frac{bd^2}{6} = \frac{210 \times 210^2}{6} = 154.35 \times 10^6 \text{ mm}^3$

2) Direct stress $\sigma_o = \frac{P}{A} = \frac{105 \times 10^3}{44100}$

$$\sigma_o = 2.38 \text{ N/mm}^2$$

1M

3) Bending stress $\sigma_b = \frac{M}{Z} = \frac{P \times e}{Z} = \frac{105 \times 10^3 \times 65}{1.5435 \times 10^6}$

$$\sigma_b = 4.42 \text{ N/mm}^2$$

1M

4) Resultant stresses.

$$\sigma_{\max} = \sigma_o + \sigma_b = 2.38 + 4.42 = 6.80 \text{ N/mm}^2$$

1M

$$\sigma_{\min} = \sigma_o - \sigma_b = 2.38 - 4.42 = -2.04 \text{ N/mm}^2 \dots (\text{Tensile})$$

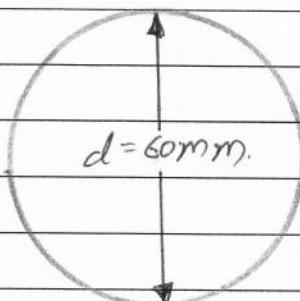
(Q5c)

→

1) Geometrical properties

i) C/S area $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 60^2$

$$A = 2827.43 \text{ mm}^2$$



ii) Section modulus $Z = \frac{\pi}{32} D^3$

$$Z = \frac{\pi}{32} \times 60^3 = 21.889 \times 10^3 \text{ mm}^3$$

1M

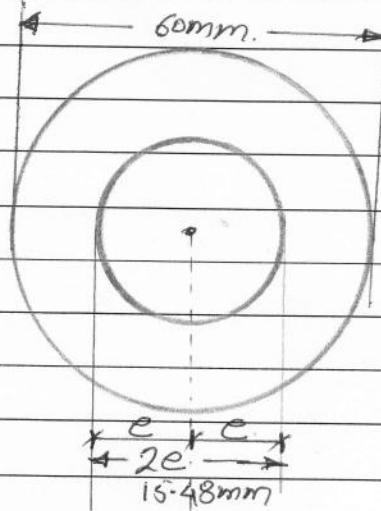
2) limit of eccentricity $e \leq \frac{z}{A}$

$$e \leq \frac{21.889 \times 10^3}{2827.43}$$

$$e = 7.74 \text{ mm.}$$

$\frac{1}{2} \text{ m}$

$$\therefore 2e = 15.48 \text{ mm.}$$



$\frac{1}{2} \text{ m}$

1m

Q5d

$$\rightarrow D = 250 \text{ mm}$$

$$d = 200 \text{ mm}$$

$$P = 20 \text{ kN}$$

$$e = 400 \text{ mm}$$

i) Geometrical properties of Section

$$\text{i) g/s area } A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} \times (250^2 - 200^2)$$

$$A = 17.67 \times 10^3 \text{ mm}^2$$

$$\text{ii) Section modulus } Z = \frac{\pi}{32} \left(\frac{D^4 - d^4}{D} \right)$$

$$Z = \frac{\pi}{32} \left(\frac{250^4 - 200^4}{250} \right)$$

1m

$$Z = 905.66 \times 10^3 \text{ mm}^3$$

$$27) \text{ Direct stress } \sigma_0 = \frac{F}{A} = \frac{20 \times 10^3}{17.67 \times 10^3}$$

$$\sigma_0 = 1.13 \text{ N/mm}^2$$

1M

$$37) \text{ Bending stress } \sigma_b = \frac{M}{Z} = \frac{P \times e}{Z}$$

$$\sigma_b = \frac{20 \times 10^3 \times 400}{905.66 \times 10^3} = 8.83 \text{ N/mm}^2$$

1M

4) Resultant stresses

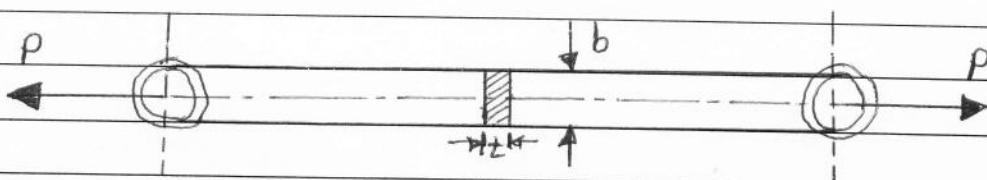
$$\sigma_{\max} = \sigma_0 + \sigma_b = 1.13 + 8.83 = 9.96 \text{ N/mm}^2$$

1/2M

$$\sigma_{\min} = \sigma_0 - \sigma_b = 1.13 - 8.83 = -7.70 \text{ N/mm}^2 \text{ (Tensile)}$$

1/2M

Q5e)



$$P = 90 \text{ kN} \quad \sigma = 65 \text{ MPa}$$

$$b = 3t$$

1) Tensile stress σ

$$\sigma = \frac{P}{A} \quad 1m$$

$$\sigma = \frac{P}{b \times t}$$

$$65 = \frac{90 \times 10^3}{3t \times t} \quad \dots \dots b = 3t \quad 1m$$

$$3t^2 = \frac{90 \times 10^3}{65}$$

$$3t^2 = 1384.61$$

$$\therefore t = 21.48 \text{ mm}$$

1M

$$\therefore b = 64.44 \text{ mm}$$

1M

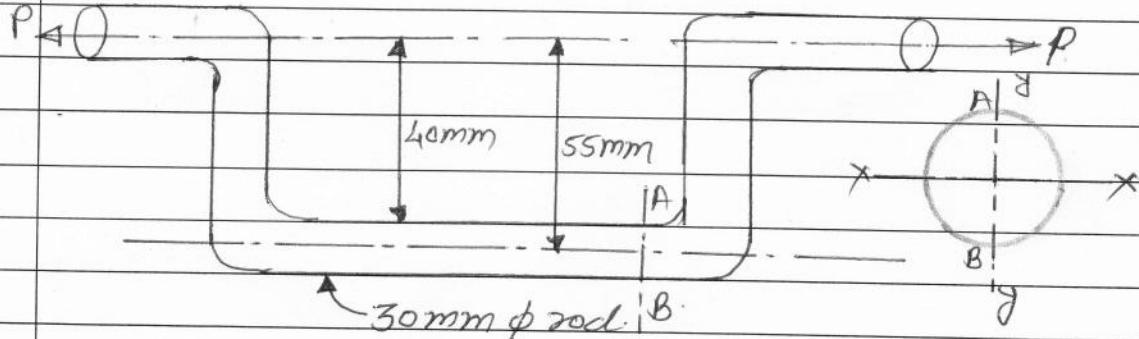
Q5f)

$$\rightarrow D = 30\text{mm}$$

$$\sigma_{\max} = 80 \text{ N/mm}^2$$

Let P be the axial force in N.

$$e = \frac{40 + 30}{2} = 55\text{mm}$$



i) Geometrical Properties of Section.

$$i) \text{ Area } A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (30)^2 = 706.85 \text{ mm}^2$$

$$ii) \text{ Section modulus } Z = \frac{\pi}{32} D^3 = \frac{\pi}{32} \times (30)^3 = 2650.71 \text{ mm}^3$$

$$iii) \text{ Direct stress } \sigma_0 = \frac{P}{A} = \frac{P}{706.85} = 1.41 \times 10^{-3} P \quad \frac{1}{2} \text{ m}$$

$$iv) \text{ Bending stress } \sigma_b = \frac{M}{Z} = \frac{P \times e}{Z} = \frac{P \times 55}{2650.71} = 20.74 \times 10^{-3} P \quad \frac{1}{2} \text{ m}$$

v) To find maximum value of P .

$$\sigma_{\max} = \sigma_0 + \sigma_b$$

$\frac{1}{2} \text{ m}$

$$80 = (1.41 \times 10^{-3} + 20.74 \times 10^{-3}) P$$

$$\therefore P = 3609.47 \text{ N.}$$

OR

$\frac{1}{2} \text{ m}$

$$P = 3.609 \text{ kN.}$$

Q6a)

→ Assumptions in theory of pure torsion.

1) Plane sections before twisting remain plane after twisting and do not twist or warp.

2) The shaft material is homogenous and isotropic.

3) The shaft is straight having uniform circular cross-section.

1m
each
foot

4) Circular sections remain circular even after twisting.

Any
four

5) Twist along the shaft is uniform.

6) Stresses do not exceed the proportional limit.

7) Shaft is loaded by twisting couples in the planes that are perpendicular to the axis of the shaft.

Q6b)

$$\rightarrow D = 200 \text{ mm}$$

$$\Theta = 2^\circ = 2 \times \frac{\pi}{180} = 0.0349 \text{ rad.}$$

$$L = 8 \text{ m.}$$

$$C = 0.8 \times 10^5 \text{ MPa.}$$

1) Using Torsional Formula.

$$\frac{T}{I_p} = \frac{C\Theta}{L}$$

1m

$$I_p = \frac{\pi}{32} D^4 = \frac{\pi}{32} \times 200^4 = 157.07 \times 10^6 \text{ mm}^4.$$

1m

$$\frac{T}{157.07 \times 10^6} = \frac{0.8 \times 10^5 \times 0.0349}{8000}$$

1m

$$\therefore T = 54.82 \times 10^6 \text{ N-mm}$$

OR

1m

$$T = 54.82 \times 10^3 \text{ N-m.}$$

Q6c)

$$\rightarrow D = 120 \text{ mm}$$

$$d = 90 \text{ mm}$$

$$T = 20 \text{ KN-m}$$

1) Using Torsional formula.

$$\frac{T}{I_p} = \frac{\tau}{R}$$

1m

$$T = 20 \text{ KN-m} = 20 \times 10^6 \text{ N-mm.}$$

$$I_p = \frac{\pi}{32} (D^4 - d^4) = \frac{\pi}{32} (120^4 - 90^4) = 13.91 \times 10^6 \text{ mm}^4 \quad 1m$$

$$R = \frac{D}{2} = \frac{120}{2} = 60 \text{ mm.}$$

1/2m

$$\therefore \tau = \frac{T \times R}{I_p}$$

$$= \frac{20 \times 10^6}{13.91 \times 10^6} \times 60 \quad 1/2m$$

$$\tau = 86.26 \text{ N/mm}^2$$

1m.

Q6d)

Let, d = Diameter of solid shaft.

$$\rightarrow d_2 = \frac{2}{3} d_1 \quad d_1 = \text{External dia. of hollow shaft.}$$

 $d_2 = \text{Internal dia. of hollow shaft.}$ For same strength Polar modulus Z_p of both the shaft must be equal.

$$Z_p \text{ of solid shaft} = Z_p \text{ of hollow shaft.}$$

1/2m

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Q6d>

Contour

$$\frac{\pi}{16} d^3 = \frac{\pi}{16} \left(d_1^4 - d_2^4 \right)$$

$$d^3 = d_1^4 - \left(\frac{2}{3} d_1 \right)^4$$

$$d^3 = \frac{d_1^4}{d_1} \left[1 - \left(\frac{2}{3} \right)^4 \right]$$

$$d^3 = 0.802 d_1^3$$

$$\boxed{d = 0.929 d_1}$$

1m

27 % saving in material

$$\% \text{ saving} = \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100 \quad \frac{1}{2} \text{M}$$

$$= \left[\frac{\pi/4 d^2 - \pi/4 (d_1^2 - d_2^2)}{\pi/4 d^2} \right] \times 100$$

$$= \left[\frac{d^2 - (d_1^2 - d_2^2)}{d^2} \right] \times 100 \quad \frac{1}{2} \text{M}$$

$$= \left[1 - \left[\frac{d_1^2 - (\frac{2}{3} d_1)^2}{d^2} \right] \right] \times 100$$

$$= \left[1 - \frac{d_1^2}{d^2} \left[1 - (\frac{2}{3})^2 \right] \right] \times 100$$

$$= \left[1 - \frac{0.555 d_1^2}{(0.929)^2} \right] \times 100 \quad \therefore d = 0.802 d_1$$

$$\% \text{ saving} = 35.70 \% \quad \frac{1}{2} \text{M}$$

Q6e)

→ Let, $D = \text{dia of solid shaft} = 80\text{mm}$

$d_1 = \text{External dia. of hollow shaft}$

$d_2 = \text{Internal dia. of hollow shaft}$

$$\therefore d_1 = 1.5d_2$$

In order that the solid and hollow shaft
Should have same strength to resist the torque,
the Polar modulus of the shaft must be equal.

Z_p of solid shaft = Z_p of hollow shaft

$$\frac{\pi}{16} D^3 = \frac{\pi}{16} \left[\frac{d_1^4 - d_2^4}{d_1} \right]$$

$$D^3 = \frac{(1.5d_2)^4 - d_2^4}{1.5d_2}$$

$$80^3 = \frac{d_2^4}{d_2} \left[\frac{(1.5)^4 - 1}{1.5} \right]$$

$$512 \times 10^3 = d_2^3 \cdot 2.708$$

$$d_2^3 = \frac{512 \times 10^3}{2.708}$$

$$d_2^3 = 189.06 \times 10^3$$

$$\therefore d_2 = (189.06 \times 10^3)^{\frac{1}{3}}$$

$$\therefore d_2 = 57.39\text{ mm}$$

$$\therefore d_1 = 1.5d_2 = 1.5 \times 57.39 = 86.08\text{ mm.}$$

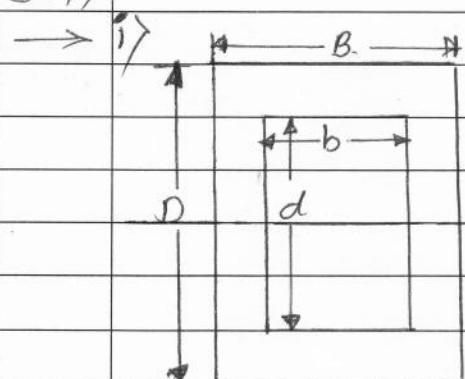
1m

1m

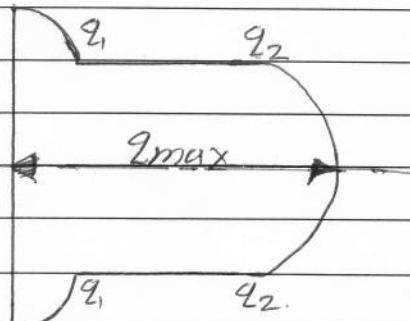
$$\therefore \text{External dia } d_1 = 86.08\text{ mm}$$

$$\text{Internal dia } d_2 = 57.39\text{ mm.}$$

Q6f)



N.A.

hollow rectangular
sectionshear stress
distribution diagram.

$$q_{\max} = q_2 + q_{\text{additional}}$$

1m

where,

q_2 = shear stress at bottom of flange by taking width as $B-b$

$q_{\text{additional}}$ = Additional shear stress due to web area above (or below) N.A.

ii) Definition:- When a tangential force is applied to a shaft at the circumference, in the plane of its transverse cross-section, the shaft is said to be subjected to a twisting moment or torque which is equal to the product of the force and the radius.

1m

S.I. Unit of Torque,

$$\text{Torque} = \text{Force} \times \text{Radius}$$

$$\text{Torque} \quad \text{N} \times \text{m}$$

$$\therefore \text{S.I. Unit of torque} = \text{N} \cdot \text{m.}$$

1m