

MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION (Autonomous)

(ISO/IEC - 27001 - 2005 Certified)

Subject Code: 17105 Summer-2013 Page No: 1/20 **Model Answers Important Instructions to examiners:** 1) The answers should be examined by key words and not as wordto-word as given in the model answer scheme. 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate. 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English Communication Skills). 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn. 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer. 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding. 7) For programming language papers, credit may be given to any other program based on equivalent concept.



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		(Autonomous) (ISO/IEC - 27001 - 2005 Certified)		
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Que.	Sub.	Model answers	Mark	Total
No.	Que.		S	Marks
1)		Solve any TEN of the following:		
	a)	Find wife $\begin{bmatrix} 2 & 1 & x \\ 0 & 2 & 1 \end{bmatrix} = 0$		
		Find x if $\begin{vmatrix} 2 & 1 & x \\ 0 & 3 & -1 \\ 4 & 0 & 2 \end{vmatrix} = 0$		
	Ans	$\therefore 2(6-0)-1(0+4)+x(0-12)=0$		
		$\therefore 12 - 4 - 12x = 0$	1	
		$\therefore x = \frac{2}{3}$	1	02
		12 2 51	_	
	b)	Find the value of the determinant: $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 2 & -1 \\ 7 & 0 & 4 \end{vmatrix}$		
	Ans	= 2(8-0)-3(4+7)+5(0-14)		
		= 16 - 33 - 70	1	02
		=-87	1	02
	c)	If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 & 3 \\ -3 & 1 & 5 \end{bmatrix}$ find the value of 4A-3B		
	Ans	$\begin{bmatrix} 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 3 \end{bmatrix}$		
		$4A - 3B = 4\begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & -1 \end{bmatrix} - 3\begin{bmatrix} 4 & 2 & 3 \\ -3 & 1 & 5 \end{bmatrix}$		
		$= \begin{bmatrix} 4 & 8 & -12 \\ 0 & 16 & -4 \end{bmatrix} - \begin{bmatrix} 12 & 6 & 9 \\ -9 & 3 & 15 \end{bmatrix}$	1	
		$= \begin{bmatrix} -8 & 2 & -21 \\ 9 & 13 & -19 \end{bmatrix}$	1	02
	d)	$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$, find A^2		
		$A^2 = A.A$		
	Ans	$A = A.A$ $= \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$	1/2	
		$= \begin{bmatrix} -1 & -5 \\ 10 & 14 \end{bmatrix}$	1½	02
		$x^{3}+1$		
	e)	Convert the improper fraction $\frac{x^3+1}{x^2+2x}$ in to proper fraction.		
1				



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)	Ans	$x^2 + 2x \overline{\smash)x^3 + 1}$		Williams
		$x^3 + 2x^2$ $-\underline{\qquad \qquad }$ $-2x^2 + 1$	1/2	
		$-2x^2 - 4x$	1/2	
		$\frac{x^{3}+1}{x^{2}+2x} = (x-2) + \frac{4x+1}{x^{2}+2x}$	1	02
	f) Ans	If $\tan A = \frac{1}{4}$, $\tan B = \frac{1}{2}$, Find $\tan (A + B)$	1/2	
		$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $= \frac{\frac{1}{4} + \frac{1}{2}}{1 - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)}$	1/2	
	2)	$= \frac{6}{7}$ Define compound angle.	1	02
	g) Ans	If A and B are any angle then addition A+B and subtraction A-B is called compound angle.	2	02
	h) Ans	If $\sin A = \frac{2}{3}$, find $\sin 3A$ $\sin 3A = 3\sin A - 4\sin^3 A$ $(2) \qquad (2)^3$	1	
		$= 3\left(\frac{2}{3}\right) - 4\left(\frac{2}{3}\right)^{3}$ $= \frac{22}{27} = 0.81$	1	02
	i)	In any $\triangle ABC$, prove that $:\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$		



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Que.	Sub.			Total
No.	Que.	Model answers	Marks	Marks
1)	Ans	In any $\triangle ABC$ $A+B+C=180$		
		B+C=180-A		
		$\frac{B+C}{2} = \frac{180-A}{2}$	1	
		$\frac{B+C}{2} = 90 - \frac{A}{2}$		
			1/2	
		$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$	7/2	
		$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$	1/2	02
	j)	Express in to product form: $\cos 28^{\circ} + \cos 42^{\circ}$		
		$\cos 28^{0} + \cos 42^{0} = 2\cos\left(\frac{28+42}{2}\right)\cos\left(\frac{28-42}{2}\right)$	1	
	Ans	$\cos 28 + \cos 42 = 2\cos\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$	1	
		$=2\cos 35\cos \left(-7\right)$		
		$=2\cos 35\cos \left(7\right)$	1	02
	k)	Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$		
	Ans	$\pi - \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$	1	
		$=\pi-\frac{\pi}{3}-\frac{\pi}{6}$	1/2	
		$\frac{-n}{3}$ 3 6		
		$=\frac{\pi}{2}$	1/2	02
	1)	If the straight line $4px+3y+8=0$ and $3px-9y+10=0$ are		
		perpendiculer .find the values of p		
		r. r.		
	Ans	m = -4p		
		$m_1 = \frac{-4p}{3}$	1/2	
		$m_2 = \frac{-3p}{-9} = \frac{p}{3}$	/2	
		$\therefore m_1.m_2 = -1$	1/2	
	1		1	1



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		$\frac{-4p}{3} \cdot \frac{p}{3} = -1$ $\frac{-4p^2}{9} = -1$	1/2	Warks
		$\frac{-4p^2}{9} = -1$ $\therefore p^2 = \frac{9}{4}$ $\therefore p = \pm \frac{3}{2}$ Attempt any four of the following	1/2	02
2)	a)			
,	Ans	Solve by Cramer's rule $x+y+z=6, 2x+y-2z+2=0, x+y-3z+6=0$		
		$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ 1 & 1 & -3 & 1 \end{vmatrix}$ $= 1(-3+2) - 1(-6+2) + 1(2-1)$		
		= 4	1	
		$D_x = \begin{vmatrix} 6 & 1 & 1 \\ -2 & 1 & -2 \\ -6 & 1 & -3 \end{vmatrix}$		
		= 6(-3+2) - 1(6-12) + 1(-2+6)		
		=4		
		$D_{y} = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -2 & -2 \\ 1 & -6 & -3 \end{vmatrix}$	2	
		= 1 (6 - 12) - 6(-6 + 2) + 1(-12 + 2)		
		= 8		
		$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & -2 \\ 1 & 1 & -6 \end{vmatrix}$		
		= 1 (-6+2) - 1(-12+2) + 6(2-1)		
		= 12		
		By Cramers rule		
		$x = \frac{D_x}{D} = \frac{4}{4} = 1$		



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	Zusi	$y = \frac{D_y}{D} = \frac{8}{4} = 2$		112012110
		$Z = \frac{D_Z}{D} = \frac{12}{4} = 3$	1/2	04
	b)	Find the value of y by determinant method :		
	Ans	$\frac{1}{x} + \frac{3}{x} + \frac{1}{z} = 2, \frac{3}{x} + \frac{1}{y} + \frac{2}{z} = 4, \frac{9}{x} + \frac{1}{x} + \frac{4}{z} = 16$		
		Let $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$		
		$\therefore a + 3b + c = 2$ $3a + b + 2c = 4$	1	
		3a+b+2c=4 9a+b+4c=16		
		$D = \begin{vmatrix} 1 & 3 & 1 \\ 3 & 1 & 2 \\ 9 & 1 & 4 \end{vmatrix}$		
		= 1(4-2) - 3(12-18) + 1(3-9) $= 14$	1	
		$D_b = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 9 & 16 & 4 \end{vmatrix}$		
		= 1(16 - 32) - 2(12 - 18) + 1(48 - 36) =8	1	
		$\therefore b = \frac{D_b}{D} = \frac{8}{14} = \frac{4}{7}$ $\therefore y = \frac{7}{4}$	1	04
	c)	Resolve in to partial fraction: $\frac{3x+1}{(x+1)(x-4)(x+3)}$		



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	Que.	3x+1 A B C	1/2	IVIAIRS
	Ans	$\frac{3x+1}{(x+1)(x-4)(x+3)} = \frac{A}{x+1} + \frac{B}{x-4} + \frac{C}{x+3}$ $x = -1$ $3(-1)+1 = A(-1-4)(-1+3)$ $\therefore -2 = -10A$ $\therefore A = \frac{2}{10} = \frac{1}{5}$	1	04
		$x = 4$ $\therefore 3(4) + 1 = B(4+1)(4+3)$ $\therefore 13 = 35B$ 13	1	
		$\therefore B = \frac{13}{35}$ $x = -3$ $\therefore 3(-3) + 1 = C(-3+1)(-3-4)$ $\therefore -8 = 14C$ $\therefore C = \frac{-4}{7}$	1	
		$\therefore \frac{3x+1}{(x+1)(x-4)(x+3)} = \frac{\frac{1}{5}}{x+1} + \frac{\frac{13}{35}}{x-4} + \frac{\frac{-4}{7}}{x+3}$	1/2	04
		Resolve in to partial fraction: $\frac{3x^2 + 5x}{(x^2 - 1)(x + 1)}$ $\therefore \frac{3x^2 + 5x}{(x^2 - 1)(x + 1)} = \frac{3x^2 + 5x}{(x - 1)(x + 1)(x + 1)}$ $\therefore \frac{3x^2 + 5x}{(x - 1)(x + 1)^2} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$ $\therefore 3x^2 + 5x = A(x + 1)^2 + B(x - 1)(x + 1) + c(x - 1)$	1/2	



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)		$x = 1$ $\therefore 3 + 5 = A(2)^{2}$ $\therefore A = \frac{8}{4} = 2$ $x = -1$	1	
		$3-5 = c(-2)$ $\therefore C = 1$ $x = 0, A = 2, C = 1$	1	
		$\therefore 0 = 2 - B - 1$ $\therefore B = 1$	1	
		$\therefore \frac{3x^2 + 5x}{(x-1)(x+1)^2} = \frac{2}{x-1} + \frac{1}{x+1} + \frac{1}{(x+1)^2}$	1/2	
	e) Ans	If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$, show that $A^2 - 8A$ is scalar matrix.		04
		$\begin{vmatrix} A^2 - 8A = A \cdot A - 8A \\ = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} - 8 \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$		
		$\begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix}$	1	
		$ = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} $	2	
	f)	Find the values of a and b from the matrix equation $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -3 & 5 \end{bmatrix}$	1	04
	Ans	$\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -3 & 5 \end{bmatrix}$ $\begin{bmatrix} 3a+10 & 3+2b \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix}$		
		$\begin{bmatrix} 3a+10 & 3+2b \\ 4a+5 & 4+b \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -3 & 5 \end{bmatrix}$	2	
		$\therefore 3a+10=4$ $a=-2$ and $3+2b=5$	1	
		b = 1	1	04
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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)	a)	Attempt any four of the following If $A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 3 \\ 0 & -3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -1 & -1 \\ -2 & 1 & 3 \end{bmatrix}$ verify that $A(B+C) = AB + AC$		
	Ans	$A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 3 \\ 0 & -3 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & -1 & -1 \\ -2 & 1 & 3 \end{bmatrix}$		
		$A(B+C) = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \left\{ \begin{bmatrix} 0 & -2 & 3 \\ 0 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -1 \\ -2 & 1 & 3 \end{bmatrix} \right\}$	1/2	
		$ \begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} \begin{vmatrix} 2 & -3 & 2 \\ -2 & -2 & 4 \end{vmatrix} $ $ \begin{bmatrix} 6 & 1 & -6 \end{bmatrix} $	1/2	
		$= \begin{bmatrix} 6 & 1 & -6 \\ -8 & -3 & 10 \end{bmatrix}$	1	
		$AB + AC = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & -2 & 3 \\ 0 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -2 & 1 & 3 \end{bmatrix}$	1/2	
			1/2	
		$= \begin{bmatrix} 6 & 1 & -6 \\ -8 & -3 & 10 \end{bmatrix}$	1	04
	b)	If I is the unit matrix of order 3 and $A = \begin{bmatrix} 1 & 2 & 6 \\ 3 & -2 & 0 \\ 5 & 7 & 4 \end{bmatrix}$, find the value of $A^2 - A + I$		
	Ans	$A^{2} - A + I = \begin{bmatrix} 1 & 2 & 6 \\ 3 & -2 & 0 \\ 5 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 6 \\ 3 & -2 & 0 \\ 5 & 7 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 6 \\ 3 & -2 & 0 \\ 5 & 7 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1	
		$\begin{bmatrix} 37 & 40 & 30 \\ -3 & 10 & 18 \\ 46 & 24 & 46 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 6 \\ 3 & -2 & 0 \\ 5 & 7 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	2	
		$= \begin{bmatrix} 37 & 38 & 24 \\ -6 & 13 & 18 \\ 41 & 17 & 43 \end{bmatrix}$	1	04
	c)	Resolve in to partial fraction: $\frac{x-5}{x^3 + x^2 - 5}$		



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3)	Ans	$\therefore \frac{x-5}{x^3 + x^2 - 5x} = \frac{x-5}{x(x^2 + x - 5)}$ $\frac{x-5}{x(x^2 + x - 5)} = \frac{A}{x} + \frac{Bx + c}{x^2 + x - 5}$	1/2	
		$\therefore x - 5 = A(x^2 + x - 5) + (Bx + C)x$ $x = 0$ $-5 = A(-5)$ $\therefore A = 1$ $x = 1, A = 1$	1	
		$\therefore 1-5=1(1+1-5)+(B+C)$ $\therefore -4=-3+B+C \qquad(1)$ $x=-1, A=1$ $-1-5=1(1-1-5)+(-B+C)(-1)$ $-6=-5+B-C$ $\therefore B-C=-1 \qquad(2)$	1	
		$(1)+(2)$ $B+C=-1$ $B-C=-1$ $2B=-2$ $\therefore B=-1$ $\therefore C=0$	1	
	d)	$\therefore \frac{x-5}{x(x^2+x-5)} = \frac{1}{x} - \frac{x}{x^2+x-5}$ Resolve in to partial fraction: $\therefore \frac{x^4+2x}{x^2-1}$	1/2	04
	Ans	$\therefore \frac{x^4 + 2x}{x^2 - 1} = x^2 + 1 + \frac{2x + 1}{x^2 - 1}$ Consider $\frac{2x + 1}{x^2 - 1} = \frac{2x + 1}{(x - 1)(x + 1)}$ $2x + 1 \qquad A \qquad B$		
		$\therefore \frac{2x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$	1	



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore 2x+1=A(x+1)+B(x-1)$		
		x = 1		
		$\therefore 2+1=A(1+1)$	4	
		$\therefore A = \frac{3}{2}$	1	
		$\therefore x = -1$		
		$\therefore 2(-1)+1=B(-1-1)$ $\therefore -1=-2B$		
			1	
		$\therefore B = \frac{1}{2}$	1	
		$\therefore \frac{2x+1}{(x-1)(x+1)} = \frac{\frac{3}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}$	1/2	
		$\therefore \frac{x^4 + 2x}{x^2 - 1} = x^2 + 1 + \frac{\frac{3}{2}}{x - 1} + \frac{\frac{1}{2}}{x + 1}$	1/2	04
	e)	If A and B are obtuse angles and $\sin A = \frac{5}{13}$, $\cos B = \frac{4}{5}$, then		
		find the value of $\sin(A+B)$.		
	Ans	$\because \cos^2 A = 1 - \sin^2 A$		
		$=1-\frac{25}{109}=\frac{144}{169}$		
		$\therefore \cos A = \pm \frac{12}{13}$		
		$\therefore \cos A = -\frac{12}{13} \text{as A is obtuse}$	1	
		$\cos B = \frac{-4}{5}$		
		$\therefore \sin^2 B = 1 - \cos^2 B$		
		$=1+\frac{16}{25}=\frac{9}{25}$		
		$\therefore \sin B = \pm \frac{3}{5}$	1	
		$\therefore \sin B = \frac{3}{5} \text{as B is obtuse}$		
		sin(A+B)=sinAcosB+cosAsinB	1	
			1	04
		$= \frac{5}{13} \cdot \frac{-4}{5} + \frac{-12}{13} \cdot \frac{3}{5} = \frac{-56}{65}$	1	UT



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3)	f)	Prove that : $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A}$		
	Ans	$= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C - \cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A - \cos C \sin A}{\cos C \cos A}$ $= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$ $= 0$	2 1 1	04
4)	a)	Find the adjoint of matrix A, if A= $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 3 \\ 0 & 3 & 5 \end{bmatrix}$		
	Ans	matrix of minors= $\begin{bmatrix} \begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 4 \end{vmatrix} \end{bmatrix}$	1	
		$= \begin{bmatrix} 11 & 5 & 3 \\ 3 & 10 & 6 \\ 4 & 7 & 8 \end{bmatrix}$	1	
		matrix of cofactors = $\begin{bmatrix} 11 & -5 & 3 \\ -3 & 10 & -6 \\ 4 & -7 & 8 \end{bmatrix}$	1	
		$Adj.A = \begin{bmatrix} 11 & -3 & 4 \\ -5 & 10 & -7 \\ 3 & -6 & 8 \end{bmatrix}$	1	04
	b)	Solve the equation using matrix method $x + y + z + 2$, $y + z = 1$, $z + x = 3$		
	Ans	Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ Consider, $ A = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$		



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4)		=1(1-0)-1(0-1)+1(0-1)	/2	04
		$= 1 \neq 0$ $\therefore A^{-1} \text{exists}$		
		$matrix of minors = \begin{bmatrix} \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$	1	
		$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ $\text{matrix of cofactors} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ $\text{Adj.} A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$	1/2	
		$A^{-1} = \frac{1}{ A }.adj.A$ $= \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ $X = A^{-1}B$	1	
	c)	$= \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ Prove that $: \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2$	1	04



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4)	~	$\sin 3x \cos 3x = \sin 3x \cos x - \cos 3x \sin x$		
	Ans	$\frac{1}{\sin x} = \frac{1}{\cos x} = \frac{1}{\sin x \cos x}$	1	
		$=\frac{\sin(3x-x)}{}$	1	
		$\sin x \cos x$	1	
		$=\frac{\sin 2x}{\sin x \cos x}$	1/2	
		sin x cos x 2 sinxcos x		
		$=\frac{2\sin x \cos x}{\sin x \cos x}$	1	
		=2	1/2	04
		$\cos 2x$ 1 2		
	d)	Prove that : $\frac{\cos 2x}{1 + \cos 2x} = 1 - \frac{1}{2} \sec^2 x$		
		1 2		
	Ans	$\therefore 1 - \frac{1}{2}\sec^2 x$		
		$=1-\frac{1}{2\cos^2 x}$		
			1	
		$=\frac{2\cos^2 x - 1}{2}$		
		$= \frac{1}{2\cos^2 x}$	2	
		$=\frac{\cos 2x}{1+co2x}$		
		OR	1	04
		cos 2 x		
		$\therefore \frac{1+\cos 2x}{1+\cos 2x}$		
		$-\cos^2 x - \sin^2 x$	1	
		$\frac{1}{2\cos^2 x}$		
		$=\frac{1}{2}-\frac{1}{2}\tan^2 x$		
			1	
		$=\frac{1}{2}-\frac{1}{2}(\sec^2 x-1)$	1	
				04
		$=1-\frac{1}{2}\sec^2 x$	1	
	e)	Prove that $\cos 20^{\circ} \cos 40^{\circ} \cos 80^{\circ} = \frac{1}{8}$		
	Ans	$\cos 20^{\circ} \cos 40^{\circ} \cos 80^{\circ}$		
	1110	$=\frac{1}{2}(2\cos 80\cos 40)\cos 20$		
		$= \frac{1}{2} (\cos 120 + \cos 40) \cos 20$	1	



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$=\frac{1}{2}\left(-\frac{1}{2}+\cos 40\right)\cos 20$	1/2	
		$=\frac{1}{2}\left(\frac{2\cos 40-1}{2}\right)\cos 20$	1/2	
		$= \frac{1}{4} (2\cos 40\cos 20 - \cos 20)$	1	
		$= \frac{1}{4} (\cos 60 + \cos 20 - \cos 20)$	1/2	
		$=\frac{1}{8}$	1/2	
	f)	Prove that := $\frac{1 - \tan 2A \tan A}{1 + \tan 2A \tan A} = \frac{\cos 3A}{\cos A}$		
	Ans	$\therefore \frac{1 - \tan 2A \tan A}{1 + \tan 2A \tan A} = \frac{1 - \frac{\sin 2A}{\cos 2A} \frac{\sin A}{\cos A}}{1 + \frac{\sin 2A}{\cos 2A} \frac{\sin A}{\cos A}}$	1	
		$= \frac{\cos 2A \cos A - \sin 2A \sin A}{\cos 2A \cos A + \sin 2A \sin A}$	1	
		$=\frac{\cos(2A+A)}{\cos(2A-A)}$	1	
5)		$= \frac{\cos 3A}{\cos A}$ Attempt any four of the following:	1	04
	a)	Prove that : $\frac{\sin 7x + \sin x}{\cos 5x - \cos 3x} = \sin 2x - \cos 2x.\cot x$		
	Ans	$R.H.S. = \frac{\sin 7x + \sin x}{\cos 5x - \cos 3x}$		
		$= \frac{2\sin\frac{7x+x}{2}.\cos\frac{7x-x}{2}}{-2\sin\frac{5x+3x}{2}.\sin\frac{5x-3x}{2}}$	1	
		$=\frac{\sin 4x.\cos 3x}{\cos 3x}$	1	
		$-\sin 4x \cdot \sin x$ $= \frac{-\cos 3x}{\sin x}$	1/2	
		$= \frac{\sin x}{-\cos(2x+x)}$ $= \frac{\sin x}{\sin x}$	1/2	
		$= \frac{-\left[\cos 2x \cos x - \sin 2x \sin x\right]}{\cdot}$		
		$ sinx = \sin 2x - \cos 2x \cdot \cot x $	1	04



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Que.	Sub.			Total
No.	Que.	Model answers	Marks	Marks
5)	b)	Prove that $\frac{\sin A + 2\sin 2A + \sin 3A}{\cos A + 2\cos 2A + \cos 2A} = \tan 2A$		
		$\cos A + 2\cos 2A + \cos 3A$		
	Ans	$LHS = \frac{\sin A + \sin 3A + 2\sin 2A}{\sin A + \sin A}$		
		$\cos A + \cos 3A + 2\cos 2A$		
		$=\frac{2\sin 2A \cdot \cos A + 2\sin 2A}{2A}$	2	
		$2\cos 2A.\cos A + 2\cos 2A$	_	
		$= \frac{2\sin 2A.(\cos A + 1)}{2\cos 2A.(\cos A + 1)}$	1	
				2.4
		$= \tan 2A$ For the principal value evaluate,	1	04
	c)	$\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 3\sin^{-1}(-1)$		
	Ans	$\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 3\sin^{-1}(-1)$		
		$=-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\left(\pi-\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)-3\sin^{-1}(-1)$	2	
		$= -\frac{\pi}{4} + 2\left(\pi - \frac{\pi}{4}\right) - 3\frac{\pi}{2}$	1	04
		$= -\frac{\pi}{4} + 2\pi - \frac{\pi}{2} - 3\frac{\pi}{2} = -\frac{\pi}{4}$	1	04
	d)	Find x, if $\tan^{-1}\left(\frac{5}{12}\right) = \sin^{-1}x$		
	Ans	$\tan^{-1}\left(\frac{5}{12}\right) = \sin^{-1}x$		
		$\tan^{-1}\left(\frac{5}{12}\right) = A \qquad \qquad 5$		
		$\therefore \tan A = \frac{5}{12}$	1+1	
		12 A		
		$\sin A = \frac{5}{13}$		
		$\therefore A = \sin^{-1}\left(\frac{5}{13}\right)$	1	
		$\tan^{-1}\left(\frac{5}{12}\right) = \sin^{-1}\left(\frac{5}{13}\right)$		
		$\therefore x = \frac{5}{13}$	1	04



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Que No.	Sub. Que	Model answers	Mark s	Total Mark s
5)	e)	Find the equation of line passing through point of intersection of the lines $x + y = 0, 2x - y = 9$ and perpendicular to $3x - y + 4 = 0$		
	Ans	x + y = 0 $2x - y = 9$		
		3x = 9 $x = 3$	1/2	
		y = -3 slope of line $3x - y + 4 = 0$	1/2	
		$m_1 = 3$	1	
		$\therefore m_2 = \frac{1}{3}$	1	
		$\therefore m_2 = \frac{-1}{3}$ $y+3 = \frac{-1}{3}(x-3)$ $x+3y+6=0$	1	04
	f)	Find the equation of straight line which passes through the point of intersection of lines		
	Ans	2x+3y=13,5x-y=7 and the point (4,5)		
		$2x+3y=13$ $5x-y=7$ $\therefore 2x+3y=13$		
		15x - 3y = 21		
		$ \begin{vmatrix} 17x = 34 \\ x = 2 \end{vmatrix} $	1	
		$\therefore y = 3$	1	
		$\frac{y-3}{3-5} = \frac{x-2}{2-4}$ $\frac{y-3}{-2} = \frac{x-2}{-2}$	1	
		$\frac{-2}{-2} - \frac{-2}{-2}$ $x - y + 1 = 0$	1	04



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Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	Que.	Attempt any four of the following:		TYTUTES
	a)	Prove that $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$		
	Ans	$\cos A = \frac{4}{5}$		
		$\therefore \sin A = \sqrt{1 - \cos^2 A}$		
		$=\sqrt{1-\frac{16}{25}}$ $=\frac{3}{5}$		
		$-\frac{1}{5}$ and $\cos B = \frac{12}{13}$	1	
		$\therefore \sin B = \sqrt{1 - \sin^2 B}$		
		$=\sqrt{1-\frac{144}{169}}$		
		$=\frac{5}{13}$	1	
		LHS=A+B	1/2	
		Consider,		
		$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$		
		$=\frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13}$		
		$=\frac{33}{65}$	1	
		$A + B = \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13}$		
		$=\cos^{-1}\frac{33}{65}$	1/2	04
	b)	Prove that $\tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} = \cot^{-1} \frac{9}{2}$		
	Ans	LHS= $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13}$		
		$= \tan^{-1} \left(\frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{7} \cdot \frac{1}{13}} \right)$	2	



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-	Sub. Que.	Model answers	Marks	Total Marks
5) c)		$= \tan^{-1}\left(\frac{20}{90}\right) = \tan^{-1}\left(\frac{2}{9}\right)$ $= \cot^{-1}\left(\frac{9}{2}\right)$ If m1 and m2 are the slopes of two lines then prove that angles between two lines is $\theta = \tan^{-1}\left \frac{m_1 - m_2}{1 + m_1 m_2}\right $	1	04
A	Ans	Let $\theta_1 = \text{Angle of inclination of } L_1$ $\theta_2 = \text{Angle of inclination of } L_2$ slope of L_1 is $m_1 = \tan \theta_1$ slope of L_2 is $m_2 = \tan \theta_2$ from fig. $\theta = \theta_1 - \theta_2$ $\therefore \tan \theta = \tan(\theta_1 - \theta_2)$ $= \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$ $= \frac{m_1 - m_2}{1 + m_1 m_2}$ $\therefore \theta = \tan^{-1} \left(\frac{m_1 - m_2}{1 + m_1 m_2} \right)$ For angle to be acute. $\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $	1 1 1	04



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Que.	Sub.	Model answers	Marks	Total
No.	Que.		TVIGING	Marks
6)	d)	Find the acute angle between the lines $3x - y = 4$, $2x + y = 3$		
	Ans	3x - y = 4	1	
		$m_1 = 3$		
		2x + y = 3	1	
		$m_2 = -2$	1	
		$\theta = \tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $		
		$\theta = \tan^{-1} \left \frac{3+2}{1+3(-2)} \right $	1	
		$\theta = \tan^{-1}(1)$		04
		$\theta = \frac{\pi}{4}$	1	04
	e)	Find the distance between two parallel lines $3x+2y-6=0$,		
		3x+2y-12=0		
	Ans	$d = \left \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right $	1	
		$d = \left \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right $ $= \left \frac{-6 + 12}{\sqrt{3^2 + 2^2}} \right $	2	
		$=\frac{6}{\sqrt{13}}$	1	04
	f)	Find the length of memoral cules from the point (2, 2) on the line		
	1	Find the length of perpendicular from the point $(3,-2)$ on the line		
		7(x-2)=5(y+3) 7(x-2)=5(y+3)		
	Ans	7x - 5y - 29 = 0	1	
			1	
		$d = \left \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right $		
		$= \frac{7(3) + (-5)(-2) - 29}{\sqrt{7^2 + (-5)^2}}$	1	
		$= \left \frac{21 + 10 - 29}{\sqrt{74}} \right $	1	
		$=\frac{2}{\sqrt{74}}$	1	04