MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC-27001-2005 Certified)

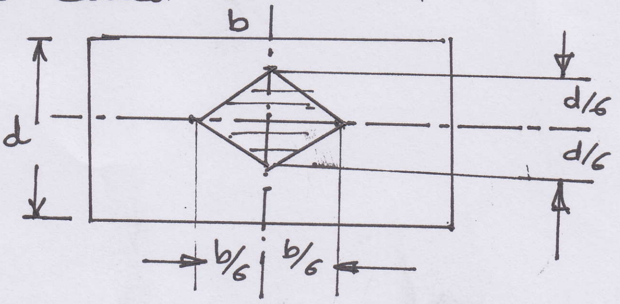
SUMMER-13 EXAMINATION

Subject Code: 12138 **Model Answer (Theory of structure)**

Q. No	Model Answer	Marks
1 A)	Attempt Any THREE	12

a) Define core of section. Sketch core of section for rectangular shape with dimensions.

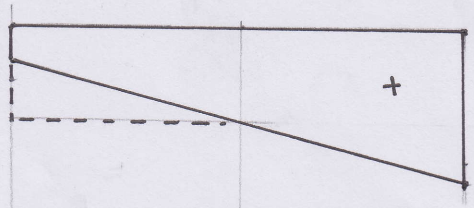
Def^n :- The central limited area of any section within which any external load acts, only produce compressive stress, then this central area is called as core of section. — 2 M



2 M

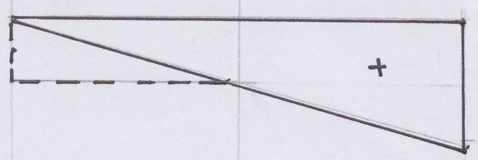
b) Sketch Resultant stress distribution diagram

i) $\sigma_o > \sigma_b$



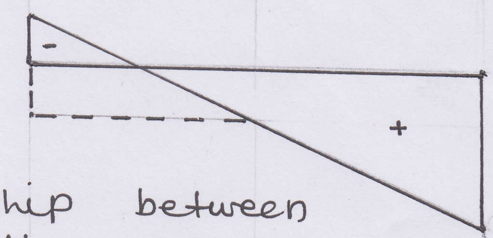
1 M

ii) $\sigma_o = \sigma_b$



1 M

iii) $\sigma_o < \sigma_b$



2 M.

c) Write the relationship between slope and deflection for cantilever and simply supported beams.

i) cantilever beam:- ⓐ In cantilever beam slope is zero at support and deflection is also zero at support. — 2 M

:- ⓑ In cantilever beam slope and deflection is maximum at free end

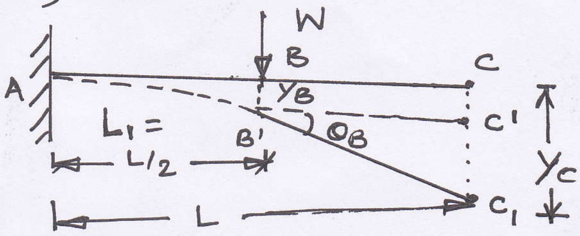
Q. NO	Model Answers	Marks.

Q1 ii) Simply supported beam :- slope is maximum at support but deflection is zero at support — 2M

:- slope is zero at centre for symmetrical loading but deflection is maximum at centre.

d) Data, span = L

pt. load = W, at centre.



1M

$$\therefore \theta_B = \frac{WL_1^2}{2EI}$$ ½M

$$\therefore Y_B = \frac{WL_1^3}{3EI} = \frac{WL^3}{24EI}$$ ½M

$\therefore \theta_c = \theta_B$ (As there is no load betn B and C) — 1M

$$\therefore \theta_c = \frac{WL_1^2}{2EI} = \frac{WL^2}{8EI}$$

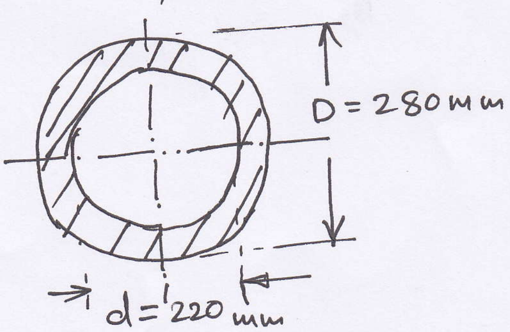
$$\therefore Y_c = cc' + c'c_1$$

$$= \frac{WL_1^3}{3EI} + \theta_B \times \frac{L}{2}$$ 1M

$$= \frac{WL^3}{24EI} + \frac{WL^3}{32EI}$$

(B) Attempt any ONE of the following.

(a)



Data

D = 280 mm

t = 30 mm

$l = 3m = 3000mm$ — 1M

one end hinged and other fixed

$\sigma_c = 5500 \ N/mm^2$

$E = 2 \times 10^5 \ N/mm^2$

*effective length. (Le)

$$\therefore Le = \frac{L}{\sqrt{2}} = \frac{3000}{\sqrt{2}} = 2121.32 mm.$$ 1M

Minimum M.I. (Imin)

$$\therefore I_{min} = I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(280^4 - 220^4)$$ 1M

$$I_{min} = 1.867 \times 10^8 \ mm^4$$

Q. No	Model Answer	Marks

Q 1 (B) (a) Eulers critical load

$$\therefore P_E = \frac{\pi^2 E I_{min}}{(Le)^2} = \frac{\pi^2 \times 2\times10^5 \times 1.867\times10^8}{(2121.32)^2}$$ 2M

$$= 8189.5810.6 \text{ N}$$

$$\therefore \text{crushing load } (P_c) = A \times \sigma_c$$

$$= \frac{\pi}{4}(280^2 - 220^2) \times 5500$$

$$= 129590.697 \text{ N.}$$ 1M

(b)

$\therefore$ Data

$\therefore$ $D = 120$ mm

$\therefore$ $d = 80$ mm

$l = 4m = 4000$ mm

fixed at both ends.

$\sigma_c = 500 \text{ MPa} = 550 \text{ N/mm}^2$

$\alpha = a = \frac{1}{1600}$

* c/s area of column (A)

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(120^2 - 80^2) = 6283.185 \text{ mm}^2$$ 1M

* Minimum M. I. (I_{min})

$$I_{min} = I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(120^4 - 80^4)$$

$$= 8168140.899 \text{ mm}^4$$ 1M

* $K_{min} = \sqrt{\frac{I_{min}}{A}}$

$$= \sqrt{\frac{8168140.899}{6283.185}}$$ 1M

$$= 36.055 \text{ mm}$$

* Effective length (Le)

$$\therefore Le = \frac{L}{2} = \frac{4000}{2} = 2000 \text{ mm}$$ 1M

* Rankines crippling load (P_R)

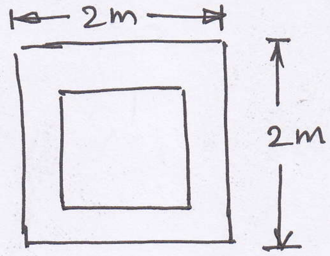
$$\therefore P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{Le}{K_{min}}\right)^2} = \frac{500 \times 6283.185}{1 + \frac{1}{1600}\left(\frac{2000}{36.055}\right)^2}$$ 2M

$$\therefore P_R = 1074735.218 \text{ N}$$

$$\therefore P_R = 1074.735 \text{ KN}$$

Q. NO	Model Answer	Marks.

2 a)



Data
$B = 2m,\ D = 2m$
$b = 1m,\ d = 1m$
$P = 1.2\ kN/m^2$
$\gamma = 20\ kN/m^3$

*Direct stress (σ_0)

$$\sigma_0 = \gamma \cdot h$$

$$\boxed{\sigma_0 = 20h.} \quad \text{-------} \ (1)$$ 2M

*finding bending stress (σ_b)

Total wind Pressure $= P \cdot B \cdot h$

$= 1.2 \times 2 \times h. = 2.4h$ 1M

Bending moment of Total wind Pressure @ base

$$M = P \times h/2$$

$$= 2.4 \times h \times \frac{h}{2}$$

$$\boxed{M = 1.2h^2}$$ 1M

Moment of Inertia (I)

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I = \frac{2^4}{12} - \frac{1^4}{12}$$

$$\boxed{I = 1.25\ m^4}$$ 1M

Bending stress (σ_b)

$$\therefore \sigma_b = \frac{M \times y}{I}$$

$$\therefore \sigma_b = \frac{1.2h^2 \times 1}{1.25}$$

$$\therefore \boxed{\sigma_b = 0.96h^2} \quad \text{-------} \ (2)$$ 1M

* for no tension condition 1M

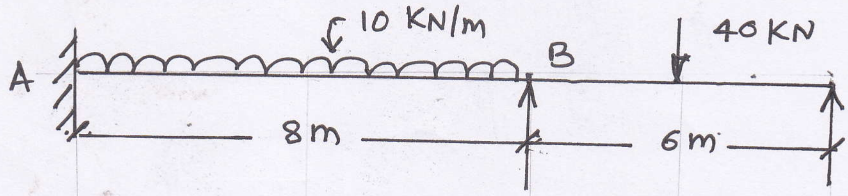
$$\sigma_0 = \sigma_b.$$

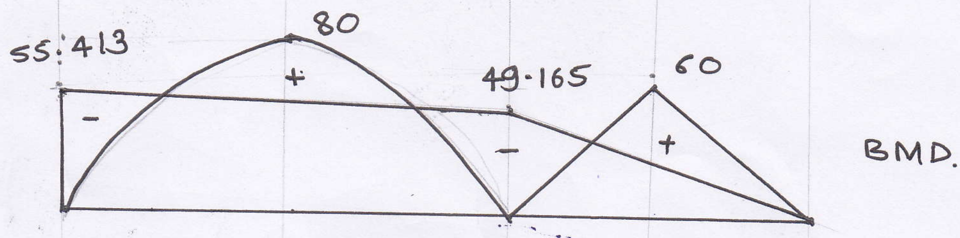
$$20h = 0.96h^2$$

$$\boxed{h = 20.833m}$$ 1M

Q. No.	Model Answer	Marks.

Q 2 b)





2M

* Assume each span of beam fixed and find fixed end moments.

span AB

$$M_{AB} = -\frac{wl_1^2}{12} = -\frac{10\times 8^2}{12} = -53.333 \text{ KN-m}$$

$$M_{BA} = +\frac{wl_1^2}{12} = -\frac{10\times 8^2}{12} = +53.333 \text{ KN-m}$$

span BC

2M

$$M_{BC} = -\frac{WL_2}{4} = -\frac{40\times 6}{4} = -30 \text{ KN-m}$$

$$M_{CB} = +\frac{WL_2}{4} = +\frac{40\times 6}{4} = +30 \text{ KN-m}.$$

* find stiffness factors and distribution factors.

Joint	Member	stiffness (K)	ΣK	D. F.
B	BA	$\frac{4EI}{L_1} = \frac{4EI}{8} = 0.5EI$	EI	0.5
	BC	$\frac{3EI}{L_2} = \frac{3EI}{6} = 0.5EI$		0.5

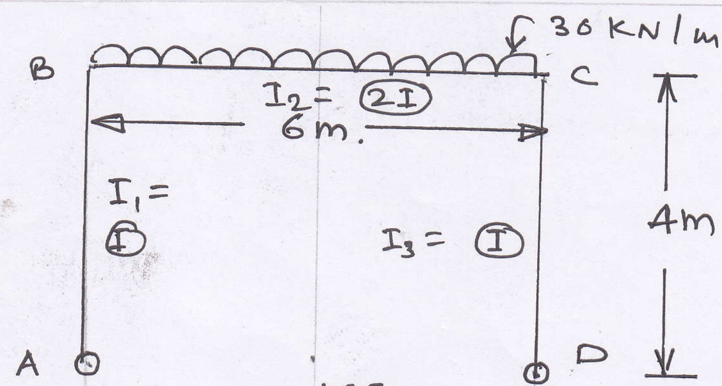
2M

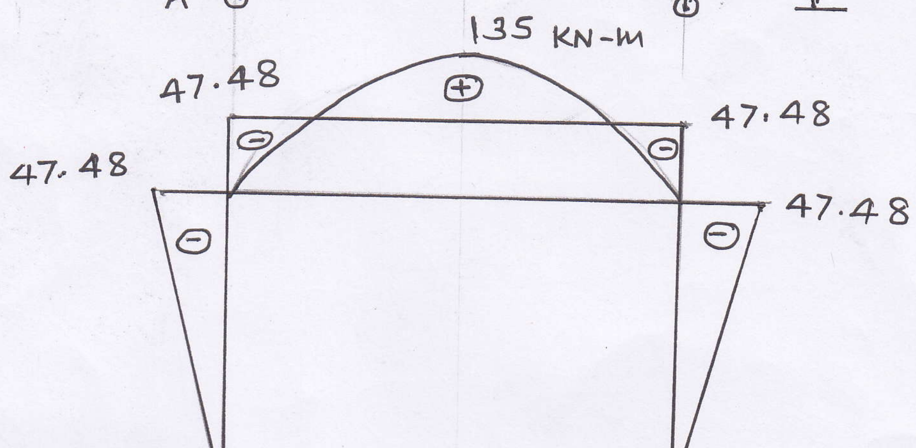
Joint	A	B		C
Member	AB	BA	BC	CB
D. F.	–	0.5	0.5	–
FEM.	-53.33	53.33	-30	30
Release c & carry over to B				-30
			-15	
Initial moment	-53.33	53.33	-45	0
Distribute		-4.165	-4.165	
carry over to A	-2.083			
final moment.	-55.413	49.165	49.165	0

2M

Q. No.

Q2 (C)





BMD

2M

* Assume all spans are fixed & find fixed end moments

SPAN AB

$M_{AB} = M_{BA} = 0$ ∴ (No load)

SPAN BC

$M_{BC} = -\frac{wL^2}{12} = -\frac{30 \times 6^2}{12} = -90$ kN-m

1M

$M_{CB} = +\frac{wL^2}{12} = \frac{30 \times 6^2}{12} = +90$ kN-m

SPAN CD $M_{CD} = M_{DC} = 0$ ∴ (No load)

* find stiffness factors.

Joint B ∴ $K_{BA} = \frac{3EI_1}{L_1} = \frac{3EI}{4} = 0.75EI$

∴ $K_{BC} = \frac{4EI_2}{L_2} = \frac{4E(2I)}{6} = 1.333EI$

Joint C $K_{CB} = \frac{4EI_2}{L_2} = \frac{4E(2I)}{6} = 1.33\,EI$

$K_{CD} = \frac{3EI_3}{L_3} = \frac{3 \times EI}{4} = 0.75\,EI$

Q.No

Model Answer

Joint	MEMBER	stiffness (k)	ΣK	D.f.
B	BA	0.75EI	2.083 EI	0.36
	BC	1.333EI		0.64
C	CB	1.333EI	2.083EI	0.64
	CD	0.75EI		0.36

2M

Moment distribution table

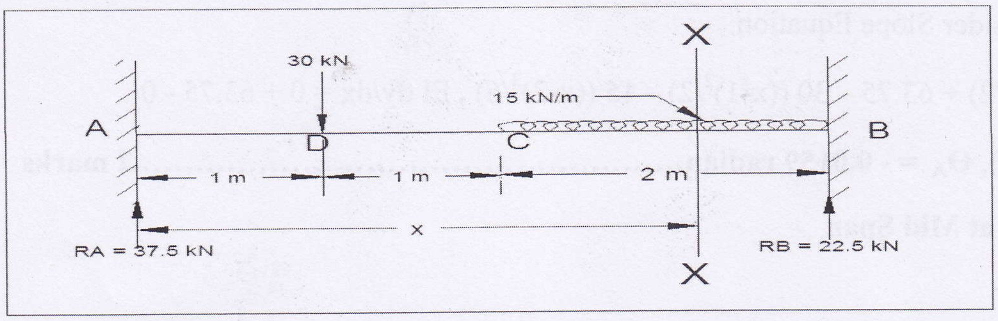
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DA
D.f.	–	0.36	0.64	0.64	0.36	–
FEM	0	0	-90	+90	0	0
Distribution		32.4	57.6	-57.6	-32.40	
carryover			-28.8	28.8		
Distribution		10.368	18.432	-18.432	-10.368	
carryover			-9.216	9.216		
Distribution		3.317	5.898	-5.898	-3.317	
carryover			-2.949	+2.949		
Distribution		1.061	1.887	-1.887	-1.061	
carry over			-0.943	0.943		
Distribution		0.34	0.603	-0.603	-0.34	
final moment		47.48	-47.48	47.48	-47.48	

3M

Q.3 Attempt any TWO of the following

(a) Given Data :-

A simply supported beam as shown in following figure



The constant EI 4000 kN/m^2

To Find

1) Slope at A, 2) Deflection at Mid Span

Support reactions :

Taking Moment @ A Determine the support reaction at B

RB x 4 = 30 x 1 + (15 x 2)x (2+2/2), RB = ((30 + 60)/4), **RB = 22.5 kN**

Substituting the above values in following equation

RA + RB = (30 + (15x2), RA = (30 + (15x2))-22.5, **RA = 37.5 kN.**

Consider a section XX at a distance x from support A in portion CB as shown in figure

EI (d_2y/dx^2) = Mx = 37.5 x-30x (x-1) –15((x-2)^2 /2)…………………Eqn 1

Integrating w. r. to x

EI dy/dx = 37.5 (x^2/2) + C1 – 30 ((x-1)^2/2) – 15 ((x-2)^3/6) ………… ..Eqn 2

Integrating again w. r. to x

EI y = 37.5 (x^3/6) + C1x + C2 – 30 ((x-1)^3/6) – 15 ((x-2)^4/24)……….Eqn 3**………2 marks**

Where C1 and C2 are integrating constant

To find C2, At A x=0, y=0

Substituting the above values in equation **3**

0= 0+0+ C2, **C2 = 0**

To find C1 , At B x=4, y=0,

Substituting the above values in equation **3**

0 = 37.5 (4^3/6) +4C1 + 0 – 30 ((4-1)^3/6) – 15 ((4-2)^4/24), **C1 = -63.75……………..1 mark**

Substituting the C1 and C2 in equation **2 and 3**

EI dy/dx = 37.5 (x^2/2) + 63.75 – 30 ((x-1)^2/2) – 15 ((x-2)^3/6) ………………Slope Equation

EI y = 37.5 (x^3/6) + 63.75 x – 30 ((x-1)^3/6) – 15 ((x-2)^4/24)…Deflection Equation……1 mark

To find Slope at A

At A x= 0 and consider Slope Equation

EI dy/dx = 37.5 (x^2/2) + 63.75 – 30 ((x-1)^2/2) – 15 ((x-2)^3/6) , EI dy/dx = 0 + 63.75 - 0

Θ_A = - (63.75/4000), **Θ_A = - 0.0159 radian……………………………………………….2 marks**

To find Deflection at Mid Span

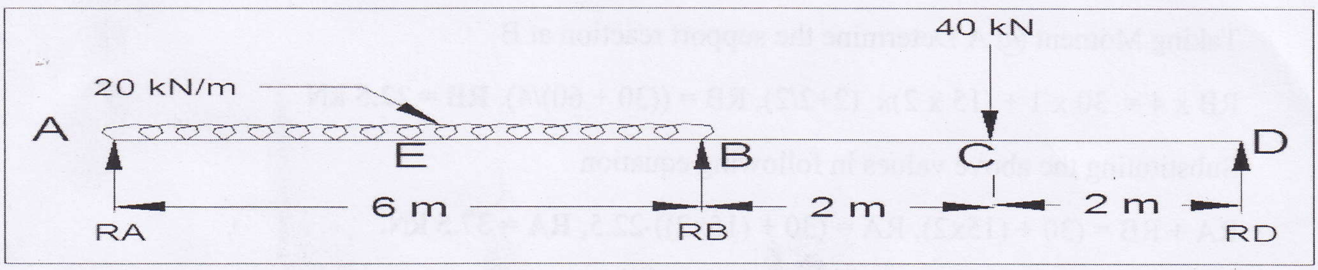
At C , X= 2

Consider deflection equation

EI y_c = 37.5 (2^3/6) + 63.75 x 2 – 30 ((2-1)^3/6) – 15 ((2-2)^4/24), EI y_c = 50 + 127.50 -5

y_c = (82.50/EI) = (82.50/4000) = -0.0206 m, **y_c = -2.06 mm…………………………2 marks**

(b)



Given Data :

Span AB = L1 = 6 m , Span BD = L2 = 4 m

To find :- M_B

Step -1 Assume the span AB and BD are as simply support and draw free Bending moment diagram

m_E = B. M. at mid Span of AB = (wL1^2/8) = (20x6^2/8) = **90 kN.m**

m_c = B. M. at mid Span of AB = (wL2/4) = (40x4/4) = **40 kN.m**

$m_{A=}$ $m_{B=}$ m_D = 0……………………………………………………………………….1 mark

Step- 2 Determine (6 $a\bar{x}$/L)

a1= 2/3 x 6x 90 = **360**, a1= 1/2 x 4x 40 = **80**

x1= L1/2 = 6/2 = **3 m** , x2= L2/2 = 4/2 = **2 m**

(6 a1$\bar{x}$1/L1) = (6 x 360x3/6) = **1080**, (6 a2$\bar{x}$2/L2) = (6 x 80x2/4) = **240…………1 marks**

Step-3 Determine the Fixed End Moment and draw fixed end moment diagram

As M_A = M_D = 0, To find M_B applying clypeyorns theorem to spans AB and BD

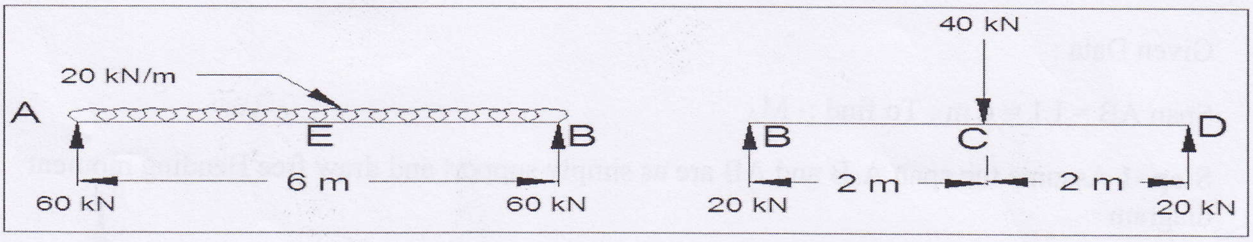
M_A x L1 +2M_B (L1+ L2)+M_D xL2 =- [(6 a1$\bar{x}$1/L1)+ (6 a2$\bar{x}$2/L2)]

0 +2M_B (6+ 4) +0 =- [1080+240], **M_B = -66 kN.m, M_B = 66kN.m (Hogging)………2 marks**

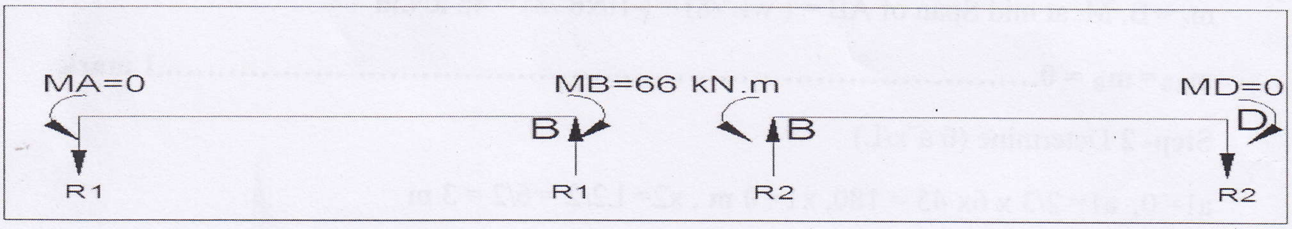
Step-4 Super impose the Fixed End moment diagram over free bending moment diagram and draw the final Bending Moment Diagram.

Step -5 Determine the support reactions

i) Reactions of Simply Supported beams



ii) Reaction Due to difference of fixed End Moments



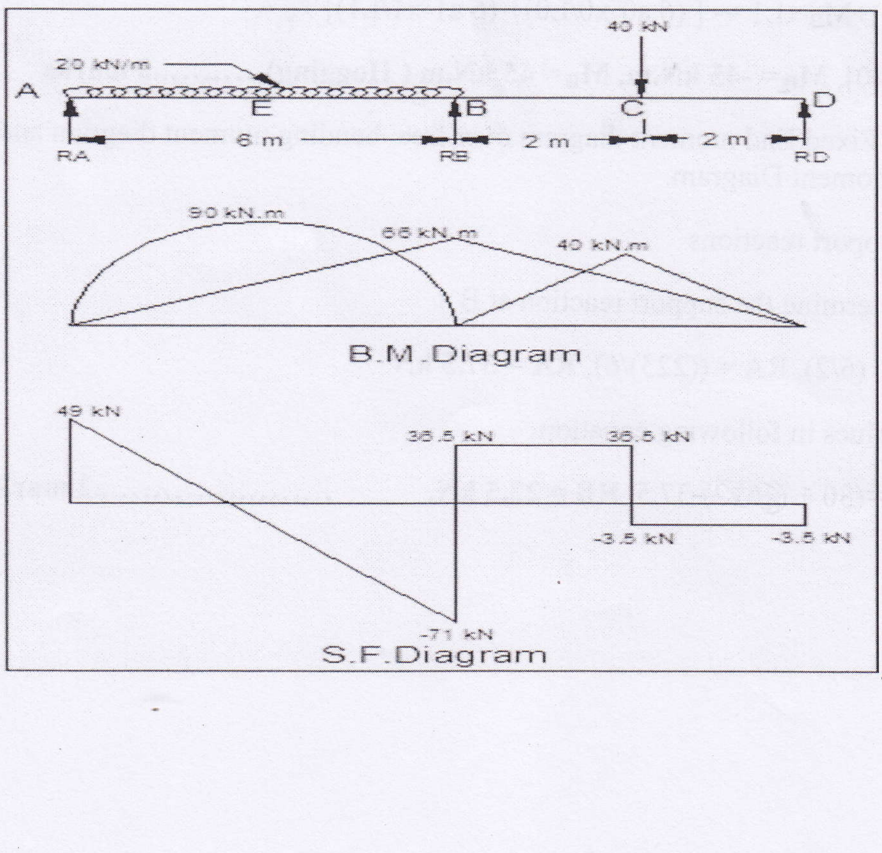
R1 = (66-0)/6 = 11 kN, R2 = (66-0)/4 = 16.5 kN

iii) Support Reactions

R_A = 60-R1 = 60-11 = **49 kN**

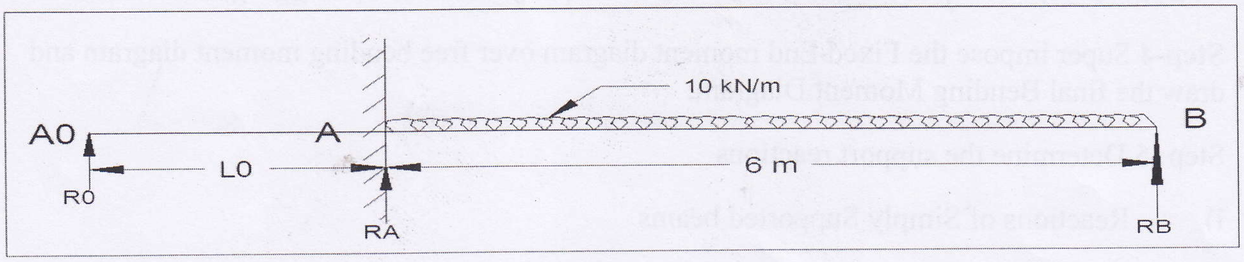
R_B = 60+20+R1+R2 = 60+20+11+16.5 =**107.50kN**

R_D = 20-R2 = 20-16.5 = **3.5 kN…………………………………………………………2 marks**



…………………2 marks

(c)



Given Data :

Span AB = L1 = 6 m , To find :- M_A

Step -1 Assume the span A_0B and AB are as simply support and draw free Bending moment diagram

M_0 = B. M. at mid Span of A_0B = **0 kN.m**

m_c = B. M. at mid Span of AB = ($wL^2/8$) = ($10x6^2/8$) = **45 kN.m**

$m_{A0} = m_B = 0$…………………………………………………………………………..1 mark

Step- 2 Determine ($6\,a\bar{x}/L$)

a1= 0, a1= 2/3 x 6x 45 = 180, x1= **0 m** , x2= L2/2 = 6/2 = **3 m**

($6\,a0\bar{x}0/L0$) = **0**, ($6\,a1\bar{x}1/L1$) = (6 x 180x3/6) = **540……………………………..1 mark**

Step-3 Determine the Fixed End Moment and draw fixed end moment diagram

As $M_{A0} = M_B = 0$

To find M_A applying three moment theorem to sapns A_0B and AB

M_{A0} x L0 +2M_A (L0+ L1)+M_B xL1 =- [($6\,a0\bar{x}0/L0$)+ ($6\,a1\bar{x}1/L1$)]

0 +2M_A (0+ 6) +0 = - [540], **M_A = -45 kN.m, M_B = 45 kN.m (Hogging)……….2 marks**

Step-4 Super impose the Fixed End moment diagram over free bending moment diagram and draw the final Bending Moment Diagram.

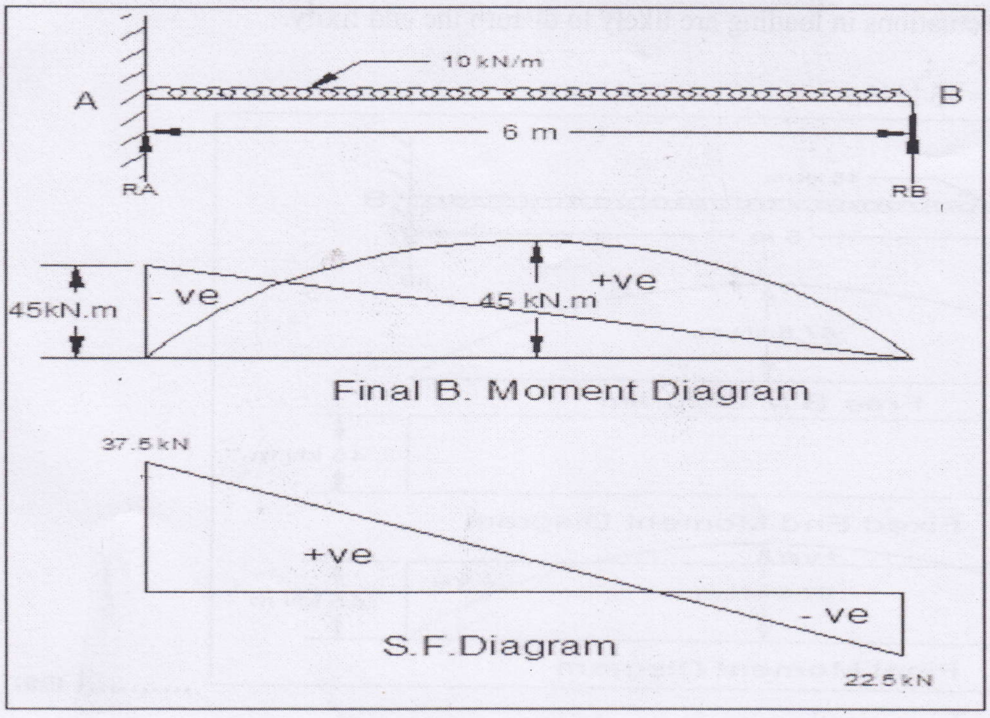
Step -5 Determine the support reactions

Taking Moment @ A Determine the support reaction at B

RA x 6 = 45 + (10 x 6)x (6/2), RA = ((225)/6), **RA = 37.5 kN**

Substituting the above values in following equation

RA + RB = (10x6), RB = 60 – 37.5, **RB = 22.5 kN.** **…………………..2 marks**



........2 marks

Q.4 (A) Attempt any three of the following

(a) Given Data:

D = 500 mm, d = 300 mm, P = 200 kN = 200x 10^3 N, e= 60 mm

To find σ max and σ min

A = (π/4($D^2 - d^2$) = (π/4($500^2 - 300^2$) = **106.350 x 10^3 mm^2**

Z= (π/32(($D^4 - d^4$)/D) =(π/32(($500^4 - 300^4$)/500) = **10.681x10^6 mm^3.....................1 mark**

σ0= P/A = 200x 10^3 /106.350 x 10^3 = **1.88 N/ mm^2**

σb = (Pxe/Z) = (200x 10^3 x60/10.681x10^6) = **1.123 N/ mm^2.......................... .....1 mark**

σ max = σ0+ σb = 1.88+1.123 = 3.003 N/ mm^2 (Compressive)

σ min = σ0- σb = 1.88-1.123 = 0.757 N/ mm^2 (Compressive)............................2 mraks

(b) The following are the advantages and Disadvantage of Fixed Beam over Simply Supported beam.

Advantages: (Any Two Two marks)

1) End slopes of a fixed beam are Zero.
2) A fixed beam is more Stiff, strong and stable than simply supported beam.
3) For the same span and loading , a fixed beam has lesser values of bending moments as compared to simply supported beam.
4) For the same span and loading , a fixed beam has lesser values of deflection as compared to simply supported beam.

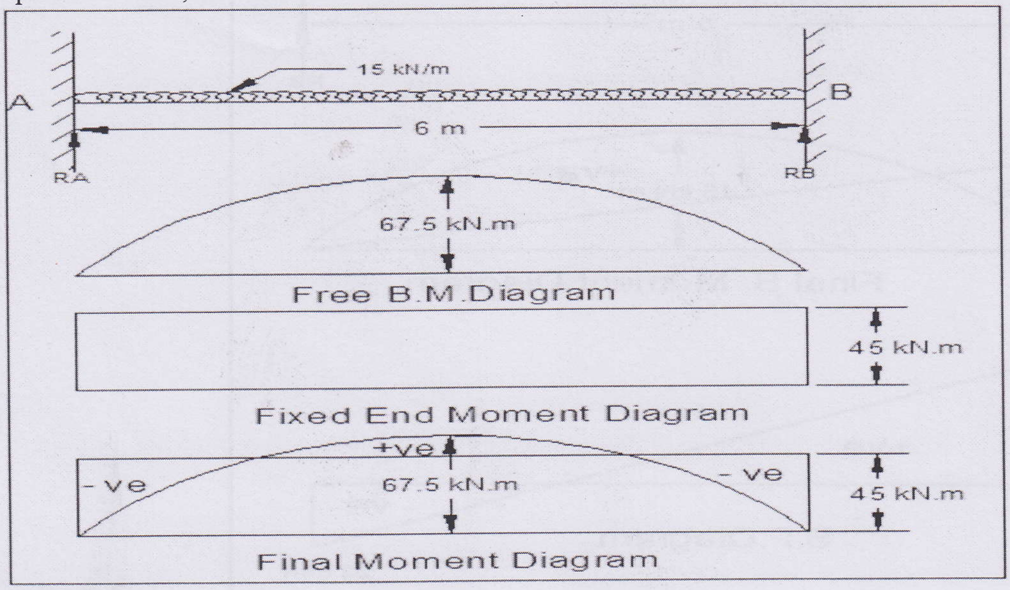
Disadvantages: (Any Two Two marks)

1) A little sinking of one support over the other induces additional moment at each end.
2) Extra care has to be taken to achieve correct fixity at the ends.
3) Due to end fixity, temperature stresses are induced due to variation in temperature.

4) Frequent fluctuations in loading are likely to disturb the end fixity.

(c) Given data:-

Sapn AB = 6 m, w = 15 kN/m



..........**1 mark**

Step – 1 To find the beam reactions

Due to Symmetry

RA = RB =(WL/2) = ((15x6)/2) = 45 kN

Step -2 To draw Free Bending moment Diagram

Free B.M. at A and B , $m_A = m_B = 0$

Free B.M. at C = $m_C = (wL^2/8) = (15x6^2/8) = 67.5$ kN.m.....................................**1 mark**

Draw parabola having maximum at center 67.5 kN.m

Step – 3 To Draw Fixed End Moment Diagram

Since MA and MB are equal Fixed End Moment diagram is Rectangle as Shown In fig.

Step – 4 To find fixed End moment digram

Using First Principle

Area of free B.M. diagram = Area of Fixed end Moment diagram

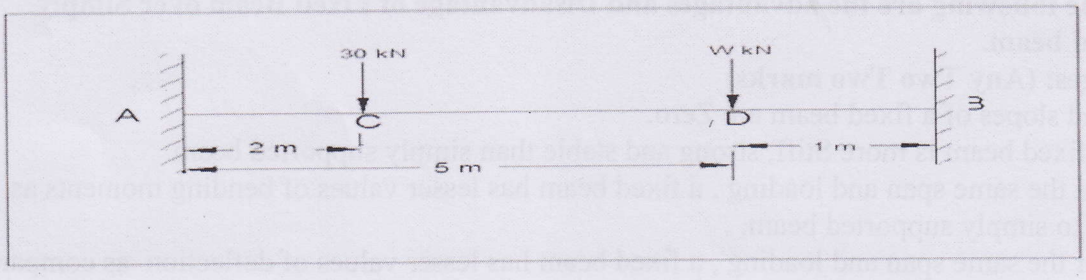
a = a'

(2/3x6x67.5) = - MA x 6

MA = - 45 kN.m...2 marks

(d) Given data:-

Sapn AB = 6 m, W1 = 30 kN , W2 = W



To find W

Step – 1 Determine the fixed end moment by standard formulae

$MA = ((W1\ a1\ b1^2)/L^2) + ((W2\ a2\ b2^2)/L^2)$

$MA = ((30x2x4^2)/6^2) + ((Wx5x1^2)/6^2)$

MA = 26.67 + 0.139W………………………………Eqn 1

MB = ((W1 $a1^2$ b1)/L^2) + ((W2 $a2^2$ b2)/L^2)

MB = ((30x2^2x4)/6^2) + ((Wx5^2x1)/6^2)

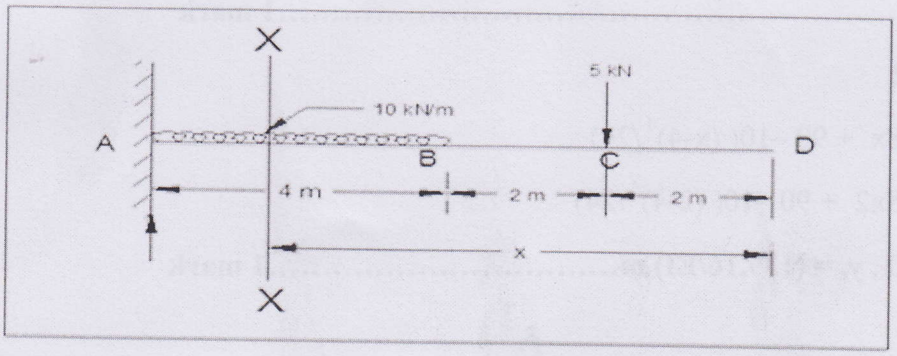
MB = 13.33 + 0.694W………………………………Eqn 2…………………………2 marks

Step – 2 To find W

As Ma and MB are equal
Equating equation 1 and 2
26.67 + 0.139W = 13.33 + 0.694W
26.67 – 13.33 = 0.694W – 0.139W
13.34 = 0.555W
W = 24.04 kN……………………………………………………………………2 marks

Q.4 (B) Attempt any One of the following

(a) Given Data:



To Find:Θ_C, y_C

Take the free end as Origin and consider asection XX at a distance x from D in the portion AB as shown in fig.

EI (d_2y/dx^2) = Mx = - 5 (x-2) –10($(x-4)^2$ /2)……………………………eqn 1

Integrating w. r. to x

EI dy/dx = - 5 $(x-2)^2/2$ + C1 –10($(x-4)^3$ /6)……………………………eqn 2

Integrating again w. r. to x

EI y = - 5 $(x-2)^3/6$ + C1x + C2 –10($(x-4)^4$ /24)……………………………eqn 3………2 marks

Where C1 and C2 are integrating constant

To find C1

At A x=8, dy/dx = 0

Substituting the above values in equation **2**

EI dy/dx = - 5 $(x-2)^2/2$ + C1 –10($(x-4)^3$ /6)

$0 = -5(8-2)^2/2 + C1 - 10((8-4)^3/6)$, **C1 = 24.58**

To find C2

At A x=8, y=0, Substituting the above values in equation 3

$EI\,y = -5(x-2)^3/2 + C1x + C2 - 10((x-4)^4/24)$

$0 = -5(8-2)^3/6 + 24.58x + C2 - 10((8-4)^4/24)$, **C2 = 90……………………………2 marks**

Substituting the C1 and C2 values in equation 2 and 3

$EI\,dy/dx = -5(x-2)^2/2 + 24.58 - 10((x-4)^3/6)$……………………Slope Equation

$EI\,y = -5(x-2)^3/6 + 24.58x + 90 - 10((x-4)^4/24)$……………………Deflection Equation

Slope at C : $\boldsymbol{\Theta_C}$

x = 2

$EI\,(dy/dx)_C = -5(x-2)^2/2 + 24.58 - 10((x-4)^3/6)$

$EI\,(dy/dx)_C = -5(2-2)^2/2 + 24.58 - 10((2-4)^3/6)$, $EI\,\Theta_C = 0 + 24.58$,

$\boldsymbol{\Theta_C}$ **= (24.58/EI) radian…………………………………………………1 mark**

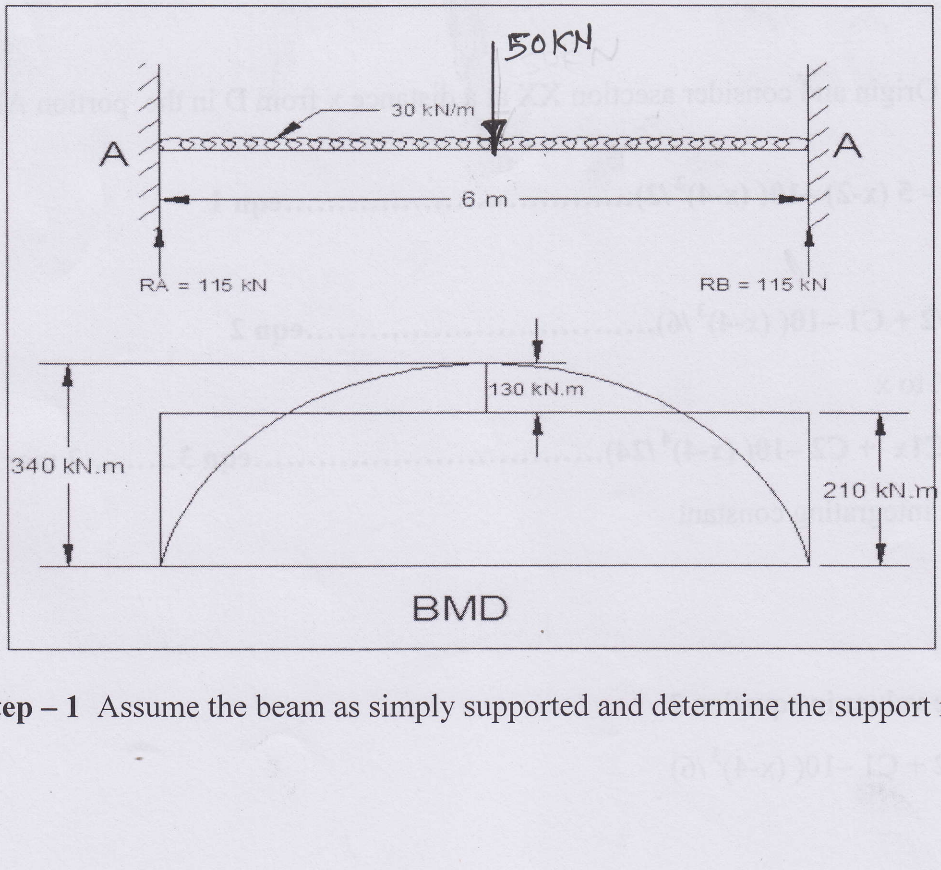
Deflection at C : $\boldsymbol{y_C}$**, x = 2**

$EI\,y_c = -5(x-2)^3/6 + 24.58x + 90 - 10((x-4)^4/24)$

$EI\,y_c = -5(2-2)^3/6 + 24.58x2 + 90 - 10((2-4)^4/24)$

$EI\,y_c = -0 + 24.58x2 + 90$, $\boldsymbol{y_c}$ **= (139.16/EI) m……………………………1 mark**

(b) Given Data:



………..2 marks

Step – 1 Assume the beam as simply supported and determine the support reaction

L= 8 m , W = 50 kN , w = 30 kN/m

VA = VB = (W/2) +(wL/2) = (50/2) +(30x6 /2) = **115 kN**

Due to symmetry ,VA = VB = 115 kN ....1 mark

Step – 2 Determine free Bending moment and Draw Diagram

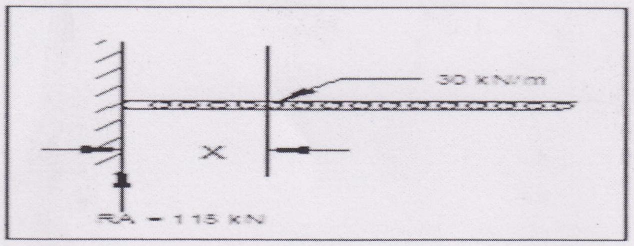
$m_A = m_B = 0$, $m_C = (WL/4) + (wL^2/8) = (50x6/4) + (30x6^2/8) = 340$ kN.m

Step – 3 Determine fixed end moment and Draw Diagram

Due to symmetry ,$M_A = M_B = -(WL/8) + (wL^2/12) = (50x6/8) + (30x6^2/12) = 210$ kN.m

$M_A = M_B$ = 210 kN.m...2 marks

Step – 4 Point of Contra flexure



B.M. at any Section XX in portion AC at a distance x from A is given by

$M_X = - M_A + 115\ x – 30\ x^2/2$, $M_X = - 210 + 115\ x – 15\ x^2$

At point of contra flexure, B.M. = 0

Equating Mx to zero, $-210 + 115x – 15\ x^2 = 0$, **x = 4.67 m** **..............................1 mark**

5a Given data

$b = 300$ mm

$d = 200$ mm

$P = 100$ kN $= 100 \times 10^3$ N

$e = 40$ mm

$A = b \times d = 300 \times 200$

$A = 60 \times 10^3$ mm^2

Direct stress $\sigma_0 = \frac{P}{A} = \frac{100 \times 10^3}{60 \times 10^3}$

$= 1.667$ N/mm^2 [comp.] 1 M

Bending stress $\sigma_b = \frac{M}{Z_{yy}} = \frac{P \times e}{\frac{db^2}{6}} = \frac{6 Pe}{db^2}$

$= \frac{6 \times 100 \times 10^3 \times 40}{200 \times 300^2}$

$= \frac{24 \times 10^6}{18 \times 10^6}$

$\boxed{\sigma_b = 1.333 \text{ N/mm}^2}$ 1 M

$\sigma_{max} = \sigma_0 + \sigma_b$

$= 1.667 + 1.333 = 3$ N/mm^2

$\boxed{\sigma_{max} = 3 \text{ N/mm}^2}$ ... Ans 1 M

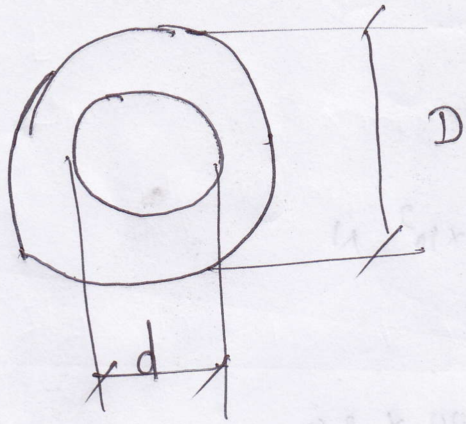
$\sigma_{min} = \sigma_0 - \sigma_b$

$= 1.667 - 1.333$

$= 0.334$ N/mm^2

$\boxed{\sigma_{min} = 0.334 \text{ N/mm}^2}$ ... Ans 1 M

5b



$$A = \frac{\pi}{4}\left[D^2 - d^2\right]$$

1) Direct stress $(\sigma_0) = \frac{P}{A}$

$$= \frac{P}{\pi/4\left[D^2 - d^2\right]}$$ 1 M

2) Bending stress (σ_b)

$$= \frac{M \times y}{I}$$

$$= \frac{(P \times e) \times \frac{D}{2}}{\frac{\pi}{64}(D^4 - d^4)}$$ 1 M

for no tension

$\sigma_0 = \sigma_b$

$$\frac{\cancel{P}}{\frac{\pi}{4}(D^2-d^2)} = \frac{\cancel{P}\times e \times \frac{D}{2}}{\frac{\pi}{64}(D^4-d^4)}$$

$$\frac{\frac{\pi}{64}(D^4-d^4)}{\frac{\pi}{4}(D^2-d^2)} = e\times\frac{D}{2}$$ 1 M

$$\frac{\frac{\cancel{\pi}}{64}\cancel{(D^2-d^2)}(D^2+d^2)}{\cancel{\pi}/4\ \cancel{(D^2-d^2)}} = e\times\frac{D}{2}$$

$$\boxed{\frac{(D^2+d^2)}{8D} = e}$$

$$\therefore\ e \le \left(\frac{D^2+d^2}{8D}\right)$$ 1 M

5C Given

$L = 5\ m = 5000\ mm$

$EI = 4 \times 10^{13}\ N\,mm^2$

$w = 10\ kN/m = 10\ N/mm$ 1 M

step I) for max. slope

$$\theta_A = \theta_B = \frac{wL^3}{24EI} \text{ radian}$$

$$= \frac{10 \times (5000)^3}{24 \times 4 \times 10^{13}}$$

$$= 0.0013 \text{ radians}$$ 1 M

$\theta_A = \theta_B =$ maximum slope at support

$= 0.0013$ radian ... Ans

step II] For max. deflection

$$y_{max} = \frac{-5\ wL^4}{384\,EI} \quad \text{... (At center)}$$

$$= \frac{-5 \times 10 \times (5000)^4}{384 \times 4 \times 10^{13}}$$ 1 M

$$= \frac{-3.125 \times 10^{16}}{1.536 \times 10^{16}}$$

$$= -2.034\ mm$$

$$\boxed{y\,max = -2.034\ mm}$$ ... Ans 1 M

$-ve$ sign indicates downward deflection

5d

$$EI \frac{d^2y}{dx^2} = M$$ 1 M

differentiate w.r.t x

$$EI\left(\frac{dy}{dx}\right) = Mx + c_1 \quad \ldots \text{[slope eqn]}$$ 1 M

differentiate w.r.t x

$$EI\, y = \frac{Mx^2}{2} + c_1 x + c_2$$ 1 M

... [deflection eqn]

where.

c_1 = slope constant

c_2 = Deflection constant

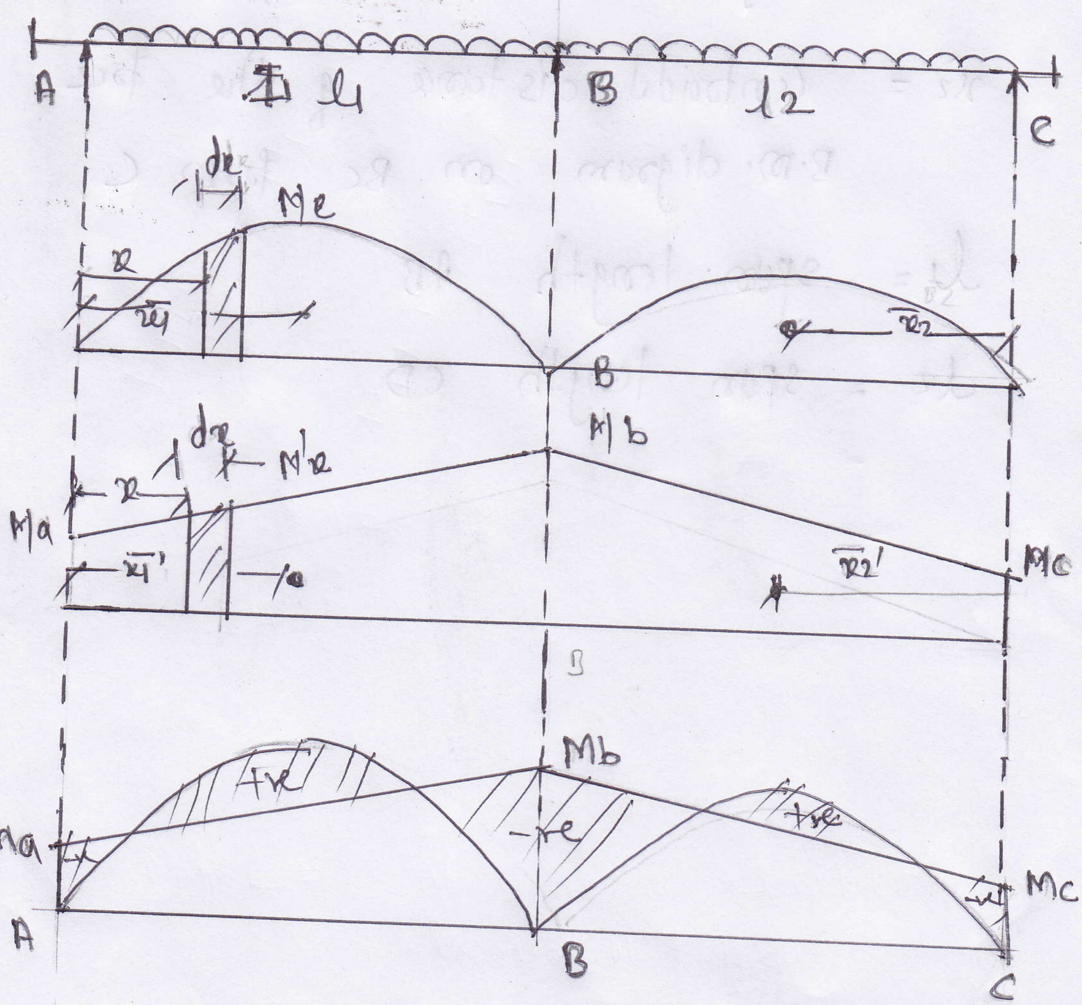
M = Bending moment

E = Modulus of elasticity of beam material 1 1m

I = M.I @ Neutral axis.

se Clapeyron's Theorem of three moment →

If AB and BC are any two consecutive span of a continuous beam subjected to an external loading, the support moment M_a, M_b and M_c at the support A, B and C are given by the relation. 1 M



$$M_a l_1 + 2M_b(l_1 + l_2) + M_c l_2$$

$$= \frac{6 a_1 \bar{x}_1}{l_1} + \frac{6 a_2 \bar{x}_2}{l_2}$$ 1 M

Where

a_1 = area of free B.M. digram for the AB

a_2 = Area of free B.M. digram for the span BC

$\bar{x_1}$ = Centroidal distance of the free B.M. digram on AB from A

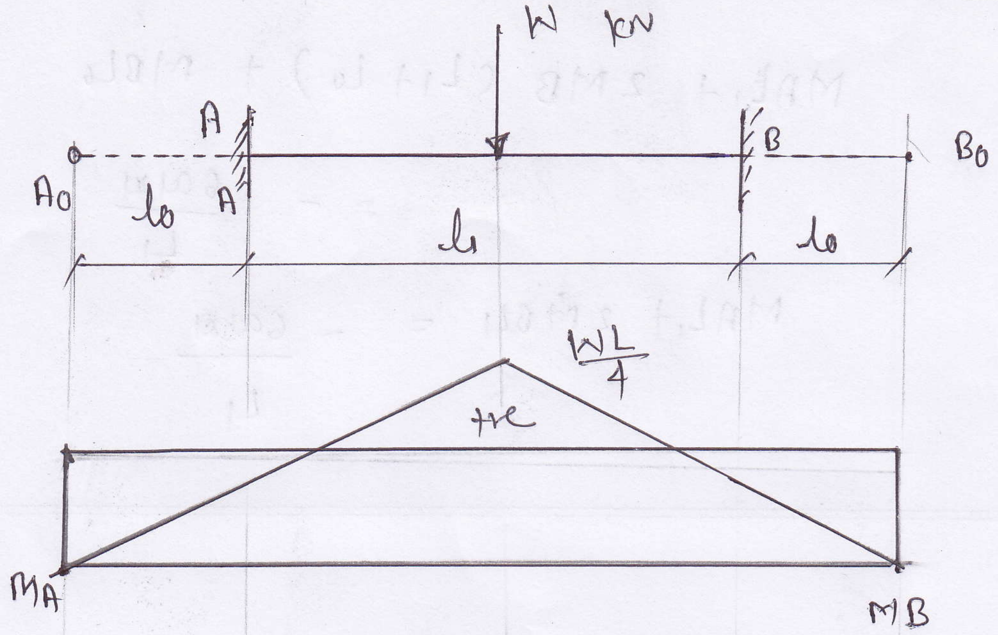
x_2 = Centroidal distance of the free B.M. digram on BC from C 1

M

l_1 = span length AB

l_2 = span length CD

6a



1 M

step (1) consider imaginary span $A_0 A$ to the left of support A

step (2) Assume span AB simply supported find max. B.M and draw BMD 1 M

step (3) Apply clapeyron's theorm for span A_0 A and AB we will get eqn (1)

$$M_0 L_0 + 2M_A(l_0 + L_1) + M_C l_2 = -\left[\frac{6 a_0 x_0}{l_0} + \frac{6 a_1 \bar{x}_1}{L_1}\right]$$

$$2 M_A L + M_C l_2 = -\frac{6 a_1 \bar{x}_1}{L_1}$$ 1 M

Step 4) consider imaginary span to the right of support B

∴ Apply clapeyron's theorm for span AB and BB_0

$$M_A L_1 + 2M_B (L_1 + L_0) + M_B L_0$$

$$= -\frac{6 a_1 \bar{x}_1}{L_1}$$

$$M_A L_1 + 2M_B L_1 = -\frac{6 a_1 \bar{x}_1}{L_1}$$

1 M

6b Define stiffness factor and distribution factor w.r.t. moment distribution method

Stiffness factor :→

The moment required at the simply supported end of a beam so as to produce unit rotation at the end without translation of the either end is called as stiffness factor. 2 M
Note that $\theta = 1$ radian. stiffness factor is generally denoted by "k"

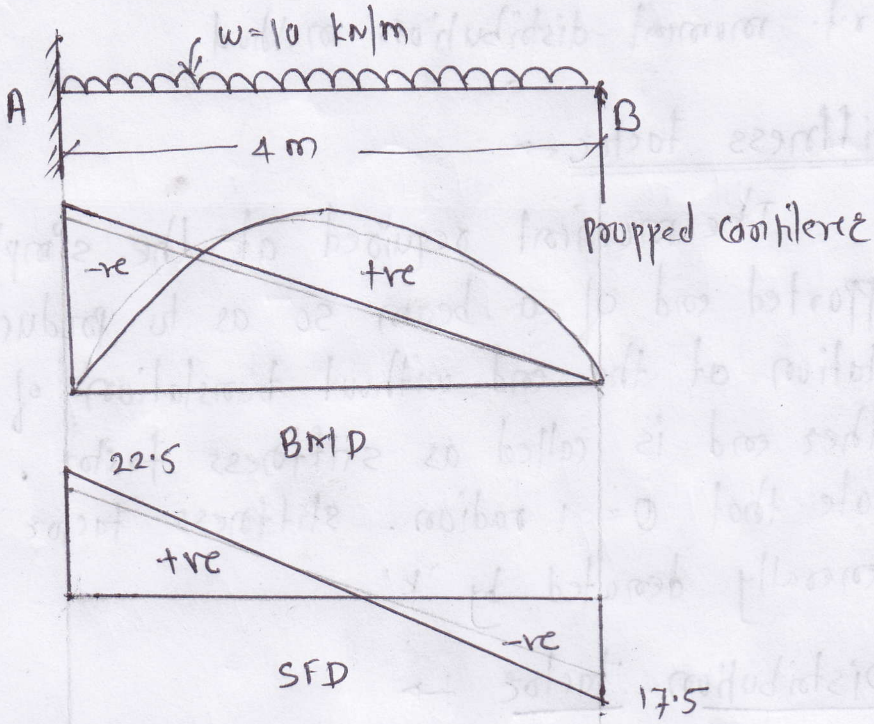
Distribution factor :→

The distribution factor for a member at a joint is the ratio of stiffness factor for that member and total stiffness of all the member meeting at a joint 2 M.

6C A propped cantilever AB of span 4m AB is fixed at A and propped at B, carrying an udl of 10 KN/m. Using moment distribution method, Calculate propped reaction and fixed end moment

Given L = span AB = 4 m

w = 10 KN/m



Step I] Find fixed end moment (Assume span AB fixed at both ends)

$$M_{AB} = \frac{-wL^2}{12} = -\frac{10 \times 4^2}{12} = -13.33 \text{ kN·m}$$

$$M_{BA} = \frac{+wL^2}{12} = \frac{10 \times 4^2}{12} = +13.33 \text{ kN·m}$$

1 M

Step II] Moment distribution table

Joint	A	B
Member	AB	BA
Fixed end moment	-13.33	+13.33
Release B & carry over to A	-6.67	-13.33
Final moment	-20	0

1 M

MA = Fixed end moment at A

= 20 kN·m (Hogging)

MB = Support moment at B = 0

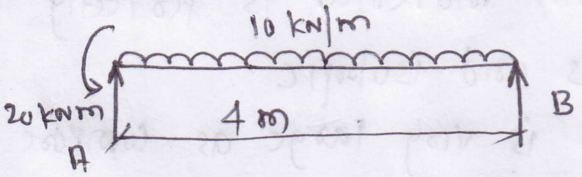
Step 3] To find out free bending moment
[Assume beam is simply supported] 1 M

MA = 0 ; MB = 0

$$M_{max} = \frac{wL^2}{8} = \frac{10 \times 4^2}{8} = 20 \text{ kNm}$$

[At centre]

Step 4] To find out support reaction



RA + RB = 10×4

$\boxed{RA + RB = 40 \text{ kN}}$ - - - (1)

Σ MA = 0

i.e. clockwise moment = Anticlockwise moment

10×4×2 = 10 + RB×4

80 = 10 + 4 RB

$$\therefore RB = \frac{80 - 10}{4} = 17.5 \text{ kN}$$

put in eqn (1)

RA + 17.5 = 40 1 M

∴ RA = 40 - 17.5

$\boxed{\therefore RA = 22.5 \text{ kN}}$

6d state and explain briefly limitations of Euler's theory for long columns

Limitation of Euler's formula

The validity of Euler's formula is subjected to the satisfaction of the assumptions mentioned below

i) The column is initially perfectly straight and its axially loaded

ii) The section of the column is uniform

iii) The column material is perfectly elastic, homogeneous and isotopic 1 M

iv) The length is very large as compare to the lateral dimensions

v) The self weigth of the column is ignorable

vi) The column will fail by buckling alone.

Further accepting the formula for crippling load

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E A k^2}{L^2} \quad \because K = \sqrt{\frac{I}{A}}$$

$$\because I = k^2 A$$ 1 M

Where K = radius of Gyration

P = buckling load at failure

E = modulus of elasticity

I = moment of inertia

L = effective length.

The stress at failure $= \frac{P}{A}$

$$= \frac{\pi^2 E}{\left(\frac{L}{k}\right)^2}$$

from the above it can be realized that the stress at failure $\left(\frac{P}{A}\right)$ according to the formula will be high when the slenderness ratio is small.

But the ~~stre~~ stress at failure cannot be greater than the crushing stress for the column material. Hence when the slenderness ratio is less than the certain limit, Euler's formula gives the value of crippling load even greater than the crushing load 1 M

~~Cons~~ for Example consider mild steel ($\sigma_c = 300$)

$$\left(\frac{L}{k}\right)^2 = \frac{\pi^2 E}{\sigma_c} = \frac{\pi^2 \times 2\times10^5}{300}$$

$$= 6579.74$$

$$\therefore \frac{L}{k} = 81.1 \cong 80$$ 1 M

Hence slenderness ratio is less than this limits for mild steel. Euler's formula will not be valid.

6 e Define effective length of column, radius of gyration, slenderness ratio and short column

Effective length of column →

The length of column which defects or bends as if it is hinged at its ends is called the effective length or equivalent length of column. It is denoted by 'L' 1 m

Effective length of column depends upon the end condition of the column.

Radius of gyration:

It is defined as the distance at which the area may be supposed to be concentrated to give same moment of inertia 1 m

$I = A k^2$

$\therefore \quad k = \sqrt{\frac{I}{A}}$ Where k = radius of gyration

I = moment of inertia

A = Area

Slenderness ratio (λ)

The ratio of effective length to minimum radius of gyration is called slenderness ratio. It is denoted by "λ" 1 m

$$\lambda = \frac{l_e}{k}$$

λ = slenderness ratio
l_e = effective length
k = radius of gyration

short column

A column in which failure occures because of crushing only i.e direct compressive stress is termed as short column 1 M

for short column slendreness ratio $\lambda < 30$