SUMMER – 2016 EXAMINATION

MODEL ANSWER

Subject: BASIC MATHEMATICS

Subject Code: 17104

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.

2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)

4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.

6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.

7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		**Attempt any <u>TEN</u> of the following:**		**20**
	(a)	Find x if $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$		
	Ans.	$\therefore \begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$		
		$\therefore 1(18-12) - x(9-4) + x^2(3-2) = 0$	1	
		$\therefore 6 - 5x + x^2 = 0$	½	
		$\therefore (x-3)(x-2) = 0$	½	
		$\therefore x = 3$ or $x = 2$		**2**

	(b)	Find the value if a and b If $\begin{bmatrix} a-4b & 5 \end{bmatrix} = \begin{bmatrix} 11 & 5 \end{bmatrix}$		
	Ans.	$\begin{bmatrix} a-4b & 5 \end{bmatrix} = \begin{bmatrix} 11 & 5 \end{bmatrix}$		
		$a - 4b = 11$	1	
		$-a + b = -5$		
		$\therefore a = 3, b = -2$	1	**2**
		---		
	c)	If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix}$ verify that $A + B = B + A$		
	Ans.	$A + B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$	1	
		$B + A = \begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$	1	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	d)	If $A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$, find whether matrix A is singular or non-singular.		
	Ans.	$A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$		
		$\lvert A \rvert = \begin{vmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{vmatrix}$		
		$= 7(6-30) - 0(3-24) + 2(5-8) = -174 \neq 0$		
		$\therefore$ Matrix A is non singular	1	
		---	1	**2**
	e)	Resolve into the partial fraction $1 + \frac{1}{x-1}$		
	Ans.	Let $\frac{1}{\quad} = \frac{1}{\quad}$		
		$\frac{1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$		
		$\therefore \quad 1 = A(x+1) + B(x-1)$		
		put $x = 1$	½	
		$\therefore 1 = A(1+1)$		
		$\therefore A = \frac{1}{2}$	½	
		put $x = -1$		
		$\therefore 1 = B(-1-1)$		
		$\therefore B = -\frac{1}{2}$	½	
		$\therefore 1 + \frac{1}{\quad_2} = 1 + \frac{1}{\quad} = 1 + \frac{\frac{1}{2}}{} + \frac{-\frac{1}{2}}{}$	½	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.				
	f)	$\sin^2 120^0 + \cos^2 120^0 + \tan^2 120^0$		
	Ans.	$\sin^2 120^0 + \cos^2 120^0 + \tan^2 120^0$		
		$\left(\frac{\sqrt{3}}{2}\right)^2 + (-\frac{1}{2})^2 + (-\sqrt{3})^2$	1	
		$= -\frac{9}{4}$ or -2.25	1	**2**
	g)	Prove that $\sin 2A = 2\sin A \cos A$		
	Ans.	L.H.S $= \sin 2A = \sin(A + A)$	½	
		$= \sin A \cos A + \cos A \sin A$	1	
		$= 2\sin A \cos A$ = R.H.S	½	**2**
	h)	$\tan A = \frac{1}{2}$ $\tan B = \frac{1}{3}$		
	Ans.	$\frac{\tan A + \tan B}{1 - \tan A \tan B}$	1	
		$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = 1$	1	**2**
	i)	Evaluate without using calculator $\frac{\tan 32^0 + \tan 88^0}{1 - \tan 32^0 \cdot \tan 88^0}$		
	Ans.	$\frac{\tan 32^0 + \tan 88^0}{1 - \tan 32^0 \cdot \tan 88^0} = \tan(32^0 + 88^0)$	1	
		$= \tan 120^0$	½	
		$= \tan(90^0 + 30^0)$	½	
		$= -\cot 30^0$		**2**
		$= -\sqrt{3}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	j)	Find the principal value of tan $(\sqrt{3})$		
	Ans.	$\tan(\sqrt{3}) = \theta$	½	
		$\sqrt{3} = \tan\theta$	½	
		since $\tan 60^0 = \sqrt{3}$	½	
		$\therefore \theta = 60^0$	½	**2**
		---		
	k)	Find the angle btween the lines $3x + 2y = 6$ and $2x - 3y = 5$		
	Ans.	$m = \frac{-3}{2}$ $m = \frac{2}{3}$		
		$\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1 m_2}\right\rvert$	1	
		$= \left\lvert\frac{\frac{-3}{2} - \frac{2}{3}}{1 + \frac{-3}{2}\times\frac{2}{3}}\right\rvert = \infty$	½	
		$\theta = \tan^{-1}\infty = 90^0 = \frac{\pi}{2}$	½	**2**
		---		
	l)	Find the range from the following data:		
	Ans	800, 725, 750, 900, 925,910, 1000, 790, 870, 920		
		Range = Largest Value – Smallest value		
		= 1000 – 725	1	
		= 275	1	**2**

2.		**Attempt any FOUR of the following:**		**16**
	a)	Solve the following equations by Cramer's rule:		
		$x + y = 3, y + z = 5, z + x = 4$		
	Ans	$D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1) + 0(0-1) = 2$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$D_x = \begin{vmatrix} 3 & 1 & 0 \\ 5 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 3(1-0) - 1(5-4) + 0 = 2$	1	
		$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(5-4) - 3(0-1) + 0 = 4$	1	
		$D = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0) - 1(0-5) + 3(0-1) = 6$	1	
		$x = \frac{D_x}{D} = \frac{2}{2} = 1$ $\frac{D_y}{D} \quad \frac{4}{2}$ $x = \frac{D_z}{D} = \frac{6}{2} = 3$		**4**
		---		
	b)	$\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $A^T \quad B^T \quad (AB)^T$ A is 2×2, B is 2×3.		
	Ans	$AB = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 6-3 & -2 & 4-3 \end{bmatrix}$	1	
		$AB = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$ $\therefore (AB)^T = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$	1	
		$B^T \quad \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \quad A^T \quad \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$	1	
		$B^T A^T = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ $B^T A^T = \begin{bmatrix} 6-3 & 3+5 \\ -2 & -1 \\ 4-3 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ -2 & -1 \\ 1 & 7 \end{bmatrix}$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	c)	Find inverse of matrix, $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$		
	Ans	Let $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ $\therefore \lvert A\rvert=\begin{vmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{vmatrix}$ $\lvert A\rvert=1(3+0)-2(-1-0)-2(2-0)$ $\lvert A\rvert=1\neq 0$ $\therefore A^{-1}$ exists	1	
		Matrix of minors $=\begin{bmatrix}\begin{vmatrix}3 & 0\\ -2 & 1\end{vmatrix} & \begin{vmatrix}-1 & 0\\ 0 & 1\end{vmatrix} & \begin{vmatrix}-1 & 3\\ 0 & -2\end{vmatrix}\\ \begin{vmatrix}2 & -2\\ -2 & 1\end{vmatrix} & \begin{vmatrix}1 & -2\\ 0 & 1\end{vmatrix} & \begin{vmatrix}1 & 2\\ 0 & -2\end{vmatrix}\\ \begin{vmatrix}2 & -2\\ 3 & 0\end{vmatrix} & \begin{vmatrix}1 & -2\\ -1 & 0\end{vmatrix} & \begin{vmatrix}1 & 2\\ -1 & 3\end{vmatrix}\end{bmatrix}$ $=\begin{bmatrix}3 & -1 & 2\\ -2 & 1 & -2\\ 6 & -2 & 5\end{bmatrix}$	1	
		Matrix of cofactors $=\begin{bmatrix}3 & 1 & 2\\ 2 & 1 & 2\\ 6 & 2 & 5\end{bmatrix}$	½	
		$\text{Adj.}A=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$ $A^{-1}=\frac{1}{\lvert A\rvert}\text{Adj.}A$ $A^{-1}=\frac{1}{1}\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$	½ 1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.	d)	$\left(\begin{bmatrix}3 & 1\\ 3 & -3\end{bmatrix}\begin{bmatrix}0 & 2\\ -5 & 4\end{bmatrix}\right)\begin{bmatrix}-1\\ 2\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}$		
	Ans	$\left(\begin{bmatrix}9 & 3\\ 9 & -9\end{bmatrix}\begin{bmatrix}0 & 4\\ -10 & 8\end{bmatrix}\right)\begin{bmatrix}-1\\ 2\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}$	1	
		$\begin{bmatrix}9 & -1\\ & \\ & \end{bmatrix}\begin{bmatrix}-1\\ 2\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix}$	1	
		$\begin{bmatrix}-9-2\\ -16-12\\ -19-34\end{bmatrix}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$	1	
		$\begin{bmatrix}-11\\ -28\\ -53\end{bmatrix}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$	1	**4**
		$x=-11, y=-28, z=-53$		
		---		
	e)	Resolve into partial fraction $\frac{x-5}{x^3+x^2-6x}$		
	Ans	$\frac{x-5}{x^3+x^2-6x}=\frac{x-5}{x(x^2+x-6)}=\frac{x-5}{x(x-2)(x+3)}$	½	
		$\frac{x-5}{x(x-2)(x+3)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+3}$		
		$x-5=(x-2)(x+3)A+x(x+3)B+x(x-2)C$		
		Put $x=0$	½	
		$-5=(-2)(3)A$		
		$\therefore A=\frac{5}{6}$	½	
		Put $x=2$		
		$-3=(2)(5)B$		

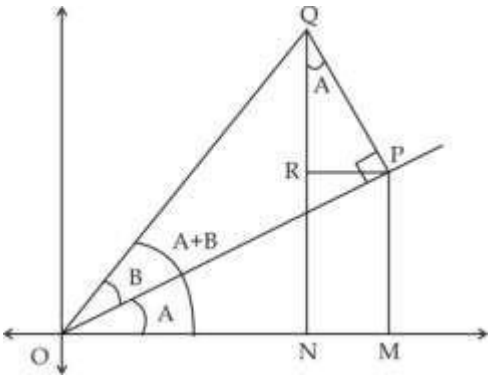
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\therefore B = \frac{-3}{10}$	1	
		Put $x = -3$ $-8 = (-3)(-5)C$ $\therefore C = -\frac{8}{15}$	1	
		$\frac{x-5}{x(x-2)(x+3)} \quad \frac{5}{6}\frac{}{x} \quad \frac{-3}{10}\frac{}{x-2} \quad -\frac{8}{15}\frac{}{x+3}$	½	**4**
		---		
	f) Ans	Resolve into partial fraction $\frac{x^3}{x-1}$ $x^2 \overline{)x^3}$ $x^3 - x$ $- \quad +$ x		
		$\therefore \frac{x^3}{x-1} = x + \frac{x}{x-1}$	1	
		$\therefore \frac{x}{x-1} = \frac{A}{x-1} + \frac{B}{x+1}$	½	
		$\therefore x = (x+1)A + (x-1)B$ Put $x = 1$ $1 = 2A$ $A = \frac{1}{2}$	1	
		Put $x = -1$ $-1 = -2B$ $B = \frac{1}{2}$	1	
		$\frac{x}{2} \quad \frac{x}{2} \quad \frac{1}{2}\frac{}{2} \quad \frac{1}{2}\frac{}{2}$	½	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		**Attempt any FOUR of the following:**		**16**
	a)	Solve the equation by inverse matrix method: $3x + y + 2z = 3\ ,\ 2x - 3y - z = -3\ ,\ x + 2y + z = 4$		
	Ans	$Let\ A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ $\lvert A \rvert = 3(-3+2) - 1(2+1) + 2(4+3)$ $\lvert A \rvert = -3 - 3 + 14$ $\therefore \lvert A \rvert = 8 \neq 0$ $\therefore A^{-1}$ exists	½	
		$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ $\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} -1 & 3 & 7 \\ -3 & 1 & 5 \\ 5 & -7 & -11 \end{bmatrix}$	1	
		$\text{Matrix of cofactors} = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$ $\text{Adj.}A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{\lvert A \rvert}\text{Adj.}A$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$A^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ & & \\ 7 & -5 & -11 \end{bmatrix}$	½	**4**
		$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$		
		$\begin{bmatrix} x \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -3-9+20 \\ 21+15-44 \end{bmatrix}$	½	
		$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix}$		
		$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$	1	
		$\therefore x = 1,\ y = 2,\ z = -1$		
		---		
	b)	Resolve into partial fractions: $\frac{3x-2}{(x+2)(x^2+4)}$		
	Ans	$\therefore \frac{3x-2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$		
		$\therefore 3x-2 = (x^2+4)A + (x+2)(Bx+C)$	½	
		Put $x = -2$		
		$\therefore -8 = 8A$		
		$\therefore A = -1$	1	
		Put $x = 0$		
		$-2 = (4)A + (2)C$		
		$-2 = -4 + 2C$		
		$\therefore C = 1$	1	
		Put $x = 1$		
		$\therefore 1 = (5)A + 3(B + C)$		
		$\therefore 1 = -5 + 3B + 3$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$\therefore 1-3+5=3B$		
		$\therefore 3=3B$		
		$\therefore B=1$	1	
		$\therefore \frac{3x-2}{} = \frac{-1}{} + \frac{x+1}{}$	½	**4**
		---		
	c)	Resolve into partial fractions $\frac{2x+1}{x(x+1)}$		
	Ans	$\frac{2x+1}{2} = \frac{A}{} + \frac{B}{2} + \frac{C}{}$		
		$2x+1 = x(x+1)A + (x+1)B + x^2C$	½	
		Put $x=0$		
		$1=(1)B$	1	
		$\therefore B=1$		
		Put $x=-1$	1	
		$\therefore -1=C$		
		Put $x=1$		
		$\therefore 3=(1)(1+1)A+(1+1)B+(1)^2C$		
		$\therefore 3=2A+2B+C$		
		$\therefore 3=2A+2-1$		
		$\therefore 2=2A$	1	
		$\therefore A=1$		
		$\therefore \frac{2x+1}{x(x+1)} = \frac{1}{x} + \frac{1}{x} + \frac{-1}{x+1}$	½	**4**
		---		
	d)	Given $\tan(A+B)=\frac{3}{4}, \tan(A-B)=\frac{77}{36}$, find $\tan 2A$		
	Ans	Let $A+B=x, A-B=y$		
		$x+y=A+B+A-B=2A$	1	
		$\therefore \tan(x+y)=\tan 2A$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$\therefore \tan 2A = \dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$	1	
		$= \dfrac{\frac{3}{4} + \frac{77}{36}}{1 - \frac{3}{4} \times \frac{77}{36}}$	1	
		$= -\dfrac{416}{87} = -4.7816$	1	**4**
		---		
	e)	$\dfrac{\pi}{4}$		
	Ans	Let $A + B = \dfrac{\pi}{4}$		
		$\therefore \tan(A + B) = \tan\dfrac{\pi}{4}$	½	
		$\therefore \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$	1	
		$\tan A + \tan B = 1 - \tan A \cdot \tan B$	½	
		$\tan A + \tan B + \tan A \cdot \tan B = 1$	½	
		$\tan A(1 + \tan B) + \tan B + 1 = 1 + 1$	1	
		$(1 + \tan B)(1 + \tan A) = 2$	½	**4**
		---		
	f)	$\tan^{-1}\left(\frac{1}{2}\right) \quad \tan^{-1}\left(\frac{1}{5}\right) \quad \tan^{-1}\left(\frac{1}{8}\right) \quad \frac{\pi}{4}$		
	Ans	$\tan^{-1}\left(\frac{1}{2}\right) \quad \tan^{-1}\left(\frac{1}{5}\right) \quad \tan^{-1}\left(\frac{1}{8}\right)$		
		$= \tan\left[\dfrac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}}\right] + \tan\left(\frac{1}{8}\right)$	1	
		$= \tan\left(\frac{7}{9}\right) + \tan\left(\frac{1}{8}\right)$	1	
		$= \tan\left[\dfrac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \times \frac{1}{8}}\right] \quad \dfrac{\pi}{4}$	2	**4**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Prove that $\cos(A+B) = \cos A \cos B - \sin A \sin B$		
	Ans.		1	

Right Angled Triangle	Acute Angle	Trigonometric Ratios
Δ OMP	∠MOP = A	$\sin A = \frac{PM}{OP}, \quad \cos A = \frac{OM}{OP}$
Δ OPQ	∠POQ = B	$\sin B = \frac{PQ}{OQ}, \quad \cos B = \frac{OP}{OQ}$
Δ PRQ	∠PQR = A	$\sin A = \frac{PR}{PQ}, \quad \cos A = \frac{QR}{PQ}$
Δ ONQ	∠NOQ = A+B	$\sin(A+B) = \frac{QN}{OQ}, \quad \cos(A+B) = \frac{ON}{OQ}$

1

$$\therefore \cos(A+B) = \frac{ON}{OQ}$$ ½

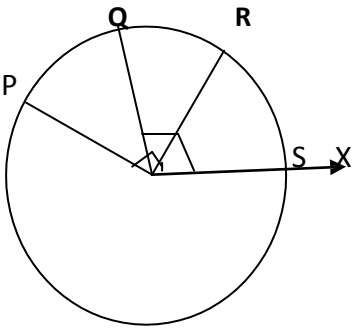
$$= \frac{OM - MN}{OQ}$$

$$= \frac{OM - PR}{OQ}$$ ½

$$= \frac{OM}{OQ} - \frac{PR}{OQ}$$

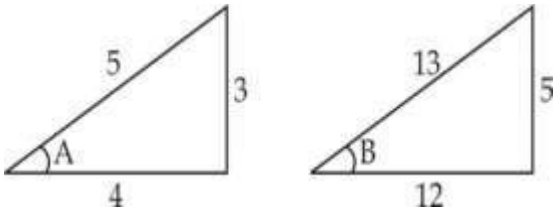
$$= \frac{OM}{OP} \times \frac{OP}{OQ} - \frac{PR}{PQ} \times \frac{PQ}{OQ}$$ ½

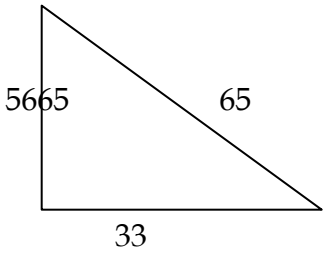
$$= \cos A \cos B - \sin A \sin B.$$ ½ **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		**OR** Consider a standard unit circle Let P,Q,R,S be points such that  $\angle XOP = A$, $\angle XOQ = B$, $\angle XOR = A - B$ From fig. $\angle POQ = A - B$ $\therefore \angle POQ = \angle XOR$ $P(\cos A, \sin A)$, $Q(\cos B, \sin B)$ $R(\cos(A-B), \sin(A-B))$, $S(1,0)$ $\therefore$ Chord PQ = Chord RS $\sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} = \sqrt{[\cos(A-B)-1]^2 + [\sin(A-B)-0]^2}$ $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = [\cos(A-B)-1]^2 + [\sin(A-B)-0]^2$ $\therefore \cos^2 A + \cos^2 B - 2\cos A \cos B + \sin^2 A + \sin^2 B - 2\sin A \sin B =$ $\cos(A-B) + 1 - 2\cos(A-B) + \sin(A-B)$ $\therefore 1 + 1 - 2(\cos A \cos B + \sin A \sin B) = 1 + 1 - 2\cos(A-B)$ $\therefore \cos A \cos B + \sin A \sin B = \cos(A-B)$ Replace B by $-B$ in above equation $\therefore \cos A \cos(-B) + \sin A \sin(-B) = \cos(A-(-B))$ $\therefore \cos A \cos B - \sin A \sin B = \cos(A+B)$ ----------	1 1 1 ½ ½	**4**
	b) Ans.	Without using calculator prove that $\cos 300^0 = \cos(360^0 - 60^0)$ $= \cos 60^0$ $= \frac{1}{2}$ $\sin 300^0 = \sin(360^0 - 60^0)$ $= -\sin 60^0$ $= -\frac{\sqrt{3}}{2}$ $\cos(-300^0) = \cos 300^0$ ... Since $\cos(-\theta) = \cos\theta$ $= \cos 30^0$ $= \frac{1}{2}$ ----------	½ ½ ½ ½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\sin(-330^0) = -\sin 330^0$ ... Since $\sin(-\theta) = -\sin\theta$	½	
		$= -(-\sin 30^0)$	½	
		$= \frac{1}{2}$		
		$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$		
		$= 1$		
		$= \text{R.H.S.}$	1	**4**
	c)	Prove that $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$		
	Ans.	$\tan 3A = \tan(A + 2A)$		
		$= \frac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$	1	
		$\frac{\frac{2\tan A}{2}}{\left(\frac{2\tan A}{1 - \tan^2 A}\right)}$	1	
		$\frac{\tan A(1 - \tan^2 A) + 2\tan A}{\frac{2}{2}}$	½	
		$= \frac{3}{1 - \tan^2 A - 2\tan^3 A}$	1	
		$= \frac{3}{1 - 3\tan^2 A}$	½	**4**
	d)	$\sin^{-1}\left(\frac{1}{2}\right) \quad \cos^{-1}\left(\frac{1}{3}\right) \quad \frac{\pi}{4}$		

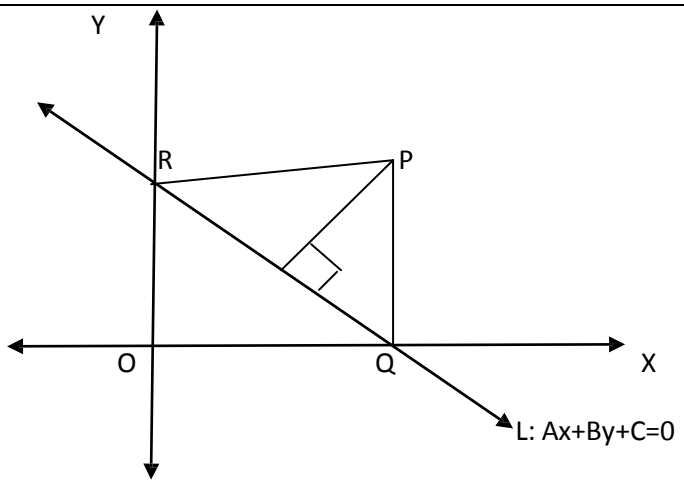
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.	Ans.	$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$	2	
		$= \frac{\pi}{4}$	1 1	**4**
	e)	Prove that $\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$		
	Ans.	$\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \frac{\sin 4A + \sin 6A + \sin 5A}{\cos 4A + \cos 6A + \cos 5A}$		
		$= \frac{2\sin 5A\cos(-A) + \sin 5A}{2\cos 5A\cos(-A) + \cos 5A}$	2	
		$\frac{\sin 5A\left[2\cos(-A)+1\right]}{\cos 5A\left[2\cos(-A)+1\right]}$	1	
		$= \tan 5A$	1	**4**
	f)	$\sin^{-1}\left(\frac{4}{5}\right)$ $\quad$ $\cos^{-1}\left(\frac{12}{13}\right)$ $\quad$ $\tan^{-1}\left(\frac{33}{65}\right)$		
	Ans.	$\sin^{-1}\left(\frac{4}{5}\right)$		
		$\therefore \cos A = \frac{4}{5}$		
		$\therefore \sin^2 A = 1 - \cos^2 A$		
		$= 1 - \frac{16}{25}$		
		$= \frac{9}{25}$		
		$\therefore \sin A = \frac{3}{5}$	1	
		$\cos^{-1}\left(\frac{12}{13}\right) = B$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \cos B = \frac{12}{13}$ $\therefore \sin^2 B = 1 - \cos^2 B$ $= 1 - \frac{144}{169}$ $= \frac{25}{169}$ $\therefore \sin B = \frac{5}{13}$ $\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$ $= \frac{4}{5}\frac{12}{13} - \frac{3}{5}\frac{5}{13}$ $= \frac{48}{65} - \frac{15}{65}$ $\frac{33}{65}$ $\therefore A + B = \cos^{-1}\left(\frac{33}{65}\right)$ $\cos^{-1}\left(\frac{4}{5}\right) \quad \cos^{-1}\left(\frac{12}{13}\right) \quad \cos^{-1}\left(\frac{33}{65}\right)$ *OR* Let $\cos^{-1}\left(\frac{4}{5}\right) = A$ $\therefore \cos A = \frac{4}{5}$ $\therefore \tan A = \frac{3}{4}$ $A = \tan^{-1}\left(\frac{3}{4}\right)$ $\cos^{-1}\left(\frac{4}{5}\right) \quad \tan^{-1}\left(\frac{3}{4}\right)$ $\cos^{-1}\left(\frac{12}{13}\right) = B$ 	1 ½ ½ 1 1	4

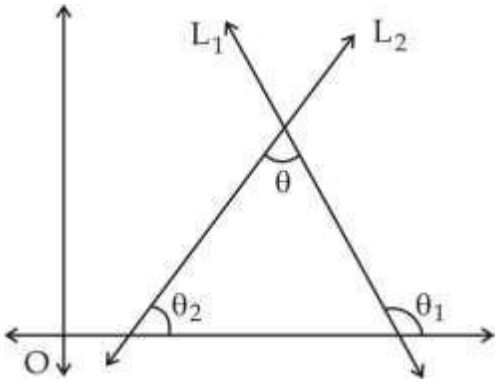
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$\therefore \cos B = \frac{12}{13}$ $\therefore \tan B = \frac{5}{12}$ $B = \tan^{-1}\left(\frac{5}{12}\right)$ $\therefore \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$	1	
		$L.H.S. = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$ $= \tan\left(\frac{\frac{3}{4}+\frac{5}{12}}{\frac{3}{4}\ \frac{5}{12}}\right)$	½	
		$= \tan\left(\frac{\frac{36+20}{48}}{1-\frac{15}{48}}\right)$ $= \tan\left(\frac{\frac{56}{48-15}}{48}\right)$ $= \tan^{-1}\left(\frac{56}{33}\right)$	½	
		Let $\tan^{-1}\left(\frac{56}{33}\right) = C$ $\therefore \tan C = \frac{56}{33}$ $\therefore \cos C = \frac{33}{65}$ $\therefore C = \cos^{-1}\left(\frac{33}{65}\right)$ $\cos^{-1}\left(\frac{33}{65}\right)$  	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		**Attempt any FOUR of the following:**		
	a) Ans.	$\sin 20^0 \sin 40^0 \sin 80^0 = \frac{3}{16}$		
		$\sin 20^0 \sin 40^0 \sin 80^0 = \sin 20^0 \sin 40^0 \cos 10^0$	½	
		$= \frac{\sqrt{3}}{2} \sin 20^0 \sin 40^0$	½	
		$= \frac{\sqrt{3}}{2} [\cos(-20^0) - \cos 60^0] \sin 80^0$	½	
		$\frac{\sqrt{3}}{4} [\sin 80^0 - \frac{1}{2}]\sin 80^0$	½	
		$= \frac{\sqrt{3}}{4} [\cos 20^0 \sin 80^0 - \frac{1}{2}\sin 80^0]$		
		$= \frac{\sqrt{3}}{4} [\frac{1}{2} 2\cos 20^0 \sin 80^0 - \frac{1}{2}\sin 80^0]$	½	
		$\frac{\sqrt{3}}{8} [\quad\quad]^0$	½	
		$= \frac{\sqrt{3}}{8} [\sin(2\times 90^0 - 80) + \frac{\sqrt{3}}{2} - \sin 80^0]$	½	
		$= \frac{\sqrt{3}}{8} [\sin 80^0 + \frac{\sqrt{3}}{2} - \sin 80^0]$		**4**
		$= \frac{3}{16}$	½	
		---		
	b) Ans.	$\cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$		
		We know that,		
		$2\sin A\cos B = \sin(A+B) + \sin(A-B)$	1	
		Put $A + B = C$		
		$A - B = D$		
		$\therefore \quad A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$	2	
		$\cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	c) Ans.	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$		
		$\therefore x = \tan^{-1}A$ and $y = \tan^{-1}B$	1	
		$\frac{\tan A + \tan B}{1 - \tan A \tan B}$	1	
		$= \frac{x+y}{1-xy}$	½	
		$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	½	**4**
		---		
	d) Ans.	Find the distance between the lines $3x + 2y = 5$ and $6x + 4y = 6$.		
		Let $L_1 : 3x + 2y = 5$		
		and $L_2 : 6x + 4y = 6 \Rightarrow 3x + 2y = 3$	½	
		$\therefore L_1 : 3x + 2y - 5 = 0$		
		$\therefore L_2 : 3x + 2y - 3 = 0$		
		$\therefore a = 3,\ b = 2,\ c_1 = -5,\ c_2 = -3$	1	
		distance between the lines is given by,		
		$d = \left\lvert \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right\rvert$	½	
		$= \left\lvert \frac{-3-(-5)}{\sqrt{3^2 + 2^2}} \right\rvert$	1	
		$= \left\lvert \frac{2}{\sqrt{13}} \right\rvert = \frac{2}{\sqrt{13}} units$	1	4
		---		
	e)	Prove that the length of perpendicular on the line $Ax + By + C = 0$ from the point $P(x_1, y_1)$ is $P = \left\lvert \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right\rvert$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	Ans.		½	
		$\left(\frac{-C}{A}\right)$ $\left(\frac{-C}{B}\right)$	½	
		$A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ \frac{-C}{A} & 0 & 1 \\ 0 & \frac{-C}{B} & 1 \end{vmatrix} = \frac{1}{2}\left\lvert x_1\left(0+\frac{C}{B}\right) - y_1\left(\frac{-C}{A}-0\right)+1\left(\frac{C}{AB}\right)\right\rvert$		
		$= \frac{1}{2}\left\lvert \frac{x_1 C}{B} + \frac{y_1 C}{A} + \frac{C^2}{AB}\right\rvert = \frac{1}{2}\left\lvert \frac{C}{AB}(Ax_1 + By_1 + C)\right\rvert$	1	
		$d(QR) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{\left(\frac{-C}{A}-0\right)^2 + \left(0+\frac{C}{B}\right)^2}$	½	
		$= \sqrt{\frac{C^2}{A} + \frac{C^2}{B}} = \sqrt{\frac{B^2C^2 + A^2C^2}{AB}}$		
		$\frac{C}{AB}\sqrt{A^2 + B^2}$	½	
		$A(\Delta PQR) = \frac{1}{2} \times d(QR) \times PM$		
		$\frac{1}{2}\frac{C}{AB}\sqrt{A^2 + B^2}$	½	
		$\therefore \frac{1}{2}\frac{C}{AB}(Ax_1 + By_1 + C) = \frac{1}{2}\frac{C}{AB}\sqrt{A^2 + B^2} \times PM$		
		$\therefore PM = \left\lvert \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right\rvert$ $\because$ distance is always positive	½	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	f)	Find equation of the line passing through the point of intersection of lines $x + y = 0$ and $2x - y = 9$ and point $(4,5)$.		
	Ans.	$x + y = 0,\quad 2x - y = 9$		
		$\therefore \quad x + y = 0$		
		$\underline{2x - y = 9}$	½	
		$\therefore \quad 3x = 9$		
		$\therefore \quad x = 3$		
		$y = -3$		
		$\therefore$ point of intersection $= (3,-3) = (x_1, y_1)$	1	
		and given point $= (4,5) = (x_2, y_2)$		
		its equation in two points form is $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$		
		$\therefore \quad \dfrac{y - (-3)}{5 - (-3)} = \dfrac{x - 3}{4 - 3}$	½	
		$\therefore \quad \dfrac{y + 3}{8} = x - 3$	1	
		$\therefore \quad y + 3 = 8(x - 3)$		
		$\therefore \quad y + 3 = 8x - 24$		
		$\therefore \quad -8x + y + 27 = 0 \quad$ or $\quad 8x - y - 27 = 0$	1	
		OR		
		$x + y = 0,\quad 2x - y = 9$		
		$\therefore \quad x + y = 0$		
		$\underline{2x - y = 9}$		
		$\therefore \quad 3x = 9$	½	
		$\therefore \quad x = 3$		
		$y = -3$		
		$\therefore$ point of intersection $= (3,-3) = (x_1, y_1)$	1	
		and given point $= (4,5) = (x_2, y_2)$		
		$\therefore$ *slope of line is* $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - (-3)}{4 - 3} = 8$	½	
		$\therefore$ *the equation is,* $y - y_1 = m(x - x_1)$	1	
		$\therefore \quad y - (-3) = 8(x - 3)$		
		$\therefore \quad y + 3 = 8x - 24$		
		$\therefore \quad -8x + y + 27 = 0 \quad$ or $\quad 8x - y - 27 = 0$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6		**Attempt any FOUR of the following:**		**16**
	a)	If m_1 and m_2 are the slope of two lines then prove that angle between two lines is $\theta = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$		
	Ans.	Let θ_1 = Inclination of L_1 θ_2 = Inclination of L_2 $\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$ Slope of L_2 is $m_2 = \tan\theta_2$	½	
			1	
		$\therefore$ from figure, $\theta = \theta_1 - \theta_2$ $\therefore \tan\theta = \tan(\theta_1 - \theta_2)$	½	
		$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$	1	
		$\therefore \tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$	½	
		Since θ is acute $\therefore \tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\|$ $\therefore \theta = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right\|$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.	b)	Find the equation of line passing through point of intersection of lines $2x+3y=13$, $5x-y=7$ and perpendicular to $3x-y+7=0$		
	Ans.	$2x+3y=13$		
		$5x-y=7$		
		$\therefore \quad 2x+3y=13$		
		$15x-3y=21$		
		$\therefore \quad 17x=34$		
		$\therefore \quad x=2,\ y=3 \quad \therefore$ Point of intersection $=(2,3)$	1	
		Slope of the line $3x-y+7=0$ is,		
		$m_0=-\frac{a}{b}=-\frac{3}{-1}=3$	1	
		$\therefore$ Slope of the required line is, $m=-\frac{1}{m_0}=-\frac{1}{3}$	½	
		$\therefore \quad y-y_1=m(x-x_1)$		
		$y-3=-\frac{1}{3}(x-2)$	1	
		$\therefore \quad x+3y-11=0$	½	**4**
		---------- ----------		
	c)	From the following data ,Calculate range and coefficient of range:		

Marks	10-19	20-29	30-39	40-49	50-59	60-69
No.of student	06	10	16	14	08	04

	Ans.	Range=Upper boundary of the last class − lower boundary of first class	
		$=69.5-9.5$	1
		$=60$	1
		$\text{coefficient of range}=\frac{\text{Range}}{\text{sum of the highest and lowest value}}$	
		$=\frac{60}{69.5+9.5}$	1
		$=\frac{60}{79}$ or 0.759	1
			4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.	d)	Find the mean deviation from median of the following distribution:		

Weight (in gms)	10–15	15-20	20-25	25-30	30-35	35-40	40-45
No. of items	07	12	16	25	19	15	06

Ans.

Class	x_i	f_i	c.f	$D_i = \lvert x_i - M \rvert$	$f_i D_i$
10-15	12.5	07	07	15.5	108.5
15-20	17.5	12	19	10.5	126
20-25	22.5	16	35	5.5	88
25-30	27.5	25	60	0.5	12.5
30-35	32.5	19	79	4.5	85.5
35-40	37.5	15	94	9.5	142.5
40-45	42.5	06	100	14.5	87
Total		100			650

2

$$Median = L + \frac{\frac{N}{2} - c.f}{f} \times h$$

$$= 25 + \frac{50 - 35}{25} \times 5$$

$$= 28$$

1

$$M.D = \frac{\sum f_i D_i}{N}$$

$$= \frac{650}{100} = 6.5$$

1

4

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e) In two factories A and B ,engaged in the same area of the industry ,the average weekly wages (in Rs.) and the S.D are as below:

Factory	Average wages	S.D
A	34.5	5.0
B	28.5	4.5

1

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.	Ans.	Which Factory A or B has greater variability in individual wages ? For Factory A $C.V_A = \frac{S.D}{x} \times 100$	½	
		$= \frac{5.0}{34.5} \times 100 = 14.49$	1	
		For Factory B $C.V_B = \frac{S.D}{x} \times 100$	½	
		$= \frac{4.5}{28.5} \times 100 = 15.79$	1	
		$C.V_A < C.V_B$ $\therefore$ Factory B has greater variability	1	**4**
		---		
	f)	Calculate the mean and standard deviation of the following frequency distribution:		

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

Ans.

C.I	x_i	Freq	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
Total		100	2500		78500

2

$\bar{x} = \frac{\sum_{i=1} f_i x_i}{N}$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
		$= \frac{2500}{100} = 25$	1	
		$S.D = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$		
		$= \sqrt{\frac{78500}{100} - (25)^2} = 12.65$	1	**4**
		OR		

C.I	x_i	f_i	d_i	d_i^2	$f_i d_i$	$f_i d_i^2$
0-10	5	14	-2	4	-28	56
10-20	15	23	-1	1	-23	23
20-30	25	27	0	0	0	0
30-40	35	21	1	1	21	21
40-50	45	15	2	4	30	60
Total		100			00	160

2

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 25 + \frac{0}{100} \times 10 = 25$$ 1

$$S.D = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$$ 1

$$= \sqrt{\frac{160}{100} - \left(\frac{0}{100}\right)^2} \times 10 = 12.65$$ **4**

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
