SUMMER – 2013 EXAMINATION

MODEL ANSWER

Subject: APPLIED MATHEMATICS

Subject Code: 12062

Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numericals shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		**Attempt any ten of the following:**		**20**
	(a)	**Evaluate** $\int \frac{1}{4-9x^2}dx$		
	Ans.	$\int \frac{1}{4-9x^2}dx = \int \frac{1}{2^2-(3x)^2}dx$	½	
		$= \frac{1}{2.2} log \left\lvert\frac{2+3x}{2-3x}\right\rvert \frac{1}{3}+c$	1	
		$= \frac{1}{12} log \left\lvert\frac{2+3x}{2-3x}\right\rvert + c$	½	**02**
		OR		
		$\int \frac{1}{4-9x^2}dx = \int \frac{1}{9\left(\frac{4}{9}-x^2\right)}dx$		
		$= \frac{1}{9}\int \frac{1}{\left(\frac{2}{3}\right)^2 - x^2}dx$	½	
		$= \frac{1}{9}\frac{1}{2.\frac{2}{3}} log \left\lvert\frac{\frac{2}{3}+x}{\frac{2}{3}-x}\right\rvert \frac{1}{3}+c$	1	
		$= \frac{1}{12} log \left\lvert\frac{2+3x}{2-3x}\right\rvert + c$	½	**02**
		Note: *In solution of integration problems, if the constant ‘c’ is not added, ½ mark* ***may be*** *deducted.*		
	(b)	**Evaluate** $\int \sin^5 x . \cos x \, dx$		
	Ans.	Put $sinx = t$	½	
		$\cos x \, dx = dt$		
		$= \int t^5 dt$		
		$= \frac{t^6}{6} + c$	1	
		$= \frac{\sin^6 x}{6} + c$	½	**02**
	(c)	Evaluate $\int x^2 e^x dx$		
	Ans.	$\int x^2 e^x dx = x^2 \int e^x dx - \int \left[\frac{dx^2}{dx}\int e^x dx\right] dx$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.		$= x^2e^x - \int 2x\, e^x dx$	½	
		$= x^2e^x - 2\int x\, e^x dx$		
		$= x^2e^x - 2\left\{x\int e^x dx - \int\left[\frac{dx}{dx}\int e^x\, dx\right]dx\right\}$	½	
		$=x^2e^x - 2\{xe^x - \int 1.e^x dx\}$		
		$=x^2e^x - 2\{xe^x - e^x\}+c$	½	**02**
	(d)	**Evaluate** $\int \frac{1}{x^2-3x+2}dx$		
		$T.T = \frac{(M.T)^2}{4\times F.T} = \frac{(-3x)^2}{4x^2} = \frac{9}{4}$	½	
		OR		
		$T.T = \left(\frac{1}{2}coeff.of\ x\right)^2 = \left(\frac{1}{2}(-3)\right)^2 = \frac{9}{4}$		
		$= \int \frac{1}{x^2-3x+\frac{9}{4}-\frac{9}{4}+2}dx$		
		$= \int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}dx$		
		$= \int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{1}{2^2}}dx$	½	
		$= \frac{1}{2.\frac{1}{2}}log\left\lvert\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right\rvert\frac{1}{3}+c$	1	**02**
		$= log\left\lvert\frac{x-2}{x-1}\right\rvert+c$		
	(e)	**Evaluate** $\int_0^\infty e^{-x}dx$		
		$= \left[\frac{e^{-x}}{-1}\right]_0^\infty$	1	
		$= -(e^{-\infty}-e^{-0})$	½	
		$= -(0-1)$		
		$= 1$	½	**02**

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>1.</td><td>(f)
Ans</td><td>Show that the differential equation.
$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$ is an exact
$M = 2xy + y - \tan y$
$\frac{\partial M}{\partial y} = 2x - \tan^2 y$
$N = x^2 - x\tan^2 y + \sec^2 y$
$\frac{\partial N}{\partial x} = 2x - tan^2 y$
$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
$\therefore Given\ equation\ is\ Exact.$</td><td>1
1</td><td>02</td></tr>
<tr><td></td><td>(g)
Ans.</td><td>Prove that $\Delta = E - 1$
Consider $\Delta f(x) = f(x+h) - f(x)$
$= Ef(x) - f(x)$
$= (E-1)f(x)$
$\therefore \Delta = E - 1$</td><td>1
½
½</td><td>02</td></tr>
<tr><td></td><td>(h)
Ans.</td><td>Find the missing term of the following data by using forward difference table.
x : 1 2 3 4 5
y : -1 -3 1 __ 51
Let $y = a \quad when \quad x = 4$
<table>
<tr><td>x</td><td>y</td><td>Δy</td><td>$\Delta^2 y$</td><td>$\Delta^3 y$</td><td>$\Delta^4 y$</td></tr>
<tr><td>1</td><td>−1</td><td>−2</td><td>6</td><td>a−11</td><td>68−4a</td></tr>
<tr><td>2</td><td>−3</td><td>4</td><td>a−5</td><td>57−3a</td><td></td></tr>
<tr><td>3</td><td>1</td><td>a−1</td><td>52−2a</td><td></td><td></td></tr>
<tr><td>4</td><td>a</td><td>51−a</td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>51</td><td></td><td></td><td></td><td></td></tr>
</table>
$\Delta^4 y = 0$
$68 - 4a = 0$
$\therefore 68 = 4a$
$\therefore a = 17$</td><td>1
½
½</td><td>02</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.	**(i)**	**Using Simpson's 1/3rd Rule,evaluate $\int_0^4 1+x^3\,dx$, h=1**		
	Ans.	Let $y = f(x) = 1 + x^3$		
		$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 0 & 1 & 2 & 3 & 4 \\ \hline y & 1 & 2 & 9 & 28 & 65 \\ \hline\end{array}$	1	
		$\int_0^4 1+x^3dx = \frac{h}{3}[y_0+y_n+2(y_2+y_4+\cdots)+4(y_1+y_3+\cdots)]$	½	
		$= \frac{1}{3}[1+65+2(9)+4(2+28)]$		
		$= 68$	½	**02**
	(j)	**If $A = \{2,3,5\}$, $B = \{3,4,6,8\}$, $C = \{2,4,7\}$ find $A \cup C$ and $B \cap C$**		
	Ans.	$A \cup C = \{2,3,4,5,7\}$	1	
		$B \cap C = \{4\}$	1	**02**
	(k)	**Evaluate $\int e^x \sec(e^x)\tan(e^x)dx$**		
	Ans.	Let $e^x = t$		
		$e^x dx = dt$	½	
		$= \int \sec(t)\tan(t)\,dt$		
		$= \sec(t) + c$	1	
		$= \sec(e^x) + c$	½	**02**
	(l)	**Find the order and degree of differential equation.**		
		$\frac{y-x\frac{dy}{dx}}{\frac{dy}{dx}} = \left(\frac{dy}{dx}\right)^2$		
	Ans.	$\frac{y-x\frac{dy}{dx}}{\frac{dy}{dx}} = \left(\frac{dy}{dx}\right)^2$		
		$= y - x\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^3$		
		Order=1	1	
		Degree=3	1	**02**

2.		**Attempt any four of the following:**		**16**
	(a)	**Verify that $y = \sin(logx)$ is a solution of D.E.**		
	Ans.	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$		
		Given $y = \sin(logx)$		
		$\therefore \frac{dy}{dx} = \cos(logx)\frac{1}{x}$	1	
		$\therefore x\frac{dy}{dx} = \cos(logx)$	½	
		$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin(logx)\frac{1}{x}$	1½	
		$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$	½	
		$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$	½	04
	(b)	$\boldsymbol{Solve\ \frac{dy}{dx} = e^{x-y} + x^2e^{-y}}$		
	Ans.	$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$		
		$= e^xe^{-y} + x^2e^{-y}$		
		$= e^{-y}(e^x + x^2)$	1	
		$\frac{dy}{e^{-y}} = (e^x + x^2)dx$		
		$e^ydy = (e^x + x^2)dx$	1	
		$e^y = e^x + \frac{x^3}{3} + c$	2	04
		Note: *In the last step, each term carries ½ marks and if all the* terms *are correct, the whole step carries 2 marks.*		
	c)	**Solve $\boldsymbol{\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}}$**		
	Ans.	$\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$		
		$Let\ \frac{y}{x} = v$		
		$y = vx$		
		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	1	
		$v + x\frac{dv}{dx} = v + \sin v$	½	
		$x\frac{dv}{dx} = \sin v$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2.		$\frac{dv}{\sin v} = \frac{dx}{x}$	½	
		$cosecvdv = \frac{dx}{x}$	½	
		$\int cosecvdv = \int \frac{dx}{x}$		
		$log\|cosecv - cotv\| = logx + c$	1	
		$log\left\|cosec\frac{y}{x} - cot\frac{y}{x}\right\| = logx + c$	½	**04**
	d)	**Solve** $\boldsymbol{\cos^2 x\frac{dy}{dx} + y = tanx}$		
	Ans.	$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{tanx}{\cos^2 x}$	½	
		$\frac{dy}{dx} + \sec^2 x\, y = tanx \sec^2 x$	½	
		Comparing with $\frac{dy}{dx} + Py = Q$		
		$\therefore P = \sec^2 x \quad Q = tanx \sec^2 x$		
		$I.F. = e^{\int pdx}$		
		$= e^{\int \sec^2 xdx}$		
		$= e^{tanx}$	½	
		$solution\ is \quad y.I.F. = \int Q\, I.F.dx + c$		
		$ye^{tanx}. = \int \tan x \sec^2 x\, e^{tanx}.dx + c$	½	
		$Put\ tanx = t \quad \therefore \sec^2 x\, dx = dt$	½	
		$ye^{tanx} = \int te^t dt$		
		$= t\int e^t dt - \int\left[\frac{dt}{dt}\int e^t dt\right] dt$	1	
		$= te^t - \int e^t dt$		
		$= te^t - e^t + c$	½	04
		$ye^{tanx} = tanx\, e^{tanx} - e^{tanx} + c$		
	e)	$\boldsymbol{Solve\ \frac{dy}{dx} + x\sin 2y = x^3\cos^2 y}$		
	Ans.	$\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2.		$\frac{1}{\cos^2x}\frac{dy}{dx}+\frac{x(2\sin y.\cos y)}{\cos y.\cos y}=x^3$		
		$\sec^2y\frac{dy}{dx}=+2x\tan y=x^3$	½	
		Put $\tan y=z$		
		$\sec^2y\frac{dy}{dx}=\frac{dz}{dx}$	½	
		This is linear differential equation of the form $\frac{dz}{dx}+Pz=Q$		
		Here $P=2x$, $Q=x^3$		
		$I.F.=e^{\int Pdx}=e^{\int 2x\,dx}=e^{x^2}$	½	
		$\therefore$ Solution is,		
		$z.I.F.=\int Q.I.F.dx+c$		
		$z.e^{x^2}=\int x^3.e^{x^2}dx+c$	½	
		$z.e^{x^2}=\int e^{x^2}x^2.xdx+c$		
		$z.e^{x^2}=\frac{1}{2}\int e^{x^2}x^2.2xdx+c$		
		Put $x^2=u$ $\therefore 2xdx=du$		
		$z.e^u=\frac{1}{2}\int e^u.u.du+c=\frac{1}{2}\int u.e^udu+c$		
		$=\frac{1}{2}\left\{u\int e^udu-\int\left[\left[\int e^udu.\frac{du}{du}\right]\right]\right\}+c$	½	
		$=\frac{1}{2}\left\{ue^u-\int e^udu\right\}+c$		
		$=\frac{1}{2}\{ue^u-e^u\}+c$		
		$=\frac{1}{2}e^u(u-1)+c$		
		$\therefore z.e^{x^2}=\frac{1}{2}e^{x^2}(x^2-1)+c$	½	
		Now put back $z=\tan y$ in above equation		

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>2.</td><td></td><td>$\therefore tany.e^{x^2} = \frac{1}{2}e^{x^2}(x^2-1)+c$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$tany = \frac{(x^2-1)}{2}+c.e^{-x^2}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$tany = \frac{(x^2-1)}{2}+c'$ since $c' = c.e^{-x^2}$</td><td>½</td><td>04</td></tr>
<tr><td></td><td>(f)</td><td>If $L=\frac{di}{dt}=30\sin(10\pi t)$.Find i terms of t, given that L=2 and i=0 at t=0</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>Given $L=2, i=0 \ at \ t=0$
$L\frac{di}{dt}=30\sin(10\pi t)$
$Ldi = 30\sin(10\pi t)\,dt$
$\int L\,di = \int 30\sin(10\pi t)\,dt$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$Li = 30\frac{-cos(10\pi t)}{10}+c$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore i=0 \ at \ t=0$
$0=\frac{-3}{\pi}cos0+c$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$=\frac{-3}{\pi}+c$
$c=\frac{3}{\pi}$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$Li == \frac{-3}{\pi}cos(10\pi t)+\frac{3}{\pi}$
$L=2$
$2i=\frac{-3}{\pi}cos(10\pi t)+\frac{3}{\pi}$</td><td>1</td><td>04</td></tr>
<tr><td>3.</td><td></td><td>Attempt any Four of the following:</td><td></td><td>16</td></tr>
<tr><td></td><td>(a)</td><td>Using Langrange's interpolation formula evaluate $f(5)$ from the following data:
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>7</td></tr>
<tr><td>y</td><td>2</td><td>4</td><td>8</td><td>16</td><td>128</td></tr>
</table></td><td></td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.	**Ans.**	$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0$ $+\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} \times y_1$ $+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_2)(x_2-x_3)(x_2-x_4)} \times y_2$ $+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$ $+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$ $y = f(x) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$	2	
		$+\frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 +$ $\frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$ $+\frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$		
		$= -0.6667 + 6.4 - 24 + 42.6667 + 8.5333$	1	
		$= 32.93$	1	**04**
	b)	**The table gives the distance of the visible horizon for the given height.** **x= height : 100 150 200 250 300 350 400** **y=distance : 10.63 13.03 15.04 16.81 18.42 19.90 21.27** **Find y when x=218 , by using Newton's forward interpolation formula.**		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.	**Ans.**			

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63	2.4	−0.39	0.15	−0.07	0.02	0.02
150	13.03	2.01	−0.24	0.08	−0.05	0.04	
200	15.04	1.77	−0.16	0.03	−0.01		
250	16.81	1.61	−0.13	0.02			
300	18.42	1.48	−0.11				
350	19.90	1.37					
400	21.27						

2

$$m = \frac{x - x_0}{h}$$ ½

$$= \frac{218-100}{50} = 2.36$$

$$f(x) = y_0 + m\Delta y_0 + \frac{m(m-1)}{2!}\Delta^2 y_0 + \frac{m(m-1)(m-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 10.63 + (2.36)(2.4) + \frac{2.36(2.36-1)}{2}(-0.39)$$

$$+\frac{2.36(2.36-1)(2.36-2)}{6}(0.15)$$

$$+\frac{2.36(2.36-1)(2.36-2)(2.36-3)}{24}(-0.07)$$

$$+\frac{2.36(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{120}(0.02)$$

$$+\frac{2.36(2.36-1)(2.36-2)(2.36-3)(2.36-4)(2.36-5)}{720}(0.02)$$ 1

$$= 10.63 + 5.664 - 0.625872 + 0.0288864 + 0.002156$$

$$= 15.70$$ ½

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

3. **Ans.** **Find f(42) from the following data:**

x:	**20**	**25**	**30**	**35**	**40**	**45**
f(x) :	**354**	**332**	**291**	**260**	**231**	**204**

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	354					
25	332	−22				
30	291	−41	−19			
35	260	−31	10	29		
40	231	−29	2	−8	−37	
45	204	−27	2	0	8	45

2

$$m = \frac{x - x_n}{h} = \frac{42 - 45}{5} = -0.6$$

½

$$f(x) = y_n + m\nabla y_n + \frac{m(m+1)}{2!}\nabla^2 y_n + \frac{m(m+1)(m+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 204 + (-0.6)(-27) + \frac{(-0.6)(-0.6+1)}{2!}(2)$$

$$+\frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(0) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(8)$$

$$+\frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)}{5!}(45)$$

1

$$= 204 + 16.2 - 0.24 - 0.2688 - 1.02816$$

04

$$= 218.66$$

½

d) **A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battery of 20 volts. Find the current in the circuit**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3.		**at any instant, if the relation between L,R and E is $L\frac{di}{dt}+Ri=E$**		
	Ans.	$L\frac{di}{dt}+Ri=E$		
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$		
		$P=\frac{R}{L}$ *and* $Q=\frac{E}{L}$		
		$\therefore IF=e^{\int pdt}=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1	
		$\therefore$ *the solution is,*		
		$i\cdot IF=\int Q\cdot IF\cdot dt+c$		
		$\therefore i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\cdot e^{\frac{R}{L}t}\cdot dt+c$		
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\cdot\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$		
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}\cdot e^{\frac{R}{L}t}+c$	1	
		At $i=0,\ t=0,$		
		$\therefore 0=\frac{E}{R}\cdot e^{0}+c$		
		$\therefore c=-\frac{E}{R}$		
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}\cdot e^{\frac{R}{L}t}-\frac{E}{R}$ *or* $i=e^{-\frac{R}{L}t}\left[e^{\frac{R}{L}t}-1\right]\frac{E}{R}$	1	
		Given $R=100,\ L=0.1,\ E=20.$		
		$\therefore i\cdot e^{1000t}=\frac{1}{5}\cdot e^{1000t}-\frac{1}{5}$ *or* $i=e^{-1000t}\left[e^{1000t}-1\right]\frac{1}{5}$	1	**04**
		Note: *In the above example, L, R, E are arbitrary constants whereas i and t are variables. Also the values of L, R, E are given in advance. Thus these values can be substituted directly in the given differential equation and then the equation can be solved as illustrated below.*		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

3.

$$0.1\frac{di}{dt}+100i=20$$

$$\therefore \frac{di}{dt}+1000i=200$$

1

$$P=1000 \quad and \quad Q=200$$

$$\therefore IF=e^{\int pdt}=e^{\int 1000dt}=e^{1000t}$$

1

04

e) **The population of a town in a decimal census is given below. Estimate the population for the year 1955.**

Year (x) : 1921 1931 1941 1951 1961

Population : 46 66 81 93 101

(in thousand)

Ans.

x	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1921	46				
1931	66	20			
1941	81	15	-5		
1951	93	12	-3	2	
1961	101	8	-4	-1	-3

1

$$m=\frac{x-x_n}{h}=\frac{1955-1961}{10}=-0.6$$

½

$$f(x)=y_n+m\nabla y_n+\frac{m(m+1)}{2!}\nabla^2 y_n+\frac{m(m+1)(m+2)}{3!}\nabla^3 y_n+\cdots$$

$$=101+(-0.6)(8)+\frac{(-0.6)(-0.6+1)}{2!}(-4)$$

1½

$$+\frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-1)$$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

3.

$$+\frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(-3)$$

$= 101 - 4.8 + 0.48 + 0.056 + 0.1008$

$= 96.8368$ 1 **04**

f) **From the following data find U_3 , if**

$U_4 = 0.35\,, U_5 = 0.88, U_6 = 1.71$ $U_7 = 2, U_8 = 8$

Ans.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
3	a	0.35−a	0.18+a	0.12−a	−0.96+a	4.05−a
4	0.35	0.53	0.3	−0.84	3.09	
5	0.88	0.83	−0.54	2.25		
6	1.71	0.29	1.71			
7	2	2				
8	4					

2

4.05−a=0 1

$\therefore a = U_3 = 4.05$ 1 **04**

OR

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	0.35	0.53	0.3	−0.84	3.09
5	0.88	0.83	−0.54	2.25	
6	1.71	0.29	1.71		
7	2	2			
8	4				

1½

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
		$m = \frac{x - x_0}{h}$	½	
		$= \frac{3-4}{1} = -1$		
		$f(x) = y_0 + m\Delta y_0 + \frac{m(m-1)}{2!}\Delta^2 y_0 + \frac{m(m-1)(m-2)}{3!}\Delta^3 y_0 + \cdots$		
		$= 0.35 + (-1)(0.53) + \frac{(-1)(-1-1)}{(2!)}(0.3) + \frac{(-1)(-1-1)(-1-2)}{(3!)}(-0.84)$	1	
		$+ \frac{(-1)(-1-1)(-1-2)(-1-3)}{(4!)}(3.09)$		
		$= 0.35 - 0.53 + 0.3 + 0.84 + 3.09$		
		$= 4.05$		
		$\therefore U_3 = 4.05$	1	**04**
4.		**Attempt any four of the following:**		**16**
	a)	**Evaluate** $\int \frac{e^x(x+1)}{\cot(xe^x)}dx$		
	Ans.	Put $t = xe^x$		
		$dt = (xe^x + e^x)dx$		
		$dt = e^x(x+1)dx$	1	
		$I = \int \frac{dt}{cot(t)}$	½	
		$= \int tant\ dt$	½	
		$I = logsec(t) + c$	1	
		$= logsec(xe^x) + c$	1	**04**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.	**b)**	**Evaluate** $\int x\tan^{-1}x\,dx$		
	Ans.	$I=\int x\tan^{-1}x\,dx$		
		$=\tan^{-1}x\int x\,dx-\int\left[\frac{d(\tan^{-1}x)}{dx}.\int xdx\right]dx$	½	
		$=\tan^{-1}x\left(\frac{x^2}{2}\right)-\int\left[\frac{1}{1+x^2}.\frac{x^2}{2}\right]dx$	1	
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$		
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$	½	
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left[\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right]dx$		
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left[1-\frac{1}{1+x^2}\right]dx$	1	
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}[x-\tan^{-1}x]+c$		
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}x+\frac{1}{2}\tan^{-1}x+c$	1	**04**
	c)	**Evaluate** $\int_0^2\frac{1}{x^2-2x+2}dx$		
	Ans.	$I=\int_0^2\frac{1}{x^2-2x+2}dx$		
		$T.T=\frac{(M.T)^2}{4F.T.}=\frac{(-2x)^2}{4x^2}=\frac{4x^2}{4x^2}=1$	1	
		OR		
		$T.T=\left(\frac{1}{2}coeff.of\ x\right)^2=\left(\frac{1}{2}(-2)\right)^2=1$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		$I = \int_0^2 \frac{1}{x^2 - 2x + 1 + 1} dx$	½	
		$= \int_0^2 \frac{1}{(x-1)^2 + 1^2} dx$		
		$= [\tan^{-1}(x-1)]_0^2$	1	
		$= \tan^{-1}(1) - \tan^{-1}(-1)$	½	
		$= \tan^{-1}(1) + \tan^{-1}(1)$		
		$= \frac{\pi}{4} + \frac{\pi}{4}$		
		$I = \frac{\pi}{2}$	1	**04**
	d)	***Evaluate*** $\int_0^{\pi} x \sin^2 x \, dx$		
	Ans.	$Let\ \ I = \int_0^{\pi} x \sin^2 x \, dx \ \ \ldots \ \ (1)$		
		$= \int_0^{\pi} (\pi - x) \sin^2(\pi - x) \, dx$ by property	½	
		$= \int_0^{\pi} (\pi - x) \sin^2(x) \, dx$ $\sin(\pi - \theta) = \sin\theta$	½	
		$= \int_0^{\pi} \pi \sin^2 x - x \sin^2 x \, dx$		
		$= \int_0^{\pi} \pi \sin^2 x \, dx - I$ $From\,(1)$	½	
		$2I = \int_0^{\pi} \pi \sin^2 x \, dx$		
		$2I = \int_0^{\pi} \pi \left(\frac{1 - cos2x}{2}\right) dx$	½	
		$= \int_0^{\pi} \left(\frac{\pi}{2} - \frac{\pi}{2} cos2x\right) dx$		
		$= \left[\frac{\pi}{2} x - \frac{\pi}{2} \frac{sin2x}{2}\right]_0^{\pi}$	½	
		$= \left[\frac{\pi}{2}(\pi) - \frac{\pi}{2} \frac{sin2(\pi)}{2}\right] - \left[\frac{\pi}{2}(0) - \frac{\pi}{2} \frac{sin2(0)}{2}\right]$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		$2I = \frac{\pi^2}{2} - \frac{\pi}{4}(0)$		
		$2I = \frac{\pi^2}{2}$	½	
		$I = \frac{\pi^2}{4}$	½	**04**
	e)	**Using integration , find the area of the circle $x^2 + y^2 = 9$**		
	Ans.	$y^2 = 9 - x^2 = 3^2 - x^2$		
		$\therefore y = \sqrt{3^2 - x^2}$		
		A_1=Area bounded by the curve $y = \sqrt{3^2 - x^2}$,X-axis and the lines $x = 0 , x = a$		
		$A_1 = \int_0^3 y dx$		
		$= \int_0^3 \sqrt{3^2 - x^2} dx$	½	
		$= \left[\frac{1}{2} x\sqrt{3^2 - x^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$	1	
		$= \left[\frac{1}{2}(3)\sqrt{3^2 - 3^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{3}{3}\right)\right] - \left[\frac{1}{2}(0)\sqrt{3^2 - 0^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{0}{3}\right)\right]$	1	
		$= \frac{9}{2}\left(\frac{\pi}{2}\right)$		
		$= \frac{9\pi}{4}$	1	
		$A_1 = Area\ of\ the\ circle = 4 \times A_1 = 4 \times \frac{9\pi}{4} = 9\pi\ sq.units$	½	
	f)	**Using integration , find the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$**		**04**
	Ans.	The two curves are $y^2 = 4x \ldots (1)$ $x^2 = 4y \ldots (2)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
		Put $y = \frac{x^2}{4}$ in equation (1)		
		$\left(\frac{x^2}{4}\right)^2 = 4x$ $\quad \therefore x^4 - 64x = 0$ $\quad \therefore x(x^3 - 64) = 0$		
		$\therefore x = 0 \ , x = 4$	1	
		When $x = 0 , y = 0$ $\therefore$ one point of intersection is $(0,0)$		
		When $x = 4 , y = 4$ $\therefore$ other point of intersection is $(4,4)$		
		From $y^2 = 4x \Longrightarrow y_1 = 2x^{1/2}$		
		From $x^2 = 4y \Longrightarrow y_2 = \frac{x^2}{4}$		
		The required area = $\int_0^4 y_1 - y_2 \, dx$		
		$= \int_0^4 2x^{1/2} - \frac{x^2}{4} dx$	1	
		$= 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$	1	
		$= \frac{4}{3}\left[4^{3/2} - 0\right] - \frac{1}{12}\left[4^3 - 0\right]$	½	
		$= \frac{32}{3} - \frac{16}{3}$		
		$= \frac{16}{3} \ sq.units$	½	**04**
5.		**Attempt any four of the following:**		**16**
	a)	**Find y′(0) from the following data:**		
		x: 0 1 2 3 4 5		
		y: 4 8 15 7 6 2		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

5. **Ans.**

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	4	4	3	−18	40	−72
1	8	7	−15	22	−32	
2	15	−8	7	−10		
3	7	−1	−3			
4	6	−4				
5	2					

2

$$h = 1, x_0 = 0, y_0 = 4, \Delta y_0 = 4, \Delta^2 y_0 = 3, \Delta^3 y_0 = -18, \Delta^4 y_0 = 40, \Delta^5 y_0 = -72$$

$$y'(0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0\right]$$

$$= \frac{1}{1}\left[4 - \frac{1}{2}(3) + \frac{1}{3}(-18) - \frac{1}{4}(40) + \frac{1}{5}(-72)\right]$$

1

$$= 4 - 1.5 - 6 - 10 - 14.4$$

½

$$\therefore y'(0) = -27.9$$

½

04

b) **Use trapezoidal rule to evaluate $\int_2^7 \frac{1}{x} dx$, divide [2,7] into 5 equal subintervals.**

Ans.

$$a = x_0 = 2, b = x_n = 7, n = 5$$

$$h = \frac{b-a}{n} = \frac{7-2}{5} = 1$$

½

x	2	3	4	5	6	7
y	0.5	0.33	0.25	0.2	0.17	0.14

1½

$$I = \int_2^7 \frac{1}{x} dx = \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$= \frac{1}{2}[(0.5 + 0.14) + 2(0.33 + 0.25 + 0.2 + 0.17)]$$

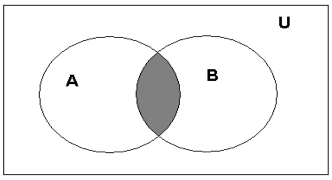
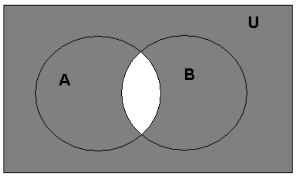
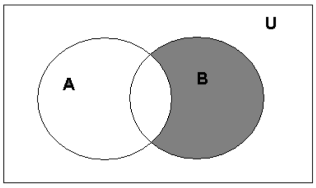
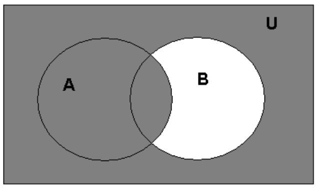
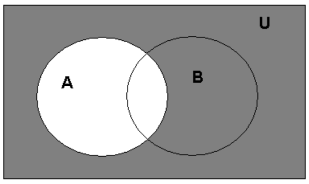
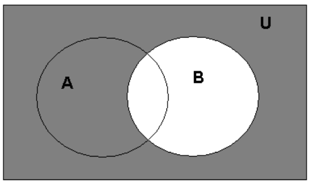
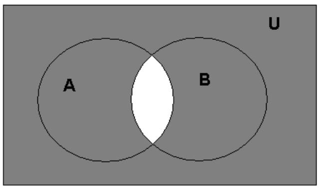
1

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5.</td><td></td><td>$I = 1.27$</td><td>1</td><td>04</td></tr>
<tr><td></td><td>c)</td><td>Using Simpson's 1/3rd rate ,evaluate $\int_0^1 \frac{1}{1+x^2} dx$, dividing the interval [0,1] into
Six equal parts.Hence find an approximate value of π</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$a = x_0 = 0\,, b = x_n = 1\,, n = 6$</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>
<table>
<tr><td>x</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{3}$</td><td>$\frac{5}{6}$</td><td>1</td></tr>
<tr><td>y</td><td>1</td><td>0.97</td><td>0.9</td><td>0.8</td><td>0.69</td><td>0.59</td><td>0.5</td></tr>
</table>
</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$I = \int_0^1 \frac{1}{1+x^2} dx = \frac{h}{3}[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$= \frac{\frac{1}{6}}{3}[(1 + 0.5) + 2(0.9 + 0.69) + 4(0.97 + 0.8 + 0.59)]$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$= \frac{1}{18}[1.5 + 2(1.59) + 4(2.36)]$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$= 0.7844$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\int_0^1 \frac{1}{1+x^2} dx = [\tan^{-1} x]_0^1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$= \tan^{-1} 1 - \tan^{-1} 0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$= \frac{\pi}{4}$</td><td>½</td><td>04</td></tr>
<tr><td></td><td></td><td>but $\frac{\pi}{4}$ =0.7844</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore \pi = 3.14$</td><td>½</td><td></td></tr>
<tr><td></td><td>d)</td><td>Apply Runge's formula of order 2, for finding approximate value of y when x=1.1</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>Given $\frac{dy}{dx} = \mathbf{3x + y^2}$ and y=1.2 when x=1</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\frac{dy}{dx} = 3x + y^2$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x_0 = 1\ , y_0 = 1.2\ \ , x_1 = 1.1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$h = x_1 - x_0 = 0.1$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$K_1 = hf(x_0, y_0) = (0.1)f(1,1.2) = (0.1)[3(1) + (1.2)^2]$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore K_1 = 0.444$</td><td></td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.		$K_2 = hf(x_0 + h, y_o + k_1) = (0.1)f(1 + 0.1,1.2 + 0.444) = (0.1)f(1.1,1.644)$		
		$= (0.1)[3(1.1) + (1.644)^2]$		
		$\therefore K_2 = 0.6003$	1	
		$K = \frac{K_1 + K_2}{2} = 0.5222$	½	
		$y_1 = y_0 + K = 1.2 + 0.5222$		
		$\therefore y_1 = 1.7222$	1	**04**
	e)	**Use Simpson's one-third rule to estimate approximately the area of the cross section of a river 80 ft.wide , the depth d(in feet) at a distance x from one bank being given by the following:**		
		x : 0 10 20 30 40 50 60 70 80		
		d: 0 4 7 9 12 15 14 8 3		
	Ans.	The area of the cross section of a river 80 ft. wide=		
		$A = \int_0^{80} d\,dx$ h=step length	1	
		$A = \int_0^{80} d\,dx = \frac{h}{3}[(d_0 + d_8) + 2(d_2 + d_4 + d_6) + 4(d_1 + d_3 + d_5 + d_6)]$		
		$= \frac{10}{3}[(0 + 3) + 2(7 + 12 + 14) + 4(4 + 9 + 15 + 8)]$	2	
		$= \frac{10}{3}[213]$		
		$\therefore A = 710\ sq.feet$	1	**04**
	f)	**Apply Runge-Kutta fourth order method , to find an approximate value of y when x=0.2 , given that**		
	Ans.	$\frac{dy}{dx} = x + y , y(0) = 1$ take h=0.2		
		$x_0 = 0 , y_0 = 1 , h = 0.2 , x_1 = 0.2$ $K_1 = hf(x_0, y_0) = (0.2)f(0,1) = (0.2)[0 + 1]$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5.		$\therefore K_1 = 0.2$	½	
		$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.2)f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right) = (0.2)f(0.1,1.1)$		
		$= (0.2)(0.1 + 1.1)$		
		$\therefore K_2 = 0.24$	1	
		$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.2)f(0.1,1.12) = (0.2)(0.1 + 1.12)$		
		$K_3 = 0.244$	1	
		$K_4 = hf(x_0 + h, y_0 + k_3) = (0.2)f(0.2 + 1.244)$		
		$\therefore K_4 = 0.2888$	½	
		$K = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$		
		$= \frac{0.2 + 2(0.24) + 2(0.244) + 0.2888}{6}$		
		$= 0.2428$	½	
		$y_1 = y_0 + K = 1 + 0.2428$		
		$\therefore y_1 = 1.2428$	½	**04**
6.		**Attempt any four of the following:**		**16**
	a)	**Apply Runge –Kutta fourth order method to find the value of y whenx=1.Given that**		
		$\frac{dy}{dx} = \frac{y - x}{y + x}$ **and** $y(0) = 1$		
		Let $\frac{dy}{dx} = f(x, y) = \frac{y - x}{y + x}$		
		$x_0 = 0$, $y_0 = 1$, $x_1 = 1$		
		h=1	½	
		$K_1 = hf(x_0, y_0) = (1)f(0,1)$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.		$= \dfrac{1-0}{1+0}$		
		$\therefore K_1 = 1$	½	
		$K_2 = hf\left(x_0 + \dfrac{h}{2}, y_0 + \dfrac{k_1}{2}\right) = (1)f\left(0 + \dfrac{1}{2}, 1 + \dfrac{1}{2}\right) = f(0.5, 1.5)$		
		$= \dfrac{1.5 - 0.5}{1.5 + 0.5}$		
		$\therefore K_2 = 0.5$	½	
		$K_3 = hf\left(x_0 + \dfrac{h}{2}, y_0 + \dfrac{k_2}{2}\right) = (1)f(0.5, 1.25) = \dfrac{1.25 - 0.5}{1.25 + 0.5} = \dfrac{0.75}{1.75}$		
		$\therefore K_3 = 0.4286$	½	
		$K_4 = hf(x_0 + h, y_0 + k_3) = (1)f(1, 1.4286) = \dfrac{1.4286 - 1}{1.4286 + 1} = \dfrac{0.4286}{2.4286}$		
		$\therefore K_4 = 0.1765$	½	
		$K = \dfrac{k_1 + 2k_2 + 2k_3 + k_4}{6}$		
		$= \dfrac{1 + 2(0.5) + 2(0.4286) + 0.1765}{6}$		
		$= 0.5056$	1	
		$y_1 = y_0 + K = 1 + 0.5056$		**04**
		$\therefore y_1 = 1.5056$	½	
	b)	**If $A = \{2,4,6,8\}$, $B = \{1,2,5,6,7\}$ and $C = \{0,2,6,7,8\}$ Verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$**		
	Ans.	$B \cap C = (2,6,7)$	½	
		$A \cup (B \cap C) = \{2,4,6,7,8\}$	1	
		$A \cup B = \{1,2,4,5,6,7,8\}$	½	
		$A \cup C = \{0,2,4,6,7,8\}$	1	**04**
		$(A \cup B) \cap (A \cup C) = \{2,4,6,7,8\}$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.	**c)**	**Shade the following sets using Venn diagram. (1)** $(A \cap B)'$ **(2)**$(B - A)'$ $(3)A' \cup B'$		
	Ans.	1) $A \cap B$ = 		
		$(A \cap B)'$= 	1	
		2) $B - A$ = 		
		$(B - A)'$= 	1	
		3) A' = 	½	
		B'= 	½	
		$A' \cup B'$= 	1	**04**

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.	**d)**	**Find how many integer from 1 to 200 are not divisible by 4 nor by 5.**		
		$n(X)=200$		
		$A=$ numbers divisible by 4.		
		$\therefore n(A)=\frac{200}{4}=50$	½	
		$B=$ numbers divisible by 5.		
		$\therefore n(B)=\frac{200}{5}=40$	½	
		$A\cap B=$ numbers divisible by 4 and 5.		
		$\therefore n(A\cap B)=\frac{200}{4\times 5}=10$	½	
		$\therefore n(A\cup B)=n(A)+n(B)-n(A\cap B)$		
		$=50+40-10$		
		$=80$	1	
		$(A\cup B)'=$ number not divisible by 4 nor by 5.		
		$\therefore n\left[(A\cup B)'\right]=n(X)-n(A\cup B)$	½	
		$=200-80$		
		$=120$	1	**04**
	e)	***Evaluate*** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{cotx}}$		
	Ans.	Consider $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{1+\sqrt{cotx}}$		
		$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\frac{\sqrt{cosx}}{\sqrt{sinx}}}$		
		$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx \qquad -(1)$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.		$x \to \frac{\pi}{3}+\frac{\pi}{6}-x=\frac{\pi}{2}-x\ (by\ property)$		
		$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{sin\left(\frac{\pi}{2}-x\right)}+\sqrt{cos\left(\frac{\pi}{2}-x\right)}}dx$	1	
		$\therefore I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}dx \qquad -(2)$	1	
		Adding equation (1) and equation (2)		
		$2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}dx$	½	
		$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}+\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}}dx$		
		$2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\,dx$		
		$2I=[x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	½	
		$2I=\frac{\pi}{3}-\frac{\pi}{6}$		
		$2I=\frac{\pi}{6}$		
		$I=\frac{\pi}{12}$	½	**04**
	f)	$\boldsymbol{Evaluate\ \int \frac{\sec^2 x}{(1+tanx)(2-tanx)}dx}$		
	Ans.	Put $t=tanx$		
		$dt=\sec^2 x\,dx$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6.		$Consider\ \frac{1}{(1+t)(2-t)} = \frac{A}{1+t} + \frac{B}{2-t}$	½	
		$1 = A(2-t) + B(1+t)$		
		$\therefore A = \frac{1}{3}$	½	
		$\therefore B = \frac{1}{3}$	½	
		$\therefore \frac{1}{(1+t)(2-t)} = \frac{\frac{1}{3}}{1+t} + \frac{\frac{1}{3}}{2-t}$		
		$I = \int \frac{\frac{1}{3}}{1+t} + \frac{\frac{1}{3}}{2-t} dt$	½	
		$I = \frac{1}{3} log(1+t) + \frac{1}{3} \frac{log(2-t)}{-1}$	1	
		$I = \frac{1}{3} log(1+tanx) - \frac{1}{3} log(2-tanx)$	½	**04**
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*		