SUMMER – 2013 EXAMINATION

MODEL ANSWER

Subject: ENGINEERING MATHEMATICS

Subject Code: 12013

Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numerical shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1)		**Attempt any ten of the following:**		20
	(a)	If $f(x)=x^2-2x+5$, find $f(1)+f(2)$		
	Ans.	Given, $f(x)=x^2-2x+5$		
		$\therefore f(1)+f(2)$	½	
		$=(1)^2-2(1)+5+(2)^2-2(2)+5$	1	
		$=4+5$		
		$=9$	½	02
	(b)	If $f(x)=x^2+4$, then show that $f(x+1)-f(x+1)=4x$		
	Ans.	Given $f(x)=x^2+4$		
		$\therefore f(x+1)=(x+1)^2+4$		
		$=x^2+2x+1+4$		
		$=x^2+2x+5$		
		$\therefore f(x+1)=x^2+2x+5$	1	
		Consider, $f(x+1)-f(x-1)$		
		$=x^2+2x+5-(x^2-2x+5)$		
		$=x^2+2x+5-x^2+2x-5$		
		$=4x$	1	02
	c)	Evaluate: $\lim_{x\to 4}\frac{x^2-16}{x-4}$		
	Ans.	$\lim_{x\to 4}\frac{x^2-16}{x-4}$		
		$=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}$	1	
		$=\lim_{x\to 4}(x+4)$		
		$=4+4=8$	1	02
		OR		
		$\lim_{x\to 4}\frac{x^2-16}{x-4}$		
		$=\lim_{x\to 4}\frac{x^2-(4)^2}{x-4}$	1	
		$=2(4)^{2-1}$		
		$=2(4)=8$	1	02

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.	d)	Evaluate: $\lim_{x\to 0}\frac{\tan 3x}{\tan 5x}$		
	Ans.	$\lim_{x\to 0}\frac{\tan 3x}{\tan 5x}$		
		$=\lim_{x\to 0}\frac{\frac{\tan 3x}{x}}{\frac{\tan 5x}{x}}$	½	
		$=\frac{\left(\lim_{x\to 0}\frac{\tan 3x}{3x}\right).3}{\left(\lim_{x\to 0}\frac{\tan 5x}{5x}\right).5}$	1	
		$=\frac{(1)3}{(1)5}=\frac{3}{5}$	½	02
	(e)	Evaluate: $\lim_{x\to 0}\frac{e^{2x}-e^{3x}}{x}$		
	Ans	$\lim_{x\to 0}\frac{e^{2x}-e^{3x}}{x}$		
		$=\lim_{x\to 0}\frac{e^{2x}-1-e^{3x}+1}{x}$	½	
		$=\lim_{x\to 0}\left[\frac{\left(e^{2x}-1\right)-\left(e^{3x}-1\right)}{x}\right]$		
		$=\lim_{x\to 0}\frac{e^{2x}-1}{x}-\lim_{x\to 0}\frac{e^{3x}-1}{x}$		
		$=\left(\lim_{x\to 0}\frac{e^{2x}-1}{2x}\right).2-\left(\lim_{x\to 0}\frac{e^{3x}-1}{3x}\right).3$	1	
		$=(1)2-(1)3$	½	02
		$=-1$		
	(f)	Differentiate with respect to x: $3^x+x^3+\sin x+3^3$		
	Ans.	Let $y=3^x+x^3+\sin x+3^3$		
		diff.w.t.to x		
		$\frac{dy}{dx}=3^x\log 3+3x^2+\cos x$	02	02
	g)	Find $\frac{dy}{dx}$, given that $y=x\sin x$		
	Ans	$y=x\sin x$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.		$\frac{dy}{dx} = x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x)$		
		$\frac{dy}{dx} = x\cos x + \sin x$	02	02
	(h)	Differentiate with respect to x: $y = \log(x^2 + 2x + 5)$		
	Ans	$y = \log(x^2 + 2x + 5)$		
		$\frac{dy}{dx} = \frac{1}{x^2 + 2x + 5} \cdot \frac{d}{dx}(x^2 + 2x + 5)$	1	
		$\frac{dy}{dx} = \frac{2x + 2}{x^2 + 2x + 5}$	1	02
	(i)	If $y = \frac{e^x}{x+1}$, find $\frac{dy}{dx}$		
	Ans	$y = \frac{e^x}{x+1}$		
		$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x+1)}{(x+1)^2}$		
		$\frac{dy}{dx} = \frac{(x+1)e^x - e^x 1}{(x+1)^2}$	1	
		$\frac{dy}{dx} = \frac{e^x(x+1-1)}{(x+1)^2}$		
		$\frac{dy}{dx} = \frac{xe^x}{(x+1)^2}$	1	02
	(j)	Find mean of the following data:		
	Ans	40,72,83,57,94,49,65,79,87,64.		
		N=10		
		$\therefore$ Mean, $\bar{x} = \frac{40+72+83+57+94+49+65+79+87+64}{10}$	1	
		$\bar{x} = \frac{690}{10}$		
		$\bar{x} = 69$	1	02
	k)	Following are marks of 10 students:		
	Ans	25,30,40,15,69,75,45,52,60,65.		
		Arrange the data in increasing order:		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.		15,25,30,40,45,52,60,65,69,75.	½	
		N=10=even		
		$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} \text{ place of observation} + \left(\frac{N}{2}+1\right)^{th} \text{ place of observation}}{2}$		
		$= \frac{\left(\frac{10}{2}\right)^{th} \text{ place of observation} + \left(\frac{10}{2}+1\right)^{th} \text{ place of observation}}{2}$		
		$= \frac{(5)^{th} \text{ place of observation} + (6)^{th} \text{ place of observation}}{2}$	1	
		$= \frac{45+52}{2}$	½	02
		$= \frac{97}{2} = 48.5$		
	(l)	Evaluate: $\lim_{x\to 0}(1+2x)^{\frac{1}{x}}$		
	Ans.	$\lim_{x\to 0}(1+2x)^{\frac{1}{x}}$	1	
		$= \left[\lim_{x\to 0}(1+2x)^{\frac{1}{2x}}\right]^2$		
		$= e^2$	1	02
2)	(a)	**Attempt any four of the following:**		
		If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ then show that $f(t) = x$		
	Ans.	$f(t) = \frac{t+3}{4t-5}$	½	
		$= \frac{\left(\frac{3+5x}{4x-1}\right)+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$		
		$= \frac{3+5x+12x-3}{12+20x-20x+5}$	2	
		$= \frac{17x}{17}$	1	
		$= x$	½	04

2.	b)	If $f(x)=\frac{1}{x}$, then show that $f(x)-f(x+1)=f(x^2+x)$		
	Ans	Given $f(x)=\frac{1}{x}$		
		Consider,		
		$f(x)-f(x+1)$		
		$=\frac{1}{x}-\frac{1}{x+1}$	1	
		$=\frac{x+1-x}{x(x+1)}$	2	04
		$=\frac{1}{x^2+x}$	1	
		$=f(x^2+x)$		
	c)	Evaluate: $\lim_{x\to 2}\left[\frac{1}{x-2}+\frac{6x}{8-x^3}\right]$		
	Ans.	$\lim_{x\to 2}\left[\frac{1}{x-2}+\frac{6x}{8-x^3}\right]$		
		$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{6x}{x^3-8}\right]$		
		$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{6x}{x^3-(2)^3}\right]$	½	
		$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{6x}{(x-2)(x^2+2x+4)}\right]$	1	
		$=\lim_{x\to 2}\frac{1}{(x-2)}\left[1-\frac{6x}{x^2+2x+4}\right]$		
		$=\lim_{x\to 2}\frac{1}{(x-2)}\left[\frac{x^2+2x+4-6x}{x^2+2x+4}\right]$	1	
		$=\lim_{x\to 2}\frac{1}{(x-2)}\left[\frac{x^2-4x+4}{x^2+2x+4}\right]$		
		$=\lim_{x\to 2}\frac{1}{(x-2)}\left[\frac{(x-2)^2}{x^2-2x+4}\right]$	1	
		$=\lim_{x\to 2}\frac{(x-2)}{x^2-2x+4}$		04
		$=\frac{0}{4}=0$	½	
	d)	Evaluate: $\lim_{x\to 1}\frac{\sqrt{x+4}-\sqrt{5}}{x-1}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	Ans	$\lim_{x\to 1}\frac{\sqrt{x+4}-\sqrt{5}}{x-1}$		
		$=\lim_{x\to 1}\frac{\sqrt{x+4}-\sqrt{5}}{x-1}\times\frac{\sqrt{x+4}+\sqrt{5}}{\sqrt{x+4}+\sqrt{5}}$	1	
		$=\lim_{x\to 1}\frac{x+4-5}{(x-1)\left(\sqrt{x+4}+\sqrt{5}\right)}$	1	
		$=\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x+4}+\sqrt{5}\right)}$		
		$=\lim_{x\to 1}\frac{1}{\sqrt{x+4}+\sqrt{5}}$	1	
		$=\frac{1}{\sqrt{1+4}+\sqrt{5}}$		
		$=\frac{1}{2\sqrt{5}}$	1	04
	e)	Evaluate: $\lim_{x\to 0}\frac{\cos x-\cos 2x}{x^2}$		
	Ans	$\lim_{x\to 0}\frac{\cos x-\cos 2x}{x^2}$		
		$=\lim_{x\to 0}\frac{-2\sin\left(\frac{x+2x}{2}\right).\sin\left(\frac{x-2x}{2}\right)}{x^2}$	1	
		$=\lim_{x\to 0}\frac{-2\sin\left(\frac{3x}{2}\right).\sin\left(\frac{-x}{2}\right)}{x^2}$		
		$=-2\lim_{x\to 0}\frac{\sin\left(\frac{3x}{2}\right)}{x}.\frac{\sin\left(\frac{-x}{2}\right)}{x}$	½	
		$=-2\left[\lim_{x\to 0}\frac{\sin\left(\frac{3x}{2}\right)}{\frac{3x}{2}}\right].\frac{3}{2}\left[\lim_{x\to 0}\frac{\sin\left(\frac{-x}{2}\right)}{\frac{-x}{2}}\right].\frac{-1}{2}$	1	
		$=-2(1)\left(\frac{3}{2}\right)(1)\left(\frac{-1}{2}\right)$	1	
		$=\frac{3}{2}$	½	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	f)	Evaluate: $\lim_{x\to 0}\frac{3^x-2^x}{\sin \pi x}$		
	Ans	$\lim_{x\to 0}\frac{3^x-2^x}{\sin \pi x}$	½	
		$=\lim_{x\to 0}\frac{3^x-1-2^x+1}{\sin \pi x}$		
		$=\lim_{x\to 0}\frac{(3^x-1)-(2^x-1)}{\sin \pi x}$	½	
		$=\lim_{x\to 0}\frac{(3^x-1)-(2^x-1)}{x}.\frac{x}{\sin \pi x}$		
		$=\left[\lim_{x\to 0}\frac{3^x-1}{x}-\lim_{x\to 0}\frac{2^x-1}{x}\right].\left(\lim_{x\to 0}\frac{\pi x}{\sin \pi x}\right).\frac{1}{\pi}$	1	
		$=(\log 3-\log 2)\frac{1}{\pi}$	2	04
		OR		
		$=\frac{1}{\pi}(\log 3-\log 2)$		
3)		**Attempt any four of the following:**		
	a)	Differantiate w.r.to x : $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$		
	Ans.	Let $y=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$		
		$y=\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$	2	
		$y=\sqrt{\tan^2 x}$		
		$y=\tan x$	1	
		$\therefore \frac{dy}{dx}=\sec^2 x$	1	04
		OR		
		Let $y=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$		
		$y=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}\times\frac{1-\cos 2x}{1-\cos 2x}}$		
		$y=\sqrt{\frac{(1-\cos 2x)^2}{1-\cos^2 2x}}$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$y=\sqrt{\frac{(1-\cos 2x)^2}{\sin^2 2x}}$	½	04
		$y=\frac{1-\cos 2x}{\sin 2x}$		
		$y=\frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$	1	
		$y=\cos ec2x-\cot 2x$		
		$\frac{dy}{dx}=-2\cos ec2x\cot 2x+2\cos ec^2 2x$	2	
	b) Ans	Diff. $\cos^{-1}\left(2x^2-1\right)$ w.r.to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$		
		Let $y=\cos^{-1}\left(2x^2-1\right)$ and $z=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$		
		Put $x=\cos\theta$		
		$\therefore\ y=\cos^{-1}\left(2\cos^2\theta-1\right)$		
		$=\cos^{-1}\left(\cos 2\theta\right)$		
		$y=2\theta$		
		$\therefore\ y=2\cos^{-1}x$	1	
		$\frac{dy}{dx}=\frac{-2}{\sqrt{1-x^2}}$	½	
		and $z=\sin^{-1}\left(2\cos\theta\sqrt{1-\sin^2\theta}\right)$		
		$z=\sin^{-1}\left(2\cos\theta\sin\theta\right)$		
		$z=\sin^{-1}\left(\sin 2\theta\right)$	1	
		$z=2\theta$		
		$z=2\cos^{-1}x$	½	
		$\frac{dz}{dx}=\frac{-2}{\sqrt{1-x^2}}$		
		$\therefore\ \frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}$		
		$=\frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore \frac{dy}{dz} = 1$	1	04
	c) Ans	If $e^y = y^x$, prove that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$		
		Given $e^y = y^x$		
		$\therefore \log e^y = \log y^x$		
		$\therefore y \log e = x \log y$		
		$\therefore y = x \log y \text{ ----------} (1)$	1	
		$\frac{dy}{dx} = x\frac{1}{y}\frac{dy}{dx} + \log y$		
		$\left(1 - \frac{x}{y}\right)\frac{dy}{dx} = \log y$		
		$\left(\frac{y-x}{y}\right)\frac{dy}{dx} = \log y$	1	
		$\frac{dy}{dx} = \frac{y \log y}{y - x}$		
		$\frac{dy}{dx} = \frac{y \log y}{y - \frac{y}{\log y}}$ ----------$(\because$ by eq.1)	1	04
		$\frac{dy}{dx} = \frac{y(\log y)^2}{y \log y - y}$	1	
		$\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$		
	e) Ans	Differantiate w.r.to $x : x \sin^{-1} x + \sqrt{1-x^2}$		
		Let $y = x \sin^{-1} x + \sqrt{1-x^2}$		
		$\therefore \frac{dy}{dx} = x\frac{1}{\sqrt{1-x^2}} + \sin^{-1} x + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}(1-x^2)$	2	
		$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x - \frac{2x}{2\sqrt{1-x^2}}$	1	
		$= \sin^{-1} x$	1	04
	f) Ans	If $13x^2 + 2x^2y + y^3 = 1$, find $\frac{dy}{dx}$ at $(1, -2)$		
		$13x^2 + 2x^2y + y^3 = 1$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
3)		$\therefore 26x+2\left(x^2.\frac{dy}{dx}+y.2x\right)+3y^2\frac{dy}{dx}=0$ $26x+2x^2.\frac{dy}{dx}+4xy+3y^2\frac{dy}{dx}=0$ $26x+4xy+\left(2x^2+3y^2\right)\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-26x-4xy}{2x^2+3y^2}=\frac{-18}{14}=\frac{-9}{7}$	2 1+1	04
4)		**Attempt any four of the following:**		
	a)	Find equation of tangent and normal to the curve $2x^2-xy+3y^2=18$ at point $(3,1)$		
	Ans	$2x^2-xy+3y^2=18$ $4x-\left(x\frac{dy}{dx}+y.1\right)+6y\frac{dy}{dx}=0$ $4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$ $\left(-x+6y\right)\frac{dy}{dx}=-4x+y$ $\frac{dy}{dx}=\frac{-4x+y}{-x+6y}$ at $(3,1)$ $\frac{dy}{dx}=\frac{-11}{3}=m$, slope of tangent $y-y_1=m\left(x-x_1\right)$ $y-1=\frac{-11}{3}\left(x-3\right)$ $11x+3y-36=0$ ------- equation of tangent $-\frac{1}{m}=\frac{3}{11}$, slope of normal $y-y_1=\frac{3}{11}\left(x-x_1\right)$ $3x-11y+2=0$ ------- equation of normal	1 1 1 1	04
	d)	Divide number 80 into two parts so that the product is maximum.		

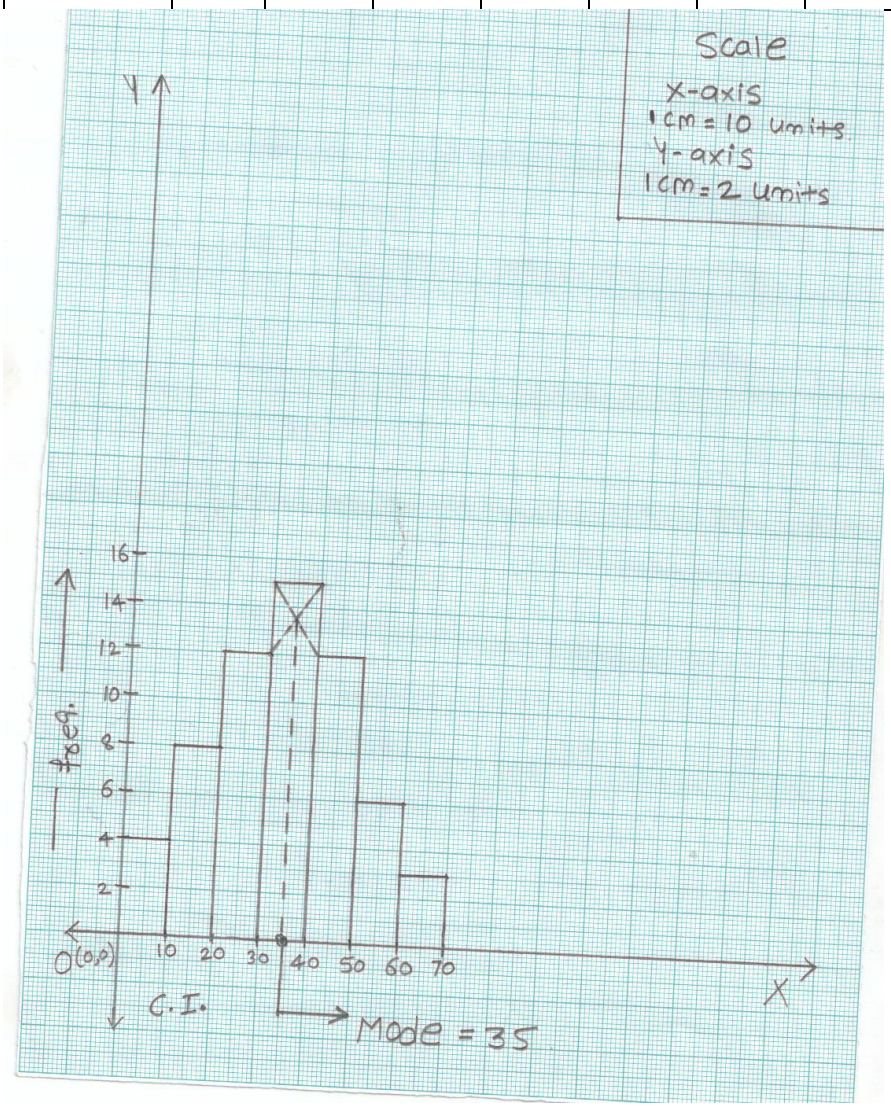
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	Ans	Consider these numbers are x and $80-x$		
		Product $P = x(80-x)$		
		$P = 80x - x^2$		
		$\frac{dP}{dx} = 80 - 2x$	1	
		Given that product is maximum		
		$\therefore \frac{dP}{dx} = 0$	1	
		$\therefore 80 - 2x = 0$	1	
		$x = 40$		
		and $80 - x = 80 - 40$	1	04
		$= 40$		
	c) Ans	Find radius of curvature of curve $y = x^3$ at point (1,1)		
		Given $y = x^3$		
		$\therefore \frac{dy}{dx} = 3x^2$		
		$\frac{d^2y}{dx^2} = 6x$	1	04
		At (1,1):		
		$\frac{dy}{dx} = 3$		
		$\frac{d^2y}{dx^2} = 6$	1	
		$\therefore$ Radius of curvatureis, $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)		$\rho = \frac{\left[1+(3)^2\right]^{\frac{3}{2}}}{6}$	1	
		$\rho = \frac{(10)^{\frac{3}{2}}}{6}$		
		$\rho = 5.27$	1	04
	d)	An open box is made up of rectangular sheet of card board measuring 16 cm×10 cm by cutting of equal squres from the corners and turning up the sides. Find the sides of the squres so that volume of box is maximum.		
	Ans	Let length of square $= x$		
		$\therefore$ length of box $= 16-2x$		
		breadth of box $= 10-2x$		
		Volume V=$l \times b \times h$		
		$= (16-2x)(10-2x)x$ $\quad (\because h = \text{length of square})$	1	
		$V = 160x - 52x^2 + 4x^3$		
		$\therefore \frac{dV}{dx} = 160 - 104x + 12x^2$	1	
		Given that Volume is maximum		
		$\therefore \frac{dV}{dx} = 0$	1	
		$160 - 104x + 12x^2 = 0$		
		$3x^2 - 26x + 40 = 0$		
		$x = \frac{20}{3}$ OR $x = 2$	1	04
		$\therefore$ Sides of square is either 20/3 or 2		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	e)	Find mode graphically for the following:	04	04

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Students	4	8	12	15	12	6	3

Ans



Marks distribution: *2 marks for plotting points and drawing histogram correctly. 1 mark for drawing line of mode to X-axis. 1 mark for value of mode. Note the value 35 is approximate value. Difference of 0.5 or -0.5 is acceptable in case of graph.*

(If class 0-10,10-20,20-30…etcare written, no marks to be given).

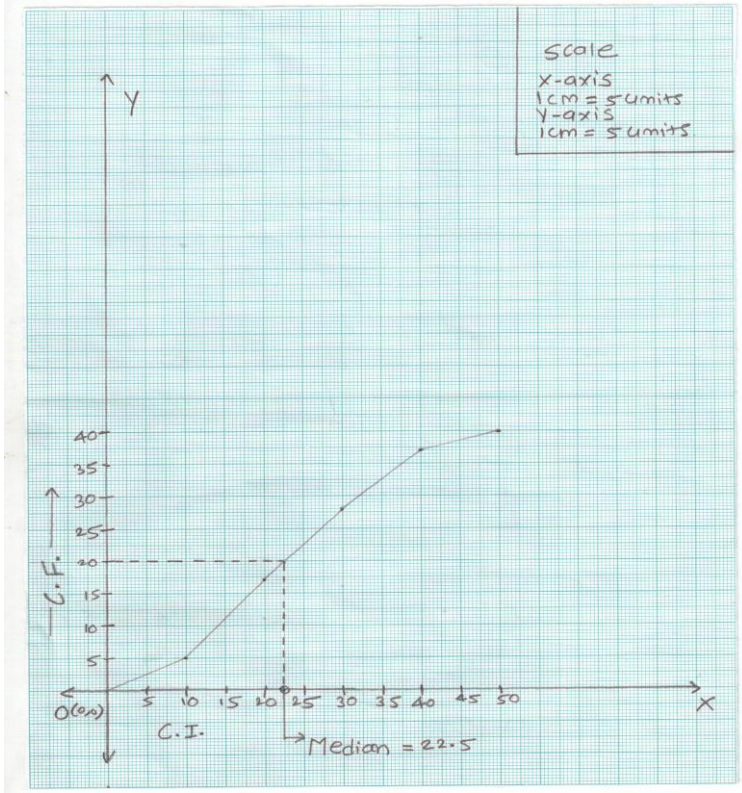
Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4)	f)	Find median for the following data by Ogive curve method:		

Score	0-10	10-20	20-30	30-40	40-50
frequency	5	12	16	4	3

Ans

C.I.	Freq.	C.F.(less than)
0-**10**	5	5
10-**20**	12	17
20-**30**	16	33
30-**40**	4	37
40-**50**	3	40
	N=40	
N/2=20		

1



03

Marks distribution: *1 mark for C.F.table (less than or more than).*
1 mark for plotting points and drawing curve correctly. 1 mark for drawing line of median to X-axis.
1 mark for value of median. Note the value 22.5 is approximate value. Difference of 0.5 or -0.5 is acceptable in case of graph.

04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		**Attempt any four of the following:**		
	a)	Find mode of the following;		

C.I.	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Freq.	3	5	9	15	20	16	10	2

Ans

Class interval	Frequency
0-5	3
5-10	5
10-15	9
15-20	15 **f1**
20-25	20 **fm**
25-30	16 **f1**
30-35	10
35-40	2

$$\text{Mode} = L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right) \times c$$

$$= 20 + \left(\frac{20 - 15}{40 - 15 - 16} \right) \times 5$$

$$= 22.778$$

3
1

04

(b) Find mean of following data by step deviation method:

Wt in kg	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No of pers.	16	21	20	28	10	3	1	1

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

5) Ans.

C.I.	xi	fi	$d_i = \frac{x_i - A}{c}$	fidi
10-20	15	16	-3	-48
20-30	25	21	-2	-42
30-40	35	20	-1	-20
40-50	45 (A)	28	0	0
50-60	55	10	1	10
60-70	65	3	2	6
70-80	75	1	3	3
80-90	85	1	4	4
		$\sum f_i = 100$ =N		$\sum f_i d_i = -87$

02

$\therefore$ Mean, $\bar{x} = A + \left(\frac{\sum f_i d_i}{N}\right) \times c$ 1

$= 45 + \left(\frac{-87}{100}\right) \times 10$

$= 36.3$ 1

c) Which of the following set is more consistent?

set	Mean	Standard deviation
I	80	6.0
II	60	7.2

Ans

$C.V.(I) = \frac{\sigma}{\bar{x}} \times 100 = \frac{6}{80} \times 100$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		$C.V.(I) = 7.5$	1	04
		$C.V.(II) = \frac{\sigma}{\bar{x}} \times 100 = \frac{7.2}{60} \times 100$	1	
		$C.V.(II) = 12$		
		$\therefore C.V.(I) < C.V.(II)$	1	
		$\therefore$ Set I is more consistent.	1	
	d)	Find median of the following;		

Mid value	115	125	135	145	155	165	175	185	195
Freq.	6	25	48	72	116	60	38	22	3

Ans

Class	fi	C.F.(less than)
110-120	6	6
120-130	25	31
130-140	48	79
140-150	72	151
150-160	**116**	**267**
160-170	60	327
170-180	38	365
180-190	22	387
190-200	3	390
	$\sum f_i = 390 = N$	

1

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)		N/2=195 $\text{Median} = L + \left(\frac{N/2 - f_c}{f_m} \right) \times c$ $= 150 + \left(\frac{195 - 151}{116} \right) \times 10$ $= 153.79$	2 1	04
	e)	The mean of 60 observations was calculated as 276 mm afterwards it was noticed that one observation was recorded wrongly as 337 instead of 373.Find correct mean.		
	Ans	Given $\bar{x} = 276, N = 60$ $\text{Mean, } \bar{x} = \frac{\sum x_i}{N}$ $276 = \frac{\sum x_i}{60}$ $\therefore \sum x_i = 16560$	1	
		$\therefore \text{Corrected Sum} = \sum x_i - \text{wrong observation+correct observation}$ $= 16560 - 337 + 373$ $= 16596$	1	
		$\therefore \text{Correct Mean} = \frac{\text{Corrected Sum}}{\text{N}}$ $= \frac{16596}{60}$ $= 276.6$	2	04
	f)	Find mean deviation about mean;		

C.I.	10-14	15-19	20-24	25-29	30-34	35-39	40-44	44-49
Freq.	4	6	10	5	7	3	9	6

04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks

5) Ans

C.I.	xi	fi	fixi	$\lvert d_i\rvert = \lvert x_i - \bar{x}\rvert$	$f_i \lvert d_i\rvert$
9.5-14.5	12	4	48	18	72
14.5-19.5	17	6	102	13	78
19.5-24.5	22	10	220	8	80
24.5-29.5	27	5	135	3	15
29.5-34.5	32	7	224	2	14
34.5-39.5	37	3	111	7	21
39.5-44.5	42	9	378	12	108
44.5-49.5	47	6	282	17	102
		$\sum f_i = 50$	$\sum f_i x_i = 1500$		490

2

Mean, $\bar{x} = \frac{\sum x_i}{N} = \frac{1500}{50} = 30$ 1

$\therefore$ M.D. $= \frac{\sum f_i d_i}{N} = \frac{490}{50} = 9.8$ 1

04

FOR CE/EJ/ME

6) **Attempt any four of the following:**

a) Find modulus and argument of complex no. $\frac{(3+i)(1-i)}{2+i}$

Ans

Let $z = \frac{(3+i)(1-i)}{2+i}$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	Ans.	$= \frac{3-3i+i-i^2}{2+i}$ $= \frac{4-2i}{2+i}$ $= \frac{4-2i}{2+i} \times \frac{2-i}{2-i}$ $= \frac{8-4i-4i+2i^2}{(2)^2-i^2}$ $= \frac{6-8i}{5} = \frac{6}{5} - \frac{8}{5}i$	1 1	
		$\lvert z \rvert = \sqrt{\left(\frac{6}{5}\right)^2 + \left(\frac{-8}{5}\right)^2}$ $= \sqrt{\frac{100}{25}} = \sqrt{4} = 2$ $x > 0, y < 0$ $\therefore \theta = 2\pi - \tan^{-1}\left(\left\lvert\frac{-8/5}{6/5}\right\rvert\right)$ $\therefore \theta = 2\pi - \tan^{-1}(4/3)$ *OR* $\therefore \theta = \tan^{-1}\left(\frac{-8/5}{6/5}\right)$ $\theta = -\tan^{-1}(4/3)$	1 1	04
	b) Ans.	Simplify: $\frac{(\cos 2\theta + i\sin 2\theta)(\cos\theta - i\sin\theta)^4}{(\cos 3\theta + i\sin 3\theta)(\cos 5\theta - i\sin 5\theta)}$ $\frac{(\cos 2\theta + i\sin 2\theta)(\cos\theta - i\sin\theta)^4}{(\cos 3\theta + i\sin 3\theta)(\cos 5\theta - i\sin 5\theta)}$ $= \frac{(\cos\theta + i\sin\theta)^2 (\cos\theta + i\sin\theta)^{-4}}{(\cos\theta + i\sin\theta)^3 (\cos\theta + i\sin\theta)^{-5}}$ $= (\cos\theta + i\sin\theta)^{2-4-3+5}$ $= (\cos\theta + i\sin\theta)^0$ $= 1$	1 2 1	04
	c)	Using Euler's formula prove that; i) $\sin^2\theta + \cos^2\theta = 1$ ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)	Ans	i) $\sin^2\theta+\cos^2\theta$		04
		$=\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2+\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2$		
		$=\frac{1}{4i^2}\left(e^{i\theta}-e^{-i\theta}\right)^2+\frac{1}{4}\left(e^{i\theta}+e^{-i\theta}\right)^2$	1	
		$=\frac{-1}{4}\left(e^{2i\theta}-2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)+\frac{1}{4}\left(e^{2i\theta}+2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)$		
		$=\frac{1}{4}\left(4e^{i\theta}e^{-i\theta}\right)=\frac{1}{4}\left(4e^{0}\right)$		
		$=1$	1	
		ii) $\cos^2\theta-\sin^2\theta$		
		$=\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2-\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2$		
		$=\frac{1}{4}\left(e^{i\theta}+e^{-i\theta}\right)^2-\frac{1}{4i^2}\left(e^{i\theta}-e^{-i\theta}\right)^2$	1	
		$=\frac{1}{4}\left(e^{2i\theta}+2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)-\left(\frac{-1}{4}\right)\left(e^{2i\theta}-2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)$		
		$=\frac{1}{4}\left(2e^{2i\theta}+2e^{-2i\theta}\right)=\frac{e^{i\theta}+e^{-i\theta}}{2}$		
		$=\cos 2\theta$	1	
	d)	Express $(1+i)$ in polar form and show that $(1+i)^8+(1-i)^8=32$		
	Ans	Let $z=1+i$		
		$\therefore x=1, y=1$		
		$r=\lvert z\rvert=\sqrt{(1)^2+(1)^2}=\sqrt{2}$		
		$\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{1}{1}\right)$		
		$\theta=\tan^{-1}(1)=\frac{\pi}{4}$		
		$z=r(\cos\theta+i\sin\theta)$		
		$(1+i)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$	1	
		$(1+i)^8=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^8$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$(1+i)^8 = 16(\cos 2\pi + i\sin 2\pi)$	1	
		Simillarly, $(1-i)^8 = 16(\cos 2\pi - i\sin 2\pi)$		
		$\therefore (1+i)^8 + (1-i)^8$		
		$= 16(\cos 2\pi + i\sin 2\pi) + 16(\cos 2\pi - i\sin 2\pi)$	1	
		$= 16(\cos 2\pi + i\sin 2\pi + \cos 2\pi - i\sin 2\pi)$		
		$= 32\cos 2\pi$		
		$= 32$	1	04
	e)	Separate into real and imaginary parts of $\frac{(1+i^3)^2}{(3-i)^3}$		
	Ans	$\frac{(1+i^3)^2}{(3-i)^3}$		
		$= \frac{1+2i^3+(i^3)^2}{(3)^3-3(3)^2 i+3(3)i^2-i^3}$	1	
		$= \frac{1-2i+i^6}{27-27i-9+i}$		
		$= \frac{1-2i-1}{18-26i}$	1	
		$= \frac{-2i}{18-26i}$		
		$= \frac{-2i}{18-26i} \times \frac{18+26i}{18+26i}$	1	
		$= \frac{-36i-52i^2}{324+676}$		04
		$= \frac{52}{1000} - \frac{36i}{1000} = \frac{13}{250} - \frac{9i}{250}$	1	
	f)	Find all roots of $(1-i)^{\frac{1}{4}}$		
	Ans.	Let $z = 1-i$		
		$\therefore x = 1, y = -1$		
		$r = \lvert z\rvert = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$	½	
		$\theta = 2\pi - \tan^{-1}\left(\left\lvert\frac{y}{x}\right\rvert\right) = 2\pi - \tan^{-1}(\lvert -1\rvert)$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$z = r\left[\cos(2\pi n+\theta)+i\sin(2\pi n+\theta)\right]$ $(1-i)=\sqrt{2}\left[\cos\left(2\pi n+\frac{\pi}{4}\right)+i\sin\left(2\pi n+\frac{\pi}{4}\right)\right]$	1	
		$(1-i)^{\frac{1}{4}}=\left\{\sqrt{2}\left[\cos\left(2\pi n+\frac{\pi}{4}\right)+i\sin\left(2\pi n+\frac{\pi}{4}\right)\right]\right\}^{\frac{1}{4}}$ $=\left(\sqrt{2}\right)^{\frac{1}{4}}\left[\cos\left(\frac{8\pi n+4\pi}{16}\right)+i\sin\left(\frac{8\pi n+4\pi}{16}\right)\right]$	1	
		Put $n=0,1,2,3$ $\therefore \left(\sqrt{2}\right)^{\frac{1}{4}}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right), \quad \left(\sqrt{2}\right)^{\frac{1}{4}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ $\left(\sqrt{2}\right)^{\frac{1}{4}}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right), \quad \left(\sqrt{2}\right)^{\frac{1}{4}}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$	1	04
		For Computer/Information Technology		
6)		**Attempt any four of the following:**		
	a)	Find the aproximate root of the equation $x^3-x-4=0$ by bisection method (3 iterations)		
	Ans	Let $f(x)=x^3-x-4$ $f(1)=-4<0$ $f(2)=2>0$ $\therefore$ root lies in $(1,2)$	1	
		$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$ $f(1.5)=-2.125<0$ $\therefore$ the root lies in (1.5,2)	1	
		$x_2=\frac{x_1+b}{2}=\frac{1.5+2}{2}=1.75$ $f(x_2)=-0.39<0$ $\therefore$ the root lies in (1.75,2)	1	
		$x_3=\frac{x_2+b}{2}=\frac{1.75+2}{2}=1.875$	1	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		OR Let $f(x) = x^3 - x - 4$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ $\therefore$ root lies in $(1,2)$	1	
		a | b | $x = \frac{a+b}{2}$ | $f(x)$ 1 | 2 | 1.5 | -2.125 1.5 | 2 | 1.75 | -0.39 1.75 | 2 | 1.875 | ---	1+1+1	04
	b) Ans	Using Regula-Falsi method, find the root of the equation $x^3 - 3x + 1 = 0$ (3 iterations). Let $f(x) = x^3 - 3x + 1$		
		$f(0) = 1 > 0$ $f(1) = -1 < 0$	1	
		$\therefore$ the root lies in (0,1)		
		$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0-(1)}{-1-(1)} = \frac{1}{2} = 0.5$	1	
		$f(x_1) = -0.375 < 0$		
		$\therefore$ the root lies in (0,0.5)		
		$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)} = 0.36$	1	
		$f(x_2) = -0.033 > 0$		
		$\therefore$ the root lies in (0,0.36)		
		$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = 0.34$	1	04
		OR $f(0) = 1 > 0$ $f(1) = -1 < 0$	1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$\therefore$ the root lies in (0,1)		

a	b	$f(a)$	$f(b)$	$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$
0	1	1	-1	0.5	-0.375
0	0.5	1	-0.375	0.36	-0.033
0	0.36	1	-0.033	0.34	---

1+1+1

04

c) Solve by Newton-Raphson method $x^3+2x-20=0$ up two iterations only.

Ans Let, $f(x)=x^3+2x-20$

$f(2)=-8<0$

$f(3)=13>0$

$f'(x)=3x^2+2$ 1

$\therefore f'(2)=14$

$\therefore$ Initial root $x_0=2$

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{(-8)}{14}=2.57$ 1½

$f(2.57)=2.11$ and $f'(2.57)=21.81$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=2.57-\frac{2.11}{21.81}=2.47$ 1½

04

d) Using Gauss elimination method, solve the following equations: $x+2y+z=2, 3x+y-2z=1, 4x-3y-z=3$

Ans

Given, $x+2y+z=2$

$3x+y-2z=1$

$4x-3y-z=3$

$x+2y+z=2$

$6x+2y-4z=2$

$-5x+5z=0$ 1+1

and

$9x+3y-6z=3$

$4x-3y-z=3$

$13x-7z=6$

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		$-7x+7z=0$ $\underline{13x-7z=6}$ $6x=6$ $\therefore x=1$ $y=0$ $z=1$	1 1	04
		Note: *In the above solution, first y is eliminated and then z is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking. Let us see, how the solution becomes by eliminating first z and then y to get the value of , as illustrated below:*		
		$2x+2y+2z=4$ $3x+y-2z=1$ $+\_\_\_\_\_$ $5x+3y=5$ and $3x+y-2z=1$ $8x-6y-2z=6$ $\underline{-\quad+\quad+\quad-}$ $-5x+7y=-5$	1+1	
		$-5x+7y=-5$ $\underline{5x+7y=5}$ $10y=0$ $\therefore y=0$ $x=1$ $z=1$	1 1	04
	e) Ans	Solve using Jacobi's method. (three iterations only) $10x+y+2z=13, 3x+10y+z=14, 2x+3y+10z=15$ $x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$ Starting with $x_0=y_0=z_0=0$ *I*) $x_1=1.3$ $y_1=1.4$ $z_1=1.5$	 1	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
6)		*I*)		
		$x_2 = 0.86$		
		$y_2 = 0.86$		
		$z_2 = 0.82$	1	
		II)		04
		$x_3 = 1.05$		
		$y_3 = 1.06$		
		$z_3 = 1.07$	1	
	f)	Solve using Gauss-Seidal method.(three iterations)		
		$10x + y + z = 12, 2x + 10y + z = 13, 2x + 2y + 10z = 14$		
	Ans	$x = \frac{1}{10}(12 - y - z)$		
		$y = \frac{1}{10}(13 - 2x - z)$		
		$z = \frac{1}{10}(14 - 2x - 2y)$		
		Starting with $y_0 = z_0 = 0$	1	
		I)		
		$x_1 = 1.2$		
		$y_1 = 1.06$		
		$z_1 = 0.948$		
		II)	1	
		$x_2 = 0.9992$		
		$y_2 = 1.02$		
		$z_2 = 0.999$	1	
		III)		
		$x_3 = 0.998$		04
		$y_3 = 1.00$		
		$z_3 = 1.00$	1	