KNAPSACK PROBLEMS

we want to pack n items in knapsack.
* The ith item is worth v_i dollars and weight w_i pounds.
* Take as valuable load as possible, but can't exceed W pounds i.e. W is the knapsack capacity.
* v_i, w_i and W are integers.

There are two types of knapsack problems

→ **0-1 Knapsack Problem:** (Dynamic Programming is used)
* Each item is taken or not taken
* Cannot take a fractional amount of an item or cannot take an item more than once.

→ **Fractional Knapsack Problem:** (Greedy Algorithm is used)
* Fractions of items can be taken rather than having to make a binary (0-1) choice for each item.

Q problems on 0-1 knapsack:
Consider three items along with their respective weights and values are given:

$I = \langle I_1, I_2, I_3 \rangle$

$w = \langle 5, 4, 3 \rangle$

$v = \langle 6, 5, 4 \rangle$

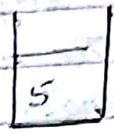
The knapsack has the maximum weight capacity $W = 7$.
we have to fill the knapsack according to greedy strategy such that it can have optimum value.

→ **Solution:**

I	w	v
I_1	5	6
I_2	4	5
I_3	3	4

1st choice: we arrange the items with decreasing values

I	w	v
I_1	5	6
I_2	4	5
I_3	3	4



Item I_1 is selected first as it has the maximum value (i.e. $I_1 = 6$)
Next all other item result in overflow since $W=7$

$w_1 + w_2 = 5+4 = 9$

$w_1 + w_3 = 5+3 = 8$

Here we can't take fractional values.
So, we get optimum value is 6 having total weight is 5 which is less than actual weight capacity.

2nd choice: Arrange the items in increasing weights

I	w	v
I_3	3	4
I_2	4	5
I_1	5	6

Here the first item with least weight is selected and the remaining weight is $7-3=4$
Next the 2nd item with weight 4 is selected and the total weight = Knapsack capacity.
The optimum value is $4+5 = 9$.

→ problems on fractional Knapsack:
Consider 5 items along their respective weights and values are given:

$I = \langle I_1, I_2, I_3, I_4, I_5 \rangle$

$w = \langle 5, 10, 20, 30, 40 \rangle$

$v = \langle 30, 20, 100, 90, 160 \rangle$

The capacity of Knapsack is $W = 60$. Find the Solution to the fractional Knapsack problem.

Solution:

Item	w_i	v_i
I_1	5	30
I_2	10	20
I_3	20	100
I_4	30	90
I_5	40	160

In fractional knapsack, we have to find out value per weight ratio i.e. $P_i = V_i/w_i$

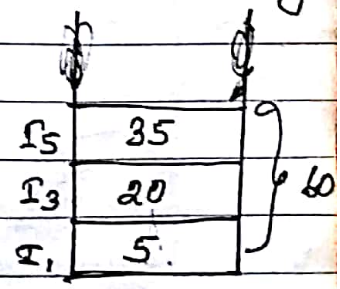
Item	w_i	v_i	$P_i = v_i/w_i$
I_1	5	30	6.0
I_2	10	20	2.0
I_3	20	100	5.0
I_4	30	90	3.0
I_5	40	160	4.0

Now, arrange the value of p_i in decreasing order

Item	w_i	v_i	$P_i = V_i/w_i$
I_1	5	30	6.0
I_3	20	100	5.0
I_5	40	160	4.0
I_4	30	90	3.0
I_2	10	20	2.0

Now, fill the knapsack according to the decreasing value of p_i.

→ first we choose item I_1 whose weight is 5, then choose item I_3 whose weight is 20.



Now the total weight of knapsack is $5+20=25$

Now, the next item is I_5 and its weight is 40, but we want only 35. so, we choose fractional part of it i.e.

The value of fractional part of I_5 is $(160/40)\times 35 = 140$

Thus, the maximum value of knapsack is:

$30+100+140 = 270$

Q Find optimal sol^n of the knapsack. Instance given $n = 7$ and $W = 15$.

Sol^n

I	profit	W	p_i
I_1	10	2	5
I_2	5	3	1.66
I_3	13	5	2.6
I_4	7	7	1
I_5	6	1	6
I_6	18	4	4.5
I_7	3	1	3

I	profit	W	p_i
I_5	6	1	6
I_1	10	2	5
I_6	18	4	4.5
I_7	3	1	3
I_3	13	5	2.6
I_2	5	3	1.66
I_1	7	7	1

By weight $= 1 + 2 + 4 + 1 + 5 + 2$ (out of 3) $= 15$

By value $= 6 + 10 + 18 + 3 + 13 + \frac{5}{3} \times 2 = 53.3$

$\left(\because \frac{5}{3} \times 2 = 3.33\right)$ or (2×1.66)

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