-	18 4 - 1 - 10 - 10 - 10 - 10 - 10 - 10 - 1
1.10	MATRIX CHAIN MULTIPLICATION
١	supose we have a collection of n two dimensional matrices
William Street, Street	for which we wish to compute the product.
	$A = A_1 \cdot A_2 \cdot A_3 \cdot \cdots \cdot A_n$
14 () () A	we will evaluate product of o roatrices by using standard
	algorithms for multiplying pour of matrices to resolve all
4/21 23	ambiguities on how the matrixes are multiplied together.
4 miles	matrin multiplication is anoceative in gature.
The state of the s	$A_1(A_2 \cdot A_3) = (A_1 \cdot A_2) + B_3$
ó/ -5 °	Example: chain of exactorices is (A1, A2, A3, Ay)
	The product of 1. Az Az Ay can be fully porrenthes ted in five ways.
V 1/4	(A1(A2(A3. A4)))
	(A1 , ((A2 · A3) · A4))
7.240	((A1. A2) (A3 : A4)) and so bydoons of so the
9474	((A1.(A2. 43))Ay) . 250 10000 00 to 200 2000000000000000000000000
	(((+1.+2).+3)+4)=+================================
The same	MATRIX-MULTIPLICATION (A) B)
the B	1. if columnsEAJ + rows EBJ
100	2. then error "Incompatible dimensions".
·	3. else for c'+ 1 to most [A]
(OB)	4. John do for je 1 to columns[B]
0	5. do c[i,j] ← 0
- ,	6. for K+ 1 to Columns[A]
	7. do c[i,i] + c[i,i] + A[i,k]. B[k,I]
	8. refung (C = OI-10) F (A) T
	Total James and

-	reported.
>	The fax matrices A and B can be multiplied of they core
	Compatible, Pice the number of columns of A mount be equal
4,113	to the number of rows of B.
- >	If matrix A having dimension prq and matrix B having
1	dimension gxx, they the resulting matrix (is fxx.
→	The time to compute the number of scalar multiplications
27777	of matrix cis pgri
→	Consider 3 matrices (A,B,C) having size 5x10,10x50,50x20.
\rightarrow	We could parenthesize there 3 matrices either A. (B. c) or (A. B). C
	Computing A. (B. c) requires
	\$0 (10.5.10.20 + 10.50.20
	= 1000 + 10000 = 11000 multiplicatrons
	Computing (A.B).c requires
Se , "	5: 10:20 + 10:50 mm in in a dome mit
£.	(5-x10, 10 x50) - 50 x20
To	5. 10. 50 + 5.50.20 = 2500 + 5000 = 7500 onultiplicity
>	In matrix onultiplication problem, we are not actually
21.00	multiplying matrices. Our objective is only to determine
	an order by which matrices are multiplied at lowest cont.
	The state of the s
>	One way to solve matrix multiplication problem is to
2 5%	simply enumerate all the possible ways of
-	parenthesization of the expression set of matrices and
	determine the gumber of multiplications performed by early
-7	Let T(n) be the number of alternative parentheer intentions
	Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices.
→	Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. when n=1, there is just one matrix and there force any
→	Let T(n) be the number of alterratives parenthesizations of a sequence of n matrices. when n=1, there is just one matrix and therefore only one way to fully parenthesize the mutar's nomehouse.
→ →	Let T(n) be the number of onetteption performed by each one of a sequence of n matrices. when n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product when n'12, a fully parenthesized matrix product is the
→ →	determine the gumber of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. When n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product. When n>2, a fully parenthesized matrix product is the product of two fully parenthesized matrix entreed is the
→ →	Let T(n) be the number of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. When n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product. When n/2, a fully parenthesized matrix product is the product of two fully parenthesized matrix subproblem Subproducts and the speit before the two of the
→ →	Let T(n) be the number of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. When n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product. When n/2, a fully parenthesized matrix product is the product of two fully parenthesized matrix product is the subproducts and the spect before the two of the subproblems.
→ →	determine the gumber of multiplications performed by each one Let T(n) be the number of alteropetives parenthesizations of a sequence of n matrices. Cushen n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product is the product of two fully parenthesized matrix product is the product of two fully parenthesized matrix subproblems subproducts and the spain between the two subproblems may occur between the 1,2,3,,n-1.
→ →	determine the number of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. when n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product. when n>2, a fully parenthesized matrix product is the product of two fully parenthesized matrix subproblem subproducts and the spaint between the two subproblems may occur between the kth and (ktl)st matrices for any K=1,2,3,,n-1.
→ →	determine the number of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. when n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product. when n>2, a fully parenthesized matrix product is the product of two fully parenthesized matrix subproblem subproducts and the spaint between the two subproblems may occur between the kth and (ktl)st matrices for any K=1,2,3,,n-1.
→ →	determine the operator of multiplications performed by each one Let T(n) be the number of alternatives parenthesizations of a sequence of n matrices. Cushen n=1, there is just one matrix and therefore only one way to fully parenthesize the matrix product is the product of two fully parenthesized matrix product is the product of two fully parenthesized matrix subproblem subproducts and the spain between the two subproblems may occur between the 1,2,3,,n-1.

7 4 F	Step 1: Determine the Structure of an optimal powenthesization
√75×	Any parenthesization of the product A: A:+1 A; must splot
	the product between An and Anti for some integer is in the
Your .	range éskerj
-	For some value of 15, we first compute the matrices Aink
ta	and Axti. Then multiply them together to produce the
40.00	final product Ai
•	The cost of this parenthesization is the sthe cust of compating
	the oncetrin Aine of the cost of computing AK+1j + the
-	cost of moultiplying them together.
- 1	Step 2: A recursive solution
	For matrix chain encel+plication problem, we have to
	determine the orinionum cast of a parenthe sization of
4400	Ai Aiti- Aj for Kisj (n)
_ - >	Let on[i, j] be the oninimum number of scalar
1777	onceltiplications needed to compete the modern Ainj
317	For the feel problem, the cost of a cheapest way to
	compute A would be on[1, n].
	ove can define on [i, j] recursively as follows:
	If ie = j, the problem is torvial i.e. the chain
1000	conseists of only one metrin so no scalar multiplication
	are necessary to compute the product.
	Thus on Eini] = 0, for c'=1,2,000
MAG	If it is, we have to take the structure of an
	optimal a Solvetion 1/2 22 separated frametics to some
	Let us concerne that the optional parenthese Zation
THE WALL	splits the product Ai Ait Aj between the and
50111	Akti rochere and Ckas junionian line or wolk
	· on [i, j] is equal to the oninion um cost for
W	Competing the sub products (Ai Ax) and
	(AK+) Aj) plus the cost of oncultiplying there
	two matrices tugether
	Each matrix di is Pi-1 x Pi Scalar oncultiplications.
	computing the matrix product A AK+1 j telles
	Pi-1. Pr. P. Scalar onceltiplications.
- 8	· m[i,j] = on[i,k] + m[k+1,j] + Pi=1 PKPj

-					
× ->	Here we assume that the value of 12 is known to we				
12821	but in really we do not know the value of k. The				
	Possible Nature at 10 of tron				
→	1't mount there are only 1-1 post-of				
1	8 ince the optimal parenthesization must use one of				
. 10.	these values for k, we need only cheek them all to				
,	fund the best.				
74 - >	Thus the recursive defination for the orionionum				
1	cost of parenthesisting the product to tett				
	Cost of fare from the				
-41	or i i - 1 - 1				
2/5	min & m [i, k] + m [ic+1, i] + Pi-1 Pic Pi if ici				
0	on[i,j] = 0 on[i,j] = 0 onin { on [i,k] + ron[ic+1,j] + Pi-1 Pik Pj if icj i < k < j \$ The ron[i,i] value aires the cours of antional solutions				
-50	The solicity value aired the course of antismal solutions				
and the second	The m[i,j] value gives the costs of optional solutions to subproblems.				
. c. ->	To keep track of how to construct an optional				
A ST.	Solution, let us défine s[i,j] to be a value				
	of K at which we can split the product				
	Ai Ai+1 A; to obtain an optional parenthesization				
34	S[i,j] equals a value & such that				
S WELLS	$\sigma(i)j = \sigma(i)i + \sigma(k+1)j + pi-1pkpj$				
£					
	step 3: Computing the optimal costs in a bottom up fushion.				
-	A recursive algorithm onay encrenter each subproblem many				
	times on defferent branches of its recursing tree. This				
140	Property of overlapping subproblems is the 2nd feature				
3/010	of dynamic programming (First feature is optimal substructum)				
→	Now ove will confeelate the approprial cost by using				
a tebular, bottom up approach.					
→	Ar has dimensione pi-1xp; for i=1,2,,				
1000	The enjoit sequence is $P = \langle P_0, P_1, \dots, P_n \rangle$				
i,	ashere length[p] = n+1.				
->	11 11 11 Call (1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				
19	which index of k achieved the optional cost in				
3	compressing on[i,j] we will use the table s to				
	construct an optimal solution. The table s to				
	Scanned by CamScanner				

	MATRIX-CHAIN-ORDER (P)
	1. n < length[P]-1
	2. for e ← 1 +0 m
	3. do ontivij to
	4. for l+2 to is D l is the chain length
	5. do for ix 1 to n-l+1
-4	6. do j ← c+e-j
	$7.$ on $Ci,jJ \leftarrow \infty$
	8. for K+ i to j-1
	9. do 9 + m[i, K] + m[K+1, j] + Pi-1 PK P;
Y	if q contisis
	11. st then tooling + q
	12. SEIJJ = K
	13. return on and s [1.3] The protocopy is asks

```
Detail 8007: (30x35) (35x15) (15x5) (5x10) (10x20) (20x25)
      25 10 20 25
          Po Pr P2 P3 Py P5 P6
                 15125
              11875
                   (105m)
                      <del>(53.75)</del>
         7875×4375
                         3500
                    2500
           2625
                  750
                       1000
m[171] = m[2,2] = m[3,3] = m[4,4] = m[5,5] = m[6,6] =0
m[1,2] = on [1, 1] + on [2,2] + Por Pipa
    = 0 0 + 30x 35 x 15 = 15,750.
m[2,3] = m[2,2] + m[3,3] + p, P2 P3
     = 0 + 0 + 35x15x5 = 2625
m[3,4] = m[3,3]+m[4,4]+P2P3Py = 141100
     = 0 + 0 + 750 = 750
m[4,5] = m[4,4] + m[5,5] + fgPyP5
     0 + 0 + 5 x 10 x 20 = 1000
m[5,6] = m[5,5] + m[6,6] + Py Ps Ps
   = 00 2+ 0 + 10 x 20 x 25 = 5000
10 E1 89 = 100 ETA ( 00 [1,1] + 00 [2,3] + POPIP3
m[1,3] = onen of x 0 + + 2625 + 30x35x5 = 7875
                m[1,2]+m[3,3]+ Po P2 P3
                15,750 + 0 + 30x 15x5 = 18000
   1. m[1,3] = 7875
    m[2,2]+m[3,4]+P1P2Py
 en[2,4] = min 2 0 + 750 + 35x15x10 = 6000 ~
       m[2,3]+m[4,4]+P,P3P4
              2625 + 0 + 35x 5x 10 = 9726 4375 (Ano
  : m[2,4] = 9885 4375
```

i .	Cm[3,3] + on[4,5] + P2P3P5
-	1 154 6 4 9 10 = 2500
	m[3,5] = onin 2 0 + 1000 + 13x 3x 20
	on[3,4]+ on[5,5] + Pg Py Ps
	750 + 0 + 15 x10 x 20 = 3 750
	.'. m[3,5] = 1480 2500
_	(m[4,4] + m[5,6] + Pg Py P6
_	on[4,6] = onig 0 + 5000 + 5×10×25 = 6250
- 200	m[4,5]+m[6,6]+P3P5P6
-	1000 + 0 + 5 x 20 x 25 = 3500
	1. on [4] = 3500
	the first that the first the first that the first the first that the first t
	(m[1, 1]+ m[2,4]+ Po P, Py
	0 + 4375 + 30x 35x10=14875
	on[1,4] = mon of on[1,2]+on[3,4]+PDP2Py
Sar a-	15750 + 750 + 30x 15x 10 = 21000
	m[1,3] + m[4,4] + Po P3 P4
2 6	$7875 + 0 + 30 \times 5 \times 10 = 9375$
	-1. m[1,4] = 9375 FULL DONNE - 9375
	10 10 10 10 10 10 10 10 10 10 10 10 10 1
	m[2,2]+ m[3,5] +P, P2P5
	$0 + 25 co + 35 \times 15 \times 20 = 13 co$
	10-10 1 min 2 m 2 3 1 + m [11 c]
,	2625 + 1500 + 25
	m[2,4]+m[5,5]+P1P4P5
	4375 7 0 + 35 400
1	'. m[2:5] = 7105
	CENST 12 4 2 4 36 X 15 X 3 4 X 15 X
w.W	
-6	$m_{i,3} + m_{i,4}$
	m[3,3]+m[4,6]+ P2P3P6
	$m[3,6] = min = 0 + 8500 + 15 \times 5 \times 25 = 5375$
	$m[3,6] = min) an[3,4] + m[5,6] + P2P4Pc$ $750 + 5000 + 15 \times 10 \times 25 = 0.5$
124	750 + 5000 + 15x 10x25 = 9500
	LUID PT MO IN A Y
149	2500 101 15 + P2 P5 P
100	$m[3,5] + m[c,6] + P_2 P_5 P_6$ $25 co + 0 + 15 \times 20 \times 25 = 10000$ $m[3,5] + 53 + 5$

	Page
(on[1,1]+on[2,5]+ to P, P5	SVESTILLAR II.
0 + 7125 + 30x 35x	20 = 28125
on[1,2]+ on[3,5] + β β2 β5	(,5), ob., anon
m[1:3] = min 15750 + 2500 + 30x 15 x 20	= 27 250
mc1,3] + m[4,5] + Po P3P5	E Blad slaground 4-
7875 + 1000 + 30 x 5 x 20 =	
on [1,4] ton[5,5] + Poly Ps	1"= 0= a. C.
9375 + 0 + 30 × 10 × 20 =	-15375
on[1,5] = 11875	
The resistance of some that he should be	5 2 buch
(m[2,2] + m[3,6] + P2 P1 P2	
0 + 5375 + 35 x 15 x	
m[2,3]+m[4,6]+P1P3	
m[2,6] = meg 2625 + 3500+35 x 5 x 2	
m[2,4] + m[5,6] + P, Pq	
4375 + 5000 + 35 × 10 × 2	
on[2,5] + on[6,6] + PIPS Pe	Y
7125 + 0 + 35 x 20 x 25	= 24625
· · · · · [2,6] = 10500	A HARRY THE WAR TO SEE THE
The second secon	to be controls.
[m[1,1] + m[2,6] + PoP1 +	1. 140 120. 3
0 + 10500 + 30 x 35 x 2	5 = 36 750
m[1,2]+m[3,6]+PaP	P6 000 00 20 . R
15750 + 5375+30×15×3	25 <u>-</u> 3 2 3 7 5
m[1,6]=min m[1,3]+m[4,6]+Po.P3 f	grade adal where I
7875 + 3500 + 30 x 5 x 25	= 15125
on[1,4] + on[5,6] + Po Py +	Prince
9375 + 5000 + 30 x /0 x 25	5=21875
m[1,5]+m[6,6]+PoPsP	4 year cord ! 4-
11875 + 0 + 30 x 20 x 25:	= 26875
:. m[1,6] = 15125 S	in the state of
Sugar Course Courses of Sugar	The It plane
2 3 3	2
3 3 3	(3 : Treps (E)
3 3 3	5
2 3 4	5

1 1	
*	Ster 4: Constructing an optional solution
30 4	Ster 4: Constructing an optimal sumber of scalar multiplications This determines the optimal number of scalar multiplications
/	DPP CAN TA COLLEGE CANONINATE (1 00 100
	directly (La) hand the model Tiber
_	we are maintaining an array stimpt s[1n, 1n]
	where S[i,j] denotes k for the optimal.
	a Pallane de accordantes Registration de la Artici
	splitting on computing Ainj = Ajor Arti.
->	The array s[1.17, 1.17] can be used recurrively to recover
•	the onultiplicention sequence.
	$SCI, \eta J = (A, \cdots ASCI, \eta J) (ASCI, \eta J + I - \cdots An)$
•	S[1,SE1,n] = (A ASE1,SE1,n]) (ASE1,SE1,n]+1 ASC1,n]
Eng.	
175	200 = 4:2:4 + 0. C = 255.
	one and do it recursively until the multiplication
	Sequence is determined.
	PRINIT_OPTIMAL_PARENS(S, c, j)
	1. if i=j
	a. then print "A"
	3. else proof ("
- 202	4. PRINT-OPTIMAL-PARENS (S, i, SC:, j])
	5. PRINT-OPTIMAL-PARENS (S, SCi, j)+1, j)
4	6. porgt "y"
	71.1700
7.5°	S[2-5] - 1 S[2-5]
	Slaves on a superior
7	Expert of the frame of the first
F # 18	C = S[2,3] \ [4,5]
- 5-	Treat & Children & Children & Children
	([22] [33] 3 C [4,4] [557
y	1
	4 4 4 4 1
	(Ay # 15) * (Ay # 15)
SHUT STILL	((H2 * H3) * (Ay x A5))
1	Scanned by CamScanner
	ACAIMED BY CARMEL