A 1887	(A)
	(4) The Moster Method
-	The moster method is used for solving the following type of recorrence
Ty.	$T(n) = aT(\frac{n}{b}) + f(n) \text{with} a > 1 \text{and} b > 1$
->	In this, packlem is devoided into 'a' scopproblems, each of size
H	1/b oshere a and b are posertive constants.
	The cost of dividing the problem and combining the results of the
	subproblem is described by the fugition fon.
-	Here, 'n' is the problem size.
	The Marter Theorem
>	Let and by be constants, let fin be a function, and
	let Ton) be defined on the nonnegative ontegers by the recurrence. Ton) = a Ton/b) +fon)
	where we obserpret only to mean eather Lolbs or lolbs.
	Then Ton) can be bounded asyon protically as follows.
	1. If $f(n) = O(n^{\log_b \alpha - \epsilon})$ for some constant $e > 0$, then $T(n) = O(n^{\log_b \alpha})$
	2. If fon) = O(n 196a), then Ton) = O(n 196a 1090)
	3. If f(n) = si (n) log a + e) for some constant e > 0 and
	If a.f(n/b) < c.f(n) for some constant (<) and all
	sufficiently large n, then Ton)= O(fin)
2	. af (7/b) & c. fcn) is called the regularity condition.
1	(3) 1/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2
-	Example 1: Ten = 47 (1/2) +n (0)
	a=4, $b=2$, $f(n)=n$
	cuse: $f(n) = O(n \log_{10} \alpha - \epsilon)$
	=> f(n) = 0 (1/1/24-6) 0000 - 1000000000000000000000000000000
	=> $f(n) = O(n^{2-\epsilon})$ + $(3n^{2-\epsilon})$
The .	$= f(n) = O(n^{2-1})$
	$\Rightarrow f(n) = O(n) (proved)$
-	Hence, we can apply case I and the soketion is
	Ton)=000128) a (n/1096a) = 0 (n/1924) = 0(n2)
→ ¿	Example 2: Ten) = 47(9/2) + 92.
	$a=y$, $b=a$, $f(n)=y^2$
	Apply case 1: fcn = O(n/196a-e)
	$\Rightarrow f(n) = O(n^{\log_2 4} - \epsilon)$
	$\Rightarrow f(n) = A(n^2 - \epsilon)$
	$= f(n) = O(n^{2-\epsilon})$ $= f(n) = O(n^{2-1}) = O(n) \text{ (Not satesfied)}$
	7 7 11 0 (1) J - U(1) (Nor Sate 17 189)
- 11	

100	The state of the s
-29	Now apply are 2: fin) = A (n/ogo a)
1682	Pan - 1/2/324)
,	The columns is $f(n) = O(n^2)$ (satisfied)
25	The solution is:
- 20 per	$T(\eta) = \Theta(\eta^{\log \alpha} \log \eta)$
64 3	7001 - 101 - 10924 Jan 1
1 -	$\Rightarrow 7cn = \theta \left(n \frac{\log_2 y}{\log_2 n} \right)$
	$= \gamma - \tau(n) = O(\eta^2 / \log \eta)$
_	Barmale T. Ting - UTing to 3
7	Example 3: $T(n) = 4T(n/2) + n^3$
	$a = 4$, $b = a$, $f(n) = \eta^3$
83.13	Apply case 1: $f(n) = O(n^{\log_b \alpha - \epsilon})$
	$\Rightarrow f(n) = O(n^{\log_2 4 - \epsilon})$
	$f(n) = O(n^{2-\epsilon})$ $f(n) = O(n^{2-\epsilon})$ $f(n) = O(n^{2-\epsilon})$
2/2	=> f(n) = 0 (n2-1) = 0(n) (Not satisfied)
	Now apply case 2:
	F(n) = A (n/10924)
	fan - A(n2) (Nat Cofficient)
	Now apply case 3:
5071	$f(\eta) = 2 \left(\eta^{(0)} a + \epsilon \right)$
_	$\Rightarrow f(n) = \Omega \left(n^{\log_2 y + \epsilon} \right)$
= =	$\Rightarrow f(n) = 2(n^{2+\epsilon})$
	$\Rightarrow f(\eta) = -2(\eta^{2+1})$
	$\Rightarrow f(n) = SL(n^3) (30 + 63 + 63 + 64)$
	conduction 1 sofisfied. How cheek the 2nd conduction.
7. +	$a \cdot f(n/b) \leq c \cdot f(n)$
	$\Rightarrow 4 \cdot 9 (1/2)^3 \leq c \cdot \eta^3$
	$\Rightarrow \times \frac{\eta^3}{4 \cdot \eta^3} < \frac{1}{2} < \frac{1}{2}$
	82
104-	\Rightarrow $n^3/2 < c.n^3$ (setisfied)
7	
15	Condition a soutisfied. Both the conditions are sastisfied.
	$T(n) = \Theta(n^3)$
ere 1	(- 19 m) 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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Example 4: Solve the recurrence wing master method
            a=9, b=3, f(n)=n
  Apply case 1: fins = O(n 1096a - 1)
           => f(n) = O(n^{16g_3}9 - \epsilon)
=> f(n) = O(n^{2-\epsilon})
           => f(n) = O(n2-1) = O(n) (30+isfied)
                  T(n) = \theta(n^{\log \alpha})
  The solution is:
  >> Ten) = \(\theta\) (\(\eta\) | \(\frac{19}{9}\) \(\frac{9}{1}\)
      T(n) = O(n^2)
-> Example 5: Bolve Tin) = T(2n)+1 by master method.
  80/4+con: a=1, b=3/2, fin)=1
        n/1960 = n/99/21 = n0 = 1
    dence nove 2 applies.
  Hence, case 2 applies.
  The solution is: Tin)=A(n/ogsa/gn)
               => T(n)= A(n1. 1gn)
                  => T(n) = O(lgn)
-> Example 6: Solve the recurrence Tim = 3T (7/4) + nlgn
  solution: a=3, b=4, fin)= olgo
          n/96 a = n/943 = O(n0.793)
        fin) = 12 (n/0943+t.) where 6 = 0.2
  case 3 applies, More for regularity condition
            \alpha \cdot f(\gamma_b) \leqslant c \cdot f(n)
       => 3. f(n/4) < c. f(n)
       => 3.(m/y) 10y(n/y) < c. 210g0
     => 3 n/4 log(n/4) < 3n logn (: c=3/4)
   The solution is:
                 Tono = O(nlogn)
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Example 7: 80/ve T(n) = 16T(9/4) + n^3

80/u+t^n 9: \alpha = 16, b = 4, f(n) = n^3

1696\alpha = n^{1694} 16 = n^2

f(n) = n^3 = n^{1694} 16 + e = n^{2+1} = n^3 (*.' E = 1)

Now check regularity condition

16f(\frac{n^3}{4^3}) < c \cdot f(n^3)

\Rightarrow 16f(\frac{n^3}{4^3}) < c \cdot f(n^3)

\Rightarrow 16f(\frac{n^3}{4^3}) < c \cdot f(n^3)

\Rightarrow 16f(\frac{n^3}{4^3}) < c \cdot f(n^3)

Hence case 3 is sawisfied. So, the solution is:

T(n) = f(f(n))

\Rightarrow T(n) = f(f(n))
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