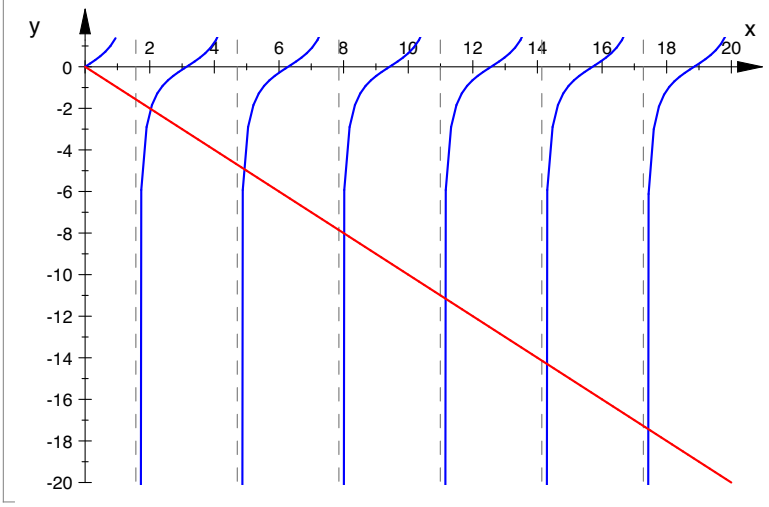
Notebook for Maths 361: Partial Differential Equations (OJM)
Lecture 9

When we do these calculations by hand we use the fact that n is a positive integer.
We can get MuPAD to assume this with the command:

```
assume(n, Type::Integer and Type::Positive)
```

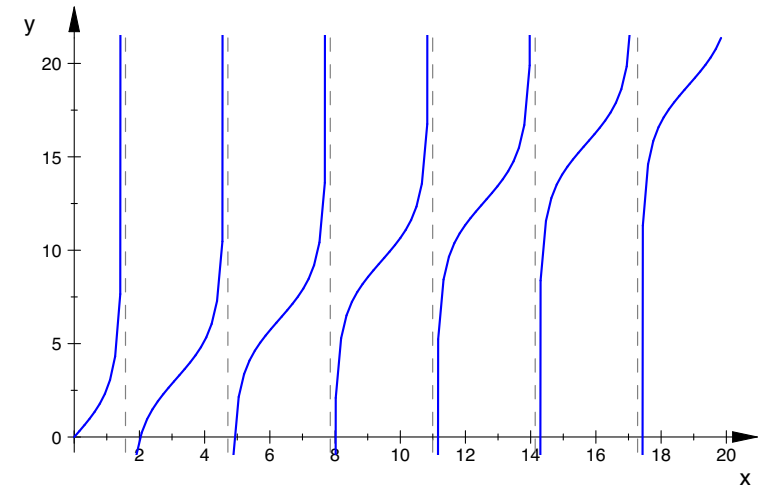
Calculate our expansion for Example 1
Let's have a look at what we're trying to solve
View 1

```
plot(tan(x),-x,x= 0..20)
```




View 2

```
plot(tan(x)+x,x= 0..20)
```




Now solve numerically over a fixed interval using random restarts

```
numeric::fsolve(tan(k)+k=0, k = 0 .. 20,Random) $ i = 1 .. 50
```

[$k$ = 2.028757838], [$k$ = 14.20743673], [$k$ = 4.913180439], [$k$ = 4.913180439],

[$k$ = 17.33637792], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 17.33637792], [$k$ = 14.20743673],

[$k$ = 2.028757838], [$k$ = 7.978665712], [$k$ = 0.0], [$k$ = 11.08553841], [$k$ = 11.08553841],

[$k$ = 17.33637792], [$k$ = 17.33637792], [$k$ = 17.33637792], [$k$ = 4.913180439],

[$k$ = 11.08553841], [$k$ = 0.0], [$k$ = 2.028757838], [$k$ = 7.978665712], [$k$ = 17.33637792],

[$k$ = 0.0], [$k$ = 7.978665712], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 0.0],

[$k$ = 17.33637792], [$k = 3.293662188\ 10^{-11}$], [$k$ = 14.20743673], [$k$ = 14.20743673],

[$k$ = 7.978665712], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 7.978665712], [$k$ = 0.0],

[$k$ = 4.913180439], [$k$ = 0.0], [$k$ = 0.0], [$k$ = 14.20743673], [$k$ = 7.978665712], [$k$ = 0.0],

[$k$ = 7.978665712], [$k$ = 17.33637792], [$k$ = 4.913180439], [$k$ = 2.028757838]

We need to just consider the unique cases (assuming no repeated roots!). Choose e.g.s

```
lam := [2.028757838^2, 11.08553841^2, 17.33637792^2]
```

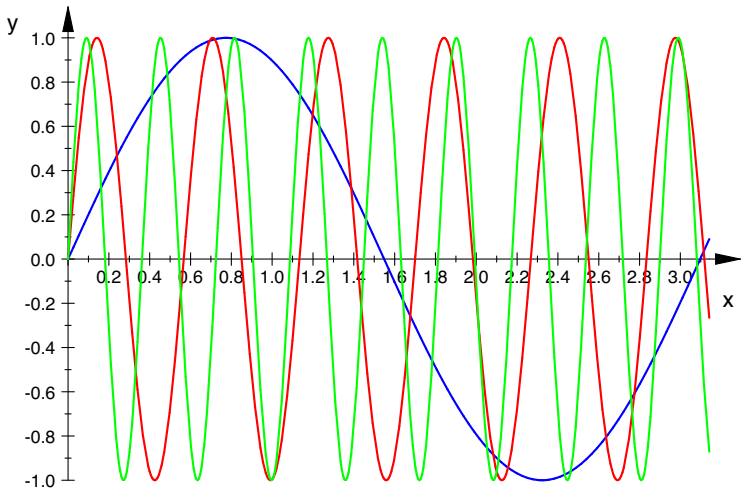
[4.115858365, 122.8891618, 300.5499994]

Our first few eigenfunctions are then

```
phi := [sin(sqrt(lam[1])*x),sin(sqrt(lam[2])*x),sin(sqrt(lam[3])*x)]
```

[sin(2.028757838 $x$), sin(11.08553841 $x$), sin(17.33637792 $x$)]

```
plot(phi[1],phi[2],phi[3],x=0..PI)
```




## Exercise!

Complete the eigenfunction expansion for f(x) = 50 (and/or some other functions) using the first few eigenfunctions. Tidy up my code if you want!