

Lecture 8 - Heat & work

- Heat transfer (v. briefly)
 - { Heat capacity }
 - Latent heat & phase transitions
 - Work: pistons containing ideal gas
 - ↳ work for pistons ($P \cdot \Delta V$)
-

Example questions (exam Qs)

- 28) The specific heat capacity of water is $4.186 \text{ J.g}^{-1}\text.{}^{\circ}\text{C}^{-1}$. How much heat is required to raise the temperature of 250 g of water from 25 to 50 $^{\circ}\text{C}$?

(1 marks)

- 31) A secret substance, called 'substance X', is being investigated for its potential as a new type of fuel. A 1.0 g sample of substance X is burned in a bomb calorimeter containing 1000 grams of water at an initial temperature of 23 $^{\circ}\text{C}$. After the reaction, the final temperature of the water is 30.1 $^{\circ}\text{C}$. The heat capacity of the calorimeter (also known as the "calorimeter constant") is 850 $\text{J}/{}^{\circ}\text{C}$. The specific heat of water is $4.184 \text{ J/g } {}^{\circ}\text{C}$ and the molar mass of the substance is 110 g/mol.

- a. Calculate the heat absorbed by the water.

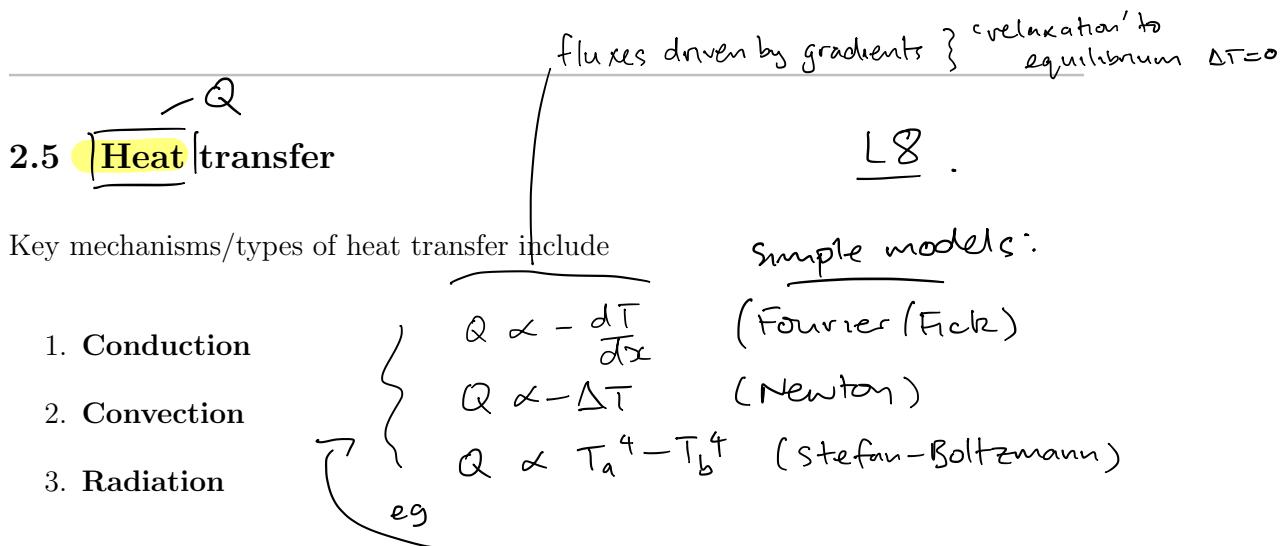
(1 mark)

- 33) A 450g sample of Nickel is heated from 25 $^{\circ}\text{C}$ to 100 $^{\circ}\text{C}$. The amount of energy required is 14.85 kJ. What is the specific heat capacity of Nickel? (2 marks)

Answer:

+ Latent heat as in Ex. Prob 5 today

Work questions ? → see tomorrow etc



Each of these has different types of constitutive equation governing the interactions between the system and its surroundings.

We won't look at these in too much detail here; we will, however, consider some constitutive (material specific) properties of substances that measure their 'capacity for receiving heat', regardless of the source or mechanism of transfer.

2.5.1 Heat capacity

The **heat capacity** of a substance measures the 'amount of heat', i.e. amount of energy transfer, required to change a given substance's temperature by a given amount

It comes in two flavours:

- 1. An extensive version C , just called *heat capacity*.
- 2. An intensive version c , called *specific heat capacity*.

big C

little c

In both cases however, you need to specify whether the heat capacity is measured at constant pressure or constant volume: otherwise it is *path dependent* and hence not a true 'state property' of the system. The symbols C_p or c_p are used for measuring heat capacity at constant pressure, while C_v or c_v are used when measuring it at constant

usually (here).

volume. Their ratio, denoted $\gamma = C_p/C_v = c_p/c_v$, is called the *heat capacity ratio* or the *ratio of specific heats*. Usually ≥ 1 , e.g. for air $\gamma = 1.4$. \leftarrow if in doubt, use this. Heat capacities can be measured using a **calorimeter**; we will return to more detailed examples of this later after we have covered a few more concepts.

Units and key equations for heat capacity

- Big C (heat capacity)

Symbol : Capital C

units : $\left\{ \begin{array}{l} J \cdot K^{-1} \leftarrow SI \\ J \cdot {}^\circ C^{-1} \end{array} \right\}$ ie energy per ΔT

\rightarrow extensive : more mass, more capacity

$C_p \leftarrow$ constant pressure
 $C_v \leftarrow$ constant volume

- Little c (specific heat capacity)

Symbol : lower-case c

units : $\left\{ \begin{array}{l} J \cdot K^{-1} \cdot g^{-1} \leftarrow SI \\ J \cdot {}^\circ C^{-1} \cdot g^{-1} \end{array} \right\}$ energy per ΔT , per mass

Key equation(s)

$$\text{Heat} \rightarrow Q = C_p \Delta T$$

↑ change in temp -

$$Q = c_p \cdot m \cdot \Delta T$$

↑ mass

} extensive C

} intensive C

Example Problems 4: Heat capacity calculations

- Given the specific heat capacity of water is $4.186 \text{ J.g}^{-1} \cdot \text{degC}^{-1}$, how much

- heat is required to raise the temperature of 200 g of water from 20 to 50 °C?
2. A 5.10 g sample of iron is heated from 36.0 °C to 75.0 °C. The amount of energy required is 89.5 J. What is the specific heat capacity of iron?
 3. A 350.0 g sample of metal is heated to 100.0 °C, then dropped into 400.0 g of water at 21.4 °C. The temperature of the water rises to 28.9 °C. Assuming there is no heat lost to the surrounds, what is the heat capacity of the metal?

Answers

1. Key equation: $Q = c_p \cdot m \cdot \Delta T$

given: $\left\{ \begin{array}{l} c_p = 4.186 \text{ J.g}^{-1} \cdot ^\circ\text{C}^{-1} \\ m = 200 \text{ g} \\ \Delta T = 30^\circ\text{C} \end{array} \right\}$ $Q = 4.186 \times 200 \times 30 \text{ (J)}$
 $= 25116 \text{ J} \approx \boxed{25 \text{ kJ}}$

2. Key equation: $Q = c_p \cdot m \cdot \Delta T$

given $m = 5.10 \text{ g}$
 $\Delta T = 75 - 36 = 39^\circ\text{C}$

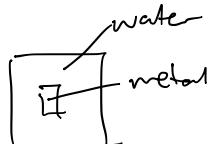
want $c_p = Q / m \cdot \Delta T = \frac{89.5}{5.1 \times 39} = \boxed{0.45 \text{ J.g}^{-1} \cdot ^\circ\text{C}^{-1}}$

3. Key equation: $Q = c_p m \Delta T$ (& or $Q = c_p \Delta T$)

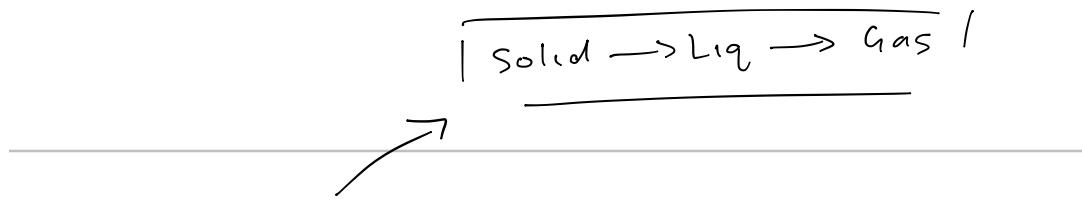
Assume metal cools to equal with water: $T_{\text{metal}} = T_{\text{water}}$
 $\& Q_{\text{metal, loss}} = Q_{\text{water, gain}}$ (no heat loss)

o water first: $Q = 4.186 \text{ (J/g.}^\circ\text{C)} \times 400 \text{ g} \times (28.9 - 21.4)^\circ\text{C} \approx \boxed{12,558 \text{ J}}$

(Careful
of
signs
 $\rightarrow c_p > 0$) { o $Q_{\text{metal}} = 12,558 \text{ J} = m c_p \Delta T$; $m = 350 \text{ g}$, $\Delta T = (100 - 28.9)^\circ\text{C}$
 $\Rightarrow c_p = \frac{12,558 \text{ J}}{350 \text{ g} \times 71.1^\circ\text{C}} \approx \boxed{0.50 \text{ J/g.}^\circ\text{C}}$



OR: $c_p = \frac{m}{73} c_p = 0.5 \times 350 \approx \boxed{177 \text{ J/g.}^\circ\text{C}}$



2.5.2 Latent heat and phase transitions

Similarly to heat capacities, we can define various types of **latent heat** as *the amount of heat/energy transfer required to completely carry out a change in phase* for a given substance. These can also be measured in extensive or intensive versions.

A change in phase is e.g. a change between solid, liquid, gas (or plasma!). For example the **latent heat of fusion** is the heat energy required to carry out melting, while the **latent heat of vaporisation** is the energy required to carry out boiling.

Some latent heat data is given in Table 1 of this section. We can use this to answer the following questions.

Substance	Specific Latent Heat of Fusion (kJ.kg ⁻¹)	Melting Point (°C)	Specific Latent Heat of Vaporisation (kJ.kg ⁻¹)	Boiling Point (°C)
Water	334	0	2264.76	100
Alcohol, ethyl	108	-114	855	78.3
Oxygen	13.9	-219	213	-183
Hydrogen	48	-259	455	-253
Carbon dioxide	184	-78	574	-57
Lead	23	327.5	871	1750

Table 1: Examples of specific latent heat of fusion and specific latent heat of vaporisation. (From notes by Thor Besier.)

Example Problems 5: Latent calculations

1. What is the energy (heat) required to convert 250 g of ice to liquid water? *melt*

2. What is the energy required to evaporate 28 g of sweat?
3. Given the specific heat capacity of water is $4.186 \text{ J.g}^{-1} \cdot \text{C}^{-1}$, what is the effective cooling effect of this evaporation for a 70 kg person?

Answers

$$\rightarrow \Delta T$$

$$1. Q = m \cdot L_f \quad \left. \begin{array}{l} \text{L is energy/mass} \\ \text{Key: units!} \end{array} \right\}$$

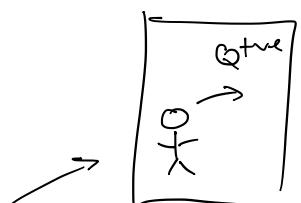
$$= 0.028 \frac{\text{kg}}{\text{kg}} \times 2264.76 \frac{\text{kJ}}{\text{kg}} = 83.5 \text{ kJ}$$

$$2. Q = m \cdot L_v$$

$$= 0.028 \frac{\text{kg}}{\text{kg}} \times 2264.76 \frac{\text{kJ}}{\text{kg}} \approx 63 \text{ kJ}$$

$$3. Q \text{ from } 2) \text{ is } 63 \text{ kJ} = 63 \times 10^3 \text{ J}$$

assume
 'person
 made of
 water'
 → how much
 does Q lower
 T ?

$$\left\{ \begin{array}{l} Q = c_p \cdot m \cdot \Delta T \\ \Rightarrow \Delta T \quad (\text{cooling effect} \rightarrow \text{give off heat via sweat}) \\ = Q / c_p \cdot m = \frac{63 \times 10^3 \text{ J}}{4.186 \text{ J} \times 70 \times 10^3 \text{ g}} \\ \approx 0.22^\circ \text{C} \quad (\text{cool by } 0.22^\circ \text{C} \\ \text{via sweating } 28 \text{ g--}) \end{array} \right.$$


/ today

2.6 Work: [pistons] and heat engines

As engineers we are very often interested in extracting work from systems, as well as in doing work on systems (e.g. to change their shape during manufacturing).

Here we'll look at two simple examples of getting work into and out of systems, and the interplay of this with heat transfer: pistons containing an ideal gas, and heat engines and their efficiency limits.

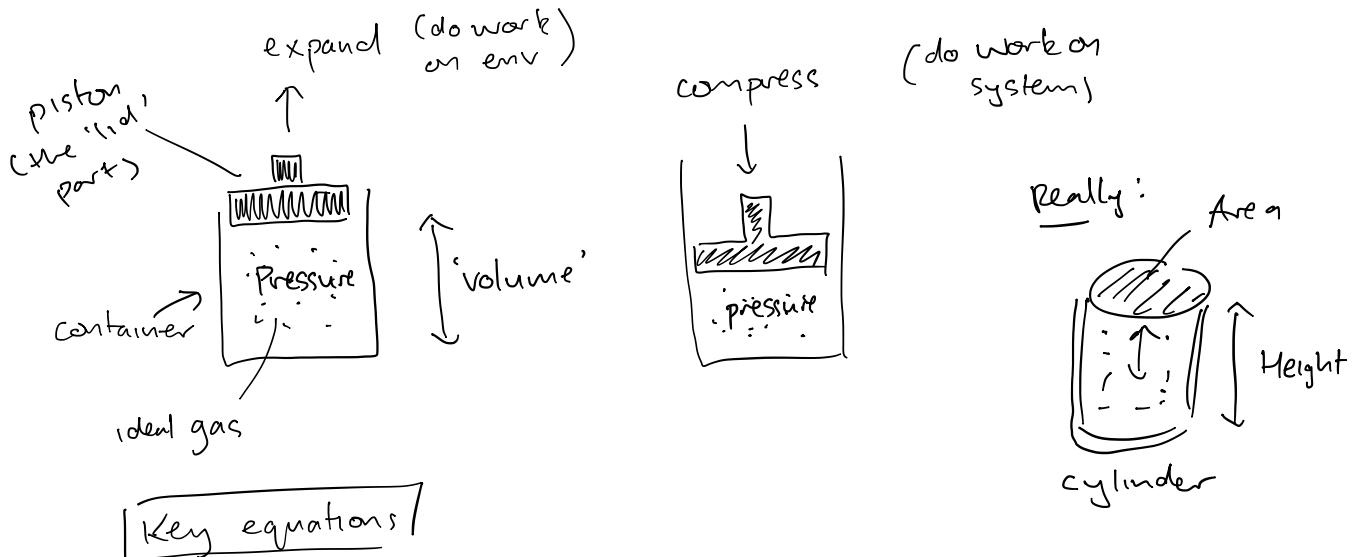
↑
today
↓ tomorrow

2.6.1 Pistons

The canonical engineering example of a thermodynamic system is a piston containing an ideal gas.

This is sketched out below.

Piston containing an ideal gas



Key equations

1. Work (reversible/quasi-equil. ← maximum available)

$$W = P \cdot \Delta V \quad \left\{ \begin{array}{l} \text{'pressure-volume' work} \\ \text{Force} \times \frac{\text{Area} \times \Delta \text{Height}}{\text{Area}} = \text{Force} \times \Delta \text{Height} \end{array} \right.$$

$$\left[\Rightarrow \frac{\text{Force}}{\text{Area}} \times \text{Area} \times \Delta \text{Height} = \frac{\text{Force} \times \Delta \text{Height}}{\text{Area}} \right] \checkmark$$

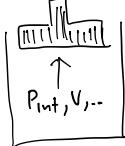
2. Ideal gas state

$$PV = nRT$$

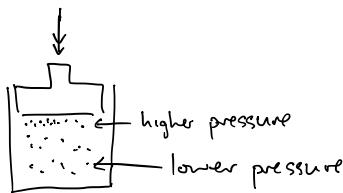
A subtle point is the difference between the **external work**, W_{ext} , and the **system work**, here $P\Delta V$: they are only equal for ideal, i.e. reversible/quasi-equilibrium processes (no entropy production - see below as well as later). This is essentially the same point as came up when discussing how to measure properties outside of equilibrium.

External vs system work

$$W_{ext} = P_{ext} \cdot \Delta V$$



Too fast:



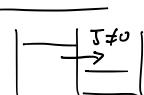
No one 'P' for gas

→ Pressure gradient \neq non-equilibrium: gradients / differences a very feature.

- External Work : $W_{ext} = P_{ext} \cdot \Delta V$
- Internal 'Work': $P_{int} \cdot \Delta V \neq$ usually what we call $P \cdot \Delta V$ work
- Slow, equilibrium
 $P_{int} = P_{ext} = P$ & so $W_{ext} = P \cdot \Delta V \neq$ we will assume!
- Fast, non-equilibrium
 $P_{int} \neq P_{ext}$ in general
so $W_{ext} \neq P_{int} \cdot \Delta V = P \cdot \Delta V$
→ e.g. accelerates then relaxes to equil.

Relaxation to equilibrium?

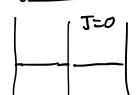
Non-equilibrium:



- gradient
- lower entropy

tends to/relaxes to if left alone

Equilibrium:



- no gradient
- higher entropy

Quasi: adjusts/relaxes 'very quickly'

e.g. increase P_{ext}

⇒ P_{int} will quickly adjust to new value to balance.

Upshot: assume $P_{ext} = P_{int}$

(if move piston too fast, gas can't adjust fast enough)

Example Problems 6: Pistons

assume reversible / quasi-equil.
ie $w = P \cdot \Delta V$

- (1) How much work is done by a gas expanding against a piston from 46 L to 65 L at a constant external pressure of 101,325 Pa (1 atmosphere)?
2. Consider a gas in a cylinder at room temperature ($T = 293$ K) with a volume of 0.065 m^3 . The gas is confined by a piston with a weight of 100 N and an area of 0.65 m^2 . The pressure above the piston is atmospheric pressure. What is the pressure of the gas?
3. The gas is heated, expanding it and moving the piston up. If the volume occupied by the gas doubles, how much work has the gas done?

Answers

$$1. \quad 1\text{ Pa} = 1\text{ J/m}^3 \quad (w = P \cdot \Delta V \rightarrow P = w/\Delta V \in 1\text{ J/m}^3)$$

$$1\text{ L} = (10\text{ cm})^3 = 1000 \times 10^{-6}\text{ m}^3 = 1 \times 10^{-3}\text{ m}^3$$

$$\Rightarrow P = 101,325\text{ J/m}^3 \quad \& \quad \Delta V = (65 - 46)\text{ L} \times \frac{1 \times 10^{-3}\text{ m}^3}{1\text{ L}} = 19 \times 10^{-3}\text{ m}^3$$

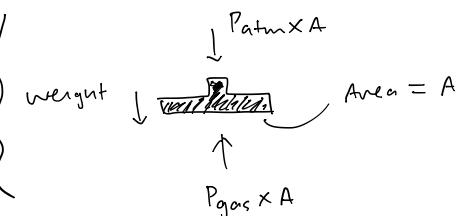
$$w = P \cdot \Delta V = 101,325 \frac{\text{J}}{\text{m}^3} \times 19 \times 10^{-3}\text{ m}^3$$

$$\approx 1925\text{ J} \quad (\approx 1.9\text{ kJ})$$

expand: do work
↑
↓

Note: w_{by} so $w_{\text{by}} = +1925\text{ J}$

2. Force balance!



$$P_{\text{atm}} \times A + mg = P_{\text{gas}} \times A$$

$$\Rightarrow P_{\text{gas}} = P_{\text{atm}} + \frac{mg}{A}$$

$$= 101,325 + \frac{100}{0.65}$$

$$\approx 101,500\text{ Pa} = 101,500 \frac{\text{J}}{\text{m}^3}$$

3. $V \rightarrow 2V$ ie $\Delta V = V$

$$W = P \cdot \Delta V = P \cdot V \quad (\text{here})$$

$$= 101,500 \frac{\text{J}}{\text{m}^3} \times 0.065\text{ m}^3$$

$$\approx 6600\text{ J} \quad (\approx 6.6\text{ kJ})$$

+ve work by system

end L8