Engsci 711

Assignment 1

Oliver Maclaren
oliver.maclaren@auckland.ac.nz

Due: Thursday 1st June (in class or online)

Question 1

Find the equilibria, classify them and sketch the phase portraits for the following systems.

a)

$$\dot{x} = x(1 - x - y)$$
$$\dot{y} = y(2 - x - y)$$

b)

$$\dot{x} = -x + 4y$$
$$\dot{y} = -x - y^3$$

For this last case prove or disprove the existence of periodic solutions.

Question 2

Consider the system

$$\dot{x} = 2x + y$$
$$\dot{y} = -y + x^2$$

- Find and classify the equilibria.
- Find the power series expansions for $W^u_{loc}(0), W^s_{loc}(0)$ up to (i.e. including) quadratic order.

Question 3

Here are two simple 'inverse problems': rather than giving you equations to determine the behaviour of, here you need to construct appropriate equations given desired solution behaviour.

a) Write down a system of differential equations

$$\dot{x} = f(x, y)$$
$$\dot{y} = g(x, y)$$

such that it has two and only two fixed points: one at $(x, y) = (2, 1)$, and one at $(x, y) = (-2, 1)$.

b) Write down a system of differential equations

$$\dot{x} = f(x, y)$$
$$\dot{y} = g(x, y)$$

such that both the x and y axes are invariant under the flow, some orbits tend to the origin as $t \to \infty$ and all other orbits have an x-component tending to $+\infty$ as $t \to \infty$.

For this last case, verify your system has these properties by plotting it using XPP. Include the nullclines (XPP can draw these for you!).

Question 4

The Brusselator is a simple model for a hypothetical chemical oscillator. One version on this model, in dimensionless form, has kinetics given by

$$\dot{x} = 1 - (b + 1)x + x^2 y$$
$$\dot{y} = bx - x^2 y$$

where $b > 0$ is a parameter and $x, y \geq 0$ are dimensionless concentrations.

- There is one equillibirum point. Find it (express it as a function of b).
- Find the Jacobian derivative, evaluate it at this point, and calculate the trace and determinant in terms of b.
- For what b values is the equilibrium stable? Show your reasoning.
- For what b values is the equilibrium a centre? Is this case structurally stable? What does 'structurally stable' mean?
- Sketch the nullclines and determine (and sketch) a trapping region for the flow. What does the existence of a trapping region indicate?

(*Hint: you may find the Lecture 6 handout with an example from Strogatz quite helpful for the above question!*)

- Plot some typical trajectories for $b = 2.2$ using XPP.