The pth-order operator appearing on the right side of (2.1.19) is known as the *formal adjoint* of L and is denoted by L^*. We can then write (2.1.19) as

$$\langle Lf, \phi \rangle = \langle f, L^* \phi \rangle, \tag{2.1.20}$$

which defines the distribution Lf in terms of the action of the distribution f on the test function $L^* \phi$. Note that the operator L^* is the one that would appear if we integrated by parts the left side of (2.1.20), treating f as an ordinary p-times-differentiable function. We always have $(L^*)^* = L$. If $L = L^*$, we say that L is *formally self-adjoint*.

Let $L = a_2(x)D^2 + a_1(x)D + a_0(x)$ be the most general second-order operator in one variable, x. Then L^* is defined from

$$L^* \phi = a_2 \phi'' + (2a_2' - a_1)\phi' + (a_2'' - a_1' + a_0)\phi, \tag{2.1.21}$$

and the necessary and sufficient condition for L^* to be formally self-adjoint is

$$a_2' = a_1. \tag{2.1.22}$$

Note that an operator L of any order in any number of variables will be formally self-adjoint if it has constant coefficients and only partial derivatives of even order.