Engsci 711

Tutorial 1: Computer Methods for Qualitative Analysis of Differential Equations

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Overview

The purpose of this tutorial is to introduce you to the software package XPP/XPPAut for carrying out qualitative analysis of differential equations and other dynamical systems. XPP can help you perform phase plane analysis, bifurcation analysis (which uses an interface to the bifurcation software 'AUTO', hence 'XPPAut') etc, and can work with ODEs, DDEs, PDEs, discrete maps and stochastic models.

XPP has a relatively simple interface and is quick to get started with. It is worthwhile learning how to code up some of the basic algorithms yourself but we'll leave that for another day.

Installation

The homepage for XPPAut is http://www.math.pitt.edu/~bard/xpp/xpp.html. The links below take you directly to the download page.

Method 1: Linux

Ideally, you should log in to linux (Ubuntu?) in the lab. Ask me if you don't know how to do this.

You can then simply download the pre-compiled binary package 'xpplinux.tgz' from

http://www.math.pitt.edu/~bard/bardware/binary/latest/.

This contains the executable 'xppaut' which you should be able to run straight away. You may need to change the permissions for this executable. To do this right-click on it, select properties then permissions, tick the option 'allow executing file as program'.

Method 2: Windows

If you really don't want to use Linux you can go to

http://www.math.pitt.edu/~bard/bardware/binary/.

or

http://www.math.pitt.edu/~bard/xpp/xpponw95.html.

You can then download the file 'winpp.zip' which contains an executable 'winpp.exe'.

Unfortunately this doesn't contain all the functionality of XPPAut, though probably enough for our purposes. It is possible to install a full version of XPPAut on Windows but (I think) requires install privileges that you don't have. The basic difficulty is the need for an 'X-windows' interface.

Method 3: Mac

Same as for linux - i.e. go to

http://www.math.pitt.edu/~bard/bardware/binary/latest/

but download 'xppaut8_osx.dmg' instead.

Exercises

Feel free to play around with the '.ode' files that come in the 'ode' folder if you want. Otherwise, you can have a go at the following.

Also - here is a basic tutorial reference which will be helpful to refer to:

http://www.math.pitt.edu/~bard/bardware/tut/xpptut.html

There are many other resources available by googling e.g. 'xppaut tutorial'.

Creating a .ode file and carrying out a simple phase plane analysis

XPP takes input files with a '.ode' extension.

Here is a simple, minimal example (which can be created in any *plain text editor* - don't use Word/Open Office etc, and try to avoid copy-paste from the pdf because this can introduce weird errors...)

```
# simple ode
dx/dt = a*x+b*y
dy/dt = c*x+d*y
par a,b,c,d
done
```

By default all parameters and initial conditions are set to zero. These can be changed after the file is read in but we can also specify other default initial conditions in the file itself. E.g.

```
# simple ode
#right-hand side
dx/dt = a*x+b*y
dy/dt = c*x+d*y
#parameters
par a=0,b=1,c=-1,d=0
#initial conditions
init x=1,y=0
#fin
done
```

Note the comment character is the hash.

Save this file as e.g. 'linear2d.ode'.

Note: you can also use syntax such as

```
x' = a*x+b*y
y' = c*x+d*y
```

to specify your system. Discrete maps and other types of equations use other intuitive syntax too (google it!).

Running in XPP

Either open a terminal and type e.g.

```
xppaut linear2d.ode
```

In this case either the executable needs to be in your path or current directory and you need 'linear2d.ode' in your current directory.

Alternatively, you can simply double click the executable 'xppaut' (or whatever it's called for you). This brings up a menu which allows you to browser your file system for a '.ode' file.

Loading this file will bring up an 'X vs T' view.

XPP allows you to use a number of simple single-letter shortcut keys to access menu items (instead of clicking). Typically this is simply the first letter of the menu item, if not it will (usually) be capitalised. Note that these shortcut keys are only available for 'full XPP' not winpp - winpp requires you to point and click.

So, to run a time simulation for 'x', try pressing 'I' then 'G'. This chooses the menu items '(I)nitialconds' and then '(G)o' (which uses the initial conditions from the file). Note that 'esc' will typically close any currently-loaded menu.

Phase plane analysis

To switch to the 'phase-plane', i.e. 'x-y', view, press 'V' then '2'. This is '(V)iewaxes' and '2D'. Choose 'X' for the Xaxis and 'Y' for the Yaxis. Change the min and max values for both to -2 and 2 respectively.

If you have previously run a time simulation you should see an ellipse - what does this represent?

Now you can try running from anywhere in the phase plane - i.e. you can evaluate the flow function $\phi(x_0, t)$. This will run forward in time until an equilibrium or other stopping event is reached. XPP uses a default parameter for how long to run - see 'numerics' below. (Note - if you want to run backwards in time you can use 'I' then 'B').

To do this press '(I)nitialconds' and 'm(I)ce'. You can then click in the phase plane as many times as you want to run forward from this starting point. What do you see? Press 'esc' to exit this mode when you are done.

Note - if you want XPP to load up the system in a particular view by default you can use add lines like

```
@xp=x,yp=y
```

after the equations and parameters and before the 'done' statement. This sets the x-axis (xp) to be the 'x' variable and the y-axis (yp) to be the 'y' variable. If you had a dependent variable called 'u' you could use e.g.

```
@xp=u
```

to put it on the x-axis and so on.

To plot the direction field/flow you can use the '(D)ir.field/flow' menu. E.g. try 'D' then 'D' and/or 'D' then 'S'. The latter option scales the velocity field so the vectors have unit length and thus only indicate the direction and not magnitude of the velocity.

Changing parameters and initial conditions

There are two basic ways to change parameters and initial conditions.

Firstly, you can use the menus at the top - click these and see if you can change the parameters and/or initial conditions.

Alternatively, the menus on the left can be accessed in the usual way - e.g. pressing 'P' brings up an interactive entry point at the top of the window. You can type the name of a parameter, press enter then choose a new value. Press enter or esc when done.

Changing the initial conditions is done similarly via 'I' then 'N'.

Graphics and view navigation

You can zoom in and out etc by using the '(W)indow/zoom' side menu.

Note that XPP does not automatically save your orbits - if you change view etc you will often lose what you have drawn.

If you have drawn something that you want to keep you need to use '(G)raphic stuff' then '(F)reeze' and then '(F)reeze' again. This saves the last curve you drawn. Note that if you use 'm(I)ce' then it will only save the very last curve drawn before you press esc.

To save a picture use 'G', 'P' then OK. This saves an image in 'postscript' format. You can also use 's(V)g' for scalar vector graphics if you want.

Note on numerics

XPP uses a number of default numerical method parameters. To inspect/changes these, use 'n(U)merics'. See if you can determine what the time step and total time are.

Nonlinear ODE

Try writing a simple 2D nonlinear ODE file and explore its phase plane. For example you could use the system we considered in class:

$$\begin{aligned}\dot{x} &= x(y-1)\\ \dot{y} &= 3x - 2y + x^2 - 2y^2\end{aligned}$$

Can you find the fixed points? You can either do it by trial and error using 'I', 'I' or '(S)ing pts' (for 'singular' points - another name for equilibria/steady-states/fixed points) followed by e.g. 'M' to use a mouse click for an initial guess.

Note that XPP gives you the eigenvalues and hence stability properties! You can use this to check you answers in future assignments :-)