Engsci 711

Assignment 1

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Due: Monday 23rd May (in class)

Question 1

- Find the fixed points of the following equations, determine their stability and sketch their phase portraits

$$
\begin{aligned}
&(a) \quad x' = -2x - y, \quad y' = x + x^3 \\
&(b) \quad x' = x + y - 2x^2, \quad y' = -2x + y + 3y^2 \\
&(c) \quad x'' + \sin x = 0
\end{aligned}
$$

Question 2

The Brusselator is a simple model for a hypothetical chemical oscillator. In dimensionless form, its kinetics are given by

$$
\begin{aligned}
x' &= 1 - (b+1)x + ax^2 y \\
y' &= bx - ax^2 y
\end{aligned}
$$

where $a, b > 0$ are parameters and $x, y \geq 0$ are dimensionless concentrations.

- Find all of the equilibrium points and use the Jacobian to classify them.
- Which equilibria would you say are 'robust' to small changes in the model itself? Which are not? What sort of stability is this called?
- Sketch the nullclines and hence determine (and sketch) a trapping region for the flow.

Question 3

Consider the system

$$
\begin{aligned}
x' &= x - y - x(x^2 + 2y^2) \\
y' &= x + y - y(x^2 + y^2)
\end{aligned}
$$

- Re-write the system in polar coordinates (r, θ) where $x = r\cos\theta, y = r\sin(\theta)$. Hint: use the identities $rr' = xx' + yy'$ and $r^2\theta' = xy' - yx'$.
- Determine a region bounded by two circles (i.e. annulus shaped), each of which are centred at the origin, such that the flow is radially outward on the inner circle and radially inward on the outer circle.
- Show that there is a periodic orbit in this region, i.e. for $r_{inner} \leq \sqrt{x^2 + y^2} \leq r_{outer}$.
- Use XPP (or Matlab or Python etc) to plot the periodic orbit (and some neighbouring orbits) in the phase plane.

Question 4

Consider the system

$$x' = -2x - 3y - x^2$$
$$y' = x + 2y + xy - 3y^2$$

- Find and classify the equilibria.
- Find the power series expansions for $W^u_{loc}(0), W^s_{loc}(0)$ up to (i.e. including) cubic order.