Notebook for Maths 361: Partial Differential Equations (OJM)
Lecture 5

When we do these calculations by hand we use the fact that n is a non-negative integer. We can get MuPAD to assume this with the command:

```
assume(n,Type::NonNegInt)
```

Let's define our function for Example 1

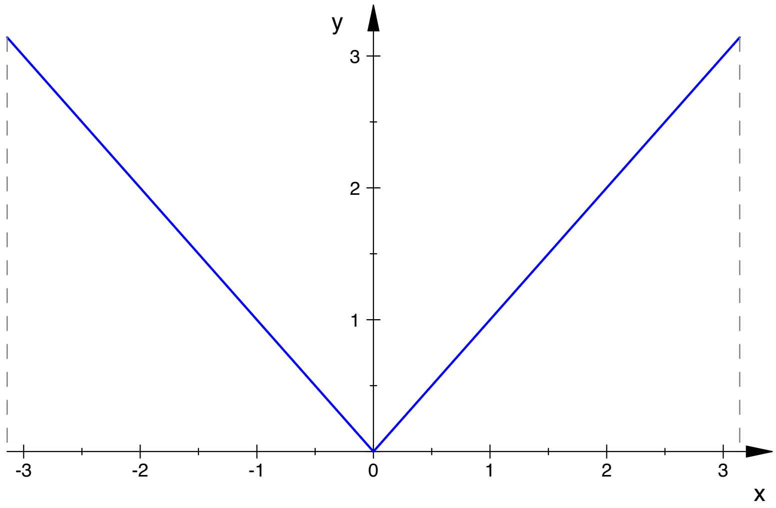
```
L:=PI
```

π

```
f:=piecewise([x>-PI and x <= 0, -x],[x<PI and x > 0, x])
```

$$\begin{cases} x & \text{if } x \in (0, \pi) \\ -x & \text{if } x \in (-\pi, 0] \end{cases}$$

```
plot(f(x),x=-L..L)
```



Now calculate the Fourier coefficients:

```
a0:=int(f(x),x=-L..L)/(2*L)
```

$$\frac{\pi}{2}$$

(Note for the following I used expand and simplify a few times to get it to output it in the form from lectures. Play with this yourself!)

```
an:=simplify(expand(simplify(int(f(x)*cos(n*PI*x/L),x=-L..L)/L)))
```

$$\frac{2\left((-1)^n - 1\right)}{n^2\,\pi}$$

```
bn:=simplify(int(f(x)*sin(n*PI*x/L),x=-L..L)/L)
```

0

We can write a and b as functions of an integer.

```
a:=m->subs(an,n=m);
a(0):=a0;
b:=m->subs(bn,n=m)
```

$m \rightarrow \text{subs}(\text{an}, n = m)$

$\frac{\pi}{2}$

$m \rightarrow \text{subs}(\text{bn}, n = m)$

Let's write a function S for the sum of the first (2N+1) terms of the Fourier series:

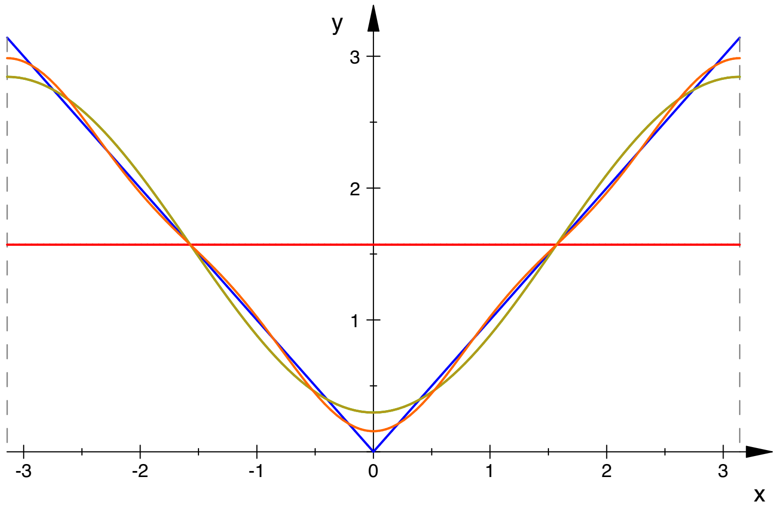
```
S:=N->a(0)+sum(a(n)*cos(n*PI*x/L)+b(n)*sin(n*PI*x/L),n=1..N)
```

$$N \rightarrow a(0) + \left(\sum_{n=1}^{N} \left(a(n) \cos\left(\frac{n \pi x}{L}\right) + b(n) \sin\left(\frac{n \pi x}{L}\right)\right)\right)$$

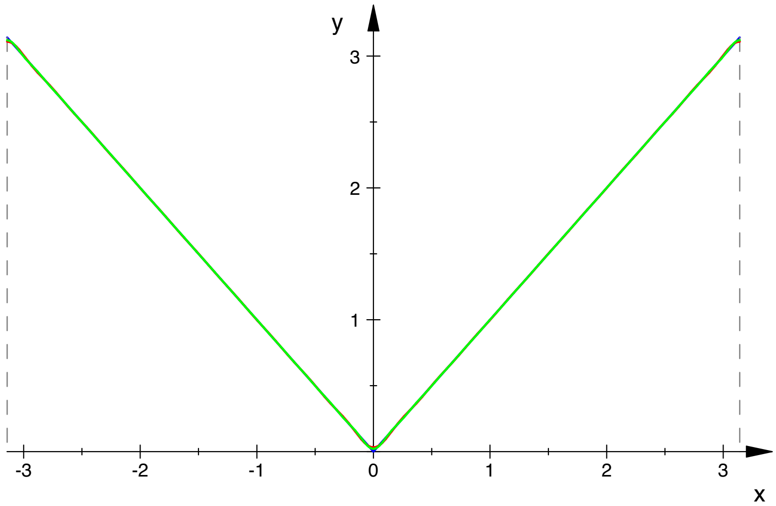
```
S(4)
```

$$\frac{\pi}{2} - \frac{4 \cos(3\,x)}{9\,\pi} - \frac{4 \cos(x)}{\pi}$$

```
plot(f,a(0),S(1),S(2),S(3),x=-L..L)
```

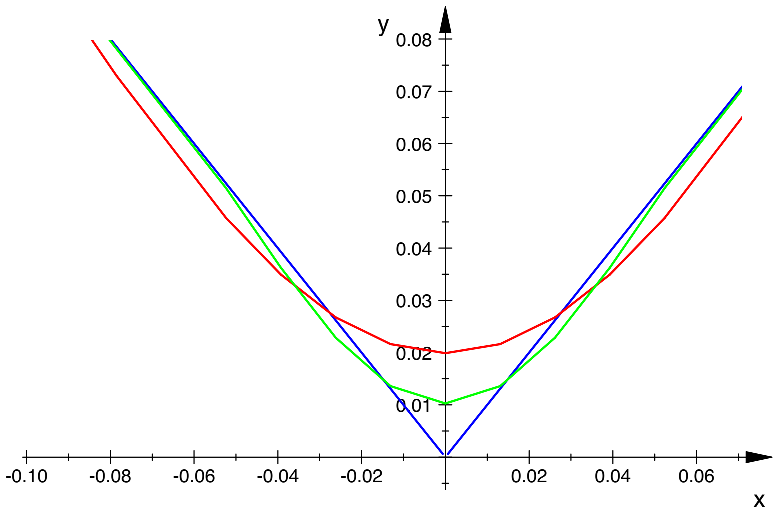



```
plot(f,S(20),S(30),x=-L..L)
```



Let's zoom in a bit (click on figure and use zoom option from menu)

```
plot(f,S(31),S(61),x=-L..L)
```

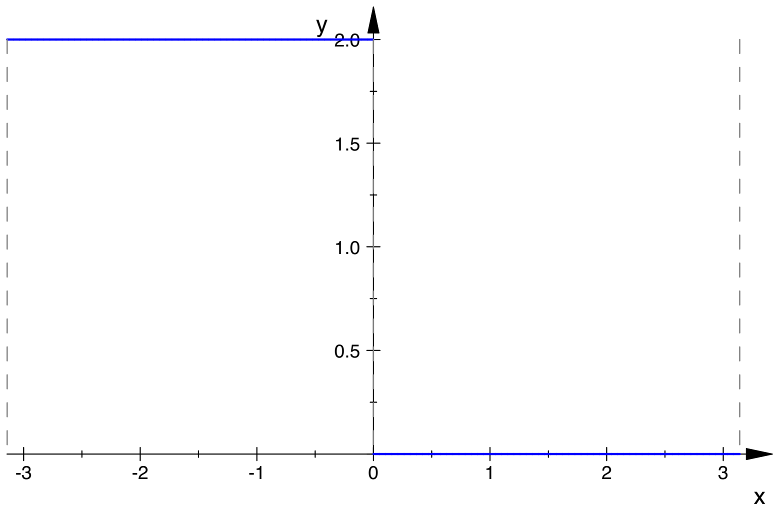


Let's run this for Example 2 instead

```
f:=piecewise([x>-PI and x <= 0, 2],[x<PI and x > 0, 0])
```

$$\begin{cases} 0 & \text{if} \quad x \in (0, \pi) \\ 2 & \text{if} \quad x \in (-\pi, 0] \end{cases}$$

```
plot(f(x),x=-L..L)
```



Now calculate the Fourier coefficients:

```
a0:=int(f(x),x=-L..L)/(2*L)
```

$$1$$

```
an:=simplify(int(f(x)*cos(n*PI*x/L),x=-L..L)/L)
```

$$0$$

```
bn:=simplify(int(f(x)*sin(n*PI*x/L),x=-L..L)/L)
```

$$\frac{2\left((-1)^n - 1\right)}{n\,\pi}$$

We can write a and b as functions of an integer.

```
a:=m->subs(an,n=m);
a(0):=a0;
b:=m->subs(bn,n=m)
```

$m \rightarrow \mathrm{subs}(\mathrm{an},\ n = m)$

1

$m \rightarrow \mathrm{subs}(\mathrm{bn},\ n = m)$

Let's write a function S for the sum of the first (2N+1) terms of the Fourier series:

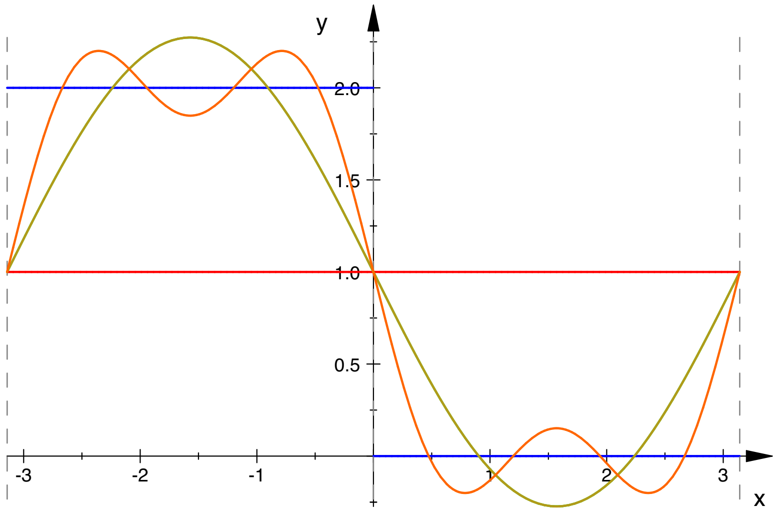
```
S:=N->a(0)+sum(a(n)*cos(n*PI*x/L)+b(n)*sin(n*PI*x/L),n=1..N)
```

$$N \rightarrow a(0) + \left(\sum_{n=1}^{N} \left(a(n)\cos\left(\frac{n\,\pi\,x}{L}\right) + b(n)\sin\left(\frac{n\,\pi\,x}{L}\right)\right)\right)$$

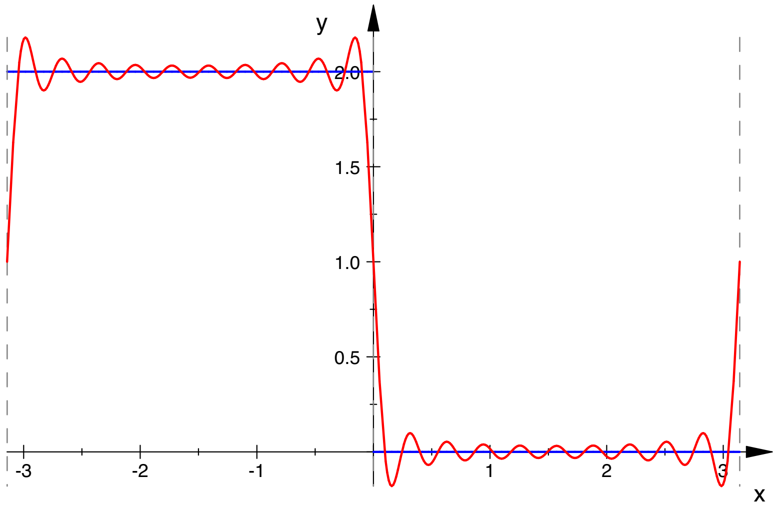
```
S(4)
```

$$1 - \frac{4\sin(x)}{\pi} - \frac{4\sin(3\,x)}{3\,\pi}$$

```
plot(f,a(0),S(1),S(2),S(3),x=-L..L)
```




```
plot(f,S(20),x=-L..L)
```



Note the Gibbs phenomenon (overshoot near the discontinuity)!
Let's take even more terms and zoom in to see how it persists

(click on figure and use zoom option from menu)

```
plot(f,S(31),S(61),x=-L..L)
```