Lecture 9 : Work & Efficiency

- Work in different process types
  - isobaric
  - isochoric
  - isothermal
  - adiabatic
- Efficiency of heat engines } using heat to do work
  - L Carnot & a first encounter with the second law.

Example questions:

26) One mole of an ideal gas, initially at $T_1 = 330\,K$, $P_1 = 101.3$ kPa and $V_1 = 27.1 \times 10^{-3}$ m^3, undergoes the following series of processes in a closed system:

Process I: Isovolumetric cooling to $T_2 = 300$ K, $P_2 = 92.0$ kPa
Process II: Isobaric expansion to $V_3 = 28.1 \times 10^{-3}$ m^3, $T_3 = 311$ K
Process III: Adiabatic compression back to its initial state.

a. Draw these three processes on a pressure-volume graph, label the axes, identify each state, process and path.

(4 marks)

Answer:

....

b. Calculate the work (W) for each process and hence the net work done. Assume an adiabatic constant γ of 1.4. Show your working.

(4 marks)

27) What is the relationship between Internal Energy (ΔU), Heat (Q), and Work (W) in an adiabatic process.

(1 mark)

Answer:

2. A reversible heat engine operates between a hot reservoir at 900K and a cold reservoir at 500K.
   a. Calculate the efficiency of the engine.
   b. The temperature of one of the heat reservoirs can be changed by 100 degrees kelvin up or down. What is the highest efficiency that can be achieved by making this temperature change?

δ : 'small amount' of process var / interaction

$$W = \int_{path} P\,dV \quad \text{or} \quad \delta W = P\,dV$$

d: small change in a state var.

2.6.2 Processes and PV diagrams

In general to calculate the work we need to *integrate the force times displacement along the full path.* This will also depend on the particular material or substance we are studying.

For an ideal gas in a piston, however, we can save some effort by identifying some special cases. These are

iso: same

1. **Isobaric** processes: constant *pressure*
2. **Isochoric** processes: constant *volume*
3. **Isothermal** processes: constant *temperature*
4. **Adiabatic** processes: *no heat transfer*

for P-V systems (usually ideal gas)

Let's derive formulae for the work done during these process (note: in general you don't need to re-derive these - you can just use the results).

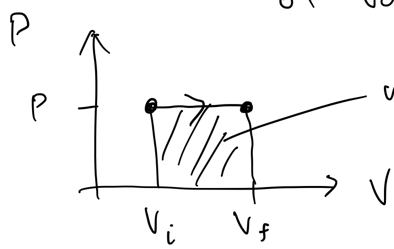
Isobaric work

$$W = \int_{V_i}^{V_f} P\,dV = P\int_{V_i}^{V_f} dV = P\cdot(V_f - V_i)$$

pressure independent of volume

upshot:

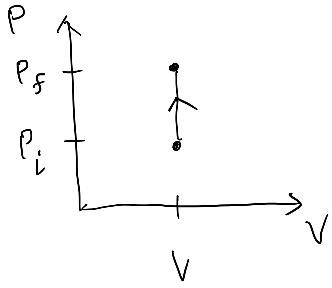
$$\boxed{W = P\cdot\Delta V} \quad \text{isobaric}$$



work = area under P-V path = P·ΔV

Isochoric work

constant volume:



$\boxed{\text{No work}}$

- zero area under curve
- zero displacement/change in vol.

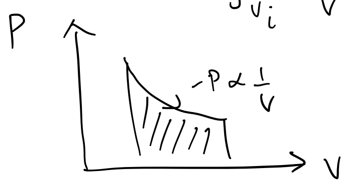
$$\delta W = p\,dV,\ dV = 0$$
$$\Rightarrow \boxed{\delta W = 0}$$
$$\Rightarrow \boxed{W = 0}$$

Isothermal work

Constant temperature expansion ($+W^{by}$) } Note: usually requires heat transfer to keep const. temp.!

$$W = \int_{V_i}^{V_f} P \cdot dV \quad \& \quad P = \frac{nRT}{V}$$

$$\Rightarrow W = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV = \underbrace{nRT}_{\text{const. temp.}} \int_{V_i}^{V_f} \frac{1}{V}\,dV = nRT\left[\ln V\right]_{V_i}^{V_f} = nRT\left[\ln V_f - \ln V_i\right]$$



Upshot: $\boxed{W = nRT \ln \frac{V_f}{V_i}}$ for isothermal

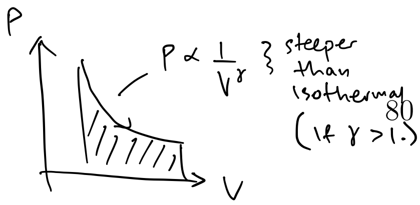
Adiabatic work

No heat transfer. Note: may change temperature! Adiabatic $\neq$ Isothermal

Can show $PV^{\gamma} = \text{constant} = A$ (say); γ = ratio of heat capacities

Integrate: $W = \int P\,dV = \int \frac{A}{V^{\gamma}}\,dV = \frac{A\left(V_f^{1-\gamma} - V_i^{1-\gamma}\right)}{1-\gamma} = \frac{AV_f^{1-\gamma} - AV_i^{1-\gamma}}{1-\gamma}$

So: $P_i V_i^{\gamma} = A$ & $P_f V_f^{\gamma} = A$



$$= \frac{\left(P_f V_f^{\gamma}\right) V_f^{1-\gamma} - \left(P_i V_i^{\gamma}\right) V_i^{1-\gamma}}{1-\gamma}$$

$$= \frac{P_f V_f - P_i V_i}{1-\gamma} \longrightarrow$$

Upshot: $W = \frac{P_f V_f - P_i V_i}{1-\gamma}$ or $\boxed{W = \frac{P_i V_i - P_f V_f}{\gamma - 1}}$ for adiabatic

(since γ usually ≥ 1.

Warning: *These formulae and derivations are only valid for ideal gases in pistons undergoing reversible processes.*

Example Problems 7: Work and different process types

1. How much work is done by 1.88 mols of a gas expanding isothermally at 298 K against a piston from 46 L, at 1.013 $\times 10^5$ Pa, to 65 L?

2. How much work is done by 1.88 mols of a gas expanding adiabatically at 298 K against a piston from 46 L, at 1.013 $\times 10^5$ Pa, to 65 L and 6.24 $\times 10^4$ Pa?

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3. One mole of an ideal gas, initially at $T_1 = 25°C$, $P_1 = 101.3$ kPa and $V_1 = 24.5 \times 10^{-3}$ m^3, undergoes the following mechanically reversible processes in a closed system:

   (a) **Process I**: Adiabatic compression to $T_2 = 473°C$, $P_2 = 505$ kPa and $V_2 = 7.65 \times 10^{-3}$ m^3. $> P_2$ $> V_1$

   (b) **Process II**: Isobaric cooling to $T_3 = 25$ °C and $V_3 = 4.89 \times 10^{-3}$ m^3 $< V_2$

   Draw these two processes on a pressure-volume graph, label the axes and identify each state, process and path.

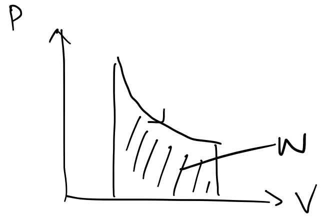
Solutions

ideal gas constant $= 8.314 \frac{J}{mol \cdot K}$

1. Isothermal work $W = nRT \ln\left(\frac{V_f}{V_i}\right)$ } note: on ratio of vols important so units don't matter here!

$\Rightarrow W = 1.88\,mol \times 8.314 \frac{J}{mol \cdot k} \times 298\,k \times \ln\left(\frac{65}{46}\right)$

$\simeq \underline{1610\,J}$ (+ve W^{by})



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2. $W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$, $\gamma = 1.4$ for air

$P_i = 1.013 \times 10^5\,\underset{\sim}{Pa}$ J/m^3

$P_f = 6.24 \times 10^4\,J/m^3$

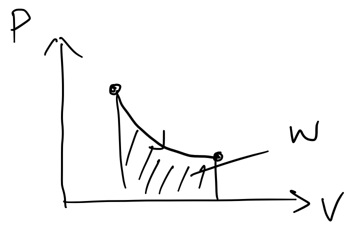
$V_i = 46 \times 10^{-3}\,m^3$

$V_f = 65 \times 10^{-3}\,m^3$

$(1L = 10^{-3}\,m^3)$

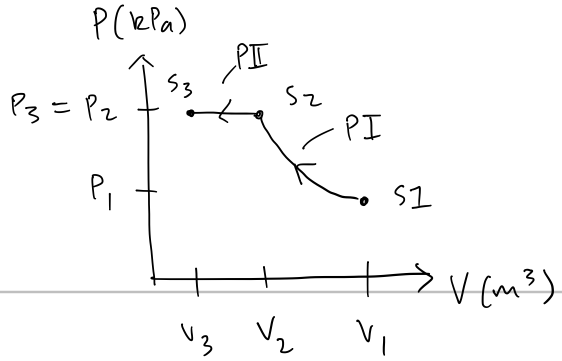
$= \frac{1.013 \times 10^5 \frac{J}{\cancel{m^3}} \times 46 \times 10^{-3}\,\cancel{m^3} - 6.24 \times 10^4 \times 65 \times 10^{-3}}{1.4 - 1}$

$\simeq \underline{1510\,J}$



(note: steeper than isothermal $\Rightarrow$ less area $\Rightarrow$ less work).

3.



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naive today.

things (energy, order etc.) tend to 'spread out'

2.6.3 Heat engines, efficiency limits and the second law

{ see next lecture, mainly.

The second law of thermodynamics arose out of the practical question in the nineteenth century: *how much useful work can we get from steam engines?* This is the topic of **heat engines**. Here 'useful' means, in essence 'macroscopic', 'bulk' or 'organised' work, i.e. usually not including e.g. microscopic 'random jiggling' of atoms.

Heat engines:

1. **Convert** thermal energy and/or heat input into useful work
2. Usually carry out a **cycle** or series of cycles.
3. Have **limits on** their efficiency.

next lecture: entropy!

Spreading out:

Macro
bulk
(concentrated)
'lose to'
Micro
vibration
(more spread out)

The discovery of the *existence of efficiency limits* on heat engines was in fact the **first discovery of the second law of thermodynamics**. By now we know, however, that the idea applies more generally than just the practical topic of 'steam engines'!

} Here: naive.

The more basic idea is that *macroscopic mechanical work can be completely converted to heat transfer/thermal energy, but not vice-versa.* Intuitively, when trying to convert thermal energy or heat input, i.e. microscopic mechanical energy, to macroscopic mechanical work, some of that energy is always 'lost' to, or retained as, microscopic mechanical energy, i.e. thermal energy.

idea! heat flows hot → cold temp.

Carnot efficiency of heat engines

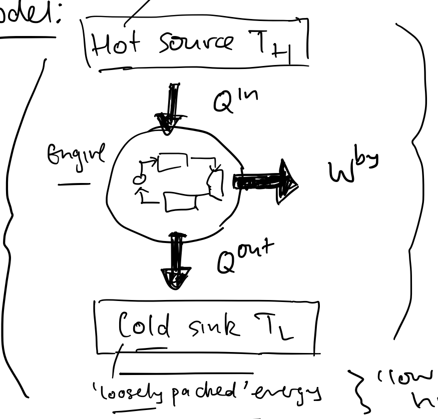
Heat engine: $T^{Hot} \xrightarrow{Q} T^{cold}$, $\downarrow W$?

Eg.

- Take heat generated by burning fuel] Q^{in}
- Convert part of this to work] W^{by}
- Expel the rest as 'waste' to the environment so that system returns to initial state & hence performs a cycle.] Q^{out}
- ie we analyse on per cycle basis.

model:

'tightly packed' energy } 'high quality' low entropy



'loosely packed' energy } 'low quality' high entropy

Hot
Q 'use' W
Cold

Heat flow { Hot → Cold; Tight → Loose. (2nd Law!); Low entropy → High entropy

1. First law & cycle.
   - System returns to initial state: $\Delta U = 0$ (cycle)
   - First law & ΔU: $W^{by} = Q^{net} = \boxed{Q^{in} - Q^{out}}$ ('added' 'waste')
   - $W^{by}_{max} = Q^{in}$, $Q^{out} = 0$. (min. waste). } just first law so far (W, Q)

2. Efficiency defn.

$$\text{Eff.} = \frac{\text{Out}}{\text{In}} = \eta = \frac{W^{by}}{Q^{in}} = \underbrace{\frac{Q^{in} - Q^{out}}{Q^{in}}}_{\text{1st law}}$$ } defn of efficiency

(output; input)

3. Carnot efficiency limit — early version of 2nd law (Note: involves Q & T)

$$\eta = \frac{Q^{in} - Q^{out}}{Q^{in}} \leq \boxed{\frac{T^{in} - T^{out}}{T^{in}} = \eta_{max}}$$ } Carnot efficiency, max efficiency give T^{in}, T^{out} $\boxed{\eta_{max}}$

Carnot's Theorem: to beat η_{max} would require: $T_{Hot} \xleftarrow{Q} T_{Cold}$

(**Upshot:** Once macroscopic mechanical energy is $\boxed{\text{'lost'}}$ from the 'macroscopic' mechanical world to the 'microscopic mechanical world' (e.g. the jiggling of atoms) it can never be fully recovered as macroscopic mechanical energy. Of course conservation of total energy - including thermal and mechanical energies - still holds! The second law is a statement about **efficiency** of the conversion of one type of energy to another.)

} macro: concentrated; micro: spread out.

or 'spreading out' of energy.

Example Problems 8: Heat engines

$\eta = \eta_{max}$

1. A reversible heat engine operates between a hot reservoir at 900 K and a cold reservoir at 500 K.
   (a) Calculate the maximum efficiency of the engine.
   (b) The temperature of one of the heat reservoirs can be changed by 100 K up or down. What is the highest efficiency that can be achieved by making this temperature change?
2. A startup company claims to have designed a new heat engine that has 42% efficiency, using at heat source at 500 K and a cold sink at 300 K. Is this reasonable?

Answers

1. a) $\eta_{max} = \dfrac{T^{in} - T^{out}}{T^{in}} = \dfrac{900-500}{900} = \underline{0.44}$

b) $T^{in} \uparrow$ or $T^{out} \downarrow$? (usually $T^{out} \downarrow$)

<u>Check</u>: Case 1, $\eta_{max} = \dfrac{1000 - 500}{1000} = \underline{0.5}$

Case 2, $\eta_{max} = \dfrac{900 - 400}{900} \approx \underline{0.56}$ ← best $\}$ <u>cold ↓</u>

2. $\eta_{max} = \dfrac{T^{in} - T^{out}}{T^{in}} = \dfrac{500-300}{500} = \underline{0.4}$

↓

claim $\eta = \dfrac{W^{by}}{Q^{in}} = \underline{0.42 > \eta_{max}}$ × <u>No</u>!

→ violates Carnot efficiency limit (hence 2nd Law).