## Overview L5

- Life & transport in fluids

→ Drag

→ Reynolds number

→ Scaling.

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## Example Qs

### Exam

(assume shape drag)

**10)** The work required to overcome air resistance ($W_{air}$) plays the biggest role in the total energy budget of a vehicle. The NZ Government are thinking about decreasing the speed limit from 100 km/h to 80 km/h. Estimate the effect this would this have on fuel consumption.

**(2 marks)**

**Answer:**

### Other

→ You want to experience what it is like for a bacterium swimming in water. Briefly explain how you could design an experiment to do this.

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L5



Article by Purcell (1977)

## 1.7 Case study: Life at low Reynolds number

L5 —

Also 'Life in moving fluids' (Book) by Vogel.

### Problem formulation

| |
|---|
| 'What the hell is water?' — A fish, according to David Foster Wallace. |

We are surrounded by fluid – air & water. This is more apparent when designing cars, boats & airplanes & other ways of moving through these fluids.

– eg air resistance, lift, pressure etc are all important things to think about

Let's consider life in fluids

→ Do bacteria swim in the same way as fish? why/why not?

→ How can we travel through fluids more efficiently, & what does this depend on?

In particular we'll think about drag forces & associated energy transfers when moving through fluids.

↳ want to find the forces of fluid on object in fluid

Note: In more advanced courses on fluid mechanics you'll encounter the Navier-Stokes equations:

$$\underbrace{\rho\left(\frac{\partial v}{\partial t} + (v\cdot\nabla)v\right)}_{\text{"ma"}} = \nabla P + \eta\nabla^2 v$$

partial differential eqn for fluid flow

"ma" = pressure force + viscosity force = $\Sigma F$

Here we'll do some naive calculations instead!

**Calculations**

Types of drag : Here × 2 .

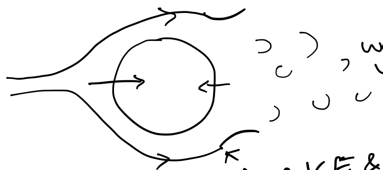
'stickiness'

- ultimately both arise from 'viscosity' (& 'ideal' fluid with no viscosity predicts no drag !)
  - ↳ but sufficiently different in scaling etc that we split into:

separation-based

"flow separation"

① "shape drag" or "pressure drag" or "inertial drag"

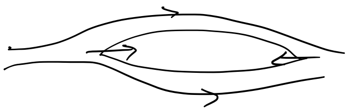


wake of vortices : low pressure region at rear

net pressure gradient ⟶

lose KE & 'detach'

'inertial'



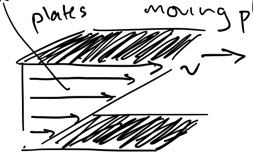
less flow separation

"streamlined"

- less pressure drop.
  - ↳ KE & pressure energy are 'interconverted'

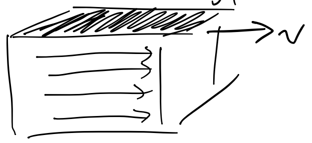
direct 'friction' based

② "viscous drag" or "skin friction"



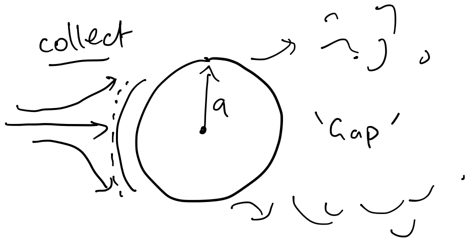

'sticks' to fixed plate

→ 'friction'




'frictionless' → freely slips over fixed plate.

separation : a simple model as 'inertial' effect




- Transfer KE to fluid : $\propto \rho a^3 v^2$ (density: $\rho$; velocity: $v$)
  'collected up'

  & $W = F \cdot a$ } work over one length

  $\Delta KE_{air} = \frac{1}{2}\rho a^3 v^2 = F \cdot a$

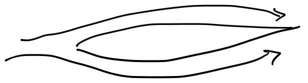
  $\Rightarrow \boxed{F = \frac{1}{2}\rho a^2}$

  $a^2$ : Frontal area

Motivates 'inertial' shape drag constitutive eqn.

$$\boxed{F_{Shape} = C_D v^2 \sim v^2}$$

$C_D$ depends on each shape, fluid properties etc.



- Fluid moves around
  → less is 'collected up'

} Less shape/pressure/separation-based drag.

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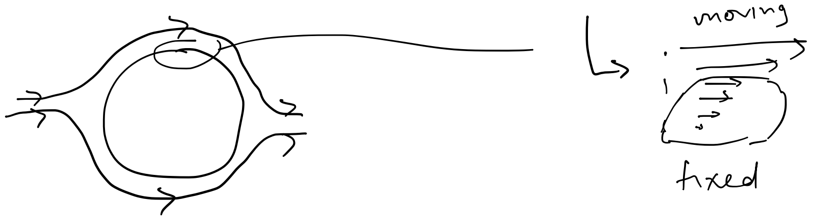
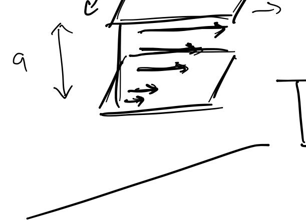
Non-separated : streamlined OR slow flow } can 'navigate' around
→ model of frictional effect




} 'shear', frictional dissipation } generates heat $\Delta KE \rightarrow$ heat




$$\frac{F}{A} = \mu \frac{v}{a}$$

$\boxed{\mu : \text{viscosity}}$

A : surface area $\sim a^2$

Newtonian, viscous drag constitutive eqn.

$$\boxed{F_{viscous} = \mu \cdot a v \sim v}$$

units:

$Pa \cdot s = \frac{N \cdot s}{m^2}$

$W = F_{vis.} \times a$ : work over one length.

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**Calculations** Which is more important?

- Inertial $\sim \rho a^3 v^2$ (KE of fluid [=] J)
- Viscous $\sim (\mu \cdot a \cdot v) \cdot a$ (viscous work [=] J : $\frac{N \cdot \cancel{s}}{\cancel{m^2}} \times \cancel{m^2} \cdot \frac{m}{\cancel{s}} = N \cdot m$) ✓

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Ratio : $\frac{\rho \cdot a^{\cancel{3}} v^{\cancel{2}}}{\mu \cdot \cancel{a^2} \cdot \cancel{v}} = \frac{\rho \cdot v \cdot a}{\mu}$

Defines Reynolds number $\boxed{Re := \frac{\rho \cdot v \cdot a}{\mu}}$ → $\boxed{\text{dimensionless ratio}}$

fluid density, velocity, characteristic length

fluid viscosity

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$\boxed{Re << 1}$ => viscous effects dominate & $\boxed{F_{drag} \sim v}$

↳ KE is quickly 'dissipated' to heat by 'friction'

↳ Stop actively swimming, stop almost instantly!

Scaling experiments: Match Re to give 'similar' effect.

↳ Imagine yourself swimming in super thick fluid ie $\mu \uparrow \Rightarrow Re \downarrow$

↳ or, imagine very small organisms, $a \downarrow$, in water → feels like honey etc to them!

↳ swim by 'dragging' or 'corkscrewing'!

↳ almost like a solid environment.

$\boxed{Re >> 1}$ => inertial effects dominate & $F_{drag} \sim v^2$

↳ swim by kicking' & transferring momentum to fluid.

↳ can 'coast' after stroke (inertia of body is important too)

## Conclusions

- Engineers need to design efficient transport methods in fluid environments
- To do this, need to understand dominant types of drag under diff. conditions
- The balance between 'inertial'/shape/pressure & frictional effects determines what it's 'like' in a given fluid, eg the dominant drag type
- This is quantified by the Reynolds number

Drag scaling.

Low Re $\rightarrow$ Drag $\propto v$

High Re $\rightarrow$ Drag $\propto v^2$

## Table of examples (Vogel)

TABLE 5.1 A SPECTRUM OF REYNOLDS NUMBERS FOR SELF-PROPELLED ORGANISMS.

| | *Reynolds Number* |
|---|---|
| A large whale swimming at 10 m s$^{-1}$ | 300,000,000 |
| A tuna swimming at the same speed | 30,000,000 |
| A duck flying at 20 m s$^{-1}$ | 300,000 |
| A large dragonfly going 7 m s$^{-1}$ | 30,000 |
| A copepod in a speed burst of 0.2 m s$^{-1}$ | 300 |
| Flapping wings of the smallest flying insects | 30 |
| An invertebrate larva, 0.3 mm long, at 1 mm s$^{-1}$ | 0.3 |
| A sea urchin sperm advancing the species at 0.2 mm s$^{-1}$ | 0.03 |
| A bacterium, swimming at 0.01 mm s$^{-1}$ | 0.00001 |

High Re $\uparrow$

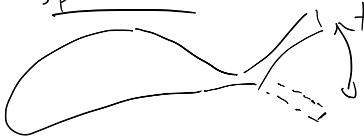
low Re $\downarrow$

Scaling: change $\rho, \mu, l$ etc to match Re & get 'equivalent' phenomenon. } experimental design.

$\rightarrow$ Balance/ratio is what matters

## Swimming Styles: Different Re

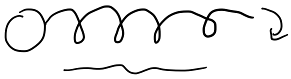
sperm whale



transfer momentum. symmetric, 'up/down' motion

Re >> 1

sperm



flagellum.

Corkscrew, using friction. Asymmetric 'winding'

Re << 1.

## Engineering & Cars (human transport in fluids!)

Drag $\propto v^2$ (High Re)

- suppose $v \downarrow$ from $\underbrace{100 \text{ km/hr to } 90 \text{ km/h}}_{10\% \downarrow \text{ in } v}$

$$\Rightarrow \frac{\text{Drag}_1}{\text{Drag}_2} = \frac{90^2}{100^2} = 0.81 \text{ ie } 19\% \downarrow \text{ in drag}$$ (& eg energy loss)

$\rightarrow$ nonlinear effect.