Overview L2

1. Some key quantities

- Force, energy, power, pressure
  - ↳ Units
  - ↳ 'Typical' numerical values for 'human scale' phenomena.

2. Energy & Work

- Conservation of energy
- Various forms of energy
- Work, energy & energy conversions

Example questions (2018 Summer School)

1. Using Newton's Second Law to help you, derive an expression for 1 Newton (1 N) in terms of base SI units.

(2 marks)

Answer:

2. A single piece of Kentucky Fried…Cheese contains 2460 kJ of energy. How high up a ladder could an 80 kg person climb from the energy available from a single piece of fried cheese? Assume muscles work at 25% efficiency converting nutritional energy to mechanical energy.

(2 marks)

Answer:

L2 –

1.3 Some key quantities involved in the physics of energy

Let's take a look at some key quantities that you might encounter in the physics of energy: *force, power, pressure* and... *energy.*

We can derive the SI dimensions/units of these by considering some simple examples. In addition, some numerical examples can help give us a feel for the typical 'human' orders of magnitude of these quantities when expressed in SI units.

Units of force from Newton's second law

Force = mass × acceleration

$$[=] \underbrace{M \times \frac{L}{T^2}}_{\text{dim.}} \quad [=] \underbrace{\frac{kg \cdot m}{s^2}}_{\text{SI units}} \overset{(\text{def}^n)}{=} N$$

ie $1\,N = 1\,\frac{kg \cdot m}{s^2}$

numerical example.

Typical gravitational force on a person

$F = m \times g.$ Say 70kg ?

$= 70\text{ kg} \times 9.81\text{ m/s}^2$

$= 686\text{ kg}\cdot\text{m}\cdot\text{s}^{-2}$

$= 686\text{ N}$

$686\ \cancel{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}} \times \left(\frac{1\text{ N}}{1\ \cancel{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}}}\right)$

Units of energy from translational kinetic energy

$KE = \frac{1}{2}mv^2$

$[=]\ M\cdot\left(\frac{L}{T}\right)^2$ (dimensions)

$[=]\ \text{kg}\,\frac{\text{m}^2}{\text{s}^2}$ (SI units)

Also: $KE\ [=]\ J$ (derived) & $1\text{ J} = 1\text{ kg}\,\frac{\text{m}^2}{\text{s}^2}$ (defn) (base)

Typical kinetic energy of a person walking

$KE = \frac{1}{2}mv^2$. Say $v = 5\text{ km/hr}$.

→

$$KE = \frac{1}{2} \times 70\ \text{kg} \times \left(\frac{5 \times 10^3\ \text{m}}{\text{hr}} \times \frac{1\ \text{hr}}{60 \times 60\ \text{s}} \right)^2$$

$$= 35\ \text{kg} \times \left(\frac{50\cancel{00}}{36\cancel{00}}\ \frac{\text{m}}{\text{s}} \right)^2$$

$$\approx 67.6\ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 67.6\ \text{J}$$

Units of $\boxed{\text{power}}$ from rate of change of energy

$$\text{Power} = \text{(change in) energy per unit time} = \frac{dE}{dt}$$

$$\left.\begin{aligned} &[=]\ \text{Energy}/\text{Time} \\ &[=]\ \left(\frac{M \cdot L^2}{T^2}\right) \cdot \left(\frac{1}{T}\right) \end{aligned}\right\} \text{dimensions}$$

$$[=]\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \quad \Big\} \ \text{SI units}$$

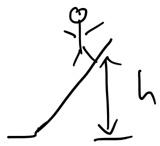
(defn)

Also, derived: Power $[=]$ Watts & $1\ \text{Watt} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$

$$= 1\ \text{J}/\text{s}$$

Typical power of a person climbing stairs

70 kg, 0.5 m/s ?



$$\frac{\Delta E}{1\ \text{se}} = \frac{m \cdot g \cdot h}{1\ \text{sec}} = \frac{70\ \text{kg} \times 9.81\ \text{m}/\text{s} \times 0.5\ \text{m}}{1\ \text{sec}}$$

$$\approx 343\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$$

$$= 343\ \text{J}/\text{s} = \underline{343\ \text{Watts}}$$

Units of $\boxed{\text{pressure}}$ from force per unit area

Pressure = Force per unit area = F/A (or $\frac{dF}{dA}$ etc)

$$\left.\begin{aligned}\text{Pressure} &\;[=]\; \text{Force}\cdot\text{Area}^{-1}\\ &= \left(M\cdot\frac{\cancel{L}}{T^2}\right)\left(\frac{1}{L^{\cancel{2}}}\right)\end{aligned}\right\}\text{dim.}$$

$$[=]\; kg\cdot m^{-1}\cdot s^{-2} \quad \Big\}\ \text{SI units}$$

Also Pressure [=] Pa
derived SI unit

& $\underline{1\,Pa = 1\,kg\cdot m^{-1}s^{-2}}$ (def^n)

& $= 1\,N\cdot m^{-2}$

Typical pressure on the soles of a person's feet

careful: $cm^2 = (cm)^2$

70kg , contact area / foot = $125\,cm^2$ $= 125\times(10^{-2}m)^2$

$\Rightarrow$ Pressure = Force / Area $= 125\times10^{-4}m^2$

$$= \frac{70kg\times9.81m/s^2}{2\times125\,cm^2} = \frac{70kg\times9.81m/s^2}{2\times0.0125\,m^2}$$

$$\simeq 27{,}440\,kg\cdot m^{-1}\cdot s^{-2}$$

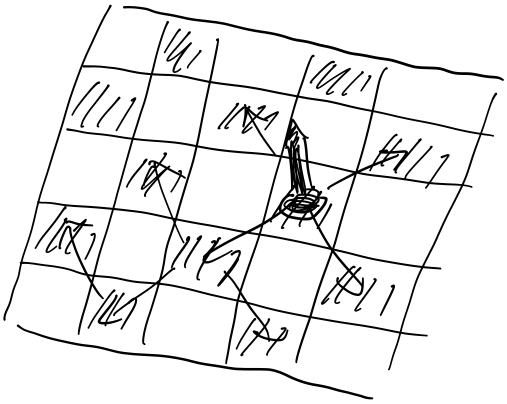
$$= 27{,}440\,Pa = \underline{27.44\,kPa}$$

1.4 Forms of energy and energy conversions

1.4.1 What is energy?

Let's ask a seemingly basic question - what is energy? To motivate our answer consider the following example.

A bishop on a chess board



Q: After N moves, what can we say?

A: Not much, except that it must be on the same colour it started on

How is this related to our question of 'what is energy?' Let's see how this example 'pops up' in the following answer to our question, given by the physicist Richard Feynman (1918-1988) in his famous *The Feynman Lectures on Physics*:

From **Section 4-1 What is Energy?** in *The Feynman Lectures on Physics* (1964, revised and expanded 2005):

There is a fact, or if you wish, a law, governing all natural phenomena that are known to date. ***There is no known exception to this law - it is exact so far as we know. The law is called the conservation of energy. It states that there is a certain quantity, which we call energy, that does not change in the manifold changes which nature undergoes.*** *That is a most abstract idea, because it is a mathematical principle; it says that there is a numerical quantity which does not change when something happens. It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number and when we finish watching nature go through her tricks and calculate the number again, it is the same...* ***Something like the bishop on a red square, and after a number of moves - details unknown - it is still on some red square. It is a law of this nature...***

...when we are calculating the energy, sometimes some of it leaves the system and goes away, or sometimes some comes in. ***In order to verify the conservation of energy, we must be careful that we have not put any in or taken any out....energy has a large number of different forms, and there is a formula for each one.*** *These are: gravitational energy, kinetic energy, heat energy, elastic energy, electrical energy, chemical energy, radiant energy, nuclear energy, mass energy.* ***If we total up the formulas for each of these contributions, it will not change except for the energy going in and out...***

re: mass on spring → include inputs/outputs to system

...It is important to realize that ***in physics today, we have no knowledge of what energy is.*** *We do not have a picture that energy comes in little blobs of a definite amount. It is not that way.* ***However, there are formulas for calculating some numerical quantity, and when we add it all together it gives "28" - always the same number.*** *It is an abstract thing in that it does not tell us the mechanism of the reasons for the various formulas.*

OK! So maybe our question is not so straightforward, though clearly it has something to do with the advice we saw earlier - *when things change, ask yourself: what stays the same?*

Here is Feynman's summary, captured in a picture taken during his original lectures (1961-1963):

Figure 1: Summary of Feynman's lectures on energy. From `http://www.feynmanlectures.caltech.edu/I_04.html`

1.4.2 Kinetic or potential energy?

Energy is sometimes classified into *kinetic* or *potential* energy categories. This isn't always so unambiguous: e.g. kinetic energy can be thought of as 'stored' in the form of translational or rotational motion energy.

1.4.3 Work and energy

A perspective on energy that is particularly helpful for an engineer is: *energy represents a capacity to do work, stored in a particular form*.

Similarly, we can say *work represents a transfer or change of energy from one form to another*.

Most importantly, it is *differences* in energy that represent either the potential to do work or the result of doing work: *energy is only defined up to an arbitrary reference value*. We will look more carefully at work and the transfer/conversion of energy in the next section when we discuss thermodynamics, where we will also discuss heat and entropy in more detail.

1.4.4 Forms of energy

With that out of the way, let's consider some basic 'forms' of (or 'ways of accounting for') energy, including how these are 'stored' and/or how they might be used to 'do work' or be converted to different forms.

Gravitational potential energy

- 'Stored' energy due to position in a gravitational field } general idea

- Particular model, or constitutive eq^n:

$$\boxed{E_p = \underbrace{m}_{\text{mass}}\,\underbrace{g}_{\text{strength of field as accel.}}\,\underbrace{(h-h_0)}_{\text{position in field relative to } h_0}}$$

(there are other models too!)

Electrical potential energy

- 'Stored' in electrical fields

- $\dfrac{\text{Potential}}{\text{Unit charge}} \overset{\text{def}^n}{=} \text{Voltage}.$

$$1 \text{ Volt} = 1 \text{ Joule}/\text{Coulomb}.$$

Chemical potential energy

- 'Stored' in bonds between atoms in compounds
  (also in eg concentration gradients)
- Can 'transform' or 'convert' to other types
  via chemical reactions
  └ breaking or forming of bonds

Example: cellular respiration $\boxed{\text{Food} \xrightarrow{\text{release}} \text{Energy}}$ (see later)

Elastic potential energy

- Recoverable ('reversible' — see later) energy stored in 'springs' & similar materials & objects

  } 'perfect' spring: get back all stored energy as work.

- Common model: $\boxed{U = \frac{1}{2}k(x-x_0)^2}$

  elastic constant; $x - x_0$: displacement rel. to ref.

  Can get from Hooke's const. law for force, eg 'do work' & 'store':

  $$\int_{x_0}^{x} F \cdot dx = \int k(x-x_0)\,dx = \frac{1}{2}k(x-x_0)^2$$

  work by stored

Nuclear energy

- 'Stored' in nuclear bonds (holding nucleus together etc)
- Splitting (fission) or fusing (fusion)
  releases a large amount of energy

Nuclear fuel ~ 1×10^6 energy density of carbon fuels

Radiant energy

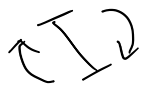
- Energy stored electromagnetic fields
- "radiation"

  comes in small 'packets' (photons)

  all travel at ... speed of light!

Kinetic energy

Translational o $\longrightarrow$ o $E_{Tr} = \frac{1}{2}mv^2$

Rotational  $E_{Rot} = \frac{1}{2} I \omega^2$

moment of inertia ('rotational mass')

angular velocity

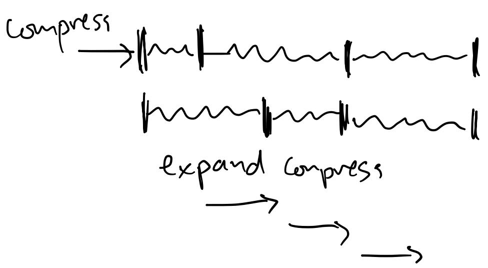
Total KE : sum, $\boxed{E_K = E_{Tr} + E_{Rot}}$

Thermal energy

- Atoms 'jiggling'
- 'Feel' via temperature
  ↑ not itself energy though!
- Thermal energy 'flows' down temperature gradients $\boxed{\text{Hot} \longrightarrow \text{Cold}}$
  heat flow.

Sound energy

- Vibrations
  ↳ mechanical
- Require a 'medium' eg 'elastic waves'



Example Problems 3: Forms of energy and work

see mechanics courses

1. Consider a cylinder of mass m, radius R and moment of inertia $I = \frac{1}{2}mR^2$ perched just at rest at the top of a flat incline, raised a distance h above the level ground. Suppose it receives a tiny nudge, just enough to cause it to begin to roll down the inclined plane. Calculate the linear speed of the cylinder when it reaches the level ground.

2. Why don't we need to consider nuclear energy in a typical mechanics problem such as the previous one?

3. Verify that the following all have units of energy:
   (a) Kinetic energy
   (b) Mechanical work (force times distance)
   (c) Pressure work
   (d) Thermal energy of an ideal gas
   (e) Energy of a light quantum (photon)
   (f) Rest mass energy

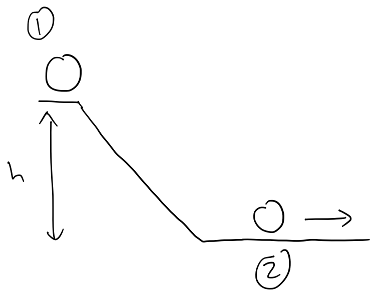
most relevant

4. (2017 Exam Q) Auckland's Sky Tower is 328m high. Determine whether the energy content of a 'Whopper' burger (from you-know-where) would be sufficient to fuel a person weighing 72kg to climb the Sky Tower. The nutrition information shows that on average a Whopper contains 2649kJ. Assume 20% efficiency in converting nutritional energy to mechanical energy.

5. What happens to the other 80% of the energy content? Does it just 'disappear'?

Answers

1. Key here is basic idea : $\boxed{\begin{matrix} 1: E_p \neq 0,\ E_K = 0 \\ 2: E_p = 0,\ E_K \neq 0 \end{matrix}}$ & $\boxed{[E_p + E_K]_1 = [E_p + E_K]_2}$



$1: E_p = mgh$

$2: E_K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

& $I = \frac{1}{2}mR^2$, $v = \omega R$

Solve

$\Rightarrow \ldots\ v = \sqrt{\frac{4}{3} \cdot gh}$

(see mechanics courses for details).

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2. $\Delta E_N = 0$ (only differences in energy are important)

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3. a) ✓

b) ✓

c) $P\Delta V \ [=]\ \frac{F}{A} \times V = \frac{F}{L^2} \cdot L^3 = F \cdot L$

see later

$[=]\ kg \cdot \frac{m}{s^2} \cdot m = kg \cdot \frac{m^2}{s^2} = J$ ✓

d) $PV = NRT$; N (number moles)

see later

R (gas constant) $[=]\ J \cdot mole^{-1}\ K^{-1}$

T (temp.) $[=]\ K$.

$\underbrace{PV}_{\text{see above } [=]\ J}$ & $NRT\ [=]\ \cancel{mole} \times \frac{J}{\cancel{mole} \cdot \cancel{K}} \cdot \cancel{K} = J$ ✓

e) $E = \hbar \nu$, $\hbar$: Planck's constant $= 6.626 \times 10^{-34}\ J \cdot s$

ν : freq. of light $[=]$ per time eg s^{-1}

$\Rightarrow E\ [=]\ J \cdot s \times \frac{1}{s} = J$ ✓

f) $E = mc^2\ [=]\ kg \cdot \left(\frac{m}{s}\right)^2 = kg \cdot \frac{m^2}{s^2} = J$ ✓

* $\boxed{4.}$ 'Raw' energy $= 2649\ kJ$

most relevant

Available for work $= 0.2 \times 2649\ kJ = 529.8\ kJ$

$\approx 530 \times 10^3\ J$

Needed to climb to height h : $mgh = 72\ kg \times 9.81\ m/s^2 \times h$

$\Rightarrow$ set mgh = available

$\Rightarrow h = h_{max} = \dfrac{529.8 \times 10^3\ J}{72\ kg \times 9.81\ m/s^2}$

$= 750\ m$

$\gg 328\ m$, so yes, can climb!

5. No! 'Converted' to thermal energy etc.

$\rightarrow$ 'heat up'.

$\rightarrow$ total energy is conserved

End L2.

1.5 Planetary energy balance

L3 —

Energy is crucial for life on Earth. Human society uses various forms of energy to - hopefully - improve quality of living. We will look at patterns of human energy consumption soon; first, we ask the more basic question: *where does the energy we use come from in the first place?*

The simple answer is: **the Sun!** It *radiates* energy generated from nuclear fusion reactions. Only a small part of this reaches us, however!