Engsci 711

Assignment 1

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Due: Monday 26th March (in class or via Canvas)

Question 1

Consider the following system

$$\dot{x} = x(1 - x - y)$$
$$\dot{y} = y(2 - x - y)$$

a) What sort of real-world system might this set of ODEs be used as a model for? Hence give an interpretation for the terms on the RHS of the equations.

b) Find the fixed points and determine their stability.

c) Draw the nullclines, determine the eigenvectors for the fixed points and sketch a qualitatively reasonable phase portrait.

d) Use Dulac's criterion to disprove the existence of periodic solutions in the positive x, y quadrant.

Question 2

Consider the system

$$\dot{x} = 2x + y$$
$$\dot{y} = -y + x^2$$

a) Find and classify the equilibria.

b) Find the power series expansions for $W^u_{loc}(0), W^s_{loc}(0)$ up to (i.e. including) quadratic order.

c) Plot some solutions using XPP (or pplane etc).

Question 3

Consider the general linear system modelling Romeo and Juliet from Lecture 6:

$$\dot{R} = aR + bJ$$
$$\dot{J} = cR + dJ$$

For the next three parts, i.e. 3a-3c, consider the case where $a = 0, b = 1, c = -1, d = 1$.

a) Give an interpretation of the 'romantic styles' of Romeo and Juliet in this case.

b) Classify the fixed point at the origin in this case. What does this imply for their love affair?

c) Sketch $R(t)$ and $J(t)$, assuming $R(0) = 1, J(0) = 0$.

The next two parts, i.e. 3d-3e, look at how Romeo and Juliet's 'love dynamics' depend on different combinations of a, b, c, d.

d) Romeo the robot: suppose nothing can change how Romeo feels about Juliet. Model this by $a = 0, b = 0$, while c and d are arbitrary. Analyse how Juliet ends up feeling about Romeo and how this depends on the signs and/or relative magnitudes of c and d.

e) Suppose Romeo and Juliet was written by 100,000 monkeys on typewriters instead of Shakespeare. Model this by simulating 100,000 realisations of a, b, c, d, each independently randomly sampled from a uniform distribution on the normalised interval $[-1, 1]$ (use Matlab/Python/R etc). Determine the frequency that each type of fixed point (saddle, stable node, centre, etc) appears, and plot this as a histogram. What is the most frequent outcome for Romeo and Juliet, as determined by your monkey simulations? Are these results surprising?

Question 4

Consider the system

$$\dot{x} = x^2 + y^2 - 2$$
$$\dot{y} = x - 1$$

where $x, y \in \mathbb{R}$.

a) Find and classify all of the equilibria of the system. You do not need to draw any pictures (yet) or find any eigenvectors.

b) Write down the equations for the $x-$ and $y-$nullclines. Sketch these in the phase plane. Include the equilibria you found above and the direction fields on the nullclines in your sketch.

c) Add some possible compatible trajectories, including compatible local behaviour near the equilibria, to your diagram. You do not need to do any further explicit calculation (e.g. you do not need to find any eigenvectors) - a qualitative sketch is enough.

d) Verify your sketch from the previous part by plotting some trajectories using XPP (or pplane etc).