FAQ

- Basic sketching for D'Alembert

given | $w_{tt} = c^2 w_{xx}$ ← note: wave speed $t>0$

$w(x,0) = f(x)$ $-\infty < x < \infty$

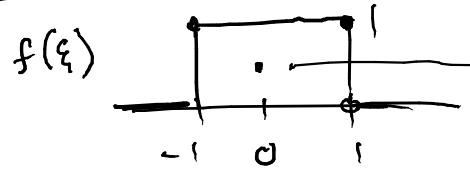
$w_t(x,0) = g(x)$ ← Integrated IC.

shifted IC

& Soln:

$$w(x,t) = \frac{1}{2}\left(f(x+ct) + f(x-ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds$$

zero



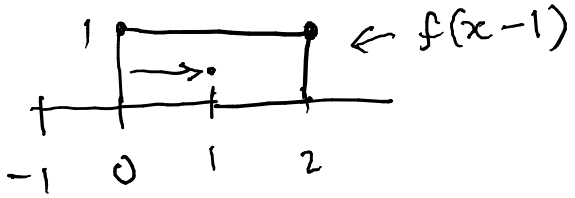
Consider $f(x-t)$ (note $c=1$)

→ f shifted to right.

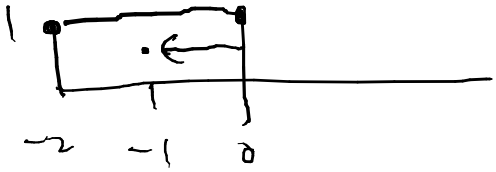
Trick: solve for new origin

$\xi = x - t = 0$

$t=1 \Rightarrow x=1$



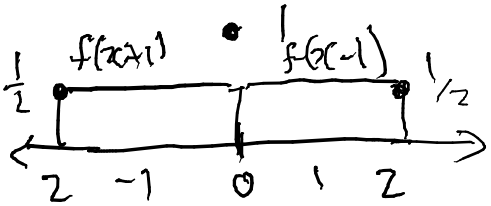
similarly $f(x+1)$ new origin $x+1=0$ $\Rightarrow x=-1$



average shifted IC

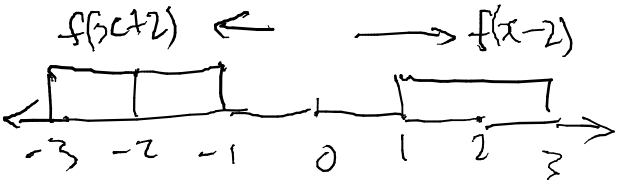
$$w = \frac{1}{2}\left(f(x-t) + f(x+t)\right)$$

$t=1$



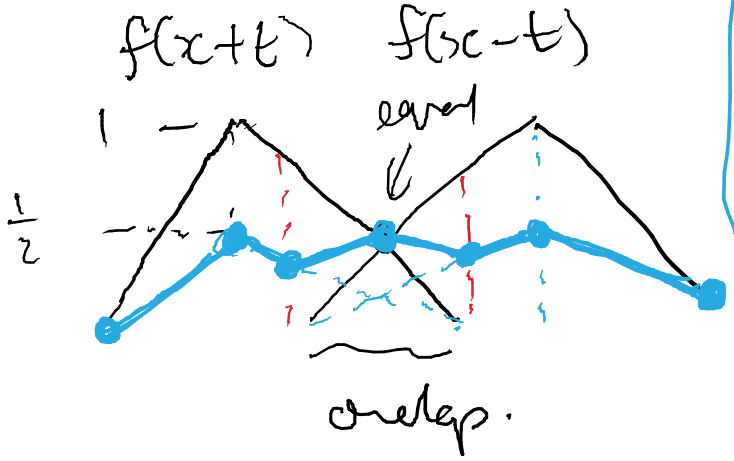
$t=2$

etc.



what about overlapping?

eg



— = $\frac{1}{2}(f(x+t)+f(x-t))$

• = 'easy points'

No sense

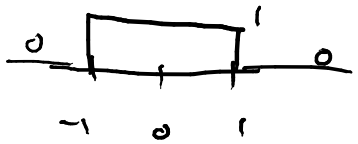
→ focus on 'easy' points '•'

(see above)

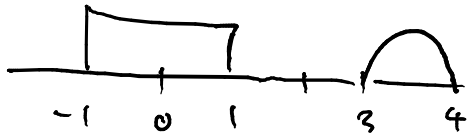
→

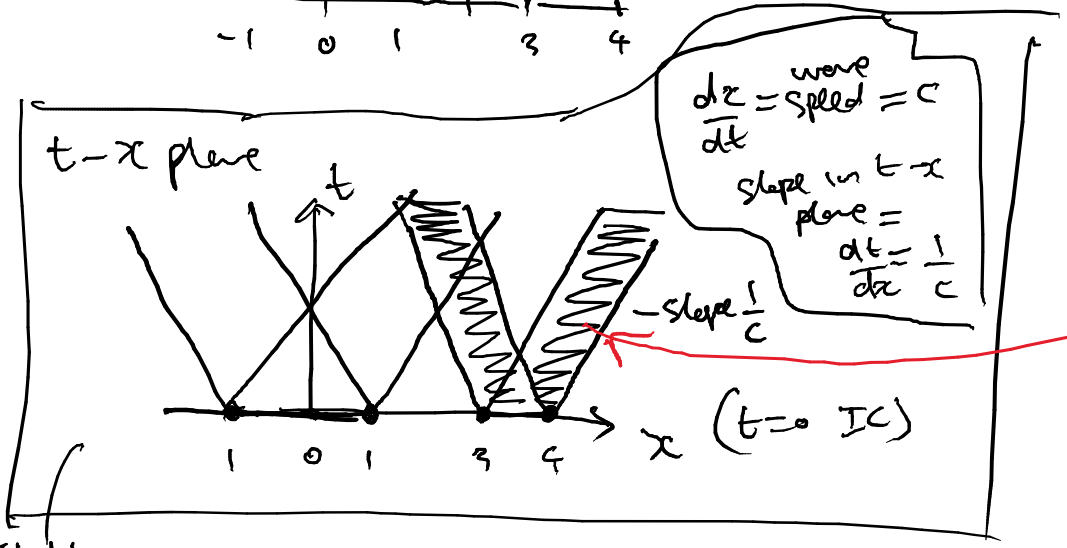
what about 2015 Q2b

Old IC:

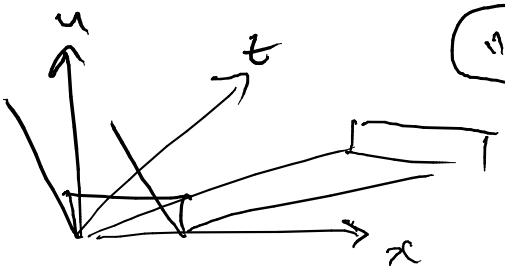


New IC:





Shaded = domain of influence of 'extra' part of IC



"where has IC travelled to?"

A: unshaded = unchanged

PIC + A enough for us!

Note: I will only ask about D'Alembert for $-\infty < x < \infty$

→ no $x=0$ BC

(eg no 2014 3b → need method of images → not covered)

Do I need to remember

$$\langle Lu, v \rangle = \langle u, L^{\dagger} v \rangle \; ?$$

$\rightarrow$ yes

Do I need to remember

$$J(u,v)\Big|_a^b = \cdots$$

$\rightarrow$ no (not for me anyway)

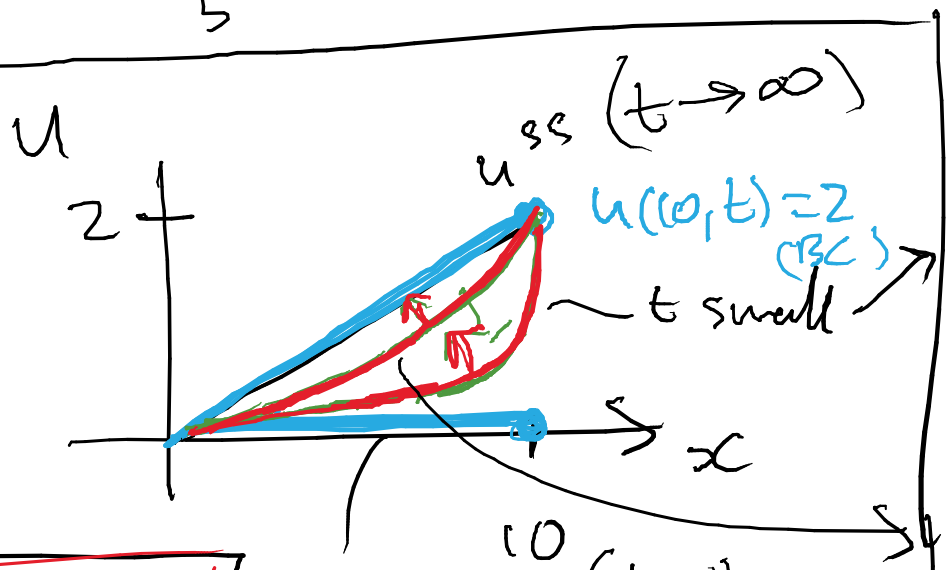
How do I sketch solns for heat eqn?

→ eg • Assignment
• 2013 1b.

Consider Q1b 2013.

Keys:
- IC
- SS soln ← from
- to
- BC ← sat. in between IC & SS

$u^{ss} = \frac{1}{5}x$; $u(x,0) = IC = 0$



• small t
→ near IC + sat BC

• large t
→ near SS + sat BC

→ note IC doesn't need to sat BC
→ defined $0 < x < 10$
→ other soln do

derivative/mixed BC

eg $u_x(0,t) + u(0,t) = 0$

⇒ write

$u_x(0,t) = -u(0,t)$

$u > 0 \Rightarrow u_x < 0$

etc.

Q: know Cauchy-Euler eqⁿ?

↳ yes

Q: know Bessel function?

↳ prob. not.

Q: calculate coeff. of eig. expansion

→ If IC is in terms of same basis, just match coeff. (orthog).

eg

$$u(x,0) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx$$

$$= 2 + 10 \cos 30x$$

$$\Rightarrow A_0 = 2,\ A_{30} = 10,\ A_n = 0 \text{ otherwise}$$