ENGSCI 213: MATHEMATICAL MODELLING 2SE

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MODULE OVERVIEW

Introduction to Probability (*Oliver Maclaren*) [9 lectures]

1. *Basic concepts* [3 lectures]

   Basic concepts of probability. Sets and subsets, sample spaces and events. Probability and counting, conditional probability, independence, Bayes' theorem. Random variables. Simple data structures for probability calculations.

2. *Discrete probability models* [3 lectures]

   Discrete random variables and discrete probability distributions. Probability mass functions and cumulative distribution functions. Binomial and Poisson distributions.

3. *Continuous probability models* [3 lectures]

   Continuous random variables and continuous probability distributions. Probability density functions and cumulative distribution functions. Exponential and Normal distributions.

LECTURE 3

Our last basic concept: random variables

Example problems for this module (see also tutorial/worksheets)

KEY IDEAS SO FAR...AND ONE MORE

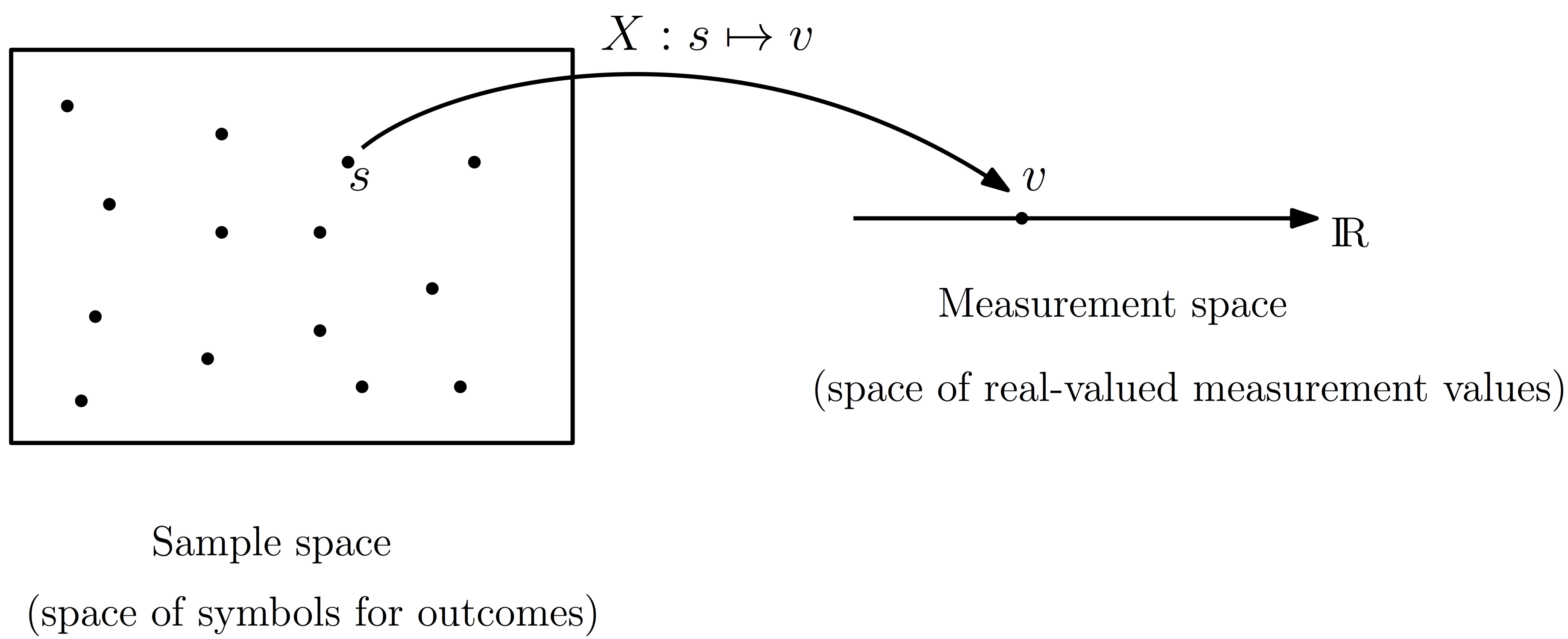
- Our 'context' is given by specifying a sample space
- Events are subsets of this sample space
- Probability measures 'randomness', 'chance' and/or 'uncertainty' by measuring the (relative) 'sizes' of sets
- If we need to 'change context' we use conditional probability

Now we introduce the idea of (real-valued) *random variables* which are essentially ways of *quantitatively labelling outcomes and events*

NOTE

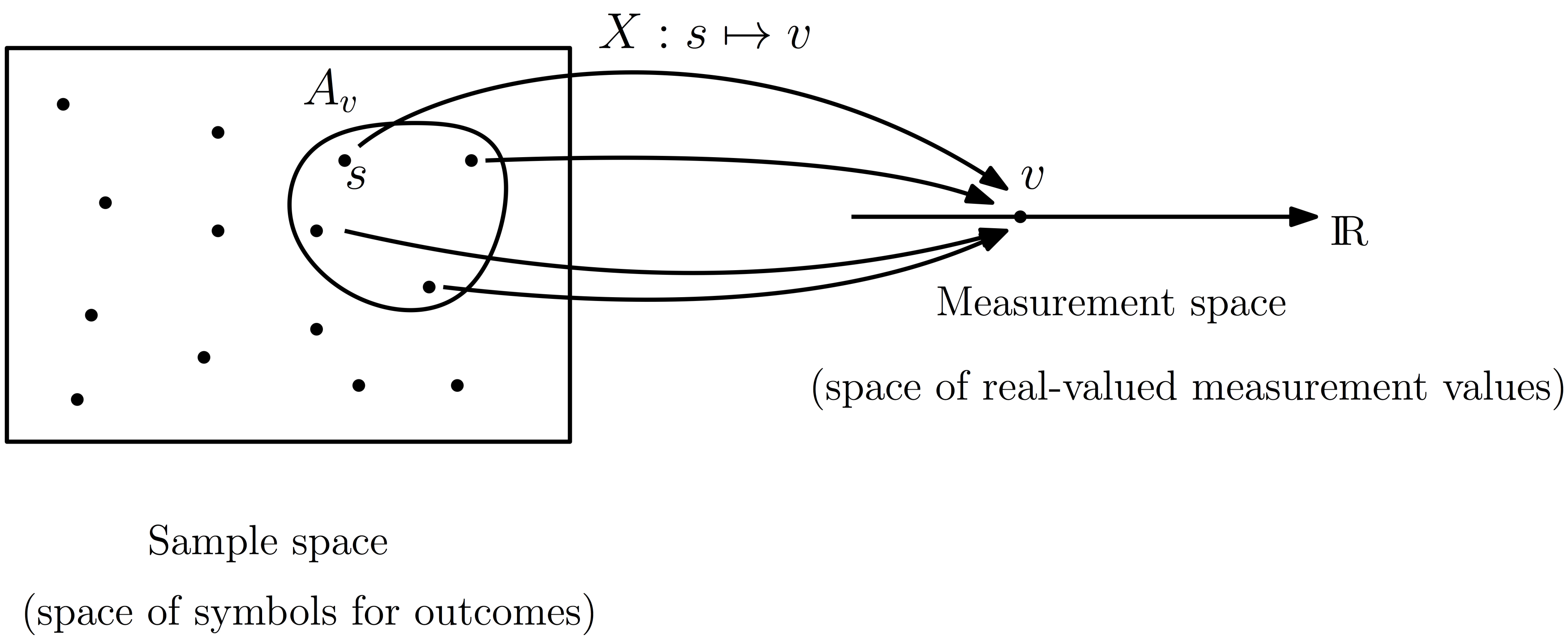
Note: here we are just *re-labelling our outcomes* according to *numerical 'properties'* of our recorded outcomes - not changing how we measure probability!

BASIC PICTURE: RANDOM VARIABLES AS FUNCTIONS FOR LABELLING *OUTCOMES*



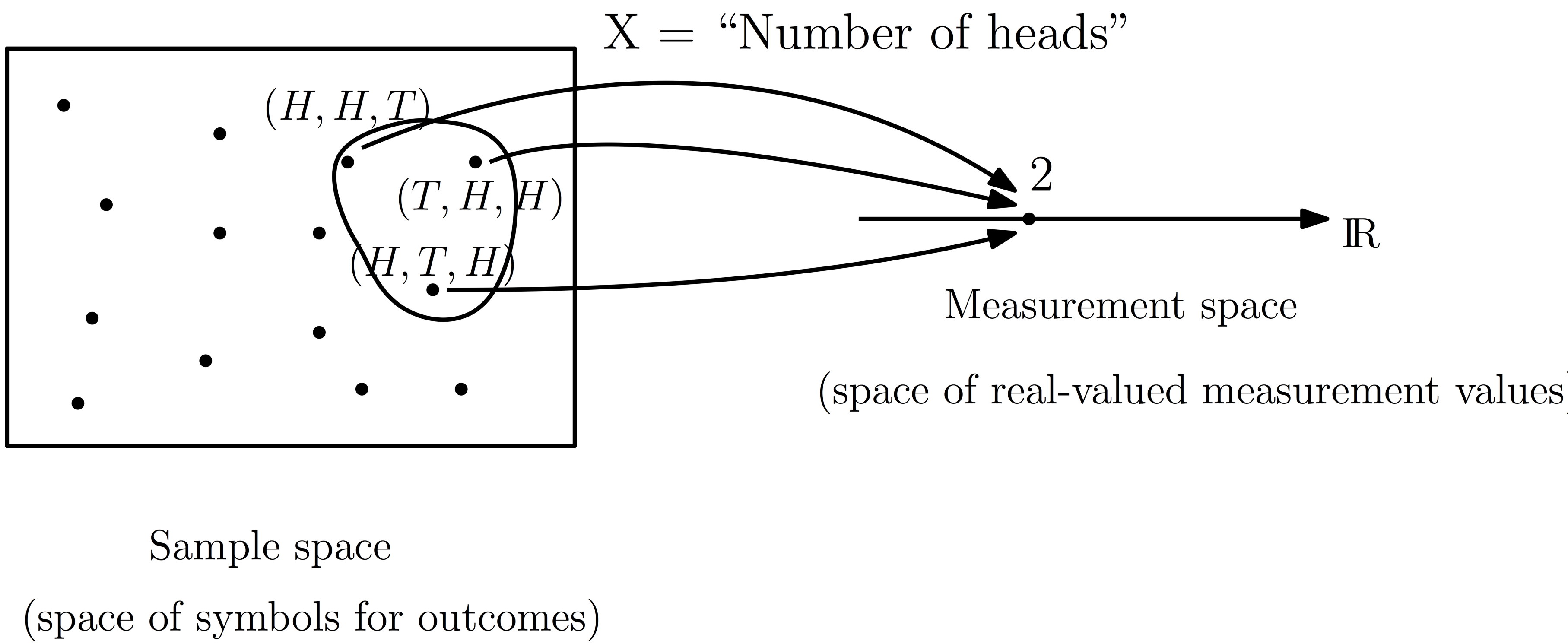
BASIC PICTURE: RANDOM VARIABLES AS FUNCTIONS FOR LABELLING *EVENTS*

$$\boxed{A_v = \{s | X(s) = v\} = X^{-1}(\{v\})}$$



BASIC PICTURE: EXAMPLE

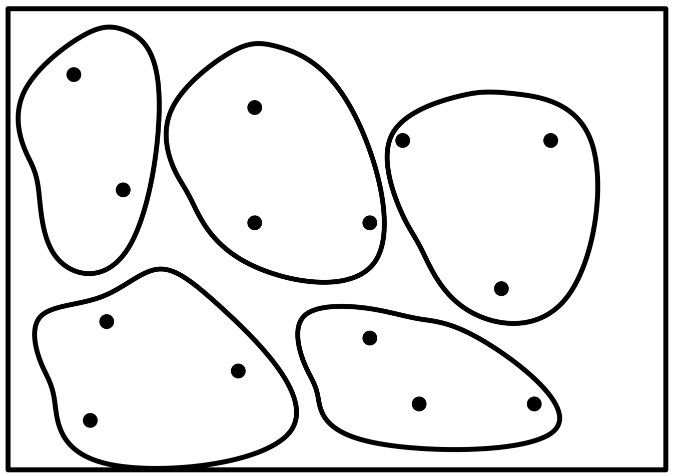
$$A_2 = \{s | \text{"Number of heads"} = 2\} = X^{-1}(\{2\})$$

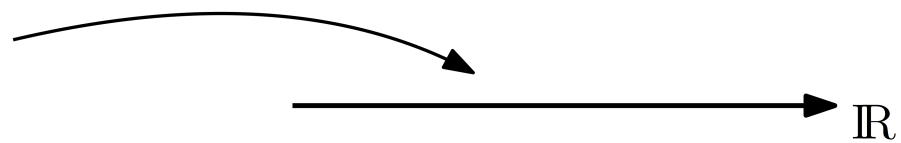


RANDOM VARIABLES AND PARTITIONS

$\{\ldots, A_1, A_2, A_3 \ldots\}$ form a partition since all s have one and only one v

$X : S \to \mathbb{R}$





Measurement space

(space of real-valued measurement values)

Sample space

(space of symbols for outcomes)

DEFINITIONS

Random variables: functions from the sample space to real numbers. Values assigned can be any real number, regardless of whether the random variable is called 'discrete' or 'continuous'.

DEFINITIONS

Discrete random variables: the sample space is a discrete set

Continuous random variables: the sample space is a continuous set

PROBABILITY FOR RANDOM VARIABLES

The probability of a random variable taking on a value is just the *probability of the associated event*

PROBABILITY FOR RANDOM VARIABLES: DISCRETE CASE

For *discrete* random variables this is easy as the *'elementary events'* - i.e. those containing a *single outcome* - have a *finite amount of probability 'attached'* and we can build everything up from these.

Each value is given a certain probability *'mass'*, i.e. a finite amount of probability. The sum of all the probability masses for each value adds to one of course.

PROBABILITY FOR RANDOM VARIABLES: DISCRETE CASE

So for *discrete* RVs we use

$$P(X = x) = P(A_x) = P(\{s \mid X(s) = x\}) = P(X^{-1}\{x\})$$

Represents the probability of the random variable taking on any particular value. $P(X = x)$ is sometimes called the *probability 'mass' function*.

PROBABILITY FOR RANDOM VARIABLES: CONTINUOUS CASE

For *continuous* random variables the *'elementary events'* defined by $X(s) = x$ have *'zero' (or 'infinitesimal') probability attached*.

Instead we define the *probability of* the random variable *taking values in finite intervals* e.g. $X(s) \in [a, b]$ OR define a *density* to integrate over to get a finite probability.

PROBABILITY FOR RANDOM VARIABLES: CONTINUOUS CASE

So for *continuous* RVs we use

$$P(a \leq X \leq b) = P(A_{[a,b]}) = P(\{s \mid X(s) \in [a, b]\})$$

$$= P(X^{-1}[a, b]) = \int_a^b f_X(x)dx$$

where $f_X(x)$ is called the *probability 'density' function*. We'll come back to this in later lectures.

WHY?

Everything we're doing with random variables *works (pretty much) the same as before* (when we were using events); we introduce them (mostly) as a *convenient way of labelling outcomes and events numerically* (and they have a nice connection to partitions).

We also need to know about them since we will be *mainly working with random variable models* (both discrete and continuous) in the next two sections of this module and for Markov processes (rather than working with general sample spaces).

EXAMPLE PROBLEMS FOR FIRST MODULE