Decision-Making & Modelling Under Uncertainty (DMU)

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[10 lectures/tutorials]

- Decision-making under uncertainty [5/10]
  - Basic concepts
  - Risk, probability, utility
  - Statistical: extended setup
    - formulation & empirical risk approx.
    - minimax & Bayes
  - Tutorial sheet
- Modelling under uncertainty {models of risk & intervention} [5/10]
  - probability, graphical models, & independence
  - causal interpretations of graphical models
  - stochastic process models (esp. Markov)
  - simulation & estimation tools
  - Tutorial sheet

Lecture 1 : Basic concepts

'Decision theory' is a multidisciplinary field, broadly concerned with formalising & analysing ways of making the 'best' or 'optimal' decision given available information.

- Decision theory informs & is informed by
  - Philosophy
  - Economics
  - Statistics
  - Psychology
  - Operations research
  - Machine learning/AI

  etc!
- Some of you may have seen elements in 255. This is not assumed! We recap the relevant material then consider different aspects.

we will look at both:

- 'simple' decision theory ①

&

- 'statistical' decision theory ②

↳ extends/incorporates above & adds in availability of new 'sample data' to inform decision

→ relevant for engineering, machine learning etc ...

Key for ① is von Neumann & Morgenstern's work on 'utility' & for ② Wald's work on 'statistical decision functions'

↳ tho ② informs modern versions of ①

Also savage's work on subjective Bayes etc.

Individuals or groups?

- The term 'decision theory' usually refers to 'single-person' decision making rather than 'group' decision making (voting etc).

- Intermediate between these is the study of games (game theory) where, eg, multiple individuals independently (simultaneously or in turn) make decisions to achieve typically conflicting goals while not knowing exactly what their 'opponent' will do.

→ we will mostly study individual decision making, tho' will also consider games in which 'nature' 'plays' too

- Decision theory can be classified as
  - normative – how 'should' people decide, 'in principle'? or
  - descriptive – how 'do' people decide, 'in practice'?

though in reality it is a mix

→ we study $\boxed{\text{normative}}$, ie idealised, decision making here.

→ ie we study 'algorithms/principles for 'optimal' decision making'

Uncertainty

- the last element of 'simple decision theory' is whether we are under conditions of:
  - $\boxed{\text{certainty}}$ (determinism)
  - $\boxed{\text{risk}}$ (probability)
  - $\boxed{\text{ignorance}}$ (ambiguity, uncertainty) ↳ (note)

Warning

- Many people read 'uncertainty' as 'related to probability',

$\boxed{\text{but}}$ in (simple) decision theory there is a long history of taking 'uncertainty' to mean ignorance (unknown prob) rather than 'risk' (known prob.)

→ Bayesians treat risk & ignorance the same (probability)
↳ others don't

→

Goal: optimal decision under different types of uncertainty

→ 'Uncertainty' types:

- Certainty : deterministic
- Risk : known probabilities
- Ignorance : unknown or partially known probabilities (or 'probability' not even well-defined)

'Uncertainty' types

- Certainty (deterministic)
  - Optimisation with no probabilities or ignorance
    → eg [linear programming or other function optimisation problems]

  } other courses/modules

- Risk (known probabilities)
  - A probability model over outcomes is taken as given/known
  - A decision needs to be made despite the 'risk' associated with the probability dist.
    → eg [Flip fair ($\mathbb{P}(H) = 0.5$, $\mathbb{P}(T) = 0.5$) coin. Guess correctly, win \$1000. Guess incorrectly, lose \$500. Or don't play. What should you do?]

- Ignorance (unknown probabilities)
  - either no probability model or a set of possible probability models is given
    → eg [same as above but now a coin with $\mathbb{P}(H) = p$, p unknown.]

Statistical decision theory

- Involves both risk & ignorance (in general, though Bayesians treat as one) as well as sample data

eg don't know θ in $\mathbb{P}(X;\theta)$ but do have a sample realisation $X=x$, generated under 'fixed but unknown' θ value θ_0, so "$X \sim \mathbb{P}(X;\theta_0)$" is known but θ_0 is not.

Example decision:

decide on estimate $\hat{\theta}$ of θ_0.

(many others)

- Another useful point of view is to see it as a game between you and 'nature'

Simple decision theory (for risk/ignorance)

Minimal elements:

- $\boxed{\text{states of nature}}$ (well-defined 'outcomes of nature')
  - ↳ risk or ignorance about which will occur
  - ↳ possible values independent of acts/decisions
  - ↳ probabilities (under risk) may depend on acts (see later)
- $\boxed{\text{Acts/decisions}}$ (your choices/actions)
  - ↳ you have control over
  - ↳ should be definite enough to determine an outcome (below) for given state of nature.
- $\boxed{\text{Outcomes}}$ (of decision)] not value see →
  - ↳ f(act, state of nature)

ie $\boxed{\text{outcome of decision} = f(\text{decision}, \text{outcome of nature})}$

→ outcome (of decision) is: 'how things turned out, in the ways you care about'

....&....

6 $\boxed{\text{Utility}}$

The final ingredient is the 'utility' of an outcome, $\boxed{u(\text{outcome})}$

[or a decision, state of nature pair, $\boxed{u(d, s)}$ → see P&I (2009) & over →]

→ essentially the 'value' you place on it

→ it is not in general 'monetary' value! How much is \$1 worth to you?

↳ millionaire vs no money & hungry?

→ it is a numerical representation of your (the decision maker's) relative preference for outcomes

ie for outcomes a, b we want

$$\boxed{\begin{array}{c} a \text{ preferred to } b,\ a \succ b \\ \text{if \& only if} \\ u(a) > u(b) \end{array}}$$

note: fancy inequality

normal inequality

→ preferences are key, utility just makes them convenient to calculate with

Note: subtle point

In statistical decision theory we often just use $\boxed{(\text{act, state of nature})}$ as the $\boxed{\text{'outcome'}}$

ie we don't explicitly mention the 'outcome' beyond just stating the act/decision & state of nature. Just $\boxed{\text{act, decision, utility}}$ (or loss →)

So we write:

$$\boxed{\text{utility(act, state of nature)}}$$

rather than

$$\boxed{\text{utility(outcome(act, state of nature))}}$$

we/I will switch between

See Parmigiani & Inoue (2009) & Schervish et. al (1990) for more

Preferences over outcomes & utility scales

→ minimal (but still controversial) assumptions on preferences allow representation theorems

↳ we can represent your preference with a utility function

eg $\left[\begin{array}{l} \forall a,b,\ a \succ b \text{ or } a \prec b \text{ or } a \simeq b \\ \forall a,b,c,\ a \succ b \,\&\, b \succ c \Rightarrow a \succ c \end{array}\right]$

where '$\succ$' means 'prefer', '$\simeq$' means 'indifferent' etc.

→ there are different types of utility scale depending on assumptions on 'preference structure' used

Eg
- ordinal: relative preference makes sense but differences not defined
  "good, OK, bad"
  <-> 1 2 3
  <-> 5 20 30
- cardinal: absolute values & differences also make sense.
  - typically $\boxed{\text{interval}}$ based for minmax regret & expected utility.
    ↳ care about relative differences in utility.

Result 4-1 ↳

Decision tables: $\boxed{\text{Ignorance}}$

- outcome table

	s_1	s_2	s_3	s_4
d_1	$o(d_1,s_1)$	$o(d_1,s_2)$	...	...
d_2	$o(d_2,s_1)$	$\ddots$		
d_3	$\vdots$			
d_4	$\vdots$			

- utility function (as table...)

	o_1	o_2	....
u	$u(o_1)$	$u(o_2)$	....

} or $u(d_i,s_j)$.... as discussed

- Utility table

	s_1	s_2	s_3	s_4
d_1	$u(o(d_1,s_1))$	$u(o(d_1,s_2))$	...	...
d_2	$u(o(d_2,s_1))$	$\ddots$		
d_3	$\vdots$			
d_4	$\vdots$			

} or $u(d_i,s_j)$

Decision tables: $\boxed{\text{Risk}}$: extra ingredient $\boxed{\mathbb{P}(s_j; d_i)}$

prob. of state s_j 'given' decision d_i

↑ not neces. conditional prob.

- outcome table with prob.

	s_1	s_2	s_3	s_4
d_1	$O(d_1,s_1)$ $\mathbb{P}(s_1;d_1)$	$O(d_1,s_2)$ $\mathbb{P}(s_2;d_1)$	...	...
d_2	$O(d_2,s_1)$ $\mathbb{P}(s_1;d_2)$			
d_3	⋮			
d_4	⋮			

(can also give prob. table separately)

- utility function (as table ...)

	o_1	o_2	....
u	$u(o_1)$	$u(o_2)$	....

$\}$ or $u(d_i, s_j)$

- utility table

	s_1	s_2	s_3	s_4
d_1	$u(o(d_1,s_1))$ $\mathbb{P}(s_1;d_1)$	$u(o(d_1,s_2))$ $\mathbb{P}(s_2;d_1)$	...	...
d_2	$u(o(d_2,s_1))$ $\mathbb{P}(s_1;d_2)$	⋱		
d_3	⋮			
d_4	⋮			

$\}$ $u(d_i, s_j)$

Note on decision-dependent probabilities

In Savage's (1954) Foundations of statistics model he assumed probabilities of states are independent of acts

→ For some problems this requires complex definitions of states

→ Alternative is to use conditional probabilities of states, ie probability of state given act (decision).

↳ eg $\boxed{\text{Jeffrey's}}$ (1965) Evidential Decision Theory

→ $\boxed{\text{we take this conditional approach as default (but see issues later)}}$ ←

see Resnik (1987) attached, 1-2a & 1-3a.

Utility & loss

- Given a utility function, we can define a 'loss' function as the negative of utility:

$$\text{loss}(\text{outcome}) = -\text{utility}(\text{outcome})$$

or, in more detail:

$$\boxed{\text{loss}(d,s) = -\text{utility}(d,s), \quad \text{for } d: \text{decision}, \; s: \text{state of nature}}$$

→ we can then aim to minimise loss instead of maximise utility (same thing).

write

$$\boxed{l(d,s) \text{ for loss function}}$$

Regret : Ignorance

- It may be the case that a decision problem always has a non-zero loss (negative utility).

-> If the decision must be made, some have argued (eg Savage) we should measure the actual loss relative to the minimum achievable loss for that state of nature

This is called the $\boxed{\text{regret}}$:

$$\boxed{\text{regret}(d,s) = \text{loss}(d,s) - \underbrace{\min_{d|s}}_{\text{minimise over } d \text{ for each } s} (\text{loss}(d,s))}$$

(easier to think of with decision, state than outcome(decision, state))

→ some require $\min_d (\text{loss}(d,s)) = 0$ & so loss = regret.] Wald often assumed implicitly.

we call a loss function $l(a,s)$ a regret loss function if it is in the above form, ie if

$$\min_d l(d,s) = 0 \text{ for each } s.$$

→ we can always convert a loss table to a regret table.

Absolute loss → relative loss (regret)

Loss

	s_1	s_2	s_3
d_1	1	0	6
d_2	3	4	5

minimum for each state & any decision → 1 0 5 $= \min_d l(d, s)$

Regret : $l(d,s) - \min_d l(d,s)$

	s_1	s_2	s_3
d_1	0	0	1
d_2	2	4	0

$\left(= \begin{array}{c|c|c} 1-1 & 0-0 & 6-5 \\ 3-1 & 4-0 & 5-5 \end{array} \right)$

Regret : risk ?

- under risk (known probabilities) we can consider the regret table under the special case (Savage!) that the state probabilities are independent of the decisions (acts)

- In this case the minimum expected regret decision is the same as the minimum expected loss decision (different loss values, same choice)

→ no real benefit to regret except measure of 'expected value of perfect information'

→ see eg Jeffrey 1965 (& example →)

So... given a decision table, how do we make a decision? Decision rules

(choose a decision, decide an act etc)

Depends!

$\boxed{\text{Ignorance}}$

- maximin utility = minimax loss
- maximin relative utility = minimax regret loss

→ same if max utility = min loss = 0 but not in general (see Resnik)

$\boxed{\text{Risk}}$

- max expected utility = min expected loss
- max expected relative utility = min expected regret loss

→ Same if state probabilities indep. of decisions

(see eg Jeffrey 1965 or Barnett 1999)

Example: loss (−utility) form

Suppose:

Loss table

	s_1	s_2	s_3
d_1	1	0	6
d_2	3	4	5

Probability table

	s_1	s_2	s_3
d_1	0.5	0.2	0.3
d_2	0.1	0.0	0.9

Regret table

	s_1	s_2	s_3
d_1	0	0	1
d_2	2	4	0

→

Minimax absolute & relative (regret) loss

Loss table

	s_1	s_2	s_3
d_1	1	0	6
d_2	3	4	5

① max loss [over states for each decision]

6

5

② min max loss: 5 (d_2)
└[over decisions]

Regret table

	s_1	s_2	s_3
d_1	0	0	1
d_2	2	4	0

① max regret

1

4

② min max regret: 1 (d_1)

→ different recommendations! Pros & cons

→ minimax regret usually less pessimistic than minimax absolute loss

→ regret implies interval utility, absolute only requires ordinal

Min expected loss / regret: varying prob??

- Loss table with prob.

	s_1	s_2	s_3
d_1	1 (0.5)	0 (0.2)	6 (0.3)
d_2	3 (0.1)	4 (0.0)	5 (0.9)

① expected loss (over states)

$0.5\times1+0.2\times0+0.3\times6=\boxed{2.3}$

$0.1\times3+0.0\times4+0.9\times5=\boxed{4.8}$

② min expected loss: 2.3 (d_1)
(over decisions)

- Regret table with probs from before?

	s_1	s_2	s_3
d_1	0 (0.8)	0 (0.2)	1 (0.3)
d_2	2 (0.1)	4 (0.0)	0 (0.9)

① expected regret

= 0.3

0.2

② min expected regret 0.2 (d_2)

Note: $\boxed{\text{not}}$ equivalent as prob. varies by state (regret not good idea?) here

→ Consider

Loss table with state prob. indep. of decision

	s_1	s_2	s_3
d_1	1 (0.5)	0 (0.2)	6 (0.3)
d_2	3 (0.5)	4 (0.2)	5 (0.3)

① expected loss

$0.5\times1+0.2\times0+0.3\times6 = \boxed{2.3}$

$0.5\times3+0.2\times4+0.3\times5 = \boxed{3.8}$

② min expected loss: 2.3 (d_1)

Regret table with state prob. indep. of decision

	s_1	s_2	s_3
d_1	0 (0.5)	0 (0.2)	1 (0.3)
d_2	2 (0.5)	4 (0.2)	0 (0.3)

① expected regret

$\boxed{0.3}$

$\boxed{1.8}$

② min expected regret: 0.3 (d_1)

→ same decision.

Note: MEL = 2.3 (min exp. loss)

MER = 0.3 (min exp. reg.)

& $\sum P_{s_i}\cdot \min_d l(d,s_i) = 2.0$

= expected loss given perfect information (EL|PI)

	s_1	s_2	s_3
d_1	1 (0.5)	0 (0.2)	6 (0.3)
d_2	3 (0.5)	4 (0.2)	5 (0.3)

EL|PI = MEL − MER } or MER = MEL − EL|PI

= "Expected loss of PI"

= "unavoidable loss"

Decision rules: summary (loss/regret cf utility)

Minimax loss:

$$\min_d\left[\max_{s|d}\left[l(d,s)\right]\right]$$

Minimax regret loss

$$\min_d\left[\max_{s|d}\left[l(d,s)-\min_{d|s} l(d,s)\right]\right]$$

Minimum expected loss

$$\min_d\left[\mathbb{E}_{P_{s;d}}\left[l(d,s)\right]\right]$$

(utility = −loss, min → max)

Resnik 1987

Content warning.

- I've removed an example dealing with a difficult topic that some might find uncomfortable (p.16).
- Also, at times the language & other examples can be a little outdated

Chapter 1
INTRODUCTION

1-1. What Is Decision Theory?

Decision theory is the product of the joint efforts of economists, mathematicians, philosophers, social scientists, and statisticians toward making sense of how individuals and groups make or should make decisions. The applications of decision theory range from very abstract speculations by philosophers about ideally rational agents to practical advice from decision analysts trained in business schools. Research in decision theory is just as varied. Decision theorists with a strong mathematical bent prefer to investigate the logical consequences of different rules for decision making or to explore the mathematical features of different descriptions of rational behavior. On the other hand, social scientists interested in decision theory often conduct experiments or social surveys aimed at discovering how real people (as opposed to "ideally rational agents") actually behave in decision-making situations.

It is thus usual to divide decision theory into two main branches: normative (or prescriptive) decision theory and descriptive decision theory. Descriptive decision theorists seek to find out how decisions *are* made—they investigate us ordinary mortals; their colleagues in normative decision theory are supposed to prescribe how decisions *ought* to be made—they study ideally rational agents. This distinction is somewhat artificial since information about our actual decision-making behavior may be relevant to prescriptions about how decisions should be made. No sane decision analyst would tell a successful basketball coach that he ought to conduct a statistical survey every time he considers substituting players—even if an ideally rational agent acting as a coach would. We can even imagine conducting research with both normative and descriptive ends in mind: For instance, we might study how expert business executives make decisions in order to find rules for prescribing how ordinary folk should make their business decisions.

Recently some philosophers have argued that all branches of the theory of rationality should pay attention to studies by social scientists of related behavior by human beings. Their point is that most prescriptions formulated in terms of ideally rational agents have little or no bearing on the question of how humans should behave. This is because logicians, mathematicians, and philosophers usually assume that ideally rational agents can acquire, store, and process un-

limited amounts of information, never make logical or mathematical mistakes, and know all the logical consequences of their beliefs. Of course, no humans—not even geniuses—come close to such ideals. This may favor studying more realistic models of rational agents, but I do not think that we have grounds for dismissing the usual theories of rationality altogether. The ideals they describe, although unattainable in practice, still serve to guide and correct our thinking. For example, we know that perfect memories would help us make better decisions. Instead of settling for poor decisions, we have tried to overcome our limitations by programming computers—which have larger and better memories than we do—to assist us in those tasks whose success depends on storing and retrieving large quantities of information.

Another problem with putting much weight on the distinction between normative and descriptive decision theory is that some abstract decision models have been introduced with neither normative nor descriptive ends in mind. I am thinking of the concept of *rational economic man* used in economics. This hypothetical being is an ideally rational agent whose choices always are the most likely to maximize his personal profit. By appealing to a hypothetical society of rational economic men economists can derive laws of supply and demand and other important principles of economic theory. Yet economists admit that the notion of rational economic man is not a descriptive model. Even people with the coolest heads and greatest business sense fail to conform to this ideal. Sometimes we forget the profit motive. Or we aim at making a profit but miscalculate. Nor do economists recommend that we emulate economic man: Maximizing personal profit is not necessarily the highest good for human beings. Thus the model is intended neither normatively nor descriptively, it is an explanatory idealization. Like physicists speculating about perfect vacuums, frictionless surfaces, or ideal gases, economists ignore real-life complications in the hope of erecting a theory that will be simple enough to yield insights and understanding while still applying to the phenomena that prompted it.

For these reasons I favor dropping the normative-descriptive distinction in favor of a terminology that recognizes the gradation from experimental and survey research toward the more speculative discussions of those interested in either explanatory or normative idealizations. With the caveat that there is really a spectrum rather than a hard-and-fast division, I propose the terms "experimental" and "abstract" to cover these two types of decision-theoretic research.

This book will be concerned exclusively with abstract decision theory and will focus on its logical and philosophical foundations. This does not mean that readers will find nothing here of practical value. Some of the concepts and methods I will expound are also found in business school textbooks. My hope is that readers will come to appreciate the assumptions such texts often make and some of the perplexities they generate.

Another important division within decision theory is that between decisions made by individuals and those made by groups. For the purposes of this division an individual need not be a single human being (or other animal). Corporations, clubs, nations, states, and universities make decisions *as individuals* (and can be held responsible for them) when they attempt to realize some organizational goal, such as enhancing their reputation or bettering last year's sales figures. However, when several individuals who belong to the same club, corporation, or university adjudicate differences about group goals or priorities, they are involved in making a *group decision*. We can illustrate the difference between group and individual decisions by looking at the role of United States presidents. By electing a Republican rather than a Democratic president, voters can decide general social and economic policy. Elections are thus one way for United States citizens to make group decisions. Once elected the president also makes decisions. They are in the first instance his own choices, to be sure; but, insofar as he incorporates the national will, they may be decisions of the United States as well. Thus when President Reagan elected to invade Grenada, this also became a decision of the United States. This was in effect an individual decision by a nation.

When we turn to game theory we will deal with individual decisions that at first sight look like group decisions. Games are decision-making situations that always involve more than one individual, but they do not count as group decisions because each individual chooses an action with the aim of furthering his or her own goals. This decision will be based on expectations concerning how other participants will decide, but, unlike a group decision, no effort will be made to develop a policy applying to all the participants. For example, two neighboring department stores are involved in a game when they independently consider having a post-Christmas sale. Each knows that if one has the sale and the other does not, the latter will get little business. Yet each store is forced to decide by itself while anticipating what the other will do. On the other hand, if the two stores could find a way to choose a schedule for having sales, their choice would ordinarily count as a group decision. Unfortunately, it is frequently difficult to tell whether a given situation involves an individual or a group decision, or, when several individuals are choosing, whether they are involved in a game or in a group decision.

Most of the work in group decision theory has concerned the development of common policies for governing group members and with the just distribution of resources throughout a group. Individual decision theory, by contrast, has concentrated on the problem of how individuals may best further their personal interests, whatever these interests may be. In particular, individual decision theory has, to this point, made no proposals concerning rational or ethical ends. Individual decision theory recognizes no distinction – either moral or rational – between the goals of killing oneself, being a sadist, making a million dollars, or being a missionary. Because of this it might be possible for ideally rational agents to be better off violating the policies of the groups to which they belong. Some group decision theorists have tried to deny this possibility and have even gone so far as to offer proofs that it is rational to abide by rationally obtained group policies.

It should be no surprise, then, that some philosophers have become fascinated with decision theory. Not only does the theory promise applications to traditional philosophical problems but it too is replete with its own philosophical problems. We have already touched on two in attempting to draw the distinctions

between the various branches of decision theory, and we will encounter more as we continue. However, philosophers have paid more attention to applications of decision theory in philosophy than they have to problems within decision theory. The notion of a rational agent is of primary importance to philosophy at large. Since Socrates, moral philosophers have tried to show that moral actions are rational actions, in other words, that it is in one's own best interest to be moral. Political philosophers have similarly tried to establish that rational agents will form just societies. Such arguments remained vague until modern decision theory supplied precise models of rationality and exact principles of social choice. It is now possible to formulate with nearly mathematical exactness modern versions of traditional arguments in ethics and social philosophy. The techniques of decision theory have also suggested new approaches to old ethical and moral problems.

Statisticians use decision theory to prescribe both how to choose between hypotheses and how to determine the best action in the face of the outcome of a statistical experiment. Philosophers of science have turned these techniques with good results on the problems in rational theory choice, hypothesis acceptance, and inductive methods. Again this has led to significant advances in the philosophy of science.

Decision theory is thus philosophically important as well as important to philosophy. After we have developed more of its particulars we can discuss some of its philosophical applications and problems in greater detail.

PROBLEMS

1. Classify the following as individual or group decisions. Which are games? Explain your classifications.
   a. Two people decide to marry each other.
   b. The members of a club decide that the annual dues will be $5.
   c. The members of a club decide to pay their dues.
   d. The citizens of the United States decide to amend the Constitution.
   e. Two gas stations decide to start a price war.
2. If it turned out that *everyone* believed that $1 + 1 = 3$, would that make it rational to believe that $1 + 1 = 3$?
3. Although decision theory declares no goals to be irrational in and of themselves, do you think there are goals no rational being could adopt, for example, the goal of ceasing to be rational?

1-2. The Basic Framework

A decision, whether individual or group, involves a choice between two or more options or *acts*, each of which will produce one of several *outcomes*. For example, suppose I have just entered a dark garage that smells of gasoline. After groping to no avail for a light switch, I consider lighting a match, but I hesitate because I know that doing so might cause an explosion. The acts I am considering are *light a match*, *do not light a match*. As I see it, if I do not light the match, there will be only one outcome, namely, *no explosion results*. On the other hand, if I do light the match, two outcomes are possible: *an explosion results*, *no ex-*

plosion results. My decision is not clear-cut because it is not certain that an explosion will result if I light the match. That will depend on the amount and distribution of the gasoline vapor in the garage. In other words, the outcome of my act will depend on the *state* of the environment in which the act takes place.

As this example illustrates, decisions involve three components—*acts*, *states*, and *outcomes*, with the latter being ordinarily determined by the act and the state under which it takes place. (Some outcomes are certain, no matter what the state or act. For instance, that I will either live through the day or die during it is certain whether or not I light the match and regardless of the presence of gasoline in the garage.) In decision theory we also construe the term "state" in a very broad sense to include nonphysical as well as physical conditions. If you and a friend bet on the correct statement of the fundamental theorem of calculus, the outcome (your winning or losing) is determined by a mathematical state, that is, by whether your version really formulates the fundamental theorem.

When analyzing a decision problem, the decision analyst (who may be the decision maker himself) must determine the relevant set of acts, states, and outcomes for characterizing the decision problem. In the match example, the act *do not light a match* might have been specified further as *use a flashlight*, *return in an hour*, *ventilate the garage*, all of which involve doing something other than lighting the match. Similarly, I might have described the outcomes differently by using *explosion* (*no damage*), *explosion* (*light damage*), *explosion* (*moderate damage*), *explosion* (*severe damage*). Finally, the states in the example could have been analyzed in terms of the gasoline-to-air ratio in the garage. This in turn could give rise to an infinity of states, since there are infinitely many ratios between zero and one. As I have described the example, however, the relevant acts, states, and outcomes are best taken as the simpler ones. We can represent this analysis in a *decision table*. (See table 1-1.)

1-1

		States: Explosive Gas Level	States: Nonexplosive
Acts	Light a Match	Explosion	No Explosion
	Do Not Light a Match	No Explosion	No Explosion

In general, a decision table contains a row corresponding to each act, a column for each state, and an entry in each square corresponding to the outcome for the act of that row and state of that column.

Suppose we change the match example. Now I want to cause an explosion to scare some friends who are with me. But I am a practical joker, not a murderer, so I want the explosion to be nondamaging. Then in analyzing this decision problem we would be forced to break down the explosive outcome into *damaging*/*nondamaging*. But the magnitude of the explosion would depend on

the amount of vapor in the garage, so our new analysis would also require a different division of the environment into states. This might yield a decision table such as 1-2.

1-2

		States: Amount of Gas Present X	Y	Z
Acts	Light	No Explosion	Explosion No Damage	Explosion Damage
	Do Not Light	No Explosion	No Explosion	No Explosion

In specifying a set of acts, states, and outcomes, or in drawing a decision table, we determine a *problem specification*. A moral to be drawn from the match example is that several problem specifications may pertain to the same decision situation. In such cases the decision analyst must determine the proper specification or specifications to apply. This is a problem in applied decision theory yet it may be absolutely crucial. In 1975, government health experts conducted an elaborate analysis prior to deciding to issue the swine influenza vaccine to the general public. But according to newspaper accounts, they simply never considered the outcome that actually resulted—that the vaccine would paralyze a number of people. Thus they failed to use the proper problem specification in making their decision.

For a problem specification to be definite and complete, its states must be mutually exclusive and exhaustive; that is, one and only one of the states must obtain. In the match example, it would not do to specify the states as *no vapor*, *some vapor*, *much vapor*, since the second two do not exclude each other. Lighting a match under the middle state might or might not cause an explosion. On the other hand, if we used the states *no vapor*, *much vapor*, we would neglect to consider what happens when there is some but not much vapor.

Securing mutually exclusive state specifications may require careful analysis, but we can easily guarantee exhaustiveness by adding to a list of nonexhaustive states the description *none of the previous states obtain*. Like the cover answer "none of the above" used to complete multiple-choice questions, this easy move can lead a decision analyst to overlook relevant possibilities. Perhaps something like this was at work in the swine flu vaccine case.

1-2a. Some Philosophical Problems about Problem Specifications

Selecting a problem specification is really an issue that arises in applying decision theory. We will eschew such problems in this book and, henceforth, assume that we are dealing with problem specifications whose states are mutually exclusive and exhaustive. Yet there are several interesting philosophical issues related to the choice of problem specifications. Three merit mention here.

The first concerns the proper description of states. Any decision problem involves some outcomes the decision maker regards as better than others. Otherwise there would be no choice worth making. Thus any decision might be specified in terms of the state descriptions *things turn out well*, *they do not*. Suppose, for example, that you are offered a choice between betting on the American or National League teams in the All-Star Game. A winning bet on the American League pays $5 and one on the National League pays $3. A loss on either bet costs $2. We would usually represent this decision problem with table 1-3.

1-3	American League Wins	National League Wins
Bet American	+ $5	− $2
Bet National	− $2	+ $3

Given that you are not a totally loyal fan of either league, this way of looking at the choice would lead you to choose between the bets on the basis of how probable you thought the American League to win. (Later we will see that you should bet on the American League if you think its chances are better than 5 in 12.) But suppose you use table 1-4 instead. Then you would simply bet American on the grounds that that bet pays better. You might even argue to yourself as follows: I will either win or lose. If I win, betting American is better, and if I lose, my bet does not matter. So whatever happens, I do at least as well by betting American.

1-4	I Win My Bet	I Lose It
Bet American	+ $5	− $2
Bet National	+ $3	− $2

The principle illustrated in this reasoning is called *the dominance principle*. We say that an act *A dominates* another act *B* if, in a state-by-state comparison, *A* yields outcomes that are at least as good as those yielded by *B* and in some states they are even better. *The dominance principle tells us to rule out dominated acts.* If there is an act that dominates all others, the principle has us choose it.

However, we cannot always rely on the dominance principle—as an example from the disarmament debate demonstrates. Doves argue that disarmament is preferable whether or not a war occurs. For, they claim, if there is no war and we disarm, more funds will be available for social programs; if there is a war and we disarm, well, better Red than dead. This produces decision table 1-5.

1-5	War	No War
Arm	Dead	Status Quo
Disarm	Red	Improved Society

Savage setting
see 1-3a

Given the view that it is better to be Red than dead, disarming dominates.

Hawks need not question anything that has transpired. They can simply respond that disarming makes it virtually certain that the other side will attack us but that continuing to arm makes war very unlikely. Doves have not taken this into account. They have not considered that in this case the act we choose affects the probabilities of the states. The example shows that the dominance principle applies only when the acts do not affect the probability of the states.

The same problem arises in the betting example. The probability of winning varies with the bet chosen. So reasoning according to the dominance principle does not apply to the choice between betting American or National.

One might think that all that is wrong with the betting example is a misapplication of the dominance principle, but I think there is a deeper problem here. It is a case of an *illegitimate problem specification*. In any decision table that uses states such as *I win*, or *things turn out well*, we can substitute the state description *I make the right choice* without having to change the outcomes or the effect of the various acts on the states. But it is surely pointless to use a decision table with the state headings *I make the right choice*, *I fail to make the right choice* to make that very choice. If you already knew the right choice, why bother setting up a decision table? Actually, this is a bit flippant. The real problem is that the designation of the term "right choice" varies with the act. If I bet American, I make the right choice if and only if the American League wins. Correspondingly for betting National. So the phrases *I make the right choice*, *I fail to make the right choice* cannot serve as state descriptions, since they do not pick out one and the same state no matter what the act. The same point obviously applies to descriptions such as *I win* or *things turn out well*.

It is not always clear, however, when a state description is proper. Suppose, for example, that you are trying to choose between going to law school and going to business school. It is tempting to use state descriptions such as *I am a success* or *opportunities are good*, but a moment's thought should convince you that these are variants of *I make the right choice* and, thus, improper. Unfortunately there is no algorithm for determining whether a state description is proper.

Nor are there algorithms for deciding whether a set of states is relevant. Suppose again that you are deciding between law school and business school. The states *the rainfall is above average for the next three years*, *it is not* are plainly irrelevant to your decision. But how about *there is an economic depression three years from now*, *there is not*? Perhaps lawyers will do well during a depression whereas business school graduates will remain unemployed; those states might then be relevant to consider.

I will leave the problem of state descriptions to proceed to another one, which also involves the choice of problem specifications. In this instance, however, the state descriptions involved may be entirely proper and relevant. To illustrate the problem I have in mind let us suppose that you want to buy a car whose asking price is $4,000. How much should you bid? Before you can answer that question, you must consider several alternative bids, say, $3,000, $3,500, and $4,000. Those three bids would generate a three-act problem specification. But is this the best specification for your problem? How about using a wider range of bids? Or bids with smaller increments? Those are clearly relevant questions and they bear on which decision table you ultimately use. Now if we think of the various answers to these questions as giving rise to different decision tables (e.g., the three-bid table first mentioned, another with the bids $2,500, $3,000, $3,500, $3,750, etc.), choosing the best problem specification amounts to choosing between decision tables. We are thus involved in a *second-order* decision, that is, a decision about decision problem specifications. We can apply decision theory to this decision too. For example, if the best act according to any of the specifications under consideration is *bid $4,000*, it would not matter which table you choose. And if all the tables use the same states, we can combine them into one big table whose rows consist of all the bids used in any of the smaller tables and whose columns are the states in question. The best bid according to that table will be the best for your decision. If the states vary too, the solution is not so clear. But that is a technical problem for decision theory. Let us continue with the philosophical problem. We have now formulated a second-order decision problem concerning the choice of tables for your original bidding problem. But questions may arise concerning our choice of a second-order problem specification. Should we have considered other first-order tables with additional bids or other sets of states? Should we have used different methods for evaluating the acts in our first-order tables? Approaching these questions through decision theory will lead us to generate a set of second-order tables and attempt to pick the best of these to use. But now we have a third-order decision problem. An infinite regress of decision problems is off and running!

The difficulty here can be put succinctly by observing that *whenever we apply decision theory we must make some choices*: At the least, we must pick the acts, states, and outcomes to be used in our problem specification. But if we use decision theory to make those choices, we must make yet another set of choices.

This does not show that it is impossible to apply decision theory. But it does show that to avoid an infinite regress of decision analyses any application of the theory must be based ultimately on choices that are made without its benefit. Let us call such decisions *immediate decisions*. Now someone might object that insofar as decision theory defines rational decision making, only those decisions made with its benefit should count as rational. Thus immediate decisions are not rational, and because all decisions depend ultimately on these, no decisions are rational.

Should we give up decision theory? I think not. The objection I have just rehearsed assumes that decision theory has cornered the market on rational decision making, that a decision made without its benefits is irrational. In fact, it is frequently irrational to use decision theory; the costs in time or money may be too high. If an angry bear is chasing you, it would not make sense to use decision theory to pick which tree to climb. On the other hand, it is not always rational to make an immediate decision either. You would not (or should not) choose your career, college, or professional school without weighing the pros and cons of a few alternatives.

But then how do we decide when to do a decision analysis and when to make an immediate decision? Well, we do not do it on a case-by-case basis. Each time I see a car coming at me in my lane, I do not ask myself, Should I do a decision analysis to decide between braking, pulling off the road, or continuing as I am or should I make an immediate decision? If I did I would have been killed years ago. (I would also have trapped myself in an infinite regress.) Instead, I follow an unstated policy of letting my "gut reactions" make the choice. And that is plainly the rational thing to do, since a sober, healthy, and experienced driver usually does the right thing in such situations. We live by many policies that tell us when we should make an immediate decision and when a decision analysis of some kind is required. Some of those policies are more rational than others; they lead in general to better lives. This means that it may be appropriate from time to time to reassess one or more of our policies. Of course, decision theory may help with that task.

A persistent skeptic might object that now we need a policy for reassessing policies, and another regress of decisions is in the offing. But I will leave the matter as it stands.

A final preliminary philosophical issue is illustrated by this fanciful example. Baker and Smith, competitors in the oil business, are both considering leasing an oil field. Baker hires a decision analyst to advise him, and Smith decides to base his decision on the flip of a coin. The decision analyst obtains extensive geological surveys, spends hours reviewing Baker's balance sheets, and finally concludes that the risks are so great that Baker should not bid on the lease at all. Letting the flip of a coin decide for him, Smith pays dearly for the lease. Yet, to everyone's surprise, a year later he finds one of the largest oil reserves in his state and makes piles of money. Something seems to have gone wrong. Smith never gave the matter any consideration and became a billionaire, whereas the thoughtful Baker remained a struggling oilman. Does this turn of events show that Baker's use of decision theory was irrational?

We can resolve some of our discomfort with this example by distinguishing between *right decisions* and *rational decisions*. Agents' decisions are *right* if they eventuate in outcomes the agents like at least as well as any of the other possibilities that might have occurred after they had acted. According to our story Smith made the right decision and Baker did not. If we had complete foreknowledge, individual decision theory would need only one principle, namely, *make the right decision*. Unfortunately, most of our decisions must be based on what we think *might* happen or on what we think is *likely* to happen, and we cannot be certain they will result in the best outcomes possible. Yet we still should try to make choices based on the information we do have and our best assessments of the risks involved, because that is clearly the rational approach to decision making. Furthermore, once we appreciate the unfavorable circumstances under which most of our decisions must be made, we can see that a rational decision can be entirely praiseworthy even though it did not turn out to be the right decision. (How often have you heard people remark that although it was true they had hoped for a better outcome, they made the only rational choice open to them at the time? Or that despite having made a stupid decision, someone "lucked out"?)

PROBLEMS

1. Set up a decision table for the following decision situation. Jack, who is now twenty, must decide whether to marry his true love Jill immediately or not see her again until he is twenty-one. If he marries her now then he will lose the million dollars his uncle has left him in trust. If he waits to see her until he is twenty-one, he will receive the money and can marry Jill at that time—*if* she still loves him. (Part of your problem is selecting an appropriate set of acts, states, and outcomes.)
2. Pascal reasoned that it was better to lead the life of a religious Christian than to be a pagan, because if God exists, religious Christians go to Heaven and everyone else goes to Hell, whereas if God does not exist, the life of the religious Christian is at least as good as that of the pagan. Set up a decision table for this argument and explain why the dominance principle supports Pascal's reasoning.

1-3. Certainty, Ignorance, and Risk

Sometimes we can be quite certain that our acts will result in given outcomes. If you are in a cafeteria and select a glass of tomato juice as your only drink then, ingenious pranksters and uncoordinated oafs aside, that is the drink you will bring to your table. Sometimes, however, you can know only that your choice will result in a given outcome with a certain probability. If, for instance, you bet on getting a 10 in one roll of a pair of unloaded dice, you cannot be certain of winning, but you can know that your chances are 1 in 12. Finally, sometimes you may have no earthly idea about the relationship between an act open to you and a possible outcome. If you have a chance to date a potential mate, a possible outcome is that the two of you will someday together pose for a photo with your great-grandchildren. But, offhand, it would seem impossible for you to estimate the chances of that happening if you make the date.

If you are making a decision in which you can be certain that all your acts are like the first example, decision theorists call your choice a *decision under certainty.* Here all you need to do is determine which outcome you like best, since you know which act (or acts) is certain to produce it. That is not always easy. Even a student who could be certain of getting all her courses might have a hard time deciding whether to sign up for logic and physics this term or for music and physics this term, postponing logic until next term. Or suppose you are planning an auto trip from New York to Los Angeles with stops in Chicago, St. Louis, New Orleans, and Las Vegas. You can use one of many routes, but they will differ in mileage, driving conditions, traffic, chances for bad weather, and scenery. Even supposing that everything concerning these attributes is certain, which will you choose?

The mathematical theory of linear programming has been applied to many problems concerning decisions under certainty with quantitative outcomes.

However, neither decision theorists nor philosophers have paid this subject much attention, so we will not cover it further in this book.

When, in a given decision problem, it is possible to assign probabilities to all the outcomes arising from each act, the problem is called a *decision under risk.* Choices between bets on fair coins, roulette wheels, or dice are paradigms of decisions under risk. But it is usual to classify investment decisions, bets on horse races, marketing decisions, choices of crops to plant, and many others like them as decisions under risk, because even when we cannot assign an exact probability, say, to the stock market rising or to a drought, it often pays to treat those decisions as if they were decisions under risk, pure and simple.

Finally, when it makes no sense to assign probabilities to the outcomes emanating from one or more of the acts (as in your date resulting in great-grandchildren), the decision problem is called a *decision under ignorance.* (Some decision theorists call it a decision under uncertainty.) Ignorance may be partial or total; it may be possible to assign probabilities to some of the outcomes emanating from some of the acts, but to none emanating from the other acts. We will turn shortly to techniques for dealing with decisions under ignorance, but we will treat only decisions under total ignorance in this book.

This classification of decisions as under certainty, risk, and ignorance is plainly an idealization. Many decisions do not fall neatly into one category or another. Yet if the uncertainties in a decision are negligible, such as an uncertainty as to whether the world will exist tomorrow, the problem is fairly treated as one under certainty. And if we can estimate upper and lower bounds on probabilities, we can break a decision problem into several problems under risk, solve each, and compare the results. If each solution yields *the same recommendation*, our inability to assign exact probabilities will not matter. On the other hand, if the range of probabilities is very wide, it might be better to treat the problem as a decision under ignorance.

The classification is philosophically controversial too. Some philosophers think the only certainties are mathematical and logical. For them there are few true decisions under certainty. Other philosophers—not necessarily disjoint from the first group—think we are never totally ignorant of the probabilities of the outcomes resultant from an act. Thus for them there are no true decisions under ignorance. We will learn more about this later when we study subjective probability.

1-3a. Some Details of Formulation

Suppose you are deciding whether to eat at Greasy Pete's and are concerned that the food will make you sick. The relevant outcomes associated with your act are *you get sick* and *you do not get sick*. Now if you take the states to be *Pete's food is spoiled*, *it is not*, the act of eating at Greasy Pete's under the state that his food is spoiled is *not* certain to result in your getting sick. (Perhaps you eat very little of the spoiled food or have a very strong stomach.) Yet, our use of decision tables presupposes that we can find exactly one outcome for each act-state pair. So how can we use decision tables to represent your simple problem?

We could introduce outcomes that themselves involve elements of uncertainty. For example, we could replace the outcomes of your getting (not getting) sick with the outcomes *you have a chance of getting sick*, *you do not*. This would ensure no more than one outcome per square. Or we could introduce a more refined division of the environment into states, using, for instance, *the food is spoiled but you can handle it*, *the food is spoiled and you cannot handle it*, and *the food is not spoiled* for the last example. Different decision problems will call for different combinations of these approaches. In any case, because these are problems with applying decision theory, we will assume henceforth that each act-state pair determines a unique outcome.

In view of this assumption, we can focus all the uncertainty in a decision on the states involved. If you do not know whether eating at Greasy Pete's will make you sick, we will take that to be because you do not know whether the food is spoiled or whether you can handle it. A further consequence is that, in the case of decisions under risk, probabilities will be assigned to states rather than outcomes. Again, if you eat at Greasy Pete's, the only way you can get sick is for the food to be spoiled and you to be unable to handle it. So to treat your problem as a decision under risk we must assign probabilities to that compound state and the other states. Suppose the probability of the food being spoiled is 70% and that of your being unable to handle spoiled food is 50%. Then (as we will learn later) the probability of your getting rotten food at Greasy Pete's and being unable to handle it is 35%, whereas the probability that you will get bad food but will be able to handle it is 35% and the probability that your food will be fine is 30%.

Unless some malevolent demon hates you, your choosing to eat at Greasy Pete's should not affect his food or your ability to handle it. Thus the probabilities assigned to the states need not reflect the acts chosen. This means that we can use the unqualified probability that Greasy Pete's food will be spoiled rather than the probability that it will be spoiled *given* that you eat there. On the other hand, if you are deciding whether to smoke and are worried about dying of lung cancer, your acts will affect your chances of entering a state yielding that dreaded outcome. As you know, the lung cancer rate is much higher among smokers than among the total population of smokers and nonsmokers. Consequently, the probability of getting lung cancer *given* that you smoke is much higher than the probability that you will get lung cancer no matter what you do. The latter probability is called the *unconditional* probability, the former the *conditional* probability of getting lung cancer. Plainly, in deciding whether to smoke, the conditional probabilities are the ones to use.

We say that a state is *independent* of an act when the conditional probability of the state given the act is the same as the unconditional probability of that state. Getting heads on the flip of a fair coin is independent of betting on heads. There being rotten food at Pete's is independent of your eating there. But contracting lung cancer is not independent of smoking, earning good grades is not independent of studying, and surviving a marathon is not independent of training for it.

When some of the states fail to be independent of the acts in a decision

Jeffrey

under risk we should use the probabilities of the states conditional on the acts. When all of the states are independent of the acts, it does not matter which probabilities we use; for, by the definition of independence, there is no difference between them. Since there is no harm in always using conditional probabilities, for the sake of uniformity, we will do so.

Those who prefer unconditional probabilities may find it possible to reformulate their decision problems using new states that are independent of the acts. Sav. Consider the smoking decision again. Not everyone who smokes gets lung cancer—not even those who have been very heavy smokers since their teens. It is plausible, then, that those who avoid cancer have some protective factor that shields them from smoking's cancer-inducing effects. If there is such a factor, smoking is unlikely to be responsible for its presence or absence. With this in mind, we can reformulate the smoking decision in terms of states involving this new factor. We replace the two states *you (do not) get lung cancer* with four states: *you have the protective factor and do (do not) get terminal lung cancer from nonsmoking causes*, *you do not have the protective factor and you do (do not) get terminal lung cancer from nonsmoking causes*. Then your smoking will not affect your chances of being in one state rather than another. In the original formulation, you saw smoking as actually determining whether you entered a state leading to your death from lung cancer; thus you saw smoking as affecting the probability of being in that state. On the new formulation, you are already in a state that can lead to your death from lung cancer or you are not. If you are in the unlucky state, your not smoking cannot alter that; but if you smoke you are certain to die, since you lack the protective factor. You do not know what state you are in, but if you knew enough about lung cancer and the factors that protect those exposed to carcinogens from getting cancer, you could assign *unconditional* probabilities to the four new states. For those states would be independent of the acts of smoking or not smoking.

PROBLEMS

1. Classify the following as decisions under certainty, risk, or ignorance. Justify your classifications.
   a. Jones chooses his bet in a roulette game.
   b. Smith decides between seeking and not seeking a spouse.

c. A veterinarian decides whether to put a healthy stray dog to sleep or to adopt it as his own pet.
d. A student trying to satisfy degree requirements chooses among the courses currently available.
e. A lawyer decides whether to accept an out-of-court settlement or to take her client's case to trial.

2. Set up a decision table for the last version of the Greasy Pete problem. Why is the outcome description *you do not get sick* in more than one square? What is the *total* probability that you will get an outcome so described?

1-4. Decision Trees

It is often more expeditious to analyze a decision problem as a sequence of decisions taking place over time than to treat it as a single one-time decision. To do this we use a *decision tree* instead of a table. A decision tree is a diagram consisting of branching lines connected to boxes and circles. The boxes are called *decision nodes* and represent decisions to be made at given points in the decision sequences. The circles are called *chance nodes* and represent the states relevant to that point of the decision. Each line projecting to the right of a node represents one of the acts or states associated with it and is usually labeled with an appropriate act or state description. To demonstrate these ideas, let us use the decision tree (shown in figure 1-1) to represent the disarmament problem discussed earlier. As the tree illustrates, outcomes are written at the tips of the

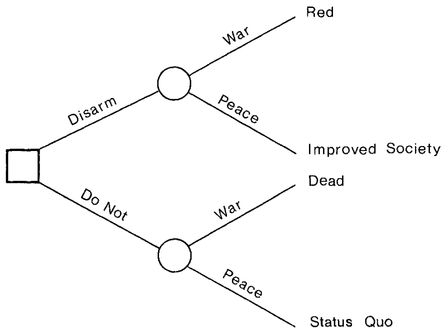

Figure 1-1

tree. This permits us to assess the possible consequences of a choice by following each of the branches it generates to its tip. Disarming, for instance, leads to an improved society if there is no war, but it leads to life as a Red if there is one.

The practical advantage of trees over tables can be appreciated by considering the following example. Suppose you must first choose between going to the seashore or staying at home. If you go to the seashore, you will wait to determine whether it is raining. If it rains, you will decide whether to fish or stay inside. If you fish and the fishing is good, you will be happy; if the fishing is not good, you will be disappointed. If you stay in, you will feel so-so. On the other hand, if there is no rain, you will sunbathe and be happy. Finally if you stay home, you will feel so-so. Figure 1-2 presents the tree for this decision.

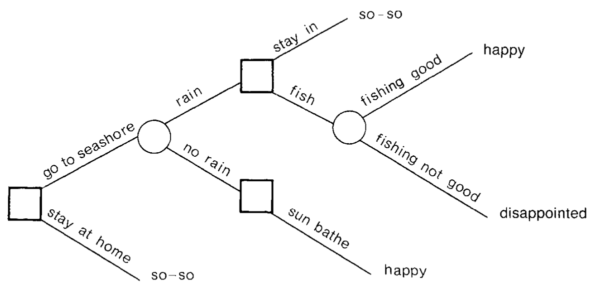

Figure 1-2

It is easy to go from a decision table to a decision tree. Start the tree with a box with one line emanating from it for each row of the table. At the end of each line place a circle with one line coming from it for each column of the table. Then at the tips of each of these lines write the outcome entries for the square to which that tip corresponds.

It is by far more interesting and important that the decision tree for any problem can be transformed into an equivalent decision table. We accomplish this by collapsing sequences of decisions into one-time choices of *strategies*. A strategy (S) is a plan that determines an agent's choices under all relevant circumstances. For example, the plan

S_1: I will go to the shore; if it rains, I will fish; if it does not rain, I will sunbathe

is a strategy appropriate for the problem analyzed by the preceding tree. The other appropriate strategies are:

S_2: I will go to the shore; if it rains, I will stay in; if it does not, I will sunbathe;

S_3: I will stay at home (under all circumstances).

We can similarly find a set of strategies that will enable us to represent any other sequential decision as a choice between strategies. This representation generally necessitates using more complicated states, as table 1-6 illustrates for the seashore problem. In case you are wondering how we obtain the entries in the second row of this table, notice that in this row S_2 is adopted. So if it rains you

1-6

	Rain & Good Fishing	Rain & Bad Fishing	No Rain
S_1	Happy	Disappointed	Happy
S_2	So-so	So-so	Happy
S_3	So-so	So-so	So-so

will stay in, and whether or not the fishing is good (for anyone), you will feel so-so. If it does not rain, you will sunbathe and be happy.

Decision theorists have presented mathematically rigorous formulations of the technique illustrated here and have proved that any decision tree can be reduced to a decision table in which the choices are between strategies. Since our interest in this book is more theoretical than practical, we will stick with decision tables and not pursue the study of decision trees further.

PROBLEMS

1. Formulate the following decision problem using a decision tree. Danny, who has been injured by Manny in an automobile accident, has applied to Manny's insurance company for compensation. The company has responded with an offer of $10,000, Danny is considering hiring a lawyer to demand $50,000. If Danny hires a lawyer to demand $50,000, Manny's insurance company will respond by either offering $10,000 again or offering $25,000. If they offer $25,000, Danny plans to take it. If they offer $10,000, Danny will decide whether or not to sue. If he decides not to sue, he will get $10,000. If he decides to sue, he will win or lose. If he wins, he can expect $50,000. If he loses, he will get nothing. (To simplify this problem, ignore Danny's legal fees, and the emotional, temporal, and other costs of not settling for $10,000.)
2. Mimicking the method used at the end of this section, reformulate Danny's decision problem using a decision table.

1-5. References

In giving references at the end of each chapter, I will refer to works by means of the names of their authors. Thus *Luce and Raiffa* refers to the book *Games and Decisions* by R. D. Luce and H. Raiffa. When I cite two works by the same author, I will either use a brief title or give the author's name and the publication date of the work in question.

The classic general treatise on decision theory is *Luce and Raiffa*. It not only surveys several of the topics not covered in this book but also contains an extensive bibliography. The reader wishing to do more advanced study in decision theory should begin with this work. *Von Neumann and Morgenstern* was

the first major book on the subject and set the agenda for much of the field. A number of journals publish articles on decision theory, but *Theory and Decision* is the principal interdisciplinary journal devoted to the subject.

Raiffa is an excellent introduction to individual decision theory for those interested in applications of the theory to business problems. This book also makes extensive use of decision trees. *Eells*, *Jeffrey*, and *Levi* are more advanced works in the subject for those with a philosophical bent. *Savage* and *Chernoff and Moses* approach the subject from the point of view of statistics.

Davis is a popular introduction to game theory, and *Sen* is a classic treatise on social choice theory. Both books contain extensive bibliographies.

For more on problem specifications, see Jeffrey's "Savage's Omelet," and *Nozick*.