Engsci 711

Assignment 1

Oliver Maclaren
oliver.maclaren@auckland.ac.nz

Due: May 27th (in class or via Canvas)

Question 1

Consider the following system

$$\dot{x} = x(4 - x - 2y)$$
$$\dot{y} = y(2 - x - y)$$

a) What sort of real-world system might this set of ODEs be used as a model for? Hence give an interpretation for the terms on the RHS of the equations. (Hint: see Lecture 5 readings.)

b) Find the fixed points and determine their stability.

c) Determine the eigenvectors of the fixed points.

d) Write down the equations for the $x-$ and $y-$nullclines. Sketch these in the phase plane. Include the equilibria you found above and the direction fields *on* the nullclines in your sketch.

e) Complete the sketch above by adding some possible compatible trajectories.

f) Use Dulac's criterion to disprove the existence of periodic solutions in the positive x, y quadrant.

Question 2

Consider the system

$$\dot{x} = x^2 + y^2 - 2$$
$$\dot{y} = x - 1$$

where $x, y \in \mathbb{R}$.

a) Find and classify all of the equilibria of the system. You do not need to draw any pictures (yet) or find any eigenvectors.

b) Write down the equations for the $x-$ and $y-$nullclines. Sketch these in the phase plane. This sketch should include: the equilibria you found above, the direction fields *on* the nullclines, and arrows indicating the qualitative flow directions in *each region* partitioned by the nullclines.

c) Add some possible compatible trajectories, including compatible local behaviour near the equilibria, to your diagram. You do not need to do any further explicit calculation (e.g. you do not need to find any eigenvectors) - a qualitative sketch is enough.

d) Verify your sketch from the previous part by plotting some trajectories using XPP (or pplane etc).

Question 3

Consider the system

$$\dot{x} = 2x + y$$
$$\dot{y} = -y + x^2$$

a) Find and classify the equilibria.

b) Find the power series expansions for both $W^u_{loc}(0)$ and $W^s_{loc}(0)$ up to (i.e. including) quadratic order.

c) Plot some solutions using XPP (or pplane etc).

Question 4

The Brusselator is a simple model for a hypothetical chemical oscillator. One version of this model, in dimensionless form, has kinetics given by

$$\dot{x} = 1 - (b+1)x + x^2 y$$
$$\dot{y} = bx - x^2 y$$

where $b > 0$ is a parameter and $x, y \geq 0$ are dimensionless concentrations.

- There is one equillibirum point. Find it (express it as a function of b).
- Find the Jacobian derivative, evaluate it at this point, and calculate the trace and determinant in terms of b.
- For what b values is the equilibrium stable? Show your reasoning.
- For what b values is the equilibrium a centre? Is this case structurally stable? What does 'structurally stable' mean?
- Sketch the nullclines and determine (and sketch) a trapping region for the flow. What does the existence of a trapping region indicate?

(*Hint: you may find the Lecture 7 handout with an example from Strogatz quite helpful for the above question!*)