ENGGEN 140
Biology and Chemistry for Engineers

Part 2 - Energy.

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Engineering Biology and Chemistry

Last updated—April 5, 2018 Part II ~17 Lectures + 3 Tut.

Contents

~5 lectures (W7–W8, L1–L5 + 1 Tut.)

Overview L1.

Split second half into

1. – Intro to energy (~ 5 lectures + 1 tut)
2. – Thermodynamics (~ 7 lectures + 1 tut)
3. – Biological energetics (~ 5 lectures + 1 tut).

Will discuss ideas

BUT

Key is solving representative examples

→ Don't panic!

→ What you need to do is relatively straightforward (I hope!)

Big picture!

Basic principles [physics] $\xrightarrow{\text{apply}}$ Chemistry → Biology

Basic principles [physics] → Biology

Basic principles [physics] → Other engineering & science areas

Chemistry, Biology, Other engineering & science areas → Society

Today

- Conservation & constitutive eq^{n}s
- Dimensions & units

Example questions (2018 summer school Exam)

1. Which ONE of the following statements is TRUE, regarding Hooke's law:

(1 mark)

A. It is an expression of conservation of energy
B. It is an expression of conservation of mass
C. It is a constitutive equation valid only for particular types of material

Answer: ____________

2. Using base SI units, show that the units for work are dimensionally consistent with the units of (translational) kinetic energy.

(2 marks)

Answer:

3. Using Newton's Second Law to help you, derive an expression for 1 Newton (1 N) in terms of base SI units.

(2 marks)

Answer:

1 Introduction to Energy

Learning Objectives

- Understand the concepts of conservation laws and constitutive equations and how these can help organise scientific and engineering knowledge.
- Understand the difference between base SI units and derived SI units. Derive correct SI units for force, energy, power and pressure and carry out basic calculations involving these quantities.
- Be aware of some of the various forms that energy comes in and how these can be interconverted. Carry out basic energy conversion calculations including accounting for efficiency limits.
- Understand the planetary energy balance and carry out basic calculations involved in this.
- Be aware of global and NZ energy consumption patterns, read basic graphs and carry out simple energy estimates and conversions.
- Apply your knowledge of energy balances and constitutive equations to simple case studies.

1.1 Conservation and constitutive equations

L1 –

One of the most important and generally applicable concepts in science and engineering is the idea of a **conservation law** or **balance equation**. The idea is roughly to ask yourself: *when things change, what stays the same?* Fundamental examples of quantities that are *conserved* during physical processes include: mass, energy and momentum. We focus primarily on the first two in this course, but the third is the basis of much of your courses in engineering mechanics. In even more fundamental physics, these ideas can be seen as different sides of the same coin: e.g. conservation of mass-energy. In 'messier' scenarios it can still be helpful to look for other approximate conserved quantities or approximate *invariants*.

In a given situation these define what processes are *possible*: violating conservation of mass, energy and/or momentum is a serious offence!

Newton's second law: $\boxed{\text{momentum}}$ and energy *

usual form: $m \times a = F_{net}$ (mass × accel. = net force).

More general form: $\frac{d}{dt}(mv) = F_{net}$ } $\boxed{\text{balance/conservation of momentum}}$

$\boxed{\text{rate of change of momentum of system} = \text{rate of transfer of momentum to system}}$

Note: if m constant, $\frac{d(mv)}{dt} = m\frac{dv}{dt} = m \times a$.

if no external forces, $\frac{d(mv)}{dt} = 0$ ie $mv = \text{constant}$

An issue arises, however: in considering the bending of a beam made of wood vs one made of steel, both satisfy the same high-level conservation principles and yet they exhibit different behaviour (e.g. different yield strengths when subject to mechanical testing). What distinguishes these if not conservation principles? While at the level of fundamental physics these are perhaps governed by the same laws of particle physics (or string theory or whatever...), at the *macroscopic* level of observation and experiment we need to introduce additional, *approximate* 'laws' or equations which describe *particular details* of the materials, substances, forces or interactions that we are dealing with.

These *force laws*, *transport laws* and/or *material property* equations 'fill in the details' of our particular application, telling us about the substances we're dealing with. Here we call these **constitutive equations**. These tell us about how *particular* substances interact and what processes *actually* occur.

L also eg 'how fast'

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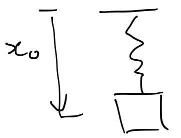
* $m\frac{dv}{dt} = F \rightarrow$ Energy? See mechanics courses....

$$\int_1^2 m\frac{dv}{dt}\,dx = \int_1^2 F\,dx \Rightarrow m\int_1^2 v\,dv = \frac{mv^2}{2}\Big|_1^2 = \int F\,dx$$

ie $\Delta KE_{1\to 2} = W_{1\to 2}$ (work-energy eq^n for a particle).

Conservation and constitutive equations: some examples

Mass on elastic spring

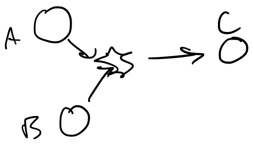


Conserved: momentum (etc) eg $ma = \sum F$

constitutive eqⁿ: Hooke's (force) law for elastic material:

$$F = k \cdot (x - x_0)$$

Chemical reaction



Conserved: mass $\frac{dA}{dt} = -J$, $\frac{dB}{dt} = -J$, $\frac{dC}{dt} = +J$

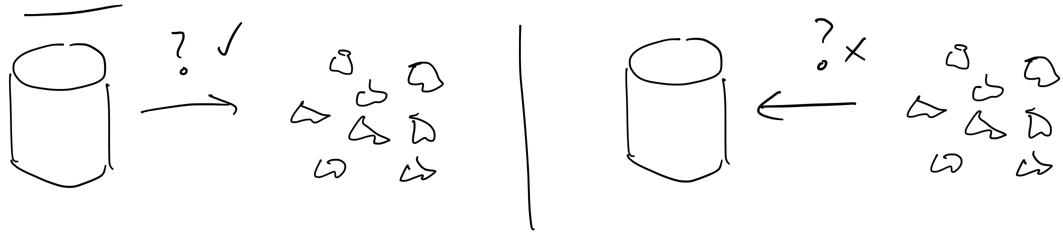
constitutive: 'law of mass action' for reaction rate:

$$J = k \cdot [A] \cdot [B]$$

An interesting case is *entropy*: as you probably already know, entropy *stays the same or increases* in an isolated system. It is *non-decreasing*. This is not quite a conservation equation in the usual sense, but it is also more general than just an ordinary constitutive equation. The non-decrease of entropy can be thought of as a **one-sided conservation law** or **directional principle**. This is essentially a statistical principle and tells us about what processes *probably* or *almost certainly* occur.

A broken glass: energy and entropy

Macro:



Micro:

→ look the 'same' zoomed in!

Upshot: it is *possible* for a broken glass to spontaneously reform (this process satisfies conservation of energy), but it *overwhelmingly unlikely* to occur (is very improbable). In more advanced courses, directional principles such as the second law of thermodynamics are often used to place constraints on constitutive equations: e.g. to ensure that diffusion occurs *down* a concentration gradient, one deduces that the diffusion coefficient in Fick's law must be positive[1]. Macroscopic diffusion is a *bulk* or *averaged* phenomenon, and can be thought of as driven by the *net entropic force* generated by imbalanced macroscopic concentrations. We will return to entropy and thermodynamics later in this course.

Example Problems 1: Conservation and constitutive equations

1. (2018 SS Exam Q) Is Hooke's law an example of a conservation equation or a constitutive equation? If the latter, to what sort of material does it apply?
2. Use appropriate conservation and constitutive equations to derive a *closed* model of the motion of a mass on a spring subject to the force of gravity.
3. What assumption have you made about entropy in the above? What effect could you include that would change this assumption? What effect would you expect this to have on the long-term dynamics of the system.

Answers

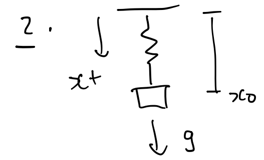
1. Constitutive force law : $F = k\cdot(x - x_0)$

(or $E = \frac{1}{2}k(x-x_0)^2 = \int_{x_0}^{x} F\,dx$)

applies to elastic materials. Constant k is different for diff. materials.

[1]Entropy can also be connected to ideas of *stability* and/or *typicality*: entropy increase 'picks out' the long-term, stable/typical - and hence *observable* - phenomena.

2.



$$\boxed{\begin{aligned} &\frac{d(mv)}{dt} = \sum F \quad \text{conservation of momentum} \\ &\downarrow + \ \sum F = mg - k(x - x_0) \quad \text{constitutive force laws (gravity, spring)} \end{aligned}}$$

&

$$\frac{d(mv)}{dt} = m\frac{dv}{dt} \quad \text{(constant mass)}$$

$$= m\frac{d^2x}{dt^2} \quad \left(\text{kinematics}: v = \frac{dx}{dt}, \frac{dv}{dt} = \frac{d^2x}{dt^2}\right)$$

$$\Rightarrow m\frac{d^2x}{dt^2} = mg - k(x - x_0)$$

$$\text{or } \boxed{\frac{d^2x}{dt^2} = g - \frac{k}{m}(x - x_0)}$$

} ODE for $x(t)$, assume all parameters known → can solve!

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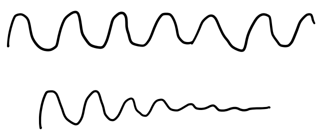
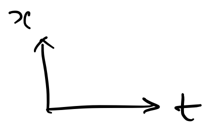
3. Assume reversisible/conservative → no friction, no entropy increase, heating etc

→ will oscillate forever!

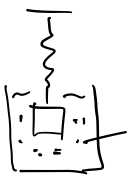
→ include friction, will 'damp' & stop moving eventually





↳ 'lose' energy to surroundings (conserved overall!)

↳ heat up.



honey : lots of friction!

1.2 Dimensions and units

You have already looked at **dimensions** and **units** in the first half of this course, but these are important enough to revisit here. We will also consider some key players in energy calculations and their associated units: *force, power, pressure* and, of course, *energy*!

The idea of a system of units for measuring different types of quantity is surprisingly subtle. We need to

- Decide what 'kinds of thing' there are.
- For each 'kind of thing', choose a 'standard' reference object of that type.

The 'kind of thing' is called the **dimension** or **type** of a quantity, while a 'standard reference' for a thing of a given type is called a **unit**.

Measuring length

Compare a 'thing' eg

to a standard 'thing' of same 'type' eg

eg. = 2.5 ×

eg = 2.5 × 1m

= 2.5m

- dimension : 'type of thing' eg length, mass, time
- unit : 'standard/reference' for a given type eg cm, kg, s

1.2.1 Notation

We will use the notation:

$$[X] \text{ means 'Units of X' or 'Dimensions of X'}$$

While we could introduce a different symbol for each meaning (units and dimensions, respectively), we prefer to let the context make it clear which is meant.

Units and dimensions of X

Example:

$$\underbrace{\text{speed} \; [=] \; \frac{\text{Length}}{\text{Time}}}_{\text{dimension}} \; \underbrace{[=] \; \frac{m}{s}}_{\text{units (eg SI)}}$$

Notes: using same symbol [=] for both coz lazy
sometimes see eg [X] for units/dim. of X.
sometimes leave out [ ] completely coz lazy

1.2.2 SI units

The International System of Units - Système international d'unités, abbreviated as SI - is the modern form of the metric system, built upon *seven base quantities and units*. The system also establishes *prefixes* to be used when specifying multiples and fractions of units, e.g. *milli*, *centi* etc. See below.

The metric system was originally conceived and introduced in 1799 as a system of measurement *based on natural phenomena*. Technical limitations at the time required the use of human artefacts to describe the metre and kilogram. However, the metre was redefined in 1960 in terms of wavelength of light and the kilogram looks set to be officially changed on 16 November 2018, to be defined relative to the equivalent energy of a photon via the Planck constant.

7 base types for SI

Table 1 summarises the base SI quantities, units of measurement and their proposed definitions. The difference ~~between base~~ and derived SI units is discussed further below.

Table 2 summarises some common SI prefixes.

type of thing

reference thing

Quantity	Dimension	SI Unit	SI Unit Symbol	SI Unit Definition
Time	T	Second	s	Defined by taking the fixed numerical value of the caesium frequency Δv_{cs}, the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.
Length	L	Metre	m	Defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m $\cdot$s^{-1}, where the second is defined in terms of the caesium frequency Δv_{cs}.
Mass	M	Kilogram	kg	Defined by taking the fixed numerical value of the Planck constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit J$\cdot$ s, which is equal to kg$\cdot$m$^2\cdot$s^{-1}, where the metre and the second are defined in terms of c and Δv_{cs}.
Current	I	Ampere	A	Defined by taking the fixed numerical value of the elementary charge e to be $1.602176634 \times 10^{-19}$ when expressed in the unit C, which is equal to A$\cdot$s, where the second is defined in terms of Δv_{cs}.
Temperature	Θ	Kelvin	K	Defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit J$\cdot$K^{-1}, which is equal to kg$\cdot$m$^2\cdot$s$^{-2}\cdot$K^{-1}, where the kilogram, metre and second are defined in terms of h, c and Δv_{cs}.
Amount	N	Mole	mol	One mole contains exactly $6.02214076 \times 10^{2}3$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities.
Luminous Intensity	J	Candela	cd	Defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{1}2$ Hz, K_{cd}, to be 683 when expressed in the unit lm$\cdot$W^{-1}, which is equal to cd$\cdot$sr$\cdot$W^{-1}, or cd$\cdot$sr$\cdot$kg$^{-1}\cdot$m$^{-2}\cdot s^3$, where the kilogram, metre and second are defined in terms of h, c and Δv_{cs}. The symbol sr stands for the dimensionless unit steradian or square radian, which is defined geometrically.

unit = reference thing

Table 1: Base SI Units and new proposed definitions. Based on `https://en.wikipedia.org/wiki/Proposed_redefinition_of_SI_base_units`.

Multiples	Prefix name		deca	hecto	kilo	mega	giga	tera	peta	exa
	Prefix symbol		da	h	k	M	G	T	P	E
	Factor	10^0	10^1	10^2	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
Fractions	Prefix name		deci	centi	milli	micro	nano	pico	femto	atto
	Prefix symbol		d	c	m	μ	n	p	f	a
	Factor	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}	10^{-18}

Table 2: SI prefixes. (From notes by Thor Besier, based on `https://en.wikipedia.org/wiki/International_System_of_Units`).

1.2.3 Conversion of units

While the dimensions of a quantity cannot be changed, *the units that we measure a quantity in are arbitrary and we can thus change between different units.*

A simple idea for changing units is to 'multiply by one', where the numerator and denominator represent the 'same thing' in the different units. This process is perhaps best illustrated via examples, as follows.

$$\_\_\_ \times \underbrace{\left(\frac{\text{Thing in units two}}{\text{Thing in units one}}\right)}_{\times 1.} = \_\_\_$$

Converting between different units

Convert 10 cm to m

use $1\ \text{cm} = 1\times10^{-2}\ \text{m}$

ie $\frac{1\ \text{cm}}{10^{-2}\ \text{m}} = \text{'one'}$ /or $\frac{10^{-2}\ \text{m}}{1\ \text{cm}} = \text{'one'}$

So $10\ \text{cm} \times \underbrace{\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)}_{\times \text{'1'}} = \frac{1\cancel{0}}{10\cancel{0}}\ \cancel{\text{cm}} \times \frac{\text{m}}{\cancel{\text{cm}}} = 0.1\ \text{m}$

Energy example

Convert 'Mtoe' (mega tonnes of oil equivalent)

to 'joules' (J)

given $1\ \text{toe} = 42\ \text{GJ} = 42\times10^{9}\ \text{J}$

So $1\ \text{Mtoe} = 1\times10^{6}\ \text{toe} = 1\times10^{6}\ \text{toe} \times \underbrace{\left(\frac{42\times10^{9}\ \text{J}}{1\ \text{toe}}\right)}_{\times\ 1}$

$= 42\times10^{15}\ \text{J}$

$(= 42\ \text{PJ})$

1.2.4 Base and derived SI quantities and units

As mentioned above, the SI system is based on the idea that we can usefully classify and measure all scientific quantities according to **seven basic types of quantity**[2]. For each of these types we define a **base SI unit** for measuring that quantity, and other **derived** units can be derived based on these.

Table 3 summarises some common derived SI units.

[2]other systems are possible, e.g. in fundamental physics Lorentz–Heaviside units, Gaussian units and/or Planck units are also commonly used.

'Build up' from base quantities/units:

Eg.

Quantity : Area [=] Length^2 } type of thing (dimensions)

SI units : m^2 } unit for measuring

Name	Symbol	Quantity	Expressed in terms of other SI units	Expressed in terms of SI base units
hertz	Hz	frequency	1/s	s^{-1}
radian	rad	angle	m/m	dimensionless
newton	N	force, weight		$kg \cdot m \cdot s^{-2}$
pascal	Pa	pressure, stress	N/m^2	$kg \cdot m^{-1} \cdot s^{-2}$
joule	J	energy, work, heat	N·m	$kg \cdot m^2 \cdot s^{-2}$
watt	W	power, radiant flux	J/s	$kg \cdot m^2 \cdot s^{-3}$
coulomb	C	electric charge or quantity of electricity	F·V	s·A
volt	V	Voltage, electrical potential difference, electromotive force	W/A J/C	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
farad	F	electrical capacitance	C/V s/Ω	$kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$
ohm	Ω	electrical resistance, impedance, reactance	V/A	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
degree Celcius	°C	Temperature relative to 273.15 K		K
sievert	Sv	equivalent dose of ionizing radiation	J/kg	$m^2 \cdot s^{-2}$

Table 3: Common derived SI units. (From notes by Thor Besier, based on `https://en.wikipedia.org/wiki/International_System_of_Units`).

Examples of derived SI units

Quantity: pressure $= \dfrac{\text{Force}}{\text{Area}}$ (definition)

$$[=] \frac{\text{Force}}{\text{Length}^2} = \frac{\text{Mass} \times \text{Accel.}}{\text{Length}^2}$$

$$= M \times \frac{L}{T^2} \times \frac{1}{L^2} = \frac{M}{T^2 \cdot L} \Big\} \text{ dimensions}$$

only need [=] once here.

SI units: $\dfrac{\text{Force}}{\text{Area}} \; [=] \; \underbrace{\dfrac{N}{L^2} = \dfrac{kg \cdot \cancel{m} \cdot s^{-2}}{m^{\cancel{2}}}}_{\text{via table 3.}} = \dfrac{kg}{s^2 \cdot m} \Big\}$ units

or eg mass × accel.

So $\underbrace{1\ Pa}_{\text{derived SI}} = \underbrace{1\ \frac{kg}{s^2 \cdot m}}_{\text{base SI}}$ ✓

Example Problems 2: Dimensions and Units

1. What is the unit of energy expressed in base SI units?
2. Show that kinetic energy and gravitational potential energy have dimensionally consistent base SI units.
3. What are the base SI units of force?
4. (2017 Exam Q) Using base SI units, show that the units for work are dimensionally consistent with the units of gravitational potential energy.

Answers

1. $KE = \frac{1}{2}mv^2 \ [=]\ M\left(\frac{L}{T}\right)^2 = \frac{ML^2}{T^2}$

$[=]\ kg \cdot \frac{m^2}{s^2}$.

2. $GPE = mgh\ [=]\ M \cdot \left(\frac{L}{T^2}\right) \cdot L = M\frac{L^2}{T^2}$ ✓

$[=]\ kg \cdot \frac{m^2}{s^2}$

3. mass x accel $[=]\ M \times \frac{L}{T^2}\ [=]\ \frac{kg \cdot m}{s^2}$

4. $W = F \times d\ [=]\ \frac{kg \cdot m}{s^2} \times m = kg \cdot \frac{m^2}{s^2}$

End L1