

Topics :

case study  
of energy  
balances (for  
car trip) } section 5.6

What you  
should be  
able to do :

- ✓ carry out simple energy accounting
- ✓ understand & calc. simple possible efficiency improvement (eg air resistance & speed etc)

Exam Q :

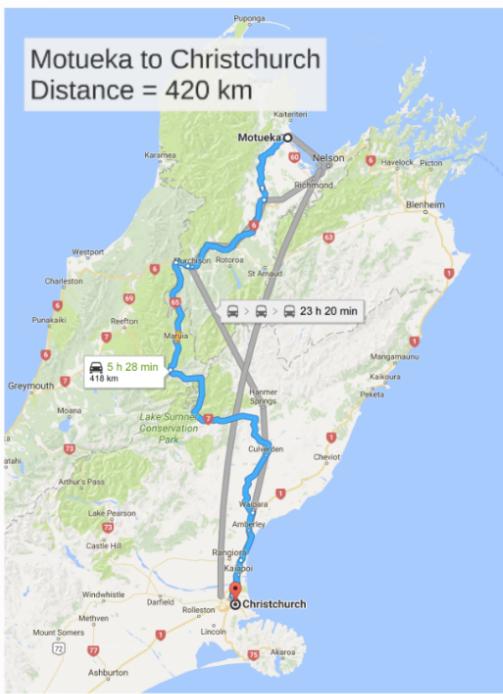
28) Simple physics provides us with ideas for reducing our transport energy cost. The work required to overcome air resistance ( $W_{air}$ ) plays the biggest role in the total energy budget of a vehicle. The NZ Government are thinking about increasing the speed limit from 100 km/h to 110 km/h. What effect would this have on the percentage increase in fuel consumption?

(2 marks)

**Answer:**

# The case study

## The trip



## The car



## The crew

4 University students  
mass = 75 kg each

## Average speed

100 km/hr

Transportation takes up a significant fraction of NZ energy consumption ( $\sim 37\%$  of consumer energy demand)

How could we reduce energy consumption?

→ let's do a case study!

$$\text{Kinetic energy} : \frac{1}{2} mn^2$$

$$\begin{aligned}\text{Total mass} &= 1400 \text{ kg} + 4475 \text{ kg} \\ &= 1700 \text{ kg}\end{aligned}$$

$$\text{Speed} = 100 \frac{\text{km}}{\text{hr}}$$

$$= 100 \frac{\text{km}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$= \frac{100 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$= 27.8 \text{ m/s}$$

$$\Rightarrow E_K = \frac{1}{2} mn^2 = \frac{1}{2} \times 1700 \times 27.8^2$$
$$\approx 657 \text{ kJ}$$

### Consumption

Petrol consumption: 17 L / 100 km.

Petrol energy density: 34.2 MJ/L

- (1) - energy 'used up' via combustion?
- (2) - energy 'available' for useful work?

(1) Total petrol consumption

$$= \frac{17 \text{ L}}{100 \text{ km}} \times \frac{420 \text{ km}}{5 \text{ journeys}}$$

$$= 71.4 \text{ L}$$

$$\times \frac{\text{Total energy consumed}}{\text{Total energy used}} \times \frac{\text{energy}}{\text{L petrol}}$$

$$= 71.4 \cancel{L} \times \frac{34.2 \text{ MJ}}{\cancel{L}}$$

$$\approx 2442 \text{ MJ}$$

(2) Assume efficiency 25%

$$\text{available} = 0.25 \times 2442 \text{ MJ} = \underline{610 \text{ MJ}}$$

Note:

used 2442 MJ

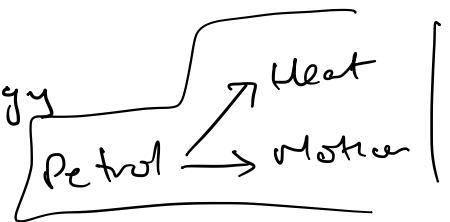
useful 610 MJ ie  $610 \times 10^6 \text{ J}$

But only have

$$E_K \approx 657 \frac{1}{2} \text{ J} \text{ ie } 657 \times 10^3 \text{ J}$$

Where's the balance??!

$\Rightarrow$  Energy 'lost' to heat energy

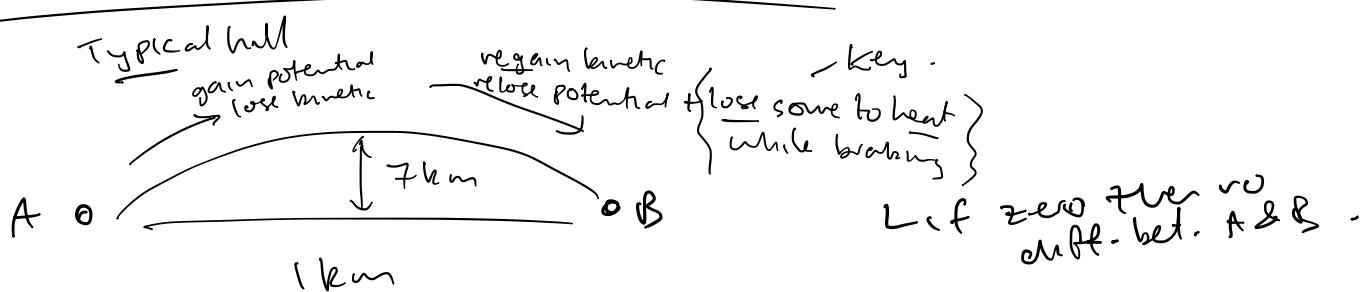


Possible: Assume climb hills & use brakes

way down

$\rightarrow$  lose, say, 50%

when braking  
down hill.



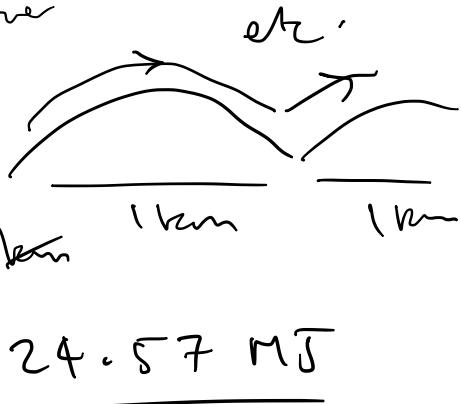
$$\text{Climb hill: } E_g = mgh = 1700 \text{ kg} \times 9.8 \text{ m/s}^2 \times 7 \text{ km} \approx 117 \text{ kJ}$$

Lose 50% to heat on way down

lose  $\sim 0.5 \times 117 \text{ kJ}$  over 7 km hill.

Worst case: hill every 1 km

$$\Rightarrow \text{lose } 0.5 \times \frac{(1700 \text{ kg} \times 420 \text{ m})}{1 \text{ km}} \approx 24.57 \text{ MJ}$$



So :

0.657 MJ to get started

(KE : once going, don't need more)

24.57 MJ to brakes

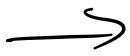
↳ friction  $\rightarrow$  heat

still << 610 MJ !

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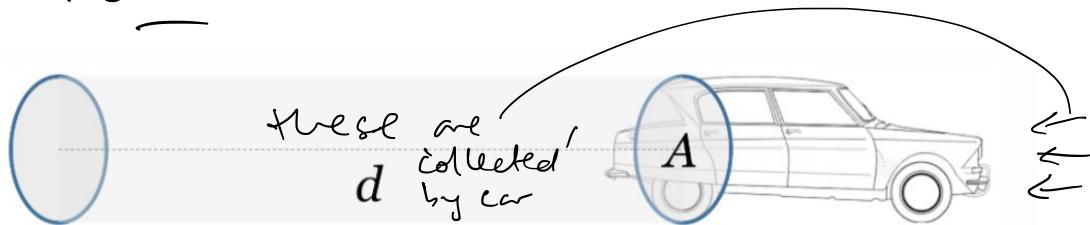
Other losses ?

- air resistance
  - tire friction
- 



## Air Resistance

Motivation:



- Car collides with molecules in tube of area  $A$  & length  $d$
- Car accelerates these molecules to velocity  $\sim \sim$

$$W_{\text{air}} = \Delta E_{\text{Kair}} \approx \frac{1}{2} m v^2$$

mass in 'tube'

$$= \rho \cdot A \cdot d$$

density  $\sim$  volume

Empirically, wel constitutive assumption

$$\boxed{W_{\text{air}} = \frac{1}{2} C_d \cdot (\rho A d) v^2}$$

drag coefficient  
(empirical, since approx.).

e.g. ↑  
'shape correction'



## Air resistance cont'd

$$\text{assume } C_d = 0.33$$

$$A = 3.2 \text{ m}^2$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$W_{\text{air}} = \frac{1}{2} C_d (A \rho) v^2$$

$$= \frac{1}{2} \times 0.33 \times (3.2 \cancel{\text{m}} \times \underbrace{420 \cancel{\text{kg}} \times 1.2 \cancel{\text{kg}}}_{d = \cancel{\text{m}}} \times \cancel{\text{m}} \times (27.8 \frac{\text{m}}{\text{s}})^2)$$

total distance travelled

$$= 206 \times 10^6 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

$$= 206 \text{ MJ}$$

What happens if increase speed from  
100 km/hr  $\rightarrow$  120 km/hr?

(reverse exam Q!)

$$\frac{W_2}{W_1} = \cancel{\frac{C_d}{C_d}} \times \cancel{\frac{A}{A}} \times \frac{v_2^2}{v_1^2} = \boxed{\frac{v_2^2}{v_1^2}} \quad \leftarrow \text{Key}$$

$$= \frac{120^2}{100^2} = 1.44.$$

$\Rightarrow$  energy lost to air resistance  $\uparrow 44\%$

$$W_2 = 1.44 \times W_1 = 1.44 \times 206 \text{ MJ}$$

$$= 297 \text{ MJ}$$

# Rolling resistance?

Car tyres on road

constitutive equation:  $\rightarrow$  in Newtons.

$$F_r = c \cdot \text{Weight}$$

$\uparrow$   
coeff. of

rolling resistance (dimensionless factor)

$c \uparrow$  with speed  
 $c \downarrow$  with tyre pressure  $\leftarrow$  keep tyre pressure  $\uparrow$ !

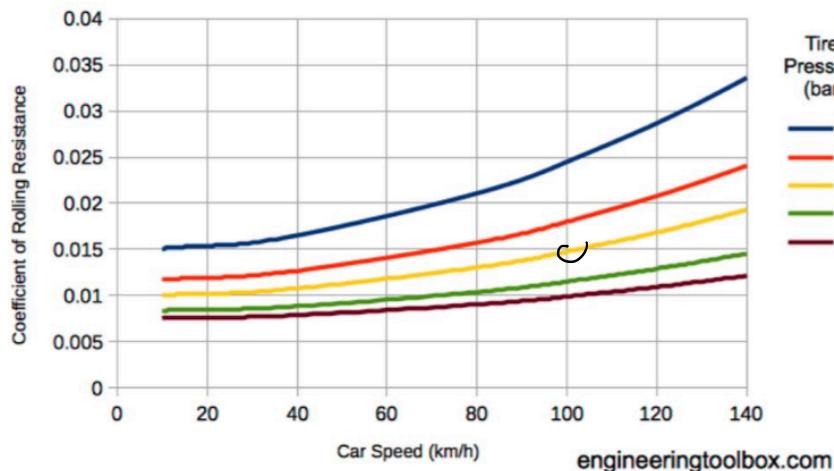


Figure 8: Coefficient of rolling resistance for car tyres.  $1 \text{ bar} = 10^5 \text{ Pa}$   
 [engineeringtoolbox.com]

$$F_r = c \cdot \text{Weight}$$

$$= c \times 1700 \text{ kg} \times 9.81 \text{ m/s}^2$$

$$\text{assume } c = 0.015$$

$$\Rightarrow F_r = 250 \text{ N}$$

$$W = F \times d = 250 \times 420 \times 10^3 = \underline{\underline{105 \text{ MJ}}}$$

## Final balance

0.6 MJ	Kinetic energy
24.6 MJ	Braking down hills
20.6 MJ	Air resistance
10.5 MJ	Rolling resistance

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336.2 MJ Total accounted for.  
(mechanical forms)

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=> Much closer to 610 MJ  
based on 25% eff.

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what is 'true' efficiency if we  
have accounted for everything?

=> Total chemical energy from  
fuel (raw) :

$$2442 \text{ MJ}$$

$$\Rightarrow \text{Efficiency of engine} : \frac{336.2}{\text{available from fuel}} \times 100 = 14\%$$

useful to drive

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