PROBABILIDADE E PROCESSOS ESTOCÁSTICOS (CKP7366)

Prof. João Paulo Pordeus Gomes

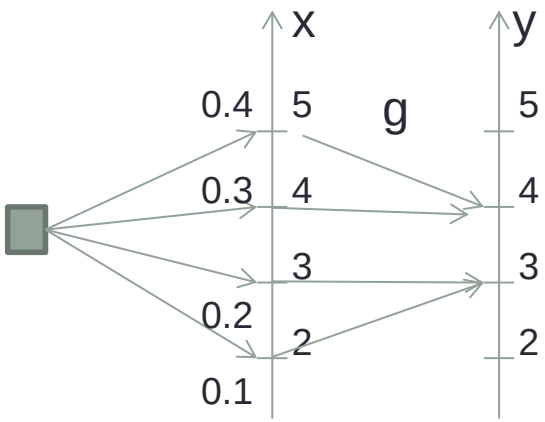
DISTRIBUIÇÕES DE FUNÇÕES DE V.A.

Distribuições de funções de v.a.

- Dada a distribuição de X, encontre a distribuição de Y=g(X)
  - Caso discreto
  - Caso contínuo
  - Caso geral
  - Funções lineares
- Dada a distribuição conjunta de X e Y, encontre a distribuição de Z=g(X,Y)

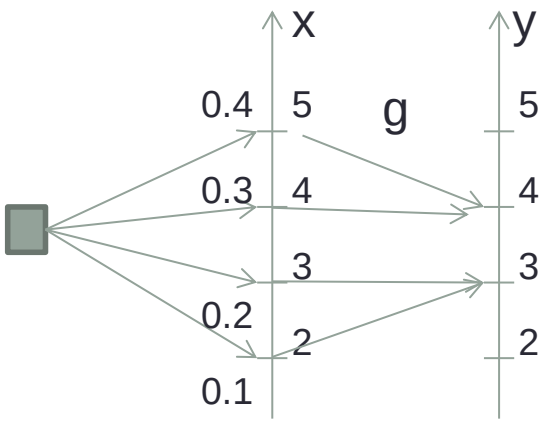
Caso Discreto

- Y=g(X)



Caso Discreto

- Y=g(X)
- $p_Y(4) = P(Y = 4) =$



Caso Discreto

- Y=g(X)
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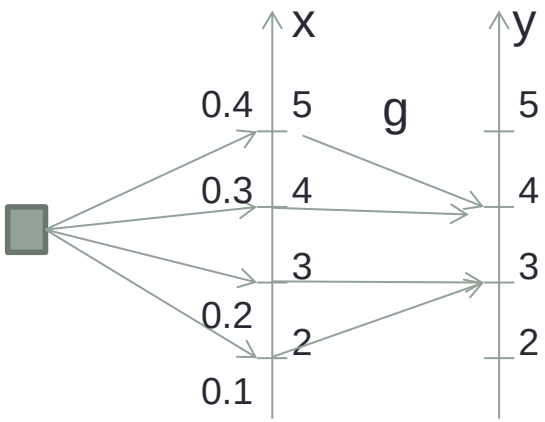
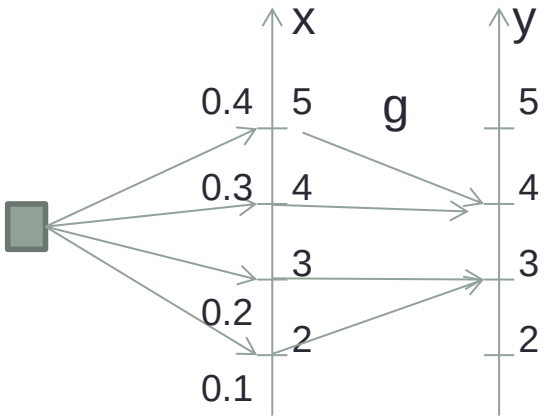


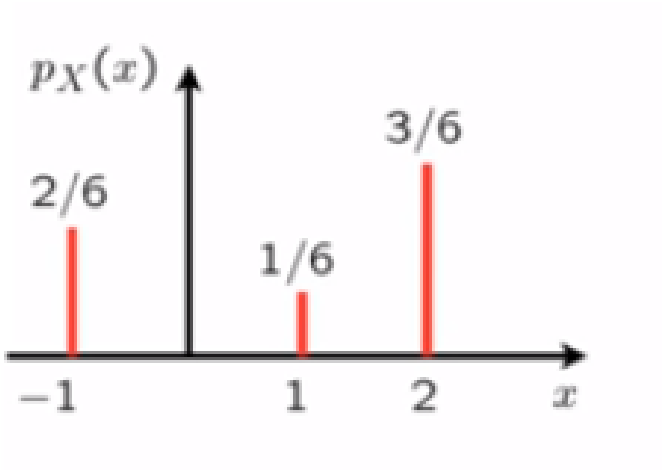
Caso Discreto

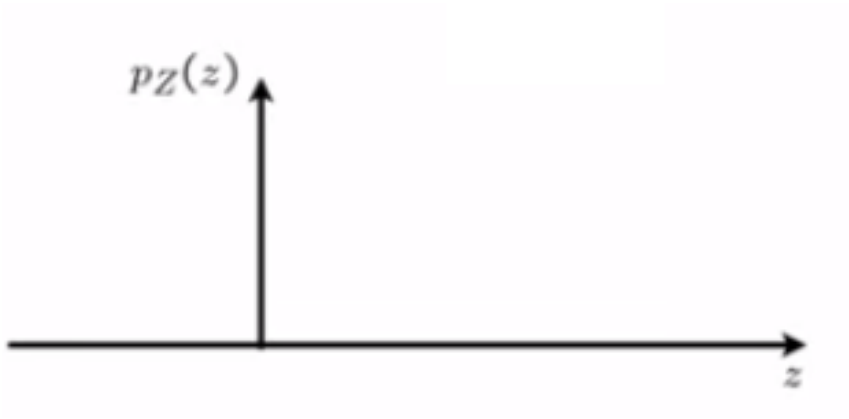
- Y=g(X)
- $p_Y(4) = P(Y = 4) = p_X(4) + p_X(5)$
- $p_Y(y) = \sum_{x:g(x)=y} p_X(x)$



Função Linear

- Z = 2X





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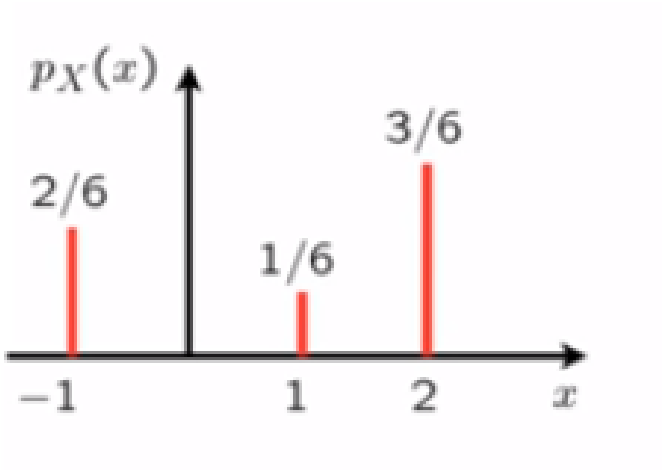

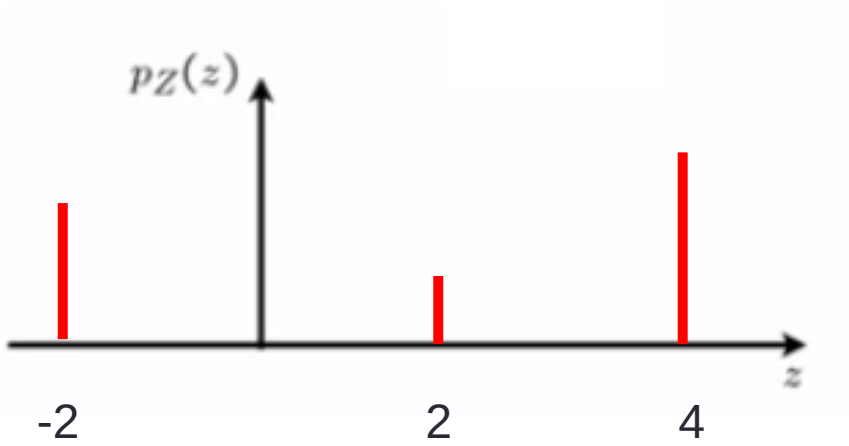

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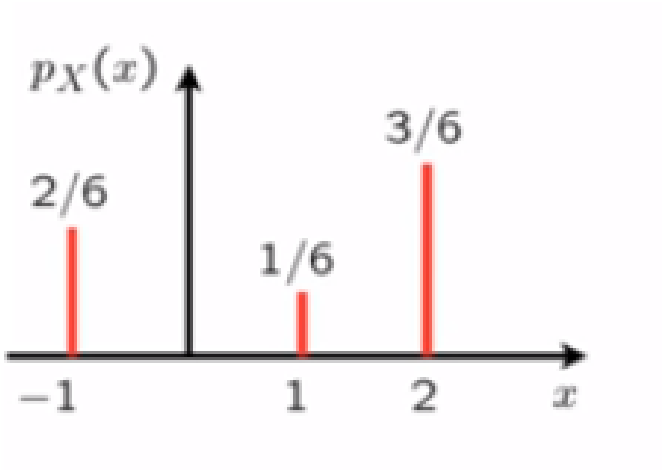

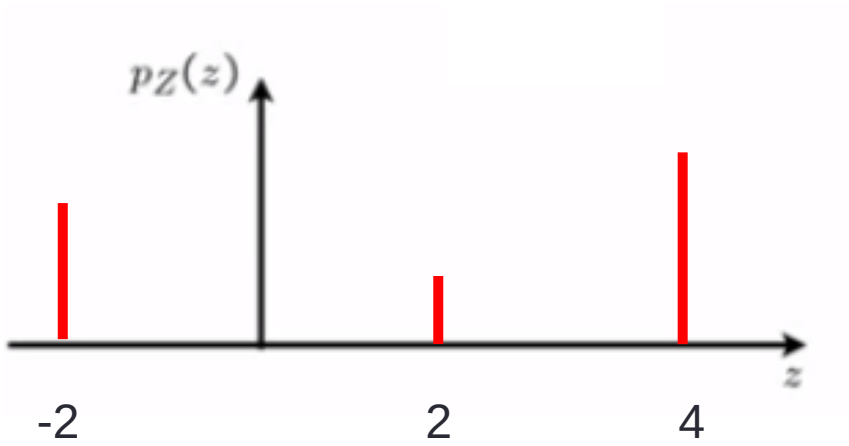

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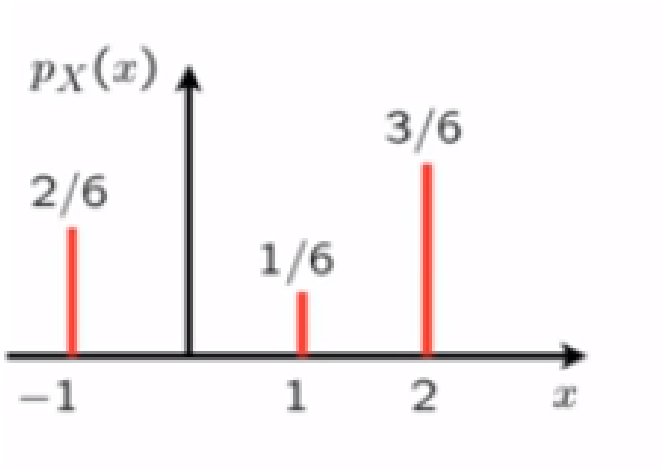

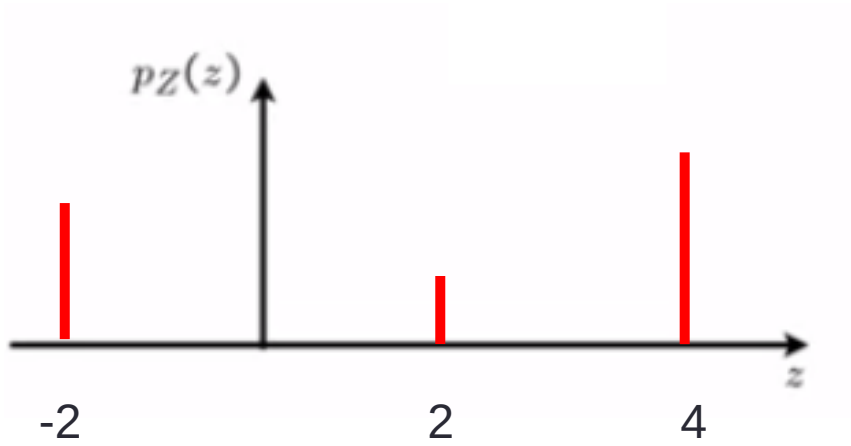

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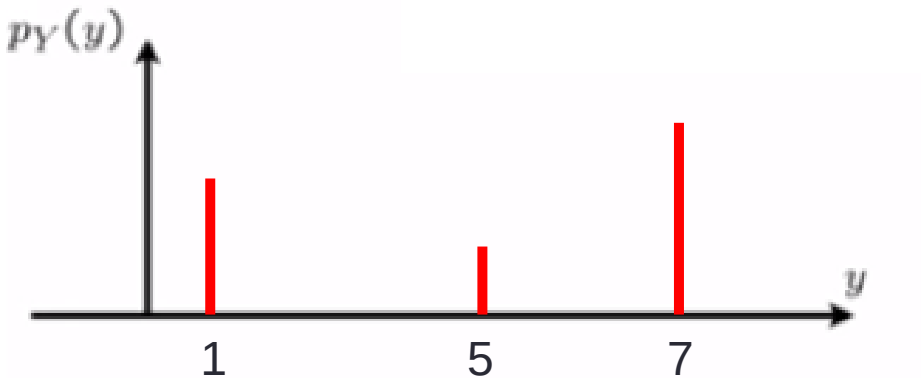

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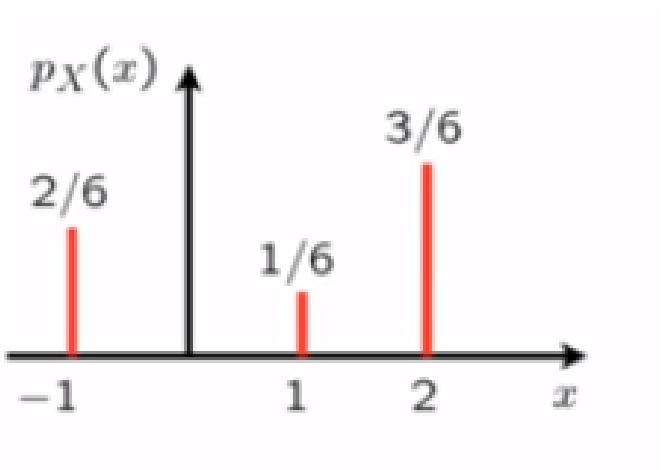

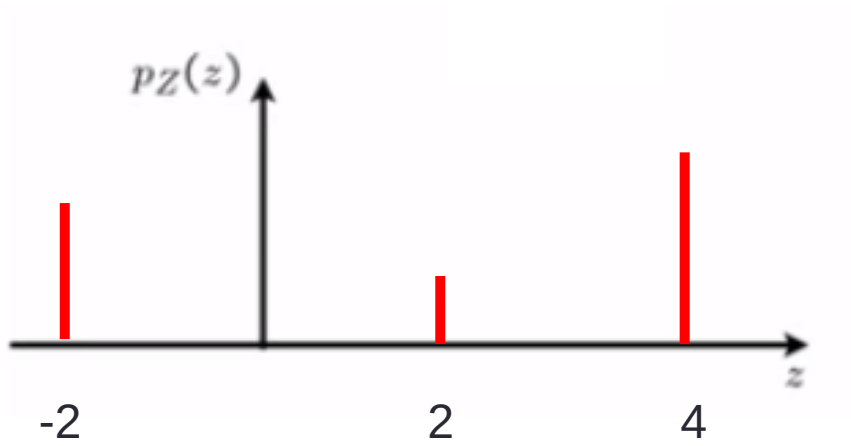

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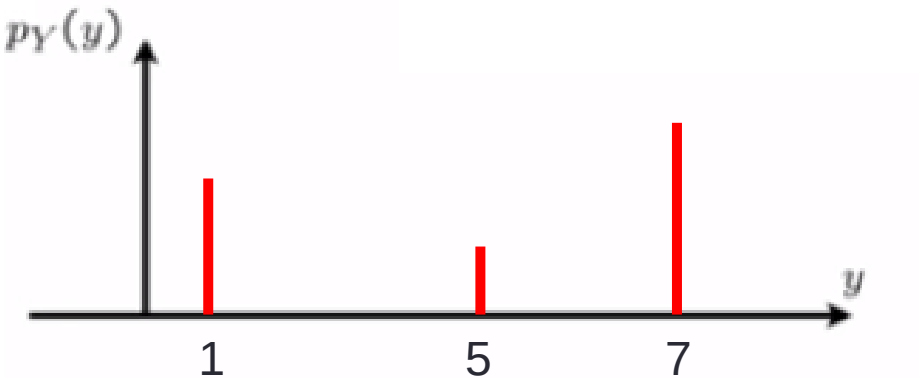

$$p_Y(y) = P(Y = y) = P(2X + 3 = y)$$

Função Linear

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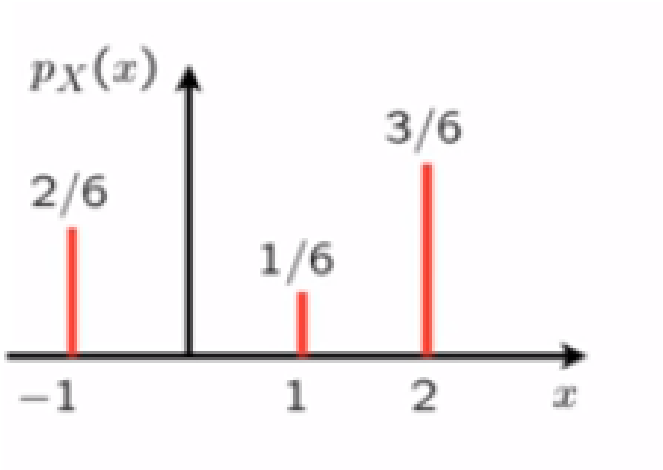

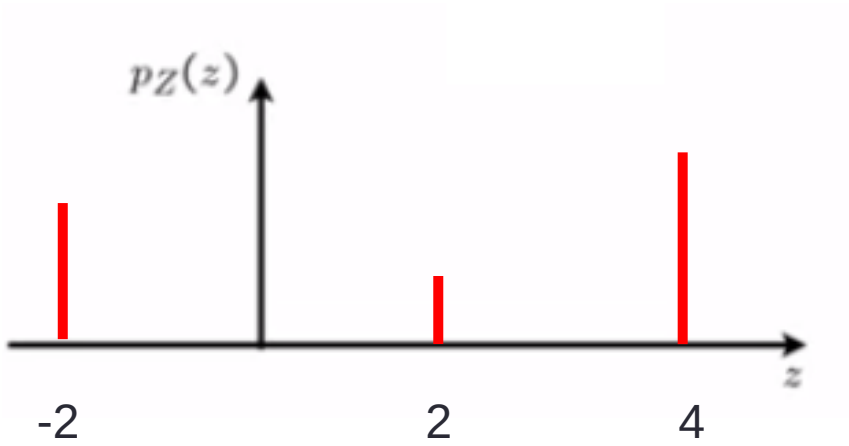

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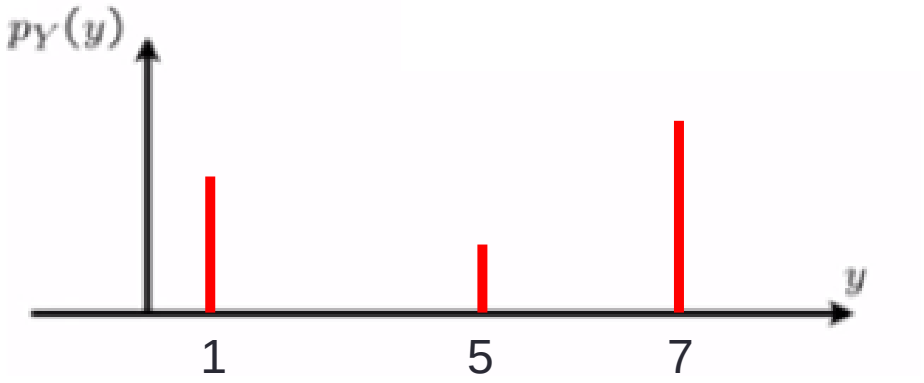


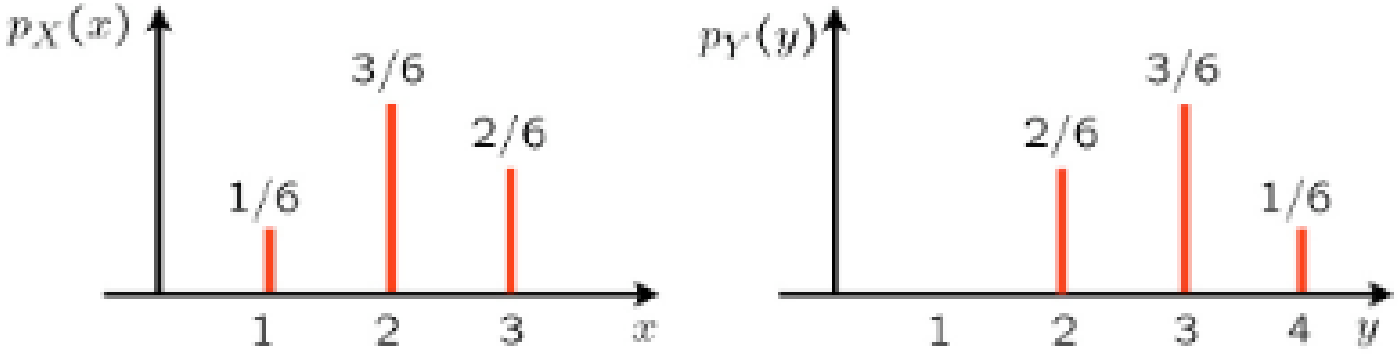
$$p_Y(y) = P(Y = y) = P(2X + 3 = y)$$

$$P\left(X = \frac{y-3}{2}\right) = p_X\left(\frac{y-3}{2}\right)$$

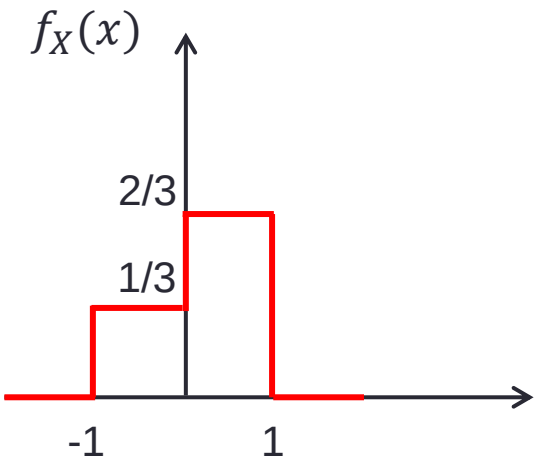
$$Y = aX + b \qquad p_Y(y) = p_X\left(\frac{y-b}{a}\right)$$

Exercício

- As variáveis aleatórias X e Y obedecem uma relação linear do tipo Y = aX + b. Dadas as seguintes distribuições, encontre os valores de a e b.



Caso contínuo (função linear)



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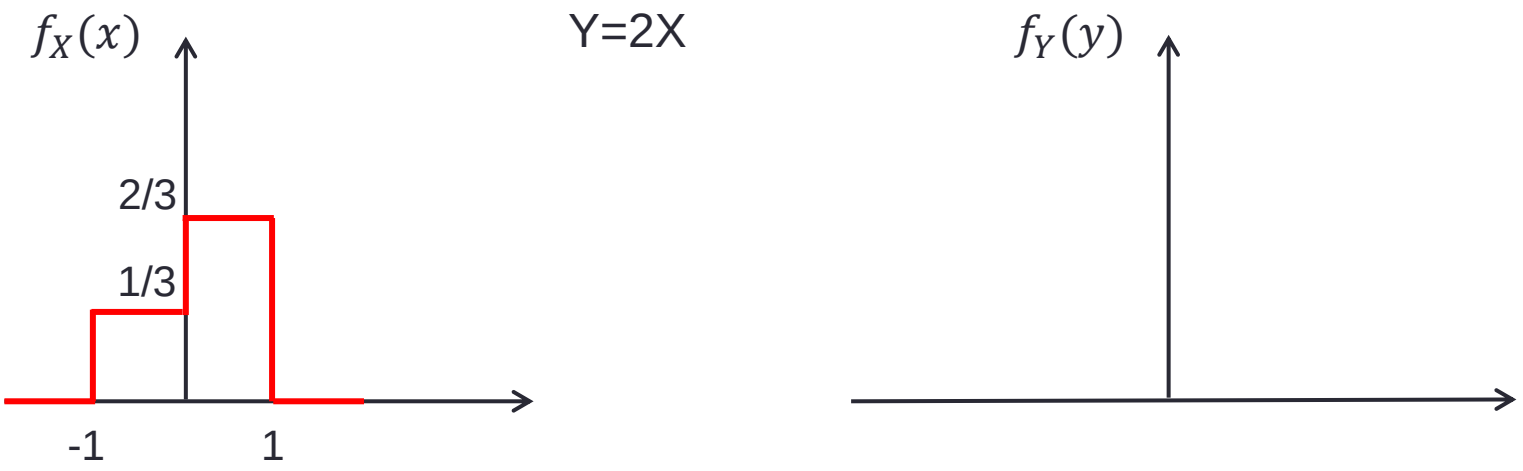

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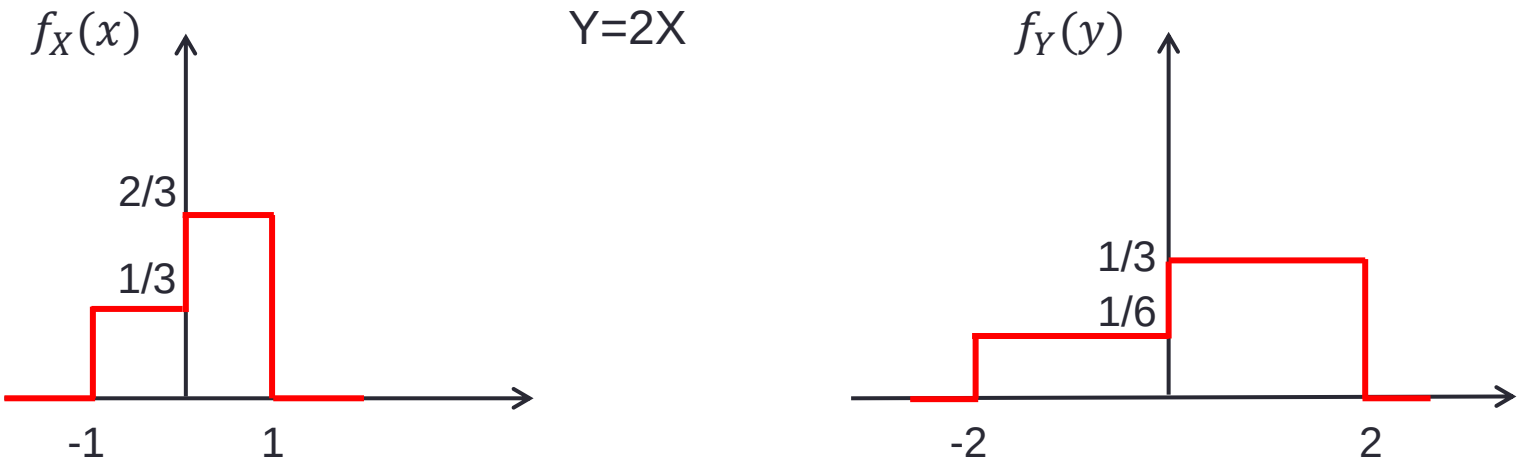

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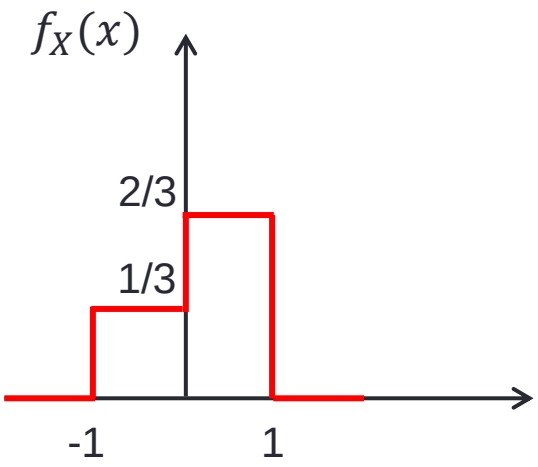

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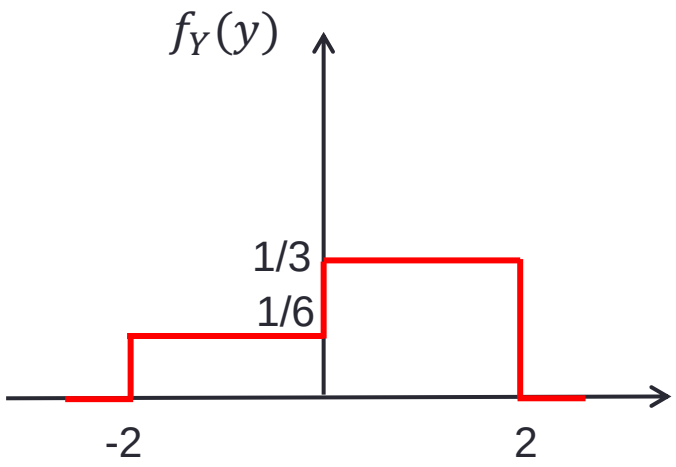

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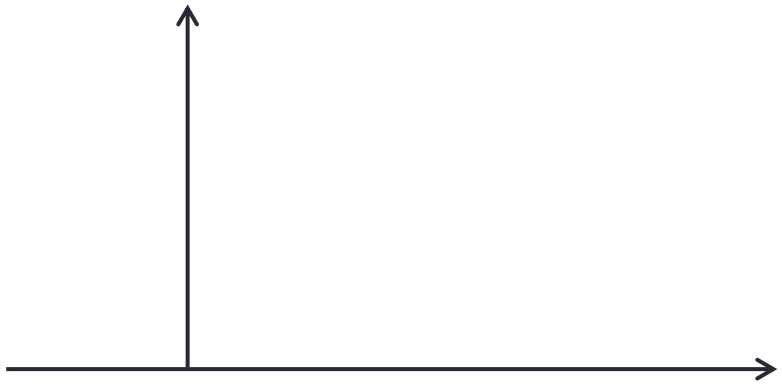

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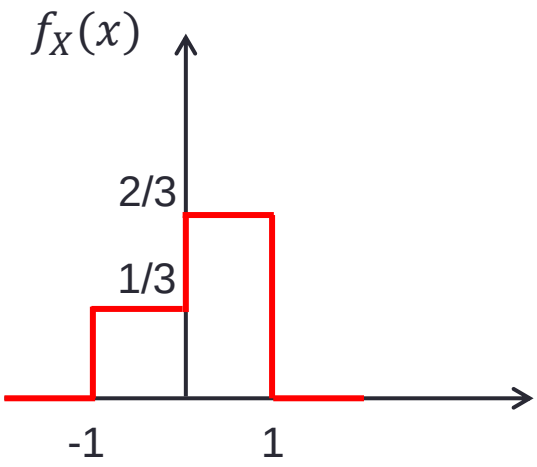

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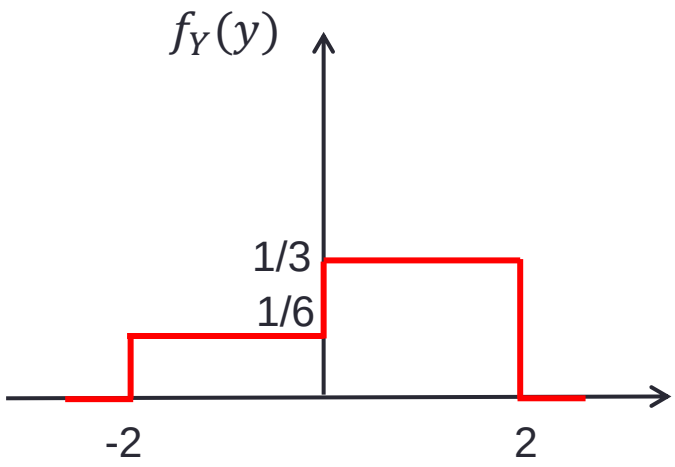

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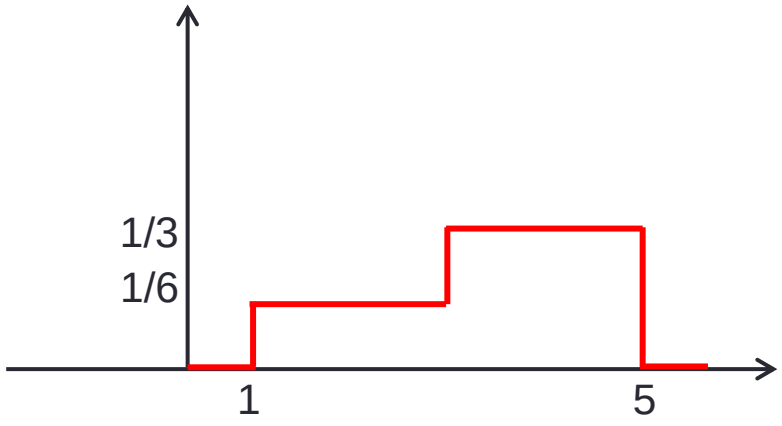

Função linear de uma v.a.

- $Y = aX + b$
- $a > 0$

Função linear de uma v.a.

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- $F_Y(y) = P(Y \leq y) = P(aX + b \leq y)$
- $P\left(X \leq \frac{y-b}{a}\right) = F_X(\frac{y-b}{a})$

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$$\frac{dF_X(x)}{dx} = f_X(x)$$

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- $F_Y(y) = F_X(\frac{y-b}{a})$
- $f_Y(y) = f_X(\frac{y-b}{a})\frac{1}{a}$

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- $a < 0$
- $P\left(X \geq \frac{y-b}{a}\right) = 1 - F_X(\frac{y-b}{a})$
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$$f_Y(y) = f_X(\frac{y-b}{a})\frac{1}{|a|}$$

$$p_Y(y) = p_X(\frac{y-b}{a})$$

Exercício

- Seja X uma v.a. exponencial e seja Y uma v.a. em que Y = aX+b. Determine para que valores de a e b, Y ainda será exponencial.

Função linear de uma v.a Normal

- $X \sim N(\mu, \sigma^2)$
- $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
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Média Variância

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Média Variância

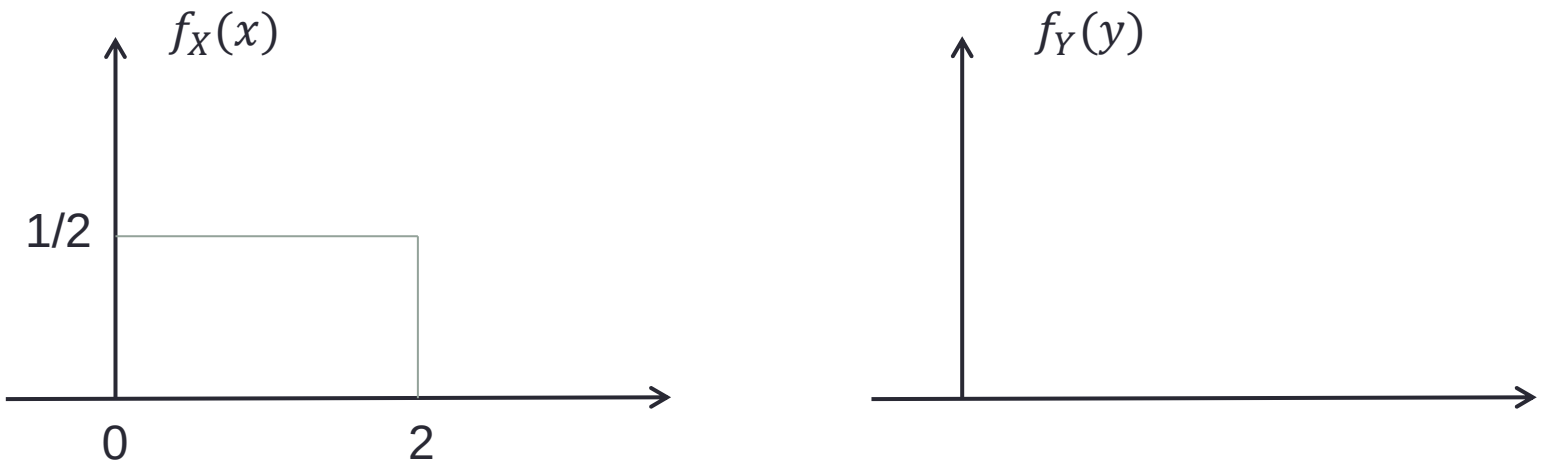
$se \quad X \sim N(\mu, \sigma^2) \quad então \quad aX + b \sim N(a\mu + b, a^2, \sigma^2)$

Uma função qualquer Y=g(X) de uma v.a.

- Processo com duas etapas
  - Encontrar a CDF de Y
  - Derivar Y para encontrar a PDF

Exemplo

- Seja $Y = X^3$; onde X é uma v.a. U(0,2)



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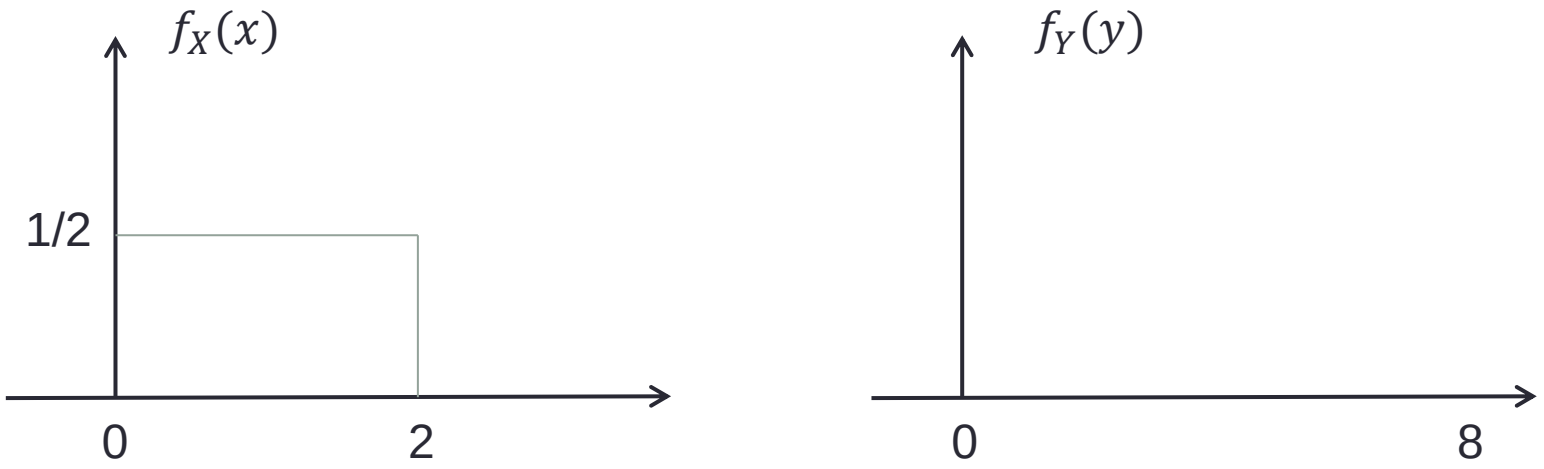

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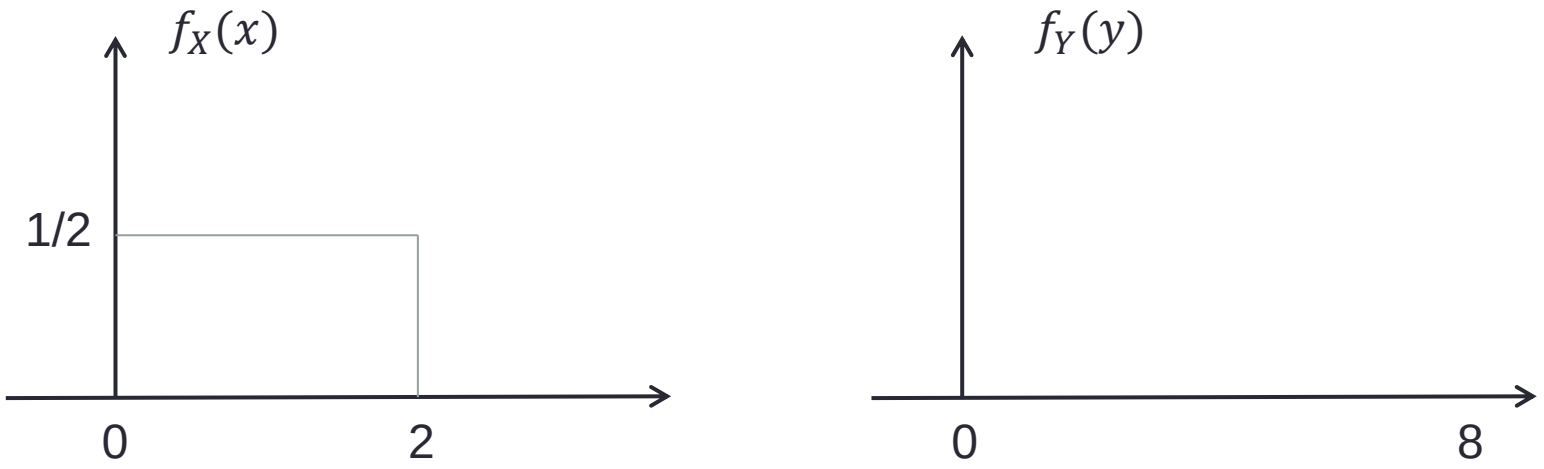

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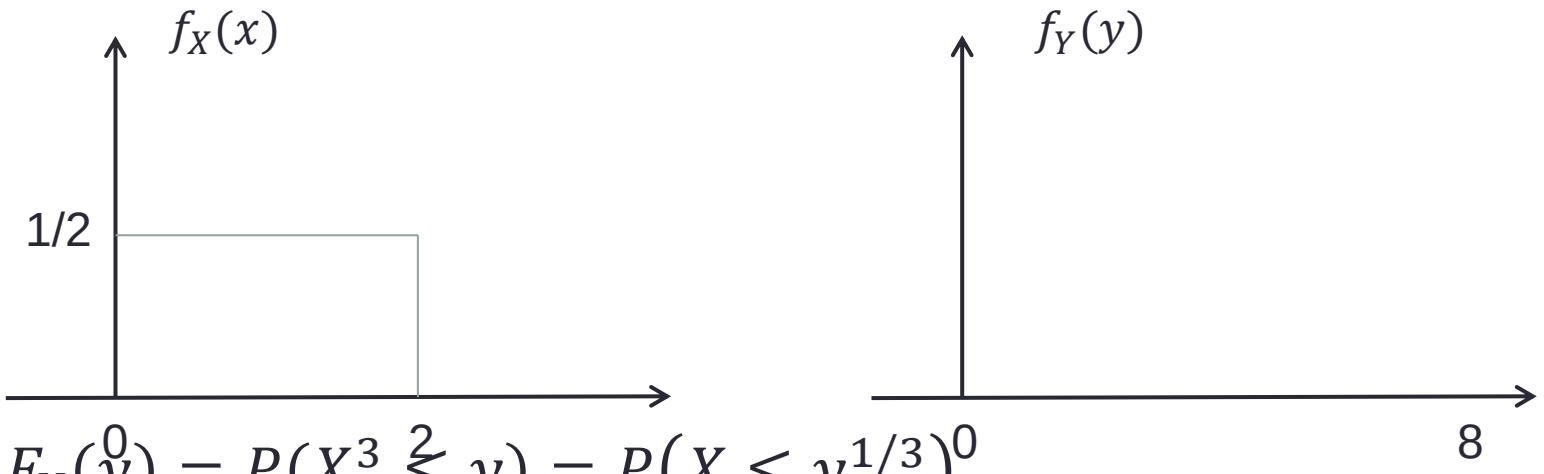

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- $F_Y(y) = F_X\left(y^{\frac{1}{3}}\right) = \frac{1}{2}y^{\frac{1}{3}}$
- $f_Y(y) = \frac{dF_Y}{dy}(y) = \frac{1}{6}y^{-2/3}$

Exercício

- Na academia você ajusta a esteira para uma velocidade entre 5 e 10Km/h de acordo com a distribuição U(5,10). Encontre a PDF do tempo gasto para percorrer 10Km.

Exercício

- Na academia você ajusta a esteira para uma velocidade entre 5 e 10Km/h de acordo com a distribuição U(5,10). Encontre a PDF do tempo gasto para percorrer 10Km.
- $f_Y(y) = \frac{2}{y^2}$

DÚVIDAS?