PROBABILIDADE E PROCESSOS ESTOCÁSTICOS (CKP7366)

Prof. João Paulo Pordeus Gomes

PROBABILIDADE CONDICIONAL E REGRA DE BAYES

Probabilidade Condicional

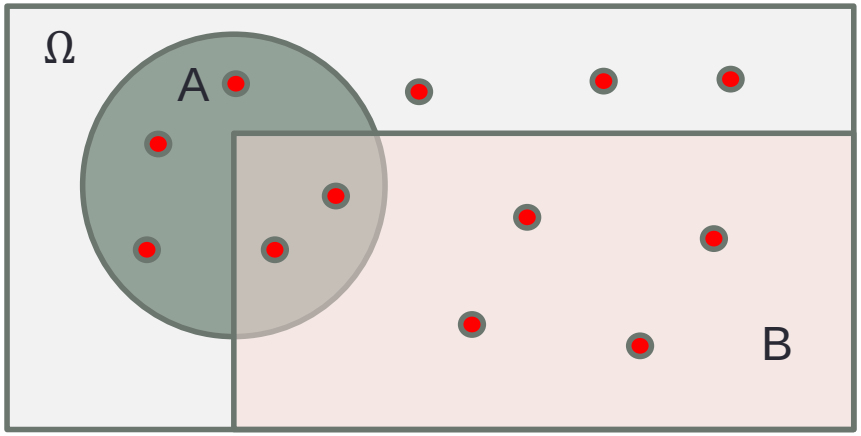
- Conhecimento sobre um evento leva a uma revisão da sua crença sobre a probabilidade de um novo evento
  - Exemplo
    - Probabilidade de uma pessoa ter mais de 25 anos

Probabilidade Condicional e Regra de Bayes

- Probabilidade Condicional
- Regras derivadas do conceito de PC
  - Regra da multiplicação
  - Teorema da probabilidade total
  - Regra de Bayes

Idéia Central

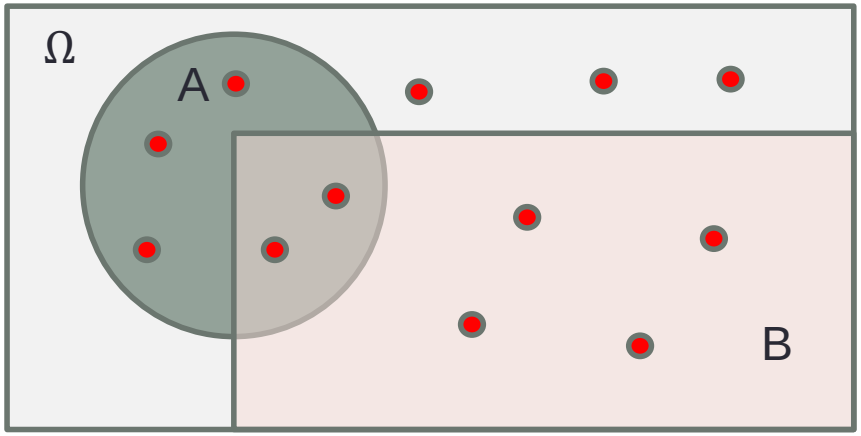
Usar nova informação para revisar a sua crença



P(A) =
P(B) =

Idéia Central

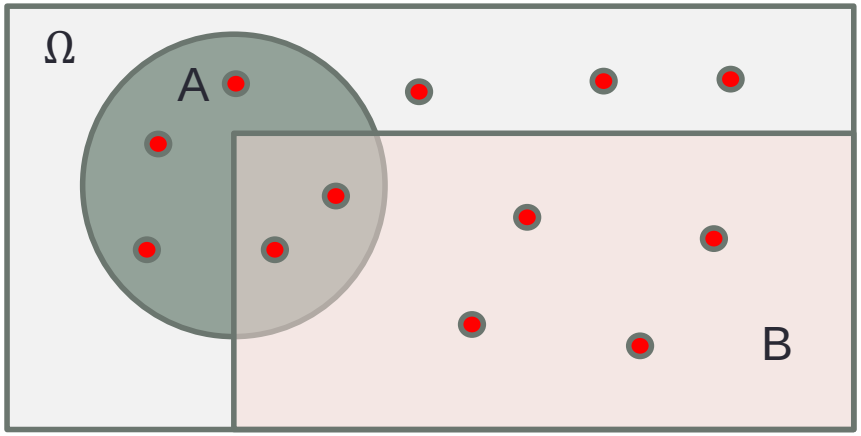
Usar nova informação para revisar a sua crença



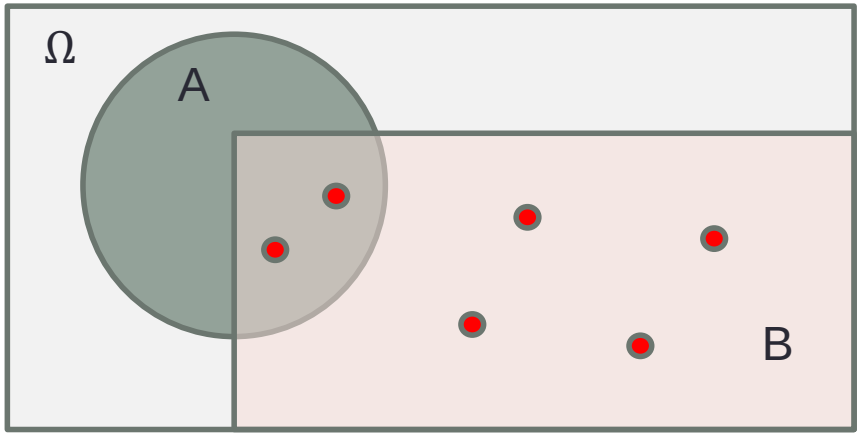
P(A) = 5/12
P(B) = 6/12

Idéia Central

Usar nova informação para revisar a sua crença



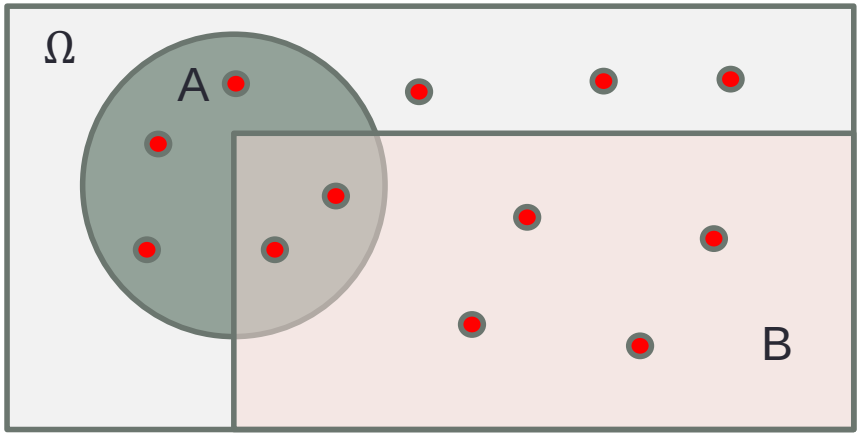
P(A) = 5/12
P(B) = 6/12

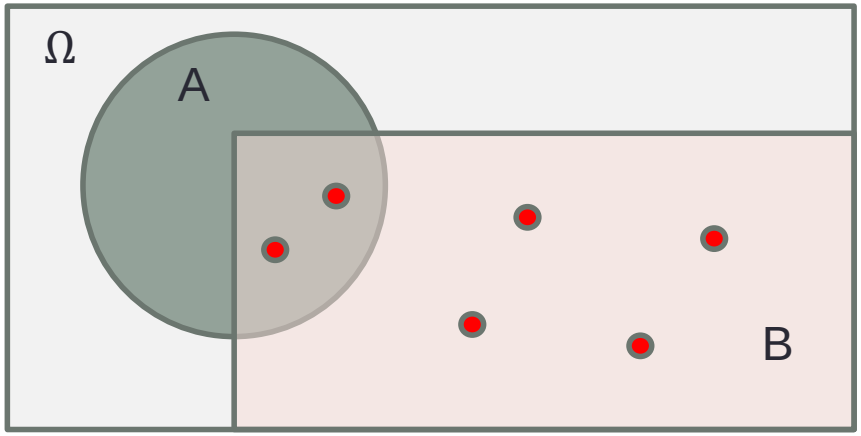


P(A) = ?
P(B) = ?

Idéia Central

Usar nova informação para revisar a sua crença



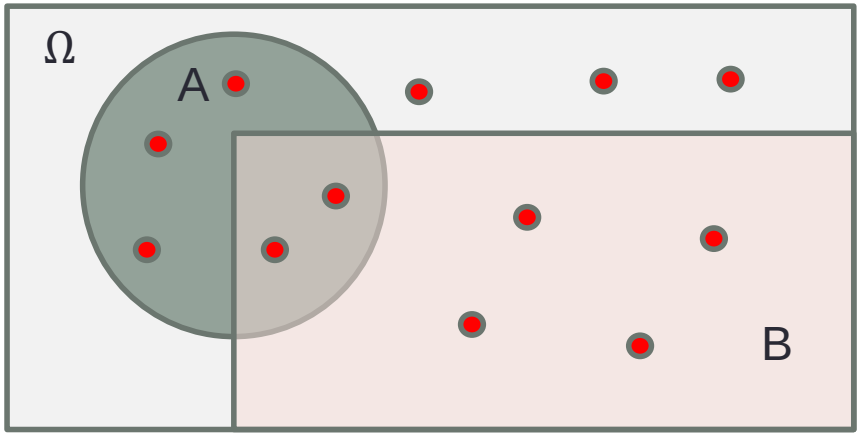


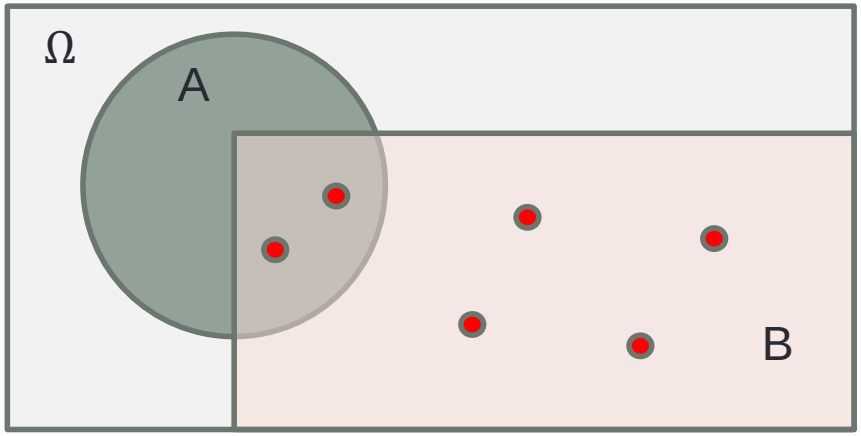
P(A) = 5/12
P(B) = 6/12

P(A|B) = ?
P(B|B) = ?

Idéia Central

Usar nova informação para revisar a sua crença

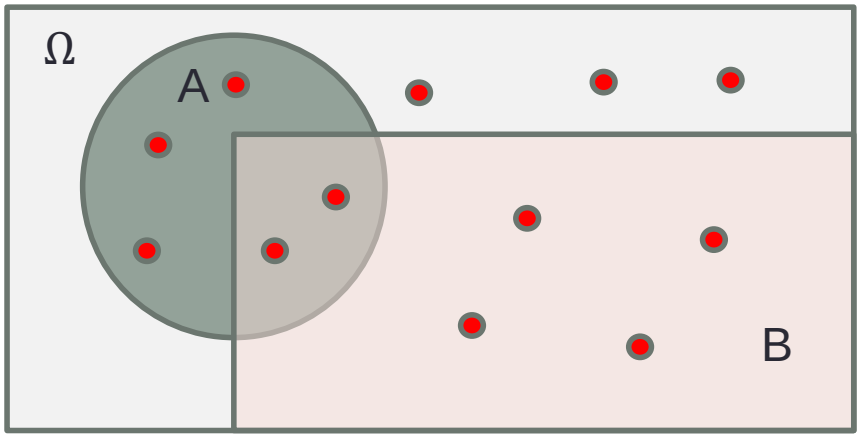


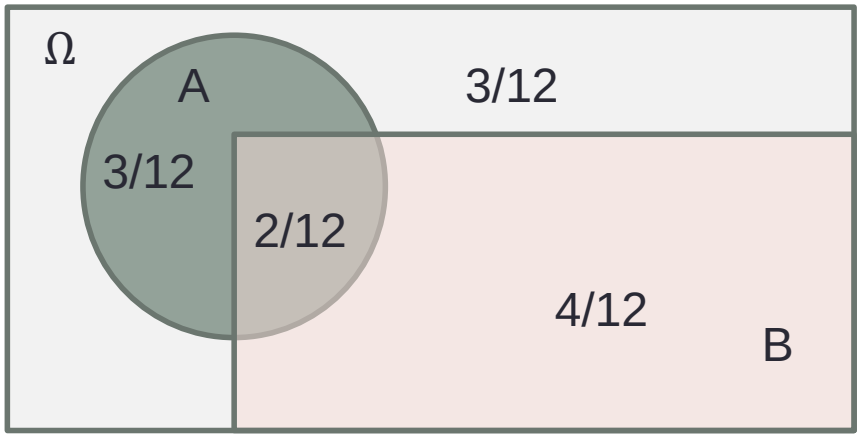


P(A) = 5/12
P(B) = 6/12

P(A|B) = 2/6
P(B|B) = 1

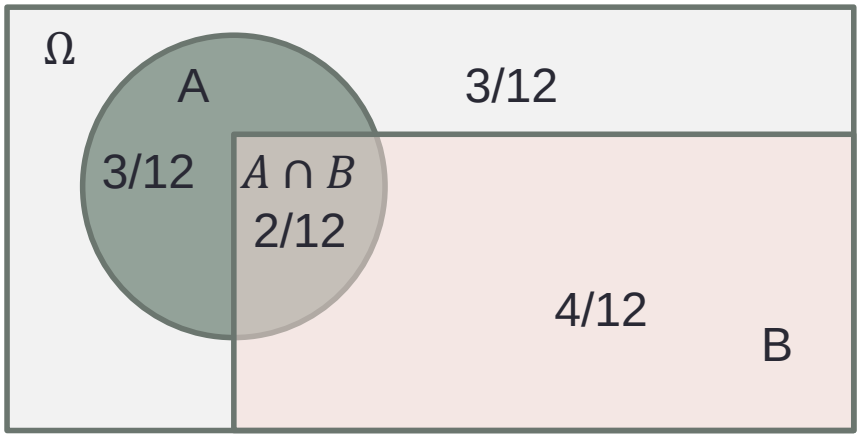
Idéia Central





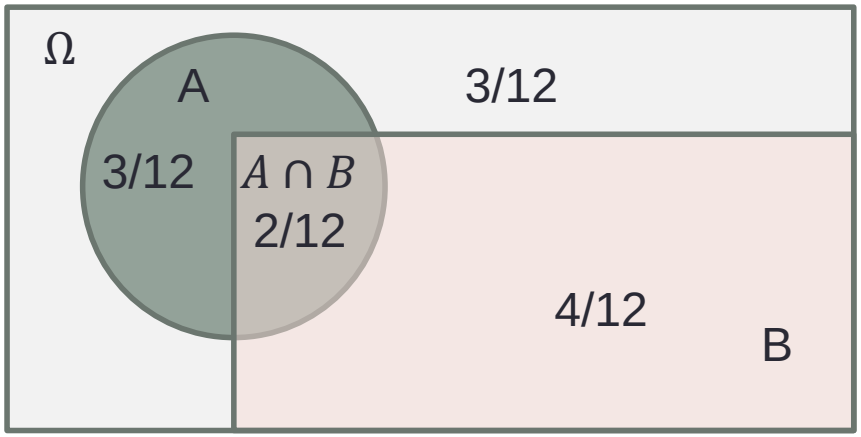
Probabilidade Condicional

- $P(A|B) =$

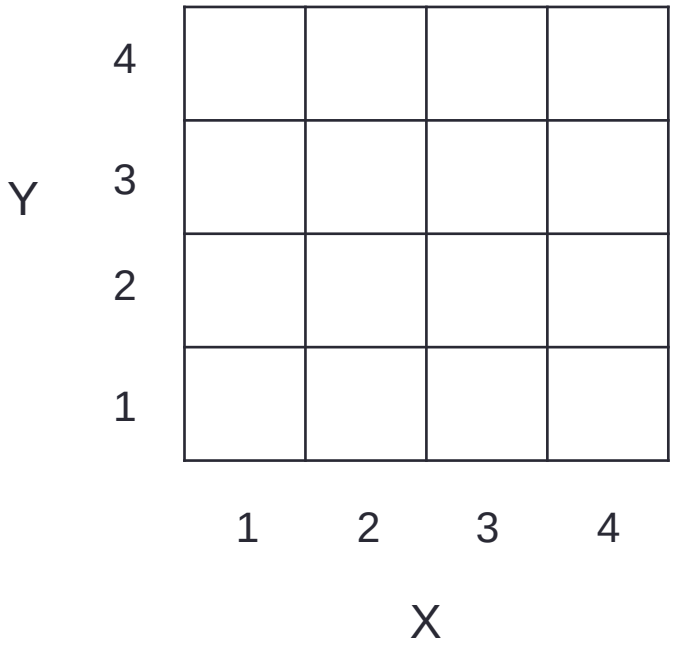


Probabilidade Condicional

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

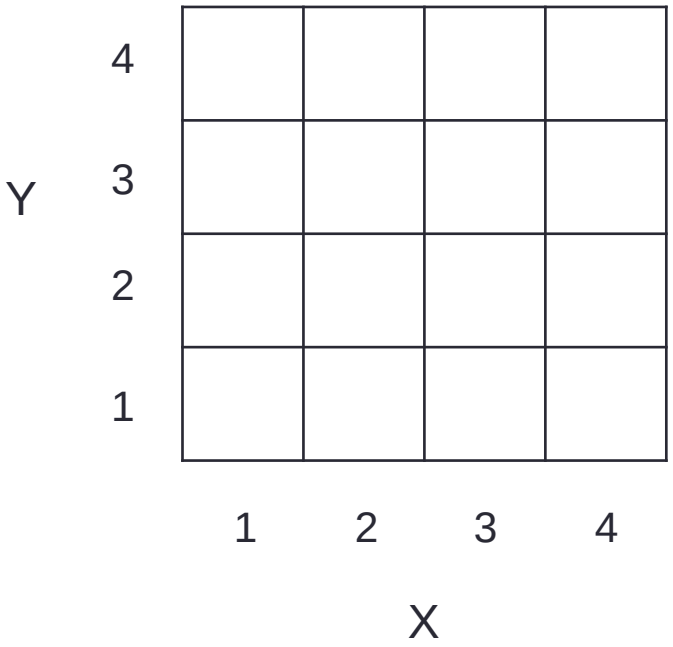


Exemplo



- *Seja B um evento : min(X,Y)=2*

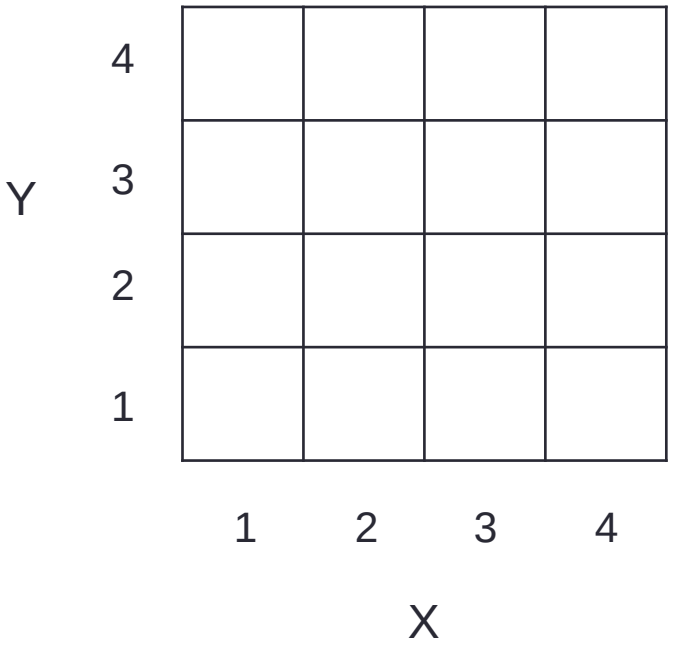
Exemplo



- *Seja B um evento : min(X,Y)=2*
- *Seja M = max(X,Y)*

- $P(M = 3|B) =$

Exemplo



- *Seja B um evento : min(X,Y)=2*
- *Seja M = max(X,Y)*

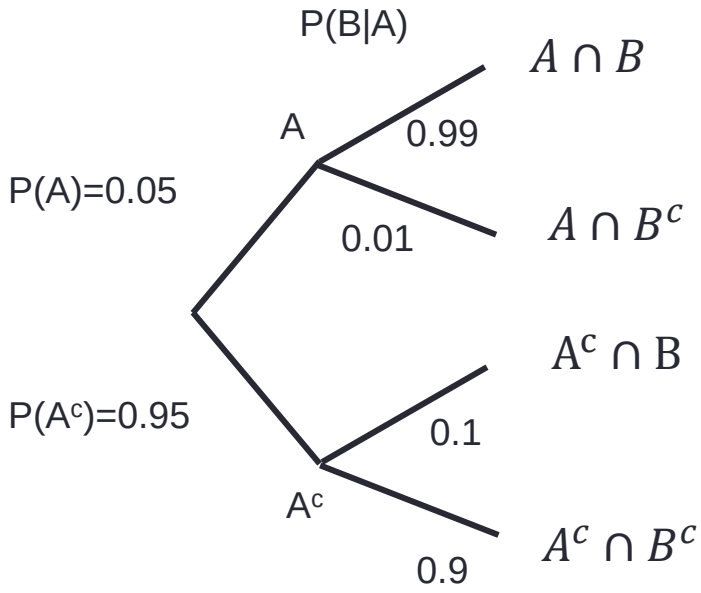
- $P(M = 3|B) = \frac{2}{5}$

Probabilidade condicional e os axiomas

- $P(A|B) \geq 0$
- $P(\Omega|B) = 1$
- $P(B|B) = 1$
- $P(A \cup C|B) = P(A|B) + P(C|B)$

Exemplo

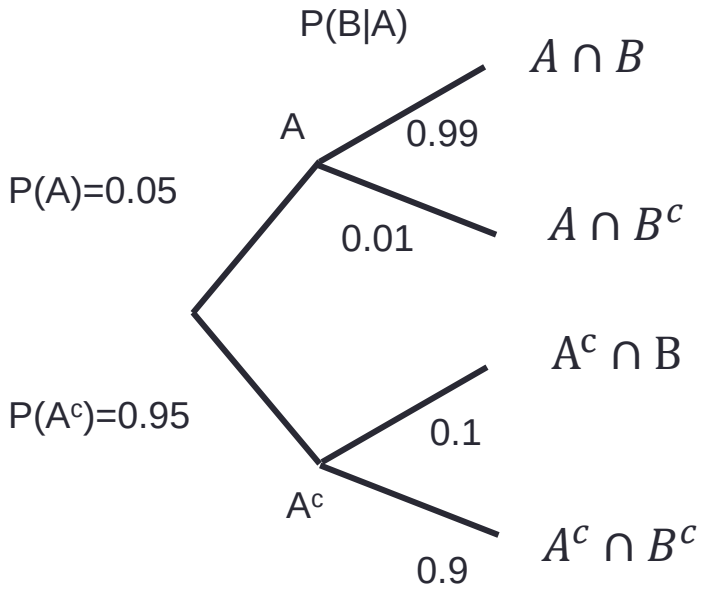
- Evento A: Avião voando
- Evento B: Algo detectado



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

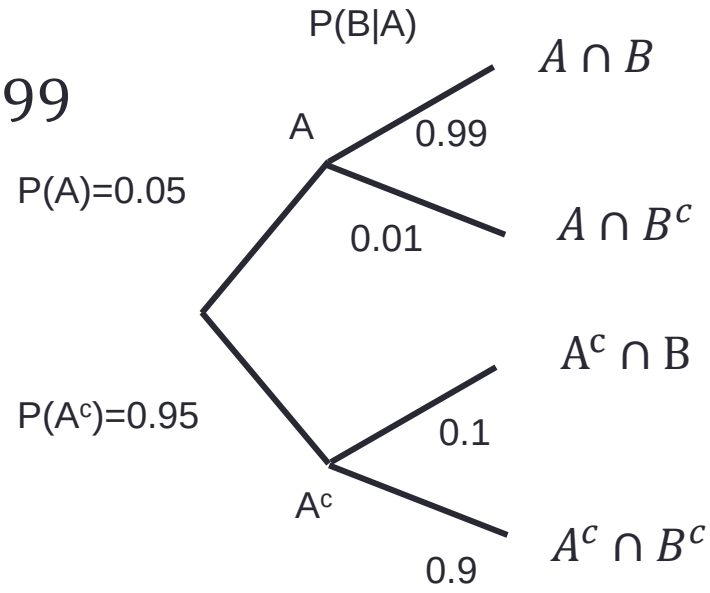
- $P(A \cap B) =$



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

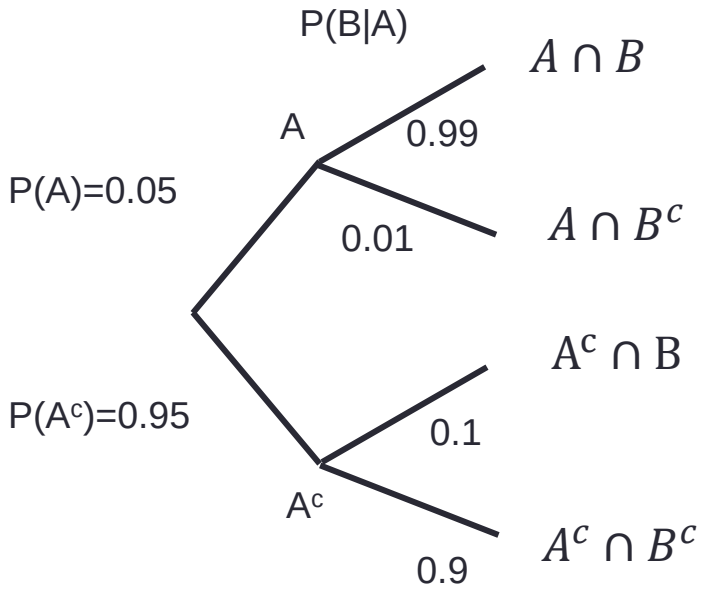
- $P(A \cap B) = 0.05 \times 0.99$



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

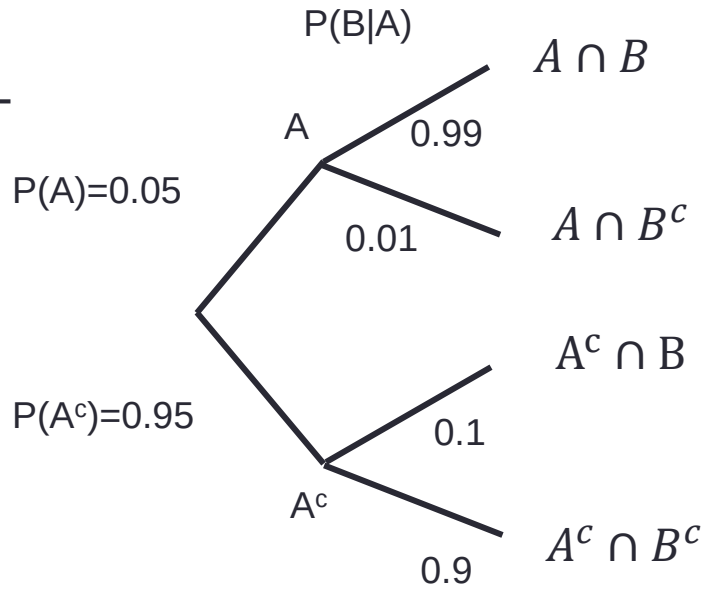
- $P(B) =$



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

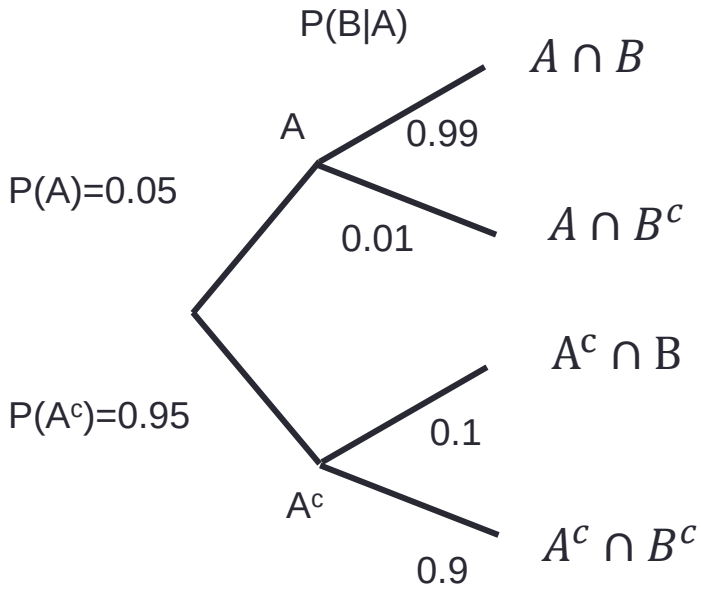
- $P(B) = 0.05 \times 0.99 + 0.95 \times 0.1 = 0.1445$



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

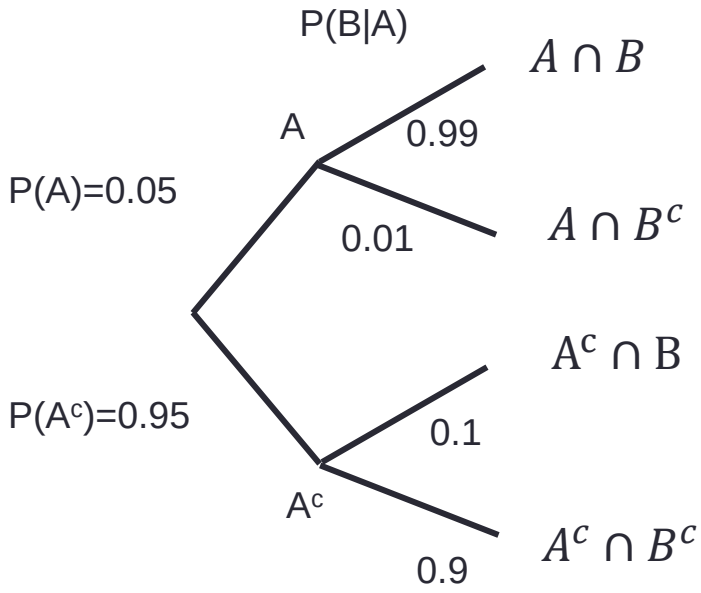
- $P(A|B) =$



Exemplo

- Evento A: Avião voando
- Evento B: Algo detectado

- $P(A|B) = 0.34$



Regra da multiplicação

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$

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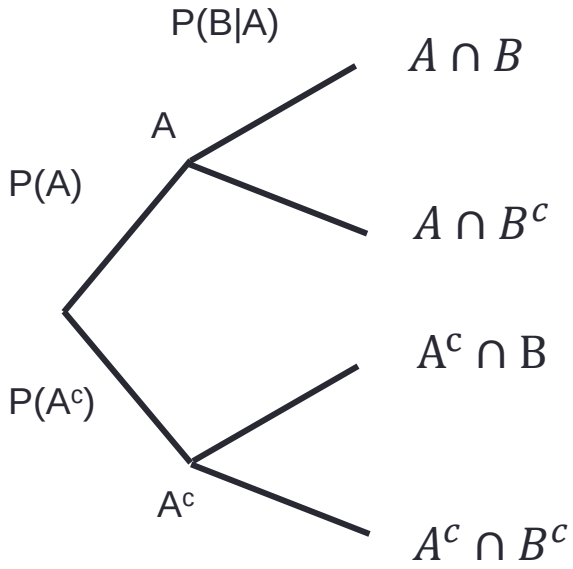
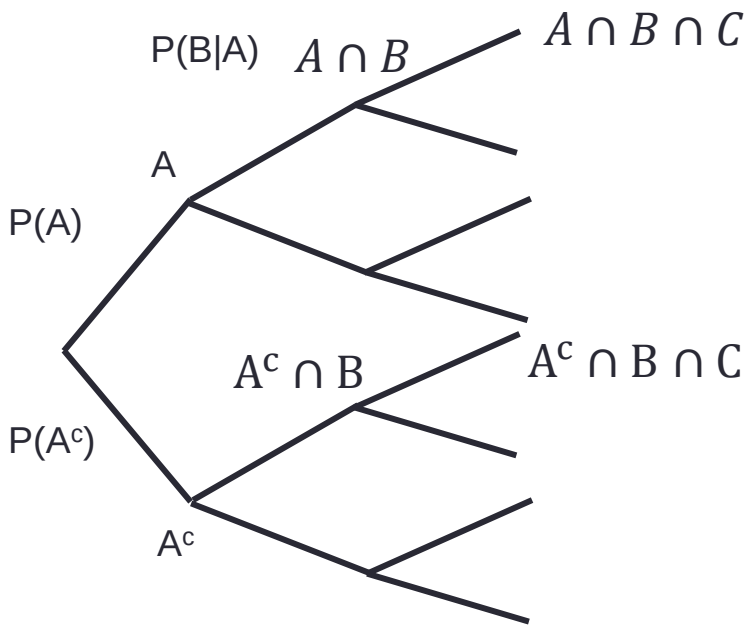


Regra da multiplicação

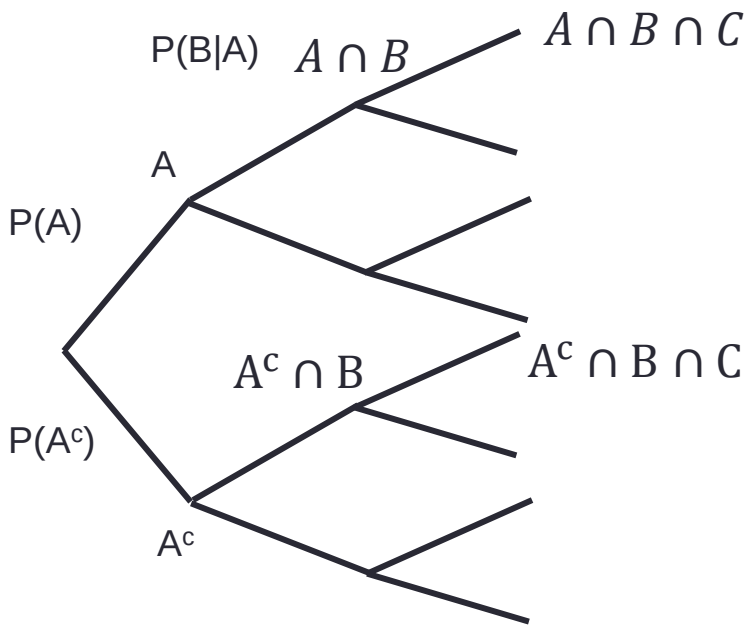
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$



Regra da multiplicação

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$

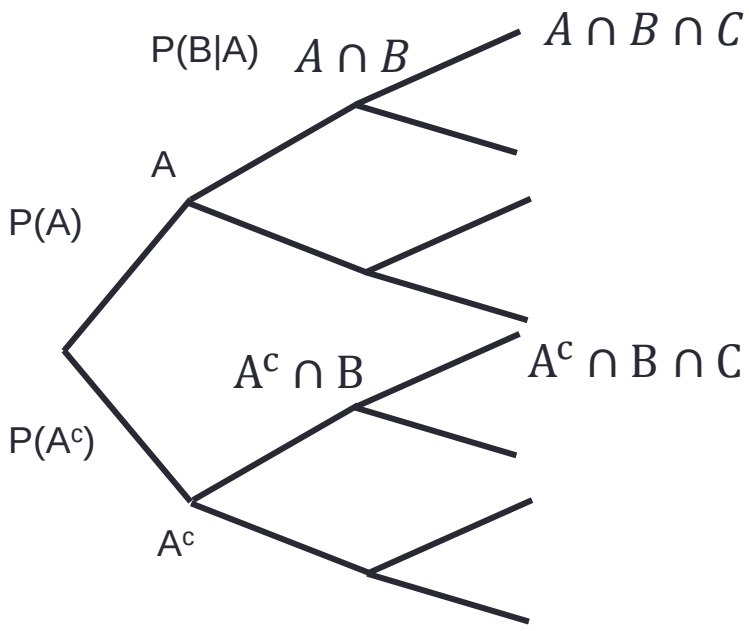
- $P(\mathrm{A^c} \cap B \cap \mathrm{C^c}) =$



Regra da multiplicação

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$

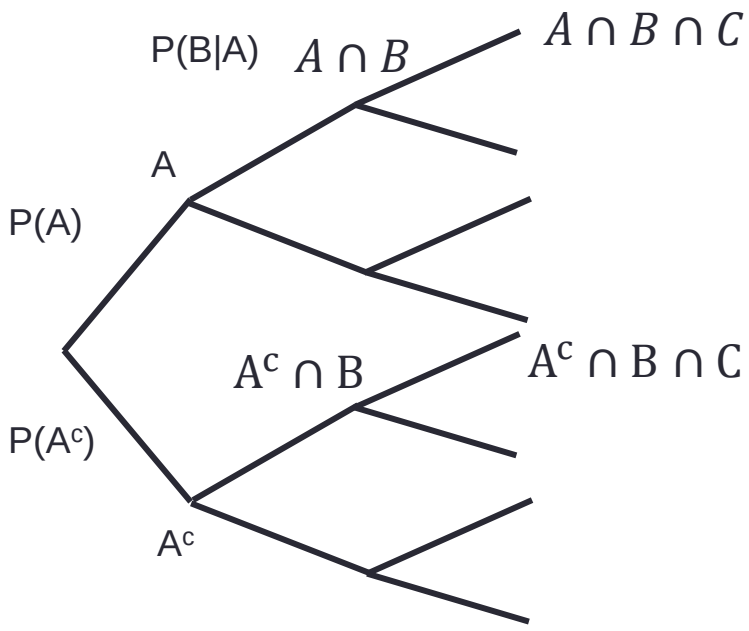
- $P((A^c \cap B) \cap C^c) =$



Regra da multiplicação

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$

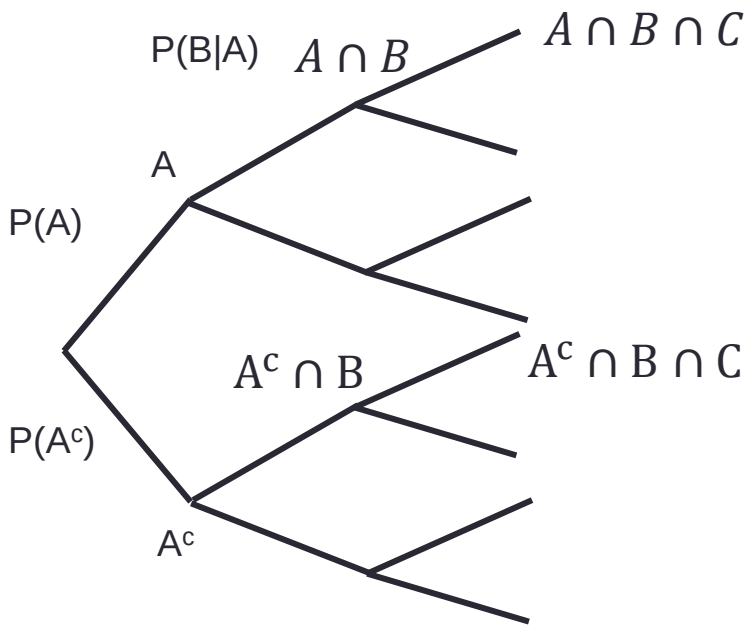
- $P((\mathrm{A^c} \cap B) \cap \mathrm{C^c}) =$
- $= \mathrm{P(C^c|A^c} \cap B)P(\mathrm{A^c} \cap B)$



Regra da multiplicação

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A|B)P(B)$

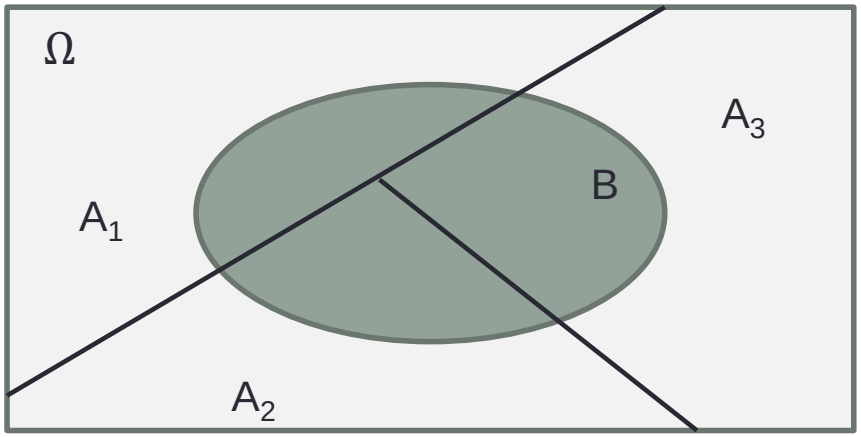
- $P((\mathrm{A^c} \cap B) \cap \mathrm{C^c}) =$
- $= \mathrm{P(C^c|A^c} \cap B)P(\mathrm{A^c} \cap B)$
- $= \mathrm{P(C^c|A^c} \cap B)P(B|\mathrm{A^c})P(\mathrm{A^c})$

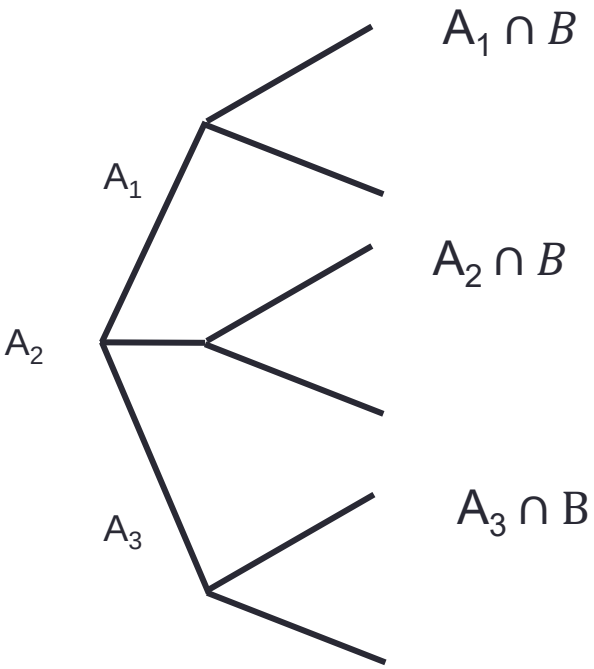


Exercícios

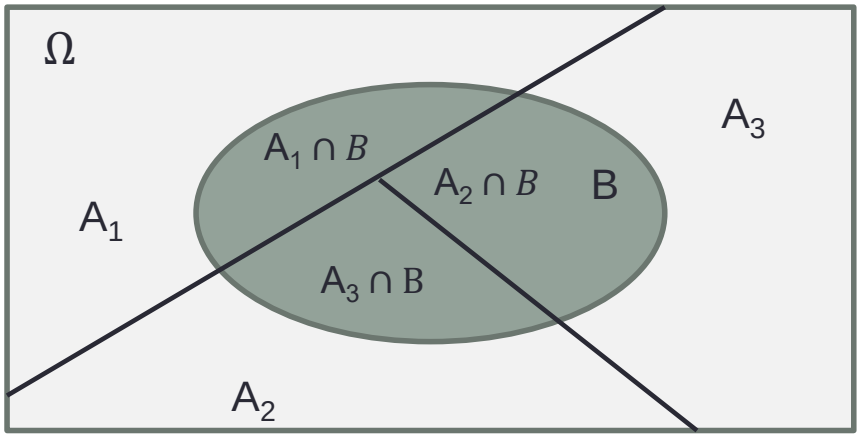
- *Mostre que:*
- $P(A \cap B|C) = P(A|C)P(B|A \cap C)$
- $P(A \cap B \cap C) = P(A)P(C \cap A|A)P(B|A \cap C)$

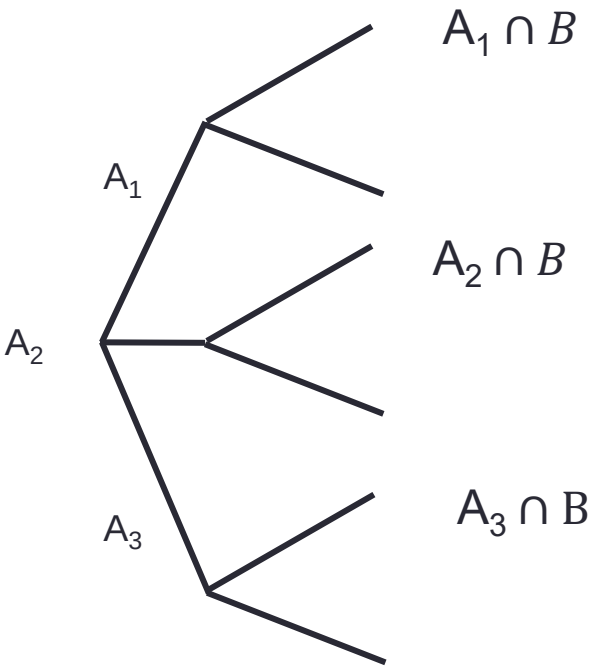
Teorema da probabilidade total





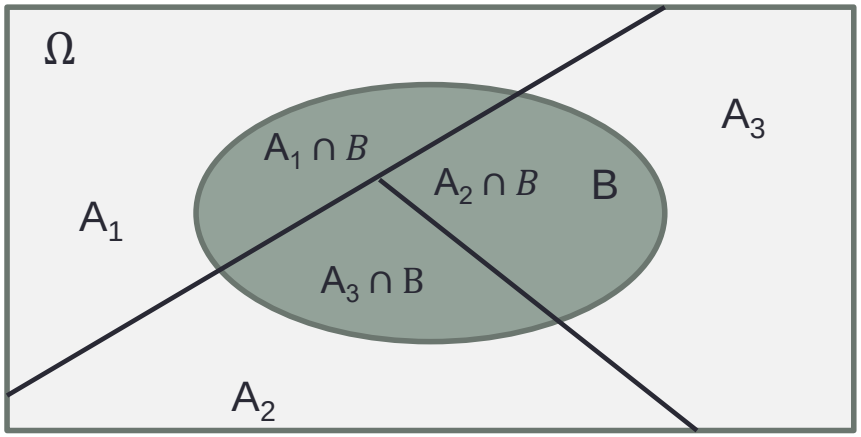
Teorema da probabilidade total

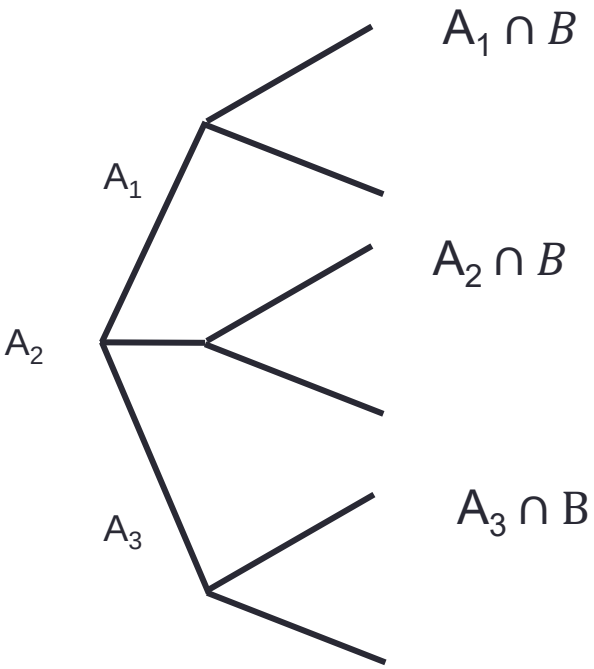




- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(B)$

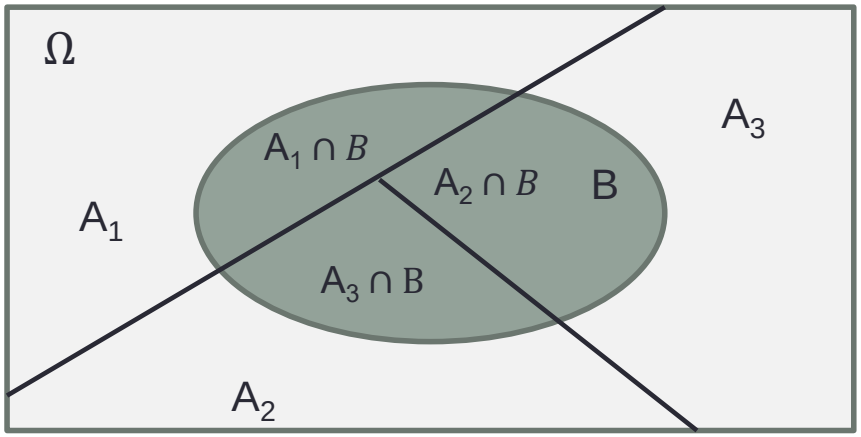
Teorema da probabilidade total

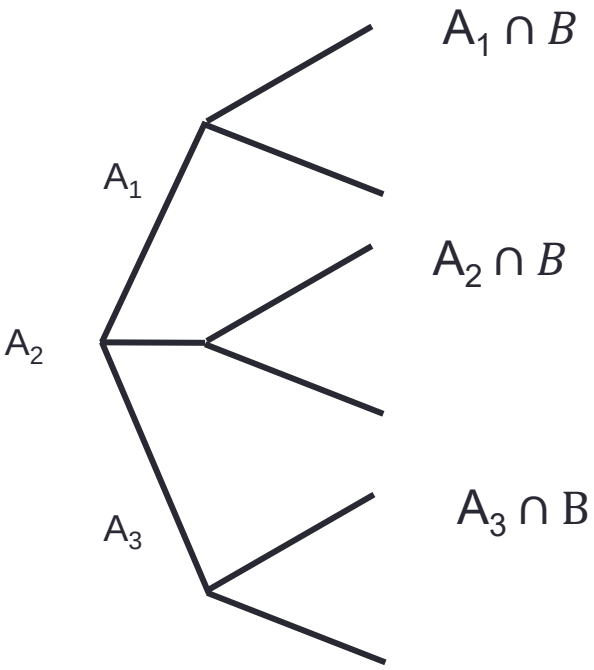




- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(B)$
- $B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$

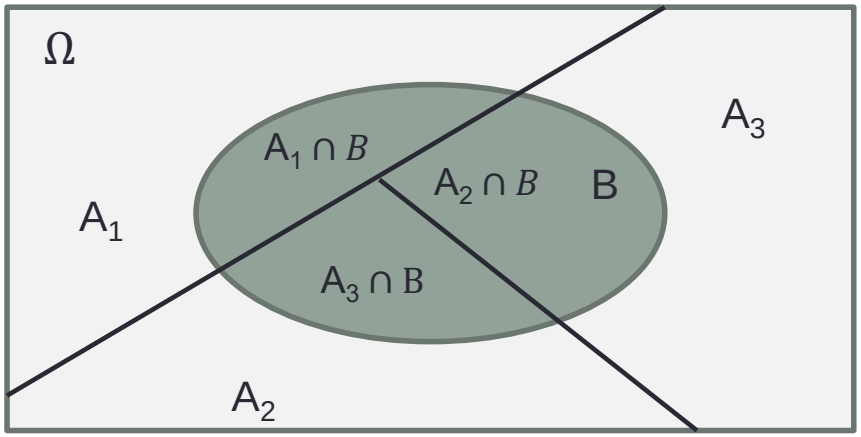
Teorema da probabilidade total

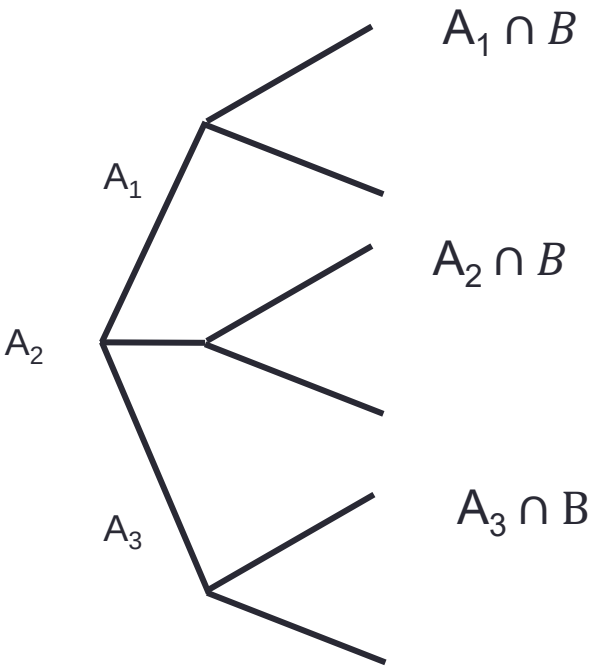




- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(B)$
- $B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$
- $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$

Teorema da probabilidade total



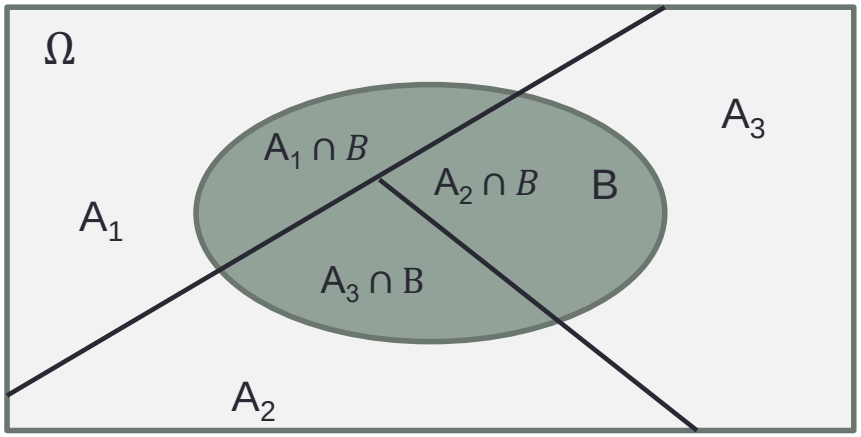


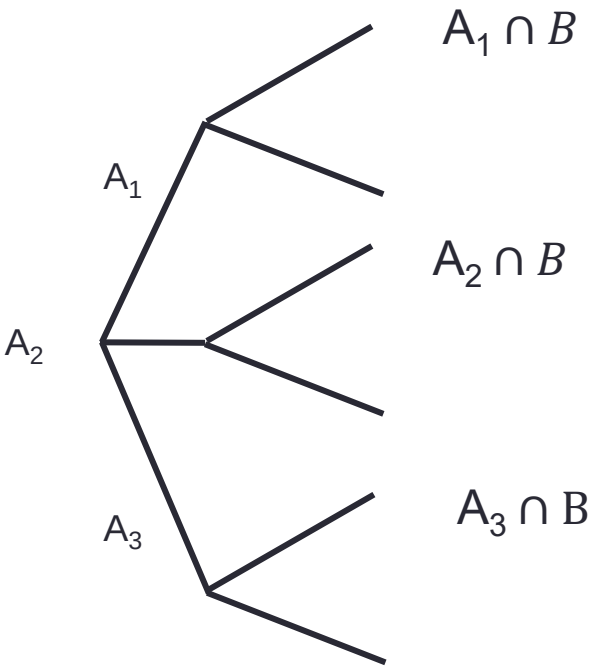
- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(B)$
- $B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$
- $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$
- $P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$

Teorema da probabilidade total

- $P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$
- $P(B) = \sum_{i=1}^{n} P(A_i)P(B|A_i)$

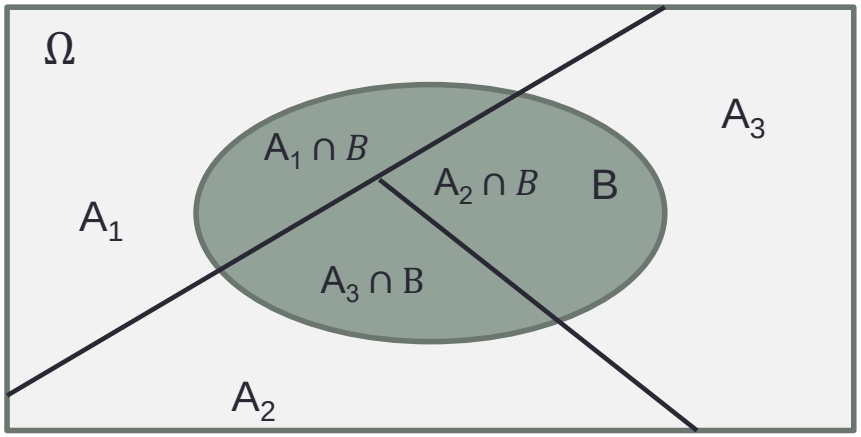
Regra de Bayes

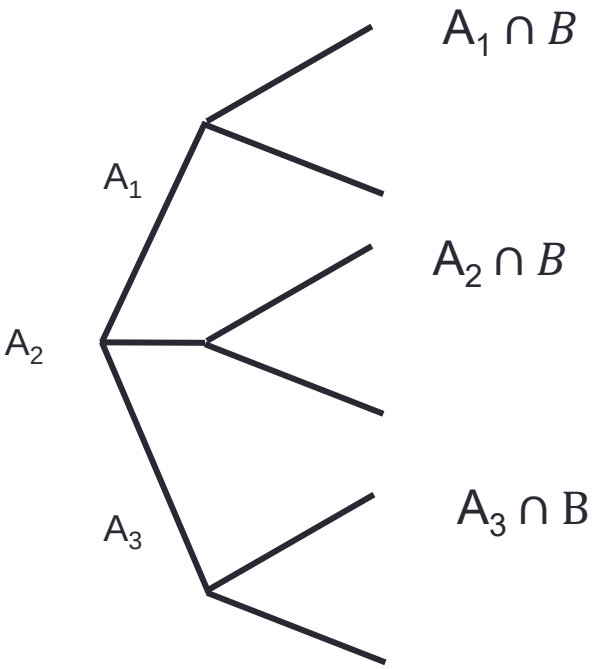




- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(A_i|B)$

Regra de Bayes





- Temos $P(A_i)$ e $P(B|A_i)$
- Calcular $P(A_i|B)$

Regra de Bayes

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Regra de Bayes

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
  - Exemplo
    - Diagnóstico

DÚVIDAS?