Project Proposal: <Eurographics 2021> *Implementation of Orthogonalized Fourier Polynomials for Signal Approximation and Transfer Using C++ and Eigen*

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Introduction

This project seeks to implement and critically assess the Orthogonalized Fourier Polynomials method introduced by Maggioli et al. (2021). In an era where shape analysis plays a foundational role in animation, scanning, and simulation, the ability to accurately transfer and approximate signals across 3D surfaces is paramount. This project explores a cutting-edge methodology—Orthogonalized Fourier Polynomials (OFPs)—which offers a principled, robust alternative to traditional signal representation frameworks. The method addresses significant challenges in the approximation and transfer of signals across three-dimensional shapes, leveraging orthonormal polynomial bases derived from eigenfunctions of the Laplace-Beltrami operator.

This is crucial in computer graphics because it enables high-fidelity surface signal processing—such as textures, descriptors, and coordinates—even in the presence of mesh irregularities or topological noise. Traditional methods using Laplacian eigenfunctions often struggle with accuracy and stability, particularly when dealing with high-frequency details and non-isometric shapes.

These limitations become critical when fine geometric features are lost during shape matching or transfer—hindering downstream tasks like shape editing or morphing. The approach presented by Maggioli et al. improves upon these methods by providing enhanced numerical stability, improved resilience to discretization artifacts, and a more detailed representation of geometric features (2021). By re-implementing this advanced method using C++ and Eigen libraries, this project aims to rigorously validate the theoretical insights and experimental results presented in the original research.

Project Objectives

1. Precisely implement the Orthogonalized Fourier Polynomials methodology as presented by Maggioli et al. (2021) using C++ and Eigen.
2. Reproduce and validate key experimental outcomes from the paper, particularly focusing on signal reconstruction, detail transfer, and spectral filtering across various shapes and signal types.
3. Perform an in-depth report on the implementation, including methods, main challenges and the solutions to overcome those challenges.
4. Deliver a comprehensive and clear presentation to facilitate peer academic discussion.

Technical and Mathematical Background

The core of this methodology involves the Laplace-Beltrami operator (Δ), fundamental to processing geometric data defined on manifolds, which are practically represented as discrete triangle meshes. The eigenfunctions (Φ_i) of this operator form the basis for constructing polynomial combinations, known as eigenproducts: $\Phi_I (x) = \Pi_{i\in I}\, \Phi_i (x)$. However, direct usage of eigenproducts encounters challenges such as linear dependency and numerical instability. To overcome these, Gram-Schmidt orthogonalization is employed, converting these eigenproducts into a stable orthonormal basis: $\{\zeta_i\}\zeta_i(x) = \phi_I i(x) - \Sigma_{(j=1)}^{(i-1)} < \phi_I i, \zeta_j > \zeta_j(x)$. Transferring signals across different shapes effectively uses the concept of functional maps, represented by matrices that encode the correspondence between eigenfunctions of source and target shapes. Maggioli et al. define a mathematically rigorous approach to constructing a stable transfer matrix: $O = R_\psi \overline{C} R_\phi^{(-1)}$ where matrices R_ψ and R_ϕ originate from the orthonormal eigenproducts computed on respective shapes.

Implementation Strategy

Our implementation strategy is guided by clarity, modularity, and scientific fidelity. Rather than simply porting existing MATLAB code, we re-engineer the pipeline from first principles, aligning each component with the core theory of the paper. This not only aids learning but ensures adaptability to broader applications.

- Computation of Eigenfunctions and Eigenproducts: Use Eigen's efficient linear algebra functionalities to compute the necessary eigenfunctions and subsequently construct the eigenproducts.

- Orthogonalization Procedure: Apply Gram-Schmidt orthogonalization to ensure linear independence and numerical stability of the Eigen product-based orthonormal basis.
- Functional Map-based Signal Transfer: Strictly adhere to the paper's methods for functional map construction and utilize these maps to accurately transfer signals between shapes.
- Experimental Validation: Replicate and validate key experiments, using established benchmarks such as FAUST and SHREC datasets, ensuring accurate and reliable outcomes that align with the original study.

Computational and Experimental Analysis

The project will involve extensive computational analysis to evaluate the efficiency and numerical stability of the proposed implementation. By employing rigorous testing methods, including reconstruction accuracy, stability under noise, and performance on non-isometric shape pairs, the project will systematically validate the theoretical advantages posited by Maggioli et al.

Project Timeline

We follow a disciplined week-by-week breakdown that ensures clear milestones for ideation, execution, and evaluation. Each phase includes internal checkpoints to track progress against both implementation goals and documentation quality.

Week	Task
TB	Comprehensive review of literature and setup of computational framework.
7	Discuss action plans; draft Project Proposal. Review data benchmarks and finalize selected shapes for evaluation.
8-9	Core implementation: eigenfunction computation, eigenproduct construction, and Gram-Schmidt basis formulation. Internal code testing.
10	Integration of functional map-based signal transfer. Begin experiments with real benchmark datasets (e.g., FAUST, SHREC'19).
11	Debugging, validation, and documentation of all results. Begin structuring seminar slides and outlining report sections.
12	Finalize project report, polish codebase for submission, rehearse seminar presentation, and prepare peer review forms.

Deliverables

- **Fully functional and well-documented C++ source code** implementing the Orthogonalized Fourier Polynomials methodology using the Eigen library.
- **Final project report**: Includes implementation details, theoretical grounding, experimental validation, and comparisons to the original MATLAB implementation.
- **Seminar slides and a 5-minute oral presentation** articulating the key challenges, results, and innovations achieved.
- **Peer review feedback form** detailing contributions and evaluations of group members.

Expected Outcomes

- **Establish concrete improvements** in numerical stability, detail preservation, and transfer accuracy—especially on non-isometric or irregularly meshed shapes—relative to conventional Laplacian bases.
- **Provide a reproducible, C++-based framework** that extends the usability of the method in real-world graphics pipelines where MATLAB dependencies are impractical.
- **Serve as a stepping-stone for future explorations** in spectral geometry processing, including compression, mesh editing, and machine learning applications on geometric data.

References

Maggioli, F., Melzi, S., Ovsjanikov, M., Bronstein, M. M., & Rodolà, E. (2021). Orthogonalized Fourier Polynomials for Signal Approximation and Transfer. *Computer Graphics Forum*, 40(2), 435-445.