Adaptive Motion Synthesis with Qualitative Control Theory

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Abstract

Generating natural-looking motions for virtual characters is a challenging research topic. It becomes even harder when generating adaptive motions interacting with the environment. Current methods are tedious, cost long computational time and fail to capture natural looking features.

This report proposes an efficient method of generating natural-looking motion based upon a new motion control theory. We propose that **only the qualitative properties of motion are controlled, adaptations are generated by perturbations of environment or body structure**, this is the **the Qualitative Control Theory (QCT)**. Inspirations for this research come from the contradiction between biological facts and current motion synthesis ideas. The biological idea we hold is that natural-looking motions mainly come from the complex interaction between the body and the environment. The natural neural system only maintains or tweaks qualitative properties of this dynamic interaction. We believe that motion is composed of many motion primitives, each motion primitive is a **structural stable autonomous system**. Motion and Adaptation can be generated without any control effort. The mathematical model we propose for the natural motion is based on the Qualitative Theory of Differential Equation. Qualitative Control is achieved through manipulating the topological structure of the dynamic system to enhance the " self-balance" ability, rather than counteracting the perturbation effects. The control method we propose is well supported by biology research.

Adaptive Motion Synthesis Method completely solving motion retargeting problem. Motion data can be directed applied for a different character in a different environment. Also this method involves with very light computational load. Further, it maintains important features of natural looking motions.

Based on such results, we discuss potential applications of this method to full body character animation synthesis, which forms the main job of the PhD phase research.

Keyword: Motion Synthesis, Qualitative Dynamic, Nonlinear Bifurcation.

Chapter 1

Introduction

1.1 Research Values and the Challenges

Character Motion Synthesis (CMS) research aims at generating motions for virtual characters. It is a valuable topic for both industry and academic community. Main applications are in the media industry, both computer games and animation films depend heavily upon character motions for storytelling. CMS also has many applications in other areas, such as user interface design, psychology, sports and medicine.

The challenge of CMS research is not to make characters move, but how to make them lifelike. This challenge comes from our human's marvellous ability of motion perception. Motions for the same task are very similar, but vary adaptively. From the variety in motion details, humans can infer the changes in mental states, health conditions or even the surrounding environment.

Nowadays in industry, high quality motions are majorly generated by manual work. For every joint of the character, animators specify a series of positions over the motion time. Corresponding mathematical model is the parametric trajectory with joint position as the value and time as the parameters. In applications, most characters are very complicated and contain a large number of joints, making animation a tedious work. Making things worse, it is difficult to reuse motion animation. Reusing motion animation for a different scenario is prone to artefacts. For this situation, high level animation tools are badly needed.

It is believed that motion can be synthesised by simulating the dynamics of body and environment and the functions of neural control system. With the advance in Mechanics Simulation, current research is trying to make virtual characters dynamically interact with their environment and adapt motions realistically. Typical research topics include locomotion, object manipulation and posture control. However the complexity of body structure proposes many problems for simulation methods. For example, the human skeleton is made up of more than 200 bones driven by more than 600 muscles. From the mechanical viewpoint, this mechanical system is full of redundant **degree of freedom** (DOF)s, which not only increase the computational load, but also make the solution nondeterministic. For a specific motion task like picking up an apple, there exist many different ways of arm motion. Only a few ways seem natural, and the believable motions vary adaptively for environmental or inner reason.

1.2 Natural Motion Features

To synthesis natural looking motion, we must understand the features of natural motion. This quesition is covered in biological **Motor Control** research. Three features are considered in CMS research but are hard to achieve at the same time by current methods.

Adaptive and Robust Natural motions are adaptive to the changes in the environment or body conditions. A common example is human locomotion. The walking motion changes on different terrains while the balance is maintained and different perturbations will generate different reactions.

Agile Some motions of animals are very fast, honeybirds may vibrate their wings in kHz. agility not only refers to the speed of motion, more puzzling is that the neural system can solve the complex motion control problem in a very short time. When an animal avoids obstacles at very high running speed, it must continue its running motion, make a turning and keep balance at the same time. It seems very easy for the neural system to find such complicate motion solution.

Energy Efficient Natural Motions are energy efficient. In theory, this idea is supported by Darwin's Theory of Evolution. Animals spent far less energy than our expectation. An evidence is that the energy consumed by human walking is only 10% of that for a robot of the same scale.

1.3 A Simple Illustration of Qualitative Control Theory

The main content of this research is the **Qualitative Control Theory(QCT)**. It is not only a new CMS technology, more important, it can also been seen as a different theory for understanding of the biological motor control system. To understand QCT, one first has to abandon the traditional parametric trajectory model for motion. In our perspective, this idea is the origin of many limitations of current research methods. A full discussion of such limitations is in section 2.5. The intuitive explanation of Qualitative Control Theory is that the trajectory of motion is not important, only some qualitative properties of the final result matters. In QCT, the objective of motion control is some qualitative property of the final motion. Further we propose that motion adaptation involves little control effort. Adaptation is generated by the intrinsic property. In mathematics, a natural motion is modeled as a **structural stable autonomous system**.

Although QCT is new in CMS research, the principal is intuitive and common in our daily life. It can be illustrated by the following example of a ship floating in the sea.

Adaptive Motion of the Ship When a ship is floating in the sea, it will move up and down along with the wave. In our perspective, this phenomenon is in essence the same with motion adaptation Such movements are not generated by control efforts; they are generated by the ship's intrinsic property and perturbations from the wave movement. To make the problem simpler, we restrict our discussion to motion of the ship in two dimensions, as shown in figure 1.1. The swing motion of the ship can be described by the following equation

$$I\ddot{\theta} + d\dot{\theta} = T_G + T_B + T_F = (Gl_g - Bl_b)sin(\theta) + T_F$$

θ is the swaying angle, I is the inertia, d is the damping coefficient, T_G is the torque of gravity, and T_B is the Torque of buoyancy. T_F is the external control torque. if $T_F = 0$, external control force is applied, the system is an **autonomous system**.

A graph is used to show the motion of the ship, in Figure 1.2, we plot the states of the ship over time. The horizontal axe represents the θ, the vertical axe represents the $\dot{\theta}$. Different waves push the ship into states with different angle θ. Because the torque of gravity and buoyancy can't compensate each other, it will begin to move. The ship with different initial states will move in different ways, thus result in two different motion

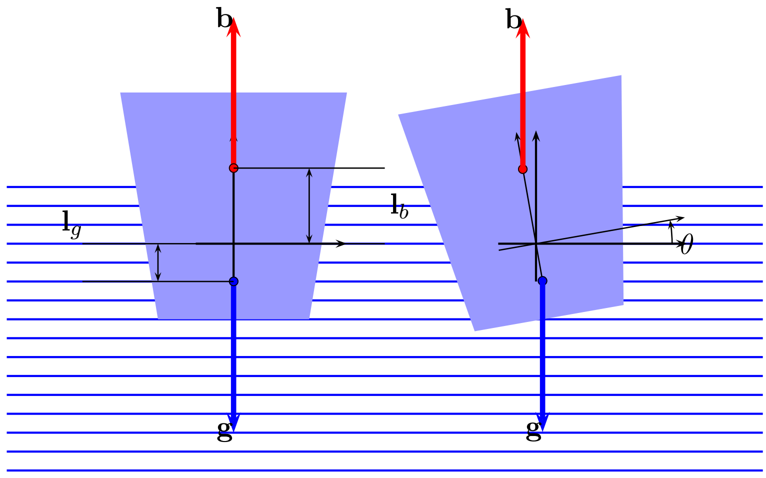


Figure 1.1: Illustration of the Ship Movement

curves. Motions are generated without any control; they can be seen as passive effects. It is a property of the autonomous system. The curves are different but the final state is fixed.

The variety of curves shows the adaptation of the ship. In the Qualitative Control Theory, the adaptation of natural motions in essence is the same with the adaptation of ship motion. In this process, neural control system may take little effort. Adaptive motions can be generated as a passive effect because of the autonomous system.

Maintain the Posture For Modern Ships, the height is much longer than the width. Special measures are needed to keep the ship from toppling sideways. This problem is solved by structure design, not by control strategies. A simple measure is to keep the centre of gravity below the centre of buoyancy. For the ship, there are only two posture where the buoyancy torque and gravity toque compensate each other, this is shown in Figure 1.3

But two postures are different, The left one is stable or attractive. If the ship is in a slightly different posture, it will return to the stable posture. On the graph, curves that starts from points in the neighborhood will approach to the center. as shown in Figure 1.4

The right one is unstable or repelling, if the ship is in a posture slightly different from the the right one. It will turn away from it. On the graph, curves that start from points in the neighbourhood seem to be repelled from the centre. as shown in Figure 1.5.

When connect Figure 1.5 and Figure 1.4together, we will see the motion curves depart from the repelling posture and approach to the attractive one. as shown in Figure 1.6. This property means the ship can automatically keep itsefl from toppling, which is deter-

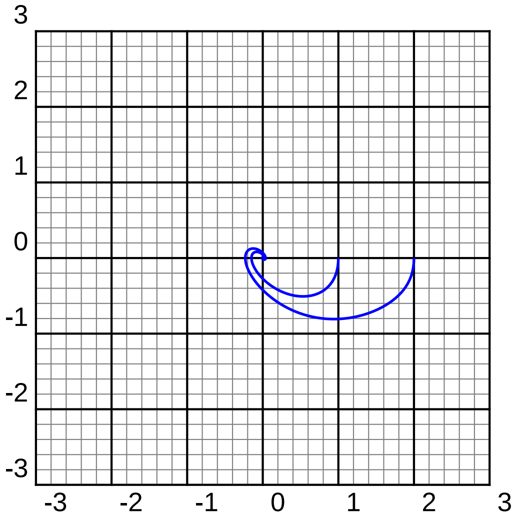


Figure 1.2: Plot the motion of the Ship. horizontal axe for θ, vertical axe for $\dot{\theta}$

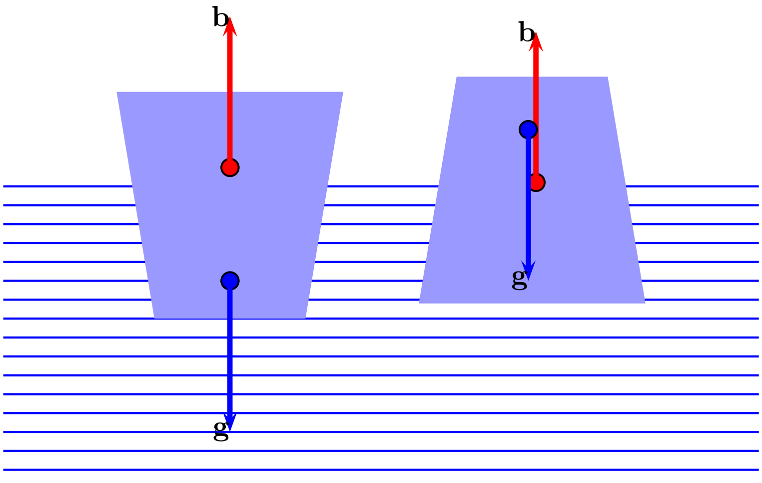


Figure 1.3: Two Equiblirum Posture of the Ship

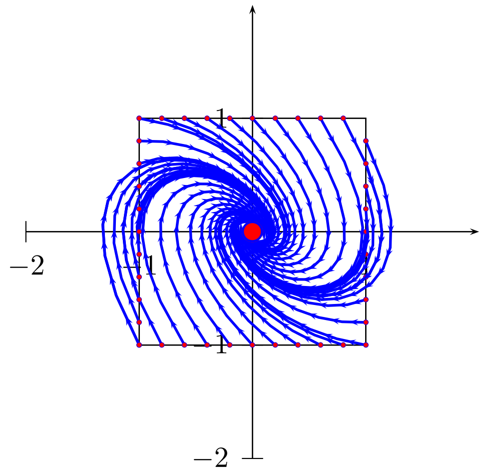


Figure 1.4: The Attractive Posture. The Center Red Dot for Posture

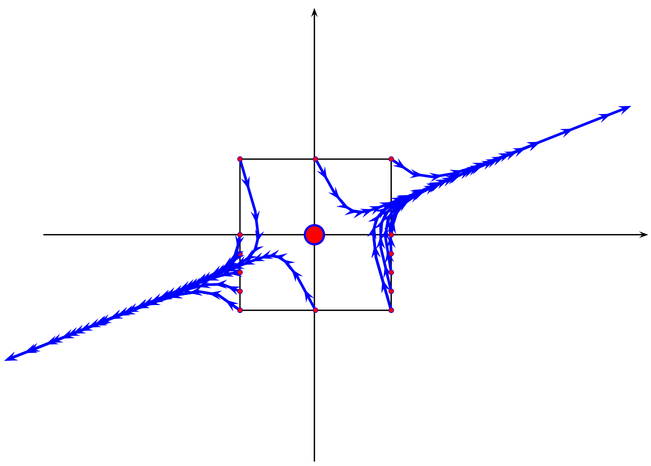


Figure 1.5: The repealling Posture

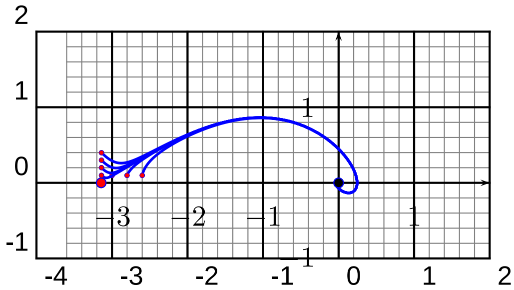


Figure 1.6: Motion Curves depart from the repelling posture and approach to the attractive posture

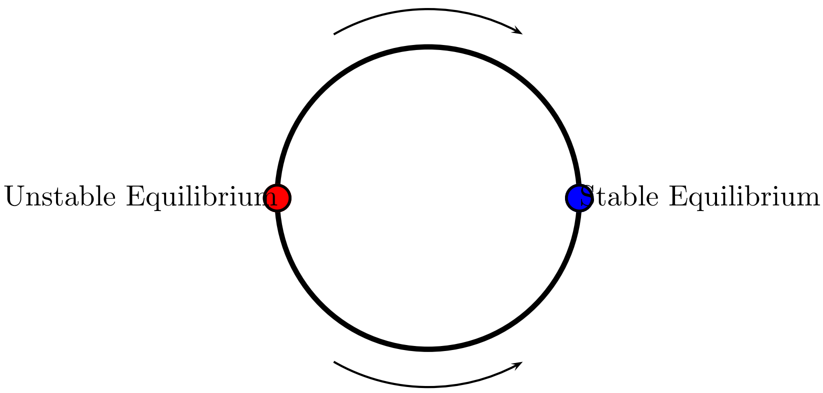


Figure 1.7: The Topology of Motion of the Ship

mined qualitatively by the structure. The variety of ship designs may change the motion details or the shapes of the curves on the graph, but will not qualitatively change the self balance ability. On the graph, it means it will not change where the motion curves start and terminate. The qualitative property can be interpreted as the topology of the graph, which is shown in Figure 1.7

For the states of the ship, $\theta \in [0, 2\pi)$, which can be presented by a line segment $\mathbf{D^1}$, for the 2π and 0 presents the same posture, we can define $0 = 2\pi$, In geometry, it means connecting the two endpoints of a line segment $\mathbf{D^1}$ together,which forms a circle $\mathbf{S^1}$. In theory, $\dot{\theta} \in (-\lim, \lim)$,$\dot{\theta}$ should be presented by a line $\mathbf{R^1}$, but in real life, the ship sways slowly, we suppose $\dot{\theta} \approx 0$,we simplify the line by a dot $\mathbf{S^0}$. The original space of the states $[\theta\dot{\theta}]$ is $\mathbf{S^1} \times \mathbf{R^1}$,which has the shape of a cyliner; The approximation $\dot{\theta} \approx 0$,simplify the cylinder into a circle, which is shown in Figure 1.7.

On this circle, two points represent the equilibrium posture; the red one is the repelling posture at $\theta = \pi$, the blue one is the attractive posture at $\theta = 0$, all the other postures are not stable, the ship will move away from the repealing one and approach the attractive one. Qualitatively we know **no matter what the initial posture of the ship is, the ship will finally stop at the attractive position.** This conclusion is based on several qualitative properties.

the state space is a circle

there are only two equilibrium points on the circle

one is repealling, one is attractive

the two equlibrium points are connected

In mathematical research, such properties are topological property, Figure 1.7 is the topological structure for the autonomous system of the ship.

For different ships, as long as the center of buoyancy is above center of gravity, the corresponding autonomous systems will possess the same topological structure, thus keep themselves from toppling. The idea of posture maintenance can also explain the posture of many swimming animals.

In Qualitative Control Theory, we take the topological structure as the most important control handle. For an automous system, if the topological structure is known, the qualitative properties of the final motion is determined. Also, if we can modify the topological structure, we can qualitatively change the behavior of the system.

For the ship example, if we make the repelling posture becomes attractive, and the attractive one repelling. We can turn the ship bottom up. This can be achieved by many methods, as long as the center of gravity is lifted above the center of bouyancy. But we are not sure how the ship will topple, we don't know the motion curve of the ship or even the direction of the toppling, whether clockwise or anticlockwise. In qualitative control theoy, we take such uncertainty as an advantage. How the ship will topple is determined by the ship,control measure and the environment perturbation. This provides a mechanism for adaptation of motion.

Facing Bigger Waves When bigger waves come, it becomes more challenging. It is impossible to measure the strength of coming wave and counteract it. Methods taken are based on changing the ship's parameters.

One method is to turn the ship around, let the head facing the wave as show in Figure 1.8

After the turn, for waves of the same strength, amplitudes of vibration are smaller. This posture with the one before the turn are shown in Figure 1.9 The equations for the two postures of the ship are nearly the same; the only difference is the inertia. The inertia I of the right posture is larger than the left. The motion curves of the two system are compared in Figure 1.10

The right graph of new posture motion is like the right one after being scaled down. The the stir measure can be geometrically interpreted as a scale deformation. This idea of adjusting parameters can explain many natural motions. When a human is walking on a narrow bar, he naturally extends his arm to prevent toppling sideways.

In Qualitative Control Theory, we propose that animals use the same idea to maintain motion stability. When moving in an environment of perturbations, the motion is maintained not by feedback control, but by the selection of a more stable posture.

A further question is maintaining the posture when waves come one after another. In real life, one method is vibrating the ship in synchronise with the wave. This method is important, in our research; it is the foundations of many periodic motion behaviours like walking locomotion. we put the discussion later for more mathematical background is needed.

Agility and Energy Efficiency Little energy and no complicate computation are needed to maintain the posture of the ship, thus this example is energy efficient and agile. The

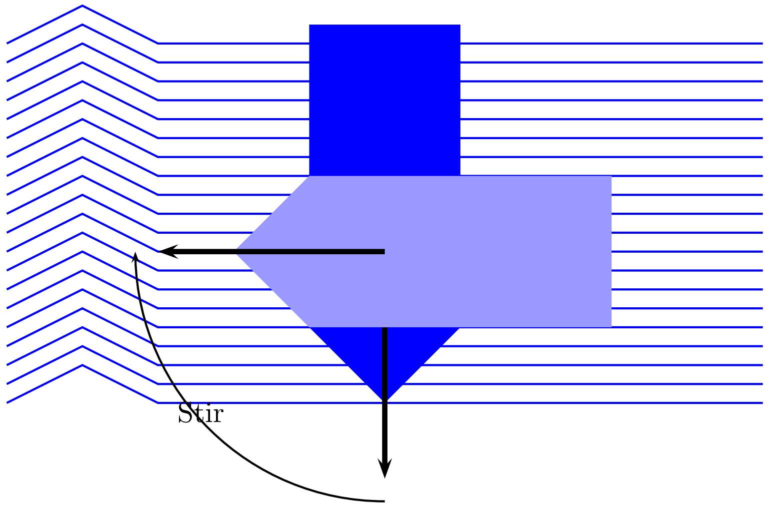


Figure 1.8: Topview of The Stir

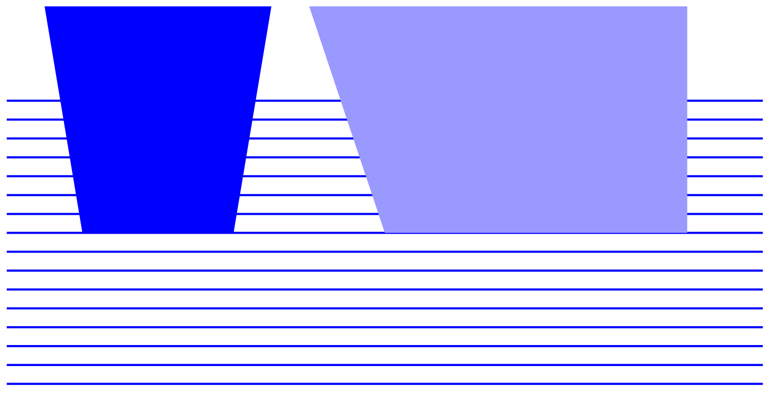


Figure 1.9: After the stir, the inertia becomes bigger

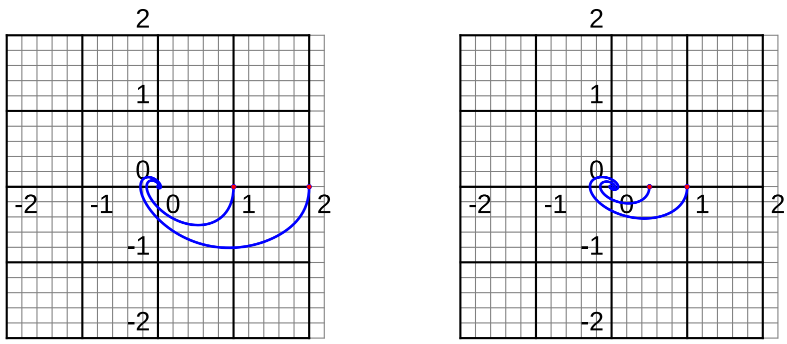


Figure 1.10: motion curves of two postures

reason is that the control objectives are met by the intrinsic properties of the autonomous system. The Qualitative Control Theory adopt the idea utilize the intrinsic properties of autonomous systems without much neural control effort.

Qualitative Control Theory suggests that the principal of animal's motions is the same as the motions of the floating ship. The interactions between body and environment form many autonomous dynamic systems, which are the foundations of natural motions. Many motion objectives are based on the qualitative properties of such autonomous systems, thus can automatically maintained without any effort. Neural system adjusts the body posture to adjust the parameters of autonomous systems for specific purpose. Complex motion is basically a sequence of basic motion primitives.

1.4 The Novelty and Contribution of the Proposed Research

Our research will contribute in both biology theory and CMS research. For biology theory, the proposed research further completes the theory of **Motor Control** . We introduce the mathematical knowledge of qualitative dynamic theory into the field. This may help us have a better understanding of animals' body and neural system. For CMS research, this research introduces a novel method aiming at generating adaptive motions. In application, it will solve the problem of reusing motion data thus greatly reduce the animation work. It especially suits repetitive and low energy motion tasks which are most challenging in current CMS research. It also has the advantage of computational efficiency and can be accelerated by GPU and may run in real-time in future.

Qualitative Control Theory has three major differences from current CMS ideas.

Qualitative and Quantitative Properties Motion Controllers in current research mainly focus on eliminating the difference between simulation results and the predefined motion curves. The difference is a local quantitative property; while QCT concerns with the qualitative properties which are determined by the topological structure of the dynamic system. Topological Properties are global and qualitative.

Active and Passive In current research, adaptation is generated by active control effort, While in our framework, adaptive motions are generated without any control effort, thus can be seen on passive effect.

Inverse and Forward Simulation Current control methods synthesis motions by solving an inverse problem. QCT is based on modifying the parameters of the system and forward simulation, thus are more efficient.

1.5 The Report Structure

This report is organized as follows: Chapter 1 introduces the application and our research objective. In Chapter 2, we review current CMS research. The focus of discussion is the limitation of the trajectory based motion model.

Motion synthesis is a typical interdisciplinary research topic. The Qualitative Control Theory this research proposed is based on many research in physical science. In Chapter 3, we provide a detailed discussion about the biology and mathematical background of the new theory. In Section 3.1 discussed the supporting biological view, it can serve as a justification of the theory and the answer to the questions why Qualitative Control Theory has the potential for natural looking motion. In Section 3.2, we build the mathematical model for this method, which serves as the foundation for computer implementation.

In Chapter 4, we discuss application of Qualitative Control Theory in CMS. In Chapter 5, some testing results for simple character are shown. Based on these results, research work in phD time is discussed in Chapter 6.

Chapter 2

A Review Of Previous of CMS Research

2.1 The Key Questions of CMS Research

Motion Synthesis is a research topic that aggregates many different ideas. At first, we discuss the common questions before comparing the variety of ideas. From our point of view, two key questions are shared by all the CMS research: The model and the algorithm.

For any CMS research, Motion needs a mathematical description. Basically we have three models for motion.

Parametric Curve Motion can be described as a parametric curve with time as the parameter and position as value. For example, and arm movement can be described by positions of all the joints during the time of motion.

State Machine Basically, complex motion and be divided into several primary motions. Motion can be described by discrete model such as state machine, like reach the apple and then pick it up.

Objective Motion can be simple described by the objective, like picking up an apple.

Another question shared by CMS research is the algorithm for searching motion solution. Current idea separates this task into two levels: The high level problem is motion planning, its responsibility is to finds a solution. Low level problem is motion maintenance or stability control. The task is maintaining motion under perturbations. Efficiency is always the concern in algorithm research.

It is impossible to discuss algorithms and model separately. The model should be treated as the foundation of a CMS research. It determines quality and difficulty. Usually, model without detailed information like state machine may ease the searching problem for motion solution but limited information will make the low level control nondeterministic; while a detailed motion model will make high level planning more difficult, more information may help low level control, but may also be over specific that no solution can be found. There is an elaborate balance.

For most current research, Motion is described as trajectories or parametric curves. Motor control is transformed into two tasks: find a trajectory and follow it. Based on different input information, trajectory based methods can be classified in two groups, **Data Driven** and **Procedural**.

2.2 Data Driven Methods

Data driven methods are based on ready motion data which are generated by Key-Frame or Motion Capture(Mocap). In practice, motion data are segmented into short time clips. An animation is generated by selecting motion clips and connecting them together[Parent, 2002].

Like other example based methods in Computer Graphics, data driven methods can generate good results if similar motion clips can be found. However it is difficult to generate novel motions, it is also difficult to reuse the motion data, whether modifying the motion data for a different character or a different scenario. This is the "motion re-targeting" problem.

In practice, the management of large motion data is another big challenge. Although some solutions like the Annotation Database [Arikan et al., 2003] and the Motion Graph [Kovar et al., 2008] were proposed, cataloguing and searching of motion data are not trivial tasks and remain an open problems.

2.3 Procedural Methods

For procedural methods, pre-recorded motion data are not needed. The motions are formulated as functions of various parameters, as shown in equation 2.1.

$$F : P \mapsto M \tag{2.1}$$

where M is the valid motion space of characters, P is the parameter space.

Different models may have different choices of parameters P and mapping function F. Currently, P and F are mainly based on rules of physical science, and such procedural methods are often called "Simulation Methods".

For kinematic methods, P is mapped to the kinematic properties like position and shape. F is based on kinematic laws. Position errors such as foot sliding on the ground can be easily eliminated by kinematic methods.

For dynamic methods, P is expanded to include dynamic properties like mass, energy and force, F is based on mechanic theory of Newton or Lagrange. Since natural object movement is governed by mechanics, this research direction has the potential to generate more realistic motions. For dynmaic methods, a dynamic model is needed for the character body. A common model is a linked rigid body mechanism. After rigid body dynamic simulation methods[Baraff, 1994, Mirtich, 1996, Stewart and Trinkle, 2000] were proposed, comes the prosperity of dynamic CMS research.

Rigid body simulation is just one of the many problems CMS faces. The bodies of animals are actuated by muscles under the control of neural system. Control system design is the most difficult problem.

Some early research applied classical control methods like PD controller [Raibert and Hodgins, 1991] for locomotion synthesis. Later research [Hodgins et al., 1995] applied the same method for different tasks like running, bicycling, vaulting and balancing. Such methods are based on simplified models which relieved the controllers from the problem of redundant DOFs, but important motion details were also neglected.

2.4 Optimization Methods

Because of the redundant DOFs in the body structure, in most cases, there exist many motion solutions for one task. Optimization methods have been applied to solve the nondeterministic problem. Among all the solutions in possible motion space, the "best" one is chosen as the proper solution:

$$\arg\max_{x} V(x), x \in F(q) \tag{2.2}$$

Where x is a solution in the solution space $F(q)$, V is the value function specified by animators. The function V in practice depends on the application requirement. For data-driven methods, V may be designed to choose the sequence with most smooth transition. For kinematic methods, V is designed to select the posture that least violates position constraints[Boulic and Mas, 1996]. With some special energy cost function V and constraints, kinematic methods have been used to retarget motion data[Gleicher and Litwinowicz, 1998, Gleicher, 1998].

For dynamic methods, a reasonable value function V is the energy cost.

$$\mathbf{V} = \int_{t0}^{t1} F_a(x)^2 dt \tag{2.3}$$

where F_a is the active force generated by actuators like motors or muscles. This is introduced to CMS research as the influential Spacetime Constraints[Witkin and Kass, 1988]. It is based on the hypothesis that the natural looking trajectory costs minimum energy. It is related to the idea of Darwin's Theory of Evolution and the principle of Natural Selection. In many cases, these methods produced very believable motions. Jain et al. [2009] provides an example of locomotion; Macchietto et al. [2009] find a method for balance maintaining movement. Liu [2009] proposed a method for object manipulating animation.

One shortcoming of Spacetime Constraint is the efficiency. Spacetime Constraint in nature is a variational optimization problem. It takes prohibitive long time to simulate complex musculoskeletal structure[Anderson and Pandy, 2001]. Optimization techniques like time window and multi-grid techniques are proposed by Cohen [1992] and Liu et al. [1994]. Finally, Popović and Witkin [1999] proposed a method based on Spacetime Constraint for full body dynamic animation.

2.5 Limitations of Trajectory based Model

All the methods mentioned in the previous section are based on the same motion model. In practice, they usually work together. For example, a reaching motion can be generated in the following steps: (1) motion capture data are selected as the motion reference; (2) Inverse Kinematic methods are used to fix the endpoint error; (3) Inverse dynamic and optimization are applied to determine the active forces of muscles. (4) Forwarding simulation is carried out to add responsive effect for perturbations. However the methods based on trajectory model are still suffering from the following problems.

Computational Complexity There is no efficient numeric method for variational optimization like Spacetime Constraint. Current numeric methods are very sensitive to model accuracy and initial conditions, and converge slowly. In many cases, it will take very long time and there is no guarantee the optimal solution can be achieved.

Over Specific Most controllers built are only capable of generating animation for a specific motion task under a specific environment condition. Current methods only cover a small portion of motion repertoire. Motions like heart beating, breathing, or motions of other animals such as the swimming of fish and jellyfish, flying of birds have not been synthesized with dynamic methods. There is no a general way to model the body and the environment effects. It seems that endless controllers need to be designed to tackle all the combinations of motions and environment.

Artefacts in Low Energy Motion Liu [2005] points out that spacetime constraint methods only suit high energy motions like jumping and running. The results for low energy motion tasks like walking don't look natural. The reason is well understood in biomechanics research. Muscles, an elastic structure, can store energy during collision impact and release the energy for activating bodies later. For low energy motions, this elastic effect can help to save a large portion (as much as 40%) of energy. However because of its complex nonlinear characteristics and large number of muscles involved, this elastic effect is usually ignored in CMS research, mainly for the computation cost reason.

In our view, these are serious limitation and hinder the progress of CMS research in many ways. Animations for more detailed anatomical structures, such as muscles and tendons will result in prohibitive long computational time. Expanding moion repertoire will facing the difficulties of modeling fluid and elastic dynamic behaviour and much more computational load. Following this idea, many common daily motion tasks like pouring water into a cup, will be formidable difficult to synthesize dynamically, while it is an easy task for human. All these problems, in our view come from the motion model. It can not be overcome by improving algorithms. Further progress of CMS needs a different motion model.

2.6 Methods based on Different Models

Some CMS research produced good motion results, but have different model for motion.

Motion Signal Processing Motion signal processing(MSP)[Bruderlin and Williams, 1995] transformed the motion data into frequency domain and introduced the signal processing techniques to CMS research. MSP is example based and thus shares the disadvantages of other data-driven methods. A special advantage of this method is the ability to generating emotion expressive motions.

MSP may shed light on the motion perception puzzle. Humans have a great ability in frequency analysis; the evidences are our sensitivity to colour and musical sound. A bold hypothesis we suggest is that maybe the motion perception and even motion control closely relates to our frequency perception power.

Limit Circle Limit Circle Control(LCC) [Laszlo et al., 1996] provides an alternative method for lower energy locomotion animation. The LCC theory has been used in explaining passive mechanics. Compared with Spacetime optimization, LLC methods is more computational efficient method for low energy motion.

However in our perspective, the current LLC method has not exhibited its full potential power in theory. In current researches[Coros et al., 2009, Laszlo et al., 1996], the limited circle is fixed and specified by the animators. The control strategies are simplified as a state machine controller following a predefined limited circle. When Limit Circle is fixed, limited circle control falls into trajectory based methods. A prefixed limited circle can be seen as a predefined motion curve of different parameters and value function. Fixed limited circle deprives limit circle control of adaptive motion under perturbance.

Evolution and Neural Network Methods Inspired by the Theory of Evolution and Neural Network, some researches[Sims] build a simple biology system and simulate the evolution process. After enough trial and error, reasonable motion controller can be developed. But the result is unpredictable. Animators have no way to tune the animation. An impressive idea in this research is not only the controller but also the body structure should be evolved.

Our research started from improving the MSP and LLC methods. The research idea is to make the LLC adaptive with perturbations. In Qualitative Control Theory, we propose that the limit circle is a qualitative property of the dynamic system. It is also determined by the topological structure and can be qualitatively controled. Another research idea is to enhance MSP method with dynamic motion synthesis capacity. Although our research is biology grounded and involves neural activity modeling, it is not a artifical neural network based method, in qualitative control theory, we model the dynamic behavior of neural signal and utilize the qualtiative dynamic properties. From the evolution based research, we don't use such computational method, but we accept the idea that the role of body structure is important in motion.

Chapter 3

The Qualitative Control Theory

In this chapter, we present the Qualitative Control Theory. In Qualitative Control Theory, the motion trajectory is not the concern, only the qualitative properties of final motion result are concerned. Also high precision computation may not be needed. Two questions arise when one applies the Qualitative Control Theory for Motion Synthesis.

The first question is why animations generated by this method can be natural looking. Section 3.1 will focus on this question. We point out that the Qualitative Control Theory owes its origin to biological research; Qualitative Control Theory is a more update idea of current biological Motor Control research, it may be more close to the real biological motor control system.

The second questions is how to mathematical describe this intuitive idea. This question is crucial for implementation of this idea on computer. We adopted the **Qualitative Theory of Differential Equation** as the foundation theory for explaining natural motion. In Section 3.2 We propose a rigid explanation of natural motion properties from mathematical viewport.

3.1 Support from Biology Research

3.1.1 Key Questions in Motor Control

The foundation of Motion Synthesis is our understanding of natural animals' motor control system. The major research force on the topic that how animals control their motion is the biological motor control research group. The biological understanding of the several questions shapes the current CMS research.

High Efficiency One well agreed characteristic of natural-looking motion is the high energy efficiency[Alexander, 2003]. It is well supported by both the evidence and theory of evolution.

Current CMS research achieves high energy efficiency by the optimization control method. The biological research agrees with the high energy efficiency feature but questioned the sensitivity to perturbation and high computational cost.

Control Job Distribution The major subsystems involved in motor control are the neural system and the musculoskeletal system. A straightforward idea of task distribution is that neural system serves as the controller in charge of motion planning; the musculoskeletal system serves as mechanical apparatus, which executes the motion command from the neural system. This think-plan-action framework [Albus et al., 1989] forms the foundation of current mainstream CMS methods. This idea will impose too much computational burden on the neural control system.

Motion Perception Mechanism The motion features we notice may be the important variables in motor control. The understanding of motion perception will greatly affect motion synthesis research. Besides in computer graphics,"real" doesn't necessary mean physically accurate as long as the audience could not notice the artefacts. Many motions which look natural for audience may violate the physics law. Human's ability in motion perception interests both the neurobiology and motor control research. In real life, motion perception cost little conscious effort. Limited by the human sensing speed and precision, it is not possible for human to judge motion by caculating the energy cost. A possible answer is that human motion perception is based on some motion features which can be easily identified.

Current explanations for the above topics are far from complete and satisfactory, that may the reason why motion synthesis problem is still unsolved. The foundations of trajectory motion model in fact are questioned by current biological research idea and cause many problems in CMS research. An update idea of biological motor control system may help to establish a better CMS framework. That is the work we attempt in this research.

3.1.2 Biological Research ideas about Computational Method

In both CMS and biologcal motor control research, one most noticeable question is the computational efficiency. More questions arise after more knowledge of the biological computer, the neural system, has been obtained. Nowadays we can provide a detailed map of the anatomical structure of neural system of human, and are also clear about the biochemistry behind generating and transmitting signals. Although the mechanism behind information processing remains obscure, some characteristics of biological information processing are well agreed. These characteristics make optimization control methods questionable. Here we list several major questions[Glynn, 2003].

Accessibility of Sense and Control For dynamic CMS research, the pitfall is to tackle motor control problem in purely mechanical viewport. In biological viewport, Motor Control is a complex process involves many chemical, electrical and mechanical effects. Pure mechanical viewport is narrow-minded and lacks the ability in explaining the complexity of natural motion. Many mechanical parameters, like force, distance and angle, can only be sensed approximately, others variable like mass and inertia, human have not direct sensing ability. For some control variables like toque, neural system has no direct control access. For the biological system, value of many crucial parameters and variables are not accessible.

Time Delay and Slowness Neural signal transmitting speed is slow; and there is a long delay between neural signal firing and force generation in muscles. It is impossible for neural system to carry out complex computation for optimization in real-time time.

Nonlinear, Time Varying and Noisy Besides the delay and slowness, the neural signals are also noisy. The body structure and environment are also nonlinear, noisy and time varying. So methods that are sensitive to model accuracy are not proper for the natural neural control system.

Limited Neural Activity Current research evidences and common life experience show that motor control involves little control effort. Many experiments show motion can happen even without brain input.

Besides the questions from neural science research under microscope, there are also questions from evolution and development. Following the idea of optimization control, an animal living in a more complex environment and with a more complex body form must possess a much great computation power. In the mechanical view, the dynamics of fluid environment and deformable body structure are more expensive to compute. But most primitive life forms live in the sea and have limited intelligence. And many animals include human exhibit complex motion behaviours at very young age, before the intelligence system is fully developed. If we expand our viewport, many complex motion abilities like breathing,heat beating and child bearing are inborn. There is no need for learning or intelligent effort.

Despite the complexity of body structures and environment, the natural motor control strategy seems relatively simple, involves little computational work, and outperforms optimization methods. The current idea of biology research is that motor control is a low level intelligent activity and can be controlled with primitive neural structure. In many animals, the active neural structure in motor control is the Central Pattern Generator(CPG) which generates rhythmic signals. Current biological idea is motor control based on low level sensorimotor activity rather the complex reasoning ability. There are many experimental researches in robotics and biomechanics succeeded in controlling some motion with very simple strategy[Nishikawa et al., 2007]. But at current, we lack a theory and it is questionable whether this idea is scalable for more complex structure and motion tasks.

3.1.3 Biological idea about Motion Model

Biomechanics research shows many interesting mechanical characteristics of natural motions. The experiment results and their explanations shed light on the mechanisms of natural motor control system. Some important researches challenge the idea that trajectory motion model.

Uncontrolled Manifold Hypothesis The observation of blacksmith's hammering motions show that even under the same conditions, the motions still vary. An explanation is the neutral system doesn't control all the DOFs. Some DOFs are not controlled and freely influenced by the environment. This is the Uncontrolled Manifold Hypothesis(UMH)[Latash, 2008]. In this viewpoint, the result of motion planning is not a trajectory, but a space of valid trajectories. An important question facing the UMH is how to find the proper manifold.

Equilibrium Point Hypothesis Equilibrium Point Hypothesis(EPH)[Feldman, 1986]can be seen as a further development of UMH. This idea comes from properties of differential equations. For a dynamic system

$$\dot{q} = H(q)$$

the equilibrium points q_e satisfy the condition $H(q_e) = 0$. For a stable system, over the time the state q will approach to the equilibrium point q_e and finally stays at q_e. EPH suggests that what the neural systems controls is not trajectory, but the equilibrium points. This idea is very attractive for several reasons.

1. Environment perturbation will affect the motion trajectory but will not affect the final motion result. The method is adaptive and will not suffer from the sensing noise.
2. For EPH, motion is a passive effect of equilibrium point shift. The passivity provides a different approach of high energy efficiency. When moving passively, the motion will follow the lowest energy path.

3. Solving q_e involves much less computation work. So the method is more computational efficient.

Impedance Control Hypothesis Impedance Control [Hogan, 1985] refines the idea of EPH by providing an explanation for effects of the extra DOFs. At an equilibrium point q_e,

$$H(q_e) = 0$$

Impedance Control proposed that the extra DOFs provide a way to control the stability and admittance of the equilibrium point q_e. The mathematical presentation is

$$H(q_e + Er) = K \tag{3.1}$$

where Er is the offset error vector, K is stiffness matrix or impedance. If K positive, q_e is unstable, characters will change his posture; if K is negative, q_e will be stable, posture can be maintained. if the value of K is large, the posture will be more stiff and rigid. if K is small, posture will be more gentle, and perturbations will cause a large offset error. Neural system will tune the direction of K according to the motion purpose, such as avoiding obstacles and risks. Experiment [Franklin et al., 2007] shows that the matrix K has anisotropic properties.

UMH,EPH and IMH are efficient at explaining some arm motion and object manipulation tasks, but the theory is incomplete. In real life, the properties of H and equilibrium point q_e are more complex. H may change over time because of the environment; the motion is unstable in nature and $q(t)$ doesn't terminate, even the number of equilibrium points q_e is infinite. The relationship between equilbirum points q_e and motions are not clear.

But we still can borrow some ideas from these hypothesises.

1. Motions are not simple trajectories.

2. Motions can be generated in a passive way, thus become efficient.

3. Stability is a key factor and is also controlled; adaptation can be seen as a side effect of stability.

A generalization theory is proposed as Morphological Computation Theory(MCT)[Nishikawa et al., 2007, Pfeifer and Iida, 2005], it has a different idea about the role neural system plays in motor control. The idea is both the body structure and the environment play a crucial role in motor control, they can be treated as a physical computer which solve low level control problem. It is not necessary for neural system to plan motion in every details, the neural system only needs a strategy to utilize intrinsic properties. For some motion tasks, basic motion patterns are generated by body and environment, the neural systems only maintains or tweaks such motion patterns.

In biological research, one important evidence of MCT comes from the fact that animals moves in the same environment and in a similar way have similar body shape despite their different position in the evolutionary chain. For example, the shape of whales are very similar to fishes.

3.2 Mathematical Foundations for Qualitative Control Theory

MCT can explain many natural motions, but it lacks a mathematical model and can not be used for engineering purpose. The Qualitative Control Theory inherits the ideas of MCT, but completes the intuitive ideas with precise mathematical description. thus made it possible for engineering purpose.

In qualitative control theory we believe that the interaction between the body and environment provide some basic pattern of motion, the interaction is **motion primitive**. What makes motion primitives special is that motion primitives have special properties which automatically generate motion adaptions and maintain motion stability.

Before we privide further mathematical defination, we discuss some intrinsic properties of motion primitives.

Adaptation is a complex motion property, understanding the adaptation ability of motion primitive is the key to understand biological motor control. A classification of the adaptation may help to solve the problem in the dived and conquer manner. From our perspective, motion adaptation can be seen as two sub capacity.

State Adaptation State Adaptation is the properties of motion adaptive to impulse perturbations, like a push when walking. When adaptive behaviour happens, we assume the body structure and the environment are constant. when the system is described as an equation, such perturbations affects the state variables.

Structure Adaptation Structure Adaptation is the property of being adaptive to body and environment changes, for the walking example, such perturbations include cripple leg or the change of the slope. It has long time effect. When the system is described as an equation, structural perturbation affects the coefficient of equations.

We propose that structure adaptation is more fundamental for biological system, but neglected by current CMS research.

Further we have to understand the the mechanism motion primitives achieve adaptation. There are two possible methods to achieve the adaptation.

Active Feedback Method The first idea is to detect the perturbation and counteract it. This method is widely adopted by many current CMS research. Feedback control will cost extra energy and computational time, besides feedback method depends on the sensing accuracy.

Passive Stability Impedance Control provides an alternative idea of control by change the body structure. With higher degrees of freedom, neural system can choose a posture that limits the effects of perturbation. Body can maintain its motion passively without any extra control effort. The adaptive motion can be seen as a passive effect, thus is energy efficient and agile.

Given properties of neural system, we believe the passive method is more fundamental, an active method should be based on the passive method. It is not possible for neural system accurately sense and counteract the perturbation. What is possible for neural control system to do is to select a more stable posture.

In Qualitative Control Theory, motion primitives are dynamic system that can achieve structural and state adaptation via passive stabiltiy. In mathmatics, such systems are **structural stable autonmous systems**

3.2.1 Mathematical Model Overview

This section develops the mathematical conceptualization of Qualitative Control Theory. A clear mathematical definition of adaptation will help to identify the passive stable system and identify the key factors for stability control.

Some mathematical background is needed in this discussion. Typical discussion of dynamic system is based on the analytical based; this discussion is more in a geometrical viewport, and relies on topology concepts. This idea is usually referred to as "qualitative dynamics", This idea can be traced back to Poincare[Poincaré and Magini, 1899, Poincaré, 1885] and recently developed by the Smale School. Please refer to other books and lectures such as [Abraham and Marsden, 1978]for introduction in details.

The logical flow is as follows, motions are modelled as differential equations. Dynamic equations can be transformed into differentiable manifold. By analyzing the topological structure of the differentiable manifold, we get the qualitative properties of motion. At first basic concepts are introduced for following discussions.

3.2.2 Basic Concepts

The configuration of system is described using state value in the state space. we represent the state of a system as a vector q, M is the state space, which is a manifold. The motion trajectory over time is $q(t)$.

For a dynamic system, $q(t)$ is not in an explicit form, usually in the form of ordinary differential equation. For motion synthesis, the equations usually take the following form.

$$\dot{q} = F_u(q) = F(q, u), q \in M \tag{3.2}$$

where u is the control effort. F is determined by the system's natural property. If $u = 0$, no control effort is applied. Such systems are **autonomous systems**.

For every point $q \in M$, F and u determines a derivative vector $\dot{q}$. All the vectors over the full space of M form the **vector field** V.

Equation (3.2) can be transformed into a geometry structure. We rename the state space as phase space. F will determines a geometry structure, a differentiable manifold.

The motion trajectory can be found by apply the integral operation on the vector filed V.

$$I : M \times V \rightarrow M$$

The result trajectory is defined as **flow** Φ, all the flows form another geometry structure, the **phase portrait**, which illustrates all the possible motions of the dynamic system.

A different u will result a different differential equation F_u, thus a different vector field and phase portrait. Thus u **has a global effect**. In classical control theory,we usually neglect effects of u on the curves far way from the predefined one, thus it is local method.

An illustrative example repeatedly used in this report is the mass-spring system. After linear transformation, a linear mass spring system can be described in canonical form (3.3)

$$\ddot{x} + x = 0. \tag{3.3}$$

where x is the position of the mass, $\dot{x}$ is the speed, and $\ddot{x}$ is the acceleration of mass.

If we chose the state variable $q = [x, \dot{x}]$, the ODE model should be

$$\dot{q} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} q$$

The phase portrait is shown in Figure 3.1

On the phase plane, for most area, the flows will not intersect. Flows can only intersect at some special position. Basically, the intersection may have following property.

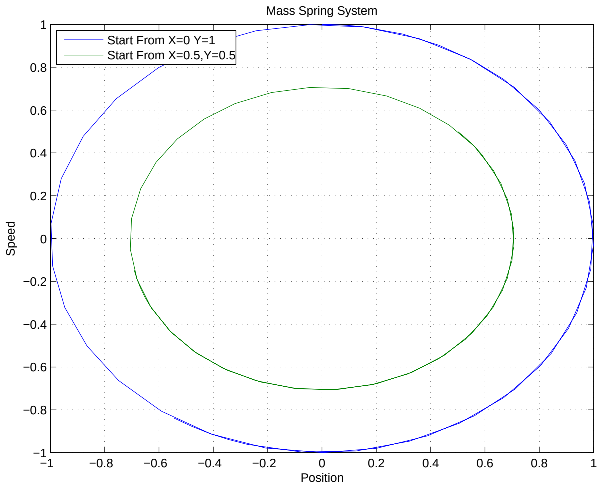


Figure 3.1: Phase Portrait of the Mass Spring System with only two flows.

Fix Point The first type of intersection is fix point or equilibrium pointq_e.

$$H(q_e) = 0$$

Period Flow Another type of intersection is a periodic flow. For any point q on the circle.

$$H(q(0)) = H(q(T))$$

Chaos The structure of such intersection is very complex, maybe be have fractural structure. Because chaos have little application value for motion, we neglect chaos in the following discussion.

Intersections like fixed point and are also called **equlibria**, if we want to include the chaos, insectional position is also called nonwandering points

At each **equilbria**, the local space can be divided into three subspace of submanifold: centre submanifold, stable manifold, and unstable submanifold.

centre submanifold If a flow θ pass through a point m on centre submanifold W_c, flowθ will remain on the Centre Manifold

$$\theta_c(t) \in W_c, t \in R$$

An equilibria must be on center manifold.

stable submanifold For the flow θ_s passes through a point m on stable submanifold W_s, the flow will finally converge to a nonwandering point on centre submanifold.

$$\theta_s(+\infty) = \theta_c$$

unstable submanifold For the flow passes through a point m on unstable submanifold W_u, the flow will be repelled from the nonwandering points on centre manifold. An alternative perspective is the inverse of the flow converge to nonwandering point.

$$\theta_u(-\infty) = \theta_c$$

The size and dimension of each submanifold varies. For some cases, the W_s (W_u) may not exist, this can be seen as the dimension of $W_s(W_u)$ is 0. **Attractors** are the equilbria where the whole local space is stable, the dimension of unstable submanifold is zero $\mathbf{dim}(W_u) = 0$. **Repellors** are the equilibria where the whole local space is unstable,the dimension of stable submainfold is zero $\mathbf{dim}(W_s) = 0$. For nonlinear system, globally, the shape of stable and unstable submanifold may be bending and connect with itself or each other. The unstable manifold of one equilibria may be the stable submanifold of another. The equilibra and its connectivity sub mainifold form a topological structure. Thus the phase plane will be divide into different regions,result in a celluar structure. there is only one attractor, all the flow in this region will converge to the attractor. and the corresponding region is called basin of attraction.

3.2.3 Redundant Problem of motion and Rank of Mapping

Given a differentiable manifold, finding the flow ϕ is the "Forward Problem". $q(t)$ can be calculated by applying numeric method iteratively, which has determined solution and low computation load.

Motor Control is a typical inverse problem. In the trajectory planning framework for CMS, motor control is to find proper $u(t)$ that can generate the desired $q(t)$. This problem can be formulated as finding a control function C.

$$u = C(q(t)) \tag{3.4}$$

Which means given a flow ϕ , try to find the proper differentiable manifold, this can be defined as the following mapping process.

$$C : M \to M \times V \times T$$

C can be seen as the inverse of integrator mapping I.

In practice, the inverse problem is normally solved by iteratively linearizing the original system:

$$\dot{q} = F_l(q)u$$

where $F_l(q)$ is in the form of a matrix. Thus, we can have a linearized solution u_{fi}

$$u_{fi} = \dot{q}(t) * F_l^{-1}(q) \tag{3.5}$$

Nondeterministic solution happens when $F_l q$ is ill conditioned, This will result in either no control solution or too many solutions. Optimization method can be used to overcome the ill condition of F_q. When there is no possible solution, optimization tries to find the solution with least error.

$$\arg\min_u (q(t) - F(q, u))^2$$

When there are too many solutions, we try to find the solution with least energy cost

$$\arg\min_u E(u), E = \int_{t0}^{t1} u^2 dt.$$

However this method lacks stability, it is sensitive to errors. The variety and uncertainty of perturbation will affect the Fq in an unexpected way.

3.2.4 Adaptation and Stability

The passive stability is the foundation of Qualitative Control Theory. Clear description of passive stability is crucial for the proposed QCT. A mechanical system can be extremely stable without any control effort. A simple example is the mass-spring system with damping effects, which can be transformed into equation(3.6).

$$\ddot{x} + \dot{x} + x = 0 \tag{3.6}$$

For system of equation(3.6), no matter what kind of perturbation on the initial conditions, it will continue the rhythmic motion and will eventually stop at the position $x = 0$, $\dot{x} = 0$. This kind of stability is close related to the **state adaptation**. For the mass-spring system, this is because of the topological structure. On the phase plane, there is only one attractor and its basin cover the whole state space.

In fact, the damping mass-spring system is "a super stable system". It can still keep stable even when you change system's parameters: change the coefficient of stiffness or damping may change the motion, but will not change the final state of the mass. In face, the stability of mass-spring system is rough stability or structure stability [Andronov and Pontryagin, 1937]. **structural stability** is the reason behind **structural adaption**.

structure stability In mathematics, structural stability is a fundamental property of a dynamical system which means that the qualitative behavior of the trajectories is unaffected by C^1 small perturbations.

Andronov-Pontryagin criterion points out that structural stability is property of topological structure. It is better to understand such rough stability or structure stability by investigating the topology structure of the system[Jonckheere, 1997]. For system (3.6), the important qualitative properties are that all the flows on the manifold ends at the position $x = 0, \dot{x} = 0$. The corresponding topology structure is that there exists only one attractive equilibrium position on the planner plane manifold.

The mass-spring system (3.3) show in figure 3.1 have a different topology structure. We can plot many trajectories; this system has the following qualitative properties:

1. all the trajectories are circle.

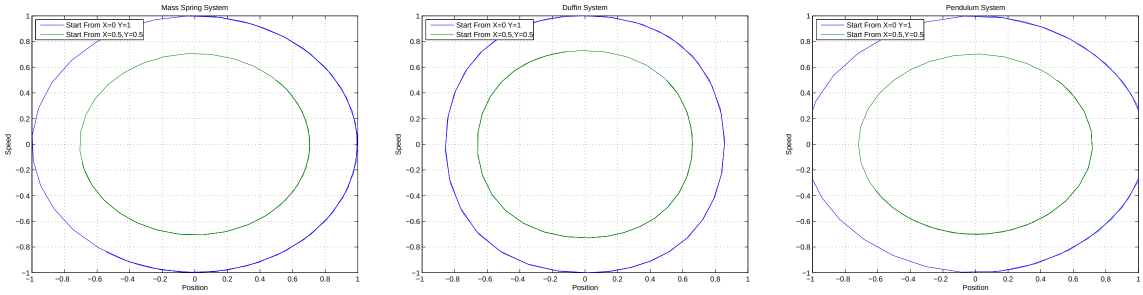
2. no two trajectories will intersect.

The corresponding topological structure is that there exist an infinite number of periodic flows and they are neither attractive nor repelling.

Motions vary greatly, this makes it difficult to define motion and tell it from another. For the walking motion, we must find a mathematical method to identify the similarity between a fast walk and slow walk and separate slow running from walking. In sports games, walking and running are separated by qualitative criteria: when heel strike happens, two feet on the ground is walking, else is running.

In Qualitative Control Theory, motion should be defined by the the topological structure. From topology viewport, motion adaptation can be modelled as homeomorphism. Homeorphic flows can be generated if the differentiable manifolds are homeomorphic, which means they share the same topological structure.

Structure stable autonomous systems have the ability to maintain its topology structure under perturbations. They generated homeorphic flows while keep the qualitative property, which means motion is adaptive but qualitatively maintained.



(a) $\ddot{x} + x = 0$ (b) $\ddot{x} + x + x^3 = 0$ (c) $\ddot{x} + sin(x) = 0$

Figure 3.2: Nonlinear distortion effects on the basic mass spring system, the topological properties are kept

In fact, the effects of control and perturbation can be modelled in the same way. A autonomous dynamic system is represented as,

$$\dot{q} = H(q)$$

Considering the control and perturbation, the system will be changed into

$$\dot{q} = H^{'}(q) = H(q) + f(q,t)$$

Where $f(q,t)$ is used to model both the control and perturbation.

This phenomenon is illustrated in Figure 3.2. Figure 3.2(a) is the original linear system of mass-spring, Figure 3.2(b) shows the effect of distortion signal x^3, Figure 3.2(c) shows the effect of perturbation signal $sin(x) - x$.

These three system ahve different flow shapes and different manifolds. But on the key domain $D^1 \times D^1, D = [-1,1]$, all the three systems share the same topological structure.

Geometrically, the effect of $f(q,t)$ can be seen as a deformation to the trajectory $q(t)$ on the phase plane. We can define a nonlinear transformation of the coordinate system.

$$q^{'} = \Theta(q)$$

such that

$$H^{'}(q) = H(\Theta(q)) = H(q^{'})$$

Figure 3.2 can be seen as an illusion of simple motion adaption.

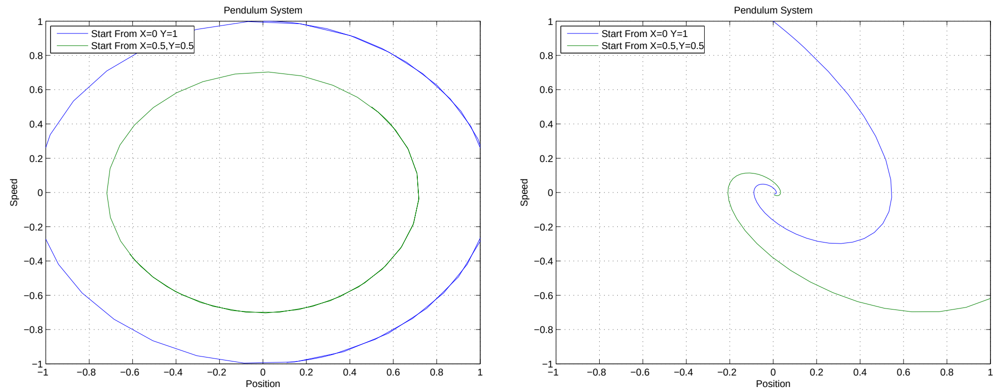
Thus we reach our conclusion. Motion can be defined precisely by the qualitative property which is determined by the topology structure. Agile and energy efficient properties can be maintained by forming autonomous system, the motion properties are generated by topological structure. Adaptation can be generated by structure stability, which is also determined by the topology structure. Topology structure of the body environment interaction is the most important properties of motion.

3.2.5 Motion Pattern Shift and Bifurcation

Besides maintaining motion, CMS also needed a mechanism to shift motion pattern. A human needs the ability to maintain walking, he also needs the ability to stop walking.

when control is applied, the system is transformed into

$$H^{'}(q) = H(q) + U(q,t)$$



(a) original system

(b) controlled system

Figure 3.3: The adaptive effects generated by topological manipulation.

Motion will be determined by $H^{'}$.

Within the Qualitative Control Theory, The concern of controller is not to follow the trajectory q(t), but to modify the system H into a system $H^{'}$ which possesses desired topological structure. This can be achieved by changing the control parameters qualitative, in mathematics, this is the **Bifurction** phenomenon.

One example of topology manipulation can be seen in Figure 3.3. If controller adds damping effects to the system, it will turn the original system

$$\ddot{x} + \sin(X) = 0$$

into

$$\ddot{x} + \dot{x} + \sin(x) = 0$$

By changing a parameter slightly, the two systems have very different structure. Result in qualitative different flow curve shape.

3.2.6 A Mathematical Description of Qualitative Control Theory

Small Control Perturbation has the effect change the area and position of the basic of attraction as shown in Figure 3.2. Big Change in parameters may have the effect of qualitatively change the topological structure as shown in Figure 3.3. Based on these two facts, we can give a detailed explanation about Qualitative Control Method.

In Qualitative Control Theory, only final motion results are concerned, In mathematical viewport, only the attractors of flows are controlled, flow is not concerned in motion control. Also, according two the type of attactors, motion can be group into two groups.

Reach Motion Such motions have fixed attractors, typical motion include posture control and picking up motion of the arm.

Peridotic Motion Such motion have peridotic attractors, typical motion include walking, running and heartbeating.

Motions are made up of motion primitives. Neural control system tweaks the basic motion primitives to achieve specific objective. From mathematical viewport, neural tweaking affects the autonomous system in three ways.

- Qualitatively change the topology structure of the autonomous system shift motion primitives.
- Change the position of the attractors for final motion result.
- Change the coverage of the attractors basins for tolerance of motion.

The motion control task is divided into the following way.

High Level Control The high level control in charge of selected a sequence of posture or primary motions for the motion task.

Low Level Control The level controller shifts the stability of postures or primary motions, ensure the final task can be finally finished.

Motion Details Motion details are not planned or controlled, they vary according to the perturbation.

Qualitative Control Theory preserve the natural motion features.

Adaptive Using this method, different perturbation will result in different motions. Motion will vary with the environment change.

Enegy Efficient Motion will be generated passively and follow the least energy path.

Easy Control QCT does not rely on high precise calculation. It is possible to manipulate the topologcal structure with some very simple methods.

3.2.7 Explanation of Natural Motion with Qualitative Control Theory

It is reasonable for natural animals to rely on structural stable autonomous system. In natural environment, perturbation and uncertainty are everywhere. In many cases, if neural system can't respond quickly enough, the better solution is to select a more structure stable configuration before action.

Qualitative Control Theory may help us understand the evolution of locomotion and neural control, which is discussed in subsection 3.1.2. Animals shift from the sea to the land. From quantitative computing viewport, the natural dynamics of the body and fluid environment are hard to predict or compute precisely. From the qualitative viewport, fluid is continuous and uniform, the topology structure is very simple. It is similar to the spring system with only one equilibrium point, and the structure stability is very good. With little neural control fish can maintain its posture. On the other side, for human walking, although the rigid like environment can be calculated precisely, the topology structure is much more complex. On the phase plane, there exist many equilibrium points, the structure of stable submanifold and unstable submanifold are more complex. The control system has to be more complex to control the more complex qualitative properties.

Qualitative theory can also help to explain another fact of biological motion system. Animals that live in similar environment and moves in a similar manner usually have similar body structure, in spite of their different position on evolution chain. This is because animals moving in a similar is based on the same motion primitive, similarity in body structure promise the dynamic systems have the same topological structure.

3.3 Modeling the Neural Control System

The qualitative control theory allows more freedom for controller design, but it does not necessarily promise natural looking . For motion synthesis application, it is a good method to mimic the behavior of biological neural control system.

In nature, an animal's body and environment can be extremely complex. it leads to high dimensional manifolds with complicated topological structure. For CMS application, one question we want to ask is the whether complex system can be controlled with a simple method. Biology Research suggested that the motor is mainly controlled by the Central Pattern Generator, which is a small autonomous network that generating rhythmic signals. The existence of CPG is very common, primitive animals like lamprey and fish, to high level animals like bird, mammal and human[Cohen, 1988]. It is found that when child learns to walk, the crucial knowledge is the rhythm and strength of the walk.

The idea of control motion by rhythmic signals can be modelled as entrainment [González-Miranda, 2004]. In this section, we provide the understanding of biological entrainment in the Qualitative Control Theory.

3.3.1 Natural Motion and Periodic Vibration

The ship example are good at explaining continues dynamic system with attractive fixed attractor. While in nature, the dynamic system may be time varying, discontinuous.

A simple example is a ball bouncing on the ground. The motion of the ball can be separated into two phases: the flying phase and the bounding phase. The flying phase can be described as a free falling motion.

$$\ddot{x} = g, x > 0$$

where x is the vertical height, and g is the gravity. When the ball hit the ground, we need a different equation,

$$\dot{x}_{+} = -e\dot{x}_{-} \tag{3.7}$$

By equation(3.7), we suppose that collision happens instantly, $\dot{x}_{-}$ is the speed before collision, $\dot{x}_{+}$ is the speed after collision, e is the collision efficient, which will be less than one.

Figure 3.4 shows the position and phase plot of the bouncing ball.

Bouncing ball is a simple example, but it captures the complexity of dynamics when animal interacts with the environment.

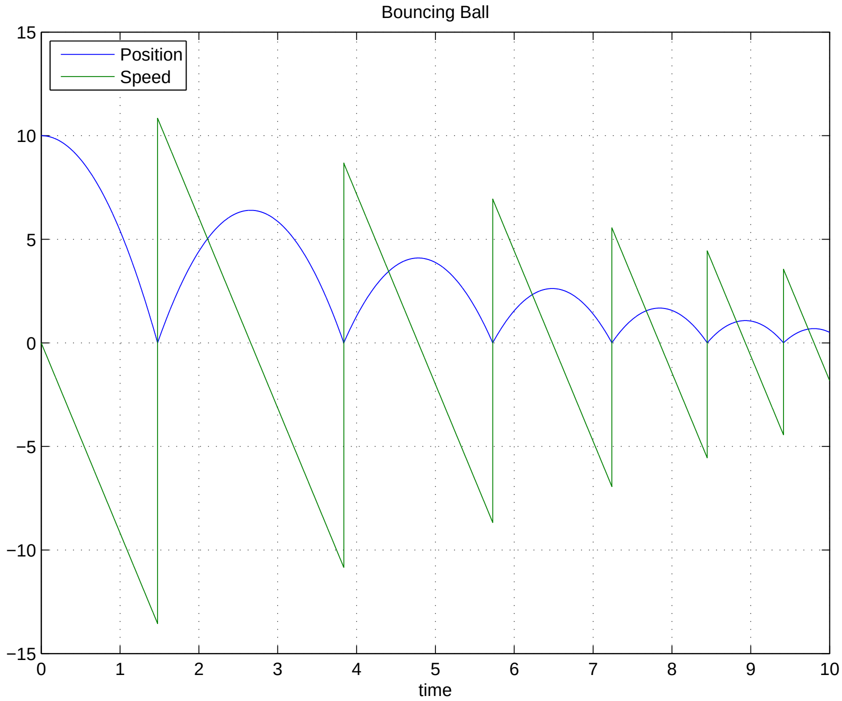
In previous section, we discussed two types of attractors: fix point and Limited Circle. We propose that limit circle comes from the time varying system are treated as the major concern of this research. This is mainly for two reasons

1. Periodic behaviour is very common in biological systems. Besides the periodic motion in swimming and running, heart beating, wake and sleep also show periodic behaviour. A periodic system has the potential to integrate with other bio system simulation to explore other motion features.

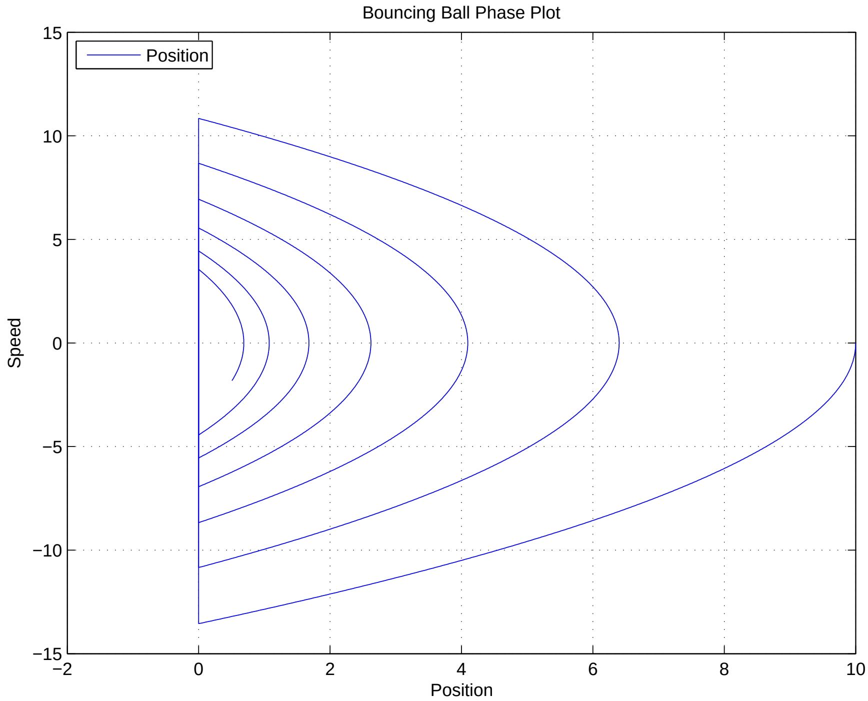
2. Periodic motion has the same effect of terminated motion when the amplitude of limited circle is very small. For CMS research, both type of motion trajectory can be simulated with periodic motion.

3.3.2 Biological based Qualitative Control Method

Modelling the body and environment is the prerequisite of control design. Current CMS method relies on model accuracy and can only be done manually, that's why they are tedious in applica-



(a) Position



(b) Phase Plane

Figure 3.4: Bouncing Ball

tion. In Qualitative Control Theory, the idea of structure stability releases the controller design from high precision. Thus it can be achieved with a concise structure based on entrainment.

Entrainment is the phenomenon that two coupled oscillator systems oscillate in a synchronize way. Although the mechanism can be very complex, the phenomenon is universal. The classic example shows individual pulsing heart muscle cells. When they are brought close together, they begin pulsing in synchrony.

Entrainment will happen when coupling two oscillators with similar oscillation frequencies but with very different characteristics. A simple explanation is that energy fluctuates between the two oscillating system.

Given two systems,

$$\dot{x} = f(x)$$

$$\dot{y} = g(y)$$

when two oscillation systems couple, the behaviour of both systems will change. The coupled system can be presented with the following equation

$$\dot{x} = f(x) + m * g(y) \tag{3.8}$$

$$\dot{y} = g(y) + n * f(x) \tag{3.9}$$

where m,and n are coupling coefficients. When m is large, the behaviour of equation(3.8) is more dominated by the second term $m * g(y)$. This can be seen as a forced oscillation system. Even m is small, behaviour of equation(3.8) can be changed qualitatively. For some cases, stability can be enhanced and chaoic behavior can be suppressed.

The entrainment model can be applied to CMS, if we took the neural oscillator as $f(x)$ and body as the mechanical oscillator $g(x)$. The properties of mechanical oscillator can be controlled by the oscillation property of neural system, the oscillation of mechanical system can be controlled by the oscillation of neural system. If we increase n, we can expect that mechanical oscillators shows the behaviour of neural oscillators. Entrainment can help to boost the stability of the mechanical oscillation. When the mechanical oscillation is disrupted but the neural oscillation remains, after the perturbation is removed, the undisrupted neural oscillation can drive the disrupted mechanical oscillation back to normal.

The phenomenon of entrain can be closely related to frequency domain method. To control entrainment synchronization, the control parameters are the oscillation frequency and couple coefficient, which are the frequency and amplitude control in MSP research.

3.3.3 The Structural Stable Oscillation of the Neural System

Although it is difficult for neural system to carry out complex computation, it is easy to build oscillator structure with neurons. It only needs two neurons with mutual inhibitive property.

The dynamic of a neuron can be model as a dynamic equation:

$$\dot{S} = L(S) + I(u) \tag{3.10}$$

Where S is the output electrical signal of neuron and u is the input signal from other neuron. An inhibit input I has the property that when $u1 > u2, I(u1) < I(u2)$.

One extensively studied oscillation model is developed by Matsuoka [1985]. The mathematical presentation is as follows:

$$\tau_1 \dot{x_1} = c - x_1 - \beta v_1 - \gamma [x_2]^+ - \sum_j h_j [g_j]^+ \quad (3.11)$$

$$\tau_2 \dot{v_1} = [x_1]^+ - v_1 \quad (3.12)$$

$$\tau_1 \dot{x_2} = c - x_2 - \beta v_2 - \gamma [x_1]^- - \sum_j h_j [g_j]^- \quad (3.13)$$

$$\tau_2 \dot{v_2} = [x_2]^+ - v_2 \quad (3.14)$$

$$y_i = \max(x_i, 0) \quad (3.15)$$

$$y_{out} = [x_1]^+ - [x_2]^+ = y_1 - y2 \quad (3.16)$$

where x and v are state variables of the oscillator, τ,c,β,γ are parameters of the oscillator.

Matuoka oscillator is an autonomous oscillator; it can begin to oscillator without any control effort. Figure 3.5 shows the natural oscillator output.

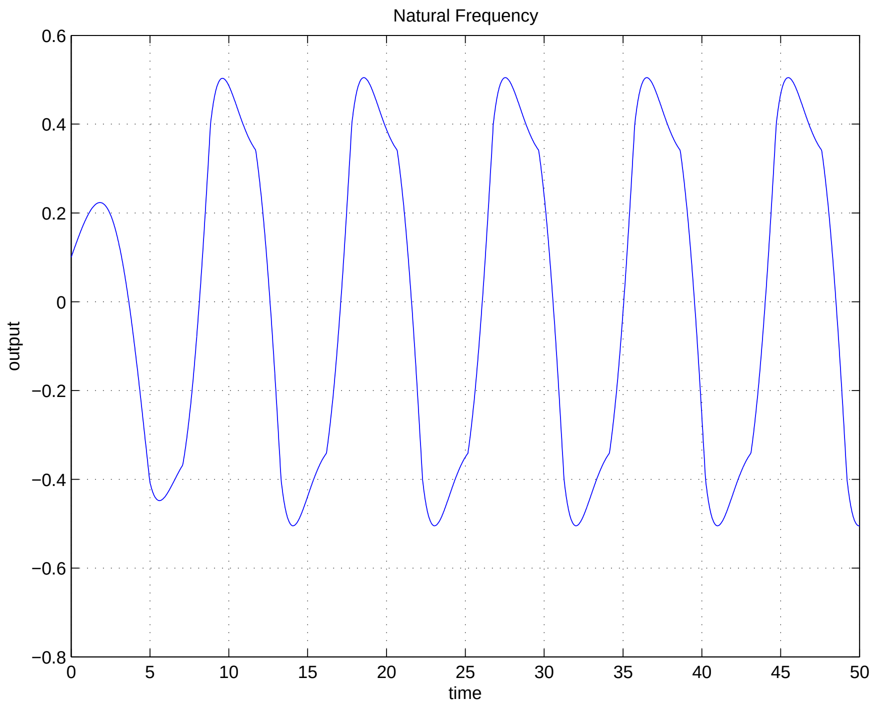


Figure 3.5: Natural Oscillation

It is also adaptive; entrainment behaviour can happen between one Matuoka oscillator and different oscillators. Figure 3.6 shows the entrain oscillation, where the oscillation Matuoka oscillator synchronise with the input signal. But because of the nonlinear properties, its behavior is not completely understood. Matsuta[Matsuoka, 1987] explains the adaptive properties from the location of the roots of characteristic equation. Wilimas[Williamson, 1998] explains the properties in frequency domain.

In our research, we find some important properties of neural oscillator by investigating simulation results.

From our simulation, we investigate the topological structure. Basically, neural oscillator shows three important properties:

- Simple Topological Structure. The topology structure of neural oscillator is simple, it includes one attractive limit circle and one fix repellor.

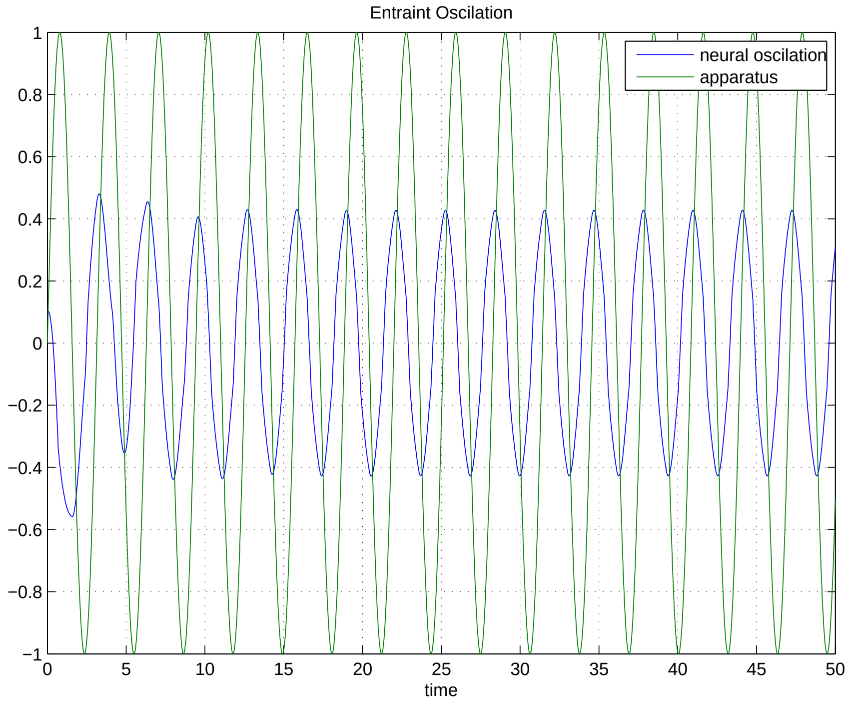


Figure 3.6: Entrainment Oscillation

- Large Basin of Attraction. All the simulations we carried out converged to the same limited circle.
- Fast Converging Speed. In most of the case, the flow will converge to the limit circle within one period time.

Features above are shown in Figure 3.9.

The large area of basin of attraction means the final behavior is totally determined by parameters. Initial condition will have no effects on the oscillator final output. The converging speed can be seen as quick recovery ability. When an impulse perturbation happens, it will recover in one period time. These properties are very valuable in CMS research. For locomotion, besides the self-adjust ability and immune to initial state perturbation, the stable locomotion is unique and controlled by the parameters. Additional properties is high converge speed, Suppose human walking have such properties, it means after perturbation, people will return to normal walking within one step.

Matuskota Oscillator is a structure stable autonomous oscillator. An question we want to further ask is if we couple the neural oscillator with body and environment, can the couple system maintained the structural stable property by entrainment?

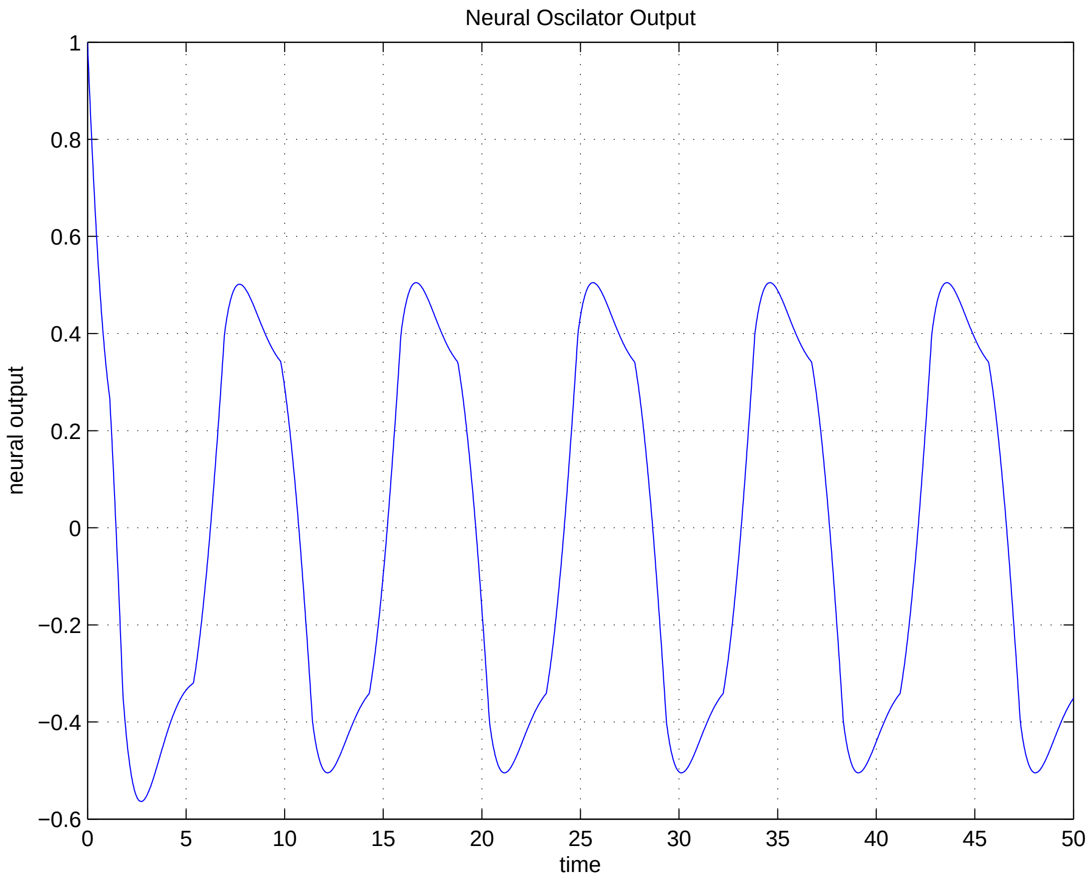


Figure 3.7: The states of neural oscilator over Time

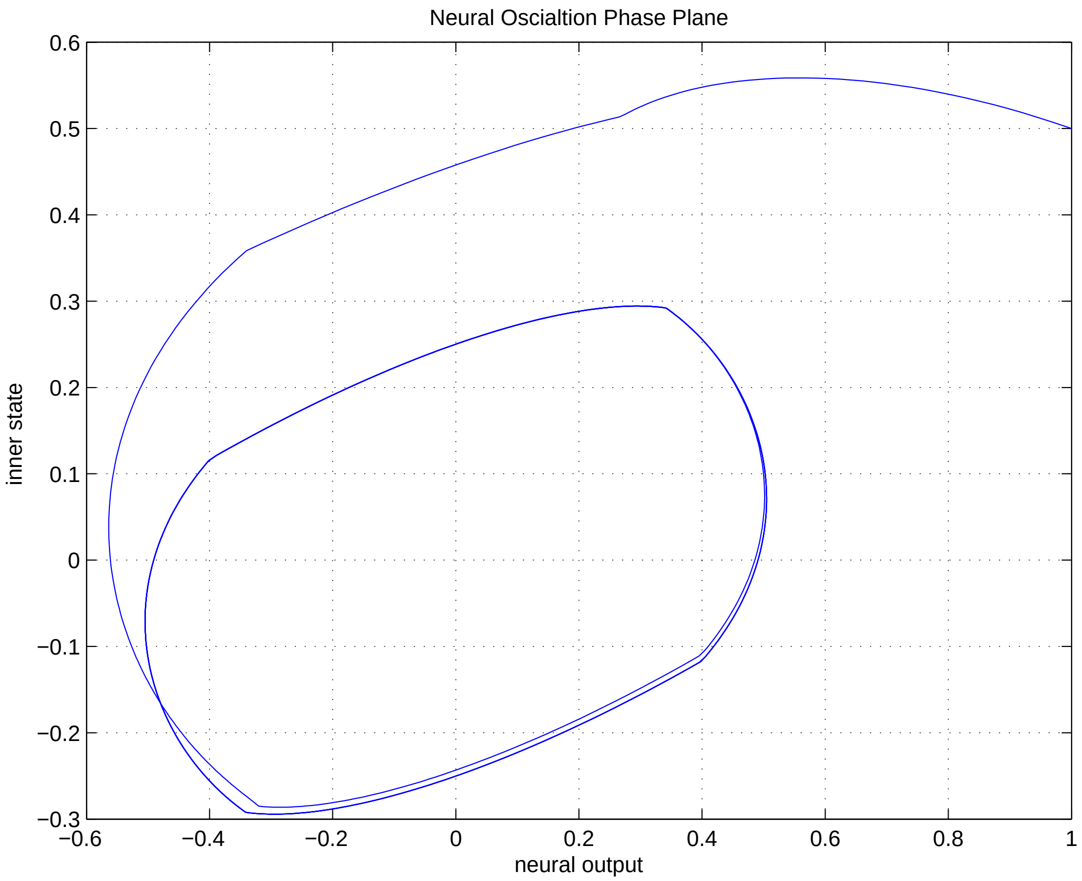


Figure 3.8: The phase portrait of Neural Oscilators

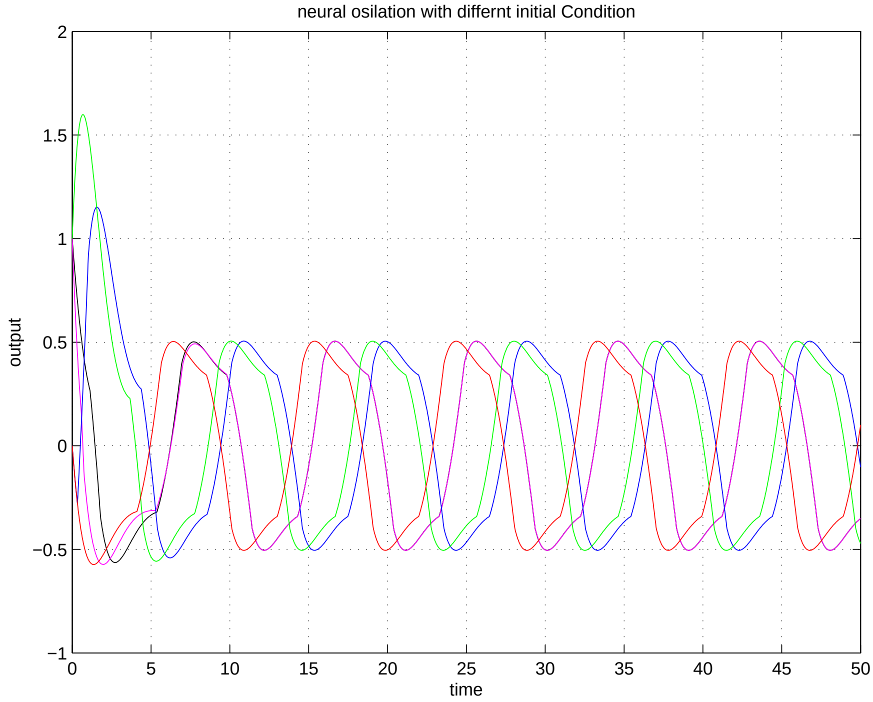


Figure 3.9: Neural output with different initial position

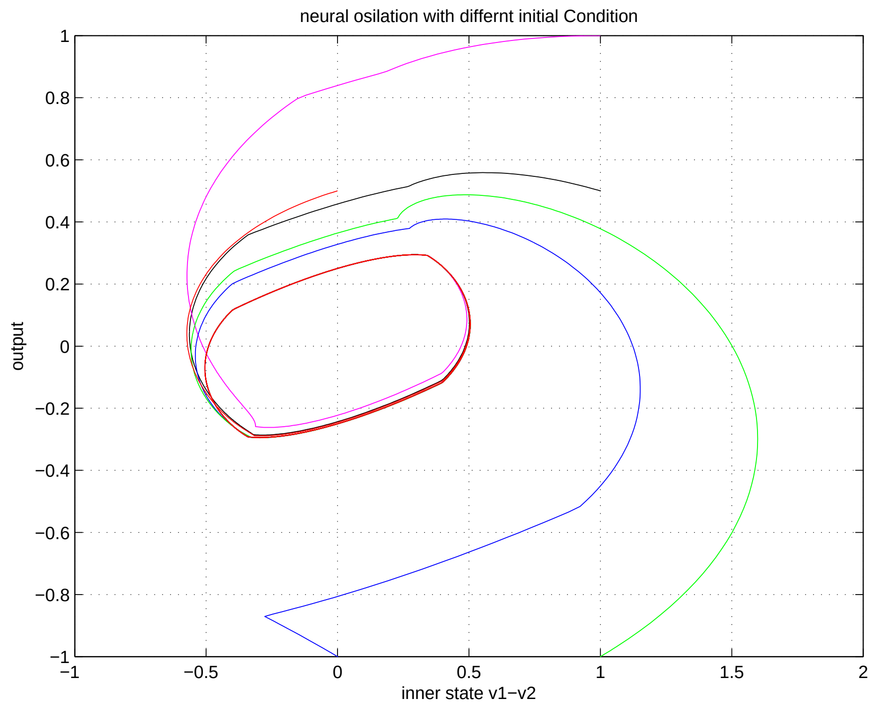


Figure 3.10: Phase plot of oscillation with different initial condition

Chapter 4

Application of Qualitative Control Theory For Character Motion Synthesis

In this chapter, we will discuss how to appy the qualitative control theory to synthesize dynamic adaptive motions for character. The concern is the balance between motion details and computational complexity. Implementation the Qualitative Control Theory for dynamic CMS is a complex task.

A full functional physics-based CMS system includes two parts.

1. Mechanical Simulation

2. Controller Designed

Besides software engineering should also be considered to ease the development and future function extension.

4.1 Mechanical Simulation

4.1.1 Body Modelling

In the QCF framework, the body modelling includes bone modelling and muscle modelling. Bones are modelled as rigid bodies. A rigid body's inherent property is determined by two values m and I, where m is the mass, I is the moment of inertia. The state of a rigid body is determined by four values

1. the position p

2. orientation q

3. the linear velocity $\dot{p}$

4. angular velocity $\dot{q}$

In biomechanics research, muscle shows complex characteristics. Muscle force contains two components: (1) active force (2) passive force[Zajac, 1989]. There are many ways to model the muscle characteristics. The four most popular models are: (1) Joint torque model[Pollard and Zordan, 2005]; (2) Linear force model[Delp and Loan, 1995]; (3) Curve force model[Sueda et al., 2008]; (4) Finite element method model[Ng-Thow-Hing, 2001].

In CMS, joint torque model and linear model are commonly used. We adopted the angular torque model for simplified motion and linear model for detailed motion. For both methods, muscle is modelled as spring. It can be controlled via two parameters, the stiffness k and rest length l.

The force model defines the relationship between muscle force and muscle parameters.

$$F_m = F(k, l) \tag{4.1}$$

To verify the adaption of the proposed research method, we will investigate two force model.

Linear Force model

$$F = k(x - l)$$

Exponent Force model

$$F = k(e^{x-l} - 1)$$

.

4.1.2 Rigid Body Simulation

Characters are modelled as linked rigid body system. There are two basic methods for simulation:

Reduced Degree of Freedom Reduced Degree of Freedom methods eliminate the degree of freedom that has been constrained. The motion equation takes the following form:

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} = F \tag{4.2}$$

where q is the vector of reduced DOFs of the mechanical system. Reduced degree of freedom method can reduce the dimension of problem and ease computational difficulties. However this method can only be used for simple mechanical structure, and only equality constraint can be well modelled[Goldstein et al., 2002].

Constraint Force Method Constraint force method does not eliminate the constrained DOFs. Instead, extra forces are added to present the constraints' effect. The equation of constraint force method takes the following form.

$$\begin{aligned} M\ddot{q} + N(q, \dot{q}) &= F + r & (4.3) \\ C(q, \dot{q}) &= 0 & (4.4) \end{aligned}$$

where r in equation 4.3 is constraint force which can be derived from the constraint equation 4.4. Constraint force method results in a high dimension equation and will bring numeric errors. But it provided a general solution for different mechanic structure.

In our research, both methods are adopted. The character is simulated using reduced degree of freedom method to explore the dynamic topological structure, and build necessary data for controller design. The environment is modelled using constraint force method.

4.2 Qualitative Control Theory Implementation

In our control framework, the concern is not the motion trajectory, but the topological structure. All the points sharing the homeomorphic flows form a region P. For periodic motion, P is the basin of attraction. Because the relationship between the parameters of neural oscillators and topology structure are not clear, we propose a data-driven based method, phase plane colouring. We pre-computed several neural control parameters, and store the resulting topological information. For one state q, under different control parameters, it belongs to different regions $P1$, $P2$, $P3$. The motion control system just picks one and set corresponding parameters for neural oscillator. Two types of techniques are adopted to solve this problem. By utilizing the periodic property, we adopted the Poincare map [Goldstein et al., 2002]method, turning continuous problem into discrete problem. Another method is cell to cell mapping[HSU, 1980] which utilize the continue property of the phase plane.

Poincaré map Periodic trajectories will repeat itself after a period of T. We can use the Poincaré map to reduce the dimension of the problem. We define a section that is orthogonal to all the trajectories and each trajectory passes it once per period. In this way, the continuous problem is turned into a discrete problem.

The system equation

$$\dot{q} = H(q)$$

can be turned into a discrete equation.

$$q_{n+1} = S(q_n)$$

Cell to Cell Map Since flows pass points in neighbourhood will show similar properties. So we can divide the phase plane into different small regions that are called cells. We assume that properties of every point in the cell can be represented by its centre. Forward simulation is carried out for all the centre points of cells.

Chapter 5

Application and Results

To verify the proposed Qualitative Control Theory, we test our method for motion synthesis.

At current, the objective is not generating a fully detailed character animation, but to develop practical routines and tools for full body animation.

Within QCF framework, this test includes three steps:

1. identify the topological structure of the motion
2. test the structure stability.
3. change the controller parameters, build a repertoire of parameters

In the following sections, three examples are shown. The first two are one degree systems, the mass spring system and bouncing ball system. One degree system is the building block for complex systems. The two examples are the simplest continuous system and discontinue system. We show that after coupling with the neural oscillator, we change the original system into structural stable autonomous system.

The third example is more complex and meaningful. It is a 2D biped walking example. The coupling of neural oscillator turned the original passive walking into a more stable autonomous system. Natural looking gaits with adaptation are generated by this method.

5.1 One Degree Of Freedom Systems

The first system is the mass-spring-damping system controlled by a nonlinear oscillator. For CMS, this example captures the essence of interaction between bones, muscles and neural systems. In this system, bones are simplified as mass; muscles are modelled as springs with damping effects. Neural oscillator simulates the effects of neural system. This idea can be extended to deal with more complex situations. Mass spring system can be used to model system with continuous dynamic properties. Typical example includes arm swing and posture control.

5.1.1 Simulation Method of Mass Spring System

Mathematically, Forced mass spring system is formulated as equation (5.1)

$$\ddot{x} + K(x - x_{os}) + D\dot{x} = 0 \tag{5.1}$$

It is a forced spring damping system. The driven force x_{os} is determined by the neural oscillator.

$$x_{os} = H_{out} * y_{out} \tag{5.2}$$

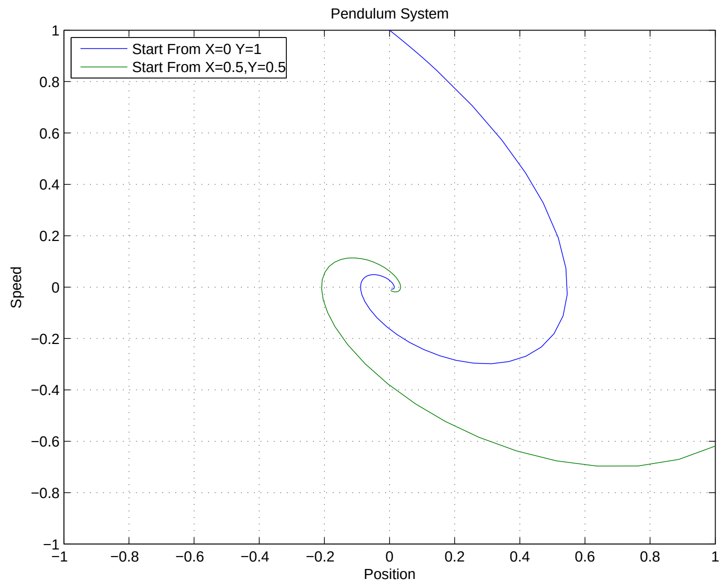


Figure 5.1: the trajectory of the damping system

where H_{out} is the couple coefficient between neural oscillator output and the mass-spring input. The neural oscillator is defined by equation (3.16).

The state values of spring damping system will be sent back to the neural oscillator as described in the following equation.

$$g_1 = h_{i1} * x \tag{5.3}$$

$$g_2 = h_{i2} * \dot{x} \tag{5.4}$$

where h_{i1} and h_{i2} determines the coupling between mechanical system and controller, g_1, g_2 are feed to the neural oscillator.

The uncoupled spring damping system shows nearly periodic behaviour. Its trajectories terminate at the position $(0, 0)$ as showed in Figure 5.1.

In our simulation, the motion equation above will be discretized and integrated with runge-kutta method[Press et al., 1992].

5.1.2 Simulation Result

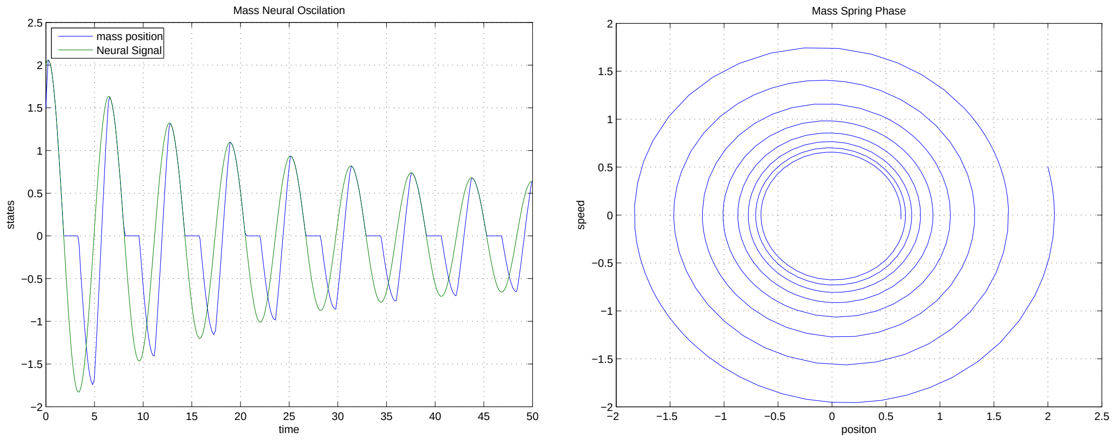
After being coupled with the neural oscillator, this mass spring system shows a different topology structure. This gives the system stability and adaptive power. Motion trajectories with different initial condition converged to the same limited circle. The trajectory of the limited circle resembles to that of the original motion. Figure 5.2(a) shows the states of the system overtime, and Figure 5.2(b) shows the phase plot.

From Figure 5.3, we found that the limit circle is attractive and basin of attraction of this system covers a large area.

5.1.3 Effects of Parameter Turning

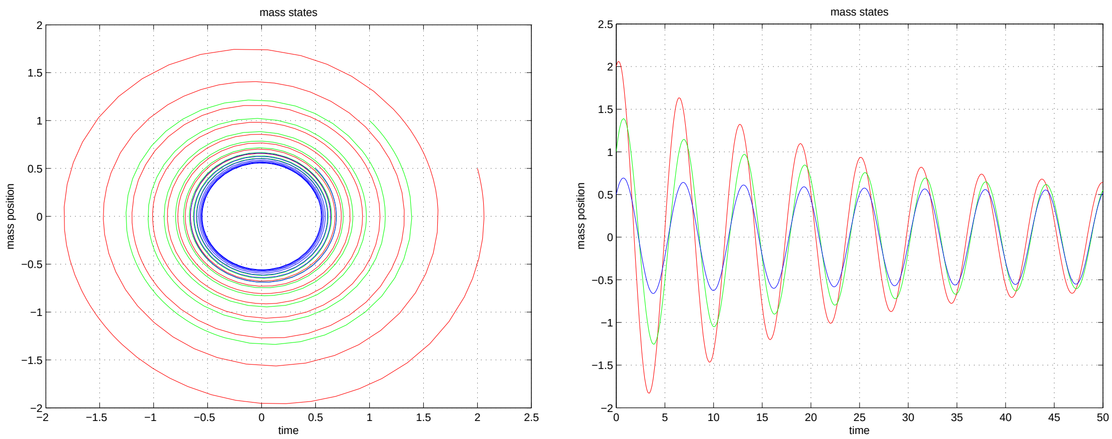
Increasing sensing coefficient H , the final oscillation will be close to mechanical oscillation.

Increasing H_{out} , the final oscillation frequency will turn close to neural oscillation.



(a) state plot (b) Phase Plane

Figure 5.2: Mass spring system coupled with a neural oscillator



(a) state plot (b) Phase Plane

Figure 5.3: Attractive Limited Circle

Increasing c of the neural oscillator , this will increase oscillation amplitude. It will not affect the frequency.

Increasing τ_1 , the frequency of oscillation will be changed. And there exists an optimal frequencyτ_{max}. No matter the frequency becomes larger or smaller than that optimal value, the oscillation magnitude will be lower.

5.1.4 Bouncing Ball System Simulation

The second example is a bouncing ball driven by a neural oscillator. This example captures the essence of complex interaction between body and environment. In CMS, the idea can be extended for simulating throw and catch, jumping, running, jiggling and walking. In our research, we found out that the neural oscillator is a very simple but robust way to control motion.

When the ball is flying in the air, the motion of the ball can be easily described with

$$\ddot{x} = g, x > x_{ground}$$

When ball hit a moving object, we assume that collision happens in short time. We apply an impulse to the ball.

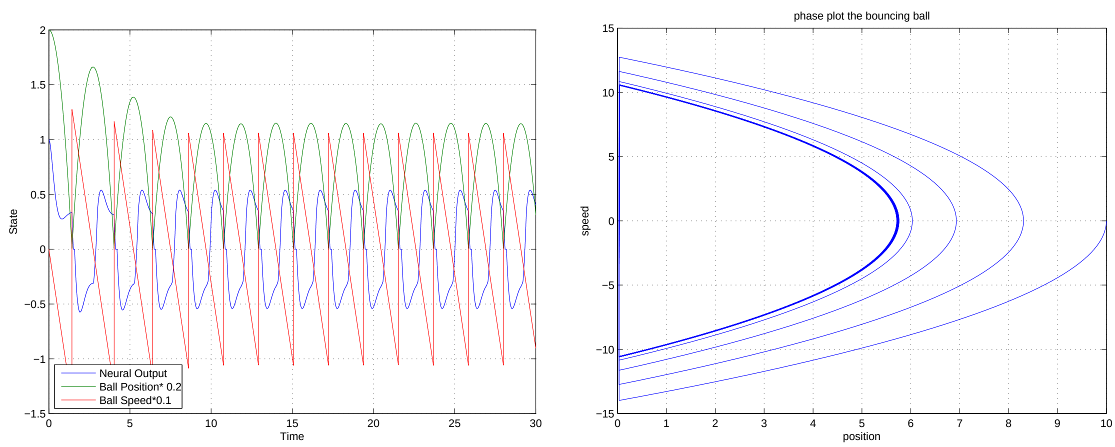
$$\dot{x}_{+} = -e(\dot{x} - \dot{x}_{ground}) + \dot{x}_{ground}$$

Because the ground is fixed, the natural topological structure shows nearly periodic behaviour showed in Figure 3.4

5.1.5 Adaptive Motion for Bouncing Ball System

Although these bouncing ball system and mass spring system are very different, bouncing ball system has the same topological structure with the damping spring mass system. Thus the bouncing ball system can be seen as a nonlinear mass spring.

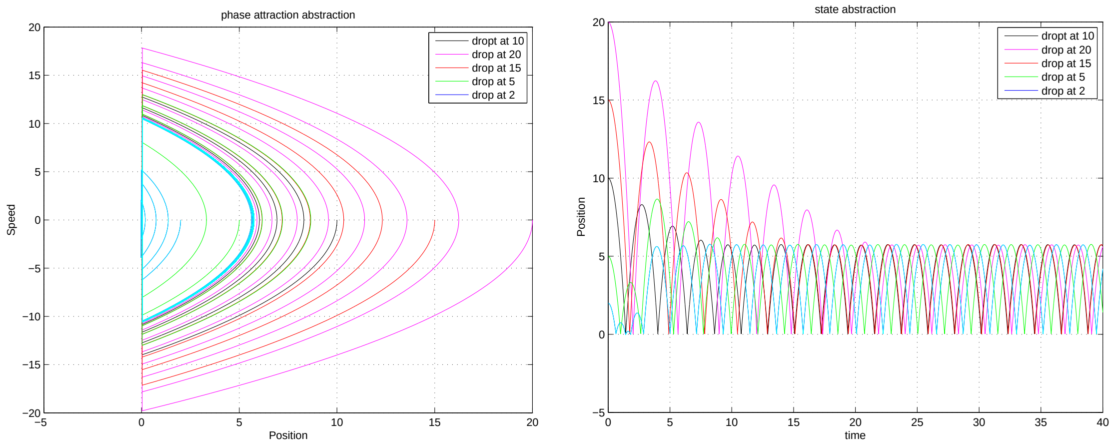
When it is coupled with a neural oscillator, the ground position x_{ground} is driven by the neural oscillator, and the speed $\dot{x}$ will be feedback to the neural oscillator as input signal. Similar to the mass spring system, the bouncing ball system converge to an attractive limit circle. The entrainment behaviour is plotted in Figure 5.4.



(a) state plot (b) Phase Plane

Figure 5.4: Attractive Limited Circle

The bouncing ball system also shows a large area of attraction. In Figure 5.5 the trajectory of the bouncing ball system converges to the same limited circle. This means if we drop the ball from different height, after several period, all the balls from different drops will bouncing in the same period and reach the same height.



(a) state plot (b) Phase Plane

Figure 5.5: Attractive Limited Circle

5.2 Passive-Based 2D walking

Bipedal walking is a common motion task and has been well studied by many research communities, including robotic Raibert et al. [1986], artificial intelligence and biology as well as computer graphic research.

From experience, walking involves little reasoning activity, this idea is supported by the biology research that the number of neurons that take part in the lower limb control is very limited, much less than arm, hand and even tongue.

While for artificial system, robust bipedal walking is difficult to achieve. Many control method has been tried, but none of them shows comparable performance with human walking. Among all the methods, there are two very important works:

ZMP ZMP[Sugihara et al., 2002] method plans the motion curve by maintain the toppling toque zero. Stable but slow walk has been achieved. The motion seems very rigid and looks unnatural. Further research shows that ZMP is not necessary or sufficient condition for walking[Tedrake et al., 2008].

Passive Walking There have been a series of passive dynamic walking machine[McGeer, 1990a,b]. If we put a passive walking machine on a slope, without any effort, it can walk down the slop. These systems exhibit very natural looking gaits. However the stabilities are very fragile. Passive walking can only be maintained when walking down a specific slope under specific condition.

From the viewport of Qualitative Control Theory, the reason why passive walking machines can walk down the slope is because that there exists a limit circle for the dynamic interaction between body and ground. The fragile stability means the basin of attraction covers only a smal area on the phase plane. We plan to boost the stability of the passive walking machine by neural oscillation entrainment. Our method is passive-based bipedal walking.

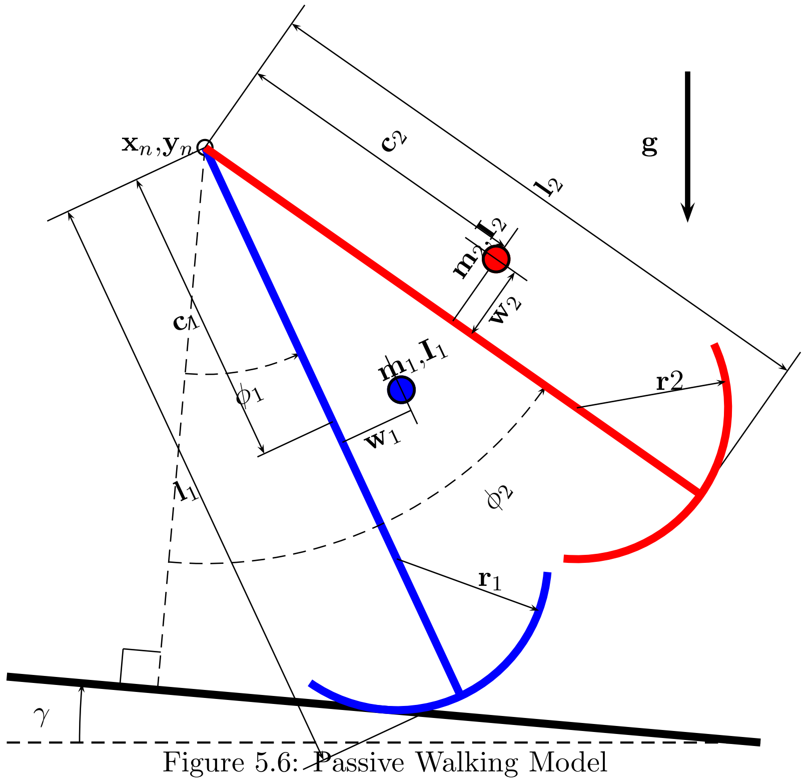


Figure 5.6: Passive Walking Model

5.2.1 2D Passive Walking Machine

The mechanical model we adopted is illustrated in Figure 5.6. Both the model and simulation method are from the paper[Wisse and Schwab, 2005]. We only give brief description to make the content self-contained.

Key parameters of the passive walker are listed below.

g	$gravity$	9.81
γ	$slopeangle$	0.0

The parameters of the environment is

leg length	l	$0.4m$
foot radius	r	$0.1m$
Com Location	c	$0.1m$
	w	$0m$
mass	m	$1kg$
inertia	I	$0.01kgm$

In simulation, the following states are concerned.

X_h	hip horizon displacement
Y_h	hip vertical displacement
ϕ_1	leg one angle
ϕ_2	leg 2 angle
xf_1	foot location of leg1
xf_2	foot location of leg2

5.2.2 Equations of Passive Walking

Passive walking is not a continuous dynamic system. We separate the motion into two phases and formulate two equations.

Leg Swing Phase During the swing phases, we suppose that one leg is fixed on the ground, the arc foot makes the passive dynamic walker rolling without sliding.

Heel Strike Phase We suppose the heel strike the ground in a short time, the angular momentum is preserved.

For a two dimensional walker, every leg is determined by three variables x_i, y_i and ϕ_i. The number of independent variables is reduced to four, xh, yh, ϕ_i, ϕ_2 for the hinge joint at hip reduced two dofs. Then we can have the following transform or Jacobin matrix T.

$$X = [x_1, y_1, \phi 1, x_2, y_2, \phi 2]^T$$
$$Q = [x_h, y_h, \phi 1, \phi 2]^T$$
$$X = \begin{bmatrix} x_h + c_1 * sin(\phi_1) + w_1 * cos(\phi_1) \\ y_h - c_1 * cos(\phi_1) + w_1 * sin(\phi_1) \\ \phi_1 \\ x_h + c_2 * sin(\phi_2) + w_2 * cos(\phi_2) \\ y_h - c_2 * cos(\phi_2) + w_2 * sin(\phi_2) \\ \phi_2 \end{bmatrix} \tag{5.5}$$
$$T_{i,k} = \frac{\delta X_i}{\delta Q_k} \tag{5.6}$$
$$T = \begin{bmatrix} 1 & 0 & c_1 * cos(\phi_1) - w_1 * sin(\phi_1) & 0 \\ 0 & 1 & c_1 * sin(\phi_1) + w_1 * cos(\phi_1) & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & c_2 * cos(\phi_2) - w_2 * sin(\phi_2) \\ 0 & 1 & 0 & c_2 * sin(\phi_2) + w_2 * cos(\phi_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.7}$$
$$\tag{5.8}$$

By Newton and Euler's Law,

$$F = M\ddot{x} \tag{5.9}$$
$$M = diag[m_1 m_1 I_1 m_2 m_2 I_2] \tag{5.10}$$
$$F = Fg + Fc + F_{cons} \tag{5.11}$$

combining the equations above, we have

$$T^T(F - M\ddot{(x)}) = 0$$

$$\ddot{x} = \dot{T\dot{q}} = T\ddot{q} + \dot{T}\dot{q}\dot{q}$$

$$
\begin{aligned}
g(x) &= \dot{T}\dot{q}\dot{q} && (5.12)\\
&= \begin{bmatrix} (-c_1 * sin(\phi_1) - w_1 * cos(\phi_1)) * \dot{\phi_1}^2 \\ (c_1 * cos(\phi_1) - w_1 * sin(\phi_1)) * \dot{\phi_1}^2 \\ 0 \\ (-c_2 * sin(\phi_2) - w_2 * cos(\phi_2)) * \dot{\phi_2}^2 \\ (c_2 * cos(\phi_2) - w_2 * sin(\phi_2)) * \dot{\phi_2}^2 \\ 0 \end{bmatrix} && (5.13)
\end{aligned}
$$

If we ignore the foot contact constraint, the equation of swinging is

$$\bar{M}\ddot{q} = \bar{f}$$

$$\bar{M} = T^T M T$$

$$\bar{f} = T^T[f - Mg]$$

Now considering the contact constraint, the position of the foot is on the ground

$$g_y = y_h - (l - r) * cos(\phi) - r = 0$$

and there is no slippery

$$g_x = x_h + (l - r) * sin(\phi) + r * \phi - x_f$$

$$D(x) = [g_x g_y]^T = 0 \tag{5.14}$$

After the first differential we get

$$\dot{D}\dot{x} = 0$$

$$\dot{D} = \begin{bmatrix} 1 & 0 & (l_1 - r_1) * cos(\phi_1) + r_1 & 0 \\ 0 & 1 & (l_1 - r_1) * sin(\phi_1 & 0 \end{bmatrix}$$

After the second time differential, we have

$$\dot{D}\dot{q} + \ddot{D}\dot{q}\dot{q} = 0$$

$$\ddot{D} = \begin{bmatrix} -(l_1 - r_1) * sin(\phi_1)\dot{\phi_1}^2 \\ (l_1 - r_1) * cos(\phi_1)\dot{\phi_1}^2 \end{bmatrix}$$

Putting them together, the equation for leg swing when one leg is fixed on the ground is

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} \tag{5.15}$$

During the heel strike phase, the equation of motion is as below

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \dot{q}^+ \\ f_c \end{bmatrix} = \begin{bmatrix} \bar{M}\dot{q}^- \\ 0 \end{bmatrix} \tag{5.16}$$

where $\dot{q}_+$ is the state variable after the collision, $\dot{q}_-$ is the state variable before the collision.

5.2.3 Passive Dynamic Walker Driven by Neural Oscillator

Although the equation for 2D walking is more complex, in principle, 2D passive dynamic walking resembles the bouncing ball system. Before the leg strikes the ground, it is moving passively. Only the gravity has effects. This is the same with ball flying in the air. When the leg strikes the ground, an impulse is generated. The state of the system changed instantly.

The input of neural oscillator is defined by the difference angle between the two legs.

$$G_{input} = \phi_1 - \phi_2$$

Neural output will drive the biped walker. After adding the neural control, the equation of the dynamic system is

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} + \begin{bmatrix} \bar{U} \\ 0 \end{bmatrix} \tag{5.17}$$

Neural oscillator output is applied at the hip joint to actuate the two legs towards different directions

$$U = [0, 0, 1, -1] * G_{out}$$

5.2.4 Adaptive 2D Passive-Based Walking Motion

When the passive walker walks down a slope, for every step, there is energy input from the potential energy, and there is also energy loss because of heel strike. There must be an equilibrium condition when the energy lost is equal to the energy input. If natural looking motion is energy efficient, such passive walking motion can be expected to be natural looking. Because there is no extra control energy input, such motion is the most energy efficient.

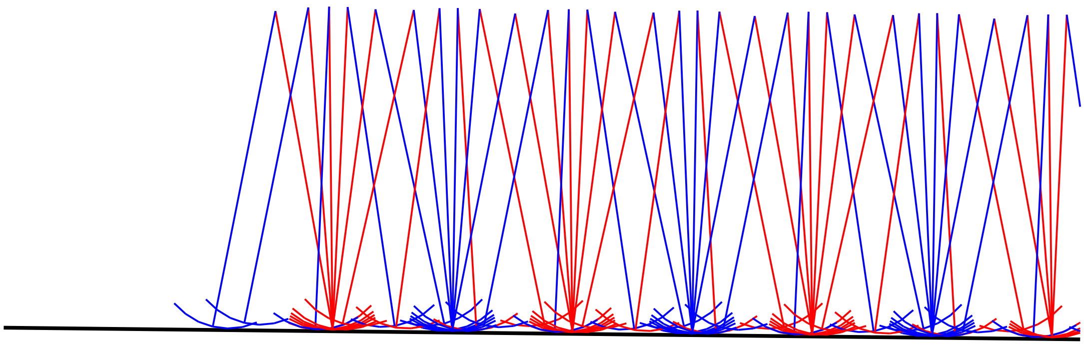


Figure 5.7: Stable passive walking gait

Figure 5.7 shows the gait of the passive walker. After coupling the neural oscillator, the basic pattern is not changed as shown in Figure 5.8.

However the stability is fragile. On a plane ground, there is no such limited circle any more. The passive walker can't walk on plane. The step size will decrease after each step, and finally it will stop or fall over as illustrated in Figure 5.9.

After coupled with the neural oscillator, this walking machine can walk on plane, and exhibits gait similar to the passive dynamic walker. Figure 5.10 shows the gait. From the state plot Figure 5.11(a), and phase plot Figure 5.11(b), we can see that the gait converged to a stable limit circle.

To verify the structural stability, we introduce a variety of perturbations to the passive walker. These perturbations include different initial condition, different slopes, different leg mass and different leg length.

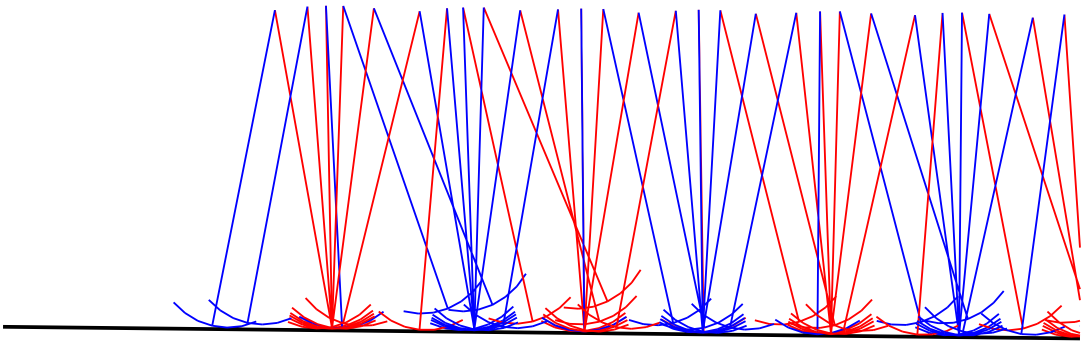


Figure 5.8: Walk down the same slop when actuated

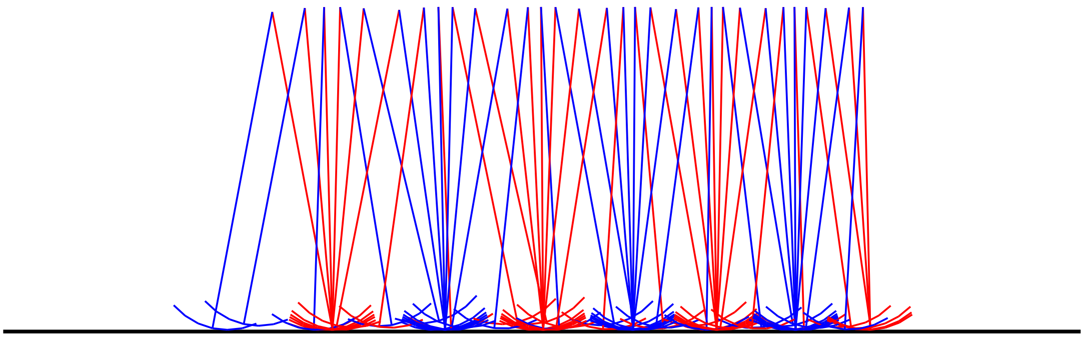


Figure 5.9: Passive walking gait can't be maintained on plane

Different Initial Condition The original passive walker is not very stable. A slight change in initial condition will result in walking failure. While after coupled with neural oscillator, the basin of attraction has been enlarged. A different initial condition can still lead to a stable gait, as show in Figure 5.12. Natural looking gait is maintained.

Walking On Different Slopes Another parameter we change is angle of the walking slope. When we increase the down slope, stable walking motion can still be maintained, as shown in figure 5.13. An important discovery is that although the walkers can walk on various down slopes, it can not walk up slope,no matter how control parameters are changed. It can't walk up slope and will fall backward after several steps. We suggest that this is because the proper limit circle does not exist in the dynamic system when walking up slope. This finding may help us to understand the upper body effect in walking.

Leg Mass Variation We add mass on one leg to 50% and find the stability of the gait is still maintained. The step length and swing period of the two legs are different, this gait is similar to that with a crippled leg, see figure5.14.

Leg Length Variation The last parameter we change is the leg length. We change the leg length to 1/8 shorter. And we find the stability of the gait is maintained, see figure5.15

The method of nonlinear entrainment did boost the structure stability and can result in stable motion under different environment change. By making walking an structure stable autonomous system, natural looking and adaptive motion are generate, without planning the motion curve.

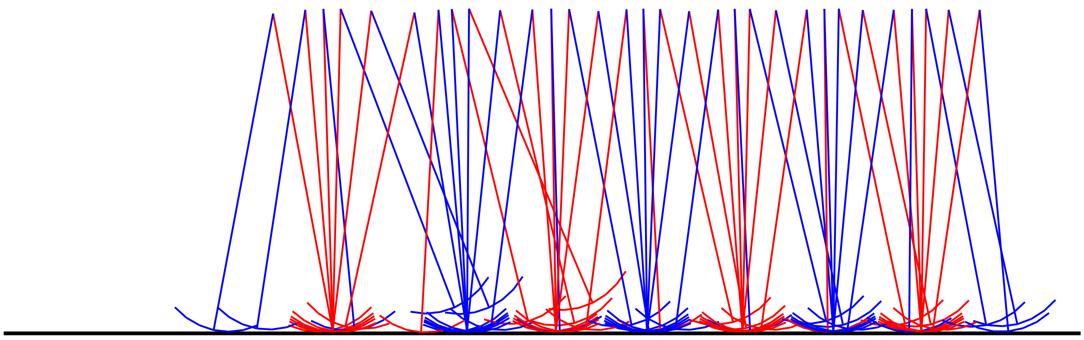
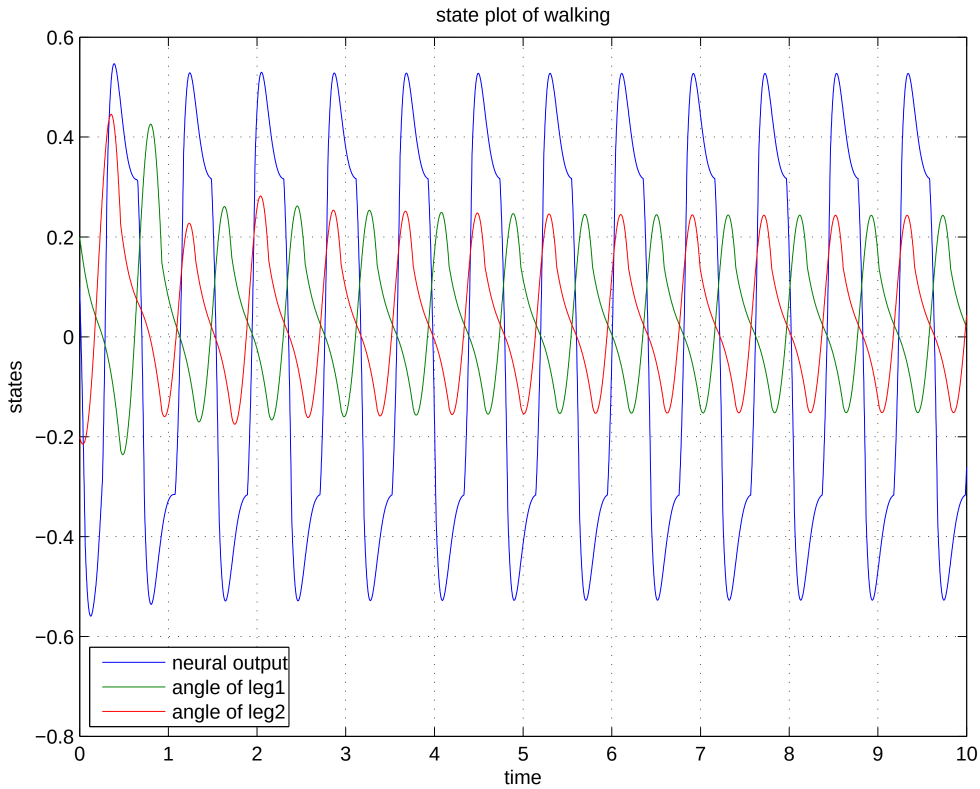
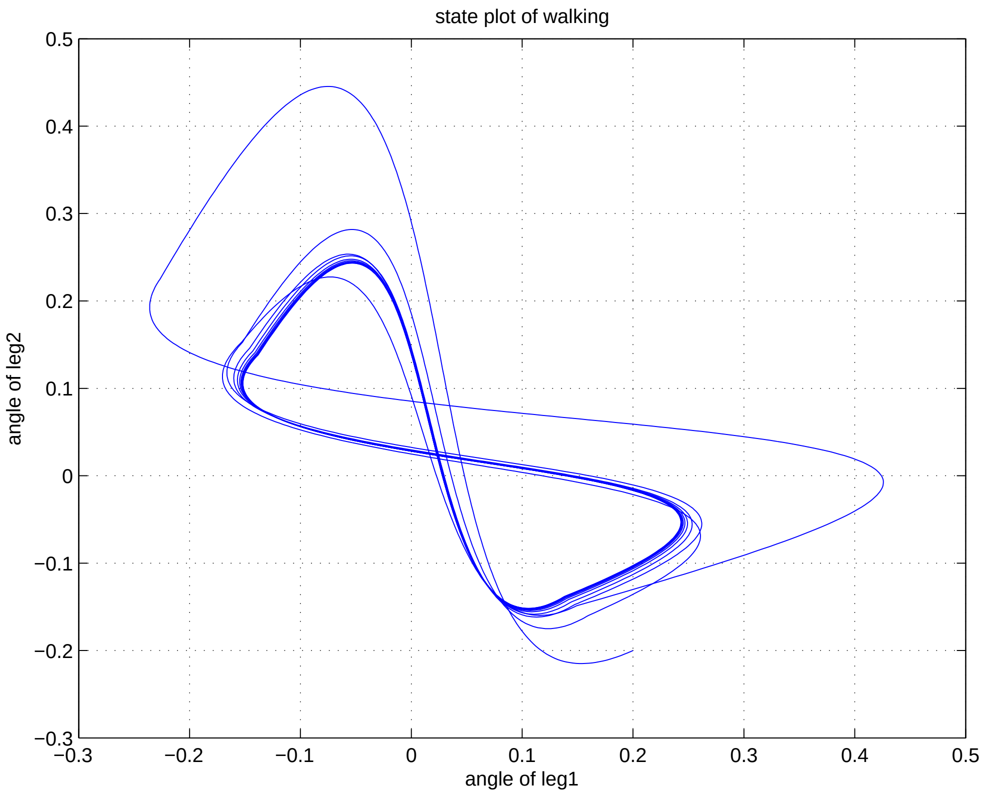


Figure 5.10: Walking on plane under neural control

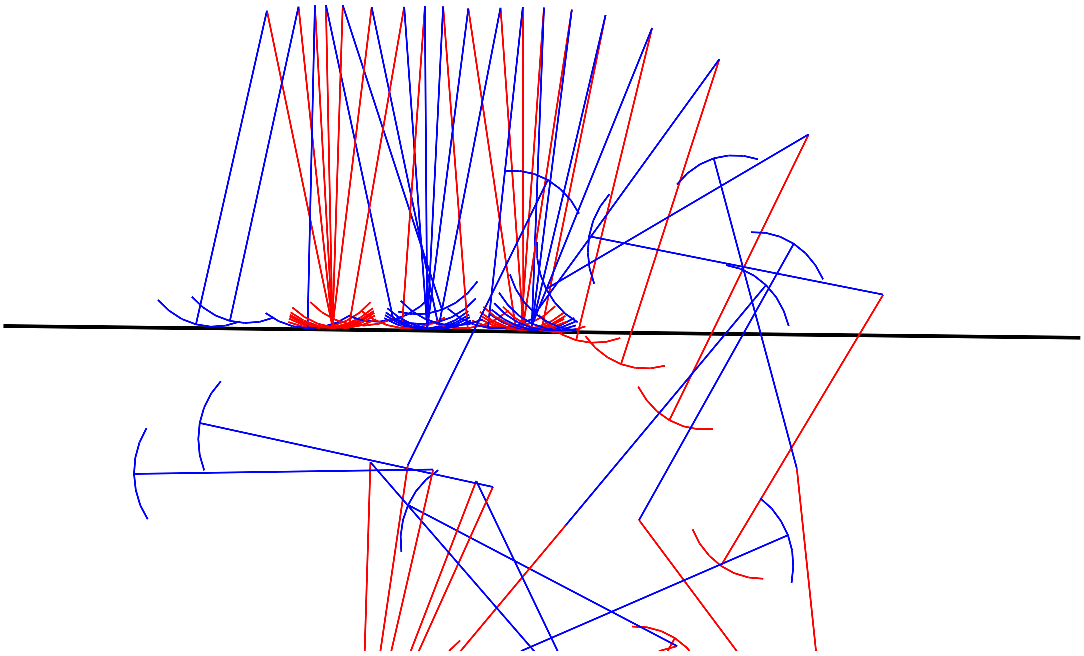


(a) State Plot

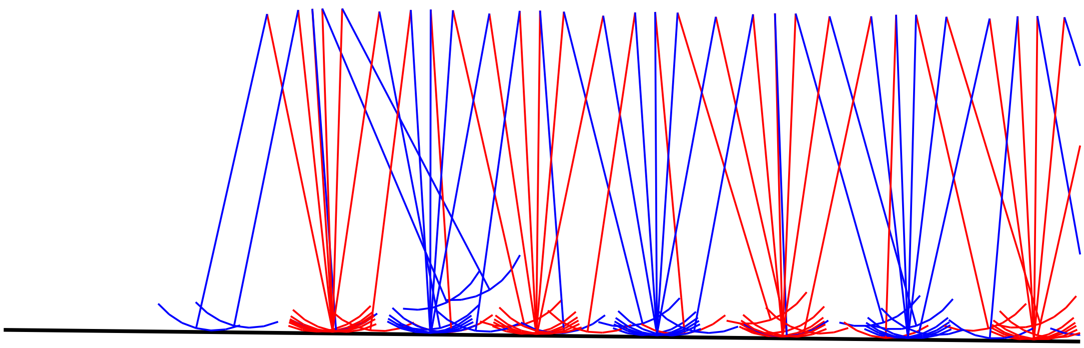


(b) Phase Plot

Figure 5.11: Walking on a plane converges to a stable limited circle

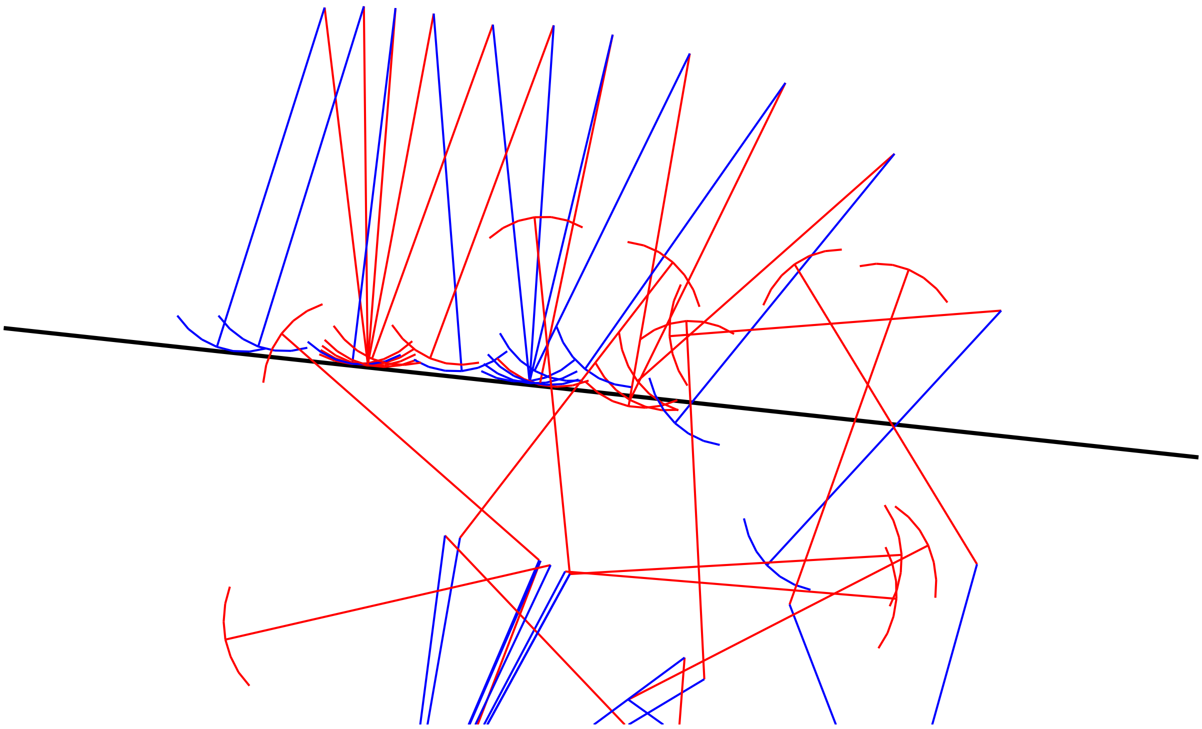


(a) without control

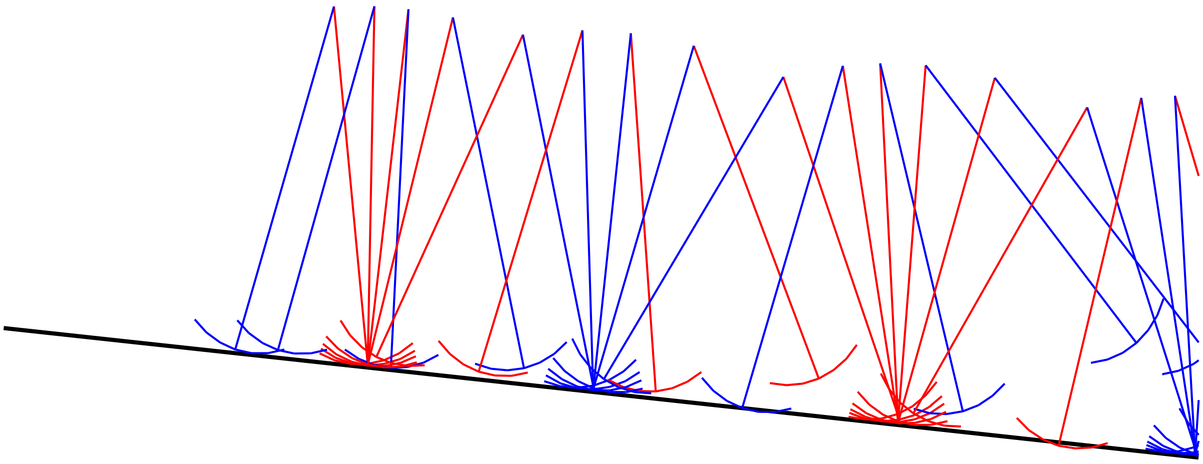


(b) with neural control

Figure 5.12: Walking with different Initial condition

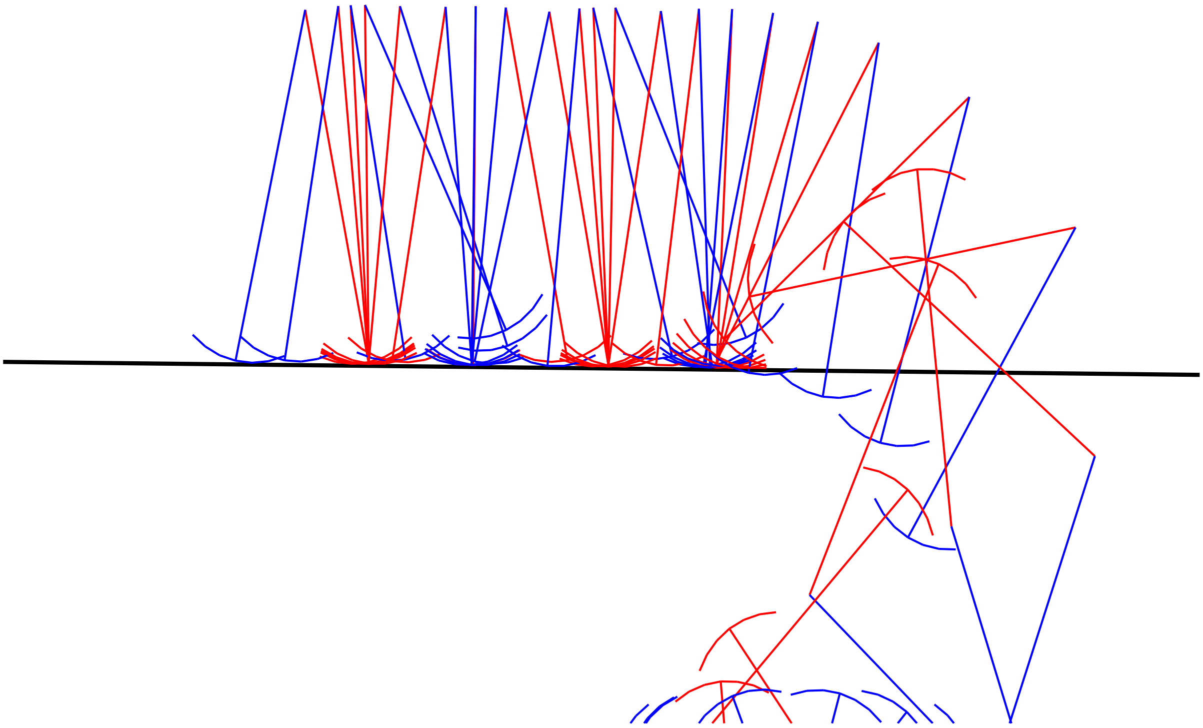


(a) without control

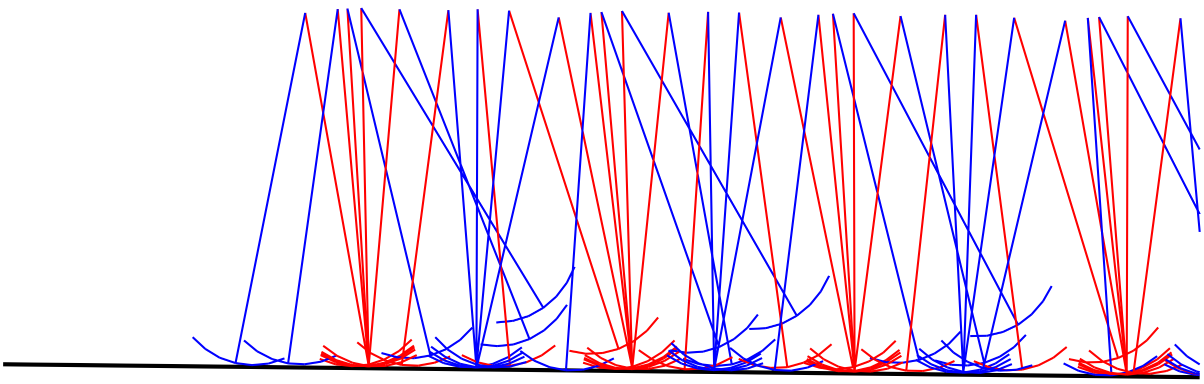


(b) with neural control

Figure 5.13: Walking with different slope angle

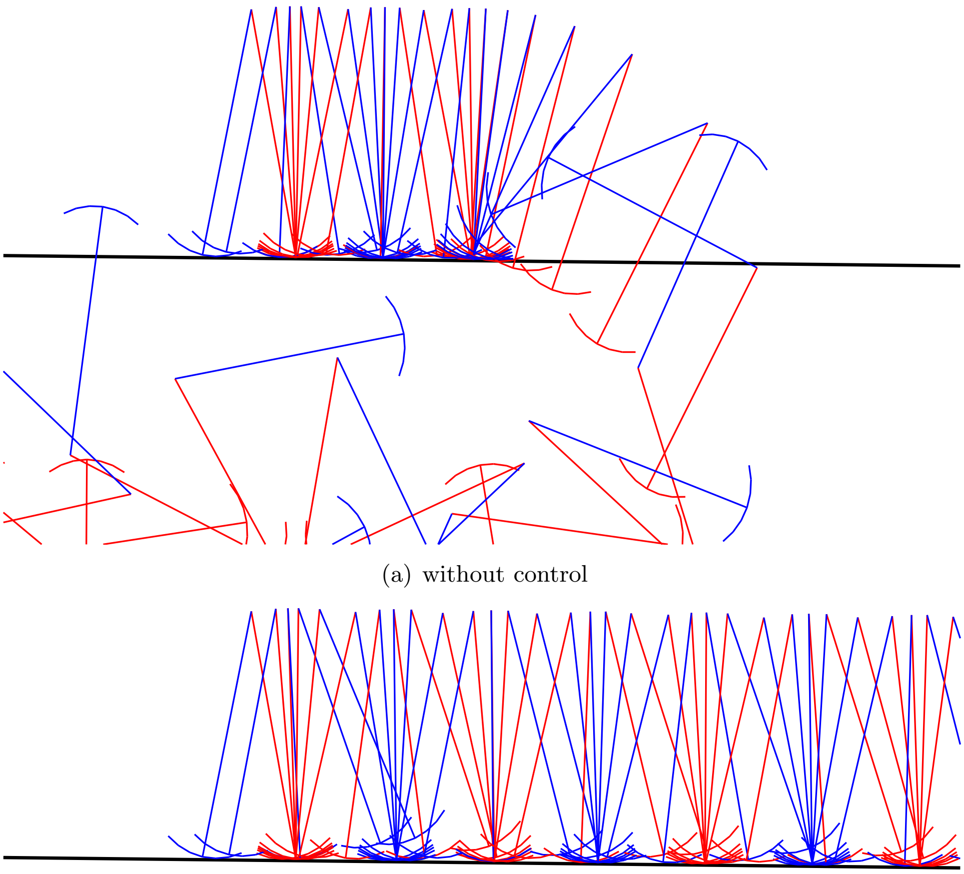


(a) without control



(b) with neural control

Figure 5.14: Walking with legs of different mass



(a) without control

(b) with neural control

Figure 5.15: Walking with shorter Legs

Chapter 6

Future Work and Time Table

More work is needed to apply the new Qualitative CMS framework to full body character animation. It is the main task of my research in PhD time. In this chapter, we will discuss several important CMS applications questions.

6.1 Full Body Character Motion

The first topic should be a solution for the redundant degree freedom problem. This will invoke the potential problem of coordination. Qualitative Control Theory provides a view for alternative methods. We propose that coordination can happen through physical coupling[Williamson, 1999]. The idea is that coordination is not generated by neural control, it happens automatically because of the intrinsic properties of body dynamics. One example is the coordination of arm swing movement with walking gaits. The natural arm swing motion can be generated without any explicit control[Collins et al., 2009], it is an effect of entrainment of mechanical coupling.

Following this idea, during a motion, all the DOFs of the full body structure can be divided into two groups, primary DOFs which are controlled by the neural system and determines the structural stability of the dynamic of motion; passive DOFs, which can moves freely and can simply be treated as perturbances. Qualitative Framework in theory provide an efficient method for dimension reduction, when only qualitative features are concerned, all the stable submanifold space can be neglected. Further extend the idea; we can develop a hybrid method to incorporate motion capture data. Physical simulation provides a rough approximation of the motion. From the motion captured data, we can find the detailed motion which is most similar to the physics simulation.

6.2 Adapt Motions to Purpose and Constraints

Another topic is how to adjust the moton to suit special purpose. For the walking example, it incudes how to avoid an obstacles while walking. In Qualitative Control Theory, we will use a different method, motion purpose is achieved by adjust ing the positions the equilibria or changing the shape of attractors. This can be achieved by changing the system's parameters. To avoid obstacles, we can put the position of the repellor at the obstacles. To reach a position, neural system put an attractor there.

6.3 Parameters Tuning and Motion Style

In our research, we've found that fixed parameters can maintain the structure stability and generate adaptive motions while, because of the structure stability, we also found similar motion behaviours can be generated by different control parameters. In natural, animals may tune the control parameters with the increase of the practise of motion. We wonder whether this method can be used for mapping different motion styles to different setting of control parameters, which is still a very difficult problem in current CMS research. Since tuning the control parameters has direct effect on the motion frequency and amplitude, a simple way is to incorporate the research result in MSP into our framework.

6.4 Future Time Table

July 2010 to December 2010 Full body character motion synthesis.

January 2011-March 2011 Modeling motion purpose and Constraints.

March 2011-September 2011 Thesis Writing.

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