Adaptive Motion Synthesis by Motion Invariant

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Figure 1: *Spring Training 2009, Peoria, AZ.*

Abstract

Physically Based Motion Synthesis is challenging, mainly because of Freedom Curse, which usually involves lots of computational work. In this paper, we develop a new method based on the biological idea of motion primitive. Motion Synthesis and Motion Retargeting are achieved by modifying the basic motion primitives. Our Method separate stability control and motion style in two decoupled problem. In mathematical viewport, the method control the topology and space of flow independently. Our method can generate adaptive motion, is applicable for a large number of system and is computational efficient.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Radiosity;

Keywords: Motion Synthesis, Physically Based,CPG, Lie Group

Links: DL PDF

1 Introduction

It is important to know the things being animated when we develop animation system. Physically based character motor synthesis includes modelling both the body mechanics and the neural system behaviour. While techniques for simulating mechanics of body is sophisticated, little is known about the biological motor control principles. In mathematical view port, the key problem in motion synthesis comes from the redundant degrees of freedom. For a motion task like picking up an apple, human has many different ways to finish it. Many biology researches suggested that natural motion is energy efficient, thus current Researches follow the pioneering work by Wikin(spacetime constraint), formulating the motion synthesis as constrained variational optimization. However, spacetime constraint entails large computational burden and sensitive to errors, makes it highly unlikely the paradigm for biological motor control. Besides energy efficient result, biological motor control also has to be agile, adaptive. After putting weight on shoulder, human adjust his gait immediately, without thinking for long. Also it is highly unlikely that human detect motion artefacts in this manner, human detects motion artefacts immediately, without long computation.

In this paper, we propose a motion synthesis framework based on a different biological motor control paradigm. Some Biology Research suggested that motor repertoires are composed of a number of basic elements, motion primitives. Neural system tweaks the motion primitives for environment constraint or special motion purpose. Complicate motions are formed by connecting different motion primitives together, just like combining alphabet into sentences. The new framework can generate adaptive and energy efficient motion with low computational cost.

The mathematical framework we developed is based on group action and invariants. New motion $m_n$is treated as an element m that modified by a group action g formulated as Equation (1))

$$m_n = g(m) \tag{1}$$

The important property of g is that some motor features are kept invariant, which are motor invariants I. Actions g and m keeps motor invariant form the action space G and motion space M as Equation (2). To synthesize target motion m_n , we need to find $g \mid m_n = g(m)$.

$$I(g(m)) = I(m), g \in G, m \in M \tag{2}$$

For animation purpose, I should contain features that natural looking. A biological hypothesis is that when I is violated, human will detect artefacts in motion. Mathematics provides some deeper information about I. For physically based animation, m is the solution of a differential equation that described the dynamics. In this paper, we propose two important motor invariants. The global motor invariant determines the qualitative properties and local motor invariant which determines the details of motion. In mathematical view, they are the Topology and Lie Group Symmetry of the corresponding differential equations that describe the dynamics of motion.

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contribution

Adaptive This framework solves the stability controls and retargeting problem in a unified manner, thus provide more types of adaptation. Traditionally, for walking example, for impulse perturbation, terrain, and crippling leg, traditional we need different controllers, while our method can generate all the adaptive motions for all conditions above.

Computational Efficient Finding element in and group involves much less computational work than optimization. The computational load of our method is extremely low; calculation only involves close form calculation. Compared with mechanical simulation, the computational load can even be neglected.

Artists Directable Usually, physically based animation, it is not very intuitive to modify control parameters modify motion. T he control parameters of our method are more intuitive use. For walking motion, we include some parameters like walking speed, step size or upslope angle.

outline

After a quick review of physically-based character motion synthesis in Section 2.1, we also reviewed some important research related to our work on biological motor control and Mechanical Symmetry. In Section 3 investigate the global motor invariant, details about its effects on motion stability and adaptation. A biological based method for keep global motor invariant is presented in section 3.4 Section 4 focuses on local motor invariant; we propose two groups that modifies the space and time properties of motion independently. A computation efficient method for control is presented in Section 4.3 Section 5 focus on combined the methods maintain global and local motor invariants together. Application and Motion Synthesis result in presented in section 5 and Section 6 discuss some features of this method and further research work.

2 Background and Previous Work

2.1 dynamic simulation

Dynamic Motion Synthesis tries to synthesize character motion through physical simulation of the mechanic structure of character body which is usually modelled as a linked rigid body system [Baraff 1994; Mirtich 1996; Stewart and Trinkle 2000]. Since many real physical properties are considered in the computation, the generated motion are normally physical feasible. However the most difficult task for those methods is to design a efficient control system to simulate the functionality of a real biological neural system. Some early research applied classical control methods like PD controller [Raibert and Hodgins 1991] for locomotion. Later research [Hodgins et al. 1995] applied the same method for different tasks like running, bicycling, vaulting and balancing. Limit Circle Control(LCC) [Laszlo et al. 1996] provides an alternative method for lower energy locomotion animation. However both the classical PD controller and Limit Circle Controller predefined motion trajectories and eliminated perturbations. This make them not good at generating motion adaptation.

Because lots of degrees of freedom are involved in the whole body simulation, in most cases, motion solutions are not unique. Many optimization methods have been applied to choose the "best" motion. For dynamic methods, a reasonable choice is to minimize the energy cost V, such that

$$\mathbf{V} = \int_{t0}^{t1} F_a(x)^2 dt$$

where F_a is the active force generated by actuators like motors or muscles. This is introduced to CMS research as the influential Spacetime Constraints[Witkin and Kass 1988], and serve as the foundation for many modern CMS research. [Jain et al. 2009] provides an example for locomotion; [Macchietto et al. 2009] find a method for balance maintaining movement. [Liu 2009] proposed a method for object manipulating animation.

2.2 Optimizaiton Based Method

Optimization provided an idea for solving the redundant freedom problem and complies with energy efficient principle of natural motion. But this method has several drawbacks for motion synthesis application.

(1) Numeric Stability and Modelling Difficulties Optimization method only grantee the energy efficiently of the motion, but nothing is about the converging speed and stability. Even the optimal solution is natural looking, it is very hard find. it can obtain the solution when the model is accurate and the initial guess is near the optimal solution. So this method is sensitive to model error and initial condition. While for motion, the accurate model is very hard to achieve and artefacts are generated. Liu2005 points out that spacetime constraint methods only suit high energy motions like jumping and running; for low energy motion tasks like walking the result doesn't looks nature. This is mainly because the muscle effects are neglected.

(2) Computational Complexity Motion Control is variational problem in nature. For complex body structure, even the state of art numeric method will take inhibitive long time. And little is know about how to reuse the computation result for motion adaptation.

(3) Limited Application Domain Optimization method nowadays only applies for very limited motion. Main for motion can by model with rigid body. A Large of motion are still uncovered. Like heart beating, breathing, fish swims and bird flying, such motion involves the soft body and fluid dynamics. Such model are computational difficult in nature, apply optimization based method for such dynamic are not computational feasible.

2.3 motion perception problem

For motion synthesis research in Graphics, we focus on generating natural looking motion rather than physically-correct motion. As long as the audience dont notice the artefacts, such result will be OK. A important biological question is how human recognize motion and detect the artefacts. Natural looking motion observed from life must be physically feasible, but it does not mean human detects motion artefacts by doing physical calculation in mind for the speed limitation, nor it is only because memory of motion for the capacity limitation.

2.4 Limit Circle Based Method

2.5 Motion Primitives

2.5.1 The Lessons from Biology Research

Motor Control from biology research is full of paradoxes. Motor Control in nature is a very complex process, it involves electrical, chemical and mechanical changes. However, for such complex problem, seems an easy task for most human and animals. From biological view port, the problems of optimization are worse. Natural motor control faces many limitation of the biological motor control system

2.2.1 Biological Constraints (1) Sensing and Control Limitation Motor control is not only a mechanical problem, it is a complex process. For the biological system, many crucial parameters and variables are not accessible. Dynamic model, force, torque, angle can only sensed with approximation, while mass, inertia, and human have not direct sense. For some control variables like toque, neural system has no direct control access.

Besides this, body and environment is noise and time varying. Methods are sensitive to errors are not suitable for biological motor control.

(2) Neural Computation The neural system is powerful but it is inferior in speed and accuracy when compared with digital computer. Neural signal transmitting speed is slow; and there is a long delay between neural signal firing and force generation in muscles. It is impossible for neural system to carry out complex computation for optimization in real-time time.

2.2.2 Biological Discoveries Limited Computation Activity Common Life experience shows motor control maybe an easy task. This is proved by some biological research. Many experiments show motion can happen even without brain input. Development Many motor ability are born, rather than developed. Eating, Breeding, Breathing and locomotion, many motor ability in animals seems is inborn rather than learned. Evolution Form Evolution view port, the motion style seems not close related to the development of neural system. Wales and Fishes swim in a similar manner. What is apparent is the body and environment. Animals with a similar body in a similar environment usually move in similar manner.

2.5.2 Different Biological Motor Control idea

Biological Research propose some different idea about motor control, which takes the biological constraints into consideration. equilibrium point hypothesis and uncontrolled manifold hypothesis An idea of motor that will simply the computation and maintain the energy efficiency is eph and ump. Some proposed that neural system doesnt control all the system, it may only control the final motion result. As long as the motion finish its task, it dont matter how it is carried out. This means control final motion position, and let motion move freely according to the environment and body condition. Which should be computational efficient and energy efficient. morphological computation and motion primitives framework. Animals dont move the way they want, but only the way they can. It is the body and environment plays the most important role in motor control, they forms the basic pattern of motion, neural system only tuning or tweak the basic pattern to fit animals special needs. More complex motions are based on combine the basic pattern together. This idea is easy to compute and can provide a way for motion perception. Basic pattern are limited and human are very familiar. When the pattern is break, human will notices the difference will result motion artefacts. 2.4 The relationship between our research and new ideas There is no a unified way to identify motion pattern and motion primitives. In our framework, we based on the idea of group and invariant. Motion Primitive in our eyes is the global qualitative properties of motion, which is capture by the topology structure of the dynamics of body and environment. This is the Global Motor Invariant. EPH are adopted as a method for tweaking basic pattern. The method we propose for equilibrium point control is based on symmetry properties of differential equation. This is the Local Motor Invariant. Because our method is based on dynamics, so it is physically-based method. The advantage of Goup and Invariant theory is that it will provide an efficient computational method.

3 MOTOR INVARIANT CONTROL

3.1 Motor Primitives

In real life, animals motions are result of the dynamic interaction between itself and environment. In general, dynamic motion can be described with differential equation

$$\dot{x} = f(x, u) \tag{3}$$

Where x is the system state, $\dot{x}$ is the derivative and u is the control signal. When u=0, such system are called autonomous system and capture the intrinsic motion property. Important properties of motion are shown on phase plot; Figure 1 shows a limit circle. All the motions converge as time goes on.

The limit of motion is called limit circle and shadow region is called basin of attraction, motion within basic attraction converge to the circle. In real life, all motions should determined limit and stable, the motion must be attractive. Basically there are two types of limits, periodic and fix point. Corresponds the period tic motion and discrete motion. A motion primitive is defined by its equilibra and its attractivity.

3.2 Motor Control

For motion control the key problem comes in two situations.

State Perturbations motion primitive is kept but the initial position is moved out of basin of attraction. For walking, a push perturbation will have such effect. In motion synthesis, this is the motor control.

Structure Perturbation Motion primitives can change its shape, type or even attractivity. comes from changing f. for walking example it may comes from the change of body weight or terrain. This problem in nature is motion retargeting. Most current idea applying a control to push state back to boa, such controller only applies to S1.

our method is different and separated in two steps. (1) Maintain the attractor type and its attractively. The shape is not considered. As in figure 2

(2)Transform the motion primitive on phase plane, to include the state X inside the basin of attraction. As in Figure 3

Thus our framework solved the stable control and motion retargeting problem in a unified way.The shape change of attractor means more adaptive motion results.

In mathematically, the first step is keep the topology of motion primitives or Global Motor Invariant. The transform available is close related to the properties of f, which is the lie symmetry, or the Local Motor Invariant.

Figure 2 Global Invariant Control

Figure 3 Local Motor Invariant Controls and Transformation

3.3 Motion Transition

When an character transform from one motion into another, character should in the state space $(q, \dot{q})$ For when motion is translated from motion ma to motion mb. The state of

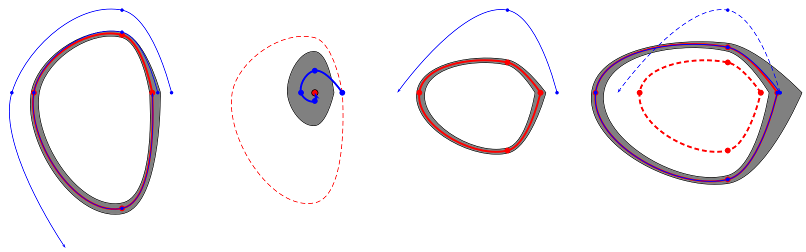

Figure 2: *Comparison of the results predicted by our models against video of a human iris. (left) One frame of an animation simulating the changes in pupil diameter and iridal pattern deformation. (center) One frame from a video of a human iris. (right) Graph comparing the measured pupil diameters from each individual frame of a nine-second-long video sequence (green line) against the behavior predicted by our model (red line). The gray bars indicate the periods in which the light was kept on and off. The complete video sequence and corresponding animation are shown in the accompanying video.*

4 Global Motor Invariant and Control

4.1 the Qualatitve Property of Motion

When generating adaptive motion, one key question is what can be adaptive while what must be maintained. For walking example, people walking in different manner and different way. But still some key properties are kept unchanged.

A method for modelling the qualitative property can be identified by the topology of phase plot. For the walking example, some researchers argue that, the key properties of walking or any other kinds of locomotion, is the periodic change of the body shape. On the phase plot, this means the topology of the phase plot is the same to that of the circle. The topology of the phase plot is defined as the Global invariant, which determines the qualitative properties of motion.

When keep the topology against the change in system dynamic, this is the structuable stability control, which in nature solves the motion retargeting problem. When the topology is kept against the perturbation in state, this is the stability control, Which in nature solves the stalibty control in motion synthesis.

Different kinds of perturbation will generate different shape of phase plot. This is allowed in our framework and even seen as an advantage, because the shape changed reflect the adaptation in motion.

By the topology, basically motion can be separated into two group. 1 periodic motion Some motion show pepetive pattern, from geometrical viewport, on the phaseplane, the attractor of curve form the shape of a circle. 2 discrete motion Some motion is terminated, from geometrical viewport, on phase phaseplane, a ttractor is a point.

In this research, we only consider periodic motion. This for two reason, the first one is that some biological research believe that life system itself is peroditic in nature. From application viewport, fix point can be approximate with a limit circle with a small amplitude.

4.2 Entrainment

For periodic motion, global controller dont care about the shape, of oscillation, it only cares about the topology of the phase plot. The method we prosposed is based on the entrainment. The basic idea is couple two oscillator together. The dynamic system form one oscillator, while the control system form the other oscillator. The control system system is topological structural stabile, and oscilate with the mechanical one in an similar manner, When the mechanical oscillator is disrupted, whe control oscillator will drive the mechanical oscillator return to oscillation model. Whe control oscillatior we prosed is based on the matusta oscillator. With the equation of following form.

Stability of Control Oscillation. Automotous Oscillation From initial position, matsushiata oscillator all converge to the same limit circle. Entrainment Oscillation Matsuta oscillation can coupble with a various oscillator.

$$\tau_1 \dot{x_1} = c - x_1 - \beta v_1 - \gamma [x_2]^+ - \sum_j h_j [g_j]^+ \quad (4)$$

$$\tau_2 \dot{v_1} = [x_1]^+ - v_1 \quad (5)$$

$$\tau_1 \dot{x_2} = c - x_2 - \beta v_2 - \gamma [x_1]^- - \sum_j h_j [g_j]^- \quad (6)$$

$$\tau_2 \dot{v_2} = [x_2]^+ - v_2 \quad (7)$$

$$y_i = \max(x_i, 0) \quad (8)$$

$$y_{out} = [x_1]^+ - [x_2]^+ = y_1 - y2 \quad (9)$$

5 Symmetry and Local Motion Control

5.1 Symmetry of Motion

For Physically-based animation, Motion is usually described by the differential equation (1)

$$\dot{X} = F(x, u) \quad (10)$$

Physically possible motion is the solution of the equation. An important property from one solution x, with a group action g, we ca get another solution x_a

$$x_a = g_a(x). \quad (11)$$

for example , We have

So the group action is

For equation (1), the group action g_a satisfy the symmetry property (2). (3) This provide us an idea about motion synthesis.Given an original motion m, and the corresponding group g, a new motion is generated by g(m).

For every group G, we can find an function I(x) unchanged by the group action G, I(x) are called local motion signature. For mechanical system, Lie Group and Symmetry has important physically meaning. I(x) corresponding to the Conservative Law, like energy or angular momentum.

5.2 Control Symmetry

For motion synthesis, usually the desired motion is ma and original motion m is known, but the corresponding group action g_a is not satisfied by differential equation. For such situation, control input u is added, which modify the original equation to allow the designed G, this is called Controlled Symmetry.

Most dynamic motion can be modelled as an Lagrange System.

$$L = K(\dot{(q)} - V(q). \tag{12}$$

And the desired action G must keep the L invariant.

The original m is defined by the eural langrage equation (4)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{13}$$

The modified system is (5)

$$\frac{d}{dt}\frac{\partial L}{\partial G(\dot{q})} - \frac{\partial L}{G(\partial q)} = 0 \tag{14}$$

Which is equal the controlled dynamic system (6)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = u \tag{15}$$

(5) and (6) are the equivalent equation, by comparing equation (5) and (6), we can get u

5.3 Examples

Offset

$$G_r(x) = (q + r, \dot{\text{q}})$$

Which keep speed, but modify the pos. thus keep the K but modify V

$$u = \frac{\partial V(g_r(q)) - V(q)}{(\partial q)} \tag{16}$$

on phase space, if q is the horizontal axis, and $\dot{q}$ is the vertical axis, this has the effect of moving the phase plot right and right.

Time Scalling

$$g_s t(q, dotq) = (q, st * dotq) \tag{17}$$

$$u = (st^2 - 1)\frac{\partial V(q)}{(\partial q)} \tag{18}$$

on phase space, this has the effect strength the phase plot in the vertical direction

Energy Scalling

For some system moving the the conservtime field with constant mass matrix. The energy is preserved and different motion present different level of energy. For such system, we have the For such

$$g_e(q, \dot{q}) = (e^2 * q, e * \dot{q}).$$

U can be developed by applying the pos scaling and time scaling in a combined manner.

On phase plot, this has the effect enlarge the phase portrait.

5.4 Symmetry of CPG

For a dynamic system L, we have global invarant controller Ugc, and local Invariant Controller Ulc. A question is how to combine them together. Our method is combine the different controller following the same symmetry. When a system L has a symmetry of G_a, G_a is applied also tow $G_g l$ The Simmetry of Global Oscilator. Neural oscillator are couple with mechanical oscillator in the following manner.

$$\dot{x} = F(x, u1)\dot{x_c} = S(x_c, u2); u1 = ho * O(x_c)u2 = hi * I(x) \tag{19}$$

Pose offset: When the original $G_o f f$ is apply to the x, If we keep $I(x)$ is the invariant of $G_o f f$, then the Symmetry of the how system will be kept. Pose scalling group When $G_s cal$ is applied to the x, if we $hi = hi/scal$. Then the symmetry is kept. Time scalling If the scale factor is ts, we keep modify modify the original equation by hou=hou*ts*ts. Time offset This involves modify the state on the limit circle.

6 Application

6.1 Boucing Ball

Dynamic Model

Bouncing ball is system bouncing by moving a pedal, a system with simple dynamic but difficult to control. While this example capture the complexity of human interatction with the environment and object. And can be the basic model for many motion tasks, like catch and throw, ball playing, and even walking.

Hybrid dynamics, in incoperate two phase,

Equation (1), is the flying phase equation, equation(2) is the bouncing equation The -1¡e¡0.

Basically, the ball will continue bouncing with smaller height.

Global Invariant Control

couple with neural oscillator boucing we get an limit circle

$$g_i n = g_i n * v_b all, (3) Pos_p edal = h * out_o scillator. (4)$$

The input of neural oscillator is the velocity of the ball multiply by the input coefficient, the output of neural oscillator drive the pedal position. An limit circle emerge as the result of entrainment. As show in figure drop from different position, all the ball will bouncing a about the same height of 5.

Local Invariant Control

Control Bouncing at different Height. For example boucing at height 10, or 15. if we define the a*5 the the boucing ball system have energy symmetry. Controlled Symmetry If x(t) is a solution, So it ax(sqrt(at)) is also a solution

Thus the neural oscillator is modified as follows. tao=tao*sqrt(a) out=ta*sqrt(a) in=in/sqrt(a)

than the a*height is mapped the limit circle.

Following example shows the mapping effect, with a=1, a=3, where the bouncing height is controlled at about height =5 and height =15. The two pictures are almost the same; this is because the symmetry is perfect.

With Limit Circle At aoubt 15, a=3

With limit circle at about 5, a=1

Conclusion

No matter where the initial position is ,the limit bouncing height can be exactly controlled by modifying the parameters of oscillator.

6.2 Bipedal Walking

6.2.1 Dynamic Model

iure 1 shows the a two dimensional walking model. Like bouncing ball, this system is also an hybrid system. The motion include four phases. 1 in free swing phase. Mddq/dt+Cdq+N=0 2 the knee strike phase Qq=Qq 3 the knee lock phase Mddq/dt+Cdq+N=0 4 the keel strike phase. Qq=Qq this system can do passive walking under specific initial position, the walker can walk passively downstairs. And exhibit a limit circle.

6.2.2 Global Invarian Control

just like mass spring system, the basic idea of control is using the entrainment oscillator to motion adaptation. The input of the nerual oscillator is the angle between the two legs. The output of the nerual oscillator is acting on the hip joint

$$g_i n = h * (q1 - q2) yout = hout * neural.$$

the period is double and the step is not symmetrical any more. So the two legs move in a slight different manner, but still, it keeps walking.

6.2.3 Local Invariant Control

lie group symmetry provide method for adapting the walking motion for different terrain and for at different speed.

Slope Change. N=N*g. u=N(g*cos(theta)-g) Speed Change u=Ng*(speed*speed-1)

4 Walking on any slope with any speed with any walking style when Combined together, our system can walking on different terrian with any speed and any style.

6.3 Stance And Walking Transition

6.3.1 Dynamic Model

the reduced model linear bipedal model suppose when standing, we hight vaiation is almost zero. We only conside the horizontal movement

basicall the stance are divided into three phase

1 Single Support Phase

$$\ddot{y} = t/ml + gy/L \tag{20}$$

2 double stance Phase

$$\ddot{y} = (TL + TR)/ML + g/l * wL(y - y_m) + g/l * wr(ym - yr) \tag{21}$$

3 fall if $ym > d$ than the stance will fall. The goal of control is confine the flow withing the safe region

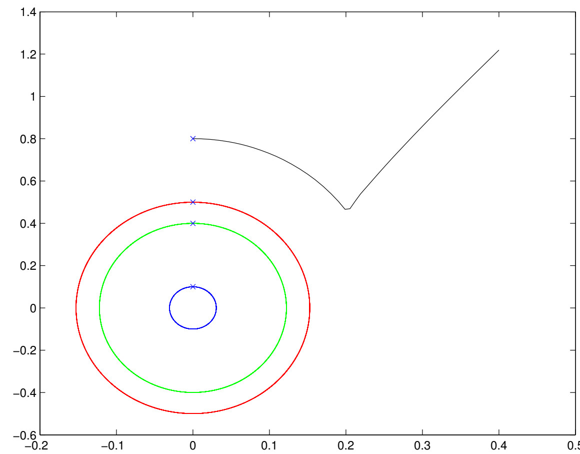

Figure 3: *Stance Dynamics*

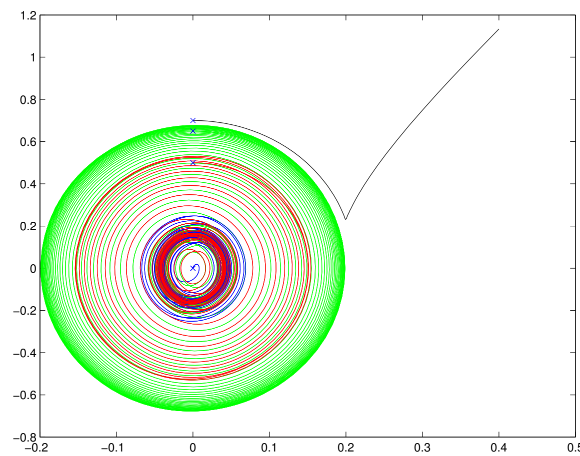

Figure 4: *Stance Dynamics*

6.3.2 Global And Local Motor Control

he original system is simmilar to mass spring system. It wil vibrate continutelly. while in our method ,by coubling neural system the the oscillator, it form a limit circle

But because of the special characteristics of the stancing model, when move out the safe region, It will fall.

2 lie group control By the speed transformation, The speed we can control the stance motion to move within the safe region.

then the stance posture is controlled

by the energy transformation, we can control the final vibration magnitude.

6.4 Walking Stance Transition

Application Walking and Stance Transformation

6.4.1 Walk Stance Transition

walk to stance transform

when transform from walk to stance, there is no heel impact. The swing leg touch the ground, it will generate force and start to bend

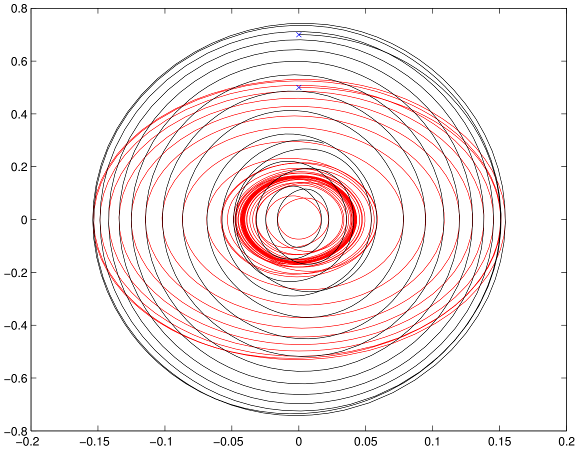

Figure 5: *Stance Dynamics*

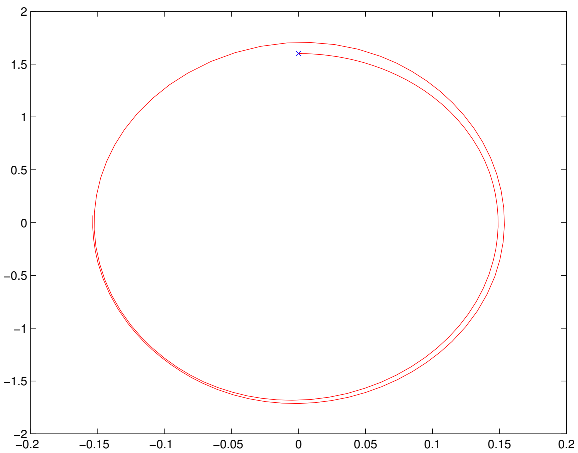

Figure 6: *Stance Dynamics*

x only depend on the stance leg there is a transform f, that transform the state from q to x.

$$x = f(q).$$

stance to walk.

Stance to walk happens , the font leg is kept strait, the back leg is becoming strait and rotate.

Walk to Stance Transform

the walk state

From Stance 2 Walk

the black one is on the limit cirle, while the red one is walking start point.

6.5 Hyper Dof and Boid System

Boid like system some animals like snake and fish have a very flexible verberate. And such system are refered to as hyper dof system. For such kind of system, motion control seems more difficult.

Based on the idea of symmetry and limit circle, we proprose an ad-hoc based idea. That every joint or every agent are controlled independently,

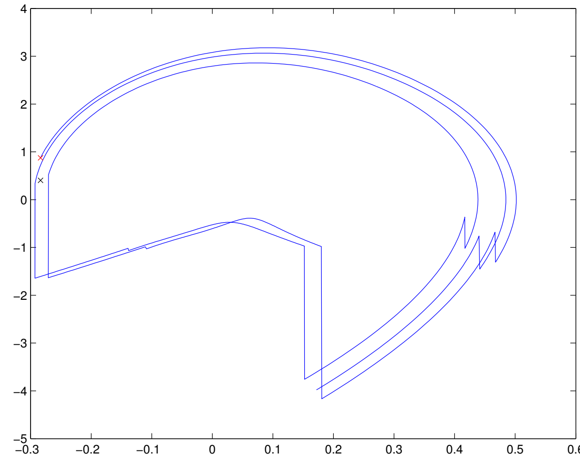

Figure 7: *Stance Dynamics*

unlike tradition boid idea, our research is based on rule based, we propose that all the agent are cotnrolled by the same neural ocillator and the same parameters.

The limit cricle will garanty that the final motion will converge to the same one, so the all the agent will move in an unifield manner. The different between agent is an phase shift. That is although all the agent are controlled by the same limit circle, but there is a phase offset between them.

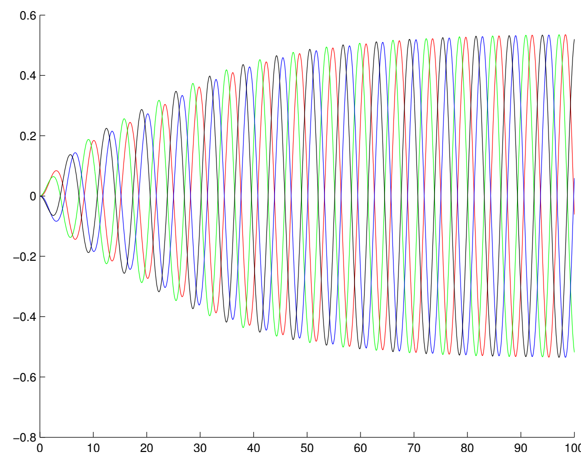

Figure 8: *Fish SWimming*

We apply this method modeling the motion of a long tail fish. And use the basic mass spring model to model the fish movement. The advantage of such system over orignal mass spring system is Unlike the origianl mass spring system, the speed and size of the vibration can be cotnrolled, also, numberic error is not a big problem. So the new system is controllable and also stable.

Figure down shows an mass spring system controlled with cpg.

7 futurework

Further Work and Discussion Motion Synthesis greatly reduced the computational load and generate adaptative motion. But since it is new and not sophisticated. Further research is needed.

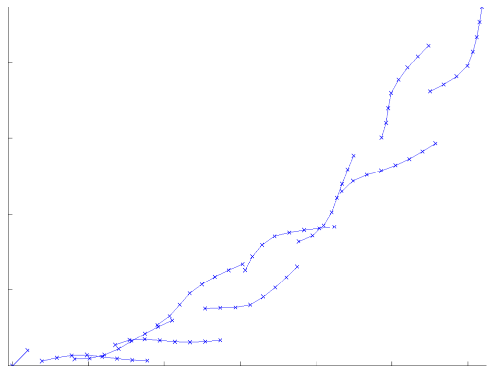

Figure 9: *Fish SWimming*

7.1 Modeling More Behaviour

This method is applicable for lots of system. But this method greatly relies on the dynamic properties of body environment. The method is not sensitive for small error, so, local modelling error maybe accepts, but the qualitative property must kept. This is not a trivial task. Some of the key features especially the qualitative dynamic property we are just begin to known. Further application, fluid dynamic of fish and squid swimming.

7.2 More Complex Systems and More Type of Symmetry

For develop lagrange, symbolic expression need to be provide, for high degree system, this may becomes problematic. Local frame based Method

7.3 Data Driven

Can we find symmetry info from data rather than symbolic differential equation?

7.4 Symmetry and Boid System

can symmetry be applied to boid system?

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To Robert, for all the bagels.

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