Adaptive Character Motion Synthesis By Qualitative Control Theory

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Abstract

*In this paper, we propose a new method for adaptive motion synthesis based upon new bioloigcal research ideas. With our method,only the **qualitative properties** of motion are controlled, adaptations comes from perturbations of environment or body structure, this is the **the Qualitative Control Theory (QCT)**.*

The biological idea we hold is that natural motions mainly come from the complex interaction between the body and the environment. The natural neural system only maintains or tweaks qualitative properties of this dynamic interaction. The mathematical model is based on the Qualitative Theory of Differential Equation. Adaptive Motion Control is achieved through manipulating the topological structure of the dynamic system to enhance the "structural stability", rather than counteracting the perturbation effects.

Adaptive Motion Synthesis Method completely solved motion retargeting problem, it is computational efficient and maintains many important natural looking features.

Index Terms

Charater Motion Synthesis, Qualitative Dynamic, Physics Based Animation

1. Introduction

The challenge of Character Motion Synthesis (CMS) is not to make characters move, but how to make them lifelike. This comes from our human's marvellous ability of motion perception. Motions for the same task are very similar, but vary adaptively. From the variety in motion details, humans can infer the changes in mental states, health conditions or even the surrounding environment.

Nowadays in industry, high quality motions are majorly generated by manual work. Most characters are very complex, which contain a large number of joints, making animation a tedious work. To make things worse, it is difficult to reuse animation data.

Some Researchers hold the belief that motion can be synthesised by simulating the dynamics of body, environment and the neural control system,while it is not an easy task. From the mechanical viewpoint, virtual characters are full of redundant **degree of freedom** (DOF)s, which not only increase the computational load, but also make the solution nondeterministic.

Some features are noticable in bioligical system but are hard to achieve at the same time by current CMS methods.

Adaptive Natural motions are adaptive to the changes in the environment or body conditions. A typical example is human locomotion. The walking motion changes on different terrains.

Agile Some motions of animals are very fast, more puzzling is that the neural system can solve the complex motion control problem in a realtime. It seems very easy for the neural system to solve such complicate problems.

Efficient Natural Motions are energy efficient. In theory, this idea is supported by Darwin's Theory of Evolution. Animals spent far less energy than our expectation. An example is that the energy consumed by human walking is only 10% of that for a robot of the same scale.

In QCT, the objective of motion control is some qualitative property of the motion. Adaptation involves little control effort.. In mathematics, a natural motion is modelled as a **structural stable autonomous system**. The key idea of Qualitative Control Theory is focusing on the topological structure of such dyanmic system.

For biology theory, the proposed research further completes the theory of **Motor Control** . For CMS research, this research introduces a novel method aiming at generating adaptive motions. In application, it will solve the problem of reusing motion data thus greatly reduce the animation work. It especially suits repetitive and low energy motion tasks which are most challenging to synthesize. It also has the advantage of computational efficiency and can be accelerated by GPU and may run in real-time in future.

2. Related Work

2.1. Dynamic CMS Methods

For procedural methods, motions are formulated as functions of various parameters such as

$$F : P \mapsto M$$

, where M is the valid motion space of characters, P is the control parameter space. Currently, P and F are mainly based on rules of physical science, and such procedural methods are often called "Simulation Methods". Since natural motion is governed by mechanics, dynamic simulation has the potential to generate more realistic motions. For dynamic methods, the body is usually modelled as a linked rigid body system,and simulated by realtime methods[Baraff, 1994, Mirtich, 1996, Stewart and Trinkle, 2000]. The bodies of animals are actuated by muscles under the control of neural system. Control system design is the most difficult problem.

Some early research applied classical control methods like PD controller [Raibert and Hodgins, 1991] for locomotion synthesis. Later research [Hodgins et al., 1995] applied the same method for different tasks like running, bicycling, vaulting and balancing. Such methods are based on simplified models which relieved the controllers from the problem of redundant DOFs, but important motion details were also neglected.

In most cases, motion solution are not unique. Optimization methods have been applied to solve the nondeterministic problem. Among all the solutions in possible motion space, the "best" one is chosen as the proper solution: For dynamic methods, a reasonal choice is minimize the energy cost V,such that

$$\mathbf{V} = \int_{t0}^{t1} F_a(x)^2 dt$$

where F_a is the active force generated by actuators like motors or muscles. This is introduced to CMS research as the influential Spacetime Constraints[Witkin and Kass, 1988]. In many cases, these methods produced very believable motions. Jain et al. [2009] provides an example of locomotion; Macchietto et al. [2009] find a method for balance maintaining movement. Liu [2009] proposed a method for object manipulating animation. One shortcoming of Spacetime Constraint is the efficiency. Spacetime Constraint in nature is a variational optimization problem. It takes prohibitive long time to simulate complex musculoskeletal structure[Anderson and Pandy, 2001]. Optimization techniques like time window and multi-grid techniques are proposed by Cohen [1992] and Liu et al. [1994]. Very a few research [Popović and Witkin, 1999] proposed Spacetime Constraint for full body dynamic animation.

Limit Circle Control(LCC) [Laszlo et al., 1996] provides an alternative method for lower energy locomotion animation. The LCC theory has been used in explaining passive mechanics. Compared with Spacetime optimization, LLC methods is more computational efficient method for low energy motion.

Inspired by the Theory of Evolution and Neural Network, some researches[Sims] build a simple biology system and simulate the evolution process of brain. After enough trial and error, reasonable motion controller can be developed. But the result is unpredictable.

2.2. Biological Research

The foundation of Motion Synthesis is our understanding of natural animals' motor control system. In biological viewport, motor control is an age old problem full of paradoxes. Motor Control is a complex process involves many chemical, electrical and mechanical effects. In both CMS and biological motor control research, one most noticeable question is the computational efficiency. More questions arise after more knowledge of the biological computer, the neural system, has been obtained, which makes traditional control idea questionable. Here we list several major questions[Glynn, 2003].

Time Delay Neural signal transmitting speed is slow; and there is a long delay between neural signal firing and force generation in muscles.

Noisy Besides the delay and slowness, the neural signals are also noisy. The body structure and environment are also nonlinear, noisy and time varying. So methods that are sensitive to model accuracy are not proper for the natural neural control system.

Limited Activity Current research evidences and common life experience show that motor control involves little control effort. Many experiments show motion can happen even without brain input.

Despite the complexity of body structures and environment, the natural motor control strategy seems relatively simple, involves little computational work. The current idea of biology research is that motor control is a low level intelligent activity and can be controlled with some very primitive neural structure. In many animals, the active neural structure in motor control is the Central Pattern Generator(CPG) which generates rhythmic signals. There are many experimental researches in robotics and biomechanics succeeded in controlling some motion with very simple strategy[Nishikawa et al., 2007]. And some new ideas about motor control have formed.

Uncontrolled Manifold Hypothesis The observation of blacksmith's hammering motions show that even under the same conditions, the motions still vary. An explanation is the neutral system doesn't control all the DOFs. Some DOFs are not controlled and freely influenced by the environment. This is the Uncontrolled Manifold Hypothesis(UMH)[Latash, 2008]. In this viewpoint, the result of motion planning is not a trajectory, but a space of valid trajectories.

Equilibrium Point Hypothesis EPH suggests that what the neural systems controls is not trajectory, but the equilibrium points. This idea comes from properties of differential equations. For a dynamic system in the form $\dot{q} = H(q)$, the equilibrium points $q_e$such that $H(q_e) = 0$. For a stable system, over the time the state q will approach to the equilibrium point q_e and finally stays at q_e.

Impedance Control Hypothesis Impedance Control [Hogan, 1985] refines the idea of EPH by providing an explanation for effects of the extra DOFs. Impedance Control proposed that at an equilibrium point q_e such that $H(q_e) = 0$, the extra DOFs provide a way to control the stability and admittance of the equilibrium point q_e. The mathematical presentation is

$$H(q_e + Er) = K \tag{1}$$

where Er is the offset error vector, K is stiffness matrix or impedance, which determines the stability and gentleness of the equlibrium point. Neural system will tune the direction of K according to the motion purpose, such as avoiding obstacles and risks. Experiment [Franklin et al., 2007] shows that the matrix K has anisotropic properties.

Morphological Computation Theory UMH,EPH and IMH are efficient at explaining some arm motion and object manipulation tasks, but the theory is incomplete for more complex motion. A generalization theory is proposed as Morphological Computation Theory(MCT)[Nishikawa et al., 2007, Pfeifer and Iida, 2005]. The idea is both the body structure and the environment play a crucial role in motor control, they can be treated as a physical computer. For some motion tasks, basic motion patterns are generated by body and environment, the neural systems only maintains or tweaks such motion patterns.

3. Qualitative Dyanmic For CMS

3.1. Qualtiative Mechanical Theory

The Qualitative Control Theory is a mathematical description of the MCT In qualitative control theory the basic patterns of motion are called **motion primitive**. In mathematic terms, motion primitives are **structural stable autonomous systems**

3.1.1. Basic Concepts of Qualitative Dynamics.

Qualitative Dynamics can be traced back to Poincare[Poincaré and Magini, 1899, Poincaré, 1885] and recently developed by the Smale School. Please refer to other books and lectures such as [Abraham and Marsden, 1978]for introduction in details.

The configuration of system is described using state value in the state space. we represent the state of a system as a vector q, M is the state space, which is a manifold. The motion trajectory over time is $q(t)$. For a dynamic system, $q(t)$ is usually in the form of ordinary differential equation.

$$\dot{q} = F_u(q) = F(q, u), q \in M \tag{2}$$

where u is the control effort. F is determined by the system's natural property. If $u = 0$, no control effort is applied. Such systems are **autonomous systems**. For every point $q \in M$, F and u determines a derivative vector $\dot{q}$. All the vectors over the full space of M form the **vector field** V. There is a corresponding geometry structure for Equation (2), a differentiable manifold. The motion trajectory can be found by apply the integral operation on the vector field. The result trajectory is defined as **flow** Φ, all the flows form another geometry structure, the **phase portrait**, which illustrates all the possible motions of the dynamic system.

On the phase plane, Flows can only intersect at some special position.

Fix Point The first type of intersection is fix point or equilibrium point q_e.

$$H(q_e) = 0$$

Period Flow Another type of intersection is a periodic flow. For any point q on the circle, we have

$$H(q(0)) = H(q(T))$$

Intersections like fixed point and are also called **equlibria**,

At each **equilbria**, the local space can be divided into three subspace of submanifold: centre submanifold, stable manifold, and unstable submanifold.

centre submanifold If a flow θ pass through a point m on centre submanifold W_c, flowθ will remain on the Centre Manifold

$$\theta_c(t) \in W_c, t \in R$$

An equilibria must be on center manifold.

stable submanifold For the flow θ_s passes through a point m on stable submanifold W_s, the flow will finally converge to a nonwandering point on centre submanifold.

$$\theta_s(+\infty) = \theta_c$$

unstable submanifold For the flow passes through a point m on unstable submanifold W_u, the flow will be repelled from the nonwandering points on centre manifold. An alternative perspective is the inverse of the flow converge to nonwandering point.

$$\theta_u(-\infty) = \theta_c$$

For nonlinear system, globally, the shape of stable and unstable submanifold may be bending and connect with itself or each other. The equilibra and its connectivity of sub manifolds form a topological structure. The phase plane is divide into different regions,result in a cellular structure. In each region,there is only one attractor, all the flow in this region will converge to the attractor. and the corresponding region is called **basin of attraction**.

3.1.2. Motion Adaptation and Stability.

A mechanical system can be extremely stable without any control effort. This kind of stability is rough stability or structure stability [Andronov and Pontryagin, 1937]. Rough stability or structure stability is determined by the topology structure of the system[Jonckheere, 1997].

Motions vary greatly, this makes it difficult to define motion and tell it from another. In Qualitative Control Theory, motion should be defined by the topological structure of the coresponding differetial equation. From topology viewport, motion adaptation can be modelled as homeomorphism. Homeorphic flows can be generated if the differentiable manifolds are homeomorphic, which means they share the same topological structure,but different shape.

Structure stable autonomous systems have the ability to maintain its topology structure under perturbations. They generated homeorphic flows while keep the qualitative property, thus the resulting motion is adaptive but qualitatively maintained.

In fact, the effects of control and perturbation can be modelled in the same way. An autonomous dynamic system is represented as,

$$\dot{q} = H(q)$$

Considering the control and perturbation, the system will be changed into

$$\dot{q} = H^{'}(q) = H(q) + f(q,t)$$

Where $f(q,t)$ is used to model both the control and perturbation.

Geometrically, the effect of $f(q,t)$ can be seen as a deformation to the trajectory $q(t)$ on the phase plane.

$$q^{'} = \Theta(q)$$

such that

$$H^{'}(q) = H(\Theta(q)) = H(q^{'})$$

In Qualitative Control Theory, only final motion results are concerned, In mathematical viewport, only the attractors of flows are controlled, while the flow itself is not concerned in motion control. Also, according two the type of attractors, motion can be group into two groups.

Discrete Motion Such motions have fixed attractors, typical motions include posture control and picking up motion of the arm.

Peridotic Motion Such motion have periodic attractors, typical motion include walking, running and heartbeating.

Motions are made up of motion primitives. Neural control system tweaks the basic motion primitives to achieve specific objective.

Qualitative Control Theory preserve the natural motion features.

Adaptive Using this method, different perturbations will result in different motions. Motion will vary with the environment change.

Efficient Motion will be generated passively and follow the least energy path.

Agile QCT does not rely on high precise calculation. Topological structure can be manipulated and maintained by some very simple methods.

3.2. CPG and Entraintment

In nature, an animal's body and environment can be extremely complex. It leads to high dimensional manifolds with complicated topological structure. For CMS application, one question is the whether complex system can be controlled with a simple method. Biology Research suggested that the motion is mainly controlled by the Central Pattern Generator, which is a small autonomous network that generating rhythmic signals. The existence of CPG is very common, primitive animals like lamprey and fish, to high level animals like bird, mammal and human[Cohen, 1988]. The idea of control motion by rhythmic signals can be modelled as entrainment [González-Miranda, 2004]. In this section, we provide the understanding of biological entrainment in the viewport of Qualitative Control Theory.

3.2.1. The Biological Entraintment. Entrainment is the phenomenon that two coupled oscillator systems oscillate in a synchronize way. Although the mechanism can be very complex, the phenomenon is universal. The classic example shows individual pulsing heart muscle cells. When they are brought close together, they begin pulsing in synchrony.

Entrainment will happen when coupling two oscillators with similar oscillation frequencies but with very different characteristics. A simple explanation is that energy fluctuates between the two oscillating system.

Given two systems,

$$\dot{x} = f(x)$$
$$\dot{y} = g(y)$$

when two oscillation systems couple, the behaviour of both systems will change. The coupled system can be presented with the following equation

$$\dot{x} = f(x) + m * g(y) \quad (3)$$
$$\dot{y} = g(y) + n * f(x) \quad (4)$$

where m,and n are coupling coefficients. When m is large, the behaviour of equation(3) is more dominated by the second term $m * g(y)$. This can be seen as a forced oscillation system. Even m is small, behaviour of equation(3) can be changed qualitatively. For some cases, stability can be enhanced and chaotic behavior can be suppressed.

The entrainment model can be applied to CMS, if we took the neural oscillator as $f(x)$ and body as the mechanical oscillator $g(x)$. The properties of mechanical oscillator can be controlled by the oscillation property of the neural system; the oscillation of mechanical system can be controlled by the oscillation of neural system. If we increase n, we can expect that mechanical oscillators shows the behaviour of neural oscillators. Entrainment can help to boost the stability of the mechanical oscillation. When the mechanical oscillation is disrupted but the neural oscillation remains, after the perturbation is removed, the undisrupted neural oscillation can drive the disrupted mechanical oscillation back to normal.

3.2.2. The Structural Stable Oscillation of the Neural System. One extensively studied oscillation model is developed by Matsuoka [1985]. The mathematical presentation is as follows:

$$\begin{aligned}
\tau_1 \dot{x_1} &= c - x_1 - \beta v_1 - \gamma [x_2]^+ - \sum_j h_j [g_j]^+ &(5)\\
\tau_2 \dot{v_1} &= [x_1]^+ - v_1 &(6)\\
\tau_1 \dot{x_2} &= c - x_2 - \beta v_2 - \gamma [x_1]^- - \sum_j h_j [g_j]^- &(7)\\
\tau_2 \dot{v_2} &= [x_2]^+ - v_2 &(8)\\
y_i &= \max(x_i, 0) &(9)\\
y_{out} &= [x_1]^+ - [x_2]^+ = y_1 - y2 &(10)
\end{aligned}$$

where x and v are state variables of the oscillator, τ,c,β,γ are parameters of the oscillator.

Matuoka oscillator is an autonomous oscillator; it can begin to oscillator without any control effort. It is also adaptive; entrainment behaviour can happen between one Matuoka oscillator and different oscillators. But because of the nonlinear properties, its behavior is not completely understood. Matsuta[Matsuoka, 1987] explains the adaptive properties from the location of the roots of characteristic equation. Wilimas[Williamson, 1998] explains the properties in frequency domain. In our research, we find some important properties of neural oscillator by investigating the phase portrait.

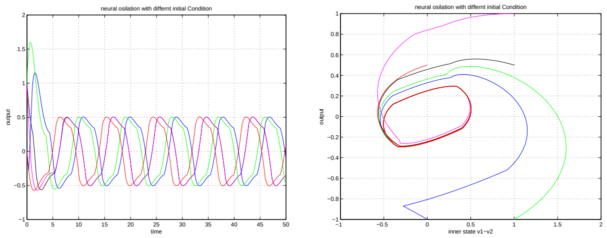
Basically, neural oscillator shows three important properties:

Simple Structure The topology structure of neural oscillator is simple, it includes one attractive limit circle and one fix repellor.

Large Basin of Attraction All the simulations we carried out converged to the same limited circle.

Fast Converging Speed In most of the case, the flow will converge to the limit circle within one period time.

Features above are shown in Figure. The large area of



(a) time state (b) Phase Portrait

Figure 1. Matsuta Oscilator

basin of attraction means the final behavior is totally determined by parameters. Initial conditions will have no effects on the final oscillation. The converging speed can be seen as quick recovery ability. When an impulse perturbation happens, it will recover in one period time. These properties are very valuable in CMS research. Matuskota Oscillator is a structure stable autonomous oscillator. An intuitive idea is that we couple the neural oscillator with mechanical oscillator of body and environment, thus tune the motion a structual stable one

4. Application to Bipedal Walking

Bipedal walking is a common motion task and has been well studied by many research communities, including robotic Raibert et al. [1986], artificial intelligence and biology as well as computer graphic research. From experience, walking involves little reasoning activity, this idea is supported by the biology research that the number of neurons that take part in the lower limb control is very limited, much less than arm, hand and even tongue. While for artificial system, robust bipedal walking is difficult to achieve. Many control method has been tried, but none of them shows comparable performance with human walking. In dynamic research,natural looking gaits can be generated by passive method. There have been a series of passive dynamic walking machine[McGeer, 1990a,b]. If we put a passive walking machine on a slope, without any effort, it can walk down the slop. However the stabilities are very fragile.

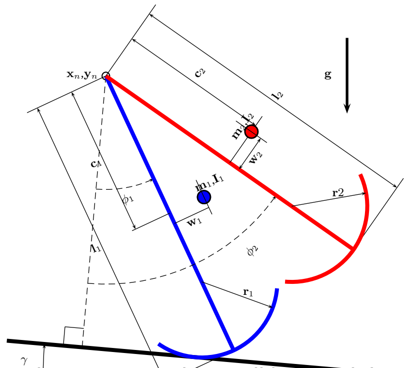


Figure 2. Passive Walking Model

Passive walking can only be maintained when walking down a specific slope under specific condition.

From the viewport of Qualitative Control Theory, the reason why passive walking machines can walk down the slope is because that there exists a limit circle for the dynamic interaction between body and ground. The fragile stability means the basin of attraction covers only a small area on the phase plane. For natural looking walking motion,We plan to boost the stability of the passive walking machine by neural oscillation entrainment.

4.1. 2D Passive Walking Model

The mechanical model we adopted is illustrated in Figure 2. Both the model and simulation method are from the paper[Wisse and Schwab, 2005]. We only give brief description to make the content self-contained.

Passive walking is not a continuous dynamic system. We separate the motion into two phases and formulate two equations.

Leg Swing Phase During the swing phases, we suppose that one leg is fixed on the ground, the arc foot makes the passive dynamic walker rolling without sliding. The equation is

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} \tag{11}$$

Heel Strike Phase We suppose the heel strike the ground in a short time, the angular momentum is preserved. The Equation is as below

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \dot{q}^+ \\ f_c \end{bmatrix} = \begin{bmatrix} \bar{M}\dot{q}^- \\ 0 \end{bmatrix} \tag{12}$$

where $\dot{q}_+$ is the state variable after the collision, $\dot{q}_-$ is the state variable before the collision.

In the equations above,

$$X = [x_1, y_1, \phi 1, x_2, y_2, \phi 2]^T$$
$$Q = [x_h, y_h, \phi 1, \phi 2]^T$$
$$T_{i,k} = \frac{\delta X_i}{\delta Q_k}$$
$$g(x) = \dot{T}\dot{q}\dot{q}$$
$$M = diag[m_1 m_1 I_1 m_2 m_2 I_2]$$
$$\bar{M} = T^T M T$$
$$\bar{f} = T^T [f - Mg]$$
$$g_y = y_h - (l-r) * cos(\phi) - r = 0$$
$$g_x = x_h + (l-r) * sin(\phi) + r * \phi - x_f$$
$$D(x) = [g_x g_y]^T = 0$$

The input of neural oscillator is defined by the difference angle between the two legs.

$$G_{input} = \phi_1 - \phi_2$$

Neural output will drive the biped walker. After adding the neural control, the equation of the dynamic system is

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} + \begin{bmatrix} \bar{U} \\ 0 \end{bmatrix} \quad (13)$$

Neural oscillator output is applied at the hip joint to actuate the two legs towards different directions

$$U = [0, 0, 1, -1] * G_{out}$$

4.2. Adaptive Walking Motion

Passive Walking When the passive walker walks down a slope, for every step, there is energy input from the potential energy, and there is also energy loss because of heel strike. There must be an equilibrium condition when the energy lost is equal to the energy input. If natural looking motion is energy efficient, such passive walking motion can be expected to be natural looking. Because there is no extra control energy input, such motion is the most energy efficient.

Figure 3 shows the gait of the passive walker. After coupling the neural oscillator, the basic pattern is not changed as shown in Figure 4.

Walking On Plain However the stability is fragile. The passive walker can't walk on plane. The step size will decrease after each step, and finally it will stop or fall over as illustrated in Figure 5.

After coupled with the neural oscillator, this walking machine can walk on plane, and exhibits gait similar to the passive dynamic walker. Figure 6 shows the gait. From the state plot Figure 7(a), and phase plot Figure 7(b), we can see that the gait converged to a stable limit circle.

To verify the structural stability, we introduce a variety of perturbations to the passive walker. These perturbations include different initial condition, different slopes, different leg mass and different leg length.

Different Initial Condition The original passive walker is not very stable. A slight change in initial condition will result in walking failure. While after coupled with neural oscillator, the basin of attraction has been enlarged. A different initial condition can still lead to a stable gait, as show in Figure 8. Natural looking gait is maintained.

Walking On Different Slopes Another parameter we change is angle of the walking slope. When we increase the down slope, stable walking motion can still be maintained, as shown in figure 9. An important discovery is that although the walkers can walk on various down slopes, it can not walk up slope,no matter how control parameters are changed. It cant walk up slope and will fall backward after several steps. We suggest that this is because the proper limit circle does not exist in the dynamic system when walking up slope. This finding may help us to understand the upper body effect in walking.

Leg Mass Variation We add mass on one leg to 50% and find the stability of the gait is still maintained. The step length and swing period of the two legs are different, this gait is similar to that with a crippled leg, see figure10.

Leg Length Variation The last parameter we change is the leg length. We change the leg length to 1/8 shorter. And we find the stability of the gait is maintained, see Figure 11

5. Conclusion

Qualitative Control Theory have the potential to relieve Character Motion Synthesis for the heavy computational work of Spacetime Constraint. It provide a new method for synthesize adaptive motion efficiently.

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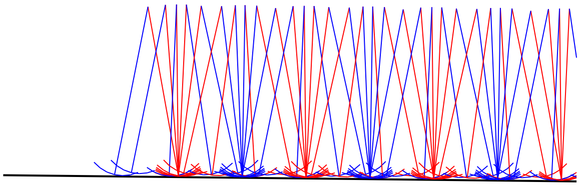


Figure 3. Stable passive walking gait

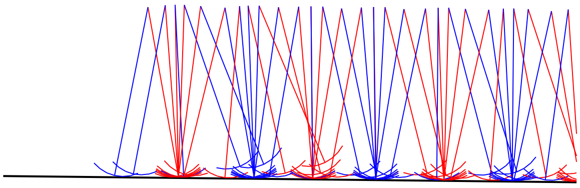


Figure 4. Walk down the same slop when actuated

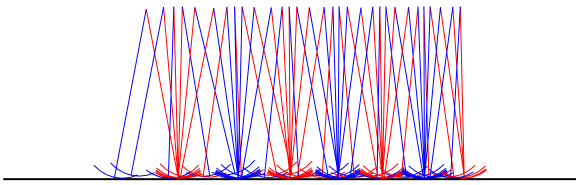


Figure 5. Passive walking gait can't be maintained on plane

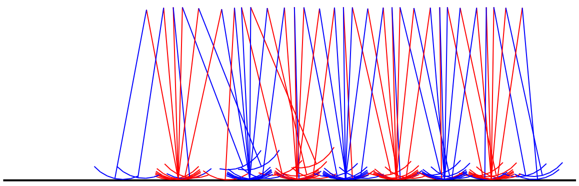
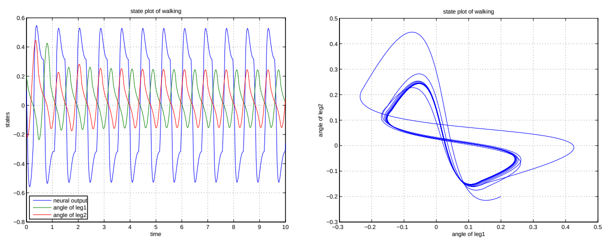


Figure 6. Walking on plane under neural control



(a) State Plot (b) Phase Plot

Figure 7. Walking on a plane converges to a stable limited circle

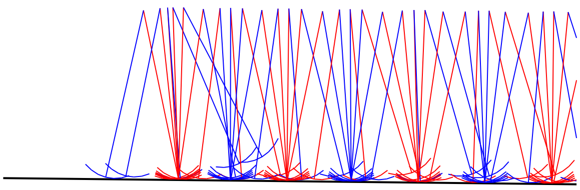


Figure 8. Walking with different Initial condition

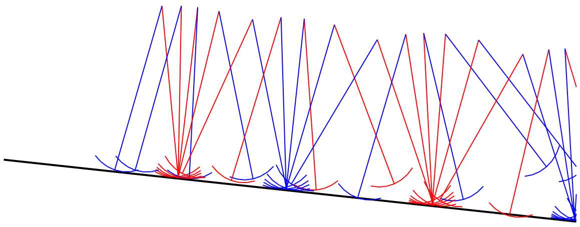


Figure 9. Walking with different slope angle

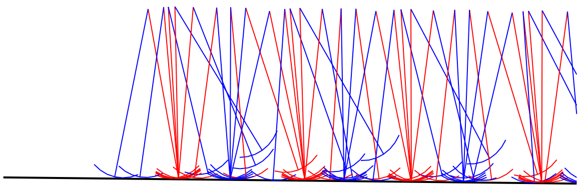


Figure 10. Walking with legs of different mass

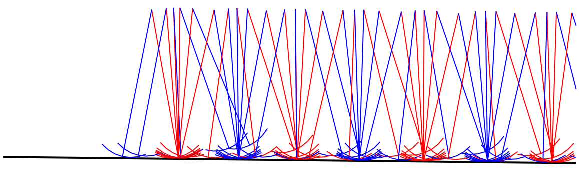


Figure 11. Walking with shorter Legs