Adaptive Motion Synthesis and Motor Invariant Theory

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Yet to be decided

Note:Topology Conjugacy

Note:Symmetry shoud ref discrete symmetry writing

Note:System Affine Transformation

Note:Kangroo example? how to add leg swing? Running how to switch leg?

Note:appendiex mathematical

Note:uncanny valley

Abstract

Generating natural-looking motions for virtual characters is a challenging research topic. It becomes even harder when generating adaptive motions interacting with the environment. Current methods are tedious, cost long computational time and fail to capture natural looking features.

This report proposes an efficient method of generating natural-looking motion based upon a new motor control theory. The principal idea is motor repertoire is made up of a limited number of elements. Motor control basically connect the basic motion primitives together just like connecting alphabets into sentences.

we propose principle of motor control is not feedbackbased, they should by model as topology conjugacy. During motion adaptation, neural system tweaking the basic mechanical system to form a analogous dynamic system that meet constraints and purpose.

When animals adapt their motion, some properties are maintained, which are called motor invariant. Motion Primitives are identified by the qualitative properties, for which we use the mathematical tools of differential topology. Tweaking of the motion primitives is model as Symmetry Preserved Transformation, for which we use lie group theory.

Following our idea, we generate adaptive natural looking motion with very little computational costs.

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Nomenclature

Roman Symbols

h_{i}	Input Coefficient of the Neural Oscilator
h_{o}	Output Coefficent of the Neural Oscilator
$\dot{q}$	Gneralized Velocity
$\simeq$	Topology Conjungacy
$\mathbf{x}$	State Variable
u_{i}	Input Signal to the Neural Oscilator
u_{o}	Output of the Neural Oscilator
A	Attractor
$B(A)$	Basin of Attration of A
G	A Lie Group
g_a	an elemetn in Lie Group G with parameter a
$I(x)$	Invariant Function of x
M	State Space Manifold
Q	Configureation Space or Configuration Manifold
q	Generalized Coordinates

S	Neural Oscilator
TM	Tangent Boundle Manifold
TQ	Tangenet Bundle of Q
u	Control Input

Chapter 1

INTRODUCTION

Character Motion Synthesis (CMS) research aims at generating motions for virtual characters. It is a valuable topic for both industry and academic community. Main applications are in the media industry, both computer games and animation films depend heavily upon character motions for storytelling. CMS also has many applications in other areas, such as user interface design, psychology, sports and medicine.

The challenge of CMS research is not to make characters move, but how to make them lifelike. This challenge comes from our human's marvellous ability of motion perception. Motions for the same task are very similar, but vary adaptively. From the variety in motion details, humans can infer the changes in mental states, health conditions or even the surrounding environment.

Note:uncanny valley

Nowadays in industry, high quality motions are mainly generated by manual work. In applications, most characters are very complicated and contain a large number of joints, making animation a tedious work. Making things worse, it is difficult to reuse motion animation. Reusing motion animation for a different scenario is prone to artefacts. For this situation, high level animation tools are badly needed.

1.1 Rethink about Motor Control

The foundation of these problems is our misunderstanding of the biological motor control system. Although motion of animals have fancied us for thousands years, some basic questions of motor control and motion perception remain open. And answers to such questions become even more valuable nowadays. Advance in this topic will greatly influence the biology, robotic engineering even intelligent research. Note:The difference between biological motor system

The paradoxes is even human are good at motor control and motion perception; human still dont have an idea of how we move and how we perceive motion. Before going into details into the research ideas, we first review some puzzles troubles the foundation of CMS.

- **degrees of freedom (DOF)**. Unlike the artificial system, biological systems have many more (DOF). Artificial ship is a fixed rigid body, while fish has a very flexible vertebrate, which is of tens of DOFs. In principle, we know that more degrees of freedom allow animals to generate motion variation and make motion adaptive to the environment. But for control system, extra degrees of freedom propose a challenging problem, for human example, it is difficult to answer how the neural system controls more than 200 bones and 600 muscles to walk one step.

- **dexterity** Human can finish much more motion task than the artificial system. Beside the walking, swimming and object manipulation, human can also utilize a large number of artificial tools, driving a car, skate, cycling, and playing tennis, and even some function the feeding, breading, language, vision all depends on motor control. The question arise how much resources are needed for so many motion abilities.

- **perception** We dont judge motion by check the physically correctness. For some artefacts, we will identify them instantly, while for some impossible motions, we dont notice the mistakes. If we acquire many motion tasks through learning, what we see is closely how we learn our motion ability, then here follows the question do we ignore some artefacts because they are not important for motor control?

After a close look, we have to recognize we know little about the biological motor control.

For computer animation research, the key principle is we should know the things we animate. Natural motion system has many valuable properties which are not captured by current motion synthesis methods.

- **Adaptive and robust** Natural motions are adaptive to the changes in the environment or body conditions. A common example is human locomotion. Walking on different terrains will exhibit different gait while the balance is maintained.

- **real-time performance** Some motions of animals are very fast, honey birds may vibrate their wings in kHz. The astonishment is to the speed of motion, more puzzling is that the neural system can solve the complex motion control problem in such a short time. When an animal avoids obstacles at very high running speed, it must continue its running, make a turning and keep balance at the same time. It seems easy for the neural system to plan complicate motions.

- **Energy Efficient** Natural Motions are energy efficient. In theory, this idea is supported by Darwin's Theory of Evolution. But animals spent far less energy than our expectation. An example is that the energy consumed by human walking is only 10% of that for a robot of the same scale.

1.2 A Different Motor Control Idea

When design a motion synthesis framework, the key is the several decision making.

- **Memory or Computation** The first question is how we achieve our motor ability. Some argue that it depends on our motion memory. Given the uncountable variations of motion, it seems impossible for us to remember possible motion. Some argues motion is based on computation or our reasoning power; it will put heavy burden on our neural system and we need think hard to walk.

- **Feedback or Feedforward** The second is which control strategy plays more important role in motor control, feedback or feed forward. Artificial control theory is feedback based, if so, human must have a power sensing system which is accurate and fast, and move in a careful and nervous manner. A different control idea is feed forward based control, if we can predict something, we can take some measures to prevent motion failure, and we can extend our arms when walking or change the shoes if tomorrow will snow. Note:analogous system Feedforward will make the task easier, it free human of the sensing requirement in accuracy and speed, which our human are not good, it depends on the prediction and experience, which our human beings are fond of since the old day.

- **Disadvantage or Advantage** Maybe the most important problem is our attitude of the body structure. The body structure is the product of natural selection in millions of years. A complex system is not necessarily difficult to use. Seen in this way, the body structure should be a heritage rather than a burden. We prefer to think the body structure as an over powerful tool beyond our current comprehension. It is not that the nature makes a big mistake; it is we dont understand the great design yet.

In this thesis, we propose different idea towards motor control and motion synthesis. In this research, we propose a different motion synthesis method based on a different motor control theory.

An insightful discovery is that motor control can be easy. For some situation, some tasks mainly explore the properties of the body and environment and can be achieved with little control effort. In nature, we dont finish difficult motion tasks, we select many easy motion tasks that we are good at, connect or modify them for our special purpose.

The easy tasks are called motion primitives; they are the basic elements of our motor ability. When we modify the motion primitives, some valuable properties of motion primitives are kept unchanged, and the maintained properties are called motor invariants.

The inspiration of our idea comes from related biological research, which covers biomechanics and neural science.

1.3 llustrate principle ideas through simple examples

Although some obscure knowledge is needed to for the mathematical model, the principle is illustrative through some daily example. Here provides two examples about our idea of the easy motion and how can we tweak them.

The world is complex, the first question is how much information we need for accomplish a motor task. Our discovery that some motion task is so easy that e we need no control effort and blindness to some information doesnt matter. This idea is illustrate by the ship floating example

1.3.1 Ship Floating

Dynamics

The first example is the ship floating. In real life, usually the height of the ship is much larger than the width. When wave comes, also comes the question how the ship maintains its posture. To our surprise, we find maintain posture is easy.

The side sway motion as shown in Figure 1.1 is described by the dynamic 1.1

$$I_{netia}\ddot{q} + d\dot{\theta} = T_G + T_B + T_F = (Gl_g - Bl_b)sin(\theta) + T_F \tag{1.1}$$

q is the swaying angle, I_{netia} is the inertia, d is the damping coefficient, T_G is the torque of gravity, and T_B is the Torque of buoyancy. T_F is the external control torque. if $T_F = 0$, external control force is applied, the system is an **autonomous system**.

Equilibrium Posture

Ship will only rest at postures when external torques are zero. There are only two postures that the external torque is zero as show in Figure 1.2

The two postures are qualitative different. The left posture is attractive, if small perturbation moves the ship away from the left equilibrium posture, then it will return to equilibrium posture.

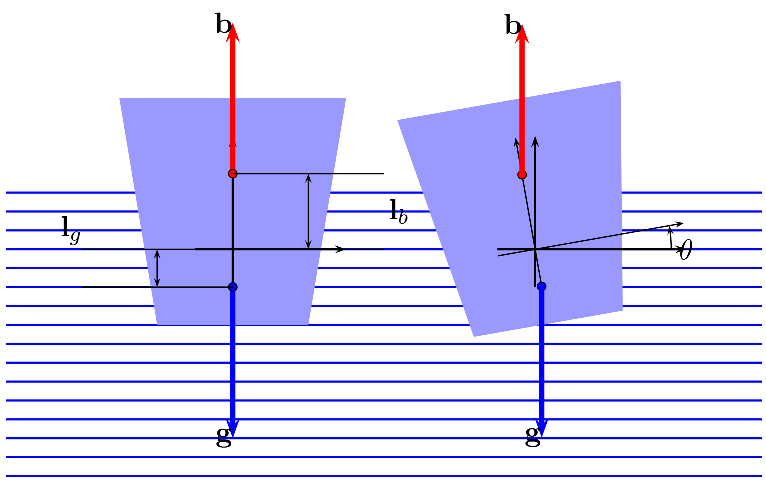


Figure 1.1: Floating Ship Example

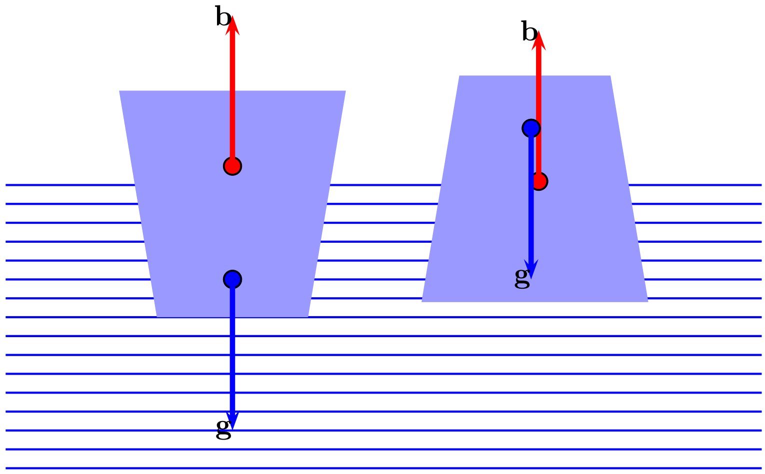


Figure 1.2: Two Equilibrium Posture

The right posture is repelling. If a small perturbation moves the state of the ship away from the equilibrium posture, it will move away from the posture.

The differences of the two postures can be illustrated with the **phase plot**. On phase plot, x axis is the angle, y axis is the angle velocity. Then the movement of the ship can be presented by a curve. Figure 1.3 shows motion about the left equilibrium posture, they will automatically move to the left equilibrium posture. Figure 1.4 shows the ship motion about the right equilibrium posture.

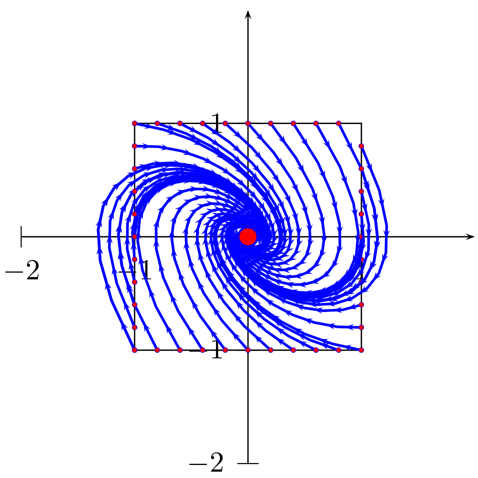


Figure 1.3: StablePosture

"Easiness"

If we plot all the possible motion of the ship, we get the **phase portrait** of the ship. We find out that all the curves will move away from the repelling posture to the attractive posture. Several curves are show in figure 1.5

Thus we come to the conclusion, the ship can automatically control maintain its left equilibrium posture. Ship can keep its upside up as long as the centre of

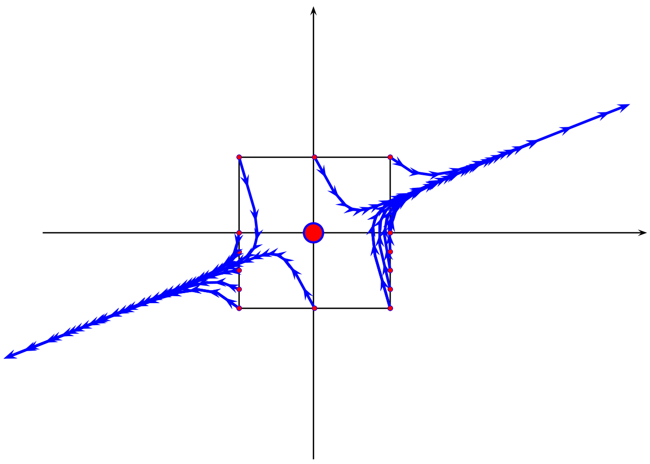


Figure 1.4: Unstable Posture

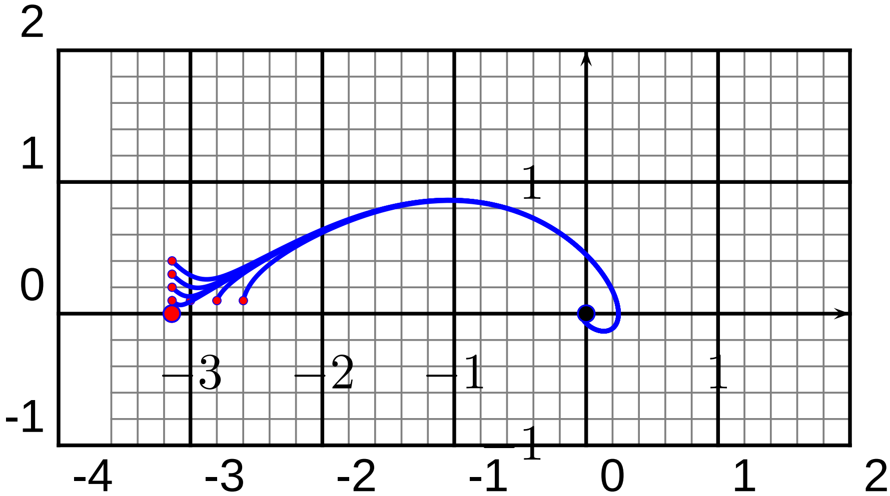


Figure 1.5: Several Possible Moiton of Ship

buoyancy force is above the centre of gravity. Maintaining posture of the ship is veryeasy.

Different Ships

Something interesting in our analysis is the conclusion is independent of the detail information about the size, weight and design of the ship. It is obvious different ship will maintain its posture with different motions. Or put in a different phrase, ship will adapt its motion during maintain posture when we change the ship parameters.

As long as the centre of buoyancy is above the centre of gravity, maintaining posture is easy. One phase plot, all the ships share some properties: one repealing point, one attractive point and all motion curves moves away the repelling point to the attractive point. As how in Figure 1.6

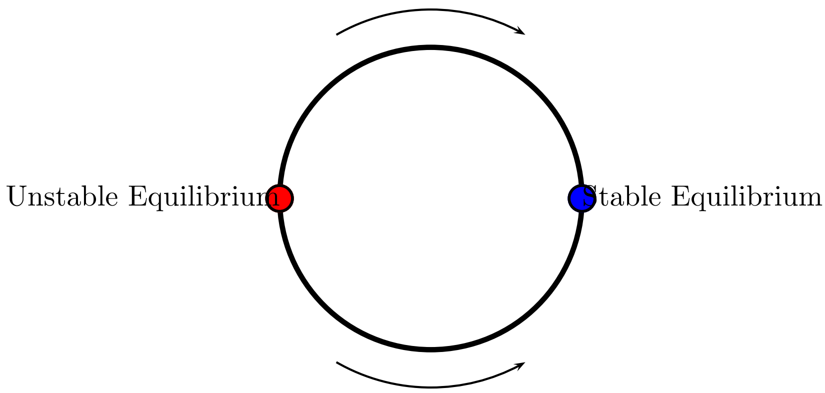


Figure 1.6: The Topology Structure

Such properties are topological property. As long as the topology is maintained, for different kinds of ship,maintaining posture is easy.

The dynamic system of two ships are presented by two equation $x = F_1(x)$ and $x_= F_2(x)$, if they share the same topology, we can say they are topological equivalant,represent by the symbol $F_1 \simeq F_2$ this kind of relationship is **topology conjugacy** and F_1 , F_2 are called analogous system.

Global Motor Invariant and System Adaptation

In our motion synthesis framework, the topology is called **global invariant**, and they encapsulate the qualitative properties of a dynamic system.

For easy task, no control is need. Following this idea, to find the easy motion task, we should investigate the topology of the phase portrait.

Another important idea is motion will adapt when we change the parameters of the dynamic system,which is called **System Adaptation**

1.3.2 The Mass-Spring Vibration

Human can finish motion tasks require high accuracy. The question is how neural systems solve the complex dynamic problem instantly with high accuracy. The biological motor control is very complex; it involves chemical, neural, electrical and mechanical process.

An alternative idea is we dont find the solution by solving the dynamics; we only need to know how to transform one solution into another. This idea is illustrative in the following mass spring example

Dynamics

Although simple, this system in Figure 1.7 captures some of the important properties of motor control system. The biological motor actuation or muscle works more like spring rather than the artificial electrical motor. Neural control adjusts spring rather applying force direct at the mass (which model the skeleton).

The canonical equation of mass spring system is equation (1.2)

$$\ddot{q} + q = 0. \tag{1.2}$$

In a similar manner, we can draw the state of mass spring system on the phase plot, as show in Figure 1.8

Symmetry and Transformation

Even it is simple, it is highly unlikely the knowledge of the mass spring is encoded in our brain in the form of equation (1.2). Very possible we know a possible motion

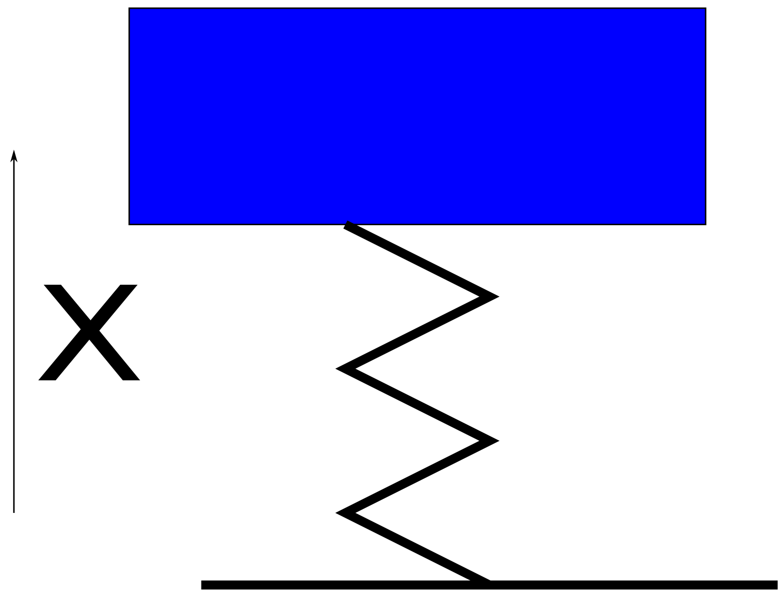


Figure 1.7: Mass Spring

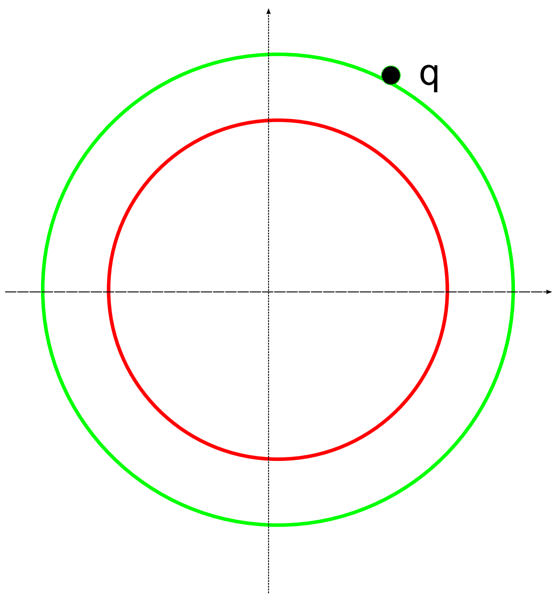


Figure 1.8: Mass Spring Phase Plot

like the red one. Given a state $[q, \dot{q}]$, we can easily work out the motion curve, it is on the green circle that shares the centre with the red one, but has a bigger radius. We can say the green curve is a scale red curve. The property that we can transform one motion into another is called ”symmetry”.

Using transformation, the new motion can be calculated easily. In fact we can ignore the complexity of the dynamic system, as long as we know the ”symmetry” property of a dynamic system. Given one motion, we can work out all the possible motions.

Dynamic Encoding

The dynamic system can be encoded in a different manner,a possible way is store a motion curve and its symmetrical property.

Impact of this idea for motor control is far reaching.

- it greatly reduces the computational cost in motor control.
- It also provides us with an idea of motion perception. In fact we dont calculate the details dynamic of motion; we can check the ”symmetry”. If we can transformed the observed motion(green) into our memorized one (red), than we think the motion are realistic, otherwise we detect artefacts.

Local Motor Invariant and Transform Adaption

When comparing the observed motion and the memorized motion, there is a better method than work out the transformation directly. Some property should be kept invariant under such transformation, for the mass spring system, we can say the shape is kept during scale transformation. From Differential Geometry view port, we can say the curvature is kept. From mechanical view, we can say for each motion, the energy is kept along the curve, and different motion is of different energy.

Energy and Curvature are called **Local Invariant.** And the transformation from one motion to another motion is called **transform adaptation**.

1.4 Overview of the Thesis

Global Motor Invariant contains the qualitative properties while Local Motor Invariant contains quantitative properties. For our motion synthesis method, the design idea is to keep the both motor invariants. Motion Adaptation is achieved through System Adaption(change system parameters) and Transform Adaptation. In application, this method easy to compute and result realistic motion adaptation.

This thesis is organized as follows.

In Chapter 2, we will discuss some previous research work of motion synthesis and biological motor control. These are the motivation and justification of our ideas.

In Chapter 3, we will focus on the Global Motor Invariant. We try to identify the qualitative properties of motion and investigate method maintaining the global motor invariant based on some biological ideas.

In Chapter 4, we discuss on the idea of Local Motor Invariant and Symmetry. Mathematical tools are developed and we show how to apply control effort to ensure symmetry. We show the computational complexity is greatly reduced by symmetry.

In Chapter 5, we discuss how the Global Motor Invariant and Local Motor Invariant Controller work together. Simple example is included to discuss the mathematical idea. Also we give an idea about how to connect motion primitives together to form more complex motion behaviour.

Chapter 3,4,5 lay the theory foundation for the motion synthesis control and be treated as the Motion Invariant Theory.

In Chapter 6, we focus on synthesizing motion of one motion primitives we apply our method for one of the most interesting and challenging motion synthesis research topic, bipedal walking. We show how our method main the stability and adaptive to different walking situation.

In Chapter 7, we will discuss how to connecting motion primitives together. New Motion Primitives (the balancing) is developed. We show how we an approximate with discrete motion with periodic motion. We show how different motion primitives connected together by switching the motion from stance to walk and from walk to stance.

In Chapter 8, we discuss about extend the basic idea to more complex system, or scale our method to address system with more difficult motion tasks. Three possible ideas discussed are the reduction, mechanical coupling and ad-hoc manner. The three ideas apply to different types of biomechanical system. Hopefully they will help us to synthesis motion we have not achieved yet.

In Chapter 9, we will discuss about some future work. After a retrospective discussion, we proposed some new question and ideas for graphics and neural science for further research.

Chapter 2

BACKGROUND

2.1 Motion Synthesis Research

We should know the things we animate. For CMS research, the basic challenge comes from our misunderstanding of the biological motor control system. Some reseachers following the memory based idea of motor control and adopted the data driven method. For reseachers following the computational based motor control,they adopted the procedure animation method.

2.1.1 Data Driven

Data-driven methods are based on ready motion data which are generated by Key-frame or Motion Capture(Mocap). In practice, motion data are segmented into short time clips. An animation is generated by selecting motion clips and connecting them together[Parent, 2002].

Like other example based methods in Computer Graphics, Data driven methods can generate good results on if similar motion clips can be found, but it is difficult to generate new motion. At current, it is also difficult to reuse the motion data, whether adapting the motion data for a different character or a different scenario. This is usually referred to the motion re-targeting problem.

Besides the difficulties in generating new motion, management of large motion data is another problem in practice. The Annotation Database [Arikan et al., 2003] and the Motion Graph [Kovar et al., 2008]are proposed. But because there is no efficient algorithm that understands motions, catalogue and search of motion data are not trivial task and are still open questions.

2.1.2 Procedure Method

Currently, for physics based motion synthesis research basically have three different ideas.

- **PD controller.** Some early research applied classical PD controller [Raibert and Hodgins, 1991] for locomotion synthesis. Later research [Hodgins et al., 1995] applied the same method for different tasks like running, bicycling, vaulting and balancing. PD control need a reference motion, so the motion is not adaptive

- **Limit Circle** Limit Circle Control(LCC) [Laszlo et al., 1996] provides an alternative method for lower energy locomotion animation. The LCC theory has been used in explaining passive mechanics. Compared with Spacetime optimization, LLC methods is more computational efficient method for low energy motion.

  In current researches[Coros et al., 2009; Laszlo et al., 1996], the limited circle is fixed. The control strategies are simplified as a state machine controller following a predefined limited circle. Like pd based controller, it is not adaptive

- **Optimization** Because of the redundant DOFs in the body structure, in most cases, there exist many motion solutions for one task. Optimization methods have been applied to solve the nondeterministic problem. Among all the solutions in possible motion space, the "best" one is chosen as the proper solution: For dynamic methods, a reasonable method is try to find the motion cost least energy E.

  $$\mathbf{E} = \int_{t_0}^{t_1} f_a(t)^2 dt \tag{2.1}$$

  where F_a is the active force generated by actuators like motors or muscles. This is introduced to CMS research as the influential Spacetime Constraints[Witkin and Kass, 1988]. It is based on the hypothesis that the natural looking trajectory costs minimum energy. It is related to the idea of Darwin's Theory of Evolution and the principle of Natural Selection. In many cases, these methods produced very believable motions. Jain et al. [2009] provides an example of locomotion. Macchietto et al. [2009] find a method for balance maintaining movement. Liu [2009] proposed a method for object manipulating animation.

Drawbacks of Optimization

Optimization is the current mainstream method for physics based animation. It generated the best motion results in current research. But this method have several drawbacks.

- **Numerical Stability and Modeling Difficulties:** Optimization can only guarantee the energy efficiency of the resulting motion, but cannot control convergence speed and stability. Even if the optimal solution is natural looking, it can be very hard to find. Finding the optimal solution depends largely on the accuracy of the model and the proximity of the initial conditions to the final solution. For motion synthesis, an accurate model is very difficult to build, which results in artefacts in the solutions. Liu [2005] points out those spacetime constraint methods only suit high energy motions, like jumping and running. For low energy tasks (such as walking) the results do not look natural, mainly because muscle effects are neglected.
- **Computational Complexity:** Optimization with spacetime constraints is a variational problem by nature. For a complex body structure, the performance of even current state of the art numeric methods is prohibitively slow, limiting the application domain of problems to those which are computationally feasible. In addition, little is known about how to reuse a computation result for motion adaptation.

2.2 Biological Motor Control

The theory from tradition artificial systems such as PD or optimization, are highly unlikely the idea for biological system. This is because they neglected the biological constraints. Recently biological research has a different idea about motor control.

2.2.1 Biological Constraints

Although the mechanism behind information processing remains obscure, some characteristics of biological information processing are well agreed. These characteristics make optimization control methods questionable. Here we list several major questions[Glynn, 2003].

- **Sensing and Control Limitations:** Motor control is not only a mechanical problem, but a complex process. Many crucial parameters and variables of the biological system are inaccessible to the neural system (such as mass, inertia, force) and can only be approximated. For important control variables (such as torque), the neural system has no direct control. In addition to this, body and environmental measurements are noisy and time varying, making methods that are sensitive to errors unsuitable for biological motor control.

- **Neural Computation:** The neural system is powerful, but is inferior in speed and accuracy when compared with a digital computer. It can only generate signals at hundereds of hz. Signal transmission speeds are slow there is a long delay between firing a neural signal and generating force in the muscles . For seeing an object to force is generate in arm, it may cost a bout haf a second. This makes it impossible for the neural system to carry out the complex computation necessary for realtime optimization. Humans body structure goes through big change through lifetime.

  Following the idea of optimization control, the dynamics of fluid environment and deformable body structure are more expensive to optimize.

  But most primitive life forms live in the sea and have limited intelligence.

- **Memory Capacity:** Some people argue that motion is not computed, but we store all the possible motor control ability in our memory, then when execute a motor task, we just access the memory for the proper motor control command. This idea may helps to drop the question of computation speed, but it faces another problem, the memory capacity. Motion varies greatly, if we store the motion in our brain, the problems is the memory capacity.

2.2.2 Motion Primitives

And many animals include human exhibit complex motion behaviours at very young age, before the intelligence system is fully developed. If we expand our view port, many complex motion abilities like breathing, heat beating and child bearing are inborn. There is no need for learning or intelligent effort. Also we find out that the motion style is not changed by the evolution of the neural system, after all whale swim more like fish than other mammals

Many researchers propose an alternative idea that animals dont move the way they want, but rather the way they can. The body and the environment play the most important role in motor control, as they form the basic pattern of motion [Nishikawa et al., 2007]. These basic patterns are called motion primitives [Poggio and Bizzi, 2004].

The number of motion primitives are the elements of motion and the number of it is limited. Complex motions are combinations of motion primives, just like we connect alphabets into sentences.

Neural Control Effects

Computer Graphic researchers find motion planning is a challenging task, while human motor control involves little mental work. A question is how much effor human take in motor control. The current idea of biology research is that motor control is a low level intelligent activity and can be controlled with primitive neural structure even without brain input. Despite the complexity of body structures and environment, the natural motor control strategy seems relatively simple, involves little computational work, and outperforms optimization methods.

Two ideas propose by bioligcal motor research build different model for neural effects.

- In many animals, the active neural structure in motor control is the Central Pattern Generator (CPG) which generates rhythmic signals. Cohen [1988] argues that human locomotion is the result of the interaction between neural and mechanical oscillators via a process called **entrainment**. Neural systems modify the motion by changing frequency and amplitude of the neural signal.
- Some research find motion will change in an uniform manner[Viviani and Stucchi, 1992],and propose tweaking effects of motion can be treated transformation[Flash and Handzel, 2007]. This idea not only explain the motor control problem, it also proide a clue for motion perception.

2.2.3 Motor Control Objective

The remaining big question is what is the control objective of motor control. The idea of neural system control movement to follow a trajectory is questioned by the biomechanical research.

The observation of blacksmith's hammering motions show that even under the same conditions, the motions still vary. An explanation is the neutral system doesn't control all the DOFs. Some DOFs are not controlled and freely influenced by the environment. This is the Uncontrolled Manifold Hypothesis(UMH)[Latash, 2008]. In this viewpoint, the result of motion planning is not a trajectory, but a space of valid trajectories. As long as the motion task is finished, neural system may not care how it is carried out.

Equilibrium Point Hypothesis(EPH)[Feldman, 1986]can be seen as a further development of UMH. This idea comes from properties of differential equations. For a dynamic system

$$\dot{x} = F(x)$$

the equilibrium points x_e satisfy the condition $F(x_e) = 0$.

Equilibrium point is the final position of the motion curve. EPH suggests that what the neural systems controls is not trajectory, but the equilibrium points.

Impedance Control [Hogan, 1985] refines the idea of EPH by providing an explanation for effects of the extra DOFs. At an equilibrium point x_e,

$$F(x_e) = 0$$

Impedance Control proposed that the extra DOFs provide a way to control the stability and admittance of the equilibrium point x_e. The mathematical presentation is

$$F(x_e + E_r) = KE_r \tag{2.2}$$

where E_r is the offset error vector, K is stiffness matrix or impedance. If K positive, x_e is unstable, characters will change his posture; if K is negative, x_e will be stable, posture can be maintained. if the value of K is large, the posture will be more stiff and rigid. if K is small, posture will be more gentle, and perturbations will cause a large offset error.

Neural system will tune the direction of K according to the motion purpose, such as avoiding obstacles and risks. Experiment [Franklin et al., 2007] shows that the matrix K has anisotropic properties.

2.3 Evidences from Bionomic Robotic Research

Biological research idea greatly inspired the engineering experiment. Some researches begin to focus on utilizing the natural dynamic and use as little control as possible with the hope that this method will produce more efficient robots. And some significant result has been reported

- **Passive Walking** A very important discovery is the bipedal walking can happen without any control[McGeer, 1990]. When putting a mechanical toy with human like body structure, it can walk down slope without any control effort. And based on this idea, new mechanical system is designed that can walk on plane with simple control[Collins et al., 2005].

- **CPG in locomotion** Also the CPG based entrainment is applied for robotic research[Williamson, 1999], the finding results show the CPG will boost the system stability and can maintain motion in unpredictable situation.

- **Symmetric based Control** The idea of Symmetry is also well exploit in Mechanical research. For mechanical view port, Symmetry has more concrete meaning. Like energy preserving or momentum preserving. The idea of Symmetry is also used in control robotics[Spong and Bullo, 2005]; some techniques are called Energy Shaping.

2.4 Our Research

This biological research idea greatly influenced our motion synthesis research. While in our research, we unified the different method under a new framework. We argue that the motion primitives and equilibrium point are closely connected; basically, we can identify motion primitives by exploring the equilibrium point type of the underlying dynamic equation.

While for tweaking, CPG and Transform have different role. CPG will qualitatively modified the motion, while transformation for quantiative motion constraints. The role of CPG is providing stability and maintains the equilibrium type, transformation will have no effect on stability, and its role is place the equilibrium point at proper position.

A question is how the CPG and boost stability. Our finding is that motion primitives have a special equilibrium type rather the one we see in the ship example. The equilibrium point of motion primitives is a periodic circle, called limit circle. Entrainment can maintain its oscillating behaviour thus maintains the qualitative property.

The idea impedance control also included in our research, original method is based on finding the required Jacobin matrix, which is computational expensive, while we found the symmetry based method can have the same effect.

Chapter 3

GLOBAL MOTOR INVARIANT

3.1 Introduction

Motion varies greatly, different people walk with different gait. A question is why the different motions of different people are all called walk. Our answer is walk is not determined by the details how it is carried out. Walk capture the qualitative properties, and we agree on the walk becomes it is a property encoded in all our body, we all have the walking ability inborn, so thats the reason why we can all identify it.

Our basic idea is motion primitives are "easy" to finish. In this chapter, we will try to give the easiness definition. The biological ideas can be provide a clear mathematical meaning.

3.2 Basic Concepts of Qualitative Dynamics

This section develops the mathematical conceptualization of Global Motor Invariant. Some mathematical background is needed in this discussion. Throughout this paper, we take the geometrical viewport of mechanical system. For analyzing qualitative properties, we introduce the ideas from differential topology. This idea can be traced back to Poincare[Poincaré, 1885; Poincaré and Magini, 1899] and recently developed by the Smale School[Smale, 1970]. It is impossible to put the a whole discipline in one chapter. Please refer to other books and lectures such as [Abraham and Marsden, 1978]for introduction in details.

Dynamic motions are modelled as differential equations, In the geometrical viewport, differential equation describes a differentiable manifold. Qualitative Properties can be obtained by analyzing the topological structure of the differentiable manifold. Global Motor Invariant is defined by the topology structure.

3.2.1 Dynamic System and Differential Manifold

The dynamic of a mechanical system is determined by its configuration q and generalized speed $\dot{q}$. we represent the state of a system as a vector $\mathbf{x} = [q, \dot{q}] \in M$, M is the state space, or state manifold. The motion is a trajectory $t \mapsto q(t)$ in the configuration space parameterized by time t. For a dynamic system, $q(t)$ usually is derived from the state trajectory $\mathbf{x}(t)$, which is described by differential equaiton.

For every point $x \in M$, F and u determines a derivative vector $\dot{x} in T_x M$ in the Tangent Space. All the vectors over the full space of x form the **vector field V**,describe by the differential equaiton 3.1 which described by the differential equation

$$\dot{\mathbf{x}} = F_a(\mathbf{x}, u), \mathbf{x} \in M \tag{3.1}$$

where u is the control effort. a is the system parameters F is determined by the system's natural property. If $u = 0$, no control effort is applied. Such systems are **autonomous systems**.

By solving the **intergral curve**to equation 3.1, **flow** $\Phi(\mathbf{x})$ of $\mathbf{V}$ is the **intergral curve** through $\mathbf{x}$. all the flows form the **phase portrait**, which illustrates all the possible motions of the dynamic system. We usually visualize the differential manifold by **phase plot**.

An illustrative example repeatedly used in this report is the mass-spring system. After linear transformation, a linear mass spring system can be described in canonical form equation 1.2 where q is the position of the mass, $\dot{q}$ is the speed, and $\ddot{q}$ is the acceleration of mass.

If we chose the state variable $\mathbf{x} = [q, \dot{q}]$, the ODE model should be in the form of equation 3.2

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x} \tag{3.2}$$

3.2.2 Global Motor Invariant

Flows can only intersect at some special position called **equilibria**. Basically there are three type of equilibrium. At each **equilbria**, the local space can be divided into three subspace of sub manifold: centre sub manifold, stable manifold, and unstable sub manifold.

centre sub manifold If a flow ϕ pass through a point $\mathbf{x}_c$ on centre sub manifold W_c, flowϕ will remain on the Centre Manifold

$$\phi_c(t) \in W_c, t \in R$$

An equilibria must be on center manifold.

stable sub manifold For the flow ϕ_s passes through a point $\mathbf{x}_s$ on stable sub manifold W_s, the flow will finally converge to a no wandering point on centre sub manifold.

$$\phi_s(+\infty) = \theta_c$$

unstable sub manifold For the flow ϕ_u passes through a point $\mathbf{x}_u$ on unstable sub manifold W_u, the flow will be repelled from the no wandering points on centre manifold. An alternative perspective is the inverse of the flow converge to no wandering point.

$$\phi_u(-\infty) = \theta_c$$

The size and dimension of each sub manifold varies. For some cases, the W_s (W_u) may not exist, this can be seen as the dimension of $W_s(W_u)$ is 0. **Attractors** are the equilbria where the whole local space is stable, the dimension of unstable submanifold is zero $\mathbf{dim}(W_u) = 0$. **Repellors** are the equilibrias where the whole local space is unstable,the dimension of stable submanifold is zero $\mathbf{dim}(W_s) = 0$.

In theory, only observe the attractor of the dynamic system can be observed, motion task should be only rely on the attractor. Two types of attrator are of great interest in motor control:(1) fixed point, as show inf Figure 1.3,(2)Limit Cycle,as shown in Figure 3.1

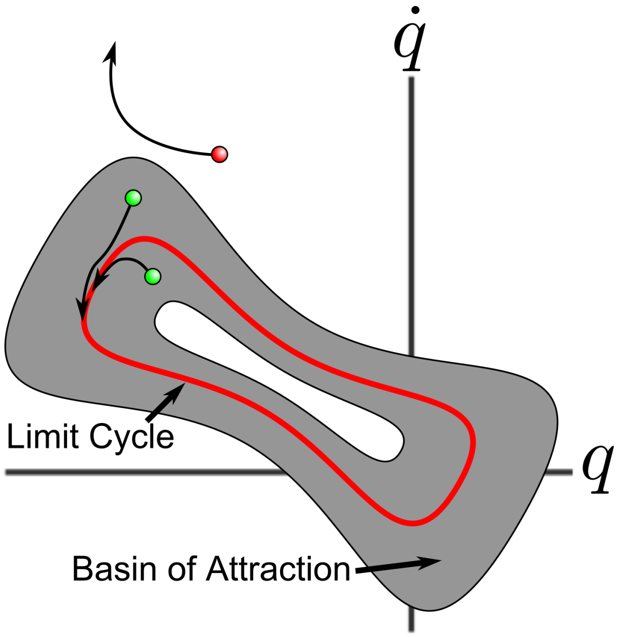


Figure 3.1: Limit Cycle

For nonlinear system, globally, the shape of stable and unstable sub manifold may be bending and connect with itself or each other. The unstable manifold of one equilibrium may be the stable sub manifold of another. The equilibra and its connectivity sub manifold form a topological structure. Thus the phase plane will be divide into different regions,result in a cellular structure. there is only one attractor, all the flow in this region will converge to the attractor A. and the corresponding region is called basin of attraction $B(A)$. as shown in figure 3.2

We can also give the biological ideas clear mathematical meaning. The UMH, the uncontrolled manifold is the basin of attraction. For EPH, the equilibrium point is the attractor. For Impendence Control, impedance control is control the shape of basin of attraction.

3.2.3 Analogous System And Topology Conjugacy

many dynamic system are have different dynamic equation, but they share the same topology. an example is the mass-spring system and the duffine system,

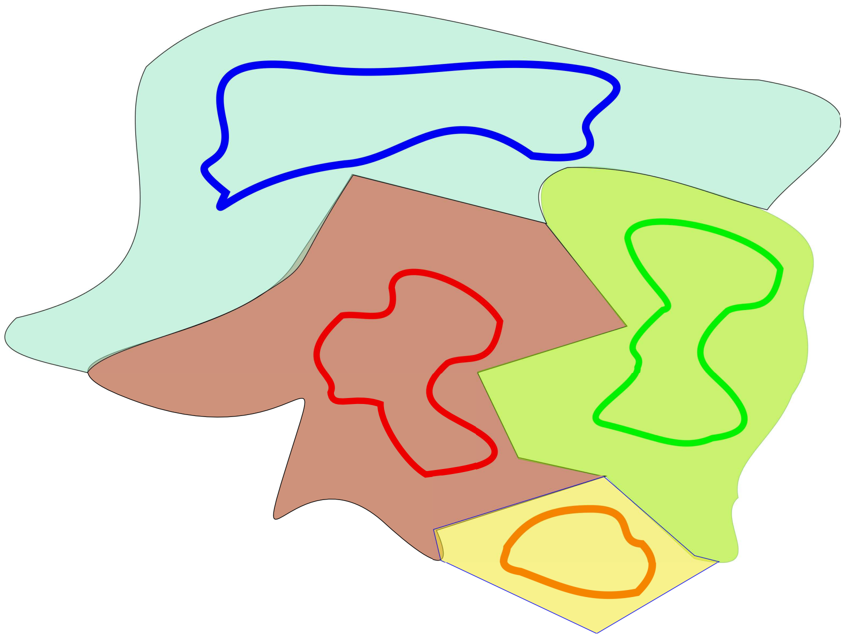


Figure 3.2: Celluar Structure of Phase Space

described by equation **??**

$$\ddot{q} + q + q^3 = 0 \tag{3.3}$$

and the phase plot of the two system are show in figure 3.3,3.4

one phase plot, the two system are similar, and we cand " deform " one into another. in mathematical term, there is an equilalence relationship for the two system. the **topological conjugacy**.

Let X and Y be topological spaces, and let $f\colon X \to X$ and $g\colon Y \to Y$ be continuous functions. We say that f is *topologically semiconjugate* to g, if there exists a continuous surjection $h\colon Y \to X$ such that $fh = hg$. If h is a homeomorphism, then we say that f and g are *topologically conjugate*, and we call h a *topological conjugation* between f and g.

if two system are topological conjugate, they are analogous systems

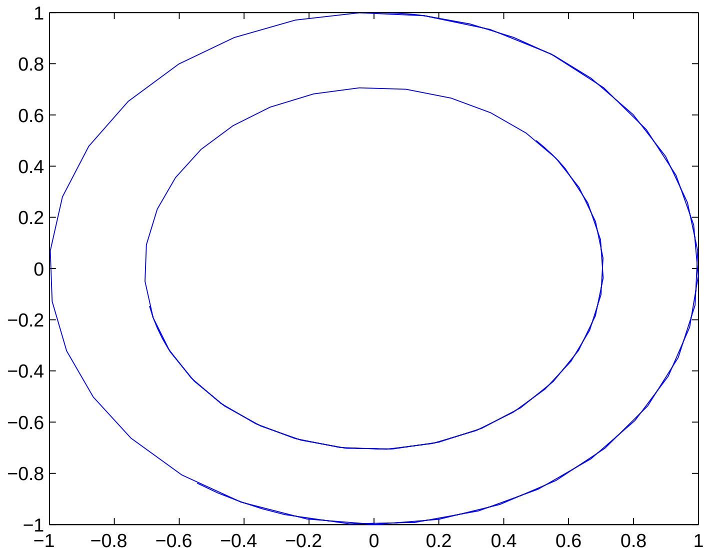


Figure 3.3: Mass Spring System

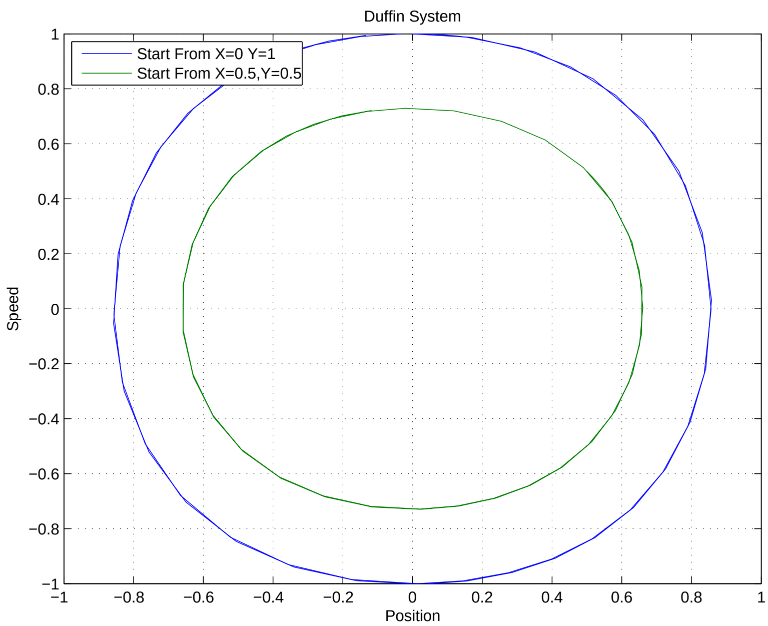


Figure 3.4: Duffin Phase Plot

3.3 Global Motor Invariant and Motion Adaptation

Global Motor Primitive is defined by the attractor type and its basin of attraction in the topology space.

Motion adaptation because of different reasons and in different situations. Such two kinds of perturbation are treated separately and result in differentiation strategy or control.

- **State perturbation**

  The perturbation that move the state off the attractor is called State Perturbation, for only the state is changed, the dynamic system underline is not changed.

  If the state is in the basin of attraction then, it will converge to the attractor. Start from different state position, it will result different flows, thus different motion.

  Such kind of motion adaptation is called Responsive Motion Adaptation. Because usually, for characters, perturbation comes from the push or pull, while the character and environment is not changed.

  To make the character more responsive without result in motion failure, Motion controller should try to enlarge the basin of attraction.

- **Structure Perturbation**

  Another type of Perturbation will affect the dynamic system; such kind of perturbation is called Structure Perturbation. Such kind of perturbation happens commonly in our daily life, when a man put a heavy box on his shoulder, it will result a change in the dynamic system.

  Structural Perturbation will change the phase portrait; some perturbation will make the system into an analogous system. As a result, the even the current state is on the attractor, motion will change, this kind of motion adaptation is called system adaptation, and one important application is

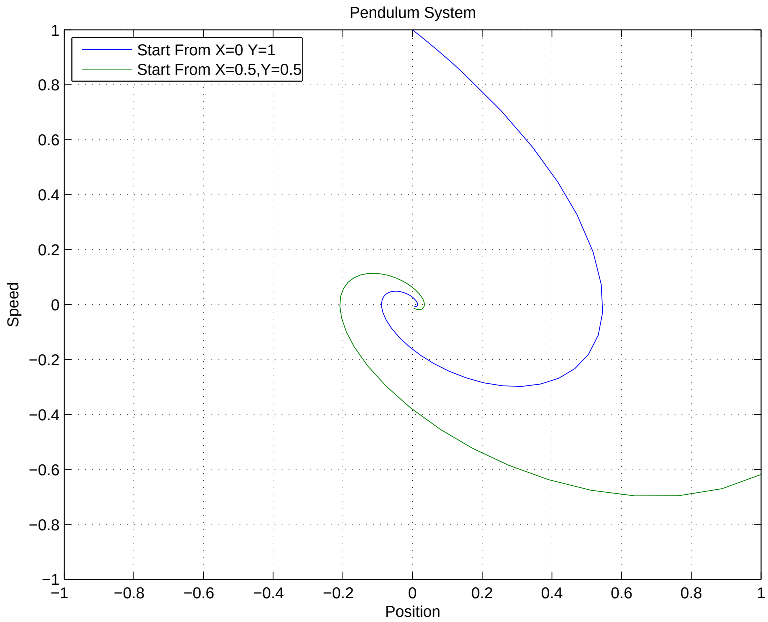


Figure 3.5: damping perturbation onmass spring system

dynamic motion retargeting, when you change the character, the dynamic system changed.

Sometimes it will change the topology the underlying dynamic system, such effects are called bifurcation. an example is the damping effects on the mass spring system. as show in figure 3.5

If ability of a dynamic system maintains its topology structure is structural stability. To make characters more adaptive to environment and body change, motor controller should boost the structural stability of the motion. Control effort should be prevent bifurcation.

3.3.1 remarks on biological motor control

Structural Stability is neglected in CMS research. It is reasonable for natural animals to rely on structural stable autonomous system. In natural environment, perturbation and uncertainty are everywhere. In many cases, if neural system cant respond quickly enough, the better solution is to select a more structure stable motion primitives.

Structural Stability and Qualitative Idea provides a better explanation for some motion phenomena than quantitative theory like optimization and feedback shooting. Qualitative Control Theory may help us understand the evolution of locomotion and neural Control. Animals shift from the sea to the land. From quantitative computing viewport, the natural dynamics of the body and fluid environment are hard to predict or compute precisely. From the qualitative viewport, fluid is continuous and uniform, the topology structure is very simple and stable, thus with little neural control fish can maintain its posture. On the other side, for human walking, although the rigid like Environment can be calculated precisely, the topology structure is much more complex. Onthe phase plane, there exist many equilibrium points, the topology structure is more complex and unstable. The control system has to be more complex to control the more complex qualitative properties.

Qualitative theory can also help to explain another fact of biological motion system. Animals that live in similar environment and moves in a similar manner usually have similar body structure, in spite of their different position on evolution chain. This is because animals moving in a similar is based on the same motion primitive, similarity in body structure promise the topologial conjugacy.

This idea may help us to understand the body and environment effect in morphological computation theory. For specific environment, body evolves to provide us with a structural stable dynamic system, thus save lots of control effort for neural control.

We can also know motion primitives are not only defined by the body, it also defined by the environment, for motion in a specific environment, the topology should be fixed, thus the number of motion primitives is quite limited.

3.4 Global Motor Invariant Control

3.4.1 CPG and Entrainment

In nature, an animal's body and environment can be extremely complex. It leads to high dimensional manifolds with complicated topological structure, which provides many motion primitives for our use. For CMS application, one question we want to ask is the so many motion primitives can be controlled with a simple method.

We propose that even there are many type of motion primitives, the type of attractor is limited. Basically there are only two types of attrator, limit circle and fix point. Even the dimension of dynamic system maybe large, the dimension of the attrator is known. For fix point, its attractor is of dimension zero. For limit circle, it is of dimension one. Thus we can only focus on the type of attractor.

Biology Research suggested that the motor is mainly controlled by the Central Pattern Generator, which is a small autonomous network that generating rhythmic signals. The idea of control motion by rhythmic signals can be modelled as entrainment [González-Miranda, 2004]. When coupling two oscillation system together, entrainment can happen when two system oscillator in synchronize. This effect will enhance the oscillation and also know as resonant.

In previous section, we discussed two types of attractors: Fixed point and Limited Cycle. It is a still open question which type is more important and serve as the foundation as motor control[Degallier and Ijspeert, 2010]. One idea limit circle is a necessary, fix point can be controlled with by (1) terminate a circle, (2)a different controller, (3)approximate by a limit circle with small amplitude or damping the limit circle, (4) change the limit circle into a fix point through by bifurcation.

In this paper,

- **P**eriodic behaviour is very common in biological systems. Besides the periodic motion in swimming and running, heart beating, wake and sleep also show periodic behaviour. A periodic system has the potential to integrate with other bio system simulation to explore other motion features.

- **P**eriodic motion has the same effect of terminated motion when the amplitude of limited circle is very small. For CMS research, both type of motion trajectory can be simulated with periodic motion.

3.4.2 Neural Oscillator Stability

Although it is difficult for neural system to carry out complex computation, it is easy to build oscillator structure with neurons. It only needs two neurons with mutual inhibitive property. One extensively studied oscillation model is developed by Matsuoka [1985]. The mathematical presentation is as follows:

$$\tau_1 \dot{s_1} = c - s_1 - \beta l_1 - \gamma [s_2]^+ - \sum_j h_j [w_j]^+ \quad (3.4)$$

$$\tau_2 \dot{l_1} = [s_1]^+ - l_1 \quad (3.5)$$

$$\tau_1 \dot{s_2} = c - s_2 - \beta l_2 - \gamma [s_1]^- - \sum_j h_j [w_j]^- \quad (3.6)$$

$$\tau_2 \dot{l_2} = [s_2]^+ - l_2 \quad (3.7)$$

$$y_i = \max(s_i, 0) \quad (3.8)$$

$$y_o = = [s_1]^+ - [s_2]^+ = y_1 - y2 \quad (3.9)$$

c,β,γ are parameters of the oscillator,in our research are kept constant. τ constrolls the oscilation frequency.

Matuoka oscillator is an autonomous oscillator; it can begin to oscillator without any control effort. Figure 3.6 shows the natural oscillator output.

It is also adaptive; entrainment behaviour can happen between one Matuoka oscillator and different oscillators. Figure 3.7 shows the entrain oscillation, where the oscillation of Matuoka oscillator synchronise with the input signal.

But because of the nonlinear properties, its behavior is not completely understood. Matsuta[Matsuoka, 1987] explains the adaptive properties from the location of the roots of characteristic equation. Wilimas[Williamson, 1998] explains the properties in frequency domain.

In our research, we find some important properties of neural oscillator by empirical.

From our simulation, we investigate the topological structure. Basically, neural oscillator shows three important properties:

- Simple Topological Structure. The topology structure of neural oscillator is simple, it includes one attractive limit circle and one fix repellor.

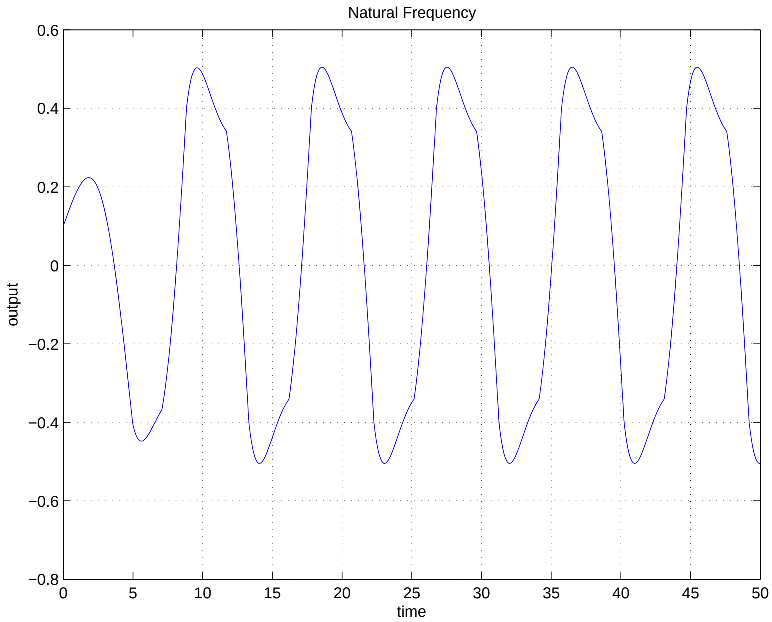


Figure 3.6: Natural Oscillation

- Large Basin of Attraction. All the simulations we carried out converged to the same limited circle.
- Fast Converging Speed. In most of the case, the flow will converge to the limit circle within one period time.

Features above are shown in Figure 3.10.

The large area of basin of attraction means the final behaviour is totally determined by parameters. Initial condition will have no effects on the oscillator final output. Thus we treat matsuta oscilator as in a simple one input, one out put system, controlled by three parameter and input signal. we usually reformed equation 3.9 in the simplified form

$$u_{\mathrm{o}} = S_{[h_{\mathrm{i}}, h_{\mathrm{o}}, \tau]}(u_{\mathrm{i}}) \tag{3.10}$$

where $u_{\mathrm{i}} = \sum_j h_j[w_j] = hw$,$u_{\mathrm{o}} = h_o y_o$

The converging speed can be seen as quick recovery ability. When an impulse perturbation happens, it will recover in one period time.

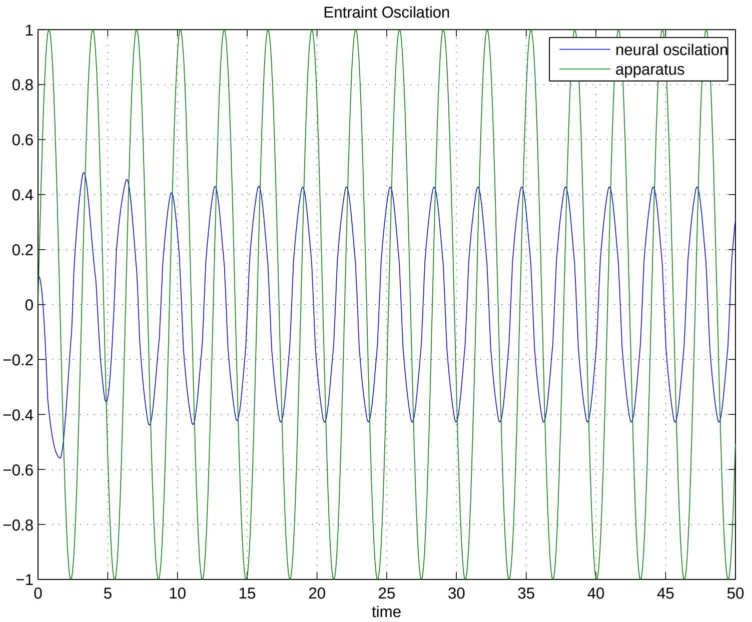


Figure 3.7: Entrainment Oscillation

3.5 Example:Maintain Bouncing Height

Bouncing ball is system ball bouncing by moving a pedal, a system with simple dynamic but difficult to control with optimizaiton or pd. While this example capture the complexity of human interatction with the environment and object. And can be the basic model for many motion tasks.

We show in this example how neural oscillator can turn the bouncing ball system into motion primitive.

Dynamics

Hybrid dynamics, in incoperate two phase,

$$\begin{aligned} \ddot{q} &= -g && \text{if } q > 0 \text{ (free flying)} \\ \dot{q}^{+}_{\text{ball}} - \dot{q}^{+}_{\text{paddle}} &= \epsilon(\dot{q}^{-}_{\text{ball}} - \dot{q}^{-}_{\text{paddle}}) && \text{if } q \leq 0 \text{ (paddle strike)} \end{aligned}$$

Basically, the ball will continue bouncing with smaller height,as show in Fig-

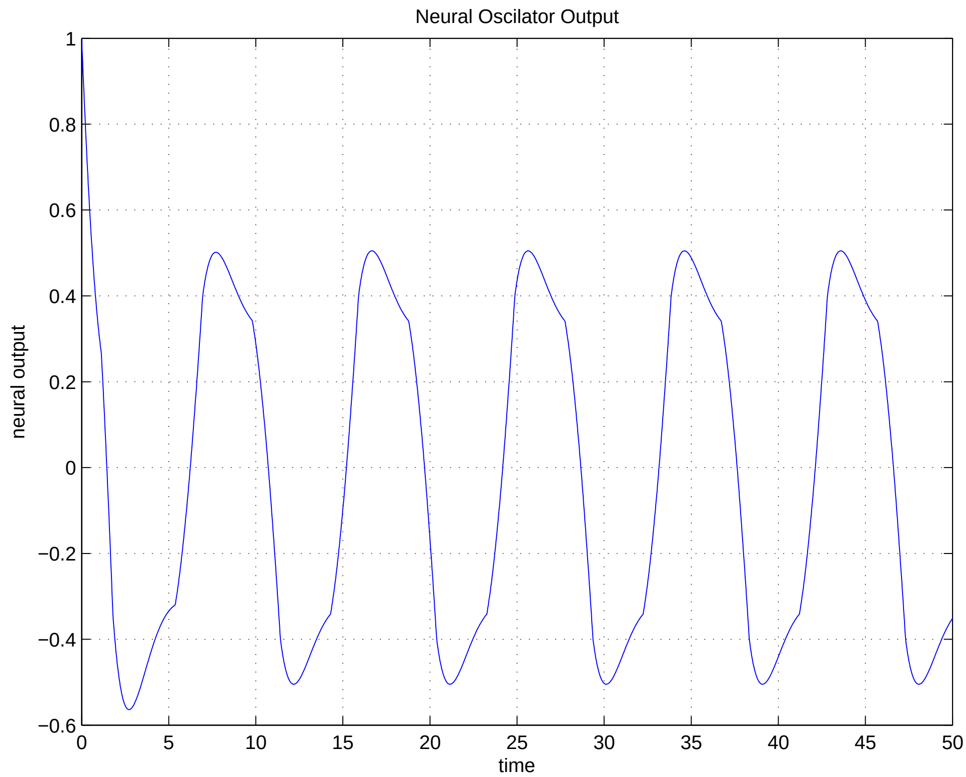


Figure 3.8: The states of neural oscillator over Time

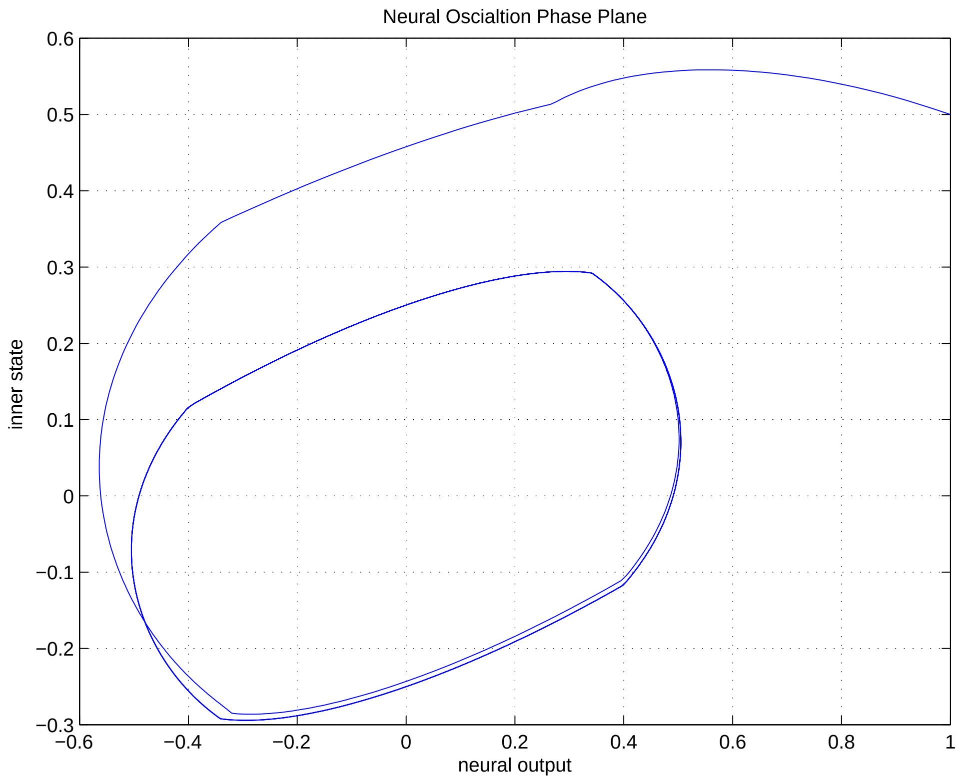


Figure 3.9: The phase portrait of Neural Oscillators

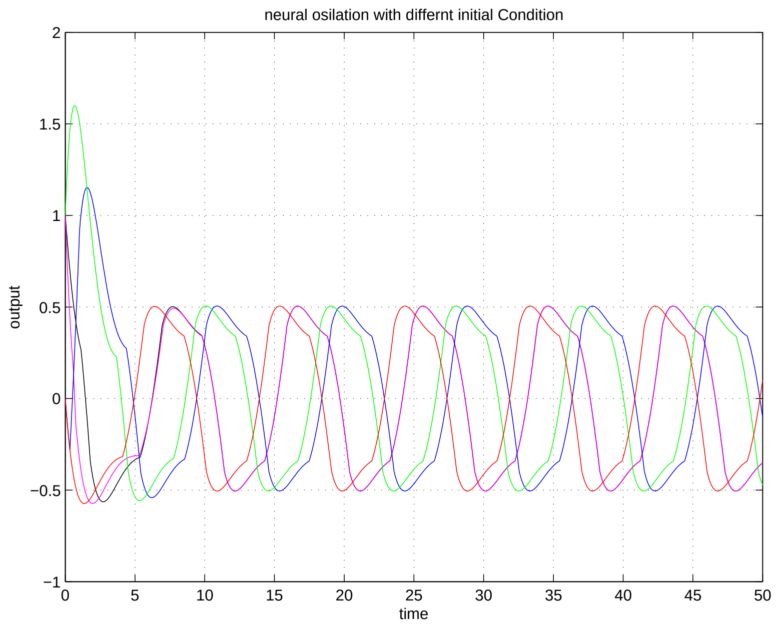


Figure 3.10: Neural output with different initial position

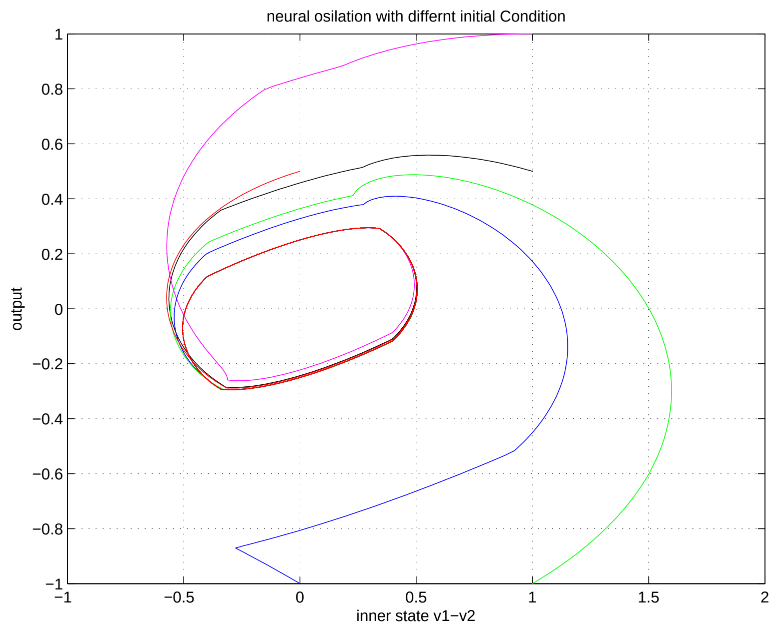
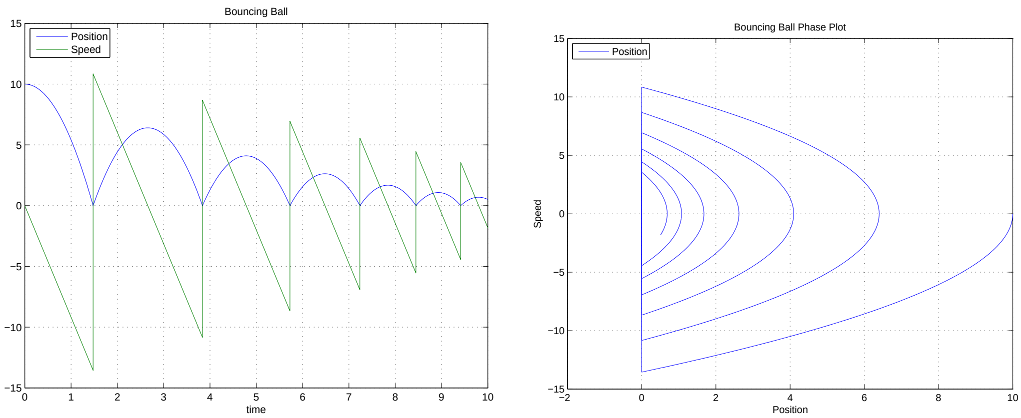


Figure 3.11: Phase plot of oscillation with different initial condition

ure 3.12.

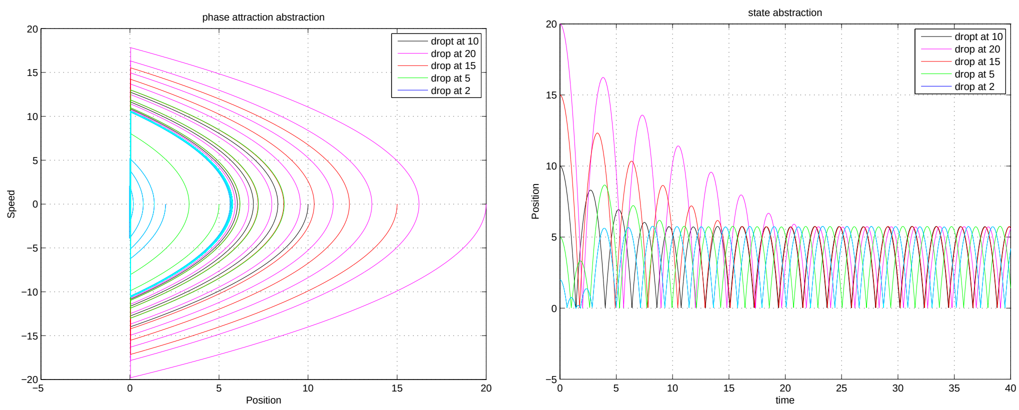


(a) state plot (b) Phase Plane

Figure 3.12: Original Bourncing Ball System

Emergence of Limit Cycle

couple with neural oscillator boucing we get an limit circle The input of neural oscillator is the velocity $u_{\mathrm{i}} = \dot{q}_{ball}$, the output of neural oscillator drive the pedal position $q_{pedal} = u_{\mathrm{o}}$. An limit circle emerge as the result of entrainment. As show in figure drop from different position, all the ball will bouncing a about the same height of 5,as show in Figure 3.13



(a) state plot

(b) Phase Plane

Figure 3.13: Attractive Limited Circle

Chapter 4

LOCAL MOTOR INVARIANT

4.1 Introduction

Global Motor Invariant Control keeps the qualitative properties of motion primitives. For animal motion is also of high accuracy. In this chapter we focus on the control on the quantitative properties of motion. In our research, we try to limit the computational cost.

The discovery is that motion of natural system will change in a uniform way. The method we proposed exploring the symmetry properties of dynamics system. The symmetry property of a dynamic system is called local motor invariant. The method we propose is based the lie group theory,please refer to book[Olver et al., 1986] for more details.

4.1.1 Group and Symmetry

For the geometrical viewpoint, "Symmetry" means when you transform an shape, the transform one and original one are exactly the same. For the square examples, rotation it by 90 degree will make it exactly the same with the original one.

All the action that can preserve the symmetry is defined as the set as group G. A group has the following properties.

- For any g_a, g_b in G, $g_a * g_b$ belongs to G. (The operation "$*$" is closed).

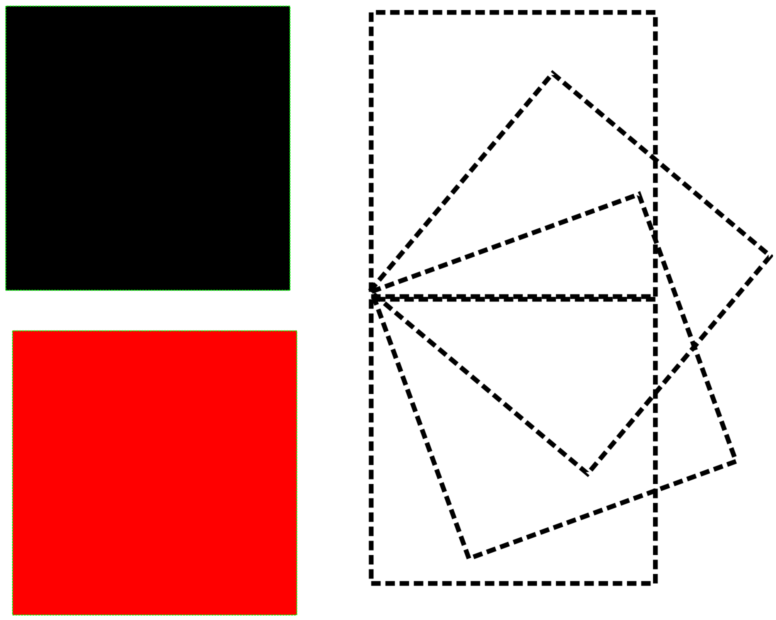


Figure 4.1: Symmetry

- For any $g_a, g_b, g_c \in G$, $(g_a * g_b) * g_c = g_a * (g_b * g_c)$. (Associativity of the operation).
- There is an element $e \in G$ such that $g_a * e = e * g_a = g_a$ for any $g_a \in G$. (Existence of identity element).
- For any $g_a \in G$ there exists an element g_h such that $g_a * g_h = g_h * g_a = e$. (Existence of inverses).

for the shape geoemtry example example g_1 is rotate 90 degree clockwise. then e is no rotation. we also know $g_2 = g_1 * g_1$ is rotate 90 degree clockwise twice.g_2 also preserve the shape.

In algebra sense, "Symmetry" means invariant, a shape can implicitly defined by an function $I(x) = 0$; The group transformation is define by $\hat{x} = g_a(x)$ If symmetry is met, we have $I(x) = I(\hat{x})$. We can say $I(x)$ is an invariant function of group G.

We have to note that not only the one shape unchanged by G. In fact ,many shape is invariant, In fact, we can pick up and two shape and combination is also an invariant shape. and form a space, the invariant space $I(x)$.

Figure 4.2: Symmetry Space

4.1.2 Lie Group and Symmetry of Dynamic System

The rotate group is discrete. if the shape is circle, than the rotation is continues, the continues group is Lie Group.

Lie Group is continues group originates from study of differential equation. For Physically-based animation, Motion is usually described by the differential equation (1)

$$\dot{\mathbf{x}} = F(\mathbf{x}) \tag{4.1}$$

Lie Group G will perserve the differential equation,for $g_a \in G$

$$Tg_a(\dot{\hat{\mathbf{x}}}) = F(g_a(\mathbf{x})$$

where Tg_a is the corresponding lift action that transform the velocity $\dot{\mathbf{x}}$. for example, if the g_a is translate transformation, velocity will not be translate, then Tg_a is identy.

Physically possible motion is the solution of the equation. An important property from one solution $\mathbf{x}(t)$. with a group action g_a, we can get another solution $g_a(\mathbf{x}(t))$

for the mass spring system 3.2 we apply the group action

$$\hat{\mathbf{x}} = g_a(\mathbf{x}) = [\alpha q, \alpha \dot{q}]$$

then the lift action is

$$\dot{\hat{\mathbf{x}}} = Tg_a(\mathbf{x}) = [\alpha \dot{q}, \alpha \ddot{q}]$$

by substitution $\mathbf{x} \mapsto \hat{\mathbf{x}}$, the original system become

$$\dot{\hat{\mathbf{x}}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \hat{\mathbf{x}}$$

which is

$$\alpha\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \alpha\mathbf{x} \tag{4.2}$$

equation 4.2 is equivalent to 3.2 thus if $\mathbf{x}(t)$ is a solution, we get another solution $\hat{\mathbf{x}}(t)$.

such transformation form a group. we define

$$g_\alpha * g_\beta(\mathbf{x}) = [\alpha\beta q, \alpha\beta\dot{q}]$$

which also satisfy the differential equation.

and

$$g_\alpha^{-1} = g_{\frac{1}{\alpha}}$$

when $\alpha \in R^+$, the group is continues, thus it is an example of Lie Group.

This provide us an idea about motion synthesis. Given an original motion $q(t)$, and the corresponding group G, a new motion is generated by transformation. For every group G, we can find an function $I(x)$ unchanged by the group action G,

$I(\mathbf{x})$ are called local motion invariant. For mechanical system, $I(x)$ has important physically meaning. $I(\mathbf{x})$ corresponding to the Conservative Law like energy or angular momentum.

4.2 Controlled Symmetry

For motion synthesis, usually the desired motion is ma For example for motio stability, we want the current state is within the basin of attraction. If we want to control the final motion style, we want the state is on the limit cycle.

For motion sysnthesis, the problem is given the system, let the original system have the desired symmetry.

and original motion m is known, but the corresponding group action g_a is not satisfied by differential equation. For such situation, control input u is added, which modify the original equation to allow the designed G, this is called Controlled Symmetry.

Most dynamic motion can be modelled as an Lagrange System.

$$L = K(\dot{(}q) - V(q).$$

And the desired action G must keep the L invariant.

The original m is defined by the eural langrage equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{4.3}$$

The modified system is

$$\frac{d}{dt}\frac{\partial L}{\partial Tg(\dot{q})} - \frac{\partial L}{\partial g(q)} = 0, \tag{4.4}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = u_{\mathrm{l}}. \tag{4.5}$$

(5) and (6) are the equivalent equation, by comparing equation (5) and (6), we can get u Some Specific example of Symmetry and Control. In the following discussion, suppose all the group element g are paramerizate by the parameter α.

Offset Action

we move the posture of the system

$$q \mapsto q + \alpha$$

then the offset transformation is

$$g_{\mathrm{f}}(\mathbf{x}) = [q + \alpha, \dot{q}]$$

$$u_{\mathrm{l}}(q) = \frac{\partial}{\partial q}\left(V(g_{\mathrm{f}}(q)) - V(q)\right). \tag{4.6}$$

if we formulate the controlled mass spring system in the followign way 4.7

$$\ddot{q} + q = u_{\mathrm{l}} \tag{4.7}$$

for the mass spring system.

$$u_{\mathrm{l}}(q) = \alpha$$

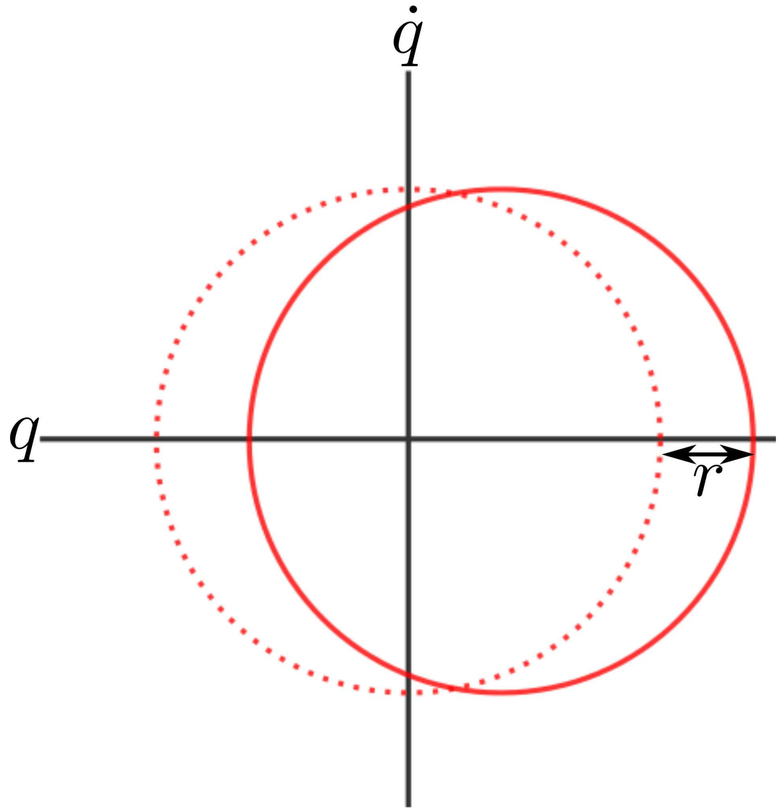


Figure 4.3: Offset Action

on phase space, if q is the horizontal axis, and $\dot{q}$ is the vertical axis, this has the effect of moving the phase plot horizontally.

Time Scalling

if we scale the time paramter

$$t \mapsto \frac{t}{\alpha}$$

we have

$$g_{\mathrm{t}}(\mathbf{x}) = [q, \alpha\dot{q}]$$

$$Tg_{\mathrm{t}}(\dot{\mathbf{x}}) = [\alpha\dot{q}, \alpha^2\ddot{q}]$$

Then the local control is

$$u_{\mathrm{l}}(q) = (\alpha^2 - 1)\frac{\partial V(q)}{\partial q}. \tag{4.8}$$

for the mass spring system in

$$u_{\mathrm{l}} = (\alpha^2 - 1)q$$

on phase space, this has the effect strength the phase plot in the vertical direction

Energy Scaling

For some system moving the the conservtive field. The energy is preserved and different motion present different level of energy. For such system, we have the the energy $E(\mathbf{x}) = K + V$, where K is the kinematic energy, V is the potential enegy.

$$E(\hat{\mathbf{x}}) = \alpha^2 E(x)$$

if the the mass of system is constant.

$$g_{\mathrm{e}}(\mathbf{x}) = (f(\alpha)q, \alpha\dot{q}).$$

where $f(\alpha)$ is a funtion of α, which depends on the shape of potential field. u_{l} can be developed by applying the pos scaling and time scaling in a combined manner.

for the mass spring system , $E = \frac{1}{2}(q^2 + \dot{q}^2)$,$f(\alpha) = \alpha$, and because the energy

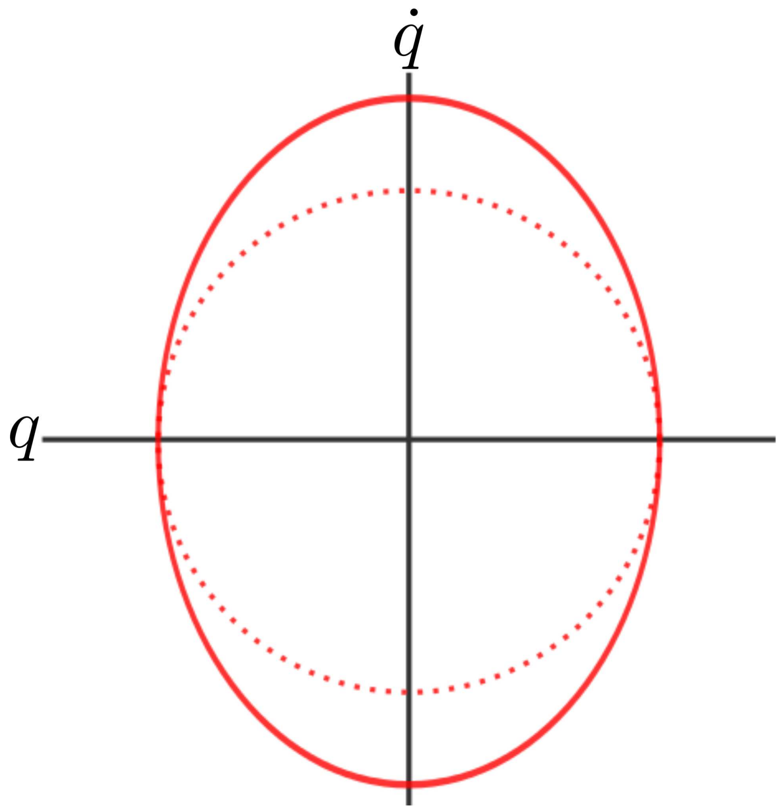


Figure 4.4: Time Scaling Action

scaling is kept by the original system, we have

$$u_l = 0$$

On phase plot, this has the effect enlarge the phase portrait.

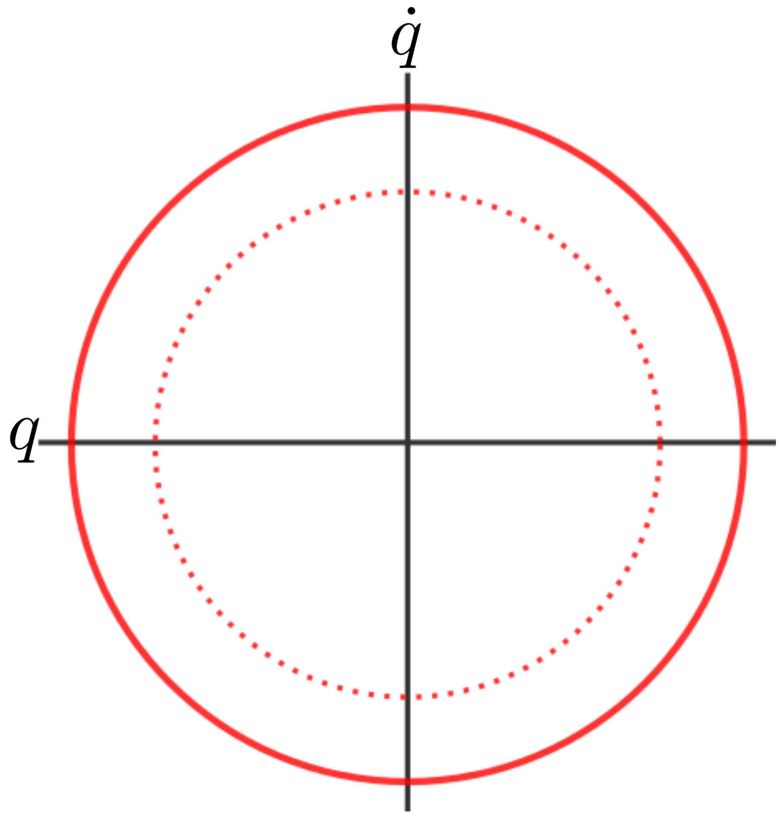


Figure 4.5: Energe Scaling Action

Time Offset

we can also offset the time t

$$t \mapsto t + \alpha$$

$$g_d(\mathbf{x}(t)) = [q(t+\alpha), \dot{q}(t+\alpha)]$$

For dynamic system, this seems obvious. And no control is need for such symmetry. For system with limit circle, this g_{d} has a special effects like phase modification.

On phase plot, this has the effect rotate on the limit circle about an angle.

4.3 Example:Bouncing Ball dropt frome Different Height

Even it is a hybrid system, The bouncing ball system has a energy scaling symmetry.

the ennegy function

$$E = g_{ravity}h + \frac{1}{2}m\dot{q}^2$$

so the

$$f(\alpha) = \alpha^2$$

then the energy scaling action is

$$g_{\mathrm{e}}(\mathbf{x}) = [\alpha^2 q, \alpha\dot{q}]$$

given the motion of droping at 5 are shown in Figure **??**.

if $\alpha = \sqrt{2}$, we get the bouncing motion drop at height 10, as show in figure **??**

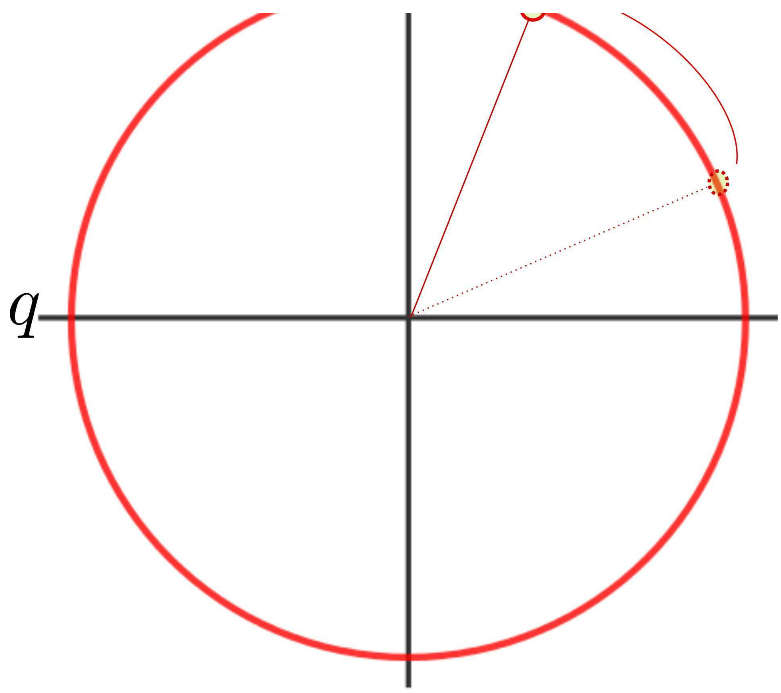


Figure 4.6: Offset Action

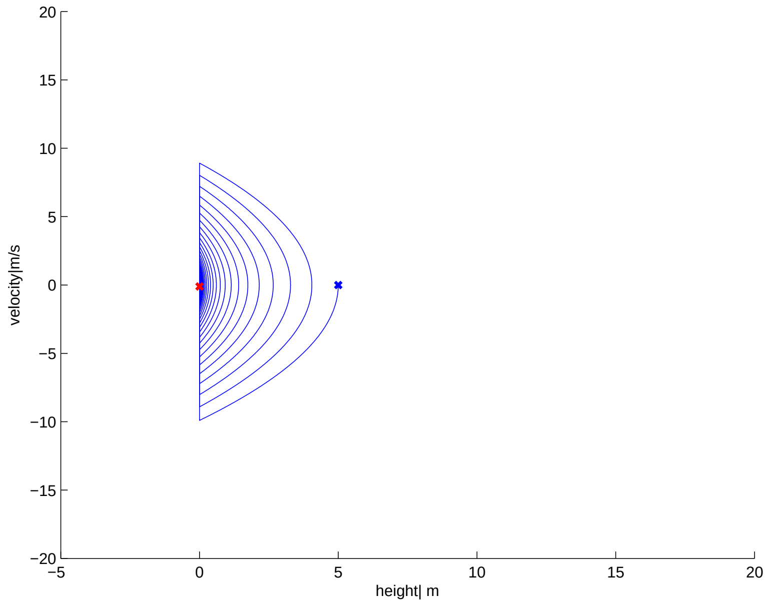


Figure 4.7: Drop at 5

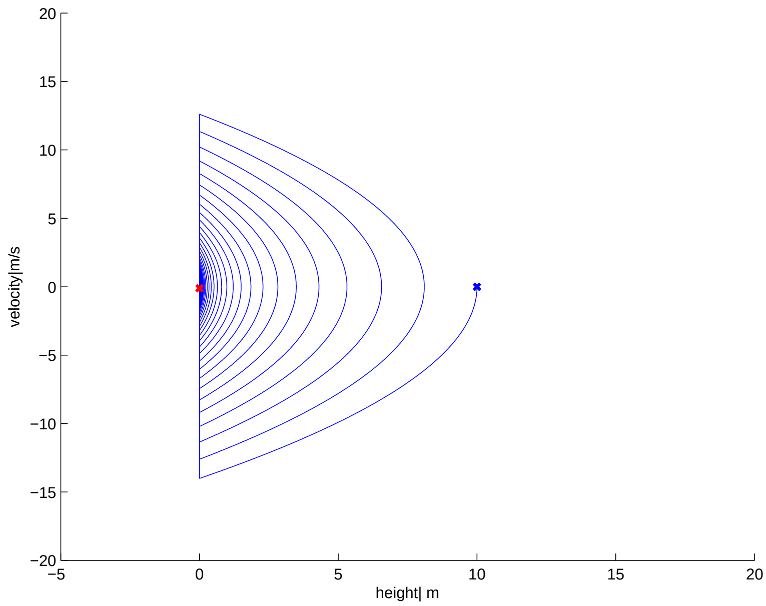


Figure 4.8: Drop at 10

Chapter 5

MOTION SYNTHESIS FRAMEWORK

Chapter 3 and Chapter 4 discussion some idea of topological conjugacy and symmetry seperately.

In this chapter ,we will dicuss how we combine the ideas full motion synthesis System. Mainly we will discuss two question,

- how global and local motor invariant controller work together.
- how combine different motion primitives together.

5.1 Combined Global and Local Motor Invariant

5.1.1 Combine Motor Invariant Control

Neural Oscillator will maintain the qualitative motion properties, and Controller Symmetry will satisfy the quantitative properties. Basically we need apply the qualitative controller first to maintain the topology against the structural perturbation, when Symmetry Controller is applied to transform the entrainment System to meet some specific user constraints.

Snimply put, we should get the qualitative right first, and then get the quantitative property . This idea is straightforward is illustrated in the following figure 5.1

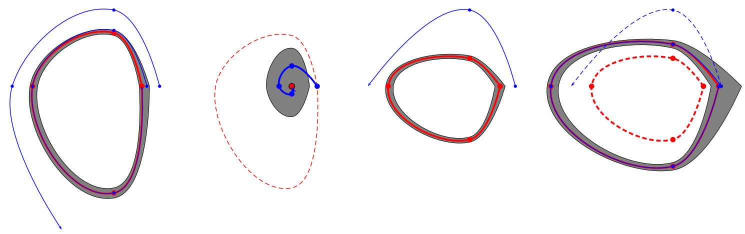


Figure 5.1: System Over View

but when applying this method, we have condition must be met.

- unlike the CPG controlled example discussed in Chapter 3. the symmetry controlled is applied, we must prove that Symmetry Controller will not violate the topology, and symmetry control is applied to the how system rather than the original system. We must prove that Lie Group Operator will not violate the topology

- In chapter 4 we discuss the controlled symmetry is applied to the original system, how ever, for motion synthesis, we must prove that the combined system also preserve the symmetry. If the original system $F(x)$ have the symmetry property, we also require the neural oscillator have the same symmetry properties. Thus we need to discuss how to transform the Neural Oscillator so the neural oscillator have the same kind of symmetry as the mechanical oscillator

the method that met the two requirement is called symmetrical entrainment control.

5.1.2 Symmetrical Entrainment Control

Theorem. *Control Symmetry is Topological Conjugasim*

Proof. by the defination of Lie Group

g is continuous and g^{-1} exist.

g is isotopy.

g is Topological Conjugasim

□

We separate the discussion

after coupling with neural oscillator, the

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

becomes a system

$$\dot{\mathbf{x}} = F(\mathbf{x}) + u_{\mathrm{o}} \tag{5.1}$$

for controlled system ,

$$Tg(\dot{\mathbf{x}}) = F(g(\mathbf{x})) + f_g(u_{\mathrm{o}}) \tag{5.2}$$

$$s\dot{t}ate = F(\mathbf{x}) + u_{\mathrm{l}} + u'_{\mathrm{o}} \tag{5.3}$$

f_g is the maping that transform u_{o} that make equation 5.1 and **??** statisfy the symmetry admited by g u'_{o} is the tranforme neural output,that makes the equation 5.2 and 5.3 are equavalient.

as show in equation 3.10. u_{o} is a fucntion of u_{i} but the system have some kinds of symmetry.

Theorem. *for time scaling the input*

$$u_{\mathrm{i}}(t) \mapsto u_{\mathrm{i}}(\frac{t}{\alpha})$$

if we modify the paramaters $\tau_{1,2}$

$$\tau_{1,2} \mapsto \alpha\tau_{1,2}$$

then

$$u_{\mathrm{o}}(t) \mapsto u_{\mathrm{o}}(\frac{t}{\alpha})$$

Proof. by substitue $t' = \frac{t}{\alpha}$,$l'\dot{,}s' = \alpha l\dot{,}s$,$\tau'_{1,2} = \alpha\tau_{1,2}$.

we can put the transformed parameters into equation 3.9, and get the same equation.

□

based on this, we can provide an scheme for modifiy the parameters $\tau_{1,2}$,h_{i},h_{o} for maintain the symmetry of the original equation.

1. modify τ by the time scaling parameter $\tau \mapsto \alpha\tau$.

2. choose input variable w and adjust the input efficient h_{i} to make sure $u_{\mathrm{i}}(t) \mapsto u_{\mathrm{i}}(\frac{t}{\alpha})$

3. ajust the prameters of h_{o} according to the type of driver. if u_{o} drive the position varaible q then, h_{o} should multilty by the postion scale value. if u_{o} drive the velocity,h_{o} multiply by the speed scale parameters, if the *hout* is force and acting on the acceleration $\ddot{q}$, then h_{o} is multilied by the acceleration scale value.

Such changing parameters is call **adjoint parameter transformation** we can prove the following therem.

Theorem. *For a transformation group G, if modified the neural socilator through adjoint parameters transformation, combined system will preserving permited by symmetry G.*

Proof. we can substitute the transformed parameters into orignal equation 5.2 and find result in a equalalient euqation of equation 5.1 □

We can further prove that

Theorem. *by apply controly symmetry and adjont parameter transformation is a topology conjugacy.*

thus we maintain the structual stability of the coupled system.

Following are some examples

Offset Symmetry.

For offset symmetry, there is no time scaling effects. to maintain u_{i} and u_{o}, we require the u_{i} and u_{o} is a function of invariant function $I(\mathbf{x})$. For example ,when walking slope changes, we choose the input of the neural oscillator to be the angle between the joints or velocity.

Time Scaling

Neural Oscillator can change its Speed by changing $\tau \mapsto \alpha\tau$. We prove change ts, we can maintain the same is maintained. if the out is force, then it acting on the $\ddot{q}$ then $h_{\mathrm{o}} \mapsto \alpha^2 h_{\mathrm{o}}$

Energy Scaling

Energy Scaling is combined action of time scaling and pos scaling. we can first modify the parameters τ and h_{o} according to the speed scaling parameters. to maintain the time scaling symmetry of u_{i}, if the input valuable is speed $\dot{q}$, then $h_{\mathrm{i}} \mapsto \frac{h_{\mathrm{i}}}{\alpha}$ where α is the speed scaling value.

5.1.3 Example: Height Control of Bouncing Ball

Bouncing Ball have energy scaling symmetry, it also can also form a limit circle. By the combined motor invariant controller, we can energy scaling the coupled system.

suppose the origianl system is bouncing at heigh of 5 for the energy scaling, for the input to the neural oscilator is $\dot{q}$, then $h_{\mathrm{i}} \mapsto \frac{h_{\mathrm{i}}}{\alpha}$, the time scaling factor is α, the $\tau_{1,2} \mapsto \alpha\tau_{1,2}$. the out is the positon of the pedal, so only the it need to be scale by the positon scale value.for $q \mapsto \alpha^2 q$, thus $h_{\mathrm{o}} \mapsto \alpha^2 h_{\mathrm{o}}$.

if we set $\alpha^2 = 3$, then the ball will boucing at height of 15, and it maintain its topological structure, still it is a limit cycle. as show in figure 5.3

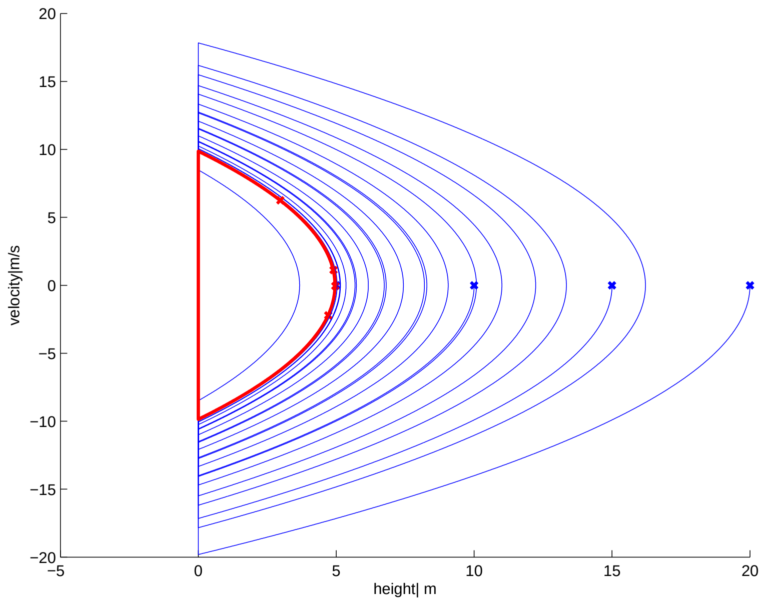


Figure 5.2: Energy Scalling

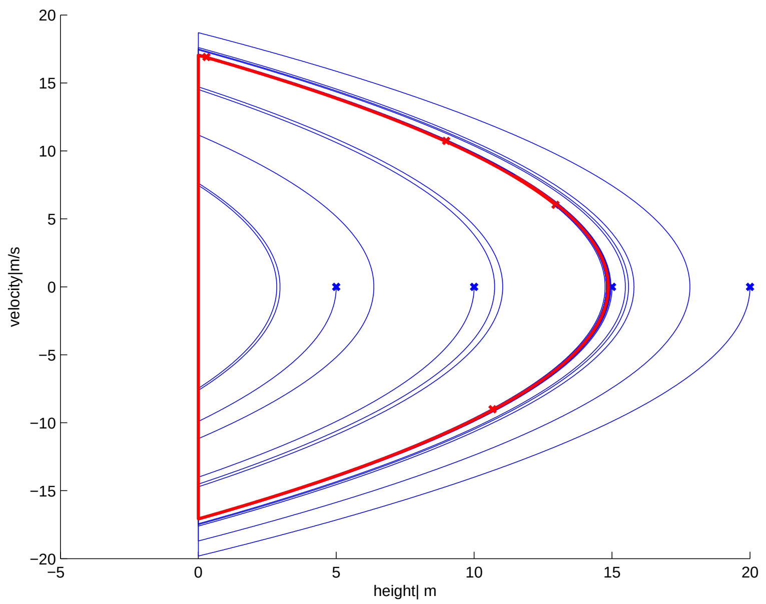


Figure 5.3: Energy Scaling

5.2 Combine Motion Primitives

5.2.1 Motion Primitives Connectivity

Motion Primitives comes from the original mechanical system, motion primitives can only transformed if they are neighbours. Following this idea, given a dynamic system we can draw a graph of motion primitives and this is called motion primitives graph. The idea is very similar to the motion graph, the difference here is in the original motion graph are handed crafted, while in our research, we propose that a motion graph of a dynamic system is fixed, at from any motion primitives, the way he can change its motion is also limited.

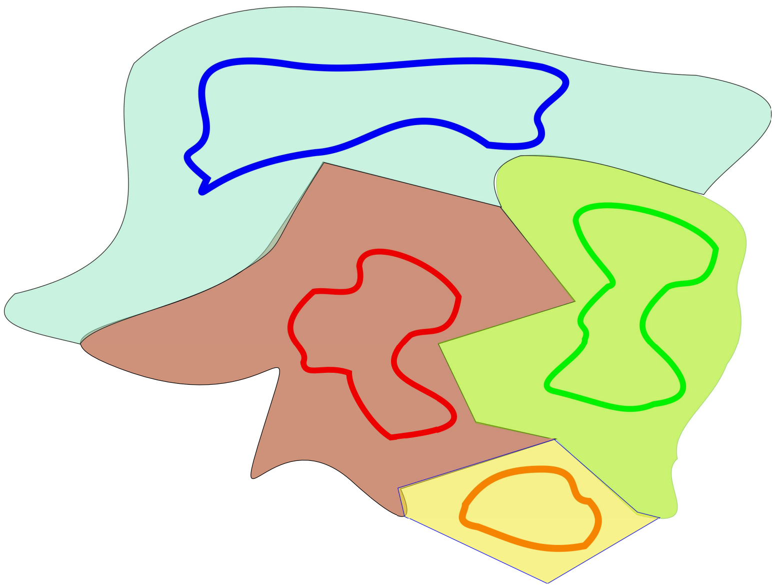


Figure 5.4: Phase Plot of Motion Primitives

5.2.2 The Motion Primitives Transition

From dynamic point of view, changing motion primitives is put the current state into basin of attraction of another attractor. As show in picture, for the uncontrolled system, the transition will not happen automatically, for the two basic of attraction will not overlap.

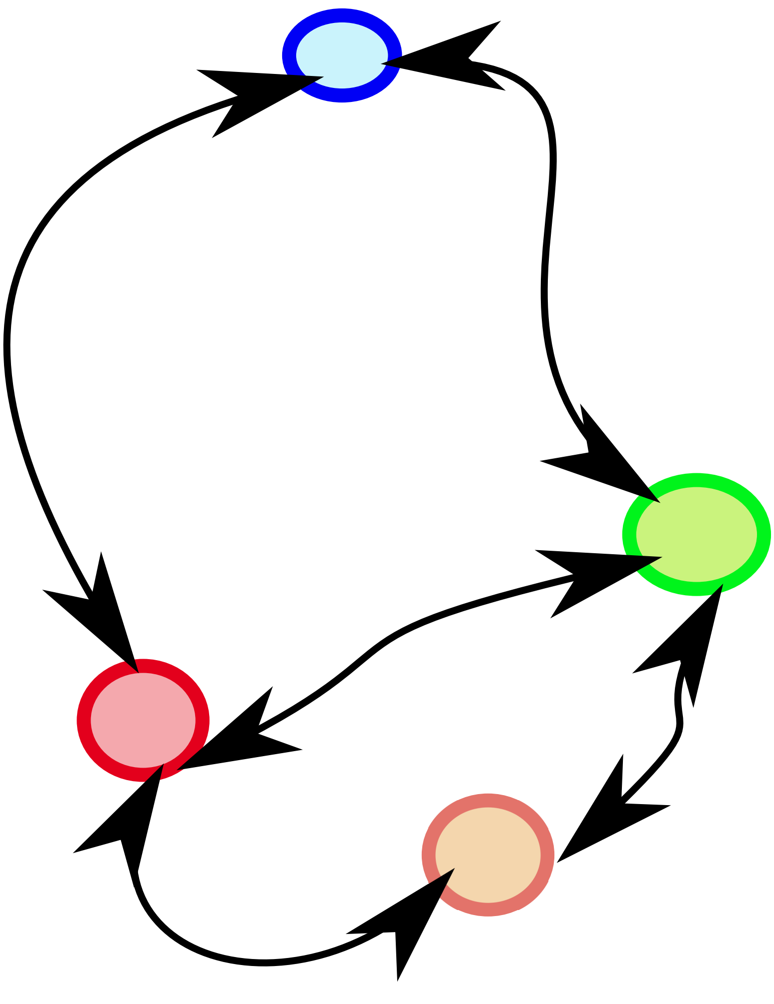


Figure 5.5: Phase Plot of Motion Primitives

Equip with controller, To put the state $\mathbf{x}$ in two different basin of attraiton, we can have basically tool method.

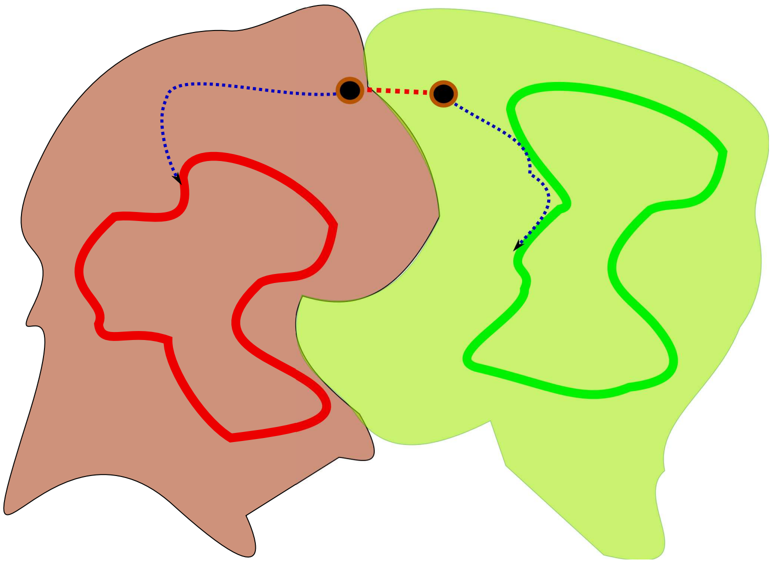


Figure 5.6: Motion Primitive Transition

- **Overlaping Entrainment** The first idea is use different CPG for different motion primitives. There is a switch mechanism of switching the CPG. If cpg a is applied to the fx, then the basic off attraction of a is enlarged, if cpg b is applied to motion primitives b, the basin of attraction is also enlarged. If a system is at a state that within the both basin of attraction, we can switch the CPG controller, we will swith the motion primitives.
- **Transform Method** Controlled Symmetr can also applied for motion state transition. For a system at $\mathbf{x}$, we can transform the phase portrait to make it which in the basic of attraction of b, this is illustrate in figure

Both the method can result in physically realistic motion transition, when the current state $\mathbf{x}$ lies a a special position. The what more problem happens when we don't know is where the current position x and where it is going. Thus we combined the two method above to achieve for a combined method, no matter where is current point is, it is going to converge to the limit cycle, thus we ask

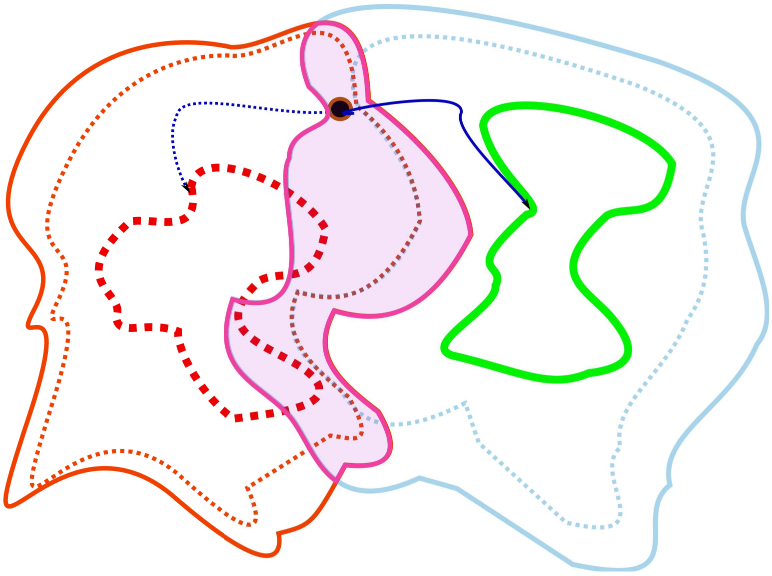


Figure 5.7: Over Lay Transition

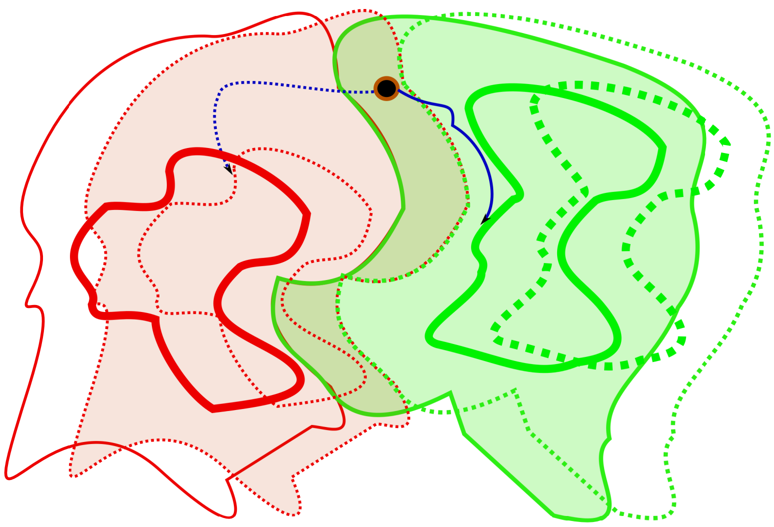


Figure 5.8: Offset Transiton

both basic of attraction cover the limit cycle, this can be achieved via using both the CPG and Transformation. And for this usually ,both motion primitives needs to be transformed, And the there is relationship between the two transformation, this relation ship is called transformation connection.

Figure download show the illustrate the idea.

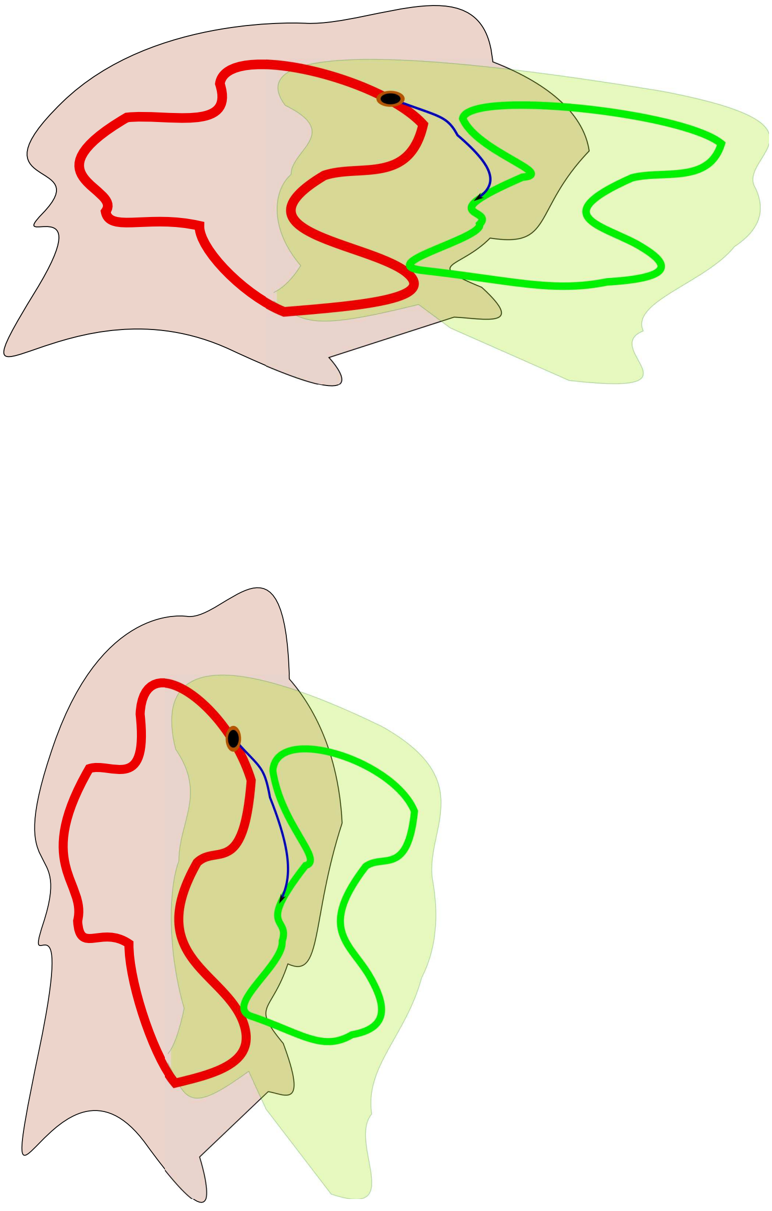


Figure 5.9: Comined Method

5.3 Motion Synthesis Framework

While this procedure may appear mathematically complex, utilizing our approach in a motion synthesis problem is straightforward. You will need:

1. a mechanical oscillator $F(\mathbf{x})$ which best describes the body and environment
2. a neural oscillator (for example, the Matsuoka oscillator in Equation **??**) and associated parameters,that form a limit cycle
3. an action $g \in G$ which adapts the problem to the current environment (we present three possible operators in Section **??**). and we apply the ajoint system transformation to the neural oscilator parameters.
4. an integrator to solve the system (we use the fourth order Runge–Kutta method provided in the MATLAB function *ode45*).

In the following chapters ,we show how this method generate adaptive motion results.

Chapter 6

MOTION PRIMITIVE TWEAKING:BIPEDAL WALKING

In previous chapters, bouncing ball and mass spring are only used for illustrate the theory, but have little value for graphic application and motor control research. In this chapter, we will discuss in details about how to adaptive motion primitve for for different environment and met user's needs. How motion primitives connected will be discussed in the next chapter.

The motion primitives we use is bipdeal walking. bipedal walking is a topic of great application value for both the graphic and robotic engineering. Currently, it is special ability robtos can't compete with human.

In the past a few decades, almost all the control method have been tried on the bipedal walking model, but still we don't achieve the human bipdeal walking ability. At the begining, researchers believe bipedal walking in nature is unstable, and developped lots of control method based on trajectory following. A great turning point is the discovery of passive dynamic walking machine, it shows walking can happens without any control effort. This idea lead us to believe that walking is an inborn ability, and most problem are solved our body mechanical structure.

In our research, we took the passive walking gait as motion primitive, we use global and local motor invariant controller to boot stability and adaptation. By this method ,we get rid of solving complex computational burden. We generate adaptive and natural looking gait in a computational efficient way.

6.1 Motion Primitives

For bipedal walking, the main motion happens in the spagittal plane. Sidesway and row motion are relative small and usually treated as secondary motion or totally neglected. In this chapter, we mainly focus on the two dimension motion, which captured most of key characterstics of motion, and maintain the completeness of our theory. For motion of more dofs, are discussed in Chapter 8, the effects of more dofs will introduce perturbation to our model, they will not change the basic motion characteris and adaptation tendency. But they will made the "symmetry" not so perfect, and may cause confusion. Thus we wil delay the discussion of fullbody in later chapter.

Figure 6.1 shows the a two dimensional walking model.

Dynamics

If put on a downslope, with some special initial condition, the passive walker can walking down the slope with a stable gait. On the phase plot, we will have a limit circle Figure 6.9. Like bouncing ball, this system is also an hybrid system,the motion include four phases[Chen, 2007].

- **Free Swing Phase** During this phase, the support leg (the blue one) is kept straight and there is no knee bending with the supporting leg. The Swing Leg is bended, the thigh and shank.

- **Knee Strike Phase** The Knee Joints has a limit, we suppose when the knee angle reach the limit,a collision happens,after that the swing leg is kept straight.

- **Knee Lock Swing Phase** During this phase, both the swing and support leg are kept straight.

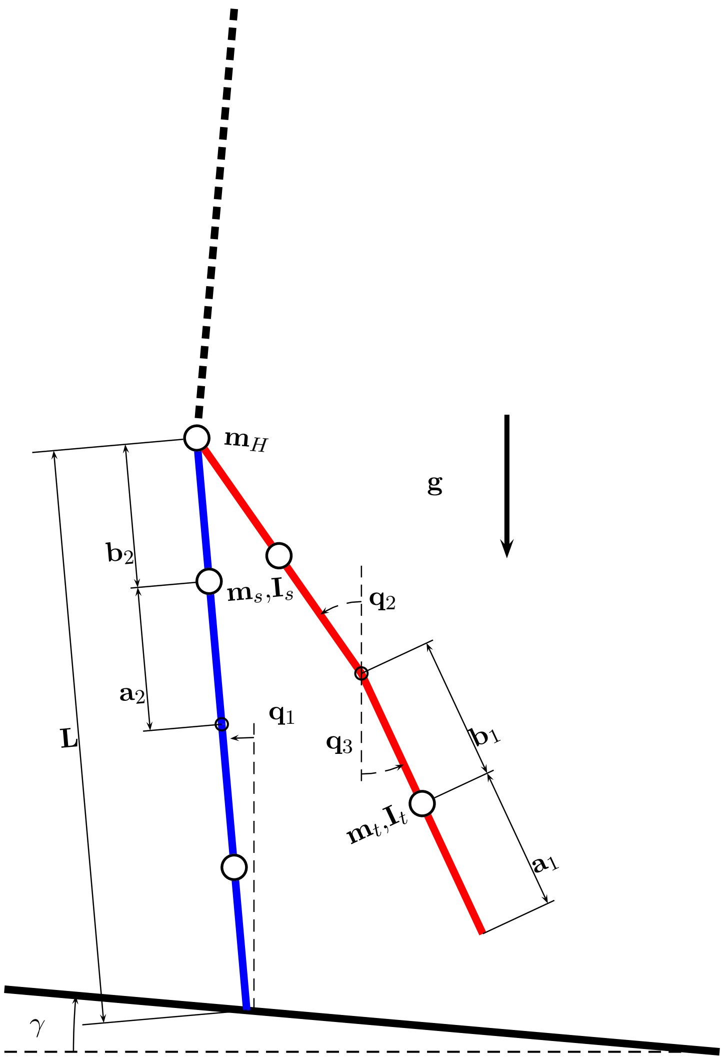


Figure 6.1: A Passive Walking Model with Knee

- **Heel Strike Phase** When Heel Collid with the ground, heel strike happens and switch the stancing and swaying leg.

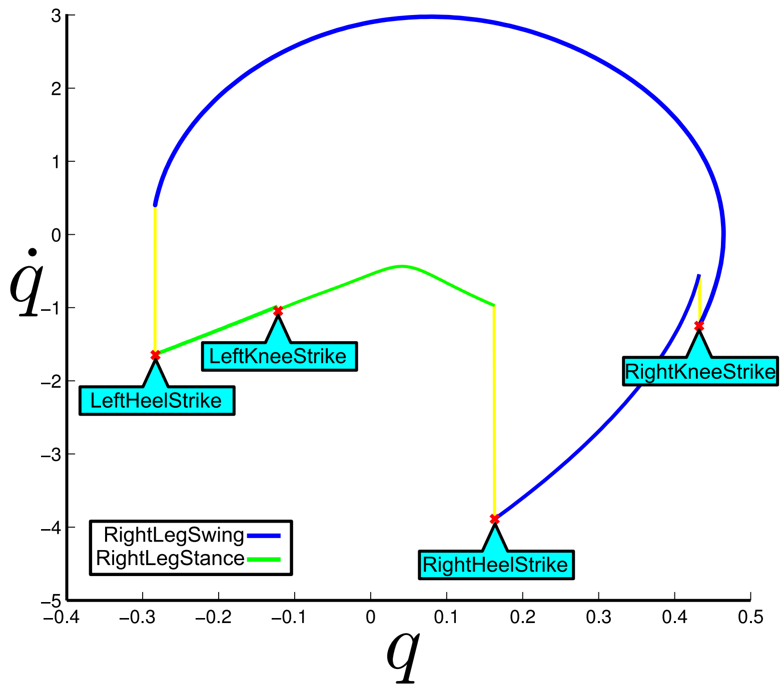


Figure 6.2: Limit Circle And Different Phase in Passive Walking

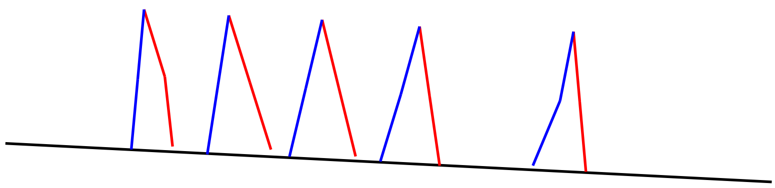


Figure 6.3: The four phases in Walking

Dynamic Equation

The passive walking are modelled as rigid body dynamics. for details of caculating the dynamic equation, please reference[Chen, 2007] The Passive Walking equation is developed based on Lagrange Mechanics.

- **flying phase** the equation are in the form of

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = 0 \tag{6.1}$$

  $q = [q_1, q_2, q_3]$,$\dot{q} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]$ where M is the initial mass matrix, C and N are the centrifugal force matrix and gravity respectively. The knee lock and contact state applies the instantaneous function for knee free a M and C are 3 by 3 Matrix, N is 3 by 1 matrix. for knee lock phase, M and C are 2 by 2 Matrix, N is 2 by 1 Matrix.

  Equation 6.1 can also in the state form, we have $\mathbf{x} = [q, \dot{q}]$. Then the function is in the form.

$$\dot{\mathbf{x}} = -\begin{bmatrix} \mathbf{1} & 0 \\ 0 & M \end{bmatrix}^{-1} \begin{bmatrix} 0 & \mathbf{1} \\ 0 & C \end{bmatrix} \mathbf{x} - \begin{bmatrix} \mathbf{0} \\ N \end{bmatrix} \tag{6.2}$$

- **The Strike Phases** When Heel Strike or Knee Strike Happens. collision equations are developed by momentum perserving laws. for the two phase, equations are off the same form.

$$Q^{+}\dot{\mathbf{q}}^{+} = Q^{-}\dot{\mathbf{q}}^{-} \tag{6.3}$$

  where Q is the mass matrix used to compute the angular momentum inetia, before and after a heel strike or knee lock For Knees Strike,Q^{-} is 3 by 2 Matrix, Q^{+} is 2 by 2 Matrix; For Heel Strike, both $Q^{+,-}$ are 2 by 2 Matrix.

For the commponents of each matrix, please refer to the appendex.

6.2 Global Motor Control And Adaptive Gaits

6.2.1 Entrainment

When Coupling Neural Oscillator with the Passive Walker, the output of neural oscillator drive the hip angel(angle between the two thighs) as shown in equation

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = Du_{\mathrm{o}} \tag{6.4}$$

for the knee lock phase $D = [1, 1]^T$. for knee free phase, $D = [1, -1, 0]^T$(acting on the difference between the two thighs, not effect the knee)

the input signal is the hip angle, we have

$$u_{\mathrm{i}} = h_{\mathrm{i}}(q_1 - q_2)$$

when the drive force is small, the entrainment system show a similar limit circle with original passive one, thus result in a similar gait. limit circle and gait are show in figure and figure

Entrainment Boost the staliby of the passive dynamic walker, the original walker can't walk on plain, when after several steps, the step size will become smaller and finally will rest or fall over. after coupline with neural oscillator, the passive walker can walk on a plane with constant stepsize. To main the energy efficient proper of natural motion, we make the limit the h_{o} to a small numer, thus the step size is very small.

Also we can inclue the state perturbation by push the walker a little when it is walking. it return to the stable walking cycle, this means the basin of attraction has been enlarged.

6.2.2 System Gait Adaptation

The passive dynamic system have many parameters, changing such parameters will generate different gait through sysmtem adaption. we can fomulate the original system as

$$\dot{\mathbf{x}} = F_a(\mathbf{x})$$

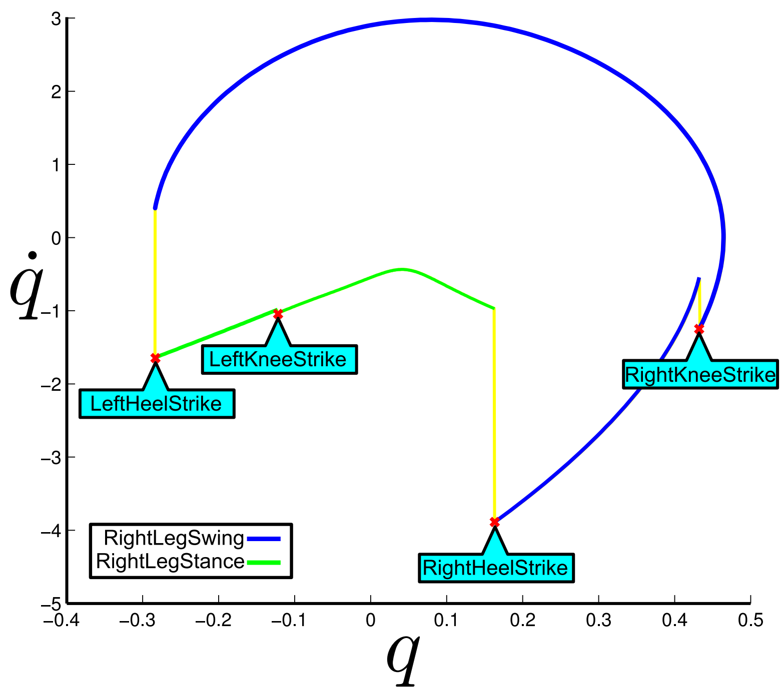


Figure 6.4: Place Holder

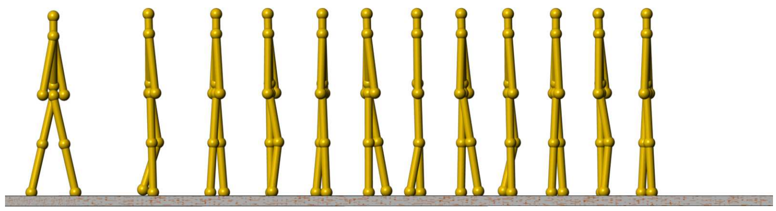


Figure 6.5: Place Holder

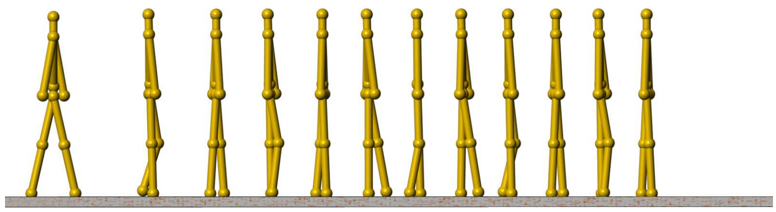


Figure 6.6: Place Holder

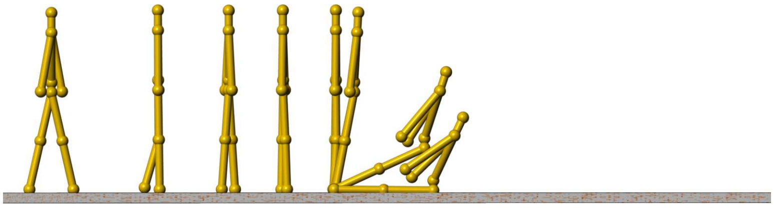


Figure 6.7: Without Neural Oscilator On Plain

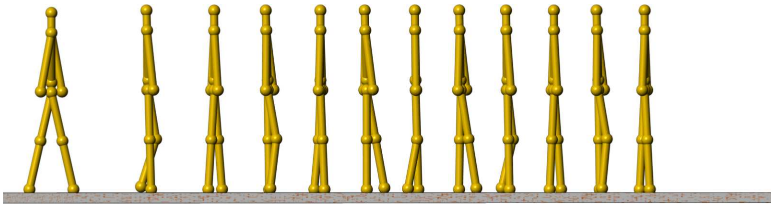


Figure 6.8: With Neural Oscilator on Plain

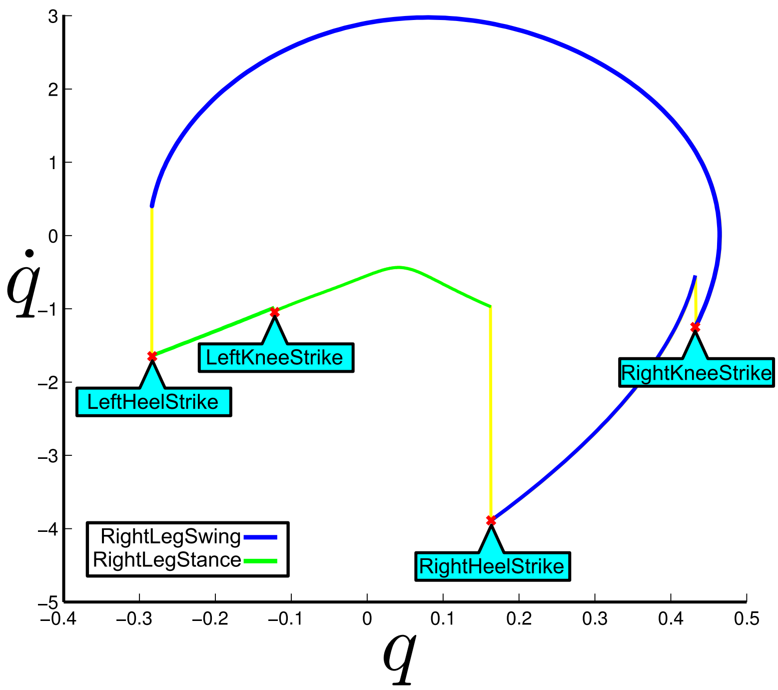


Figure 6.9: Limit Circle And Different Phase in Passive Walking

where a is the system parameters. we adjust the several parameters, neural oscillator maintain the limit cycle, but different parameters will generate different gait. Some meaningfull gait are shown. Becaus uniformally change the parameter will not effect the gaits, only will effect the period, so we most of the parameters we change is the ratio. We wil maintain the leg length L and mass sum.

Mass Distribution Change

We change the mass of the hip m_h, different m_h will generate different gaits, while bigger m_h wil result motion that are similar to burdenned gait. different Limit Cycles are show in Figure **??**

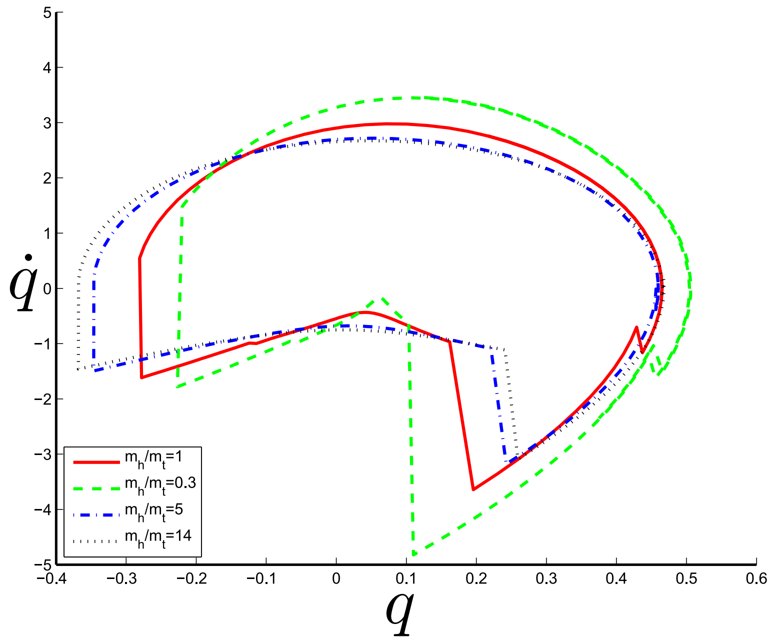


Figure 6.10: Different Gait Resulting From the Different Mass Ratio

From the phase plot, we find some interesting tendency, when the hip is heavier, it will walk with bigger step but a slow speed($\dot{q}$ is lower),and also character tend to fall backward. which the body is become litter, charater will waker with quickly($\dot{q}$ is bigger) and it tends to lean forward.

This may give us some infomation about the upper body motion. Usually, when we carry something heavier, we tend to bend the body upper forward to prevent following backward.

different gaits are show in the following pictures.

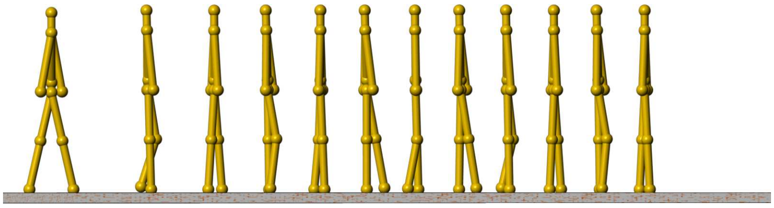


Figure 6.11: Place Holder

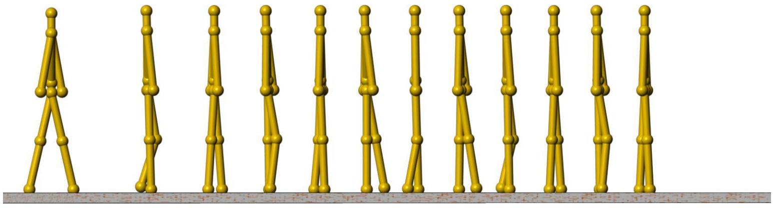


Figure 6.12: Place Holder

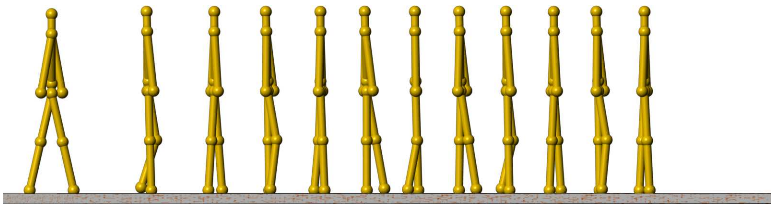


Figure 6.13: Place Holder

Leg Length Distribution Change

we keep the leg length unchanged, but alter the ration of shank and thigh. We generate gait with different leg length ratio. as shown in figure 6.27

for the limit cycle in figure 6.27, we find something interesing. Basically, the support leg motion is almost the same, while different leg length ration will result in different sway angle basically the longger the shank, thigh has to sway quickly

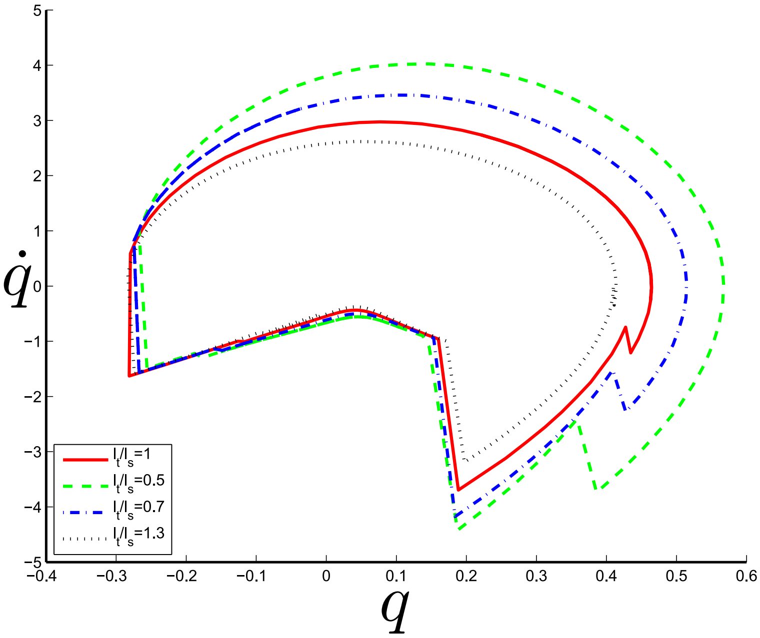


Figure 6.14: Different Gait Resulting From the Different Mass Ratio

and with bigger amplitude. There are aslo bigger impulse during the strike phase. For both the knee and heel strike, larger impulse is generated. while step size is kept. This maybe true that girls walking with tall heel will easilly get knees and heel injured and aslo will generate larger step sound.

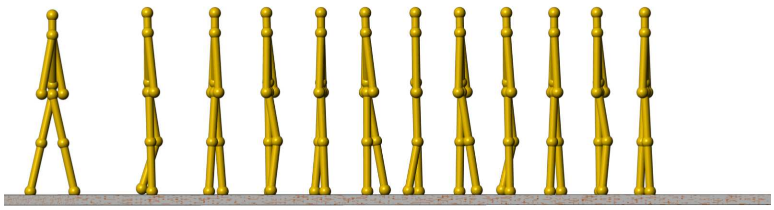


Figure 6.15: Place Holder1

SlopChange

Also we can change different downslope. For different slope, entrainment maintained the limit cycle, but limit cycle changed its shape. different stable limit

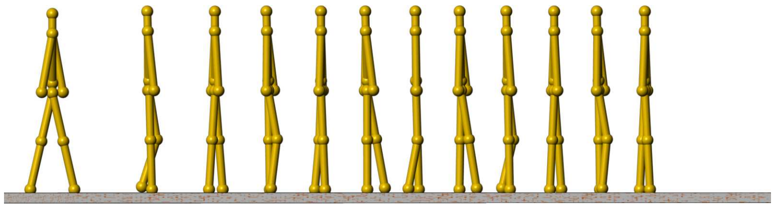


Figure 6.16: Place Holder2

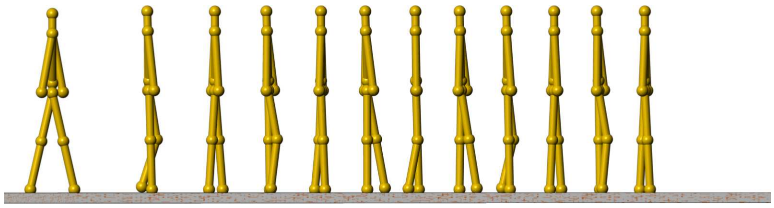


Figure 6.17: Place Holder3

circles are show in figure ?? basically ,the bigger the slope, the bigger the step size the higher the speed, and produce a gait similar to energy scaling.

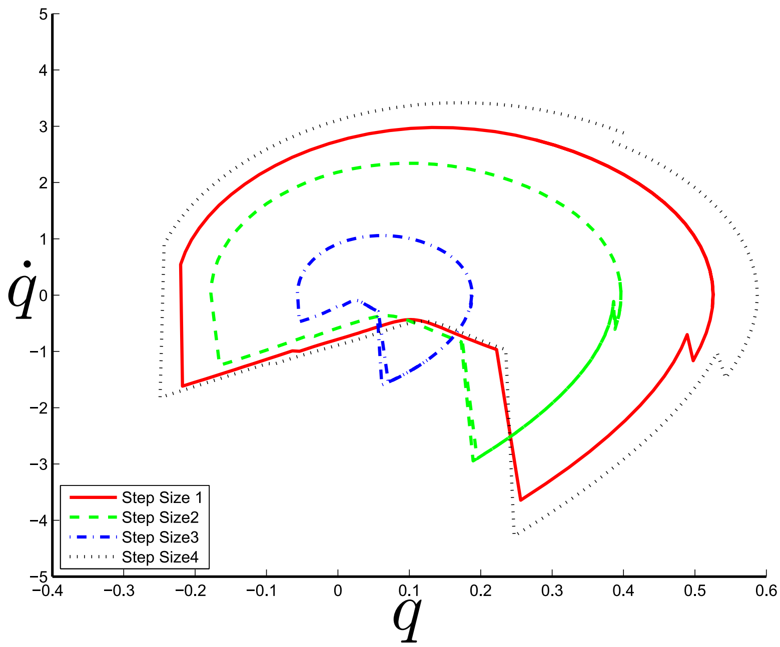


Figure 6.18: different step size walking

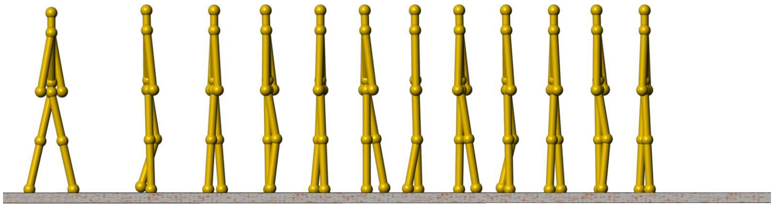


Figure 6.19: Place Holder1

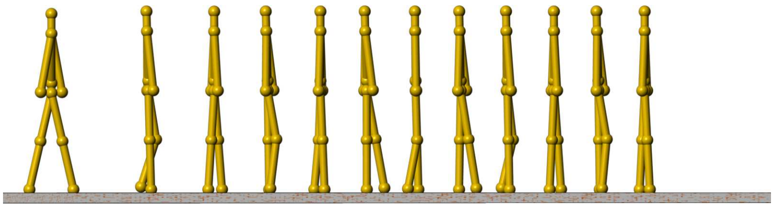


Figure 6.20: Place Holder2

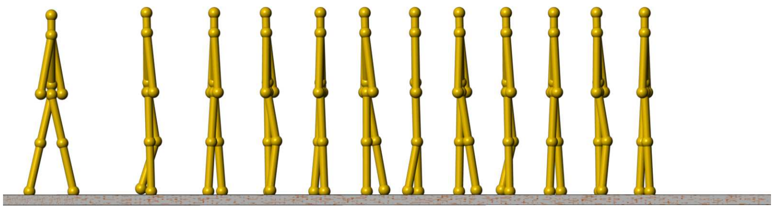


Figure 6.21: Place Holder3

Different Leg Mass

we also change the leg mass of the two leggs, make them not equal. This will result two legs sway differently and the limit circle of orignal system is doubled. Bigger difference will result in a gait looks like cripple.

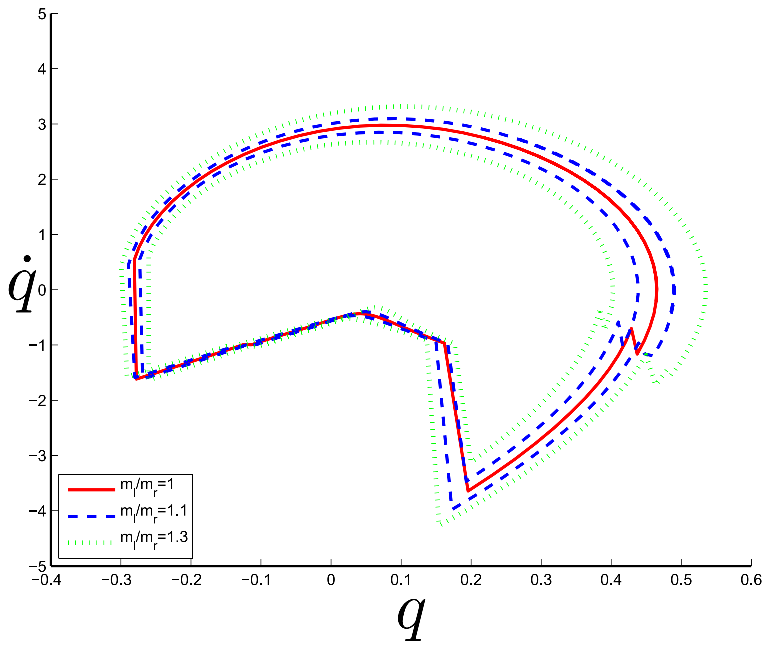


Figure 6.22: Different Leg Mass Stable Gaits

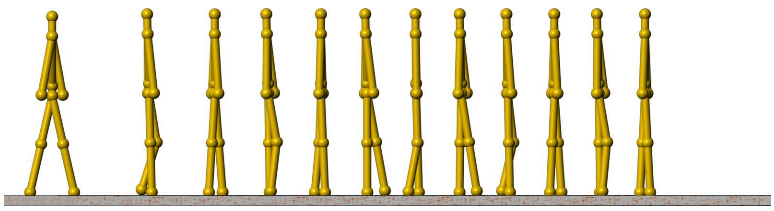


Figure 6.23: Place Holder1

the period is double and the step is not symmetrical any more. So the two legs move in a slight different manner, but still, it keeps walking.

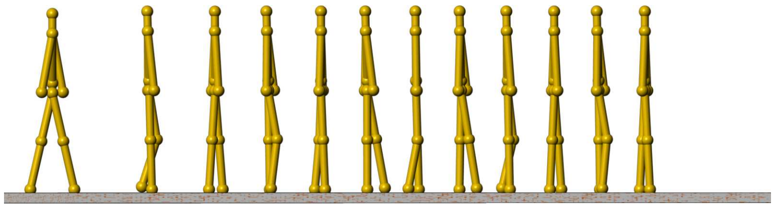


Figure 6.24: Place Holder2

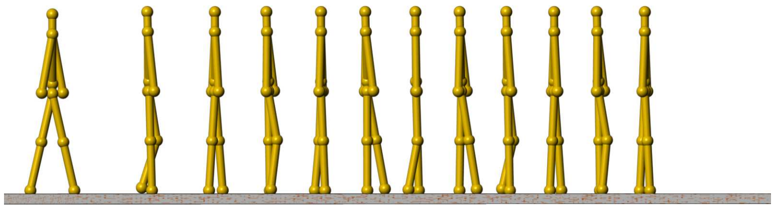


Figure 6.25: Place Holder3

6.3 Transform Gait Adaptation

Local Invariant Controller will not generate different gait style, but it will transform the gait at different speed and at different terraint. One important limitation we find of limit cycel is it can not walk up stope. This is because when walking upslope, it is symmetrical to walk down the slope in another direction, if the limit cycle is stable enough, after a few steps, it will walk backwards and walk downslope.

the failure of walking up slope is show in figure

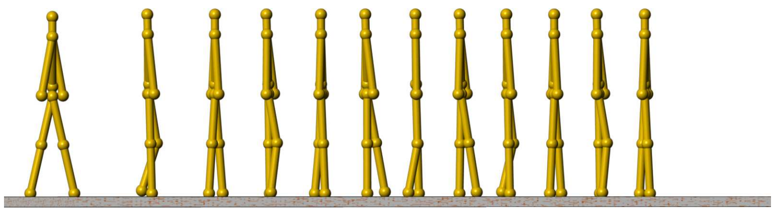


Figure 6.26: Place Holder

we apply two types of local motor invariant.

- **Slope Change**. Slope Change is incoporate through offset transformation.

$$u_1 = N\cos(\theta) - 1)$$

- **Speed Change** Speed Action will ajust the walking speed, but maintain the walking gait. the the new walking speed is s. we have

$$u_1 = N(s^2 - 1)$$

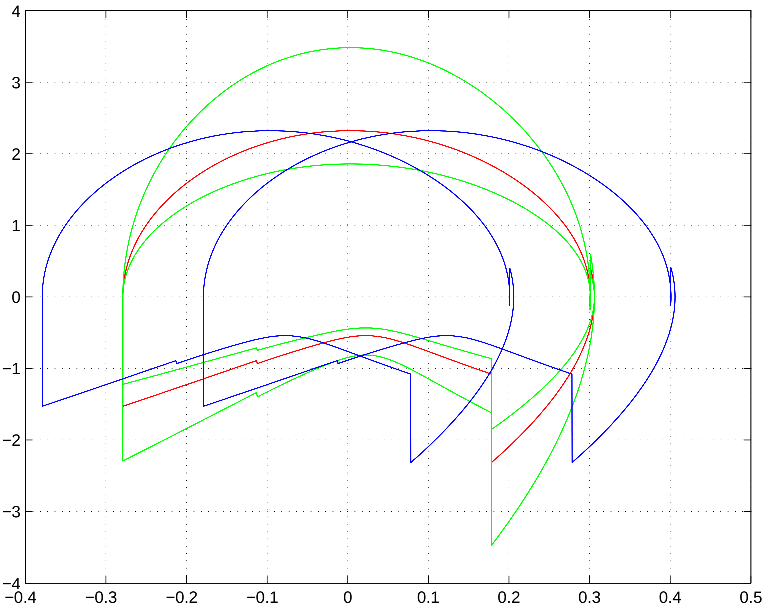


Figure 6.27: Different Leg Mass Stable Gaits

The red on is the origianl limit cricle. Green one are apply speed action blueone are applied locol stransform action

Also it can help the passive walke walk upslope. as show in figure

and on phase plot we see

and another terrain.

if the offset action is not enough, we find character can also walk up the slope, just with smaller step size. Because a smaller offset action is equalivant to walking on the same transform the gait and with the system adaption of the walking down slop. In figure 5, we show we adjust the stepsize by changing the

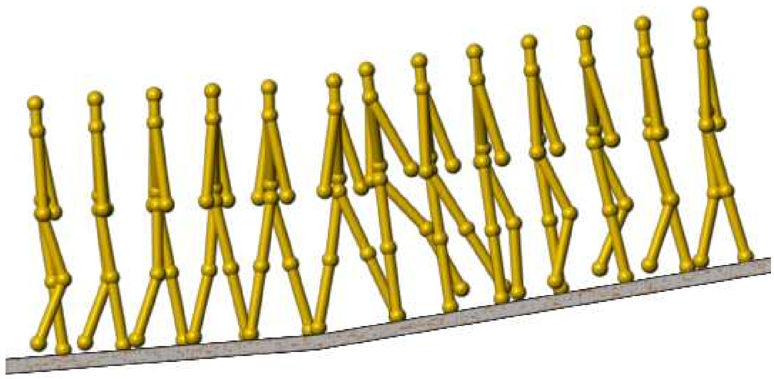


Figure 6.28: Varying Terrain

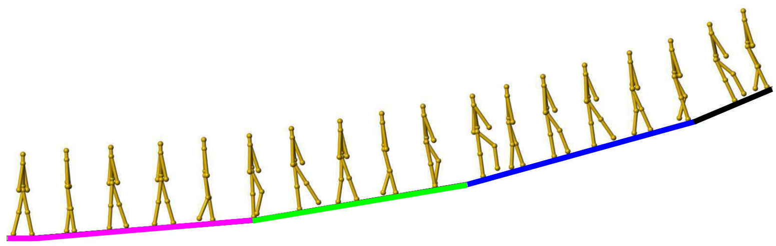


Figure 6.29: Varying Terrain

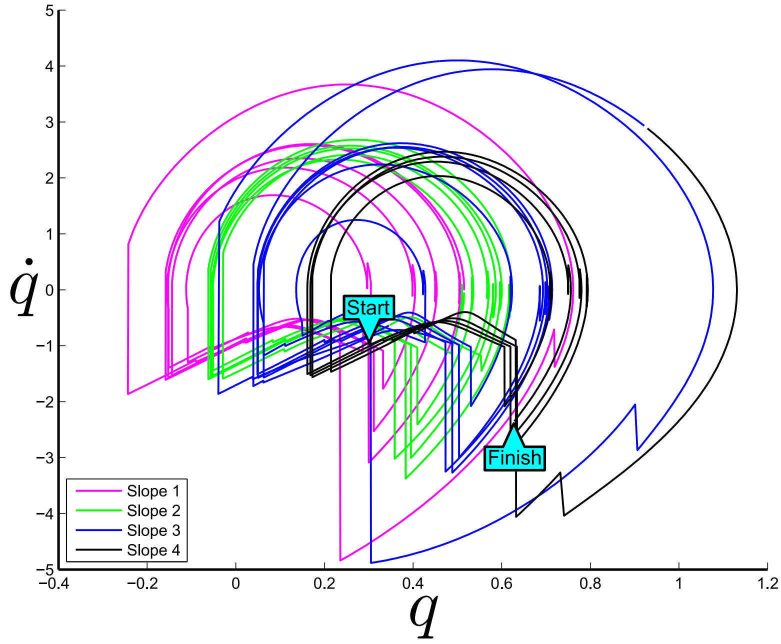


Figure 6.30: Varying Terrain

slope action when walking on plane.

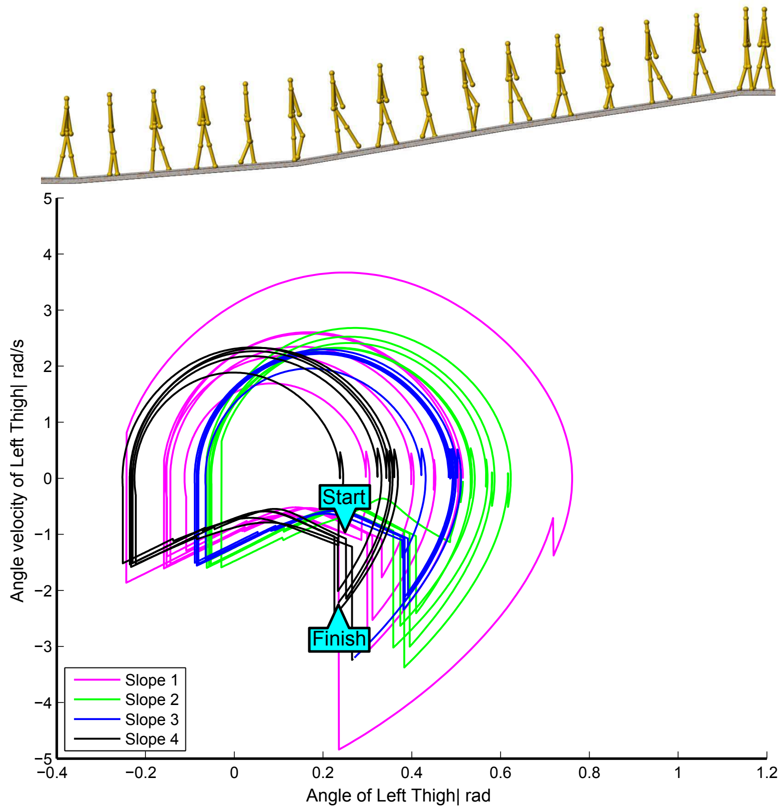


Figure 6.31: Varying Terrain

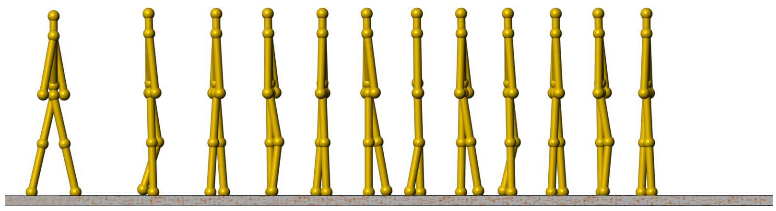


Figure 6.32: Place Holder1

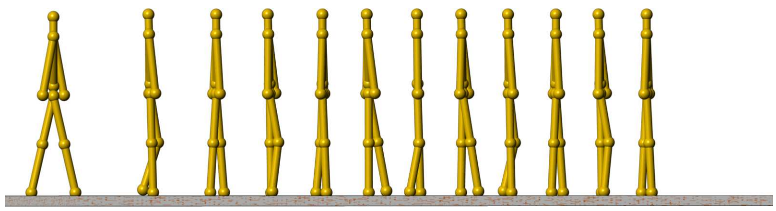


Figure 6.33: Place Holder2

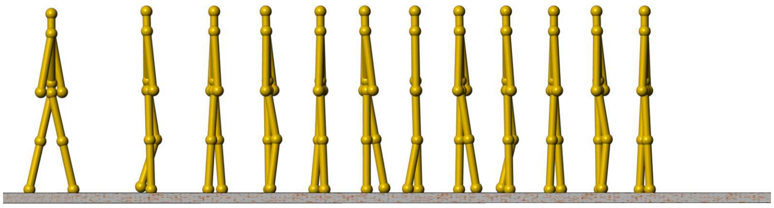


Figure 6.34: Place Holder3

Chapter 7

MOTION PRIMITIVE TRANSITION:WALK AND STANCE

7.1 Motion Primitives

For Bipedal Walking, when the heel strike happens, it the passive walker don't get enough velocity, it will stop walk and rest at the heel strike pos. This is posture is stable, then we got another motion primitive: the stance, as show in figure **??**

usually when people stands, the two legs are almost straight, and the heigt is almost constant. It is not nessary to consider the full details of four link leg model. we can simpliy the stance mode as a point supported by two straight legs. For normal human stance, high change will be less than 5%, we suppose it is constant. we only consider the horizontal displacement. this simplified model is proposed by although is simple, but capture the key characters of stancing [Stephens and Atkeson, 2009]. use this method, we show the stance motion in the following figures **??**

the stance is easy for human, but the dynamic is not trifile. stance is not continues system,the dynamic involves three phase

- **Double Support** When the horitontal displace model is small, people

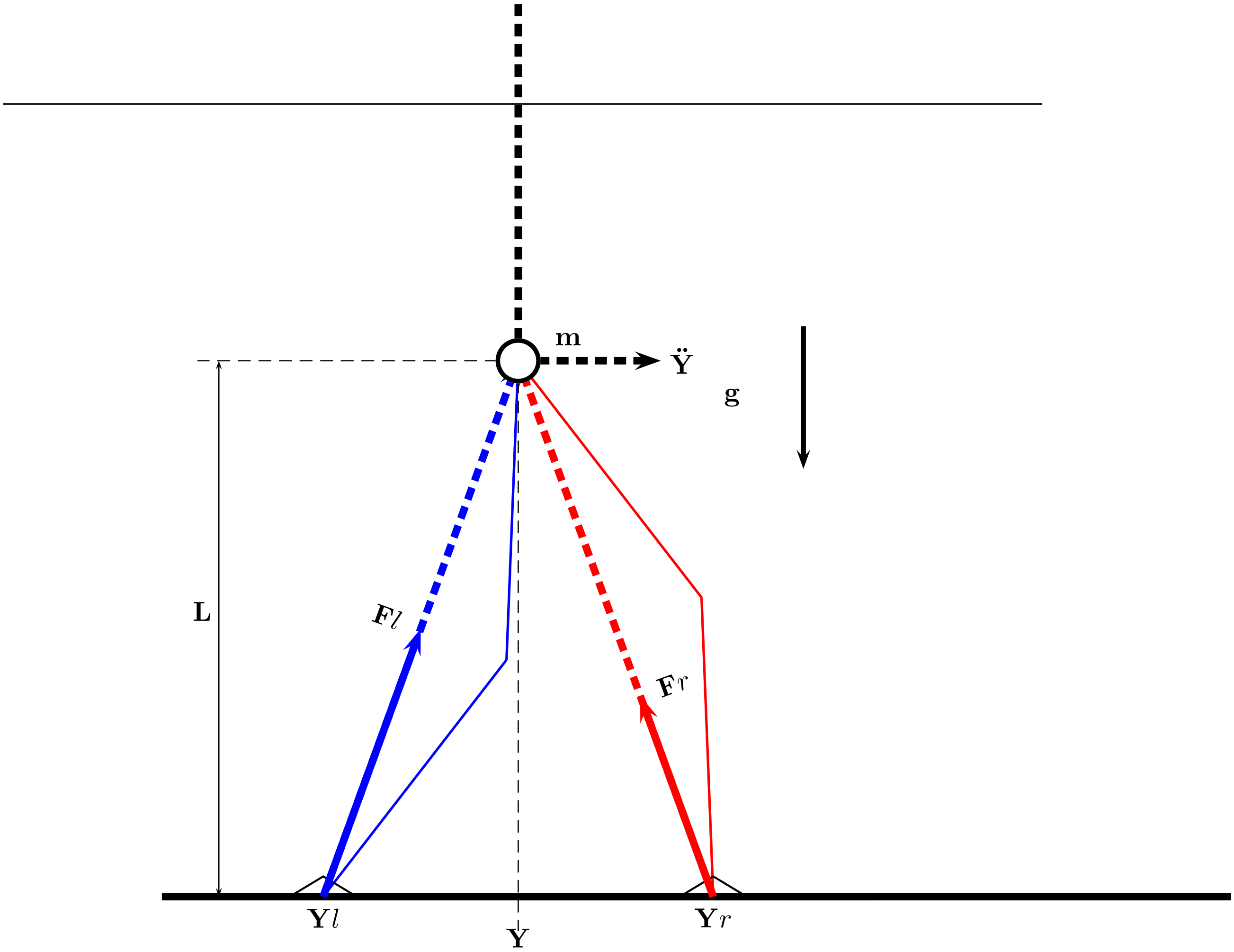


Figure 7.1: The Stance Motion Primitives

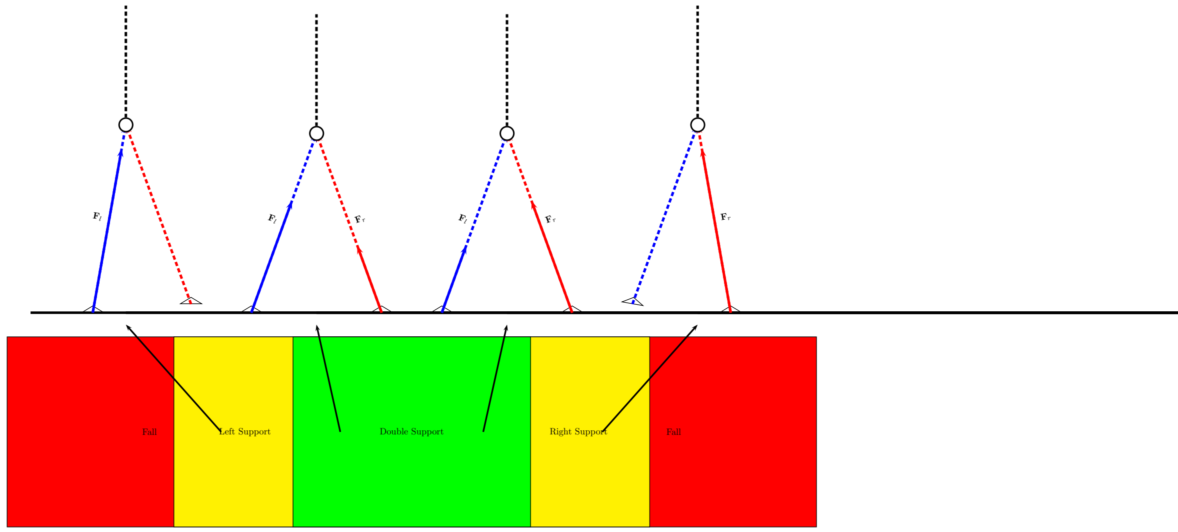


Figure 7.2: The Stance Motion Posture

stand with two legs support. the motion is governed by the gravity.

$$\ddot{q} = \frac{g}{L}w_r(y_m - y_r) + \frac{g}{L}w_l(y_m - y_l)$$

but usually some intutive strategy is applyied. if the two legs generate torque to maintain the posture, the two torques should not be euqal, if the center moving to the left, then the left torque will generate more torque. if the center moving to the right, then the right leg will generate more torque. supporse the realitionship is linear. then we get the following intutive control stance equation, which we used as the mechanical oscillator.

$$\ddot{q} = \frac{g}{L}w_r(y_m - y_r) + \frac{g}{L}w_l(y_m - y_l) + \frac{\tau_L + \tau_R}{mL} \tag{7.1}$$

- **Single Leg Support** if the there is a big horizontal displacement, there pepole will stand with single leg. passively, the equaltion is

  $$\ddot{q} = \frac{g}{L}q$$

  and intutive even controlled

  $$\ddot{q} = \frac{g}{L}q + \frac{y_{L,R}}{L}\tau_{L,R} \tag{7.2}$$

- **Fall and Walk** if the displacement is even bigger,then the walker will move out the support region. for a human, where the stance region is depends on the hight and the stepsize. And the goal of balancing is to maintain the system state within the support region.

  usually,it will result two motion sequence, wheather it will start to walk for it will fall

7.2 Stance Control

7.2.1 Entraintment

without damping effects,the original system is simmilar to mass spring system. It wil vibrate continutelly.

the phase plot is shown in

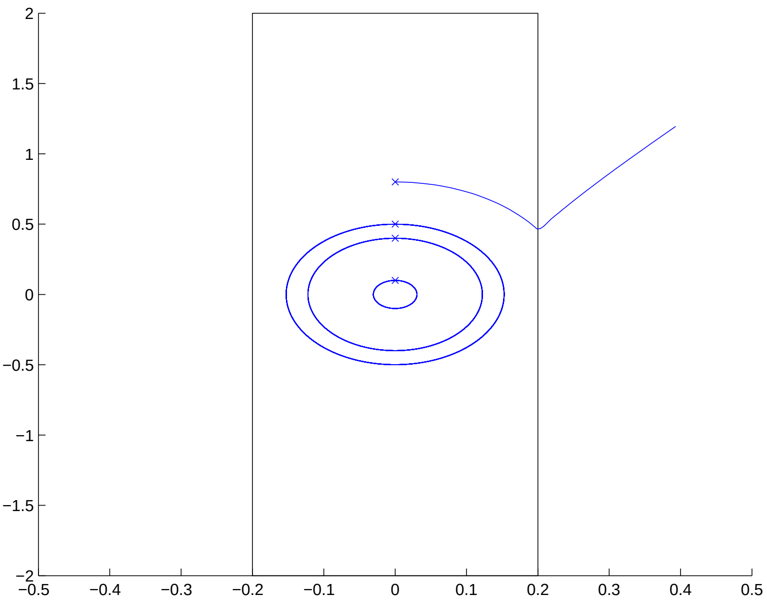


Figure 7.3: un controlled motion

if the the initial speed is higher, then it will move out the support region.

while in our method ,by coubling neural system the the oscillator, it will form a limit circle, but for this situation of standing, limit circle doesn't not boost the stablity,because the boundary is fixed, neural oscillator will not modify the boudary,and it is impossible for mechanical system to converge to the limit circle within 1/4 period. thus we make the output of the output of neural controller very small u_o.

Limit Circle is useful for it can make stance start to vibrate at a constant speed, this will be very helpful when we start from stance to walk.

7.2.2 Local Invariant Control

All the three group controller can be used,but basically only two control method are useful.

Time Scaling

The first is time scalling $G_t s$ as show in figure

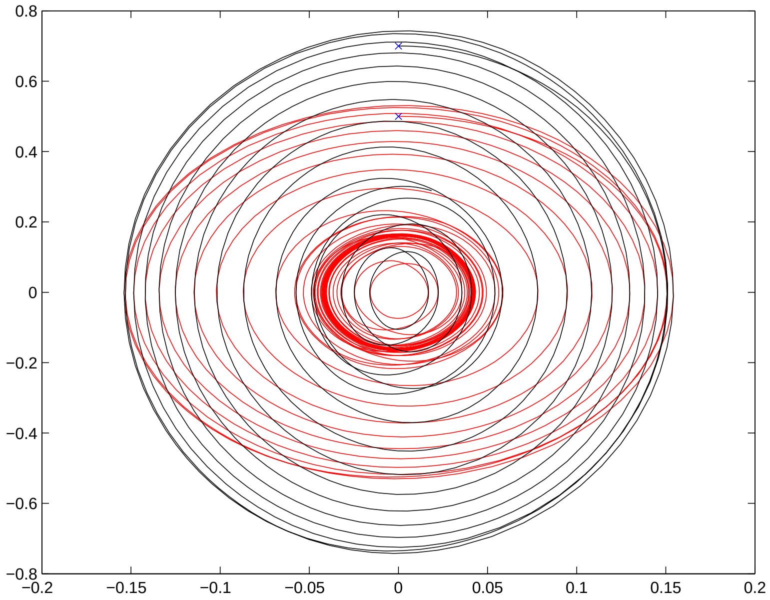


Figure 7.4: Time Scaling

The falling motio is show in the figure 7.5

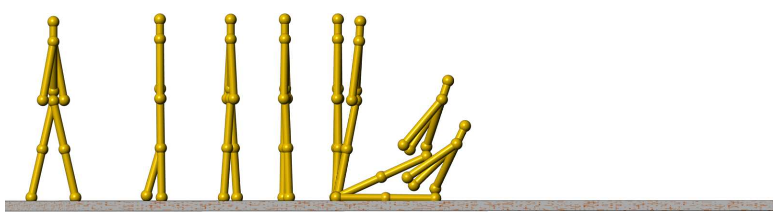


Figure 7.5: Place Holder

Energy Control

By modifying the Energy Scalling, we can modify the size of limit circle, the will adjust the oscillation amplitude. as show in Figure ??

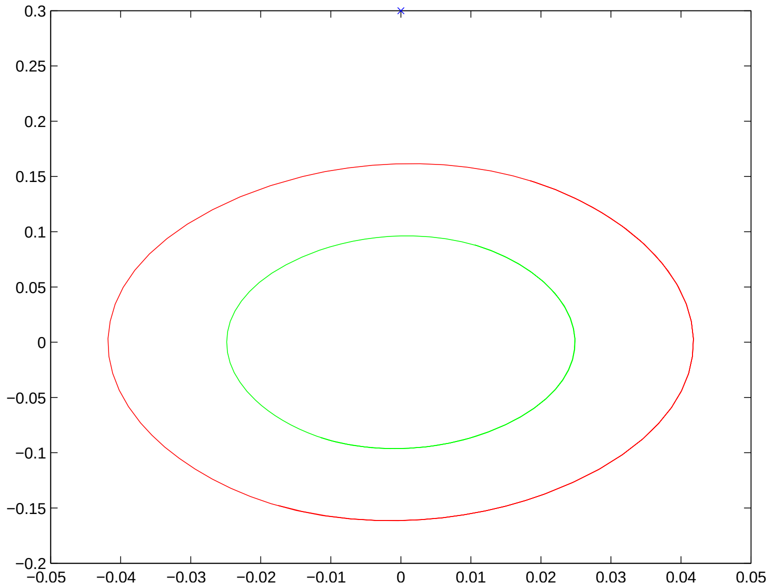


Figure 7.6: Energy Scaling

but it will also shrink the size of basin of attraction.

Fast Stable

by applying speed and energy scalling sequecely, we can make the stance pos coverge in a quick speed. as shown in figure 7.8

7.3 Walking and Stance Transition

we phase plot we show the limit circle of two motion primitives.

the phase plot here shows the supporting leg, the swing leg is show in shadow red

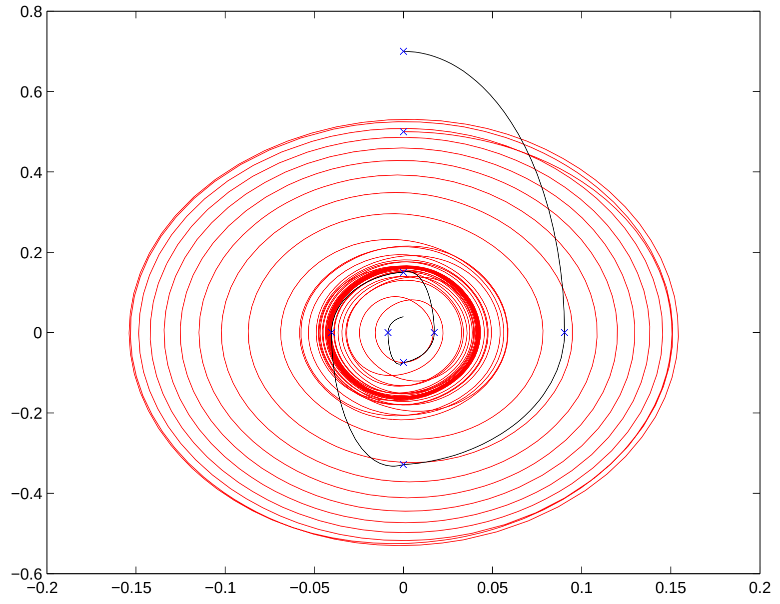


Figure 7.7: Fast Converge

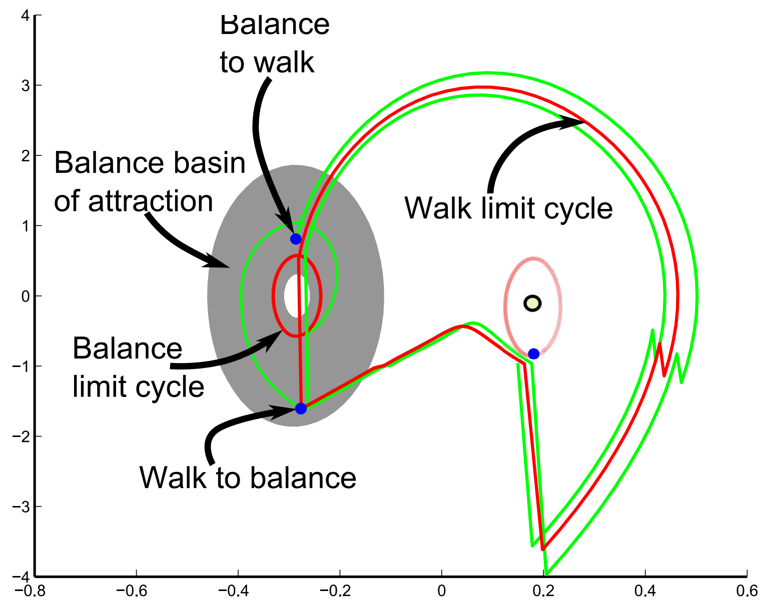


Figure 7.8: Fast Converge

7.3.1 Walk to Stance

walk to stance transition happens at the heel strike phase. if without control effort, the bipedal machine will continue two walk, while if we swich the on the stance controller, the bipedal machine will converge to a small amplitue,then both legs start oscillation, this is the stance to walk.

we show in picture

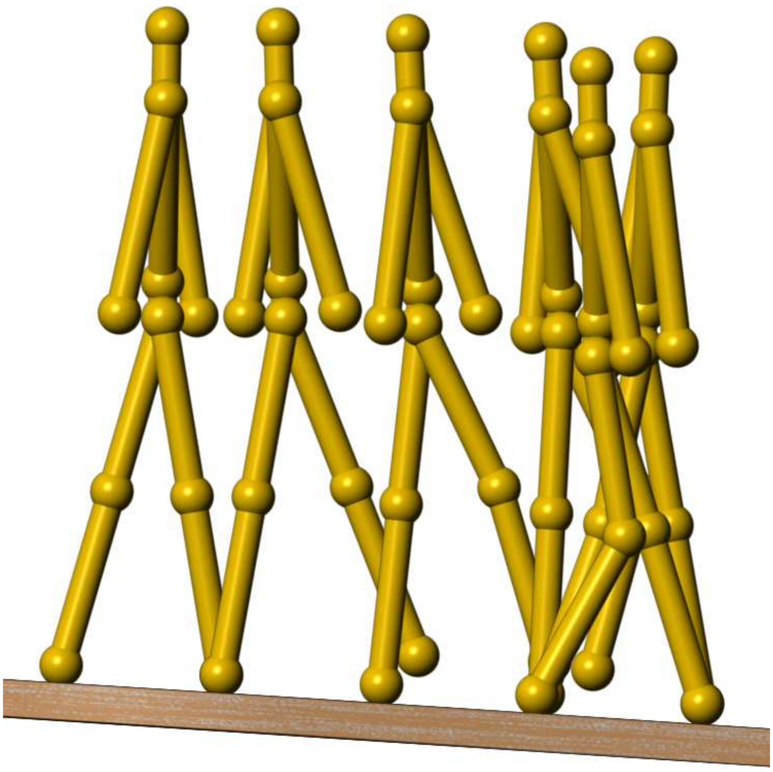


Figure 7.9: Walk to Balance

When walk to stance happens, the walker have to the stance controller must include the heel strike position.

Knee Bending Scheme

When from walk to stance, that the heel strike time, the two legs are straight, for this case, the support region is very small. Any push of the figure, it will move out of the two support region. the walkers have to bend legs and lower the height. Many possible ways can be develop for bending to lower the body height We have develop many bending scheme, have seen many possible usage of the bending

- **One Leg Bending** walker can bend one leg while keep the other leg straght.
- **Double Leg Bending** We can make the two leg bend.

it is very difficult to tell which one more realistic for human, basically, for when human walk, the knees is not necessary straight. two two scheme provide is the exeme condition.

we have walking stance transition is shown in the following figures

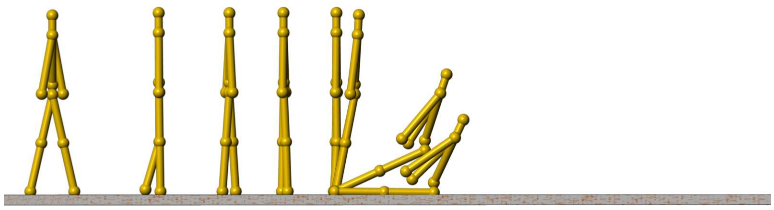


Figure 7.10: Place Holder

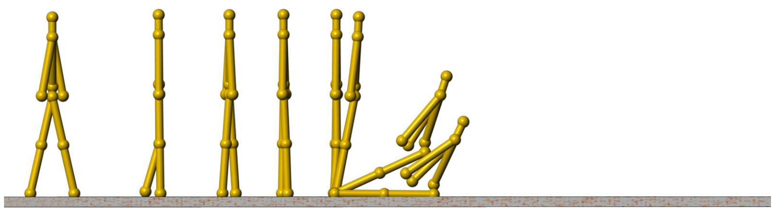


Figure 7.11: Place Holder

7.3.2 Stance to Walk

From stance to walk, we must start be making the current state close to the limit circle of walking. we should place the state of the near the position of start swing position (show in blue). On the limit circle of stancing, this is the position that the leg is moving forward at maxim speed and the positon of the hip is in the middle between the two legs. if we switch to the walker at this time, then it will begin to walk.

from walking to stance, there the height has to be increase, so there is only one scheme for straight the knees. the scheme we use is to keep the supporting leg straight then the make the swing leg from bend to straight.

as show in figure

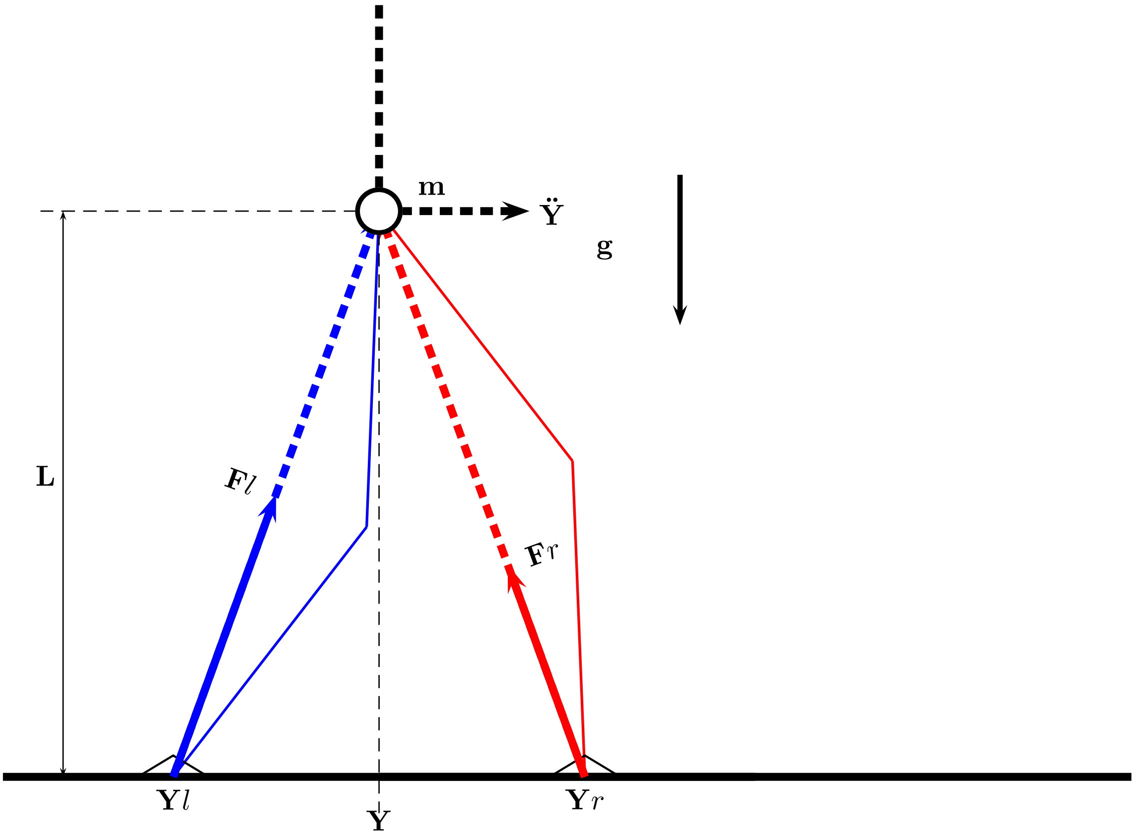


Figure 7.12: The Place Holder

A notrivial problem is when switch from stance to walk, we can't make two legs both on the limit circle. Our approach is put the stancing leg on the limit circle, for the stancing leg is more important for maintaining stability.

in the figure above, the stance leg is bit offset of the limit circle, if we put the original swing leg on the limit circl,for it is going to be the support leg in the following walkstep.

7.3.3 Speed Action Connection

When transit from walk to stance, the basin of attraction must include the heelstrike state. original basin of attraction will not include the heelstrike, a speed action is needed to enlarge the basin of attraction, There is an alternative to this, we can lower the walker speed, in this way, we lower the effort of balance cotnrol.

Also, if transition from stance to walk, if little effort is executed,the it will start at pos far from the limit cycle. then to maintain the stability walking, speed action needed to included for start walking slowing.

So the the speedaction of stancing and walking must meet some relationship as

$$S_s S_w = c$$

The pheonemon is common for our daily experience, here we give it a methematical meaning.

Chapter 8

TOWARDS HIGH DIMENSION

8.1 Introduction

Passivity

High DOF is key challenging in motion synthesis. In our research, we partly solved the problem. For our walking model, when it walk down the slop, all the dof is uncontrolled. Some now mater how many dof the walker have doesn't matter. Even controlled with neural oscilator, only one dof is controlled, Even when transformation controller is added, all the dof are controlled, but the matter will not result in underminiestic problem. For our model, even the dof is not os huge, we have extra dof problem completely.

the question arise for model of high degree system, the question is whether the dynamic system is large.

for our walking and stance example, the question is how the torso should be added, the foot, the side sway and yaw motion, and for system like fish and snake and worm, they have infinate number dof

our idea is for different model, we should take different measures,

- **Neglectable** for some dof can be neglected for they will have little effects on the qualitative property, we can simulate them a different mechanical model, our motion control method don't have to be changed. because the qualtative property and symmetry is kept.

- **Symmetry Reduction** For some DOF,like rotation or pos, the extra dof will have not effects on the dof or the effects can be transformed the dynamic system into transformed version of the orignial system, such Dofs can be reduced.
- **Mechanical Coupling** For some high dof system, we can treat them like a coupled mechanical oscillaor, the can only focus on one oscilator.
- **mimic** all the three method above are based the effects of dof are not equal, for system that all the dofs are the some, such method will not work. For dof with similar effects, we can propose a different method, the system is divided into several equal component,we only caculate one component, other mimic the strategy of one.

8.2 neglectable

Although bioliogical mechanical structure is of high degree of freedom, many dof will not have effects on the topology or symmetrical properties. For the walking example,marcke raiber point out walking is the same as a ball rolling down a slope while running is ball bounding down a slope.

we can have system from low dof to high dof as the following picture.

- **Rimless Wheel**
- **Compass Gait**
- **Arch Foot**
- **Our Model**

All the four system form the same limit circle. And the later three, the limit circle shape seems very similar.

Thus we can say, the include or knee will not affect the qualitative properties.

And all the four system, can apply the same kind of symmetry group.

This idea may be understand through the perturbation or averaging theory.

In therory all the manifolds share the shape of torus.

if the motion is relatively small,

the dynamics can be approximate as a low dimentional cycle.

as show in Figure.

Folowing this idea, although not implemeted in this research, more dofs like foot may also possible. But that's just an extra level of complexity, without modifying the prinicples.

8.3 Symmetry Reduction

Some dof will result in a different motion, but such kind of motion will have not qulatitavie effects. An example is the side sway motion.

the side and front plane is how in picuture. The side sway effects can be seen can be decoupled for spaghettal function.

This method is we can project the gravity in the spagetta plane, then we see, the way function is just an uniform of minimizing the gravity, which is the same as speed action.

With lie group operator such dof can be totally ignored.

8.4 Coupled Network

Some mechanical system can be treated as connecting many different simple components together. the different parts of motion formed mechanical entrainment.

Mechanical Coupling

In fact any mechanical system can be treated in this way, a proper method should separate different components when the coupling is weak. the weak couple joint can be found through the mechnaical strucutre.

They happens when they the mechanical have a branch structure. when mechanical have a branch strucutre, the dynamic equation will be in the following manner.

based on where the mechanical branches, we can seperate the dynamic equation into two parts, and simulate them idependently and form the mechanical entrainment network.

if we consider the difference in heel strike phase, the limit circle becomes more a bit noisy, but qualitative properties are maintained.

Torsol And Arm

using this method, we can incoroporate the arm and torso motion is our dynamic system.

A without developing the full body dynamic, we can seperate it into two parts.

the torso dynamcis.

the entrainment or torsoq.

by analysizing the equation, we have find that torso is unstable in nature, some control effort must be added. While we know little about it, in our research, we use a simple pd controller.

through the anaylis ,we know that the torso have little effects on the lower body motion, that may be why we can carry out many upper body motion while maintain our walk.

It is also possible to use an hybrid method, we can incoporate motion capture data for the upper body, and adding the effects motion to the lower body.

8.5 Boid and Adhoc

For some sepere and fish, the mechanical system is in chain and involves lots of similar joints. such system dof can't be neglected or reduced through symmetry and through mechanical coupling.

our propose idea is an adhoc method, we simulate just one dof, other dof following the solution. since the dynamics is similar, similar strategy will result in similar solution. such idea have two kinds of application.

The first idea is applying this method for the boid system. Original boid system are ruled based, but method don't promise stability. While we useing group theory for simulation boid system, if all the agent using the same neural oscilator, they will converge to the same limit circle, thus garanty the final motion of the group are in an uniform manner, the different in the agent of modeled by lie group symmetry, different symmetry applied to different agent will result in difference. on import symmetry is time offset, which will result in the same motion but of different phases.

In the following example, we have 8 ajent, the form are controlled by the CPG with the same parameters but of different initiial condition, this will result an motion of different phase .

we expend this boid example to model the fish swining. The fish is made up 8 links, and each dof is controllerd by a neural oscillator.

the same symmetry is applied to the 8 neural socilator, then it will modify the system in an unified manner.

as show in picture

8.6 From Low Dimention To High Dimension

For computer animation, even we can get high dimentional results, maybe it does not meet the animators needs. simulation result in motion can be treat as a reference, rather than put into result in deformaiton.

we can discribe the motion procecures, and using mechanical simulation a low dof to drive the high dimensional motion. in the following exmaple, we describe the jumping motion in an procedure method, and bouncing ball is used to drive the motion.

also we can using the low dimension data as a key word to search motion capture data. which will give us more realistic motion results.

Chapter 9

Conclusion And Future Work

9.1 Conclusion

Based on the idea of biology research, we implemented an new motion synthesis framework. We also introduce new mathematical tools for both the graphic motion synthesis research and biological motor control research. by the idea of topology conjungacy, we unified the different biological idea.

our idea propose an computational efficient method for graphic research.

by the topolgy and symmery property, the dynamic of system can be cataglo-ryed.

9.2 Futher Work

Our research is not finished, based on the method we found, new question arise. Which in our eyes are important for both the biological and graphic research.

9.2.1 Local Frame Based Method

In our method, the transformation is described by a fixed frame. Through the idea of differential gemometry, transformation can also be described with moving frame or local frame.

Some invariant form is easy in local frame and symmetry direction can by decoupled in local frame. this will provide us some more types of symmetry which are hard to find in fixed method.

one example is the nohonomoilic dynamic system, which are not discussed in our research.

9.2.2 Integrable System

entrainmet and CPG only get the structual stability through emerical method. For application use, we can use an analysoub but stable sytem instead of the original system.

A different idea is we start from a structural stable dynamic system and the modify it to approximate the cpg. some idea for this maybe the integrable system and perturbation theoy.

by substitute the cpg with an disturbed integrable system may provide better

9.2.3 Symmetry of PDE

we are very interesting in expand our method to the pde domain, if so, we can simulate motion involves fluid and elestic object.

9.2.4 System Transform and Muscle Actuation

One question we have not touched in our research is the muscle system.

in our research, our method is based on transform the state space. this will result in a control method that need to accurately feedback the current state.

while a different idea is transform the dynamic system. If the system is of High Dof, when we can achieve the same effect as controlled symmetry by modifying the system's parameters.

for the simple mass spring system. off set can implemented by chaning the rest length d speed action can be implemented by chaning the stiffness K and energy scaling is met by the dyanmics, control can be achieved by ajust the stiffness K and then restore it.

this method will get rid of the need of feedback and extral control effort, which is more realistic to natual motor results. In this point view, an controllable spring is avatange for energy shapping method.

this may give us some insight into our body's muscle system. A further hypothsis muscle are not used to generating force, treat them as parameters that transfrom th system they can be seen as an advantage.

also when symmetry is applied, muscle system will change in an uniform manner. this is the same with the biological idea of muscle synergy.

we try to incoporate muscles in the the following research, current problems is include more muscles will include more parameters and complicate the dynamic simulation.

9.2.5 Perception Dynamics

human motion perception is an high level capacity, basecall it is based up our object recoganition ability and our dynamic reasoning ability. When dynamic motion synthesis, we also touch the question of dynamic perception and encoding problem in intelligence.

It is not very likely that the neural system encode the dynamic system in the form of symbolic differential equation. while our approach in motor control provide two ideas for modeling dynamics in our neural system.

the first is qualitative approach. Neural system may not need to encode the details of dynamic system, neural system can form an analous dynamic system in our brain that is analogys to the real dynamics. Such model will like the details accuracy, but get the qualitatives properties right.

Another idea is we can remeber behaviour of one dynamic system, and appling transformation when we econter a different dynamic system.

we are not still not sure which method is better, but both of them are better than solving the differential equaiton in our brain. We proprose maybe a new dynamic simulator can be designed by this principles, if neural system encode the dynamic in this way, we can expected that such kind of simulator will result in belieable motion and are more intutive for animators to use.It can be aslo be used as and tools to biological research.s

9.2.6 Rethink abouth the uncanny valley

For computer graphic research, uncanny valley is the central problem. Currently little is known about over comming the valley mainly because we don't yet know the mechanism behind. although it is wiered in our research to apply to motor invaraint in our research, our research may provide some insight in to the uncanny valley.

we propose that the uncanny phenomenon is because there is a switch of perception mechanism in human perception. if we meet something not familar, our perception mechism is based analogy, which we only care about the qualitative properties,as long as the qualitative properties of it is similar to some qualitative propers of our known, we think it "belieable". This is extendion of our idea of Global Invariant.

if something is very like to something we know, we used a different perception mechanism, which is based on our experience. Maybe we will begin to analysizing some quantative properties and measure the difference between the observed and our experience. this can be seen as an extension of our idea of local invariant.

When the object is becomming more and more like in our experience, there is a switch in the perpection mechanism, the origianl qualitative analy ones become very poor alike in the quantative approach, thus wil result in the valley.

This idea is just a propose and need further proof and experiment. but we find it interesing and if so, maybe we will move into a new era of graphic research.

Appdx A

and here I put a bit of postamble ...

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