## Local Motor Invariant

## Introduction

Global Motor Invariant keeps the topology motion primitives.

For animal motion is also of high accuracy.

In this chapter we focus on the control on the quantitative properties of motion.

In our research, we try to limit the computational cost.

The discovery is that motion of natural system will change in a uniform way.

The method we proposed exploring the symmetry properties of dynamics system.

The symmetry property of a dynamic system is called local motor invariant.

The method we propose is based the lie group theory.

### Group and Symmetry

Symmetry in geometrical sense means when you transform an shape, the transform one and original one are exactly the same.

For the square examples, rotation it by 90 degree will make it exactly the same with the original one.

All the action that can preserve the symmetry is defined as the set as group.

A group has the following properties.

If g1 ,g2 in G

g1\*g2 in G

g1 \*g2=e;

In algebra, a shape can implicitly defined by an function I(x)=0;

The group transformation is define by x’=g(x),

If symmetry is met, we have I(x)=I(g(x)).

We can say I(x) is an invariant function of group G.

We have to note that f(x) is not only the shape unchanged by g.

In fact ,many shape is invariant, and form a space, the invariant space.

In fact, we can pick up and two shape and combination is also an invariant shape.

## Lie Group and Symmetry of Dynamic System

Lie Group is continues group original from study of differential equation.

For Physically-based animation,

Motion is usually described by the differential equation (1)

(1)

\dot{X}=F(x,u)

Physically possible motion is the solution of the equation.

An important property from one solution x, with a group action g, we ca get another solution x\_a

(2)

x\_a=g\_a(x).

for example

,

We have

So the group action is

For equation (1), the group action g\_a satisfy the symmetry property (2).

(3)

This provide us an idea about motion synthesis.

Given an original motion m, and the corresponding group g, a new motion is generated by g(m).

For every group G, we can find an function I(x) unchanged by the group action G,

I(x) are called local motion invariant.

For mechanical system, I(x) has important physically meaning.

I(x) corresponding to the Conservative Law like energy or angular momentum.

## Controlled Symmetry

For motion synthesis, usually the desired motion is ma

For example for motio stability, we want the current state is within the basin of attraction.

If we want to control the final motion style, we want the state is on the limit cycle.

For motion sysnthesis, the problem is given the system, let the original system have the desired symmetry.

and original motion m is known, but the corresponding group action g\_a is not satisfied by differential equation.

For such situation, control input u is added, which modify the original equation to allow the designed G, this is called Controlled Symmetry.

Most dynamic motion can be modelled as an Lagrange System.

L=K(\dot(q)-V(q).

And the desired action G must keep the L invariant.

The original m is defined by the eural langrage equation

(4)

\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q} = 0

The modified system is

(5)

\frac{d}{dt}\frac{\partial L}{\partial G(\dot{q})}-\frac{\partial L}{G(\partial q)} = 0

Which is equal the controlled dynamic system

(6)

\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q} = u

(5) and (6) are the equivalent equation, by comparing equation (5) and (6), we can get u

### Some Specific example of Symmetry and Control

### 1 Offset Action

G\_r(x)=(q+r,\d{q})

Which keep speed, but modify the pos. thus keep the K but modify V

u=\frac{\partial V(g\_r(q))-V(q)}{(\partial q)}

on phase space, if q is the horizontal axis, and \dot{q} is the vertical axis, this has the effect of moving the phase plot right and right.

## 2 Time Scalling.

g\_st(q,dot{q})=(q,st\*dot{q})

we have

u=(st^2-1) \frac{\partial V(q)}{(\partial q)}

on phase space, this has the effect strength the phase plot in the vertical direction

## 3 energy scaling

For some system moving the the conservtime field with constant mass matrix.

The energy is preserved and different motion present different level of energy.

For such system, we have the

For such

g\_e(q,\dot{q})=(e^2\*q,e\*\dot{q}).

U can be developed by applying the pos scaling and time scaling in a combined manner.

On phase plot, this has the effect enlarge the phase portrait.

## 4 time offset

Is q(t) is solution to f(q)

g\_{toff}(q(t) \dot{q}(t))=(q(t+toff),\dot{q}(t+toff))

For dynamic system, this seems obvious. And no control is need for such symmetry.

For system with limit circle, this g\_toff has a special effects like phase modification.

On phase plot, this has the effect rotate on the limit circle about an angle.

Simple Example:

Bouncing Ball

The bouncing ball system has a energy scaling symmetry, if bouncing height is q(t), \dot{q(t)}

The new system is

Eq(t) \dot{e^2q{t}}

Chapter 5 Motion Synthesis Based Motor Invariant

Chapter 3 and Chapter Discussion some important idea of natural motion control properties from the mechanical viewpoint.

In this chapter ,we will dicuss how we combine the idea from Chapter 3 and Chapter 5 into meaning full motion synthesis System.

Mainly we will discuss two question,

1 how global and local motor invariant controller work together.

2 how combine different motion primitives together.

Combined Global and Local Motor Invariant

1 motor control idea based on Global and Local Motor Invariant.

Neural Oscillator will maintain the qualitative motion properties, and Controller Symmetry will satisfy the quantitative properties.

Basically we need apply the qualitative controller first to maintain the topology against the structural perturbation,

when Symmetry Controller is applied to transform the entrainment System to meet some specific user constraints.

Simply put, we should get the qualitative write first, and then get the quantitative write .

This idea is straightforward is illustrated in the following example

but when applying this method, we have condition must be met.

1 unlike the CPG controlled example discussed in Chapter 3, when must maintain the topology when the symmetry controlled is applied.

To met this condition, we must prove that Symmetry Controller will not violate the topology, and symmetry control is applied to the how system rather than the original system.

We must prove that Lie Group Operator will not violate the topology

2 In chapter 4 we discuss the controlled symmetry is applied to the original system, how ever, for motion synthesis, we must prove that the combined system should preserve the symmetry.

If the original system $F(x)$ have the symmetry property, we also require the neural oscillator have the same symmetry properties.

Thus we need to discuss how to transform the Neural Oscillator so the neural oscillator have the same kind of symmetry as the mechanical oscillator

1 Controlled Symmetry Preserved The Topology

Theorem Controlled Symmetry is Topology Preserving

suppose the original system is $\dot{x}=F(x)$, Controlled system is $\dot{G(x)}=F(G(x))$,

proof :

by the demotion mation, G is continues and one one mapping.

The the the original system and transformed system are isotropy.

Thus Controlled Symmetry is topology Preserving

2 Symmetry of Neural Oscillator.

We separate the discussion

after coupling with neural oscillator, the $\dot{x}=F(x)$ becomes a system

\dot{x}=F(x)+u\_in$

for controlled system, when must maintain

\dot{G(x)=F(G(x)+u\_ing$

for the neural system

the the dynamic equation is

$\dot{xc}=S(x\_c)+uin$

we requires te symmetry must meet

$\dot{Gxc)=S(G(x\_c)+u\_ing$

the out put function is

O(x\_c)

for the symmetry property, we must statisfy the following following follow equation

$u\_inG=O(x\_cG)$

$u\_inG=O(xg)$

For must our research, we find that we only adjust the three parameters of the neural oscillator to meet equation above

$\tau,hi,ho$

and some examples are included.

1 offset Symmetry.

For offset symmetry, we carefully select the U\_in and U\_out acting on the Invariant variables.

For example ,when walking slope changes, we choose the input of the neural oscillator to be the angle between the joints or velocity difference,

then all the input function will not change, if the output of the neural oscillator is apply not directly the q, but to the difference of q of velocity of q,

then the out put function can is also unchanged.

2 Time Scaling

Neural Oscillator can change its Speed by changing ts.

We prove change ts, we can maintain the same is maintained.

For the dynamic system, usually control will affect the second order derivative.

Then we can u needs to be ts^2 the original system.

So we can multiply hout=ts^2 hout

3 Energy Scaling

energy scaling is G(x)=(G(q),G(\dot{q})

q=sq,

\dot{q}=t(s)q

it is equal to apply the scaling of the configure q and time scaling sequentially.

For our research, we apply the speed action to the neural oscillator first,

when scaling the h to keep the input keep constant.

4 time offset

Neural Oscillator don't have to do any thing with the state,

but it also have to rotate its state.

A simple example

Bouncing Ball

we prove the

Combine Motion Primitives

The Motion Primitives Graph

Motion Primitives comes from the original mechanical system, motion primitives can only transformed if they are neighbours.

Following this idea, given a dynamic system we can draw a graph of motion primitives and this is called motion primitives graph.

The idea is very similar to the motion graph, the difference here is in the original motion graph are handed crafted,

while in our research, we propose that a motion graph of a dynamic system is fixed, at from any motion primitives, the way he can change its motion is also limited.

The Motion Primitives Transition

From dynamic point of view, changing motion primitives is put the current state in the basin of attraction of one attractor into the basic of attraction of another attractor.

As show in picture, for the uncontrolled system, the transition will not happen automatically, for the two basic of attraction will not overlap.

To put the state x in basin of attraction, have two methods.

1 Overlaping Method.

The first idea is use different CPG for different motion primitives. There is a switch mechanism of switching the CPG.

If cpa a is applied to the fx, then the basic off attraction is enlarged, if cpg b is applied to motion primitives b,

the basin of attraction is also enlarged.

If a system is at state that within the both basin of attraction, we can switch the CPG controller.

2 Transform Method

Controlled Symmetr can also applied for motion state transition.

For a system at x , we can transform the phase portrait to make it which in the basic of attraction of b,

this is illustrate in figure

Both the method can result in physically realistic motion transition, when the current state x lies a a special position.

The what more problem happens when we don't know is where the current position x and where it is going.

Thus we combined the two method above to achieve for a combined method,

no matter where is current point is, it is going to converge to the limit cycle, thus we ask both basic of attraction cover the limit cycle, this can be achieved via using both the CPG and Transformation.

And for this usually ,both motion primitives needs to be transformed,

And the there is relationship between the two transformation, this relation ship is called transformation connection.

Figure download show the illustrate the idea.