# Adaptive Motion Synthesis and Motor Invariant Theory

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Abstract

Generating natural-looking motions for virtual characters is a challenging research topic.

It becomes even harder when generating adaptive motions interacting with the environment.

Current methods are tedious, cost long computational time and fail to capture natural looking features.

This report proposes an efficient method of generating natural-looking motion based upon a new motion control theory.Inspirations for this research come from the contradiction between biological facts and current motion synthesis ideas.

The principal idea is natural-looking motions mainly come from the complex interaction between the body and the environment.

The natural neural system only maintains or tweaks qualitative properties of this dynamic interaction.

We believe that motion is composed of many motion primitives, each motion primitive is a \textbf{structural stable autonomous system}.

Motion and Adaptation can be generated without any control effort.

The mathematical model we propose for the natural motion is based on the Qualitative Theory of Differential Equation.

Qualitative Control is achieved through manipulating the topological structure of the dynamic system to enhance the `` self-balance'' ability, rather than counteracting the perturbation effects.

For the quantitive motor, the neural system apply the symmetrical properties fo the dynamic system, which can transform one motion into another motion.

The control method we propose is well supported by biology research.

Adaptive Motion Synthesis Method completely solving the motion reuse and retargeting motion.

Motion data can be directed applied for a different character in a different environment.

Also this method involves with very light computational load.

Further, it maintains important features of natural looking features.

INTRODUCTION

Character Motion Synthesis (CMS) research aims at generating motions for virtual characters.

It is a valuable topic for both industry and academic community.

Main applications are in the media industry, both computer games and animation films depend heavily upon character motions for storytelling.

CMS also has many applications in other areas, such as user interface design, psychology, sports and medicine.

The challenge of CMS research is not to make characters move, but how to make them lifelike.

This challenge comes from our human's marvellous ability of motion perception.

Motions for the same task are very similar, but vary adaptively.

From the variety in motion details, humans can infer the changes in mental states, health conditions or even the surrounding environment.

Nowadays in industry, high quality motions are majorly generated by manual work.

For every joint of the character, animators specify a series of positions over the motion time.

Corresponding mathematical model is the parametric trajectory with joint position as the value and time as the parameters.

In applications, most characters are very complicated and contain a large number of joints, making animation a tedious work.

Making things worse, it is difficult to reuse motion animation.

Reusing motion animation for a different scenario is prone to artefacts.

For this situation, high level animation tools are badly needed.

It is believed that motion can be synthesised by simulating the dynamics of body and environment and the functions of neural control system.

With the advance in Mechanics Simulation, current research is trying to make virtual characters dynamically interact with their environment and adapt motions realistically.

Typical research topics include locomotion, object manipulation and posture control.

However the complexity of body structure proposes many problems for simulation methods.

For example, the human skeleton is made up of more than 200 bones driven by more than 600 muscles.

From the mechanical viewpoint,

this mechanical system is full of redundant \textbf{degree of freedom} (DOF)s, which not only increase the computational load, but also make the solution nondeterministic.

For a specific motion task like picking up an apple, there exist many different ways of arm motion.

Only a few ways seem natural, and the believable motions vary adaptively for environmental or inner reason.

1 the problem of motion perception and motion synthesis

How animals move is an fascinating question.

Which is old but still questions the nowadays people.

Besides our old fancy dream, it is also of great important in nowadays.

Advance in this topic will greatly influence the biology, robotic engineering and

Also intelligent research.

It is also of one of the most challenging question in computer animation research.

We are so familiar with motion of human and animals; we can easily identify the artefacts of motion but still don’t have an idea of how motion is synthesized.

Across the whole discipline, motion synthesis or motor control in biology terms is full of paradoxes.

Before going into details into the research ideas, we first review some fundamental questions in motor controller.

1. The first problems comes the dof. Unlike the artificial system, bioligcal system has a much more degree of freedom. Artificial ship is a fixed rigid body, while biological fish has much more dof than ship, the vebrae is quite flexible, and has many degree of freedom than it artifical component. In principle, we know that more degree of freedom will make the animals generate more motion and adaptive to the environment, but from the control view port, such degree of freedom propose a challenging problem to the control system, for human, how can the biological control system control more than 200 bones and 600 muscles to finish just one step walk.
2. The second problem comes from you dexourity of our motion ability.

Human can carried out many motion task than the artificial system.

Beside the walking ,swimming and object manipulation which are treated as motor motion task, the feeding, breading, language, vision also involves lots of motor control. Besides of all these, human can also utilize a large of artificial tool, driving a car ,skate, and cycling, and play tennis, all these ability depends on the motor control.

Human and animals exhibit a variety of motion tasks which will astonish the most artificial system

1. The third problem of motor control is our motor perception system. If we acquire many motion tasks through learning, what we see is closely related to what we learn and how we learn our motion ability. A interesting phenomenon in human motion perception is motion perception is quite selectable. For some minor motion mistakes we will identify them instantly, while for some impossible motions, we don’t seem to notice the artefacts. What we see is what we learn?

An important question is if the artefacts neglected are because they are not important?

We are very familiar with motion of human or other animals,

But familiarity doe not mean well understanding, if look closely,

We will have to recognize we know little about the biological motor control,

2 rethink about motion synthesis research

For computer animation research, the key principles is we should know the things we animate, if not so we will faces lots of troubles in generating animation.

Natural motion system has many valuable properties which are not captured by our current motion synthesis system

What natural motion looks like?

1 adaptive and robust

Natural motions are adaptive to the changes in the environment or body conditions.

A common example is human locomotion.

The walking motion changes on different terrains while the balance is maintained and different perturbations will generate different reactions.

2 real-time performance

Some motions of animals are very fast, honey birds may vibrate their wings in kHz.

agility not only refers to the speed of motion,

more puzzling is that the neural system can solve the complex motion control problem in a very short time.

When an animal avoids obstacles at very high running speed,

it must continue its running motion, make a turning and keep balance at the same time.

It seems very easy for the neural system to find such complicate motion solution.

3 energy efficient

Natural Motions are energy efficient.

In theory, this idea is supported by Darwin's Theory of Evolution.

Animals spent far less energy than our expectation.

An example is that the energy consumed by human walking is only 10\% of that for a robot of the same scale.

To design a motion synthesis framework, we facing the several key problem.

1 Memory or thinking

The first question is how where we get our motion task, whether because we through memory, given some much motion details, it seems impossible for us to remember all the motion data. If motion is based on thinking our reasoning power, it will put an heavy burden on our brain.

Feedback or feed forward

The second is which control strategy plays more important role in motor control, feedback or feed forward. Most artificial or artificial control theory is feedback based, if so, human motor system must have a power sensing system are accurate but also high speed, it will means that animals move in a “ careful and nervous manner”. A different control idea is feed forward based control, the idea is if we know something will happen, we can take some measures earlier to prevent the failure of motion tasks or make the motion “easier”, like change the shoes if tomorrow will snow. feed forward control can get rid of all the sensing accuracy and speed, which our human are not good, it depends on the prediction and experience, which our human beings are fond of since the old day.

Disadvantage or Advantage

Maybe the most important problem we should rethink is the attitude toward the body structure. The body structure is the result of natural selection of millions of years, seen in this way, the body structure is heritage rather than a burden, somehow, we may think the body structure is an over powerful tool beyond our current comprehension, a complex system does not necessarily difficult to use. it is not that the nature make a big mistake, it is we don’t understand the great design yet.

1. overview of a different motor control framework

in this research, we propose a different motion synthesis system based on a different motor control idea.

We have an insightful founding is that motor control can be an easy task.

For some situation, such motion tasks can be achieved without any control effort.

By exploring the properties of the body and environment, some motion tasks are simple and easy to control. We don’t really carry difficult motion tasks, we just select many easy motion tasks that we are good at and modify them for our special needs.

Such simple motion tasks are called motion primitives, when we modify the motion primitives, we must keep some valuable properties of motion primitives unchanged, and the maintained property is called motor invariant.

1. in this paper, we propose a different motion synthesis. The inspiration of our knew method comes from a different biological viewpoint upon the biological body system and neural control.
2. a simple example

Although some obscure mathematical knowledge is needed to build the mathematical model, the principle is illustrative through some daily example. Here provides two examples about our idea of the “easy motion task” and how can we tweak them.

1. the ship example for qualitative dynamics

The first example is the ship floating on the sea, we know that the high of the ship is much larger than the width. Since it is floating on the waver water, follows the question how the ship maintains its posture.

To our surprise, we find maintain posture is a very “easy” task.

Following is the dynamic equation describing the ship’s posture dynamics.

For the ship, there are only two posture that the bouyance force is equal to the gravity. As show in figure

The two postures are different, one is attractive, if small perturbation moves the ship away from the equilibrium state, then it will moves back to the equilibrium state.

The posture 2 is repelling.

If a small perturbation moves the state of the ship away from the state,

It will move away from the state.

If we plot all the possible motion of the ship, than we know all the motion will move away from the appealing posture to the attractive posture.

Thus we come to the solution, as long as the centre of buoyancy force is above the centre of gravity, the ship can automatically control maintain its posture.

So maintaining posture of the ship is very simple “task”.

Something interesting in our analysis is that we need details information about the ship, the size, weight and design. It is obvious different ship will maintain its posture with different motion.

But Such variation will not affect the “easiness” of the maintain posture task. The “easiness” comes form the “repealing state” and “attractive state” and the “connectivity”. Such properties are “topological property”.

As long as the topology is maintained, for different kinds of ship,

Maintaining posture is “easy”.

So the topology is kept invariant when we vary the ship parameters.

Thus we have an intuitive idea, the easiness of a motion task, is captured by the topology.

In our motion synthesis frame work, the topology is called global invariant, and they encapsulate the qualitative properties of a dynamic system.

Following this idea, we propose the first idea is “easy motion task”, motion primitives should be identified by their qualitative properties, or the topology of the phase portrait.

If a motion task process some great qualitative properties, no control is need to finish the motion task.

1. the mass-spring example for symmetry.

For dynamic motion synthesis, the key question facing both the computer graphic researcher and neural researchers is how the neural system solves the dynamic problem of motion. The biological motor control is so complex; it involves chemical, neural, electrical and mechanical process. The question is seems it is such a complex task, how can neural system solves it instantly.

An alternative idea is we don’t need solve the dynamic system, we only need to know how to transform one motion into another.

This idea is illustrative in the following mass spring example

A=-kx.

Although it is very simple, this example capture some of the important properties of motor control system.

The biological motor actuation or muscle works more like spring rather than the artificial electrical motor. So in this example, mass is the model of skeleton and spring is the muscle system.

In a similar manner, we can draw the state of mass spring system on the phase plot.

If we have some knowledge of the system, like a possible curve, it is highly not likely our knowledge is not about the symbolic differential equation, rather it maybe a possible motion quirve m, or q(t).

if given a different state q, we can easily work out the motion, it is on the circle that share the center, with the same shape, but a big radius.

Or we can say the new curve is scaled curve of the original one. The properity that we can transform one motion into another is called “symmetry”.

This in this manner, the motion can caculated very easily. In fact we can eaven hide the compliexity of the mass-spring system away,as long as we know the “symmetry “ properties of a dynamic system, we can work out allow the possible motion.

The dynamic system can be encoded in a more simple manner,

A possible motion curve and its symmetrical property.

This idea have two important effects for motor control.

This will help us greatly reduce the memory and computational cost.

It also provide us with an idea of motion perception. In fact we don’t calculate the details dynamic of motion, what we do is we do the inverse of the transformation to transform the observed motion into our memorized motion, if the in the memory space, motion are similar, than we think the motion are realistic, otherwise we detect artefacts.

The motion reference curve in the memory space is called motion signature. When we transform the motion signature through exploring the properties of symmetry, some properties of motion are kept invariant, there properties related to the symmetry properties, for the mass spring system, an example is the differential invariant, that is the curvature, we can acclaim that the curvature of the motion is kept during transformation. Such invariant properties are called local motor invariant.

Global Motor invariant kept the qualitative properties while local motor invariant keeps some quantitative properties.

But developing method that can maintain the global motor invariant and local motion invariant,

We provide a method that can qualitative and quantitative properties separately. we developed a new motion synthesis method which are both easy to compute and result realistic motion.

5 organization of the paper

This paper is organized as follows.

In chaper 2 we will discussed some background of motion synthesis research and motor control research. These are the motivation of our research and justification of our ideas, in chaper 3 we will focus on the Global motor invariant. We try to identy the qualitative properties of motion and investigate how to maintain the global motor invariant based on some biological ideas.

In chapter 4 we dicuss on the idea of Local Motor Invariant and Symmetry.

Mathematical tools is developed and we developed how to maintain Local motor invariant by applying control effort. We show how the computational complexity is greatly reduced by symmetry.

In chapter 4 We discuss how the Global Motor Invariant and Local Motor Invariant Controller work together. Simple example is included to discuss the mathematical idea. Also we give an idea about how to connect motion primitives together.

Chapter 3, 4, 5 lay the theory theory foundation for the motion synthesis control and be treated as the Motion Invariant Theory.

In Chaper 6, we try to use focus on synthesis motion of one motion primitives

we apply our method for one of the most interesting and challenging motion synthesis research topic, bipedal walking. We show how our method main the stability and adaptive to different walking situation.

In Chaper 7. We will dicuss how to connecting motion primitives together.

New Motion Primitives the balancing or stancing motion primitives is developed and different symmetry is investigated. We show how different motion primitives connected together by switching the motion from stance to walk and from walk to stance.

In Chaper 8 We dicuss about extend the basic idea to more complex system, or scale our method to address system with more difficult motion tasks. Three possible idea for more degree of freedom is the reduction, mechanical coupling and ad-hoc manner. The three idea applies to different biomechanical system.

An hopefully will help us to synthesis motion we have not achieved yet.

In Chaper 9 We will discuss about some future work. After a retrospective viewpoint is we are inspired by some new question and ideas for graphics and neural science.

Backgroud

We should know the things we animate. For CMS research, the basic or most challenging comes from our misunderstanding of the biological motor control system.

Current motion Synthesis basically have three different idea.

PD controller.

Some early research applied classical control methods like PD controller \citep{Raibert1991} for locomotion synthesis.

Later research \citep{Hodgins1995} applied the same method for different tasks like running, bicycling, vaulting and balancing.

Such methods are based on simplified models which relieved the controllers from the problem of redundant DOFs,

but important motion details were also neglected.

PD need a reference Curve so the motion is not adaptive

Limit Circle

Limit Circle Control(LCC) \citep{Laszlo1996} provides an alternative method for lower energy locomotion animation.

The LCC theory has been used in explaining passive mechanics.

Compared with Spacetime optimization, LLC methods is more computational efficient method for low energy motion.

However in our perspective, the current LLC method has not exhibited its full potential power in theory.

In current researches\citep{Coros2009,Laszlo1996}, the limited circle is fixed and specified by the animators.

The control strategies are simplified as a state machine controller following a predefined limited circle.

When Limit Circle is fixed, limited circle control falls into trajectory based methods.

A prefixed limited circle can be seen as a predefined motion curve of different parameters and value function.

Fixed limited circle deprives limit circle control of adaptive motion under perturbance.

Optimization

Because of the redundant DOFs in the body structure, in most cases, there exist many motion solutions for one task.

Optimization methods have been applied to solve the nondeterministic problem.

Among all the solutions in possible motion space, the ``best'' one is chosen as the proper solution:

\begin{equation}

\label{equ:max\_select}

\arg\max\_x V(x), x\in F(q)

\end{equation}

Where $x$ is a solution in the solution space $F(q)$, $V$ is the value function specified by animators.

The function $V$ in practice depends on the application requirement.

For data-driven methods, $V$ may be designed to choose the sequence with most smooth transition.

For kinematic methods, $V$ is designed to select the posture that least violates position constraints\citep{boulic1996hierarchical}.

With some special energy cost function $V$ and constraints, kinematic methods have been used to retarget motion data\citep{Gleicher1998,Gleicher1998a}.

For dynamic methods, a reasonable value function $V$ is the energy cost.

\begin{equation}

\textbf{V}=\int\_{t0}^{t1}F\_{a}(x)^2dt

\end{equation}

where $F\_{a}$ is the active force generated by actuators like motors or muscles.

This is introduced to CMS research as the influential Spacetime Constraints\citep{Witkin1988}.

It is based on the hypothesis that the natural looking trajectory costs minimum energy.

It is related to the idea of Darwin's Theory of Evolution and the principle of Natural Selection.

In many cases, these methods produced very believable motions.

\citet{Jain2009} provides an example of locomotion;

\citet{BalanceControl} find a method for balance maintaining movement.

\citet{Liu2009} proposed a method for object manipulating animation.

One shortcoming of Spacetime Constraint is the efficiency.

Spacetime Constraint in nature is a variational optimization problem.

It takes prohibitive long time to simulate complex musculoskeletal structure\citep{Anderson2001}.

Optimization techniques like time window and multi-grid techniques are proposed by \citet{Cohen1992} and \citet{Liu1994}.

Finally, \citet{Popovi'c1999} proposed a method based on Spacetime Constraint for full body dynamic animation.

Biological Motor Control

Recently biological research have a different idea about motor control.

The theory comes from tradition artificial system are highly unlikely the idea for biological system. This is because they neglected the biological constraints.

In both CMS and biological motor control research, one most noticeable question is the computational efficiency.

More questions arise after more knowledge of the biological computer, the neural system, has been obtained.

Nowadays we can provide a detailed map of the anatomical structure of neural system of human,

and are also clear about the biochemistry behind generating and transmitting signals.

Although the mechanism behind information processing remains obscure,

some characteristics of biological information processing are well agreed.

These characteristics make optimization control methods questionable.

Here we list several major questions\citep{Glynn2003}.

Controllerbility

For dynamic CMS research, the pitfall is to tackle motor control problem in purely mechanical viewport.

In biological viewport, Motor Control is a complex process involves many chemical, electrical and mechanical effects.

Pure mechanical viewport is narrow-minded and lacks the ability in explaining the complexity of natural motion.

Many mechanical parameters,

like force, distance and angle, can only be sensed approximately,

others variable like mass and inertia, human have not direct sensing ability.

For some control variables like toque, neural system has no direct control access.

For the biological system,

value of many crucial parameters and variables are not accessible.

The Speed Constraints of Neural System.

Neural signal transmitting speed is slow; and there is a long delay between neural signal firing and force generation in muscles.

It is impossible for neural system to carry out complex computation for optimization in real-time time.

The Noisy of The mechanical apparatus.

Besides the delay and slowness, the neural signals are also noisy.

The body structure and environment are also nonlinear, noisy and time varying.

So methods that are sensitive to model accuracy are not proper for the natural neural control system.

Human’s body structure goes througth big change thought lifes

Capacity Of Information

Some people argue that motion is not computed, but we store all the possible motor control ability in our memory, then when execute an motor task, we just access the memory for the proper motor control command.

This idea may helps to drop the question of computation speed, but it faces another problem, the memory capacity. Motion varies greatly, if we store the motion in our brain, the probems is the memory capacity.

The biological research finding

Evolution

Learned or Turnned.

Besides the questions from neural science research under microscope,

there are also questions from evolution and development.

Following the idea of optimization control,

an animal living in a more complex environment and with a more complex body form must possess a much great computation power.

In the mechanical view, the dynamics of fluid environment and deformable body structure are more expensive to compute.

But most primitive life forms live in the sea and have limited intelligence.

And many animals include human exhibit complex motion behaviours at very young age, before the intelligence system is fully developed.

If we expand our view port, many complex motion abilities like breathing, heat beating and child bearing are inborn.

There is no need for learning or intelligent effort.

Also we find out that the motion style is not changed by the evolution of the neural system, after all whale swim more like fish than other mammals

Computation or entrainment

Despite the complexity of body structures and environment, the natural motor control strategy seems relatively simple, involves little computational work, and outperforms optimization methods.

The current idea of biology research is that motor control is a low level intelligent activity and can be controlled with primitive neural structure.

In many animals, the active neural structure in motor control is the Central Pattern Generator(CPG) which generates rhythmic signals.

Current biological idea is motor control based on low level sensorimotor activity rather the complex reasoning ability.

Observation and Perception

For moiton perception, we have found some result. One is that all the motion seems modified in a quite regular manner.

There is an uniform change of motion, like the speed and curvature of motion curve is close related.

Some recent research identify this as some motion properties can by modelled with affine transformation.

Reseach also propose that the affine transformation has greatly related to our vision system. This means maybe our ability to identity motion is closely related to our ability to identify object in spite of its different position in space.

Biological Motor Control Hypothesis

Uncontrolled manifold hypothesis

The observation of blacksmith's hammering motions show that even under the same conditions, the motions still vary.

An explanation is the neutral system doesn't control all the DOFs.

Some DOFs are not controlled and freely influenced by the environment.

This is the Uncontrolled Manifold Hypothesis(UMH)\citep{latash2008neurophysiological}.

In this viewpoint, the result of motion planning is not a trajectory, but a space of valid trajectories.

An important question facing the UMH is how to find the proper manifold.

Equalibrium Point Hyptothesis

Equilibrium Point Hypothesis(EPH)\citep{Feldman1986}can be seen as a further development of UMH.

This idea comes from properties of differential equations.

For a dynamic system

\[

\dot{q}=H(q)

\]

the equilibrium points $q\_{e}$ satisfy the condition $H(q\_{e})=0$.

For a stable system, over the time the state $q$ will approach to the equilibrium point $q\_{e}$ and finally stays at $q\_{e}$.

EPH suggests that what the neural systems controls is not trajectory, but the equilibrium points.

Impedance Control

Impedance Control \citep{hogan1985ica} refines the idea of EPH by providing an explanation for effects of the extra DOFs.

At an equilibrium point $q\_{e}$,

\[

H(q\_{e})=0

\]

Impedance Control proposed that the extra DOFs provide a way to control the stability and admittance of the equilibrium point $q\_{e}$.

The mathematical presentation is

\begin{equation}

H(q\_{e}+Er)=K

\end{equation}

where $Er$ is the offset error vector, $K$ is stiffness matrix or impedance.

If $K$ positive, $q\_{e}$ is unstable, characters will change his posture;

if $K$ is negative, $q\_{e}$ will be stable, posture can be maintained.

if the value of $K$ is large, the posture will be more stiff and rigid.

if $K$ is small, posture will be more gentle, and perturbations will cause a large offset error.

Neural system will tune the direction of $K$ according to the motion purpose, such as avoiding obstacles and risks.

Experiment \citep{Franklin2007} shows that the matrix $K$ has anisotropic properties.

“great idea but computationa expensive”

Morphylogical Computation

A generalization theory is proposed as Morphological Computation Theory(MCT)\citep{nishikawa2007neuromechanics,Pfeifer2005},

it has a different idea about the role neural system plays in motor control.

The idea is both the body structure and the environment play a crucial role in motor control,

they can be treated as a physical computer which solve low level control problem.

It is not necessary for neural system to plan motion in every details, the neural system only needs a strategy to utilize intrinsic properties.

For some motion tasks, basic motion patterns are generated by body and environment,

the neural systems only maintains or tweaks such motion patterns.

In biological research, important evidence of MCT comes from the fact that animal’s moves in the same environment and in a similar way have similar body shape despite their different position in the evolutionary chain.

For example, the shapes of whales are very similar to fishes.

Symmetrical idea

Tfash propose a different idea about the tweaking effects of motion.

Rather than caclualition the Stiff Matrix, latex propose a method by exploring the symmetryical properties of motion.

His idea is using Lie Group, to transform the one motion into another,

This method can achieve the same effect of impedance control but is more computational efficient.

Some proof in biontonic robotic research

Biological reaserch idea greatly inspired the engineering experiment.

Some research begin to focus on utilizing the natural dynamic and use as little control as possible. Hopefully this method will generate new and more efficient robots. And some significant result has been reported

Passive Wallking

A very important discovery is the bipedal walking can happen without any control. If walking machine have a simpler body structure, it can walk without and control

CPG in locomotion

Also the CPG is applied for robotic research, the finding results show emerically the CPG will boost the system stability and can maintain motion in unpredictable situation.

While little is now how the adjust the CPG parameters and also very little is how many CPG is necessary needed.

Symmetric based Control

The idea of Symmetry is also well exploit in Mechanical research.

For mechanical view port, Symmetry has more concrete meaning.

Like energy preserving or momentum preserving.

The idea of Symmetry is also used in control, Some techniques are called “Energy Shaping”.

Our approach can be treated as a new computation theory of the EPH.

By combine CPG based method and Symmetry based method, we find the combination is signaficiant. Basical CPG and Entrainment based method will determine the qualitativey properties of motion, or the stability of motion,

While the symmery based method provides a mean for modify the quantative properties while keep the qualitative properties untouched.

The two methods don’t conflict each other and we have the good of both sides.

Chapter 4 Global Motion Invariant

Introduction

Our intuitive idea is basic motion patterns should be “easy” to finish. In this chapter, we will try to give the “easiness” of motion primitives a more definite mathematical meaning. The qualitative properties of dynamic system is defined by can be find by the topology of the

Basic Concepts.

This section develops the mathematical conceptualization of Qualitative Control Theory.

A clear mathematical definition of adaptation will help to identify the passive stable system and identify the key factors for stability control.

Some mathematical background is needed in this discussion.

Typical discussion of dynamic system is based on the analytical based; this discussion is more in a geometrical viewport, and relies on topology concepts.

This idea is usually referred to as ``qualitative dynamics'',

This idea can be traced back to Poincare\citep{Poincar'e1899,Poincar'e1885} and recently developed by the Smale School.

Please refer to other books and lectures such as \citep{abraham1978foundations}for introduction in details.

The logical flow is as follows, motions are modelled as differential equations.

Dynamic equations can be transformed into differentiable manifold.

By analyzing the topological structure of the differentiable manifold,

we get the qualitative properties of motion.

At first basic concepts are introduced for following discussions.

\subsection{Basic Concepts}

The configuration of system is described using state value in the state space.

we represent the state of a system as a vector $q$, $M$ is the state space, which is a manifold.

The motion trajectory over time is $q(t)$.

For a dynamic system, $q(t)$ is not in an explicit form, usually in the form of ordinary differential equation.

For motion synthesis, the equations usually take the following form.

\begin{equation}

\dot{q}=F\_{u}(q)=F(q,u),q\in M

\label{eq:ode}

\end{equation}

where $u$ is the control effort.

$F$ is determined by the system's natural property.

If $u=0$, no control effort is applied.

Such systems are \textbf{autonomous systems}.

For every point $q \in M$,

$F$ and $u$ determines a derivative vector $\dot{q}$.

All the vectors over the full space of $M$ form the \textbf{vector field} $V$.

Equation \eqref{eq:ode} can be transformed into a geometry structure.

We rename the state space as phase space.

$F$ will determines a geometry structure, a differentiable manifold.

The motion trajectory can be found by apply the integral operation on the vector filed $V$.

\[

I:M \times V \rightarrow M

\]

The result trajectory is defined as \textbf{flow} $\Phi$, all the flows form another geometry structure,

the \textbf{phase portrait}, which illustrates all the possible motions of the dynamic system.

A different $u$ will result a different differential equation $F\_{u}$, thus a different vector field and phase portrait.

Thus \textbf{$u$ has a global effect}.

In classical control theory, we usually neglect effects of $u$ on the curves far way from the predefined one, thus it is local method.

An illustrative example repeatedly used in this report is the mass-spring system.

After linear transformation,

a linear mass spring system can be described in canonical form \eqref{eq:mass-spring}

\begin{equation}

\label{eq:mass-spring}

\ddot{x}+x=0.

\end{equation}

where $x$ is the position of the mass, $\dot{x}$ is the speed, and $\ddot{x}$ is the acceleration of mass.

If we chose the state variable $q=[x,\dot{x}]$, the ODE model should be

\[

\dot{q}=

\left[

\begin{array}{cc}

0 &-1\\

1 &0

\end{array}

\right]q

\]

State Space

Phase Plot

Phase Portrait

Equlibrium Point

Fix point

Limit Circle

Basin of Attraction

Intersections like fixed point and are also called \textbf{equlibria},

if we want to include the chaos, inspectional position is also called nonwandering points

At each \textbf{equilbria},

the local space can be divided into three subspace of submanifold: centre submanifold, stable manifold, and unstable submanifold.

\begin{description}

\item[centre submanifold]

If a flow $\theta$ pass through a point $m$ on centre submanifold $W\_{c}$,

flow$\theta$ will remain on the Centre Manifold

\[

\theta\_{c}(t) \in W\_{c}, t \in R

\]

An equilibria must be on center manifold.

\item [stable submanifold]

For the flow $\theta\_{s}$ passes through a point $m$ on stable submanifold $W\_{s}$, the flow will finally converge to a nonwandering point on centre submanifold.

\[

\theta\_{s}(+\infty)=\theta\_{c}

\]

\item[unstable submanifold]

For the flow passes through a point $m$ on unstable submanifold $W\_{u}$, the flow will be repelled from the nonwandering points on centre manifold.

An alternative perspective is the inverse of the flow converge to nonwandering point.

\[

\theta\_{u}(-\infty)=\theta\_{c}

\]

\end{description}

The size and dimension of each submanifold varies.

For some cases, the $W\_{s}$ ( $W\_{u}$) may not exist,

this can be seen as the dimension of $W\_{s}$($W\_{u}$) is $0$.

\textbf{Attractors} are the equilbria where the whole local space is stable, the dimension of unstable submanifold is zero $\mathbf{dim}(W\_{u})=0$.

\textbf{Repellors} are the equilibrias where the whole local space is unstable,the dimension of stable submanifold is zero $\mathbf{dim}(W\_{s})=0$.

For nonlinear system, globally, the shape of stable and unstable submanifold may be bending and connect with itself or each other.

The unstable manifold of one equilibria may be the stable submanifold of another.

The equilibra and its connectivity sub manifold form a topological structure.

Thus the phase plane will be divide into different regions,result in a cellular structure.

there is only one attractor, all the flow in this region will converge to the attractor.

and the corresponding region is called basin of attraction.

Stablility and Perturbation

Basically put, it the state is in the basin of attration then, it wil l convege to the fix point. In practicle use, unstable motion comes from two reason.

1 the state is pertubtated so it moves out of basin of attraction or qualitative property of the system is change, the basin of attraction does not exist anymore.

Such two kinds of perturbation are treated separately and result in differentiation strategy or control.

State perturbation

The first type is state perturbation, and the best method to make the system more stable is to move the state into the basin of attraction, this can be done in two ways, the first is by applying force, to change the state to make it in the basin of attration(the shooting,or feedback based method) or we can move the basin of attraction to make it include the current state, this is the symmetrical method, we will discuss it later.

Structual Perturabtion

The passive stability is the foundation of Qualitative Control Theory.

Clear description of passive stability is crucial for the proposed QCT.

A mechanical system can be extremely stable without any control effort.

A simple example is the mass-spring system with damping effects, which can be transformed into equation\eqref{eq:dampgms}.

\begin{equation}

\label{eq:dampgms}

\ddot{x}+\dot{x}+x=0

\end{equation}

For system of equation\eqref{eq:dampgms},

no matter what kind of perturbation on the initial conditions,

it will continue the rhythmic motion and will eventually stop at the position $x=0$, $\dot{x}=0$.

This kind of stability is close related to the \textbf{state adaptation}.

For the mass-spring system, this is because of the topological structure.

On the phase plane, there is only one attractor and its basin cover the whole state space.

Such ablity is defined by structural stability.Which is the desered property of moon.

Stability, Retargeting and motion Adaptation

Structural stability allow the dynamic system change it parameters

Global Motor Invariant Control

CPG and Entrainment

Neural Oscillator

Neural Oscillator Stability

A simple example.

The bouncing ball

And mass spring system

conclusion

Local Motor Control

Symmetry and dynamics

Symmetry and Group.

Lie Group Symmetry And Dynamic System

Controlled Symmetry

Offset Symmetry

Time Scaling

Energy Scaling

Simple Example:

Bouncing Ball

Chapter