Kalman Filtering for Discrete Time LTI Systems with State Dependent Packet Losses

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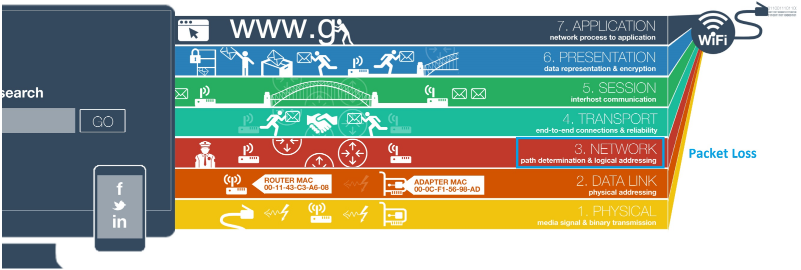
Motivation of the problem

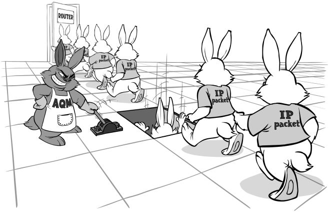
- What is a packet loss?
  - A **packet** is a unit of data, usually associated with the network layer, and transmitted as a bit stream
  - A **packet loss** occurs when one or more such packets fail to reach their intended destinations



An Open Systems Interconnection (OSI) model showing possible layers where a packet loss can take place[1]

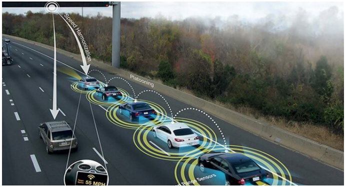
[1] *Protocol Layering And The OSI Model*, URL = `http://www.apposite-tech.com/blog/osi/protocol-layering-osi/`.

Some Applications of State Dependent Packet Losses

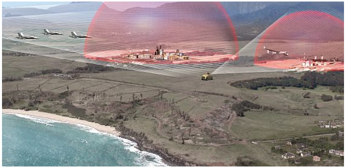


Active queue management
(Modeling internet traffic)

Aircraft tracking and Air traffic control
(Geographically separated estimators, controllers, actuators and plants)



Autonomous highways
(Wireless Networked Control Systems)



Systems in sensor denied/hostile environments
(Systems operating in sensor denied or hostile environments)

State-of-the-Art in the Literature

Methods of handling packet losses in the literature

1. *Delay exceeding some d^* is treated as a packet loss*[a,b]*:*
   - *Deterministic, constant but unknown, time varying measurement delay*
   - *Delay addressed as a stochastic variable*
2. ∞ *delay occurring according to some probability*[c]
3. *Gilbert-Elliott model as a two-state Markov process*[d]

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[a] **wang2010kalman**.

[b] Minyong Choi et al. "State estimation with delayed measurements considering uncertainty of time delay". In: *Robotics and Automation, 2009. ICRA'09. IEEE International Conference on.* IEEE. 2009, pp. 3987–3992.

[c] Bruno Sinopoli et al. "Kalman filtering with intermittent observations". In: *IEEE transactions on Automatic Control* 49.9 (2004), pp. 1453–1464.

[d] Ling Shi, Michael Epstein, and Richard M Murray. "Kalman filtering over a packet-dropping network: a probabilistic perspective". In: *IEEE Transactions on Automatic Control* 55.3 (2010), pp. 594–604.

These do not address the state dependence of packet arrival process, which is the focus of this work.

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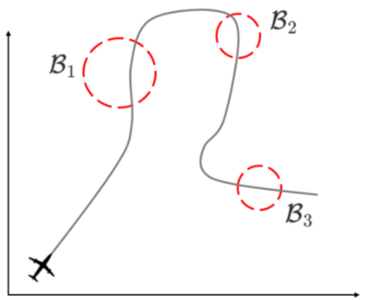
Objectives

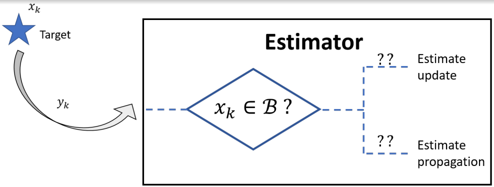
- Discrete time LTI system subject to *state dependent packet losses*.
- Losses occur in a countable number of finite 'bad regions'
- An exhaustive list of all such regions, and *a priori* belief about their spatial distributions is known
- The problem of state estimation for such a system is called the **state dependent packet loss problem**



Example: Aircraft tracking scenario with the trajectory subject to 3 radar jammers

Challenges

- Packet arrival process statistics do not have a stationary distribution
  - Since $\Pr(\mathit{packetloss})$ changes with state x_k, which varies with time k
- 'Optimal' estimator must decide at each iteration whether to perform state estimation or estimate propagation
- Numerical computations of packet loss probabilities are required when the estimator 'decides' to use the available information, or discards it
- Proving stability for such systems with multiple observation models is non-trivial, even for the stationary Markovian case

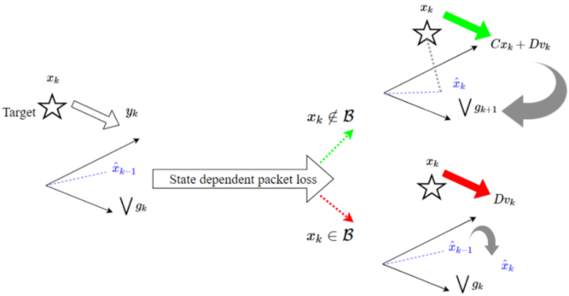


Basic idea of the estimator for non stationary packet arrival process

This thesis attempts to provide answers to these problems

Contributions

- Formulating state dependent packet loss problem, and the Markovian sensor malfunction a special case of the above
- Finding the MMSE estimator for the formulations above using Projection based methods
- Demonstration of the derived estimators for practically motivating scenarios



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System under consideration

System dynamics

- Discrete-time linear time invariant dynamics:

$$x_{k+1} = Ax_k + Bw_k$$

- continuous state: $x_k \in \mathbb{R}^n$, process noise: $w_k \in \mathbb{R}^p$
- system matrix: $A \in \mathbb{R}^{n \times n}$, process noise matrix: $B \in \mathbb{R}^{n \times p}$

Measurement model

$$y_k = \begin{cases} Cx_k + Dv_k, & \text{if } x_k \notin \mathcal{B} \qquad \text{(nominal measurement)} \\ Dv_k, & \text{if } x_k \in \mathcal{B} \qquad \text{(state dependent packet loss)} \end{cases}$$

- measurement: $y_k \in \mathbb{R}^r$, measurement noise $v_k \in \mathbb{R}^q$,
- measurement matrix: $C \in \mathbb{R}^{r \times n}$, measurement noise matrix: $D \in \mathbb{R}^{r \times q}$

Assumptions

- The process and measurement noises w_k and v_k are assumed to be i.i.d. Gaussian random vectors with zero mean and identity covariance
- The regions $\mathcal{B}$ corresponding to state dependent packet losses are assumed to be pairwise disjoint, i.e., $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset \; \forall i = j$
- The individual regions $\mathcal{B}_i$ that comprise $\mathcal{B}$ have spherical which are defined by the boundaries:

$$\mathcal{B}_i(x_k) \equiv h^i(x_k, \theta^{(i)}) = \|x_k - \theta^{(i)}\| \leq r_i, \forall i \in \{1, 2, \ldots, N\}$$

- The center of each individual region $\theta^{(i)} \in \mathbb{R}^n$ is a Gaussian random vector with a distribution $\theta^{(i)} \sim \mathcal{N}(\bar{\theta}^i, \Sigma^{\theta^{(i)}})$ and assumed to be i.i.d. with $v_k, w_l \, \forall \, k, l$

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Problem Formulation

Measurement equation

$$
\begin{aligned}
y_k &= C_{\sigma_k} x_k + D v_k \\
\text{where } C_{\sigma_k} &:= \mathbb{I}_{\mathcal{B}^c}(x_k)\, C \\
&= \left(1 - \sum_{i=1}^{N} \mathbb{I}_{\mathcal{B}_i}(x_k)\right) C = \left(1 - \mathbb{I}_{\mathcal{B}}(x_k)\right) C
\end{aligned}
$$

Here $\sigma_k \in \{1, 2\}$ is the discrete state, or *mode*, which takes discrete values according to the following state dependence:

$$
\sigma_k = \begin{cases} 2 & \text{if } \exists j \in \{1, 2, \cdots, N\} \text{ such that } \mathbb{I}_{\mathcal{B}_j}(x_k) = 1 \\ 1 & \text{otherwise} \end{cases}
$$

$\sigma_k = 1$ corresponds to a nominal sensor behavior, while $\sigma_k = 2$ to the occurrence of a state dependent packet loss

- MMSE continuous state estimate $\hat{x}_k$:

$$\hat{x}_k := E[x_k|\mathcal{I}_k]$$

- Discrete state estimate $\hat{\sigma}_k$:

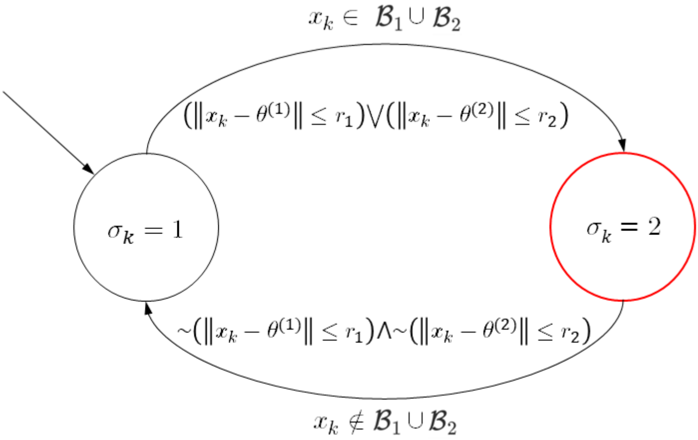
$$\begin{aligned}\hat{\sigma}_k &:= \begin{cases}1 & \text{if } \Pr\left(x_k \notin \mathcal{B}|\mathcal{I}_k\right) \geq \Pr\left(x_k \in \mathcal{B}|\mathcal{I}_k\right)\\ 2 & \text{if } \Pr\left(x_k \notin \mathcal{B}|\mathcal{I}_k\right) < \Pr\left(x_k \in \mathcal{B}|\mathcal{I}_k\right)\end{cases}\\ &= \underset{i}{\operatorname{argmax}}\left(\Pr\left(\sigma_k = i|\mathcal{I}_k\right)\right)\end{aligned}$$

- Where the information available to the estimator at each step is:

$$\mathcal{I}_k := \left\{ y_0, y_1, \ldots, y_{k-1}, \quad \underbrace{\mathcal{B}, \{\bar{\theta}^j, \Sigma^{\theta^{(j)}}\}_{j=1}^{j=N}}_{\text{new information used in this work}} \right\}$$

Problem Formulation

The estimation problem can now be written as a linear system with a two-state measurement model:



Estimator Derivation — Continuous state

The key steps & concepts involved in the estimator design are:

- Create the measurement stack available at time k as $g_k = [y_0^T, y_1^T, \cdots, y_{k-1}^T]^T$
- The MMSE estimate is found by projecting the state x_k on to the linear span of g_k as:

$$\hat{x}_k := \mathcal{P}_{\mathcal{M}_k} x_k = \mathcal{R}_{x_k g_k} \mathcal{R}_{g_k}^{-1} g_k$$

  - $\mathcal{M}_k \equiv \bigvee g_k, \mathcal{R}_a = E[aa^*], \mathcal{R}_{ab} = E[ab^*], \mathcal{P}_{\mathcal{Z}} x \equiv$ orthogonal projection of x on to $\mathcal{Z}$

- The error covariance evolution, given the orthogonality assumptions of the noise from the state, is:

$$E[\tilde{x}_k \tilde{x}_k^*] = \mathcal{R}_{x_k} - \mathcal{R}_{x_k g_k} \mathcal{R}_{g_k}^{-1} \mathcal{R}_{g_k x_k}$$

Estimator Derivation — Continuous state

- The state dependence of σ_k makes the calculation of $\mathcal{R}_{g_k}$ in $\hat{x}_k = \mathcal{R}_{x_k g_k} \mathcal{R}_{g_k}^{-1} g_k$ intractable since x_k is no longer independent of σ_k
- $\because$ true states x_k & σ_k are unknown, the problem is solved by first estimating most probable discrete state $\hat{\sigma}_k$, and applicable measurement models $\hat{C}_k$
- This *averages out the uncertainty in the measurement model*, using previous discrete state estimate
- Then the observation covariance is given by

$$
\begin{aligned}
E\left[y_i y_j^T | \mathcal{I}_k\right] &\approx \sum_{n,m=1}^{2} \left(\hat{C}_i E[x_i x_j^T | \mathcal{I}_k] \hat{C}_j^T \middle| \sigma_i = m, \sigma_j = n, \mathcal{I}_k \right) \\
&\qquad \cdot \Pr\left(\sigma_i = m, \sigma_j = n | \mathcal{I}_k\right) \\
&= \hat{C}_i E\left[x_i x_j^T | \mathcal{I}_k\right] \hat{C}_j^T \ \cdot \Pr\left(\sigma_i = 1, \sigma_j = 1 | \mathcal{I}_k\right)
\end{aligned}
$$

- This helps compute $\mathcal{R}_{g_k}$ using the most probable discrete state

Estimator Derivation — Continuous state

- $\mathcal{R}_{g_k} = \text{diag}\{\hat{C}_i\}_{i=0}^{k-1}\,[\mathrm{R}]_{k\times k}\,\text{diag}\{\hat{C}_j^T\}_{j=0}^{k-1} + I_n \otimes DD^T$ where $[\mathrm{R}]_{k\times k}$ is an $nk \times nk$ matrix whose ij^{th} entry is given by $\mathcal{R}_{x_i x_j}\mathrm{Pr}\left(\sigma_i = 1, \sigma_j = 1 | \mathcal{I}_k\right)$
- $\mathcal{R}_{x_k g_k} = \{\mathcal{R}_{x_k x_j}\mathrm{Pr}\left(\sigma_j = 1, \sigma_k = 1\right) | \mathcal{I}_k\}_{j=0}^{j=k-1}\text{diag}\{\hat{C}_j^T\}_{j=0}^{j=k-1}$
- This gives a **batch estimate** form $\hat{x}_k = \mathcal{R}_{x_k g_k}\mathcal{R}_{g_k}^{-1} g_k$
- Update the measurement stack g_{k+1} and express the new state estimate $\mathcal{R}_{x_{k+1}g_{k+1}}\mathcal{R}_{g_{k+1}}^{-1} g_{k+1}$ in terms of the previous one
- A **recursive estimate** is obtained using Woodbury's matrix lemma
- This gives the state estimate in terms of the *mode probabilities*:

$$
\begin{aligned}
\hat{x}_{k+1} &= A\hat{x}_k + AQ_k\hat{C}_k^T\mathrm{Pr}\left(\sigma_k = 1|\mathcal{I}_k\right)\Delta_k^{-1}[y_k - \mathrm{Pr}\left(\sigma_k = 1|\mathcal{I}_k\right)\hat{C}_k\hat{x}_k] \\
\Delta_k &:= \hat{C}_k Q_k \hat{C}_k^T\mathrm{Pr}\left(\sigma_k = 1|\mathcal{I}_k\right) + DD^T \\
Q_{k+1} &= AQ_kA^T + BB^T - \mathrm{Pr}\left(\sigma_k = 1|\mathcal{I}_k\right)AQ_k\hat{C}_k^T\Delta_k^{-1}\hat{C}_kQ_kA^T\mathrm{Pr}\left(\sigma_k = 1|\mathcal{I}_k\right)
\end{aligned}
$$

Estimator Derivation — Mode Probability Calculation

Mode probabilities

$$\Pr\left(\sigma_k = 1 | \mathcal{I}_k\right) = 1 - \sum_{j=1}^{N} \Pr\left(\|x_k - \theta^{(j)}\| - r_j \leq 0 | \mathcal{I}_k \right)$$

$$= 1 - \sum_{j=1}^{N} \Pr\left(\begin{bmatrix} x_k \\ \theta^{(j)} \end{bmatrix}^T \begin{pmatrix} I & -I \\ -I & I \end{pmatrix} \begin{bmatrix} x_k \\ \theta^{(j)} \end{bmatrix} \leq r_j^2 \middle| \mathcal{I}_k \right)$$

$$= 1 - \sum_{j=1}^{N} \Pr\left({\zeta_k^j}^T M \zeta_k^j \leq r_j^2 \right)$$

for $\zeta_k^j := [\hat{x}_k^T, {\theta^{(j)}}^T]^T$, and a Hermitian matrix M. This quadratic for of Gaussian random vectors is simplified using Pearson's approximation.

Estimator Derivation — Mode Probability Calculation

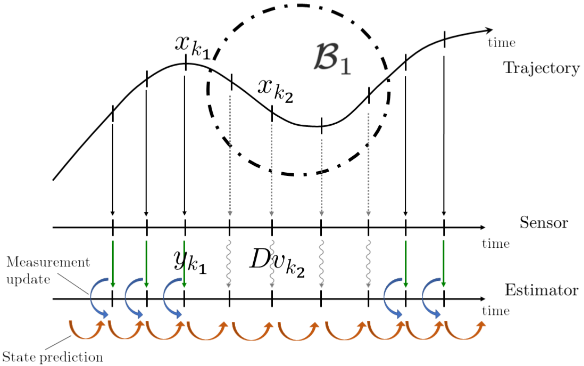
Pearson's Approximation

For a Gaussian random vector $\xi \sim \mathcal{N}(t, S)$, consider the quadratic form $\mathcal{F}(\xi) = \xi^T F \xi$ for some matrix $F = F^T$, then the following hold:

- $\mathcal{F}(\xi) = \sum_{j=1}^{n}([U]_j + [b]_j)^2$ for $U = P^T S^{-1/2}(\xi - t)$ and $b = P^T S^{-1/2} t$. $[U]_j$ & $[b]_j$ denote the j^{th} entries corresponding to the j^{th} eigenvalue λ_j of $S^{1/2} F S^{1/2}$ ($PP^T = I$, and P diagonalizes $S^{1/2} F S^{1/2}$).
- $\Pr\left(\mathcal{F}(\xi) \leq \alpha\right) \simeq \Pr\left(\chi^2_\nu \leq (\alpha - \kappa_1)\sqrt{\nu/\kappa_2} + \nu\right)$ where
$$\kappa_k = \sum_{j=1}^{n} \lambda_j^k (1 + k[b]_j)^2, \quad k = 1, 2, 3$$
$$\nu = \kappa_2^3/\kappa_3^2;$$

$$\boxed{\therefore \hat{\sigma}_k = \underset{i}{\operatorname{argmax}}\left(\Pr\left(\sigma_k = i | \mathcal{I}_k\right)\right)}$$

Algorithm



The proposed estimator performs the state estimation based update using the incoming measurements

- state prediction when it receives 'useful' information y_{k_1}
- chooses to ignore noisy measurement data

based on the probability of a state-dependent packet loss increases due to the system's trajectory entering $\mathcal{B}_1$

Algorithm

Algorithm 1: Estimator for state-dependent packet losses

Data: $A, B, C_{\sigma_j}, D, x_0, P_0, \Sigma_0, \bar{\theta}^j, \Sigma^{\theta(j)} \forall j \in \{1, 2\}$

input : y_k

output: $\hat{x}_k$

begin

$k \leftarrow 0, \hat{x}_k \sim \mathcal{N}(x_0, \Sigma_0),\ Q_k \leftarrow \Sigma_0$

$\Pr(\sigma_k | \mathcal{I}_k) \leftarrow P_0$

Filter Initialization

while $k \leq$ *runtime* **do**

$\hat{C}_k \leftarrow C_{\hat{\sigma}_k}$

$\Delta \leftarrow \hat{C}_k Q_k \hat{C}_k^T \Pr(\sigma_k | \mathcal{I}_k) + DD^T$

$\hat{x}_k \leftarrow A\hat{x}_k + \Pr(\sigma_k | \mathcal{I}_k)\, AQ_k \hat{C}_k^T \Delta^{-1}(y_k - \Pr(\sigma_k | \mathcal{I}_k)\, \hat{C}_k \hat{x}_k)$

Continuous state estimate

$Q_k \leftarrow AQ_kA^T + BB^T - AQ_k \Pr(\sigma_k | \mathcal{I}_k)\, \hat{C}_k^T \Delta^{-1} \hat{C}_k \Pr(\sigma_k | \mathcal{I}_k)\, Q_k A^T$

Error covariance update
(modified Riccati equation)

for $i \leftarrow 0$ **to** J **do**

Find Σ^{ζ} and parameters of χ^2_ν:

κ^j for $j = \{1, 2, 3\}$

$\Pr(\sigma_k = i | \mathcal{I}_k) \leftarrow$

$\Pr\left(\chi^2_\nu \leq (r_i^2 - \kappa_1)\sqrt{\nu/\kappa_2} + \nu\right)$

Discrete state estimate

$\hat{\sigma}_k = \text{argmax}_i \left(\Pr(\sigma_k = i | \mathcal{I}_k)\right)$

$\mathcal{I}_k \leftarrow \mathcal{I}_k$

Information stack update

Remarks

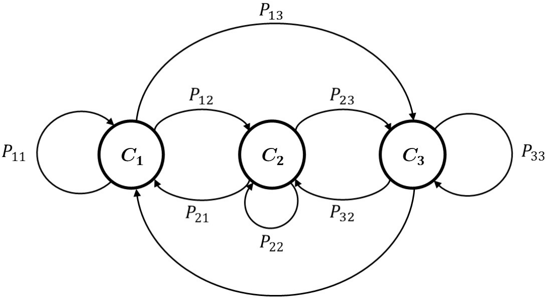
State estimation for the state dependent packet loss problem:

- The proposed state estimator for state-dependent packet loss problem has the KF structure with a stochastic Riccati equation:
  - this ensures a recursive form like the KF, and
  - the mode probabilities need to be calculated at each step
  - if $\Pr(\sigma_k = 1) \to 1$, the estimator for state dependent packet losses reduces to the KF
- The proposed estimator can effectively address the non-stationarity in the packet arrival process, unlike existing works[2,3]
- The proposed algorithm requires numerical probability calculations of the order $\mathcal{O}(N)$, in each iteration, for N number of observation denied regions in the collection $\mathcal{B}$

[2] Bruno Sinopoli et al. "Kalman filtering with intermittent observations". In: *IEEE transactions on Automatic Control* 49.9 (2004), pp. 1453–1464.

[3] Xiangheng Liu and Andrea Goldsmith. "Kalman filtering with partial observation losses". In: *Decision and Control, 2004. CDC. 43rd IEEE Conference on.* Vol. 4. IEEE. 2004, pp. 4180–4186.

Special Case — Markovian Sensor Malfunctions



Markov jump linear formulation for $J = 3$ modes of measurement equation;
$\Pi = \{P_{ij}\}$, where P_{ij} denotes the probability of going from mode i to j

- If the modes of the measurement model σ_k follow a Markov process, one can represent the system as a J-state Markov jump linear system.
- The discrete states can model Markovian sensor faults such as partial/total packet losses, multiplicative sensor degradations
  - a closed form solution for the estimator can be obtained
  - unlike the state dependent case, we can provide arguments for stability

Special Case — Markovian Sensor Malfunctions

- Due to the constant Markov state transition matrix Π, the optimal estimate can be calculated using Projection Theorem, *in a closed form*
- A recursive estimate is obtained by employing the Woodbury inversion lemma as:

$$\hat{x}_{k+1} = A\hat{x}_k + AQ_k\bar{P}_0^T\bar{\Pi}^{k^T}\bar{C}^T\Delta_k^{-1}[y_k - \bar{C}\bar{\Pi}^k\bar{P}_0\hat{x}_k]$$

$$\Delta_k = \bar{C}\bar{\Pi}^k\bar{P}_0Q_k\bar{P}_0^T\bar{\Pi}^{k^T}\bar{C}^T + DD^T$$

$$Q_{k+1} = AQ_kA^T + BB^T - AQ_k\bar{P}_0^T\bar{\Pi}^{k^T}\bar{C}^T\Delta_k^{-1}\bar{C}\bar{\Pi}^k\bar{P}_0Q_kA^T$$

This estimator is referred to as the Markovian Packet Loss Kalman Filter (MPKF).

Remarks

Markovian Packet Loss Kalman Filter (MPKF):

- If Markov state transition matrix Π is unknown, we first estimate it using Wonham filtering[4], or similar methods
- $\alpha_k = \bar{C}\bar{\Pi}^k\bar{P}_0$ captures the probabilistic switching and fuses the discrete states to obtain an equivalent measurement matrix $\bar{C}\bar{\Pi}^k\bar{P}_0$
- Exponential stability with a uniform ultimate bound exists for $k > T$, and Markov processes that attain stationary distribution at time T
- For stationary Markov processes, $\alpha_k \to \alpha^\dagger$ as $k \to \infty$
- The ultimate bound can be found by solving the discrete algebraic Riccati equation given by solving for X in

$$X = AXA^T + BB^T - AX\alpha^{\dagger*T}(\alpha^\dagger X\alpha^{\dagger^T} + DD^T)^{-1}\alpha^\dagger XA^T$$

[4] W Murray Wonham. "Some applications of stochastic differential equations to optimal nonlinear filtering". In: *Journal of the Society for Industrial and Applied Mathematics, Series A: Control* 2.3 (1964), pp. 347–369, W Murray Wonham. *Random differential equations in control theory*. 1970.

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Aircraft Tracking Scenario — State Dependent Packet Losses

- Consider a 2-D aircraft tracking scenario with $x = [\xi, \dot{\xi}, \eta, \dot{\eta}]^T \in \mathbb{R}^4$, the $\xi-$ axis pointing towards North, and the $\eta-$ axis towards East
- The nearly constant velocity model is given by:

$$x_{k+1} = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}T_s^2 & 0 \\ T_s & 0 \\ 0 & \frac{1}{2}T_s^2 \\ 0 & T_s \end{bmatrix} w_k$$

$$y_k = C_{\sigma_k} x_k + v_k$$

- Process noise w_k models small random accelerations that drive the aircraft, v_k denotes the measurement noise, and $T_s = 1s$ is the sampling time for the measurement process

Aircraft Tracking Scenario — State Dependent Packet Losses

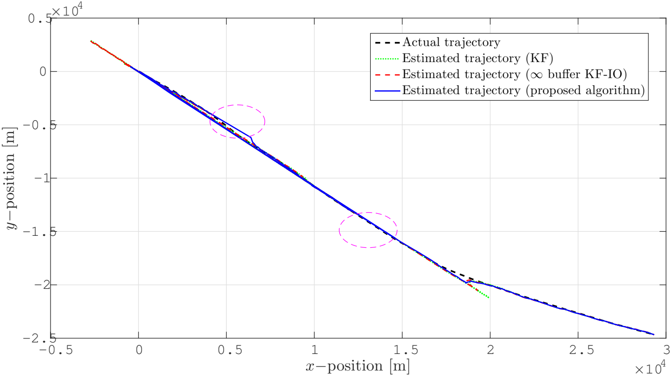
- The remaining parameters are given as: $E[w_k w_k^T] = 10 I_2$, $E[v_k v_k^T] = 10^4 I_2$, (I_n is an $n-$dimensional identity matrix) and

$$C_{\sigma_k=1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{nominal sensor behavior}}, C_{\sigma_k=2} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{packet loss}}$$

- The trajectory is subject to 2 radar jammers with radii $r_1 = 1560\, m$ and $r_2 = 1820\, m$ and $\mathcal{B}_i \equiv \|x - \theta^{(i)}\| \leq r_i,\ i \in \{1, 2\}$
- Jammer positions are given by $\theta^{(1)} \sim \mathcal{N}\left([13062.5\, m, -14867.5\, m]^T, (1.5\, m^2) I_2\right)$, and $\theta^{(2)} \sim \mathcal{N}\left([5625\, m, -4692.8\, m]^T, (1.5\, m^2) I_2\right)$
- The discrete state map is given as:

$$\sigma_k = \begin{cases} 1 & \text{, if } x_k \notin \mathcal{B}_1 \cup \mathcal{B}_2 \\ 2 & \text{, otherwise} \end{cases}$$

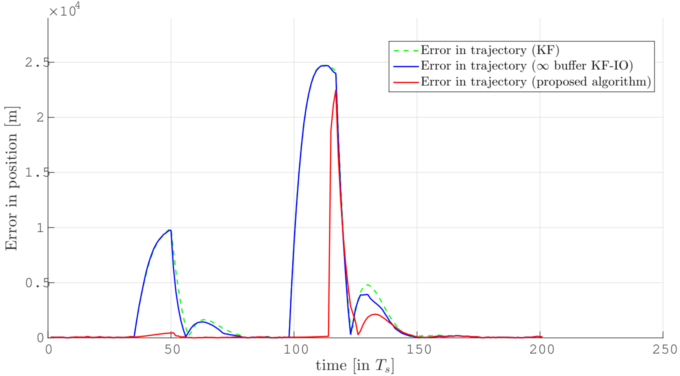
Aircraft Tracking Scenario — State Dependent Packet Losses



True and estimated aircraft trajectories for a typical run of the proposed estimation algorithm, the KF, and the KF-IO[5]

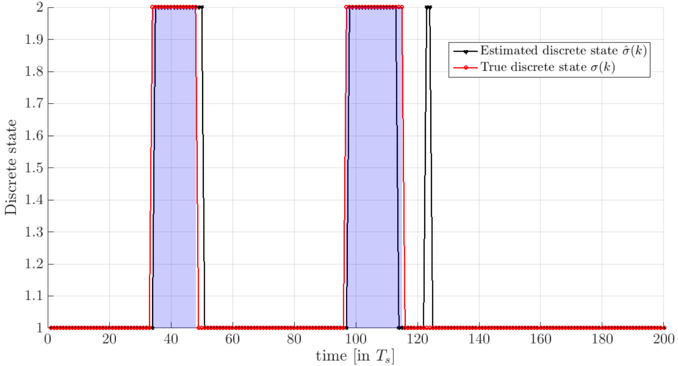
[5] Bruno Sinopoli et al. "Kalman filtering with intermittent observations". In: *IEEE transactions on Automatic Control* 49.9 (2004), pp. 1453–1464.

Aircraft Tracking Scenario — State Dependent Packet Losses



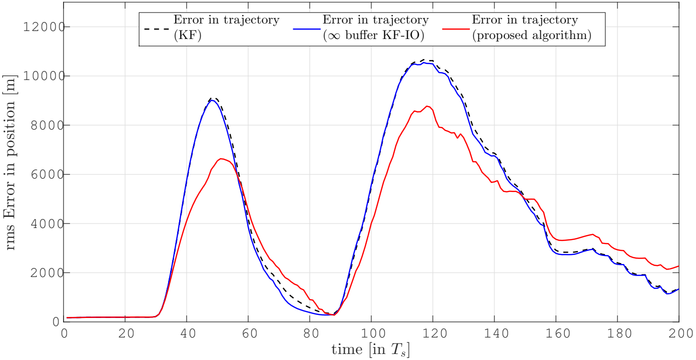
Estimation errors in the aircraft trajectory for a typical run

Aircraft Tracking Scenario — State Dependent Packet Losses



Estimated discrete state versus true discrete state for a typical run; blue regions represent the trajectory entering $\mathcal{B}_1$ and $\mathcal{B}_2$ respectively

Aircraft Tracking Scenario — State Dependent Packet Losses



Root mean squared (rms) error in the estimated position of the aircraft by the proposed algorithm, the KF, and the KF-IO, for 10,000 Monte Carlo runs

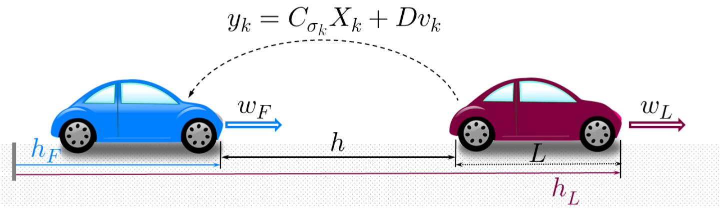
Remarks — State dependent packet losses

- KF-IO attains optimality when the packet arrival process has stationary statistics[6], which no longer holds
- The KF-IO performs almost as bad as the KF when $x_k \in \mathcal{B}$
- Since the proposed algorithm doesn't assume packet arrival to have stationary statistics, it instead 'realizes' the trajectory entering and leaving regions $\mathcal{B}$ by finding the most probable discrete state
- The evolution of the discrete state estimate $\hat{\sigma}_k$ closely follows that of the true discrete state σ_k
- The proposed estimation algorithm consistently perceives the trajectory entering and leaving the observation denied regions $\mathcal{B}$, as shown in 10,000 Monte Carlo runs

[6] Bruno Sinopoli et al. "Kalman filtering with intermittent observations". In: *IEEE transactions on Automatic Control* 49.9 (2004), pp. 1453–1464.

Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions

Consider a pair of leader-follower cars in an autonomous highway



- The combined leader-follower dynamics for a nearly-constant velocity scenario is given by $\begin{bmatrix} h_k \\ \dot{h}_k \end{bmatrix} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} X_k + \begin{bmatrix} w_L - w_F \end{bmatrix}$
- The follower measures headway distance h via a faulty sensor that can experience total packet losses, or a sensor degradation

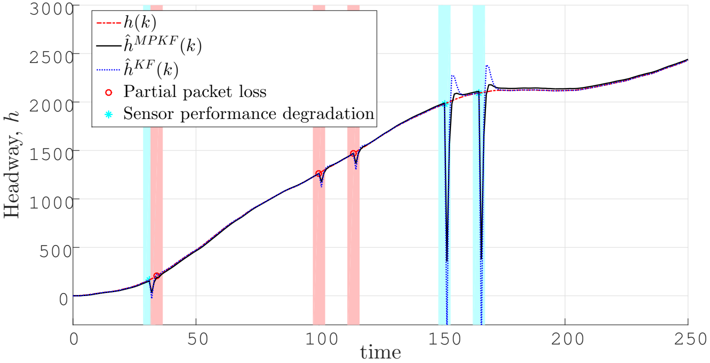
Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions

The estimator parameters are given as:

- $$\begin{aligned} C_{\sigma_k=1} &= \ \ [1\ 0] &&: \quad \text{Nominal measurement} \\ C_{\sigma_k=2} &= \eta[1\ 0] &&: \quad \text{Sensor degraded by } 1-\eta \\ C_{\sigma_k=3} &= \ \ [0\ 0] &&: \quad \text{Full packet loss} \end{aligned}$$
- Markov state transition matrix makes a packet loss occur roughly once every 50 measurements and a sensor degradation every 100 measurements (as observed from 1,000 Monte Carlo runs) as a realistic figure[7]:
$$\Pi = \begin{bmatrix} 0.9820 & 0.0120 & 0.0060 \\ 0.9940 & 0.0060 & 0.0020 \\ 0.9982 & 0.0018 & 0.0006 \end{bmatrix}$$
- Remaining parameters are $P(0) = [0.99, 0.008, 0.002]^T$, $T_s = 1$, $q_L = q_F = 0.5$, $x_0 = \hat{x}_0 = [0, 0]^T$

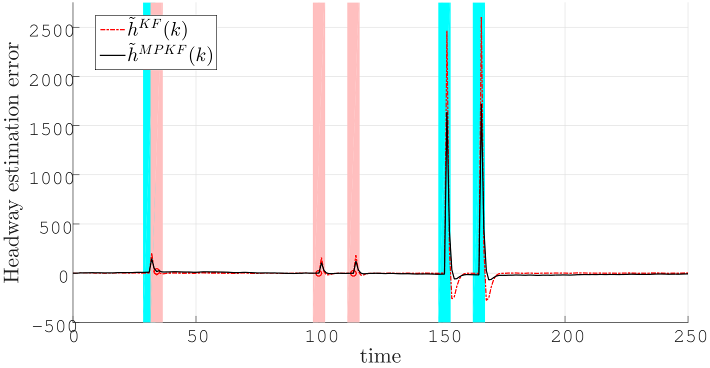
[7] Minyi Huang and Subhrakanti Dey. "Stability of Kalman filtering with Markovian packet losses". In: *Automatica* 43.4 (2007), pp. 598–607.

Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions



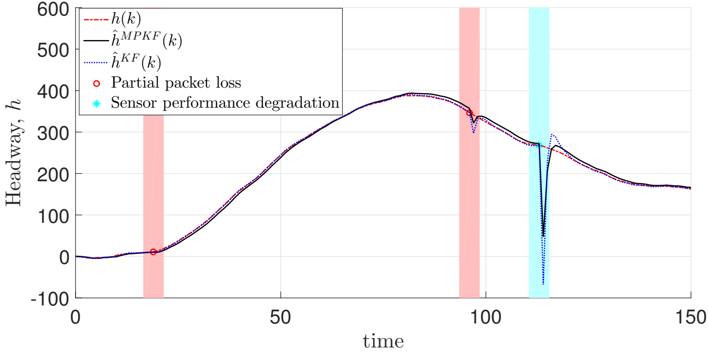
Tracking headway distance h in a V2V network – Leading vehicle gradually increasing headway h

Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions



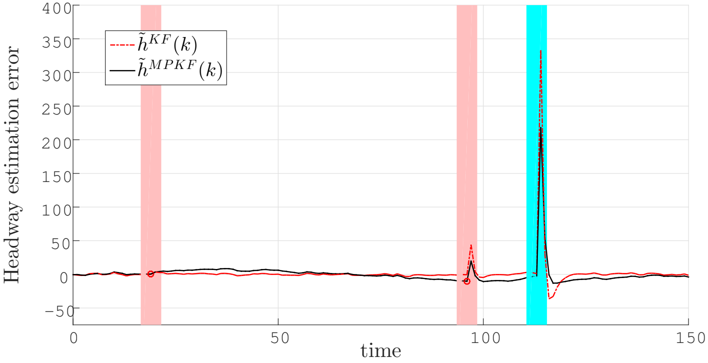
Estimation error in tracking headway h – Leading vehicle gradually increasing headway h

Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions



Tracking headway distance h in a V2V network – Leading vehicle increasing, then decreasing headway h

Aircraft Tracking Scenario — State Dependent Packet Losses



Estimation error in tracking headway h – Leading vehicle increasing, then decreasing headway h

Remarks — MPKF

- KF estimation errors show peaks while estimating the headway h during both types of malfunctions
- MPKF experiences smaller error spikes during the same malfunction events
- The MPKF recovers faster than the KF as well, with little or no overshoot in tracking the headway h
- The KF loses estimator performance during both, a packet loss event and a sensor degradation event, with a pronounced loss during the latter

Outline

- Motivation
  - Motivation of the problem
  - Applications
  - State-of-the art in the literature
- Objectives & Contributions
  - Objectives
  - Challenges
  - Contributions
- Problem Setup
  - System under consideration
  - Assumptions
- Estimator Design for State Dependent Packet Losses
  - Estimator derivation
  - Special Case — Markovian Sensor Malfunctions
- Simulation Results
  - Aircraft Tracking Scenario — State Dependent Packet Losses
  - Vehicle-to-vehicle Communication — Markovian Sensor Malfunctions
- **Conclusion & Future work**

Conclusion

- The state dependent packet loss problem is formulated as an LTI system with observation model with two operational states with the challenges:
  - non-stationary packet arrival process: handled by mode estimation using Pearson's approximation
  - finding recursive optimal state estimation: solved by Projection based approach
- This discrete state estimate is employed to get the state estimate
- A special case of the state dependent packet loss problem is presented in the form of Markovian sensor malfunctions and:
  - a recursive MMSE estimator is obtained in closed form
  - stability results and bounds are provided
- The proposed estimators are demonstrated with:
  - a vehicle-to-vehicle communication example with packet loss
  - 2-D aircraft tracking with state dependent packet loss

Future work

Related future work is to find the stability and convergence properties for the proposed estimation algorithm for state-dependent packet losses.

Additionally, we would like to test the algorithm for state estimation in a experimental sensor denied environment, in the future.

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