MMRI – Programming Stream Project

Programming with MATLAB Project – Option 1

OM PATEL (patelo7, 400378073)

Date of Submission: June 27, 2023

Problem

Given the following ODE, use MATLAB to determine whether the Euler Method or Runge-Kutta Method is better in terms of accuracy:

$$y' = 7x + y$$

Methods

Euler Method

For $y'(t) = f\big(t, y(t)\big); y_0 = y(t_0)$, using a step size of h:

- $t_{n+1} = t_n + h$
- $y_{n+1} = y_n + hf(t_n, y_n)$

Exact Solution

$$y' = 7x + y; y(0) = 0$$

$$y = 7e^x - 7x - 7$$

Runge-Kutta Method

For $y'(t) = f\big(t, y(t)\big); y_0 = y(t_0)$, using a step size of h:

- $y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
- $t_{n+1} = t_n + h$

For k_n values:

- $k_1 = f(t_n, y_n)$
- $k_2 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right)$
- $k_3 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right)$
- $k_4 = f(t_n + h, y_n + hk_3)$

Results

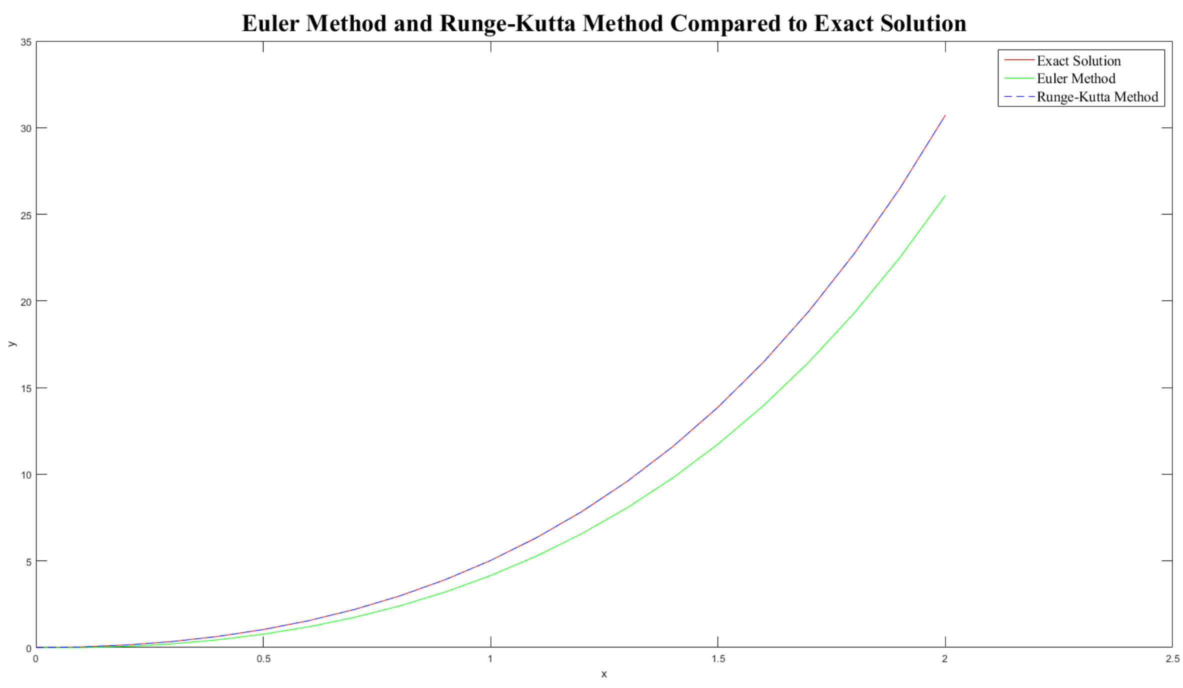

Figure 1: Using the Euler and Runge-Kutta methods to approximate an ODE and comparing it to the exact solution.

Table 1: Comparing exact y-values and Euler y-values. A step-size of 0.1 was used.

Exact y-values	Euler y-values	Percent Error
0	0	0
0.0361964265295338	0	100
0.149819307121188	0.0700000000000000	53.27705
0.349011653032021	0.217000000000000	37.82443
0.642772883488892	0.448700000000000	30.19307
1.04104889490090	0.773570000000000	25.69321
1.55483160273356	1.20092700000000	22.7616
2.19626895229334	1.74101970000000	20.7283
2.97878649944727	2.40512167000000	19.25834
3.91722177809865	3.20563383700000	18.16563
5.02797279921332	4.15619722070000	17.33851
6.32916216762503	5.27181694277000	16.70593
7.84081845915583	6.56899863704700	16.2205
9.58507667333471	8.06589850075170	15.84941
11.5863997679127	9.78248835082687	15.56921
13.8718234923665	11.7407371859096	15.3627
16.4712269707658	13.9648109045005	15.21694
19.4176317420904	16.4812919949506	15.12203
22.7475322508906	19.3194211944456	15.07025
26.5012610959549	22.5113633138902	15.0555
30.7233926925146	26.0924996452792	15.07286

Table 2: Comparing exact y-values and Runge-Kutta y-values. A step size of 0.1 was used.

Exact y-values	Runge-Kutta y-values	Percent Error
0	0	0
0.0361964265295338	0.0361958333333333	0.001639
0.149819307121188	0.149817995954861	0.000875
0.349011653032021	0.349009479437764	0.000623
0.642772883488892	0.642769680564800	0.000498
1.04104889490090	1.04104447017787	0.000425
1.55483160273356	1.55482573464353	0.000377
2.19626895229334	2.19626138617744	0.000344
2.97878649944727	2.97877694304621	0.000321
3.91722177809865	3.91720989646050	0.000303
5.02797279921332	5.02795820894616	0.00029
6.32916216762503	6.32914443041287	0.00028
7.84081845915583	7.84079707444641	0.000273
9.58507667333471	9.58505107009684	0.000267
11.5863997679127	11.5863692953481	0.000263
13.8718234923665	13.8717874094477	0.00026
16.4712269707658	16.4711844344554	0.000258
19.4176317420904	19.4175817940808	0.000257
22.7475322508906	22.7474738026825	0.000257
26.5012610959549	26.5011929120721	0.000257
30.7233926925146	30.7233133716288	0.000258

Discussion

After comparing the Euler and Runge-Kutta methods against the exact solution, it is clear that the Runge-Kutta method is superior in terms of accuracy as all entries in **Table 2** show a percent error of less than 0.01% even with $h = 0.1$ as its step-size. The y-values produced from the Euler method are significantly less accurate than those produced by the Runge-Kutta method using the same step-size.

Therefore, The Runge-Kutta method is better in terms of accuracy compared to the Euler method.