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## **MMRI – Programming Stream Project**

### **Programming with MATLAB Project – Option 1**

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## Problem

Given the following ODE, use MATLAB to determine whether the Euler Method or Runge-Kutta Method is better in terms of accuracy:

$$y' = 7x + y$$

## Methods

### Euler Method

For  $y'(t) = f(t, y(t)); y_0 = y(t_0)$ , using a step size of  $h$ :

- $t_{n+1} = t_n + h$
- $y_{n+1} = y_n + hf(t_n, y_n)$

### Exact Solution

$$y' = 7x + y; y(0) = 0$$

$$y = 7e^x - 7x - 7$$

### Runge-Kutta Method

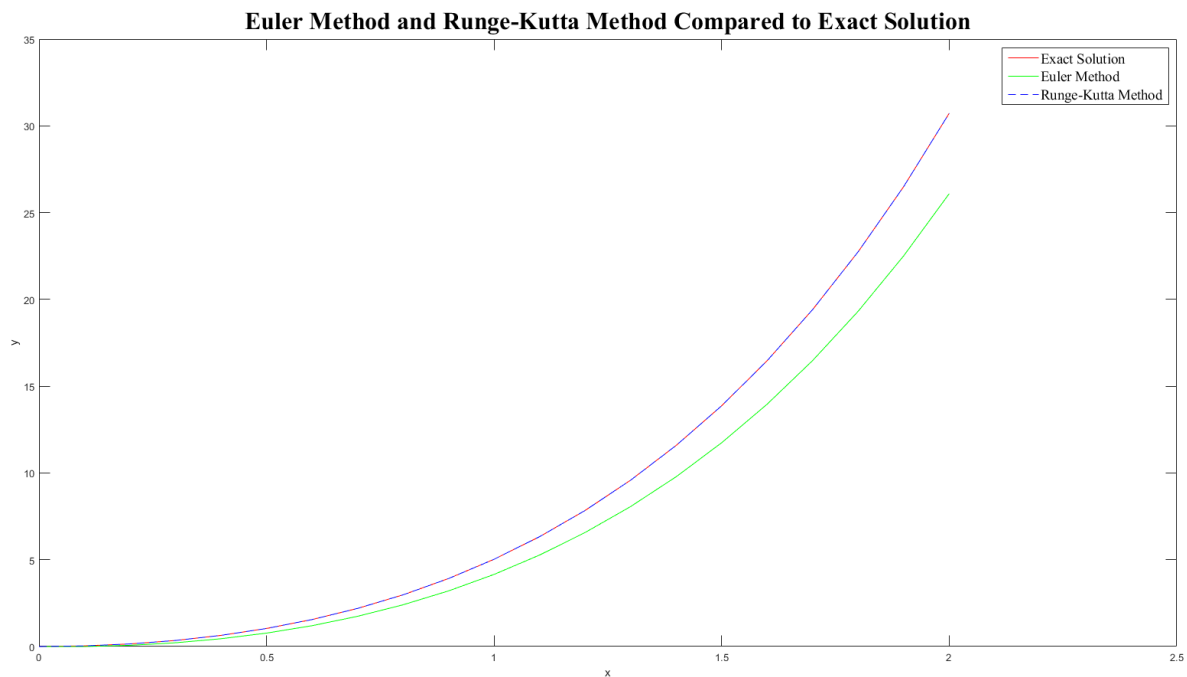
For  $y'(t) = f(t, y(t)); y_0 = y(t_0)$ , using a step size of  $h$ :

- $y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
- $t_{n+1} = t_n + h$

For  $k_n$  values:

- $k_1 = f(t_n, y_n)$
- $k_2 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right)$
- $k_3 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right)$
- $k_4 = f(t_n + h, y_n + hk_3)$

## Results



**Figure 1:** Using the Euler and Runge-Kutta methods to approximate an ODE and comparing it to the exact solution.

**Table 1:** Comparing exact y-values and Euler y-values. A step-size of 0.1 was used.

Exact y-values	Euler y-values	Percent Error
0	0	0
0.0361964265295338	0	100
0.149819307121188	0.0700000000000000	53.27705
0.349011653032021	0.2170000000000000	37.82443
0.642772883488892	0.4487000000000000	30.19307
1.04104889490090	0.7735700000000000	25.69321
1.55483160273356	1.2009270000000000	22.7616
2.19626895229334	1.7410197000000000	20.7283
2.97878649944727	2.4051216700000000	19.25834
3.91722177809865	3.2056338370000000	18.16563
5.02797279921332	4.1561972207000000	17.33851
6.32916216762503	5.2718169427700000	16.70593
7.84081845915583	6.5689986370470000	16.2205
9.58507667333471	8.0658985007517000	15.84941
11.5863997679127	9.7824883508268700	15.56921
13.8718234923665	11.7407371859096000	15.3627
16.4712269707658	13.9648109045005000	15.21694
19.4176317420904	16.4812919949506000	15.12203
22.7475322508906	19.3194211944456000	15.07025
26.5012610959549	22.5113633138902000	15.0555
30.7233926925146	26.0924996452792000	15.07286

**Table 2:** Comparing exact y-values and Runge-Kutta y-values. A step size of 0.1 was used.

Exact y-values	Runge-Kutta y-values	Percent Error
0	0	0
0.0361964265295338	0.0361958333333333	0.001639
0.149819307121188	0.149817995954861	0.000875
0.349011653032021	0.349009479437764	0.000623
0.642772883488892	0.642769680564800	0.000498
1.04104889490090	1.04104447017787	0.000425
1.55483160273356	1.55482573464353	0.000377
2.19626895229334	2.19626138617744	0.000344
2.97878649944727	2.97877694304621	0.000321
3.91722177809865	3.91720989646050	0.000303
5.02797279921332	5.02795820894616	0.00029
6.32916216762503	6.32914443041287	0.00028
7.84081845915583	7.84079707444641	0.000273
9.58507667333471	9.58505107009684	0.000267
11.5863997679127	11.5863692953481	0.000263
13.8718234923665	13.8717874094477	0.00026
16.4712269707658	16.4711844344554	0.000258
19.4176317420904	19.4175817940808	0.000257
22.7475322508906	22.7474738026825	0.000257
26.5012610959549	26.5011929120721	0.000257
30.7233926925146	30.7233133716288	0.000258

## Discussion

After comparing the Euler and Runge-Kutta methods against the exact solution, it is clear that the Runge-Kutta method is superior in terms of accuracy as all entries in **Table 2** show a percent error of less than 0.01% even with  $h = 0.1$  as its step-size. The y-values produced from the Euler method are significantly less accurate than those produced by the Runge-Kutta method using the same step-size.

Therefore, The Runge-Kutta method is better in terms of accuracy compared to the Euler method.