Introduction

Exercises

1. **Exercise 1:**
   (a) We have $2^{15} - 1 = 32767 = 1057 \cdot 31 = 4681 \cdot 7$.
   (b) We know that $32767 = 1057 \cdot 31$, we have $2^{32767} - 1$ divisible by $2^{31} - 1$ (since $2^{32767} - 1 = (2^{31} - 1)(1 + 2^{31} + 2^{2 \cdot 31} + \cdots + 2^{1056 \cdot 31})$) and $1 < 2^{31} - 1 < 2^{32767} - 1$.

2. **Exercise 2:**

n	**is n prime?**	$3^n - 1$	**is $3^n - 1$ prime?**	$3^n - 2^n$	**is $3^n - 2^n$ prime?**
2	yes	8	no	5	yes
3	yes	26	no	19	yes
4	no	80	no	65	no
5	yes	242	no	211	yes
6	no	720	no	665	no
7	yes	2186	no	2059	no?

   Conjecture 1. Suppose n is an integer, if $n > 0$ then $3^n - 1$ is not prime.

3. **Exercice 3:**
   (a) Let $m = 2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$, by theorem 3 m is prime.
   (b) Let $m = 2 \cdot 5 \cdot 11 + 1 = 111$ is not prime since $111 = 3 \cdot 37$, thus it yields two primes.

4. **Exercice 4:** Let $x = (5 + 1)! + 2 = 6! + 2$, Hence by theorem 4, we gave $722, 723, 724, 725, 726$ are not primes.

5. **Exercice 5:** We have $2^5 - 1 = 31$, Hence according to Euclid $2^{5-1}(2^5 - 1) = 496$ is perfect.

6. **Exercice 6:** Let's suppose that there exist such "triplet primes": $(n, n + 2, n + 4)$ where $n \geq 5$, n could be written as follows, either $n = 3k + 1$ or $n = 3k + 2$. If $n = 3k + 1$ then $n + 2 = 3k + 3 = 3(k + 1)$ which is divisible by 3, thus contradicting the fact that $n + 2$ is prime and our supposition. If $n = 3k + 2$ then $n + 4 = 3k + 6 = 3(k + 2)$ which is divisible by 3, thus contradicting the fact that $n + 4$ is prime. Hence in all cases of n we end up with a contradiction, we can then conclude that there are no "triplet primes" for all $n \geq 5$.