Omar Hijab

Introduction to Calculus and Classical Analysis, Second Edition

SPIN Springer's internal project number, if known

November 22, 2006

Springer

Preface

This is the second edition of an undergraduate one-variable analysis text. Apart from correcting errors and rewriting several sections, material has been added, notably in Chapter 1 and Chapter 4. A noteworthy addition is a *combinatorial* proof that the radius of convergence of the Bernoulli series is 2π (Chapter 5). What follows is the preface from the first edition.

For undergraduate students, the transition from *calculus* to *analysis* is often disorienting and mysterious. What happened to the beautiful calculus formulas? Where did ϵ-δ and open sets come from? It is not until later that one integrates these seemingly distinct points of view. When teaching "advanced calculus", I always had a difficult time answering these questions.

Now, every mathematician knows that analysis arose naturally in the nineteenth century out of the calculus of the previous two centuries. Believing that it was possible to write a book reflecting, explicitly, this organic growth, I set out to do so.

I chose several of the jewels of classical eighteenth and nineteenth century analysis and inserted them at the end of the book, inserted the axioms for reals at the beginning, and filled in the middle with (and only with) the material necessary for clarity and logical completeness. In the process, every little piece of one-variable calculus assumed its proper place, and theory and application were interwoven throughout.

Let me describe some of the unusual features in this text, as there are other books that adopt the above point of view. First is the systematic avoidance of ϵ-δ arguments. Continuous limits are defined in terms of limits of sequences, limits of sequences are defined in terms of upper and lower limits, and upper and lower limits are defined in terms of sup and inf. Everybody thinks in terms of sequences, so why do we teach our undergraduates ϵ-δ's? (In calculus texts, especially, doing this is unconscionable.)

The second feature is the treatment of integration. Since the integral is supposed to be the area under the graph, why not define it that way? What goes wrong? Why don't we define[1] the area of *all* subsets of $\mathbf{R}^2$? This is the point of view we take in our treatment of integration. As is well known, this approach remains valid, with no modifications, in higher dimensions.

The third feature is the treatment of the theorems involving interchange of limits and integrals. Ultimately, all these theorems depend on the *monotone*

[1] As in geometric measure theory.

convergence theorem which, from our point of view, follows from the Greek mathematicians' *method of exhaustion*. Moreover, these limit theorems are stated only after a clear and nontrivial need has been elaborated. For example, *differentiation under the integral sign* is used to compute the Gaussian integral.

As a consequence of our treatment of integration, uniform convergence and uniform continuity can be dispensed with. (If the reader has any doubts about this, a glance at the range of applications in Chapter 5 will help.) Nevertheless, we give a careful treatment of uniform continuity, and use it, in the exercises, to discuss an alternate definition of the integral that was important in the nineteenth century (the Riemann integral).

The fourth feature is the use of real-variable techniques in Chapter 5. We do this to bring out the elementary nature of that material, which is usually presented in a complex setting using transcendental techniques.

The fifth feature is our heavy emphasis on computational problems. Computation, here, is often at a deeper level than expected in calculus courses and varies from the high school quadratic formula in §1.4 to $\zeta'(0) = -\log(2\pi)/2$ in §5.8.

Because we take the real numbers as our starting point, basic facts about the natural numbers, trigonometry, or integration are rederived in this context, either in the text or as exercises. Although it is helpful for the reader to have seen calculus prior to reading this text, the development does not presume this. We feel it is important for undergraduates to see, at least once in their four years, a nonpedantic, purely logical development that really does start from scratch (rather than pretends to), is self-contained, and leads to nontrivial and striking results.

We have attempted to present applications from many parts of analysis, many of which do not usually make their way into advanced calculus books. For example we discuss a specific transcendental number, convex conjugates, Machin's formula, the Cantor set, the Bailey–Borwein–Plouffe series, continued fractions, Laplace and Fourier transforms, Bessel functions, Euler's constant, the AGM, the gamma and beta functions, the entropy of the binomial coefficients, infinite products and Bernoulli numbers, theta functions, the zeta function, primes in arithmetic progressions, the Euler–Maclaurin formula, and the Stirling series. Again and again, in discussing these results, we show how the "theory" is indispensable.

As an aid to self-study and assimilation, there are 350 problems with all solutions at the back of the book. If some of the more "theoretical parts" are skipped, this book is suitable for a one-semester course (the extent to which this is possible depends on the students' calculus abilities). Alternatively, covering thoroughly the entire text fills up a year–course, as I have done at Temple teaching our advanced calculus sequence.

Philadelphia, Fall 2006 *Omar Hijab*

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