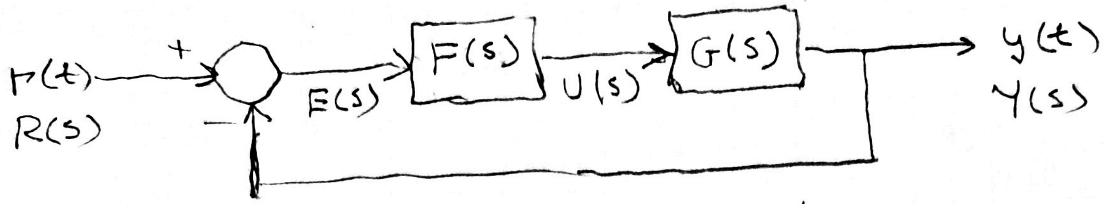
Föreläsning 3.

Allmänt återkopplat system



$F(s)$ = regulator $G(s)$ = system

$Y(s) = G(s) \cdot U(s)$, $U(s) = F(s) \cdot E(s)$

$E(s) = R(s) - Y(s) \Rightarrow$

$Y(s) = G(s) \cdot F(s) \cdot (R(s) - Y(s)) \Rightarrow$

$Y(s)(1 + F(s)G(s)) = F(s) \cdot G(s) \cdot R(s)$

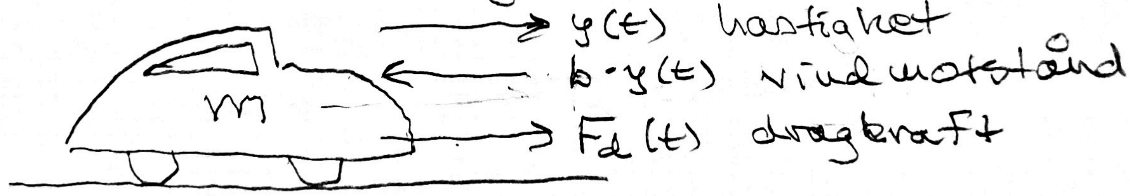
överföringsfunktion $G_{ry}(s)$ ges av

$$G_{ry}(s) = \frac{Y(s)}{R(s)} = \frac{F(s) \cdot G(s)}{1 + F(s) \cdot G(s)}$$

$F(s) \cdot G(s) = L(s)$ = kretsöverföring $\Rightarrow$

$$G_{ry}(s) = \frac{L(s)}{1 + L(s)}$$

Bilmodel med motordynamik



Kraftbalans: $m \cdot \dot{y}(t) = F_d(t) - b y(t) \Rightarrow$

$m \cdot \dot{y}(t) + b\, y(t) = F_d(t)$.

Laplace ger $m s Y(s) + b Y(s) = F_d(s)$

$(ms + b) \cdot Y(s) = F_d(s)$

Motordynamiken ges av $F_d(s) = \dfrac{K_M}{1 + T_M \cdot s} \cdot U(s)$

där $U(s)$ motsvarar gaspådraget.

$$Y(s) = \frac{1}{ms+b} \cdot \frac{K_M}{1+sT_M} \cdot U(s)$$

$$= \frac{K}{1+sT} \cdot \frac{K_M}{1+sT_M} \cdot U(s) \Rightarrow$$

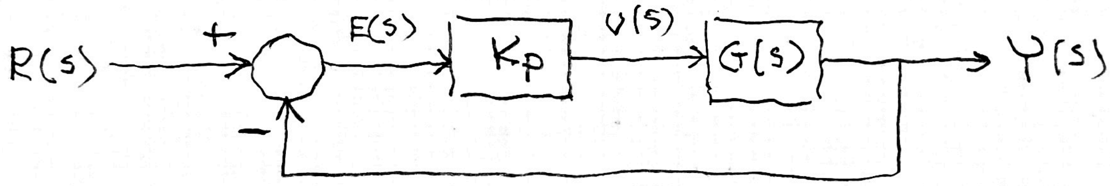
$$G(s) = \frac{Y(s)}{U(s)} = \frac{K \cdot K_M}{(1+sT)(1+sT_M)}$$

$$K = \frac{1}{b}, \quad T = \frac{m}{b}$$

$m = 10^3$ kg $\quad b = 200$ N/m/s $\quad K_M = 10$ kN/rad $\quad T_M = 1$ s $\Rightarrow$

$$G(s) = \frac{50}{(1+5s)(1+s)}$$

P-regulator



$$L(s) = K_p \cdot G(s) = \frac{50K_p}{(1+5s)(1+s)} \Rightarrow$$

$$G_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{50K_p}{(1+5s)(1+s)}}{1+\frac{50K_p}{(1+5s)(1+s)}} =$$

$$= \frac{50K_p}{(1+5s)(1+s)+50K_p} = \frac{50K_p}{5s^2+6s+1+50K_p} =$$

$$= \frac{10K_p}{s^2+1{,}2s+0{,}2+10K_p}$$

Ett system av 2:a ordningen

Allmänt system av ordning 2:

$$G(s) = \frac{K \cdot \omega_n^2}{s^2 + 2\zeta \cdot \omega_n \cdot s + \omega_n^2}$$

ω_n = odämpad resonansfrekvens

ζ = dämpkonstant

K = Förstärkning

För bilen ger detta

$$\omega_n = \sqrt{0{,}2 + 10K_p}$$

$$\zeta = \frac{1{,}2}{\omega_n} = \frac{1{,}2}{\sqrt{0{,}2 + 10K_p}}$$

$$K = \frac{10K_p}{\omega_n^2} = \frac{10K_p}{0{,}2 + 10K_p}$$

Låt nu $r(t) = r_0 \cdot \sigma(t)$ och bestäm kvarstående fel $e(\infty) = y(\infty) - r_0$

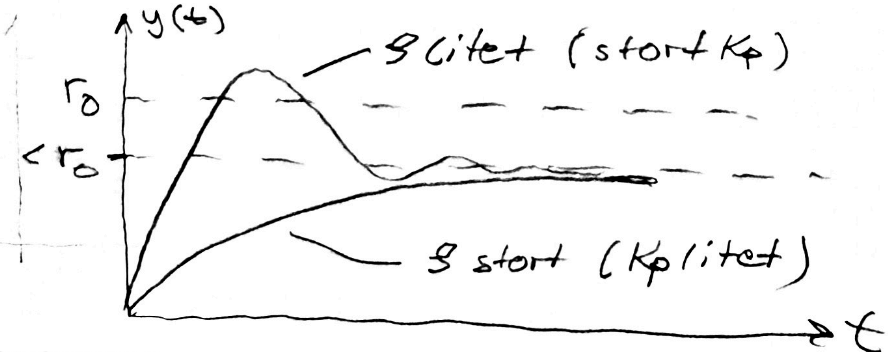
$$y(\infty) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot G(s) \cdot R(s) =$$

$$= \lim_{s \to 0} s \cdot \frac{10K_p}{s^2 + 1{,}2s + 0{,}2 + 10K_p} \cdot \frac{r_0}{s} = \frac{10K_p}{0{,}2 + 10K_p} \cdot r_0 < r_0$$

$\Rightarrow$ Kvarstående fel

$$e(\infty) = r_0 - \frac{10K_p}{0{,}2 + 10K_p} r_0 = \boxed{\frac{0{,}2}{0{,}2 + 10K_p} \cdot r_0}$$

Stegsvar för olika ζ



PI - regulator

$$F(s) = K_p\left(1+\frac{1}{T_i\cdot s}\right) = K_p\cdot\frac{1+T_i s}{T_i s} \Rightarrow$$

$$L(s) = F(s)\cdot G(s) = \frac{50}{(1+5s)(1+s)}\cdot K_p\cdot\frac{1+T_i\cdot s}{T_i\cdot s}$$

Här väljer vi $T_i = 5$ i regulatorn så att den långsamma polen $(T=5)$ i motorn kancelleras

$$L(s) = \frac{50}{\cancel{(1+5s)}(1+s)}\cdot K_p\cdot\frac{\cancel{1+5s}}{5s} = \frac{50K_p}{5s\cdot(1+s)}$$

$$L(s) = \frac{10K_p}{s(1+s)} \Rightarrow$$

$$G_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{10K_p}{s(1+s)}}{1+\frac{10K_p}{s(1+s)}} =$$

$$= \frac{10K_p}{s(1+s)+10K_p} = \frac{10K_p}{s^2+s+10K_p}$$

Låt $r(t) = r_0\cdot\sigma(t) \Rightarrow R(s) = \frac{r_0}{s}$

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot G_{ry}(s)\cdot R(s) =$$

$$= \lim_{s\to 0} s\cdot\frac{10K_p}{s^2+s+10K_p}\cdot\frac{r_0}{s} = r_0$$

Inget kvarstående fel!