# Experiment 4:

## Momentum and Impulse

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**Lab:** Section 1, Monday 9am

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# Introduction

The purpose of this lab was to measure the impulse of an object after a collision in order to demonstrate the principle of the conservation of momentum. In order to perform the experiment, a bumper and a photogate flag was attached to a glider, which was set on a leveled air track. At one end of the track, a force sensor was attached horizontally so that the bumper on the glider would strike it after traveling along the air track. A photogate sensor was placed above the air track, near the force sensor, so that the photogate flag on the glider could pass through. The force sensor and the photogate sensor were attached to a DAQ system so that data could be recorded during the experiment. For each trial, the glider was pushed toward the force sensor so that it would pass completely through the photogate, strike the force sensor, and travel back through the photogate.

The speed of the glider before and after the collision in each trial was calculated based on the amount of time that the photogate sensor was blocked by the glider's photogate flag. The analysis of changes in momentum, based on changes in velocity, gave one method for measuring the impulse of the glider. Another method of measuring impulse was to plot the force that the glider exerted on the force sensor at uniform intervals of time, and then integrate the data. The impulse calculations based on these two methods were compared for each trial to see if the values were consistent.

In order to calculate the impulse for each run based on changes in velocity, it was necessary to calculate the initial momentum "$p_i$" and the final momentum "$p_f$" of the glider. The impulse "J" was then calculated by taking the difference of the two momentum calculations, as shown in the equations below, where "M" is the glider's mass, and "$v_i$" and "$v_f$" are the initial and final velocities of the glider.

$$p_i = Mv_i \quad \text{(Eq. 1.0)}$$

$$p_f = Mv_f \quad \text{(Eq. 1.1)}$$

$$J = \Delta p = p_f - p_i \quad \text{(Eq. 1.2)}$$

$$J = Mv_f - Mv_i \quad \text{(Eq. 1.3)}$$

In order to calculate the impulse based on the collision of the glider with the force sensor, the integral of the force was taken over the time span of the collision. Since the units of force are in newtons ($\frac{kg*m}{s^2}$), the time derivative of the force gives units of momentum ($\frac{kg*m}{s}$). The impulse equation based on the integration method is shown in equation 1.4 below.

$$J = \Delta p = \int dt\, F(t) = \int_{t_1}^{t_n} F(t)\, dt \quad \text{(Eq. 1.4)}$$

The integration shown in equation 1.4 can be approximated by a Riemann Sum, as shown in equation 1.5 below, where "n" refers to the number of subintervals that are used.

$$J \approx \Delta t \sum_{i=1}^{n} F(t_i) \quad \text{(Eq. 1.5)}$$

# Experimental Results and Data Analysis

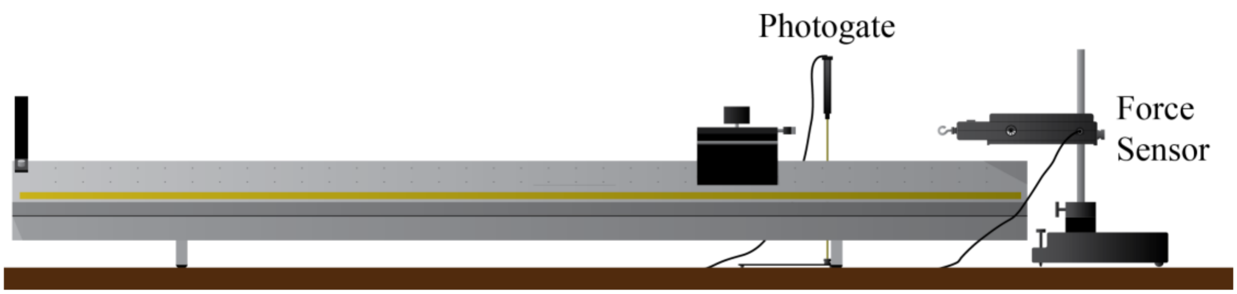


**Figure 1: Experiment Setup:** This image depicts the setup of the experiment. Each trial was initiated by pushing the glider toward the force sensor along the air track. Data was continuously recorded from the force sensor in order to show the changes in force during the collision. Each time the glider's photogate flag passed through the photogate, the sensor was blocked for a recorded length of time, which was used to calculate the glider's speed. The data from the force sensor and the photogate was used to calculate impulse in two different ways. (Source: UCLA Physics 4AL Lab Manual v20, Professor Campbell)

The photogate flag that was attached to the glider had a length of 3.8 ± 0.1 cm. Since the speed of an object can be calculated by dividing the object's change in position by the change in time, it was possible to obtain the glider's speed through the photogate by dividing the length of the photogate flag by the amount of time that the sensor was blocked for. This calculation was automatically completed twice for each trial by the Capstone software in order to produce values for speed in units of m/s.

One way to find the impulse of the glider was by calculating the change in momentum of the glider before and after the collision with the force sensor at the end of the air track. The mass "M" of the glider with the photogate flag and the bumper attached was found to be 200.1 ± 0.2 g. By using the mass of the glider and the initial and final velocity of the glider in equation 1.3, the impulse in each trial could be found, as shown in the following calculations.

$$J_{best} = M_{best} v_{f\,best} - M_{best} v_{i\,best}$$

$$\delta J = \sqrt{\left( \left| M_{best} v_{f\,best} \right| \sqrt{\left( \frac{\delta M}{|M_{best}|} \right)^2 + \left( \frac{\delta v_f}{|v_{f\,best}|} \right)^2} \right)^2 + \left( \left| M_{best} v_{i\,best} \right| \sqrt{\left( \frac{\delta M}{|M_{best}|} \right)^2 + \left( \frac{\delta v_i}{|v_{i\,best}|} \right)^2} \right)^2}$$

$$Trial\ 1{:}\ \ J = (-0.0215 \pm\ 0.0003) \frac{kg * m}{s}$$

$$Trial\ 2{:}\ \ J = (-0.0711 \pm\ 0.0003) \frac{kg * m}{s}$$

In order to calculate the impulse based on data from the force sensor, the sensor had to be calibrated. The masses of the glider (with the photogate flag and bumper attached), as well as the weights that were used to calibrate the force sensor, are displayed in table 1 below.

| | Weight 1 | Weight 2 | Weight 3 | Weight 4 | Weight 5 |
|---|---|---|---|---|---|
| Mass (g) | 2.8 ± 0.1 | 4.4 ± 0.1 | 7.6 ± 0.1 | 19.3 ± 0.1 | 33.3 ± 0.1 |

**Table 1: Masses of Weights for Force Sensor Calibration:** This table displays the masses of the weights that were used to calibrate the force sensor. The third weight's mass was found by placing two weights on the scale at the same time.

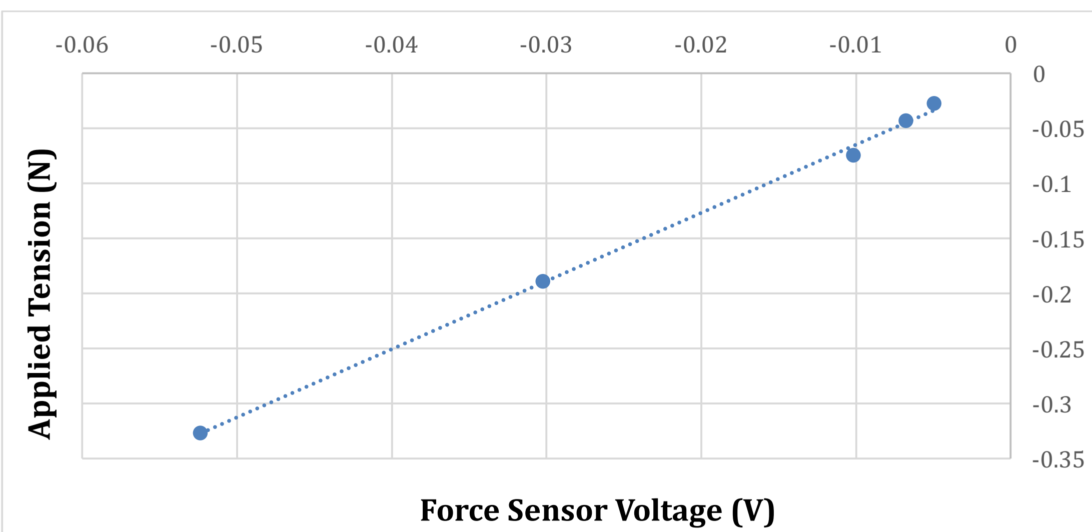


**Figure 2: Force Sensor Calibration:** 5 weights (with masses shown in table 1) were hung from the force sensor after pressing the "tare" button in order to calibrate the sensor before using it in the experiment. The mass of each weight was multiplied by the acceleration of gravity (taken as -9.81 m/s$^2$) in order to calculate the force that was applied to the sensor. These forces, along with the corresponding voltages that were measured by the sensor, were plotted on the graph. A linear trend line was fit to the points, which had the equation F = (6.19 ± 0.15 N/V) V – (0.003 ± 0.004 N). The uncertainty in the equation was determined by using Microsoft Excel's regression tool, which can be found under the "data" tab after installing the "Analysis ToolPak" add-on.

By using Microsoft Excel's regression analysis tool on the data shown in figure 2, the force sensor calibration constant "k" was found to be 6.19 ± 0.15 N/V. By multiplying this value by a voltage reading from the force sensor, the corresponding force that was applied to the sensor could be calculated. The calculated forces throughout each trial were plotted in figures 3 and 4 below.

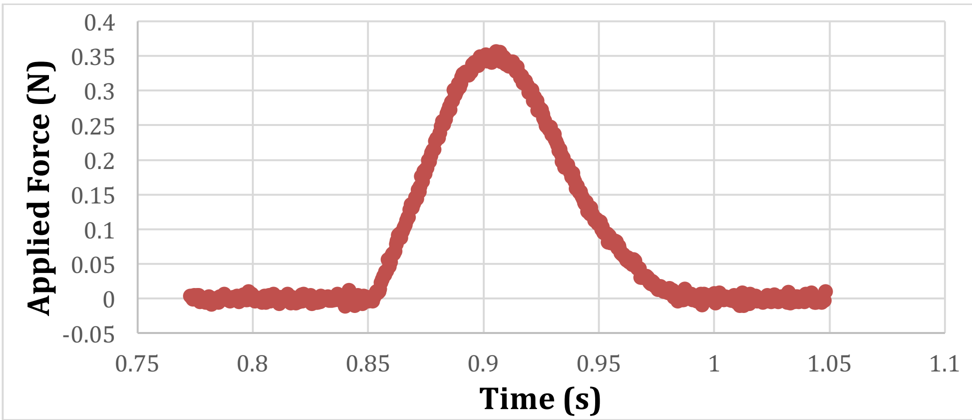


**Figure 3: Force During Collision (Trial 1):** This graph shows the amount of force that the glider exerted on the force sensor during the collision in trial 1. The average of the forces corresponding to the points on the left and right of the curve were subtracted from all of the forces in order to shift the baseline to zero. The bell-shaped curve visually shows the changing force of the glider on the force sensor during the collision. The integration of the data with respect to time, as shown in equations 1.4 and 1.5, gives a measure of the impulse of the glider.

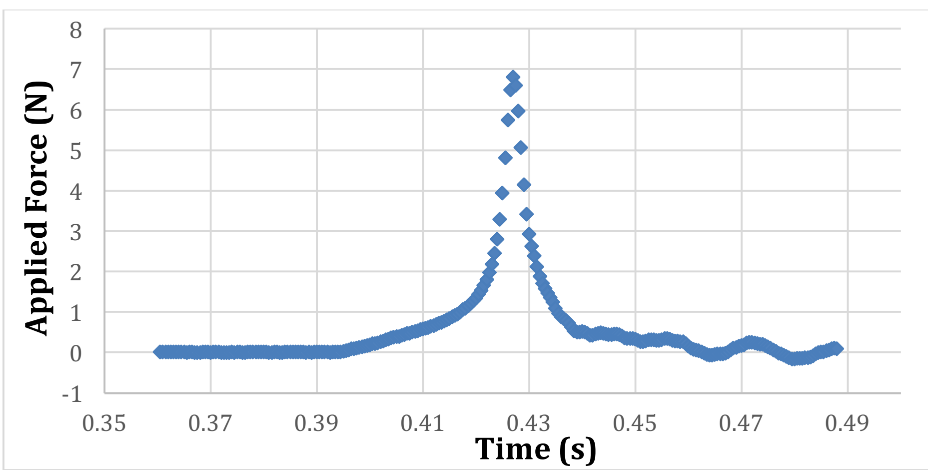


**Figure 4: Force During Collision (Trial 2):** This graph shows the amount of force that the glider exerted on the force sensor during the collision in trial 2. The average of the forces corresponding to the points on the left and right of the main curve were subtracted from all of the forces in order to shift the baseline to zero. The sharp curve visually shows the changing force of the glider on the force sensor during the collision. The oscillations that are seen to the right of the main curve are due to movement of the force sensor after the collision, and can be ignored when integrating to calculate impulse. The data can be integrated by using equations 1.4 and 1.5.

In order to determine the impulse of the glider in each trial based on the forces exerted on the force sensor, the data was integrated by using a Riemann sum, as seen in equation 1.5. This equation shows that the impulse could be calculated by multiplying the time interval between each point with the sum of the forces on the force sensor during the collision. Since the only uncertainty in the forces was due to the uncertainty of the calibration constant, the uncertainty of the integration was assumed to be the same as the uncertainty of the calibration constant.

By taking the initial direction of the glider to be positive, and using equation 1.5, the glider's impulse from trial 1 was found to be (-0.02 ± 0.15) $\frac{kg*m}{s}$, and the impulse from trial 2 was (-0.07 ± 0.15) $\frac{kg*m}{s}$.

| | Impulse Based on Δp ($\frac{kg*m}{s}$) | Impulse Based on Integration ($\frac{kg*m}{s}$) |
|---|---|---|
| Trial 1 | $-0.0215 \pm\ 0.0003$ | -0.02 ± 0.15 |
| Trial 2 | $-0.0711 \pm\ 0.0003$ | -0.07 ± 0.15 |

**Table 2: Different Methods of Measuring Impulse:** This table displays the impulse values of the glider during each trial based on two different calculation methods. In one method, the changing velocity of the glider was used to calculate changes in momentum, which gave a measurement of impulse. In the second method, voltage readings from a force sensor were converted to measurements of force based on the glider's collision. The integration of the forces during the collision gave calculations of impulse for each of the two trials. The impulse values are negative relative to the initial movement direction of the glider due to the fact that some of the glider's momentum was transferred during the collision. The impulses calculated based on integration had much higher uncertainties than those calculated with the other method due to the fact that the calibration slope had a high uncertainty value.

# Conclusion

The purpose of this lab was to calculate the impulse of an object after it was involved in a collision in order to observe the conservation of momentum. The first method of calculating the impulse involved analyzing the change in velocity of a glider before and after a collision. The second method involved integrating the forces that that the glider exerted during the collision.

One source of systematic uncertainty in the experiment was due to friction on the air track. Even when the airflow was turned to the highest setting, the glider showed a noticeable decline in speed while gliding along the air track due to friction. Since friction always impedes the glider's motion, an increase in friction would cause calculations to show a greater change in momentum during the experiment. A decrease in friction would conversely decrease the observed change in momentum. In order to decrease the effects of friction in the system, a stronger and more uniform airflow should be established throughout the air track. The glider should also be pushed straight along the air track, in line with its center of mass, in order to decrease friction due to shaking.

As seen by the results in table 2, the two methods of impulse calculation gave values that fell within each other's uncertainty ranges for each of the two trials. This indicates that the principal of the conservation of momentum was successfully demonstrated by the experiment. In order to strengthen this idea, the experiment can be repeated with more accurate tools that would lower the uncertainty involved in the calculations, especially when dealing with integration.

# Extra Credit

In order to analyze the inelasticity of collisions, two gliders with photogate flags were set on the air track, with attached bumpers facing each other. Two photogates were placed along the air track so that the gliders could collide between the two sensors. In order to perform the experiment, the gliders were placed on opposite sides of the track, and were pushed toward each other so that they would collide between the photogates and reverse directions. This was done once with the bumpers facing each other, and once with the bumpers facing away from each other.

In order to calculate the coefficient of restitution "$C_R$" for the collisions, the following equations were used, where "$v_f$" corresponds to the final velocity of the glider, and "$v_i$" corresponds to the initial velocity.

$$C_R = \frac{|Relative\ Final\ Velocity|}{|Relative\ Initial\ Velocity|} = \frac{|v_{f1}|+|v_{f2}|}{|v_{i1}|+|v_{i2}|} \qquad \text{(Eq. 1.6)}$$

$$\delta C_R = \left|C_{R_{Best}}\right| \sqrt{\left(\frac{\left(\sqrt{\delta v_{f1}^2+\delta v_{f2}^2}\right)}{|v_{f1}|+|v_{f2}|}\right)^2 + \left(\frac{\left(\sqrt{\delta v_{i1}^2+\delta v_{i2}^2}\right)}{|v_{i1}|+|v_{i2}|}\right)^2} \qquad \text{(Eq. 1.7)}$$

By using equations 1.6 and 1.7, it was found that the $C_R$ for the collision with bumpers was $0.548 \pm 0.006$, and the $C_R$ for the collision without bumpers was $0.796 \pm 0.004$.

In order to find the degree to which energy was conserved during each collision, the equation for kinetic energy was used, as shown below, where "$M_1$" and "$M_2$" correspond to the masses of the two gliders, and "$v_1$" and "$v_2$" correspond to the velocities of the gliders.

$$K_{total} = \frac{1}{2}M_1 v_1^2 + \frac{1}{2}M_2 v_2^2 \qquad \text{(Eq. 1.8)}$$

$$\delta K_{total} = \sqrt{\left(|M_{1best}v_{1best}|\sqrt{\left(\frac{\delta M_1}{|M_{1best}|}\right)^2 + \left(\frac{\delta v_1}{|v_{1best}|}\right)^2}\right)^2 + \left(|M_{2best}v_{2best}|\sqrt{\left(\frac{\delta M_2}{|M_{2best}|}\right)^2 + \left(\frac{\delta v_2}{|v_{2best}|}\right)^2}\right)^2} \quad \text{(Eq. 1.9)}$$

By using equations 1.8 and 1.9 to calculate the total kinetic energy before and after the collision, the change in kinetic energy could be found.

In the collision with bumpers, the initial kinetic energy was $0.0036 \pm 0.0003$ J, and the final kinetic energy was $0.0010 \pm 0.0003$ J, indicating a loss of $0.0025 \pm 0.0004$ J, which is about 70% of the initial energy. This shows that energy was not conserved well in the collision with bumpers.

In the collision without bumpers, the initial kinetic energy was $0.0088 \pm 0.0003$ J, and the final kinetic energy was $0.0056 \pm 0.0003$ J, indicating a loss of $0.0032 \pm 0.0004$ J, which is about 36% of the initial energy. This shows that energy was conserved better in the collision without bumpers, than in the collision with bumpers.

In order to find the degree to which momentum was conserved during the collisions, initial momentum of the gliders was compared to the final momentum of the gliders by using the equations below, where "$M_1$" and "$M_2$" correspond to the masses of the two gliders, and "$v_1$" and "$v_2$" correspond to the velocities of the gliders.

$$p_{total} = M_1 v_1 + M_2 v_2 \quad \text{(Eq. 2.0)}$$

$$\delta p_{total} = \sqrt{\left(|M_{1best}v_{1best}|\sqrt{\left(\frac{\delta M_1}{|M_{1best}|}\right)^2 + \left(\frac{\delta v_1}{|v_{1best}|}\right)^2}\right)^2 + \left(|M_{2best}v_{2best}|\sqrt{\left(\frac{\delta M_2}{|M_{2best}|}\right)^2 + \left(\frac{\delta v_2}{|v_{2best}|}\right)^2}\right)^2} \quad \text{(Eq. 2.1)}$$

In the collision with bumpers, the initial momentum was $0.0530 \pm 0.0003\ \frac{kg*m}{s}$, and the final momentum was $0.0290 \pm 0.0003\ \frac{kg*m}{s}$, indicating a loss of $0.0239 \pm 0.0004\ \frac{kg*m}{s}$, which is about 45% of the initial momentum.

In the collision without bumpers, the initial momentum was $0.0811 \pm 0.0003\ \frac{kg*m}{s}$, and the final momentum was $0.0646 \pm 0.0003\ \frac{kg*m}{s}$, indicating a loss of $0.0165 \pm 0.0004\ \frac{kg*m}{s}$, which is about 20% of the initial momentum.