Experiment 3:

Conservation of Mechanical Energy

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Lab: Section 1, Monday 9am

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Introduction

The purpose of this experiment was to determine the degree to which mechanical energy is conserved during motion. In the case of perfect harmonic motion, it is expected that kinetic energy and potential energy will vary, but will always add to the same total mechanical energy that the system started with. This experiment was designed to minimize the loss of total energy so that the effects of energy conservation could be observed.

In order to carry out the experiment, a photogate comb was placed on top of a glider that could move along a horizontal air track with minimal friction. A photogate was place above the track so that the sensor was blocked each time a tooth of the comb passed by. Two springs were then used to attach the sides of the glider to the sides of the air track so that displacement of the glider would cause oscillatory motion.

Before beginning measurements and calculations, the photogate was moved to a position just to the left of the 31st tooth on the photogate comb, so that a slight displacement of the glider to the left would trigger the photogate sensor. This was done so that pulling the glider to the right and releasing it when the comb was past the photogate would cause the 31st recorded time to correspond to zero displacement of the glider along the air track. Through the use of the photogate sensor and a DAQ system, a block count and time was recorded every time a tooth on the photogate comb passed the photogate sensor during each oscillation. The data was used to calculate the potential, kinetic, and total mechanical energy of the glider.

In order to calculate the kinetic energy of the glider, the following equation was used, where "M" corresponds to the mass of the glider with the comb attached, and "v" corresponds to the glider's velocity at a specific time.

$$K = \frac{1}{2}Mv^2 \qquad \text{(Eq. 1.0)}$$

Since velocity is based on change in position, equation 1.0 was modified to allow for calculation of kinetic energy at average positions between every two teeth on the photogate comb.

$$x_{avg} = \frac{x_i + x_{i+1}}{2} \qquad \text{(Eq. 1.1)}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} \qquad \text{(Eq. 1.2)}$$

$$K(x_{avg}) = \frac{1}{2}M(v_{avg})^2 = \frac{1}{2}M\left(\frac{x_{i+1} - x_i}{t_{i+1} - t_i}\right)^2 \qquad \text{(Eq. 1.3)}$$

In order to calculate the potential energy of the glider, the following equation was used, where "k" corresponds to the spring constant, and "x" refers to the position of the glider relative to the 31st tooth on the photogate comb.

$$U = \frac{1}{2}kx^2 \qquad \text{(Eq. 1.4)}$$

Since kinetic energy was found based on average positions between the teeth of the comb, the potential energy calculation was also changed based on the average position calculation in equation 1.1.

$$U(x_{avg}) = \frac{1}{2}k(x_{avg})^2 \qquad \text{(Eq. 1.5)}$$

Experimental Results and Data Analysis

In order to use equation 1.3 to calculate kinetic energy of the glider, it was necessary to find the mass "M" of the glider with the photogate comb attached. Through the use of a gram scale, the glider's mass was found to be 223.3 ± 0.1 g.

It was also important to calculate the spring constant of the springs that caused the glider to oscillate, since the value was needed in equation 1.5 to calculate potential energy. In order to do so, string was draped around a pulley at one end of the air track, and attached to the glider. Weights were then attached to the free end of the string, and the displacement of the glider was recorded. The setup is displayed in figure 1.

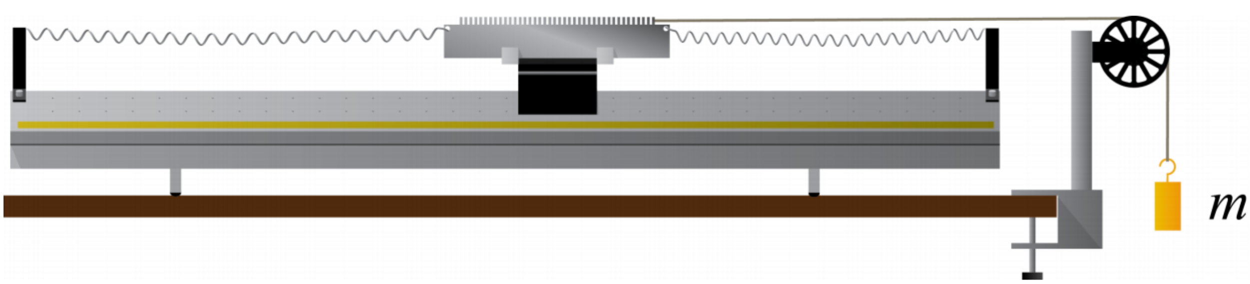

Figure 1: Experiment to Calculate Spring Constant: This photo shows the setup that was used to calculate the spring constant "k" of the two springs together. The displacement of the glider along the air track was based on the weight that was attached to the string. The recorded weights and displacements were graphed using Microsoft Excel, and the slope of the trend line gave the calculated spring constant. (Source: UCLA Physics 4AL Lab Manual, version 20)

Mass (g)	Displacement (m)	Applied Force (N)
4.4 ± 0.1	0.008 ± 0.001	0.043 ± 0.001
19.6 ± 0.1	0.031 ± 0.001	0.192 ± 0.001
60.6 ± 0.1	0.095 ± 0.001	0.593 ± 0.001
99.3 ± 0.1	0.160 ± 0.001	0.973 ± 0.001

Table 1: Data Used to Calculate Spring Constant: The data in this table was gathered by performing the experiment shown in figure 1 with 4 different weights. The masses and displacements were found by direct measurements, and the forces were calculated by using the equation F = ma, where a = 9.8 m/s². The uncertainty in the force calculations was found with equation ii.21 in the lab manual, which shows that $\delta f = |a|\delta m$.

The data from table 1 was plotted on a graph in Microsoft Excel, and a trend line was created. The slope of the trend line gave a value the calculated spring constant "k" based on equation 1.6 below.

$$F = -kx \quad \text{(Eq. 1.6)}$$

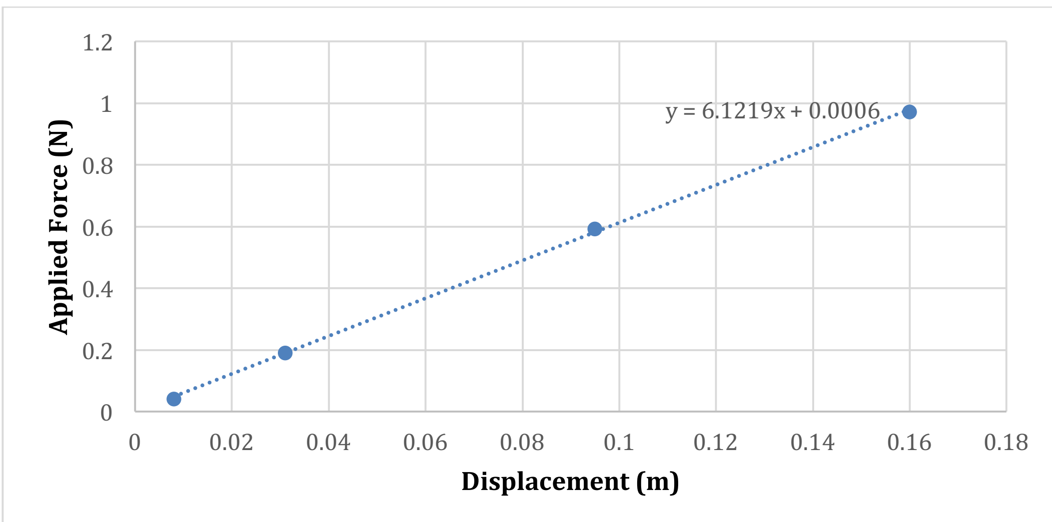

Figure 2: Graph to Calculate Spring Constant: The data from table 1 was plotted on a graph to display the relationship between the applied force on the glider and the glider's displacement. The slope of the trend line gives a value for the average force divided by the magnitude of the displacement, which yields an approximation for the spring constant based on equation 1.6. Error was determined through regression analysis in Microsoft Excel, which was done by selecting "Data Analysis" under the "Data" tab, and selecting "Regression". This showed that the spring constant was 6.12 ± 0.09 N/m.

Based on the calculated spring constant "k" of 6.12 ± 0.09 N/m (from figure 2), and the block count and time information gathered through the DAQ system, the potential energy at average positions between the teeth of the photogate comb could be calculated with equation 1.5. Since the teeth, and the gaps between the teeth, were each 2 mm wide, the block counts were converted to positions by multiplying by 4 mm.

In order to calculate the kinetic energy at the same points that were used to calculate potential energy, the average positions between the teeth of the photogate comb were used once again. The mass "M" of the glider and the average positions "x_{avg}" between the teeth, were put into equation 1.3 to determine kinetic energy.

Through these calculation methods, the values for kinetic energy and potential energy were each found based on the average positions between the teeth of the comb. These values were added together to give the total mechanical energy at the specified positions. Figure 3 below shows the calculated kinetic energy, potential energy, and total mechanical energy at each point.

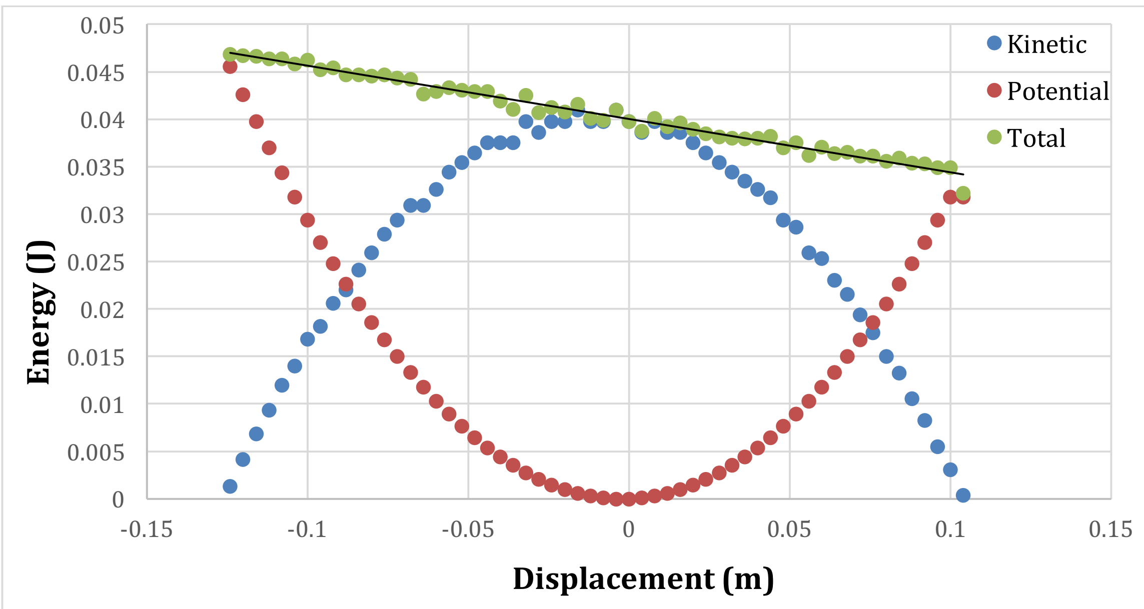

Figure 3: Energy of a Glider During Oscillation: This graph gives a visual representation of the trends in kinetic, potential, and total mechanical energy of a glider during half of an oscillation cycle. When the glider has the large magnitudes of displacement, the potential energy is high, and the kinetic energy is low. When the glider has small magnitudes of displacement, the kinetic energy is high, and the potential energy is low. The kinetic and potential energy always add to the total mechanical energy, which follows a steadily decreasing trend, indicating the loss of energy in the system. The trend line gives an equation for the total mechanical energy at a given displacement "x": E = -0.055x + 0.040.

In order to calculate the coefficient of friction of the glider on the air track, the following equations can be used, where "F_k" is force due to kinetic friction, "F_N" is normal force, and "μ_k" is the coefficient of friction.

$$F_k = \mu_k F_N \qquad \text{(Eq. 1.7)}$$

$$\mu_k = \frac{F_k}{F_N} \qquad \text{(Eq. 1.8)}$$

$$F_N = ma = Mg \qquad \text{(Eq. 1.9)}$$

$$\boldsymbol{\delta} F_N = |g|\delta M \qquad \text{(Eq. 2.0)}$$

$$\delta\mu_k = |\mu_{k_{best}}| \sqrt{\left(\frac{\delta F_k}{|F_{k_{best}}|}\right)^2 + \left(\frac{\delta F_N}{|F_{N_{best}}|}\right)^2} \qquad \text{(Eq. 2.1)}$$

The force due to kinetic friction was found by doing regression analysis on the total mechanical energy data from figure 3 with Microsoft Excel. This showed that F_k = -0.0550 ± 0.0009 N. Equations 1.9 and 2.0 were used to find that the normal force "F_N" on the glider was equal to -2.188 ± 0.001 N.

The values for μ_k and $\delta\mu_k$ were found by putting the values F_k, F_N, δF_k, and δF_N into equations 1.8 and 2.1. This yielded the value for the coefficient of friction, μ_k = 0.0251 ± 0.0004.

Conclusion

The purpose of this lab was to determine the extent to which mechanical energy is preserved during an object's motion. In order to do so, the kinetic and potential energy of a glider at regular displacement intervals was calculated and plotted on a graph. Although the kinetic and potential energy values showed an inverse relationship, the combination of the two values at each displacement showed a strong linear trend in total mechanical energy, where the standard deviation was less than 0.004 J. The fact that the two parabolic energy trends added to form a linear trend demonstrates that energy was mostly conserved during the experiment. The downward slope of the trend line for total mechanical energy was due to losses of energy due to external forces on the system.

One source of systematic uncertainty was friction. If the springs in the experiment caused the glider to tilt slightly during movement, an uneven stream of air would have slowed movement. Since friction always impedes an objects motion, an increase in friction would have caused a decrease in the system's total mechanical energy, which would cause a steeper slope in the trend line in figure 3. In order to minimize the effects of friction on the system, the air track should be properly leveled, the springs should be aligned with the glider's center of mass, and an even airflow should be established throughout the air track. It was also seen that decreasing the strength of the airflow caused a noticeable increase in friction, so it is important to have airflow that is strong enough to keep the glider levitating.

Extra Credit

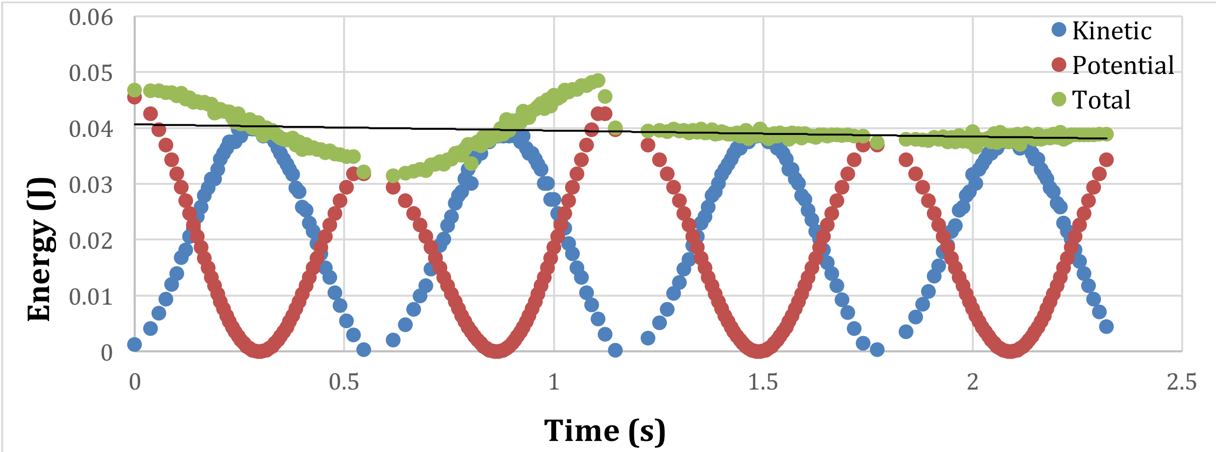

Figure 4: Mechanical Energy During Multiple Oscillations: This graph shows the kinetic, potential, and total mechanical energy of the glider at regular time intervals. Since the raw position data included positions that were always increasing, the positions had to be modified by offsets in order to cause the position to decrease when the glider switched directions. An exponential trend line was fit to the total energy data with Microsoft Excel, which had the equation E = $0.0406e^{-0.028t}$.

Based on the equation of the trend line in figure 4, E = $0.0406e^{-0.0208t}$, it is possible to find the time that it would take for the amplitude of oscillation to decrease by a factor of e. Given that the initial amplitude "A_0" is 0.124m, the time can be calculated as shown in the steps below.

$$A_f = \frac{0.124m}{e} = 0.0456m \qquad \text{(Eq. 2.2)}$$

$$U_f = \frac{1}{2}k\left(A_f\right)^2 = \frac{1}{2}\left(6.12\ \frac{N}{m}\right)(0.0456m) = 0.140\ J \qquad \text{(Eq. 2.3)}$$

$$U_f = 0.140\ J = 0.0406e^{-0.0208t} \qquad \text{(Eq. 2.4)}$$

Based on equation 2.4, it would take the glider about 59.4 seconds for the oscillation amplitude to decrease by a factor of e.