Modeling and Simulation of Planetary Orbits

Application of Ellipses and Kepler's Laws

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Table of Contents

ABSTRACT

This project aims to analyze whether it is possible to model and graphically represent the orbits of the planets in the solar system using only two key parameters: the aphelion and the perihelion. By applying the geometric properties of ellipses and incorporating fundamental physical reasoning, the study seeks to calculate essential orbital elements—such as the semi-major axis, semi-minor axis, focal distance, and eccentricity—based on these values. These parameters are then integrated into a computational model developed using the Python programming language, enabling the simulation and visualization of planetary trajectories in both Cartesian and polar coordinates.

The methodology is based on a simplified two-body approximation that considers only the gravitational interaction between the Sun and each planet, disregarding the influence of other celestial bodies. In addition to using geometric formulas, the project implements the Verlet method to simulate planetary motion, enhancing the realism of the orbital paths generated.

Although Kepler's laws are not applied directly in the modeling process, the results make evident the fulfillment of the first law, as the simulated orbits display elliptical shapes with the Sun at one of the foci. The second and third laws are taken into account from a theoretical standpoint and serve as contextual support for interpreting the variation in orbital speed and the relationship between orbital periods and distances.

INTRODUCTION

The orbits of the planets around the Sun have been the subject of study since ancient times, and their understanding has evolved significantly thanks to Johannes Kepler's laws. These laws revealed that planetary orbits are not circular, but elliptical, with the Sun at one of the foci. This orbital behavior is directly related to the study of ellipses in geometry, where it is observed how the distance from a moving point to two foci varies.

In this project, we seek to model and simulate the orbit of the planets in the solar system using the mathematical tools that describe ellipses and relying on the theoretical principles offered by Kepler's and Newton's laws. The main objective is to plot the orbit of the planets individually in Cartesian coordinates, then move to Polar coordinates, and using the Python programming language visually represent the elliptical trajectory of the planets.

We will initially focus on modeling Earth's orbit around the Sun. Once we have successfully achieved this, we will expand the representation to include the orbits individually of all the other planets in our solar system. For each planet, we will simulate at least one complete revolution around the Sun.

In the following chapters, we are going to find the equations that describe the ellipse of each planet's orbit. To do this, we used the values of perihelion and aphelion, initially expressed in astronomical units. These values were carefully researched and corroborated through various sources and studies to obtain the highest possible accuracy. Subsequently, these data were converted to kilometers to have the information in different units, but all procedures and graphs will be presented using astronomical units.

This project highlights the importance of combining concepts learned in geometry and physics with the use of computational technologies, allowing a deeper and more detailed analysis of planetary orbits. The ability to simulate the orbit also provides an intuitive visual approach to understanding the differences in orbital velocity and distance variations between a planet and the Sun at different points in the orbit.

Although this project is intended to be an accurate representation of the planetary orbits in the solar system, it is important to establish some limitations that affect the accuracy and complexity of the model. The main limitation is that we assume that the planet's orbit is a two-body problem, i.e., we only consider the gravitational interaction between it and the Sun, ignoring the influence of other planets or celestial objects in the solar system. This simplified approach allows each planet orbit to be modeled efficiently, but neglects more complex effects that arise when more bodies are introduced into the system, as occurs in the *three-body problem.*

JUSTIFICATION

The accurate modeling and visualization of planetary orbits play a crucial role in the study of astronomy, physics, and applied mathematics. Understanding how planets move around the Sun not only enables scientists to predict their positions but also provides fundamental insights into gravitational mechanics, space exploration, and celestial dynamics. However, most orbital simulations require a wide range of complex data, such as orbital inclinations, planetary masses, and velocity vectors, which can limit accessibility for students or individuals new to the exploration of orbital mechanics.

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This project is justified by the need to simplify access to scientific modeling using only essential data points—aphelion and perihelion distances—while still achieving accurate and meaningful representations of planetary motion. By relying on Kepler's laws of planetary motion and the geometric properties of ellipses, the project demonstrates how classical physics can be used effectively to reconstruct and analyze celestial phenomena with minimal input.

Moreover, this approach promotes interdisciplinary learning by combining physics, mathematics, astronomy, and computer science through a practical application.

Why Python? Python has emerged as a popular choice for scientific computing and simulations. Its clean and easy-to-understand syntax makes it accessible to researchers and scientists from various backgrounds. Additionally, Python boasts a vast ecosystem of libraries and packages, such as NumPy, SciPy, Matplotlib, and Pandas, providing powerful tools for numerical computations, data analysis, and visualization. This flexibility allows Python to be used for a wide range of tasks, from simple data manipulation to complex simulations.

OBJECTIVES

General Objective

- To determine whether it is possible to model and graph the orbits of the planets in the solar system using only the aphelion and perihelion values, by applying physical principles such as Kepler's laws and the mathematical equations of ellipses through computational simulation.

Specific Objectives

- To calculate key orbital parameters (semi-major axis, semi-minor axis, eccentricity, and focal distance) for each planet based solely on their aphelion and perihelion values using geometric formulas.

- To develop a Python-based computational model capable of graphing planetary orbits in both Cartesian and polar coordinates using the derived parameters.

- To verify whether the geometric method and the Verlet method produce comparable and accurate representations of planetary orbits, considering the different limitations.

1. THEORETICAL FRAMEWORK

1.1 Geocentric model

In ancient times, under the geocentric model proposed by philosophers such as Aristotle and developed by Claudius Ptolemy in the second century A.D., the Earth was believed to occupy the center of the universe. In this model, the Sun, Moon, planets and stars revolved in circular orbits around the Earth. However, astronomical observations of the time revealed that the planets did not follow simple trajectories; some appeared to stop in their motion and then move backward briefly before continuing on their way. This phenomenon, known as retrograde motion, could not be adequately explained by circular orbits.

To solve this problem, Ptolemy introduced the concept of epicycles. According to this model, each planet not only followed a circular orbit around the Earth (called deferent), but also described a small circle (the epicycle) as it moved through its main orbit. This combination of motions allowed Ptolemy to adjust his model to match observations, making the planet appear to move forward, stop, and move backward as it moved across the sky. Despite its complexity and the continuous corrections it required, Ptolemy's model managed to hold up for more than a thousand years, due to the lack of technology to observe the universe more accurately.

1.2 Heliocentric model

In the 16th century, the Polish astronomer Nicolaus Copernicus proposed a radical change in this view with his heliocentric model. In his best known work, *"De revolutionibus orbium coelestium"* (On the revolutions of the celestial spheres), Copernicus argued that the Sun, not the Earth, occupied the center of the solar system, and that all the planets, including the Earth, revolved in circular orbits around the Sun. This new approach greatly simplified the explanation of planetary motions. For example, the retrograde motion of the planets, which in the geocentric model required the introduction of epicycles, was naturally explained as a visual effect: when the Earth, moving in its own orbit, overtook other planets in its path around the Sun, they appeared to move backwards in the sky, an effect similar to that which we experience when overtaking a car on the road.

The model proposed by Copernicus had a considerable impact on astronomy, and in general provided a new way of understanding the cosmos because, fundamentally, his theory established that the Earth revolved on itself once a day, and that once a year it made a complete revolution around the Sun, apart from this he asserted that the Earth, in its rotating current, was tilted on its axis. However, he still maintained some principles of the ancient cosmology, such as the idea of the spheres inside which were the planets and the outer sphere where the stars were immobile. In this sense, the Copernican model was still linked to antiquity.

1.3 Johannes Kepler

The real breakthrough came in the 17th century, when the German astronomer Johannes Kepler, using the precise observational data of his mentor, Tycho Brahe, was able to correct the Copernican model. Kepler discovered that planetary orbits were not circular, but elliptical, and that the Sun was not at the center of these ellipses, but at one of their foci.

1.3.1 Kepler's First Law of Planetary Motion

This discovery was embodied in Kepler's first law, described in his 1609 book *"Astronomia Nova"*: **"The planets describe elliptical orbits with the Sun at one of their foci."** This was a crucial revelation that significantly improved the accuracy of predictions about planetary motion.

1.3.2 Kepler's Second Law of Planetary Motion

Kepler's second law, also described in his 1609 book *"Astronomia Nova"*, also known as the **law of areas**, stated that the planets move faster when they are closer to the Sun (at perihelion) and slower when they are farther away (at aphelion). This law can be expressed as follows: **"The radius vector connecting a planet to the Sun sweeps equal areas in equal times."** In other words, the velocity of a planet in its orbit is not constant, but varies depending on its distance from the Sun.

1.3.3 Kepler's Third Law of Planetary Motion

Finally, Kepler's third law, described in his 1619 book *"Harmonices mundi"*, provided a quantitative relationship between the orbital period of a planet and its average distance from the Sun. This law states that: **"The quotient between the semi-major axis of the elliptical orbit (cubed) and the period of revolution (squared) is the same for all planets."** In other words, the square of the orbital period of a planet is proportional to the cube of the length of the semi-major axis of its orbit. This relationship allowed astronomers to accurately predict the duration of planetary orbits based on their distances from the Sun.

1.4 Orbital motion

1.4.1 Orbit

An orbit is the curved path followed by an object as it moves under the influence of a gravitational force, typically around a larger celestial body. In astronomy, an orbit refers to the trajectory a planet, moon, or satellite takes as it revolves around a central mass, such as the Sun or a planet. This motion results from the balance between the object's inertia (its tendency to move in a straight line) and the gravitational pull of the larger body. Orbits can take several forms—circular, elliptical, parabolic, or hyperbolic—depending on the object's energy and initial velocity. Still, in the case of planets in our solar system, they are best described as elliptical paths, with the Sun occupying one of the ellipse's two foci.

1.4.2 Circular vs. elliptical orbits

In celestial mechanics, orbits can vary in shape depending on the energy and velocity of the orbiting body. A circular orbit is a special case in which the distance between the orbiting object and the central body remains consta nt at all times. In this configuration, the gravitational force provides the exact centripetal acceleration required to maintain uniform motion along a perfect circle. On the other hand, elliptical orbits are more common and reflect a more general and realistic model of planetary motion. In an elliptical orbit, the distance between the object and the central body changes continuously, being shortest at perihelion (the closest point) and longest at aphelion (the farthest point). The central body, such as the Sun, is located at one of the foci of the ellipse, not at the center.

1.4.3 Perihelion, aphelion, major and minor axes, focus

In an elliptical orbit around the Sun, the perihelion is the point at which a planet is closest to the Sun, while the aphelion is the point at which it is farthest away. These two positions reflect the non-uniform distance that characterizes elliptical motion. The major axis is the longest diameter of the ellipse, stretching from one end of the orbit to the other and passing through both the perihelion and aphelion. The semi-major axis is half of this distance and is a fundamental parameter used in calculating orbital periods. Perpendicular to the major axis is the minor axis, which represents the ellipse's shortest diameter. An ellipse has two fixed points known as foci (singular: focus); in the context of planetary motion, the Sun occupies one of these foci.

1.5 Ellipse

An ellipse is a closed, smooth curve that forms part of the family of conic sections—geometric figures obtained by intersecting a plane with a double-napped cone. When the cutting plane intersects the cone at an angle that is neither perpendicular to the base nor parallel to the side, the resulting shape is an ellipse. It can be defined as the set of all points for which the sum of the distances to two fixed points, called foci, is constant. This property gives the ellipse its characteristic elongated shape. In mathematical terms, ellipses are described using parameters such as the semi-major axis, semi-minor axis, eccentricity, and focal distance.

1.5.1 Ellipse Equation (Cartesian Coordinates)

The equation of an ellipse in Cartesian coordinates is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Where:

- (h,k) are the coordinates of the center of the ellipse.
- a is the length of the semi-major axis.
- b is the length of the semi-minor axis.

1.5.2 Ellipse Equation (Polar Coordinates)

Polar coordinates are a two-dimensional coordinate system used to locate points on a plane by distance and angle with respect to a fixed point called origin or pole. Instead of using rectangular coordinates (x, y) as in the Cartesian system, the polar system uses a pair of values (r, θ), where:

- r: is the radial distance from the origin to the point. It can be positive or negative depending on the direction.
- θ: is the angle measured from a reference axis, usually the positive x-axis in the Cartesian system, in a counterclockwise direction.

In polar coordinates, the equation of an ellipse where the origin is one of the foci can be expressed as:

$$r = \frac{b^2/a}{1 + \epsilon \cos(\theta)}$$

where r is the distance from the focus to a point on the ellipse, b is the semi-minor axis, a is the semi-major axis, ϵ is the eccentricity of the ellipse, and θ is the angle measured from the major axis.

This form of the equation describes how the distance between the focus and a point on the ellipse varies as a function of the angle θ.

The expression b^2/a can be replaced with r_0 known as the reference radius. This substitution simplifies the equation, so that the equation of the ellipse can be written as:

$$r = \frac{r_0}{1 + \epsilon \cos(\theta)}$$

1.6 Astronomical Units

An astronomical unit (au) is a standard measure of distance used primarily in astronomy to describe the vast distances within the Solar System. It is equivalent to 149,597,870.7 kilometers, according to the Spanish Society of Astronomy. This unit originated in the 16th and 17th centuries when astronomers like Johannes Kepler and Tycho Brahe began attempting to measure the distances between the planets and the Sun, although with rudimentary methods. At that time, the exact distance to the Sun was not known, so the average distance between the Earth and the Sun was used as a reference, which later became formally defined as the "astronomical unit." This measure has been crucial for calculating the scale of the Solar System and for establishing a standard to compare the orbits of planets and other celestial bodies.

1.7 Gravitational and Dynamic Principles

1.7.1 Universal Law of Gravitation

The motion of celestial bodies is governed by fundamental physical laws. According to Newton's Universal Law of Gravitation, every object in the universe attracts every other object with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This gravitational force provides the necessary acceleration that keeps planets in orbit around the Sun. Complementing this, Newton's Second Law of Motion states that the net force acting on an object is equal to the product of its mass and its acceleration (F = ma). Together, these principles explain how and why objects in space follow curved trajectories: their motion is the result of a continuous force—gravity—that changes their velocity and direction.

$$a = G * M/r^2$$

$$a_x = (G * centralBodymass * x_difference)/distance^3$$

$$a_y = (G * centralBodymass * y_difference)/distance^3$$

1.8 Computational Methods

1.8.1 Verlet Method

The Verlet method is a numerical integration technique widely used to simulate the motion of particles in classical mechanics, particularly in systems where acceleration depends on position, such as planetary orbits. Unlike basic methods like Euler's, the Verlet algorithm offers improved stability and energy conservation over extended simulations, making it particularly well-suited for modeling celestial dynamics. It estimates the future position of a body based on its current and previous positions, along with the acceleration at the current point, without explicitly calculating velocities at each step. This property reduces the accumulation of numerical errors and results in smoother trajectories. In the context of orbital mechanics, the Verlet method allows for an accurate approximation of planetary motion under gravitational forces, even when only limited input data—such as aphelion and perihelion distances—are available.

$$new_position = 2 * current_position - last_position + acceleration * deltaT^2$$

1.8.2 Geometric Method

Another key approach used in this project is the geometric method, which relies on the fundamental properties of ellipses to construct orbital paths from known distances—specifically, the aphelion and perihelion. By calculating parameters such as the semi-major axis, eccentricity, and focal distance, it is possible to reconstruct each orbit purely from its geometric definition. This method offers a straightforward, visual way to model planetary motion and serves as a conceptual foundation for the more advanced numerical simulations that follow. It also allows us to verify the elliptical nature of orbits in a clear and mathematically precise manner.

1.8.3 Python and Scientific Libraries

Python is a high-level, open-source programming language recognized for its clear syntax, cross-platform compatibility, and powerful capabilities in scientific computing. It has become a standard tool in research and education across disciplines such as physics, astronomy, engineering, and data science. In this project, Python was chosen to develop a computational model for simulating and graphing planetary orbits due to its flexibility and ease of use.

The implementation relied on several core scientific libraries: NumPy, which provides fast array operations and numerical functions; Matplotlib, used for generating high-quality 2D visualizations of orbital paths; and SciPy, which offers advanced tools for scientific calculations and interpolation. These libraries are part of the broader Python scientific ecosystem, widely used in both academia and industry.

Notably, Python and its libraries are also utilized by organizations such as NASA and ESA for data analysis, mission design, and visualization tasks, demonstrating the language's reliability and scalability in professional aerospace contexts. Its active global community, extensive documentation, and integration with platforms like Jupyter Notebooks make it especially effective for creating reproducible and interactive simulations.

2. Modeling Earth's Orbit

In this chapter, we will explore the orbit of Earth, the third planet in the solar system and the only known to harbor life. Although Earth is our home, there is still much we have yet to learn about our planet.

Understanding Earth's orbit is essential to appreciate its position and movement concerning the Sun and other celestial bodies in the solar system. Like the other planets, Earth follows an elliptical path, with key characteristics described by concepts such as perihelion, aphelion, semi-major axis, and semi-minor axis.

2.1 Aphelion and Perihelion

To graph Earth's orbit, and the orbits of all the other planets, we need two main values, its maximum distance from the Sun (aphelion), and its minimum distance (perihelion).

The aphelion distance in the Earth's orbit is equal to 1.017au *(Observatorio Astronómico de Quito, 2017).* In kilometers:

$$1au = 149,597,870.7km$$

$$1.017au = x$$

$$x = 1.017 \times 149,597,870.7$$

$$1.017au \approx 152,141,034.5km$$

The perihelion distance in the Earth's orbit is equal to 0.983au *(Observatorio Astronómico de Quito, 2017).* In kilometers.:

$$1au = 149,597,870.7km$$

$$0.983au = x$$

$$x = 0.983 \times 149,597,870.7$$

$$0.983au \approx 147,054,706.9km$$

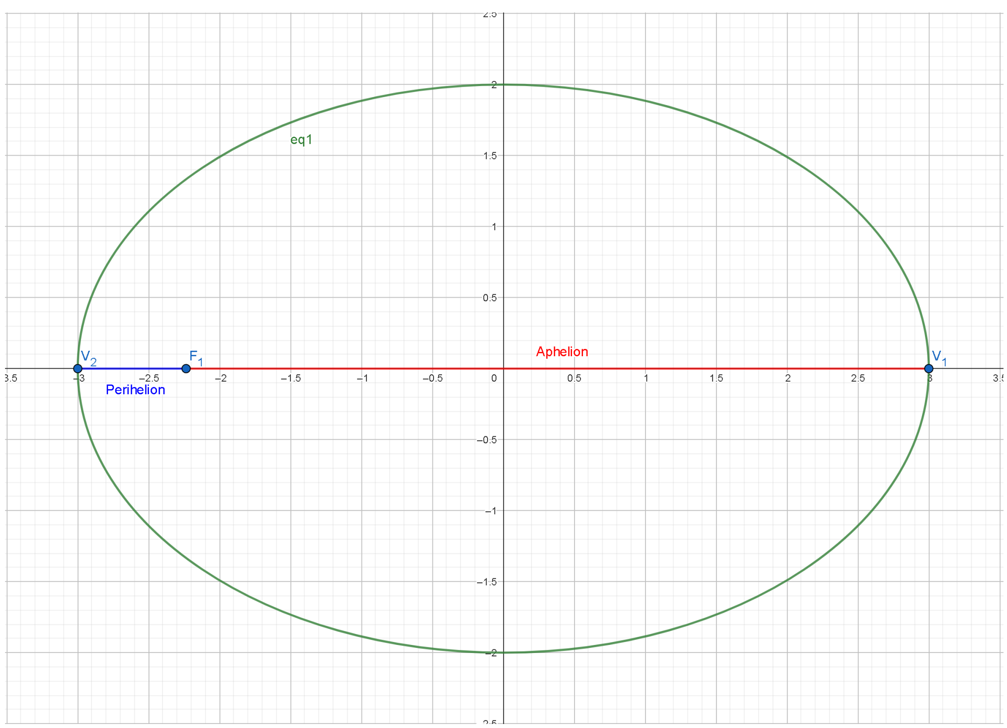
2.2 Semi-major Axis

The semi-major axis of an ellipse is the longest radius of the ellipse. It is equal to half the length of the major axis, which is the longest diameter of the ellipse. Having said that, there is a very simple way to calculate the semi-major axis of our ellipse based on the aphelion and perihelion.

In an ellipse, the points where we can find the longest and shortest distances from one focus are the vertices. Therefore, the sum of the aphelion and the perihelion is equal to two times the semi-major axis, as shown in the following image: In this way, we can find a

Figure 1

Aphelion (red line) + Perihelion (blue line) = 2a



Note. Figure created by the authors.

(semi-major axis) by doing:

$$\frac{Aphelion + Perihelion}{2} = a$$

For the Earth:

$$\frac{1.017au + 0.983au}{2} = a$$

$$\frac{2}{2} = a$$

$$1au = a$$

This tell us that the semi-major axis of the ellipse representing the orbit of the Earth is 1au.

2.3 Focal Distance

The focal distance is an important parameter in defining the shape of an ellipse. It represents the distance between the center of the ellipse and one of its two foci, which are points that influence the curvature of the ellipse. The greater the focal distance relative to the semi-major axis, the more eccentric the ellipse becomes. The focal distance between the Sun and the Earth in its elliptical orbit is the distance from the center of the Earth's orbit to the Sun, which is located at one of the two foci of the ellipse, as stated in Kepler's First Law.

We already know that the semi-major axis of Earth's orbit is 1au, in this sense we can find the focal distance (c) by doing:

$$Aphelion - a = c$$

$$1.017au - 1au = c$$

$$c = 0.017au$$

2.4 Semi-minor Axis

The semi-minor axis is one of the fundamental components of an ellipse, representing half of the shortest diameter of an ellipse. To find the length of the semi-minor axis, we can apply the Pythagorean theorem, which relates the semi-major axis, the semi-minor axis, and the focal distance. Given that the semi-major axis is the longest radius, extending from the center to the farthest point on the ellipse, and the focal distance represents the distance between the center and one of the two foci, we can use the following formula:

$$a^2 = b^2 + c^2$$

But, we already know the values of a and c, so we can rewrite the equation to solve for b:

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(1)^2 - (0.017)^2}$$

$$b = \sqrt{1 - 0.000289}$$

$$b = \sqrt{0.999711} au$$

$$b \approx 0.9998554896 au$$

Note that, for our calculations, we are going to use the exact values (in this case $\sqrt{0.999711}$) to remain as precise as possible.

This means that the difference between the semi-major and semi-minor axis of the Earth is:

$$1 - \sqrt{0.999711} \approx 0.00014 au$$

or

$$0.00014 \times 149,597,870.7 \approx 20943.7019 km$$

2.5 Eccentricity

The eccentricity (ϵ) is a measure of how much an orbit deviates from being a perfect circle. It defines the shape of an ellipse, with values: $0 < \epsilon < 1$. When the eccentricity is 0, the orbit is a perfect circle, and as it approaches 1, the orbit becomes more elongated or elliptical.

The formula to calculate the eccentricity of an orbit is the following:

$$\epsilon = \frac{c}{a}$$

So, we can plot our values for c and a to find Earth's Orbit eccentricity:

$$\epsilon = \frac{0.017}{1} = 0.017$$

This means that Earth's orbit is nearly circular, and we are going to see if that is correct.

2.6 Plotting Earth's Orbit

2.6.1 Cartesian Coordinates

Through this chapter, we've been finding some values that we are going to use to plot Earth's orbit, and now we can replace these values in the ellipse equation in cartesian coordinates.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

We can replace a and b, with our values for the semi-major axis and semi-minor axis of Earth's orbit, having the following result:

$$\frac{x^2}{1^2} + \frac{y^2}{(\sqrt{0.999711})^2} = 1$$

As part of the project, we have developed a script in Python with libraries such as numpy and matplotlib that, by entering the values of the Earth's aphelion and perihelion, is able to perform the calculations we have previously done to find the semi-major axis, semi-minor axis, focal length, etc. and thus plot the ellipse that represents the orbit of our planet around the Sun.

The Cartesian coordinate method generates 10,000 equidistant points distributed along the X-axis, representing the length of the semi-major axis. Then, the values of y per x point were calculated to generate the points on the Y-axis, by solving for y in the ellipse equation as follows:

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 \times \left(1 - \frac{x^2}{a^2}\right)$$

$$y = \sqrt{b^2 \times \left(1 - \frac{x^2}{a^2}\right)}$$

$$y = \pm\left(b \times \sqrt{1 - \frac{x^2}{a^2}}\right)$$

Thus, by calculating the points of y that belong to each point of x, we can create the ellipse that describes the orbit of the Earth.

Additionally, the position of the Sun is placed at one of the foci of the ellipse (specifically at the focal distance) and is visualized with a marker on the graph. Also, Earth's representation was added at a random position along the orbit, highlighting the dynamism of orbital motion.

2.6.2 Polar Coordinates

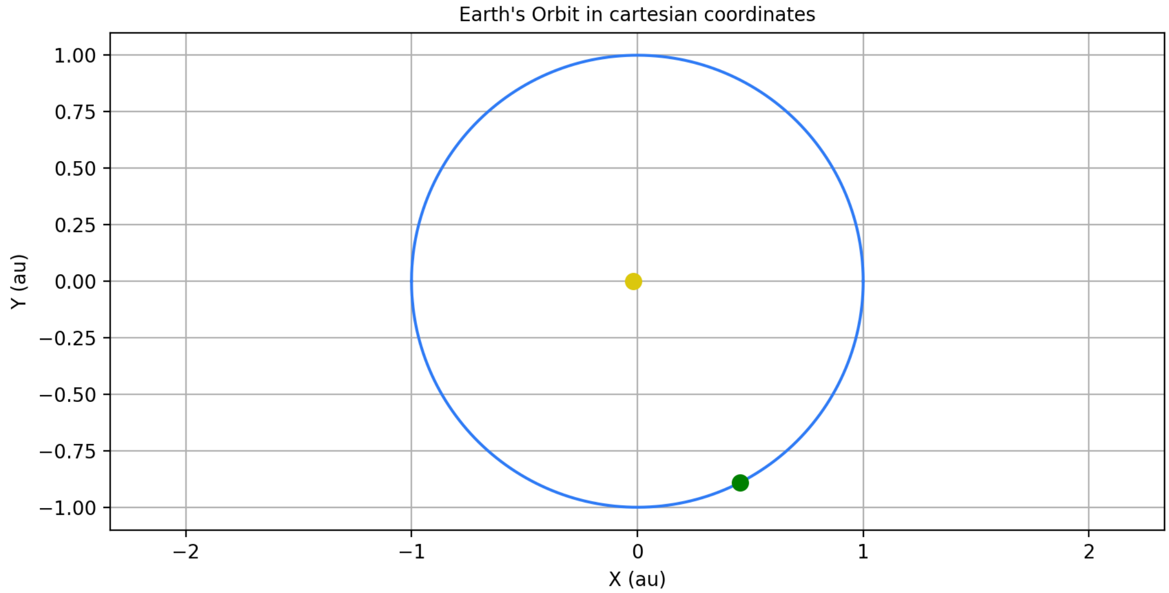
To graph Earth's orbit in polar coordinates, we can replace the values that were previously calculated into the ellipse equation:

$$r = \frac{r_0}{1 + \epsilon \cos(\theta)}$$

$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{0.999711})^2}{1} = 0.999711$$

Figure 2

Earth's Orbit representation



Note. Figure created by the authors.

Giving us the following equation:

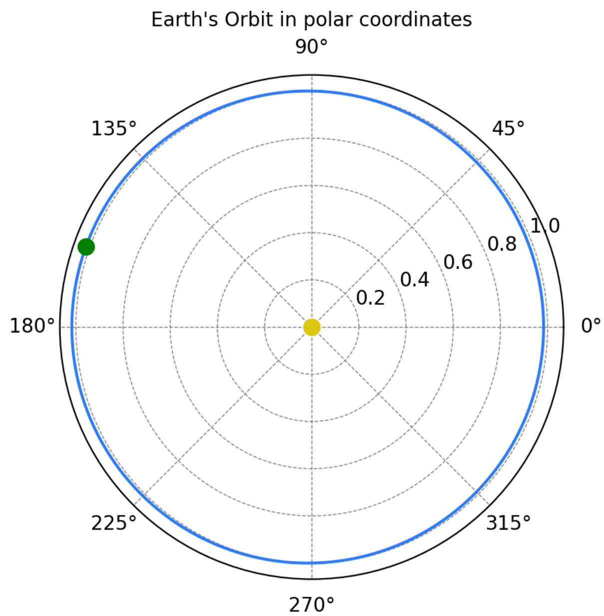
$$r = \frac{0.999711}{1 + 0.017\cos(\theta)}$$

With this in mind, we added to our Python script the possibility of plotting Earth's orbit in polar coordinates, thanks to the Matplotlib and Numpy libraries.

For this, we used numpy to generate 10,000 values between 0 and 2π (one revolution) that will represent the angle θ, and using the formula previously described, we calculated the values of r that correspond to each value of θ. Then, we plotted the Sun in the center of the graph (because the formula used, specifically works with one foci in the center of the ellipse), and added the Earth in a random point in the graph to represent the motion of the planets around the orbits. Our script plotted the following orbit:

Figure 3

Earth's Orbit in polar coordinates



Note. Figure created by the authors.

3. Modeling Mercury's Orbit

In this chapter, we will graph Mercury's orbit. Mercury was part of the Greek planets, although they were not defined the same way as they are today. The ancient Babylonians also knew it. However, the English astronomer Thomas Harriott and the Italian scientist Galileo Galilei made the first observation of the planet in the scientific era in 1610. It is one of the four inner planets of the solar system and, therefore, belongs to the group of rocky planets. It has a diameter of 4,879 kilometers at the equator.

3.1 Aphelion and Perihelion

The aphelion distance in Mercury's orbit is equal to 0.466 au *(Instituto de Astrofisica de Canarias)*. In kilometers:

$$x = 0.466 \times 149,597,870.7$$

$$0.466au \approx 69,712,607.75km$$

The perihelion distance in Mercury's orbit is equal to 0.307 au *(Instituto de Astrofisica de Canarias)*. In kilometers:

$$x = 0.307 \times 149,597,870.7$$

$$0.307au \approx 45,926,546.3km$$

3.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{0.466au + 0.307au}{2} = a$$

$$\frac{0.773}{2} = a$$

$$0.3865au = a$$

3.3 Focal Distance

$$Aphelion - a = c$$

$$0.466au - 0.3865au = c$$

$$c = 0.0795au$$

3.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(0.3865)^2 - (0.0795)^2}$$

$$b = \sqrt{0.14938225 - 0.00632025}$$

$$b = \sqrt{0.143062}au$$

$$b \approx 0.378235au$$

This means that the difference between the semi-major and semi-minor axis of Mercury is:

$$0.3865 - \sqrt{0.143062} \approx 0.008265au$$

or

$$0.008265 \times 149,597,870.7 \approx 1,236,426.401km$$

3.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.0795}{0.3865} = 0.205692108$$

3.6 Plotting Mercury's Orbit

3.6.1 Cartesian Coordinates

The formula that represents Mercury's Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 0.3865, b = \sqrt{0.143062}$$

Replacing the values gives:

$$\frac{x^2}{0.3865^2} + \frac{y^2}{(\sqrt{0.143062})^2} = 1$$

3.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon \cos(\theta)}$$

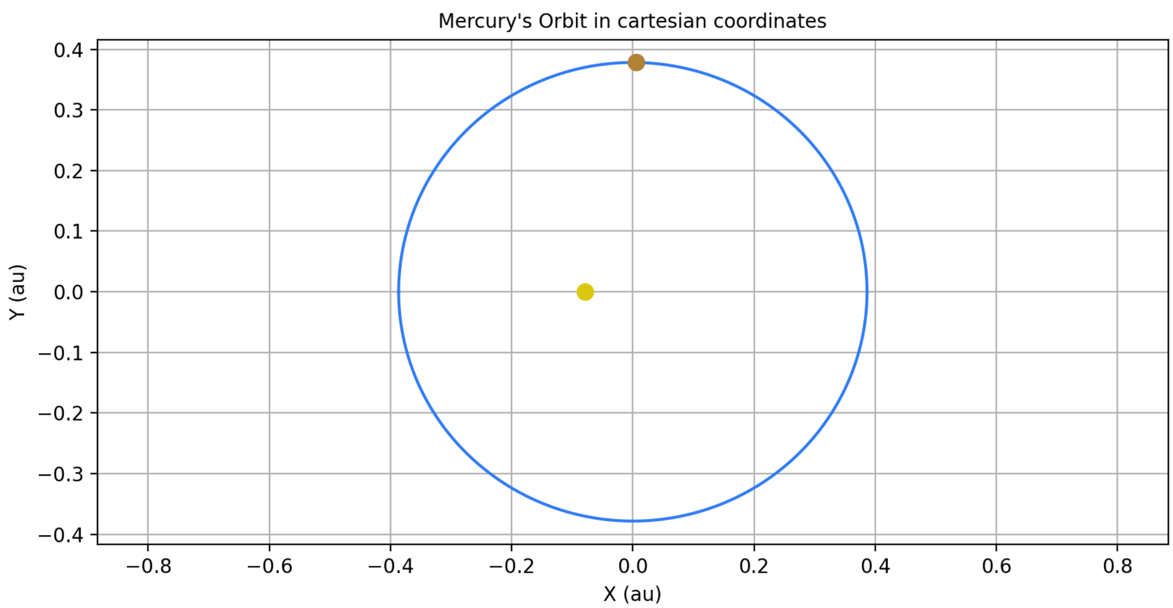
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{0.143062})^2}{0.3865} \approx 0.3701474775$$

Replacing the values give us the following equation:

$$r = \frac{0.3701474775}{1 + 0.205692108 \cos(\theta)}$$

Figure 4

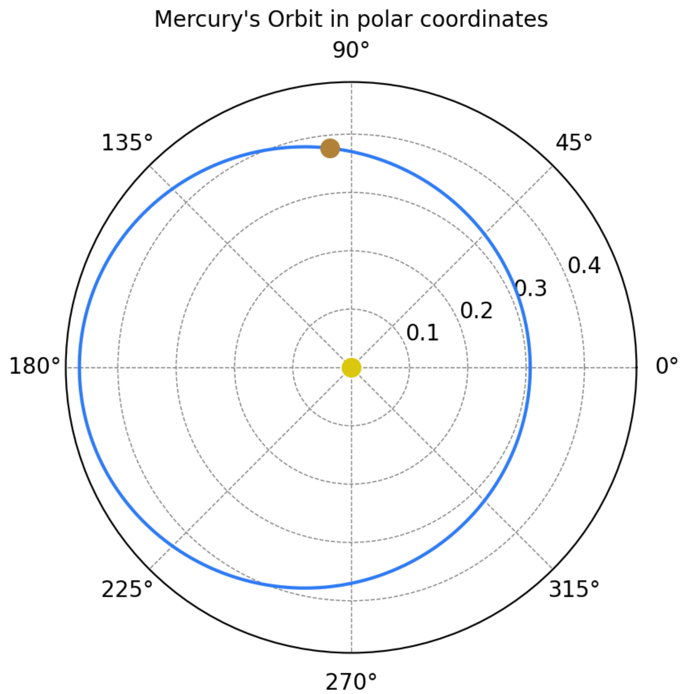
Mercury's Orbit representation



Note. Figure created by the authors.

Figure 5

Mercury's Orbit in polar coordinates



Note. Figure created by the authors.

4. Modeling Venus' Orbit

In this chapter, we are going to talk about Venus, which is the second planet from the Sun and the closest to Earth. It rotates slowly in the opposite direction to most planets and is named after the ancient Roman goddess of love and beauty, known as Aphrodite by the ancient Greeks. Venus has a structure and size similar to Earth's, and it is sometimes called Earth's evil twin. Its thick atmosphere traps heat in a runaway greenhouse effect, making it the hottest planet in our solar system, with surface temperatures high enough to melt lead. Beneath the dense and persistent clouds, the surface features volcanoes and deformed mountains.

4.1 Aphelion and Perihelion

The aphelion distance in Venus' orbit is equal to 0.728 au *(International Astronomical Union)*. In kilometers:

$$x = 0.728 \times 149,597,870.7$$

$$0.0728au \approx 108,907,249.9km$$

The perihelion distance in Venus' orbit is equal to 0.718 au *(International Astronomical Union)*. In kilometers:

$$x = 0.718 \times 149,597,870.7$$

$$0.718au \approx 107,411,271.2km$$

4.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{0.728au + 0.718au}{2} = a$$

$$\frac{1.466}{2} = a$$

$$0.723au = a$$

4.3 Focal Distance

$$Aphelion - a = c$$

$$0.728au - 0.723au = c$$

$$c = 0.005au$$

4.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(0.723)^2 - (0.005)^2}$$

$$b = \sqrt{0.522729 - 0.000025}$$

$$b = \sqrt{0.522704}au$$

$$b \approx 0.722982au$$

This means that the difference between the semi-major and semi-minor axis of Venus is:

$$0.723 - \sqrt{0.522704} \approx 0.00001729au$$

or

$$0.621765 \times 149,597,870.7 \approx 2,586.547km$$

4.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.005}{0.723} = 0.0069156293$$

4.6 Plotting Venus' Orbit

4.6.1 Cartesian Coordinates

The formula that represents Venus' Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 0.723, b = \sqrt{0.522704}$$

Replacing the values gives:

$$\frac{x^2}{0.723^2} + \frac{y^2}{(\sqrt{0.522704})^2} = 1$$

4.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon\cos(\theta)}$$

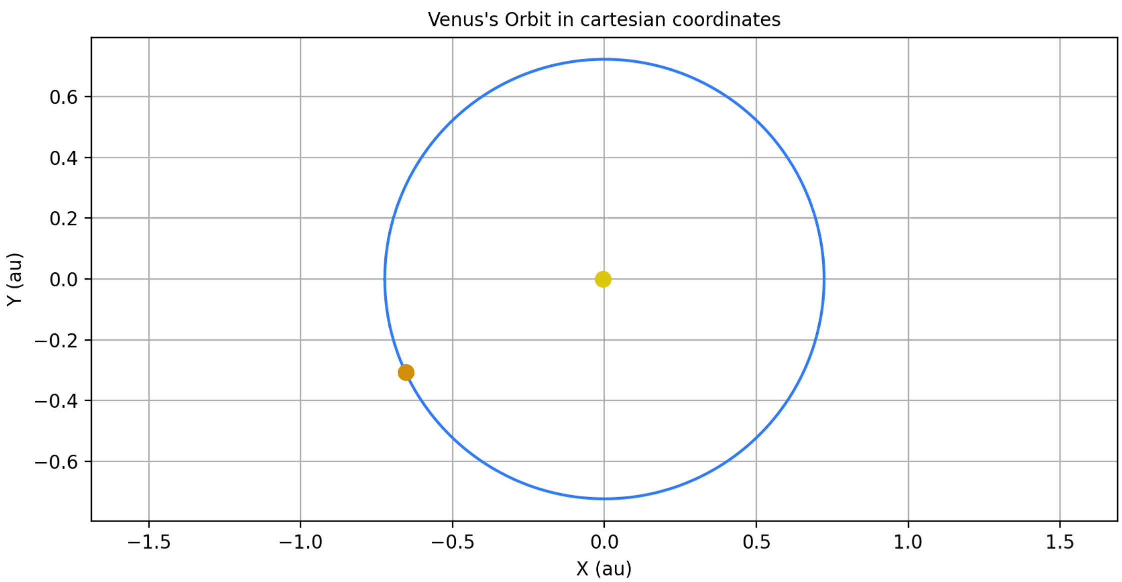
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{0.522704})^2}{0.723} \approx 0.7229654218$$

Replacing the values give us the following equation:

$$r = \frac{0.7229654218}{1 + 0.0069156293\cos(\theta)}$$

Figure 6

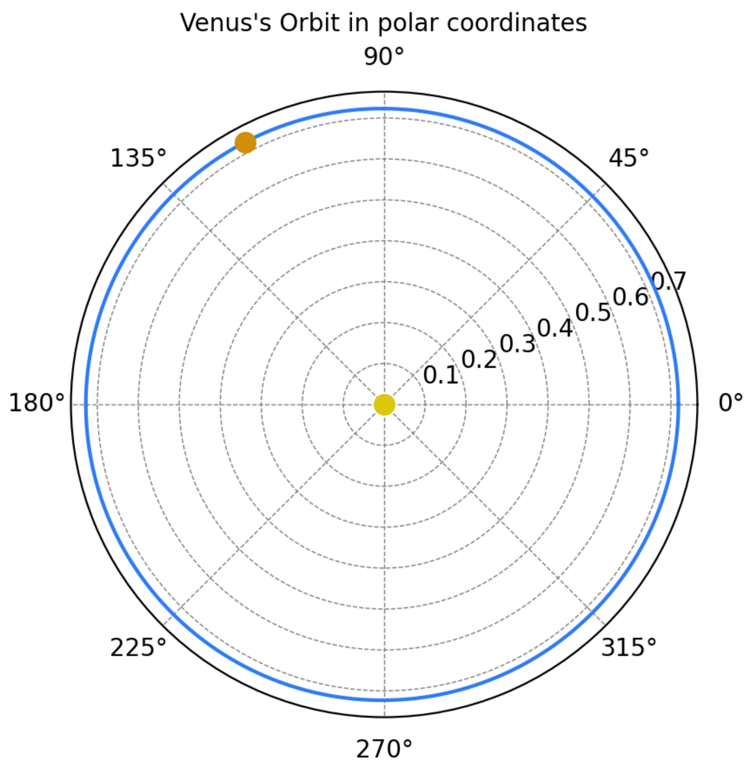
Venus' Orbit representation



Note. *Figure created by the authors.*

Figure 7

Venus's Orbit in polar coordinates



Note. Figure created by the authors.

5. Modeling Mars' Orbit

In this chapter, we will explore the orbit of Mars, one of the most explored bodies in our solar system and the only planet to which we have sent rovers. The Romans named Mars after their god of war because its reddish color reminded them of blood. The Egyptians called it 'Her Desher,' meaning 'the red one. Even today, it is often called the 'Red Planet' because the iron minerals in the Martian dust oxidize, making the surface appear red.

5.1 Aphelion and Perihelion

The aphelion distance in Mars' orbit is equal to 1,665 au *(International Astronomical Union)*. In kilometers:

$$x = 1,665 \times 149,597,870.7$$

$$1,665au \approx 249,080,454.7km$$

The perihelion distance in Mars' orbit is equal to 1,381 au *(International Astronomical Union)*. In kilometers:

$$x = 1,381 \times 149,597,870.7$$

$$1,381au \approx 206,594,659.4km$$

5.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{1,665au + 1,381au}{2} = a$$

$$\frac{3.046}{2} = a$$

$$1.523au = a$$

5.3 Focal Distance

$$Aphelion - a = c$$

$$1,665au - 1.523au = c$$

$$c = 0.142au$$

5.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(1.523)^2 - (0.142)^2}$$

$$b = \sqrt{2.319529 - 0.020164}$$

$$b = \sqrt{2.299365}au$$

$$b \approx 1.516365721au$$

This means that the difference between the semi-major and semi-minor axis of Mars is:

$$1.523 - \sqrt{2.299365} \approx 0.006634278942au$$

or

$$0.006634278942 \times 149,597,870.7 \approx 992,474.0033km$$

5.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.142}{1.523} = 0.0932370322$$

5.6 Plotting Mars' Orbit

5.6.1 Cartesian Coordinates

The formula that represents Mars' Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 1.523, b = \sqrt{2.299365}$$

Replacing the values gives:

$$\frac{x^2}{1.523^2} + \frac{y^2}{(\sqrt{2.29936})^2} = 1$$

5.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon\cos(\theta)}$$

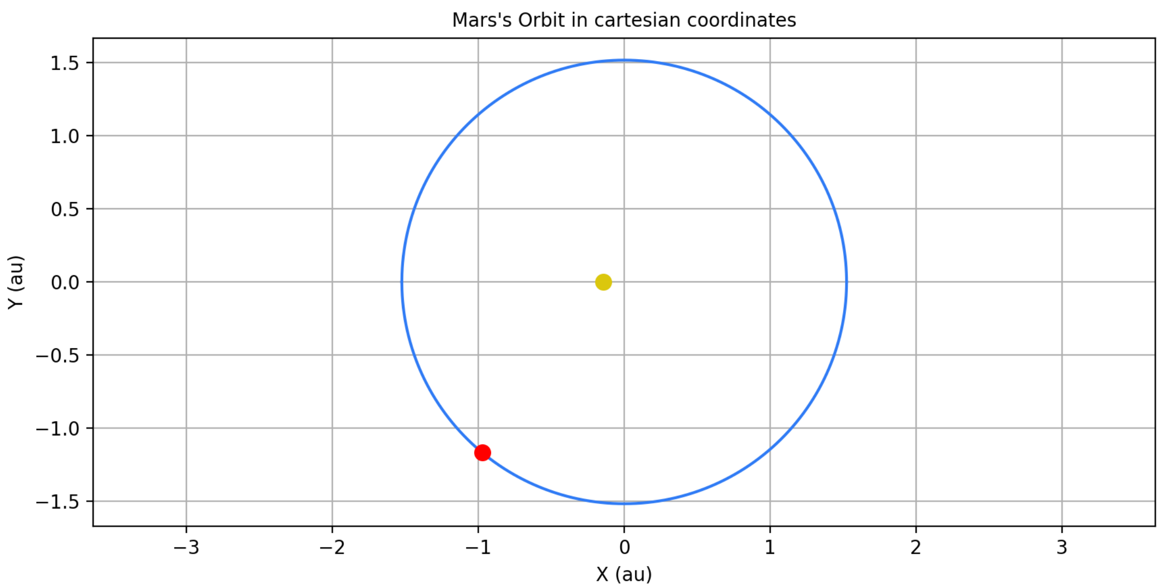
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{2.299365})^2}{1.523} \approx 1.50976034$$

Replacing the values give us the following equation:

$$r = \frac{1.50976034}{1 + 0.0932370322\cos(\theta)}$$

Figure 8

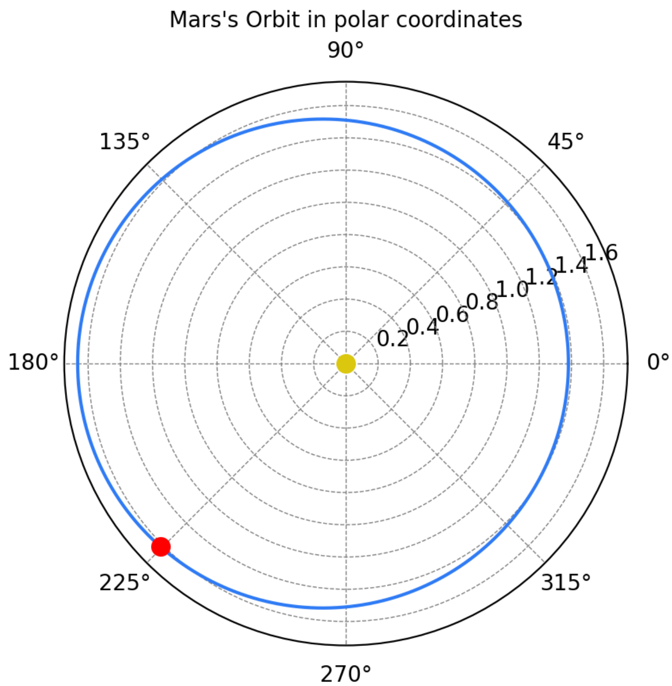
Mars' Orbit Representation



Note. Figure created by the author.

Figure 9

Mars' Orbit in polar coordinates



Note. *Figure created by the author.*

6. Modeling Jupiter's Orbit

In this chapter, we are going to talk about Jupiter, which was named after the king of the gods in Roman mythology. The characteristic stripes and swirls on Jupiter are cold, windy clouds of ammonia and water floating in an atmosphere of hydrogen and helium. The dark orange stripes are called belts, while the lighter bands are called zones, and they flow from east to west in opposite directions. Jupiter's iconic Great Red Spot is a giant storm that has been raging on the planet for hundreds of years.

6.1 Aphelion and Perihelion

The aphelion distance in Jupiter's orbit is equal to 5.458 au *(International Astronomical Union)*. In kilometers:

$$x = 5.458 \times 149,597,870.7$$

$$5.458au = 816,505,178.3km$$

The perihelion distance in Mars' orbit is equal to 4.950 au *(International Astronomical Union)*. In kilometers:

$$x = 4.950au \times 149,597,870.7$$

$$4.950au = 740,509,460km$$

6.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{5.458au + 4.950au}{2} = a$$

$$\frac{10.408}{2} = a$$

$$5.204au = a$$

6.3 Focal Distance

$$Aphelion - a = c$$

$$5.458au - 5.204au = c$$

$$c = 0.254au$$

6.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(5.204)^2 - (0.254)^2}$$

$$b = \sqrt{27.081616 - 0.064516}$$

$$b = \sqrt{27.0171}au$$

$$b \approx 5.197797611au$$

This means that the difference between the semi-major and semi-minor axis of Jupiter is:

$$5.204 - \sqrt{27.0171} \approx 0.006202389474au$$

or

$$0.006202389474 \times 149,597,870.7 \approx 927,864.2584km$$

6.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.254}{5.204} = 0.048808608$$

6.6 Plotting Jupiter's Orbit

6.6.1 Cartesian Coordinates

The formula that represents Jupiter's Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 5.204, b = \sqrt{27.0171}$$

Replacing the values gives:

$$\frac{x^2}{5.204^2} + \frac{y^2}{(\sqrt{27.0171})^2} = 1$$

6.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon\cos(\theta)}$$

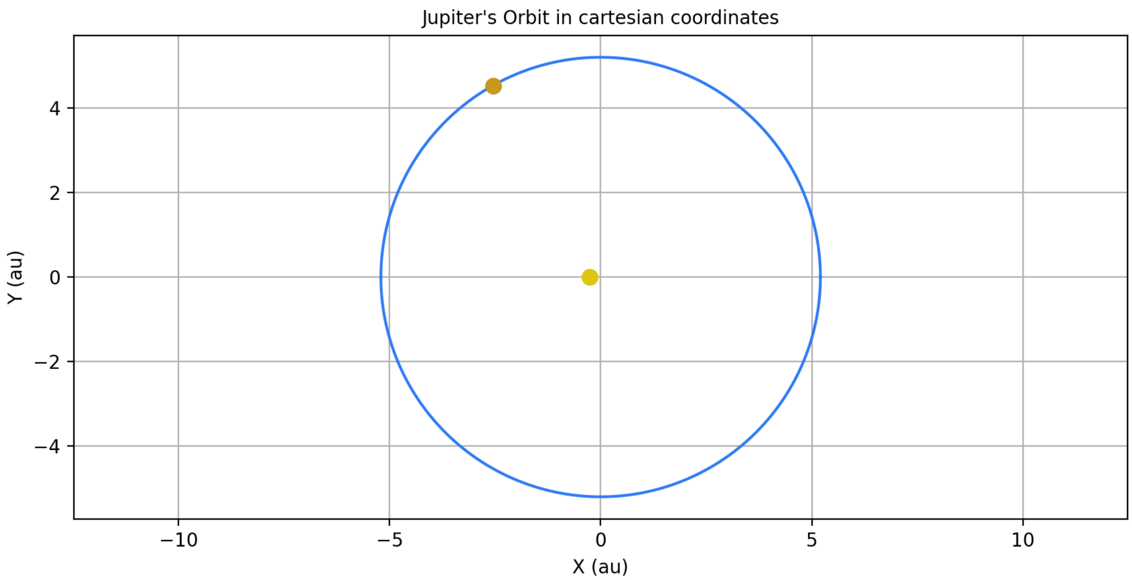
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{27.0171})^2}{5.204} \approx 5.1916026133$$

Replacing the values give us the following equation:

$$r = \frac{5.1916026133}{1 + 0.048808608\cos(\theta)}$$

Figure 10

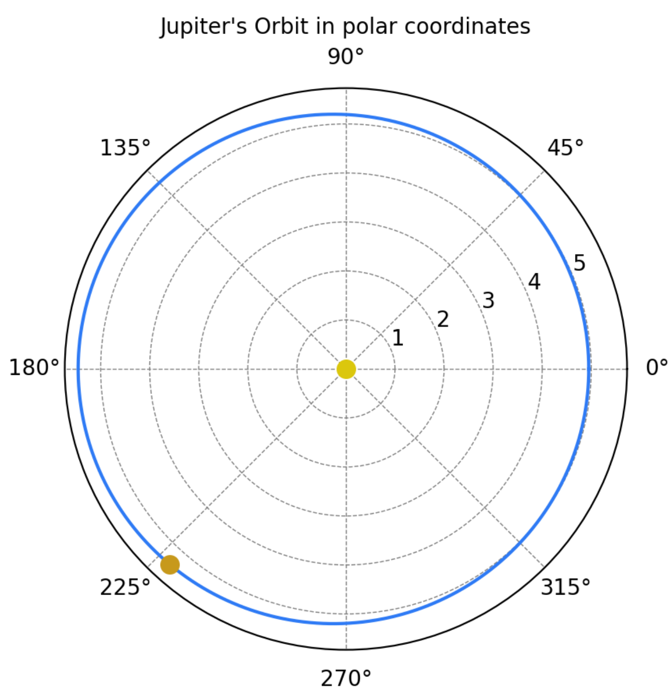
Jupiter's Orbit Representation



Note. Figure created by the authors.

Figure 11

Jupiter's Orbit in polar coordinates



Note. Figure created by the authors.

7. Modeling Saturn's Orbit

In this chapter, we are going to talk about Saturn, a massive sphere composed primarily of hydrogen and helium. Although it is not the only planet with rings, none are as spectacular or complex as Saturn's, making it easily recognizable. In addition to its impressive rings, Saturn has dozens of moons. From the water geysers erupting from Enceladus to the methane lakes on the polluted Titan, Saturn's system is a rich source of scientific discoveries and still holds many mysteries. Known since ancient times, Saturn is the farthest planet from Earth that can be observed with the naked eye, and its name refers to the Roman god of agriculture and wealth.

7.1 Aphelion and Perihelion

The aphelion distance in Saturn's orbit is equal to 10.115 au *(International Astronomical Union)*. In kilometers:

$$x = 10.115 \times 149,597,870.7$$

$$10.115au \approx 1,513,182,462.1305km$$

The perihelion distance in Saturn's orbit is equal to 9.048 au *(International Astronomical Union)*. In kilometers:

$$x = 9.048 \times 149,597,870.7$$

$$9.048au \approx 1,353,561,534.0936km$$

7.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{10.115au + 9.048au}{2} = a$$

$$\frac{19.163}{2} = a$$

$$9.5815au = a$$

7.3 Focal Distance

$$Aphelion - a = c$$

$$10.115au - 9.5815au = c$$

$$c = 0.5335au$$

7.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(9.5815)^2 - (0.5335)^2}$$

$$b = \sqrt{91.80514225 - 0.28462225}$$

$$b = \sqrt{91.52052}au$$

$$b \approx 9.566635772au$$

This means that the difference between the semi-major and semi-minor axis of Saturn is:

$$9.5815 - \sqrt{91.52052} \approx 0.01486422769au$$

or

$$0.01486422769 \times 149,597,870.7 \approx 2,223,656.81202km$$

7.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.5335}{9.5815} = 0.055680217$$

7.6 Plotting Saturn's Orbit

7.6.1 Cartesian Coordinates

The formula that represents Saturn's Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 9.5815, b = \sqrt{91.52052}$$

Replacing the values gives:

$$\frac{x^2}{9.5815^2} + \frac{y^2}{(\sqrt{91.52052})^2} = 1$$

7.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon \cos(\theta)}$$

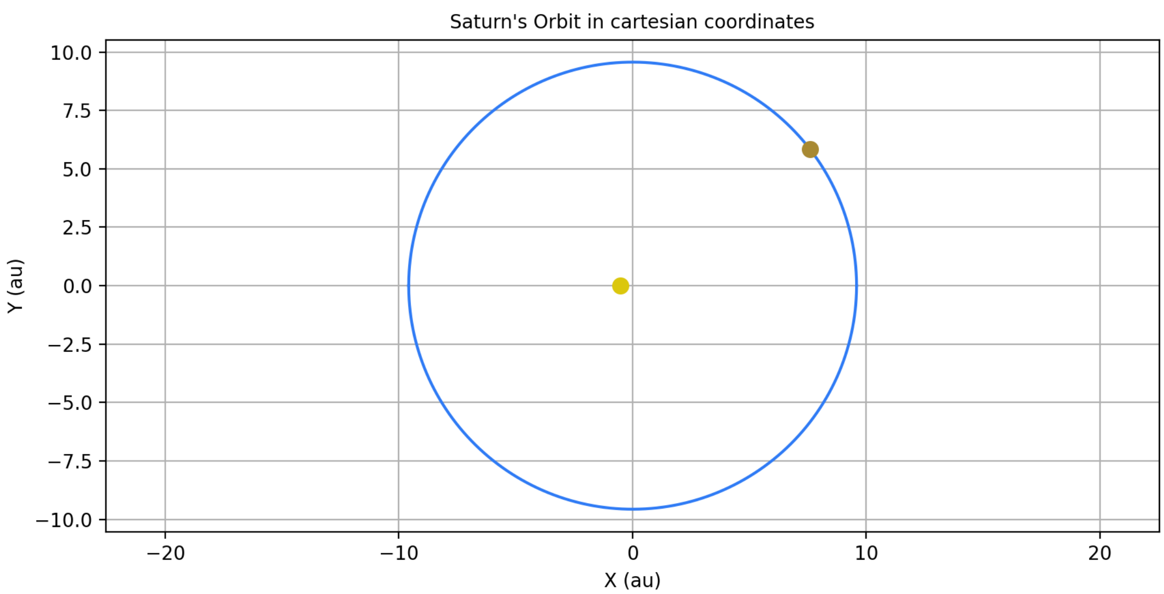
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{91.52052})^2}{9.5815} \approx 9.5517946042$$

Replacing the values give us the following equation:

$$r = \frac{9.5517946042}{1 + 0.055680217 \cos(\theta)}$$

Figure 12

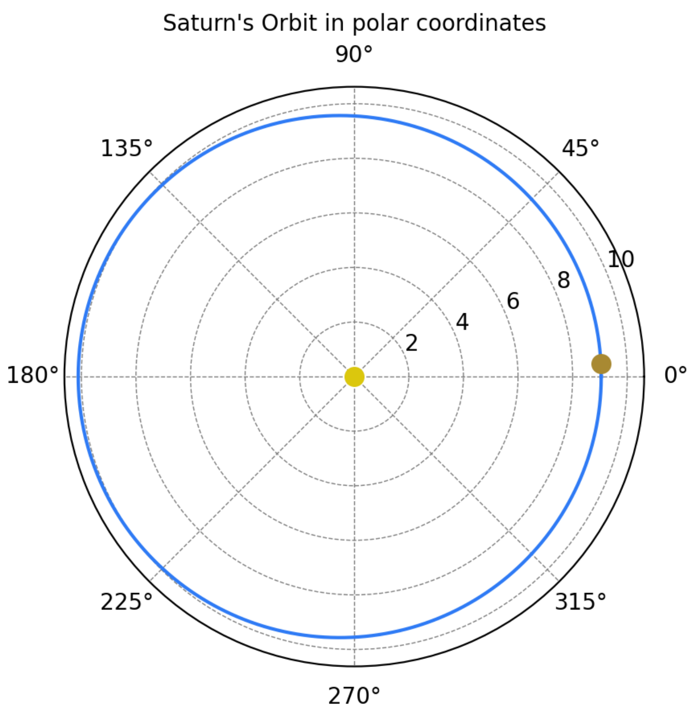
Saturn's Orbit Representation



Note. Figure created by the author.

Figure 13

Saturn's Orbit in polar coordinates



Note. *Figure created by the authors.*

8. Modeling Uranus' Orbit

In this chapter, we will explore the orbit of Uranus, a cold and windy world. This ice giant is surrounded by 13 faint rings and 28 small moons. Uranus rotates at an extreme tilt of nearly 90 degrees relative to the plane of its orbit. This unique tilt makes Uranus appear to spin on its side, orbiting the Sun like a rolling ball. Uranus was the first planet discovered with the aid of a telescope. In 1781, astronomer William Herschel identified it, although he initially thought it was a comet or a star. Two years later, thanks to the observations of astronomer Johann Elert Bode, it was universally accepted as a new planet. Herschel had unsuccessfully attempted to name the discovery Georgium Sidus in honor of King George III, but Bode proposed the name Uranus, after the Greek god of the sky.

8.1 Aphelion and Perihelion

The aphelion distance in Uranus' orbit is equal to 20.083 au *(International Astronomical Union)*. In kilometers:

$$x = 20.083 \times 149,597,870.7$$

$$20.083au \approx 3,004,374,037.2681km$$

The perihelion distance in Uranus' orbit is equal to 18.375 au *(International Astronomical Union)*. In kilometers:

$$x = 18.375 \times 149,597,870.7$$

$$18.375au \approx 2,748,860,874.1125km$$

8.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{20.083au + 18.375au}{2} = a$$

$$\frac{38.458}{2} = a$$

$$19.229au = a$$

8.3 Focal Distance

$$Aphelion - a = c$$

$$20.083au - 19.229au = c$$

$$c = 0.854au$$

8.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(19.229)^2 - (0.854)^2}$$

$$b = \sqrt{369.754441 - 0.729316}$$

$$b = \sqrt{369.025125}au$$

$$b \approx 19.21002668au$$

This means that the difference between the semi-major and semi-minor axis of Uranus is:

$$19.229 - \sqrt{369.025125} \approx 0.01897332121au$$

or

$$0.01897332121 \times 149,597,870.7 \approx 2,838,368.45312km$$

8.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.854}{19.229} = 0.04441208591$$

8.6 Plotting Uranus' Orbit

8.6.1 Cartesian Coordinates

The formula that represents Uranus' Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 19.229, b = \sqrt{369.025125}$$

Replacing the values gives:

$$\frac{x^2}{19.229^2} + \frac{y^2}{(\sqrt{369.025125})^2} = 1$$

8.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon\cos(\theta)}$$

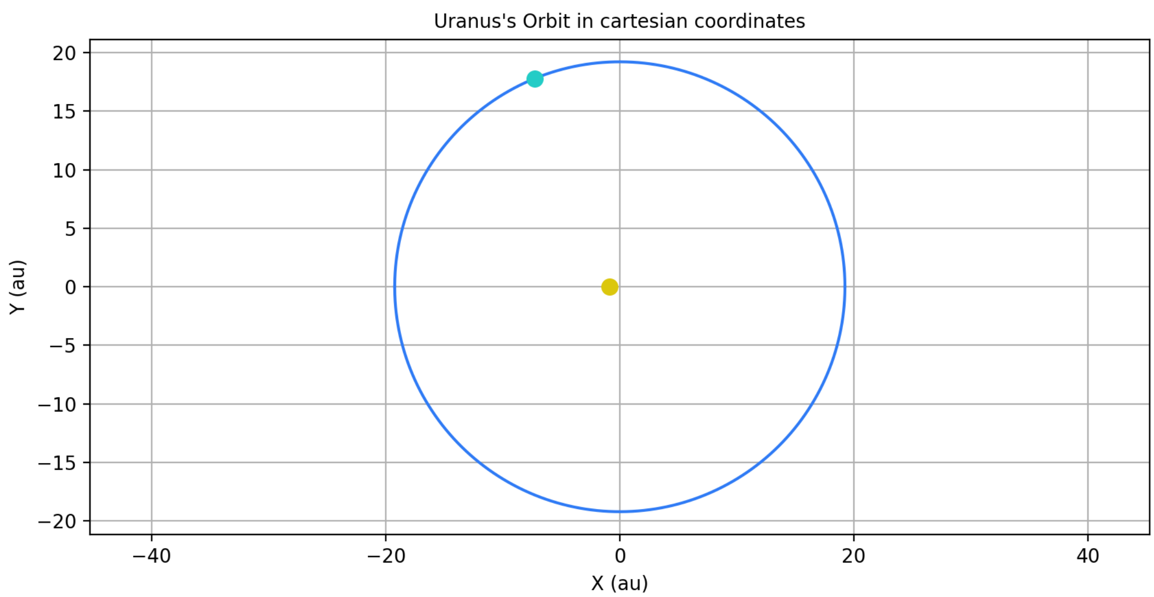
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{369.025125})^2}{19.229} \approx 19.1910720786$$

Replacing the values give us the following equation:

$$r = \frac{19.1910720786}{1 + 0.04441208591\cos(\theta)}$$

Figure 14

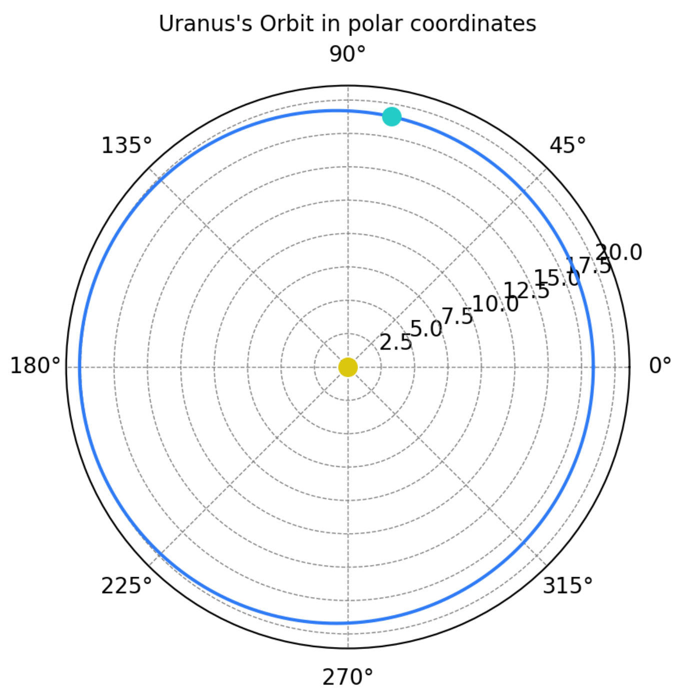
Uranus' Orbit Representation



Note. Figure created by the authors.

Figure 15

Uranus' Orbit in polar coordinates



Note. *Figure created by the authors.*

9. Modeling Neptune's Orbit

In this chapter, we will explore Neptune. Dark, cold, and battered by supersonic winds. Neptune is the only planet in our solar system that cannot be seen with the naked eye. Neptune is so far from the Sun that noon on the big blue planet would appear to us like a faint twilight. The warm sunlight we experience here on Earth is about 900 times brighter than the sunlight on Neptune. Remarkably, Neptune was the first planet to be discovered through mathematical predictions. Based on Urbain Le Verrier's calculations, Johann Galle located the planet in 1846. Neptune is named after the Roman god of the sea, as suggested by Le Verrier.

9.1 Aphelion and Perihelion

The aphelion distance in Neptune's orbit is equal to 30.441 au *(International Astronomical Union)*. In kilometers:

$$x = 30.441 \times 149,597,870.7$$

$$30.441au \approx 4,553,908,781.9787km$$

The perihelion distance in Saturn's orbit is equal to 9.048 au *(International Astronomical Union)*. In kilometers:

$$x = 29.766 \times 149,597,870.7$$

$$29.766au \approx 4,452,930,219.2562km$$

9.2 Semi-major Axis

$$\frac{Aphelion + Perihelion}{2} = a$$

$$\frac{30.441au + 29.766au}{2} = a$$

$$\frac{60.207}{2} = a$$

$$30.1035au = a$$

9.3 Focal Distance

$$Aphelion - a = c$$

$$30.441au - 30.1035au = c$$

$$c = 0.3375au$$

9.4 Semi-minor Axis

$$b = \sqrt{a^2 - c^2}$$

$$b = \sqrt{(30.1035)^2 - (0.3375)^2}$$

$$b = \sqrt{906.2207123 - 0.11390625}$$

$$b = \sqrt{906.1068061}au$$

$$b \approx 30.10160803au$$

This means that the difference between the semi-major and semi-minor axis of Saturn is:

$$30.1035 - \sqrt{906.1068061} \approx 0.0018919682au$$

or

$$0.0018919682 \times 149,597,870.7 \approx 283,034.41415km$$

9.5 Eccentricity

$$\epsilon = \frac{c}{a}$$

$$\epsilon = \frac{0.3375}{30.1035} = 0.01121132094$$

9.6 Plotting Neptune's Orbit

9.6.1 Cartesian Coordinates

The formula that represents Saturn's Orbit around the sun will be:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a = 30.1035, b = \sqrt{906.1068061}$$

Replacing the values gives:

$$\frac{x^2}{30.1035^2} + \frac{y^2}{(\sqrt{906.1068061})^2} = 1$$

9.6.2 Polar Coordinates

$$r = \frac{r_0}{1 + \epsilon\cos(\theta)}$$

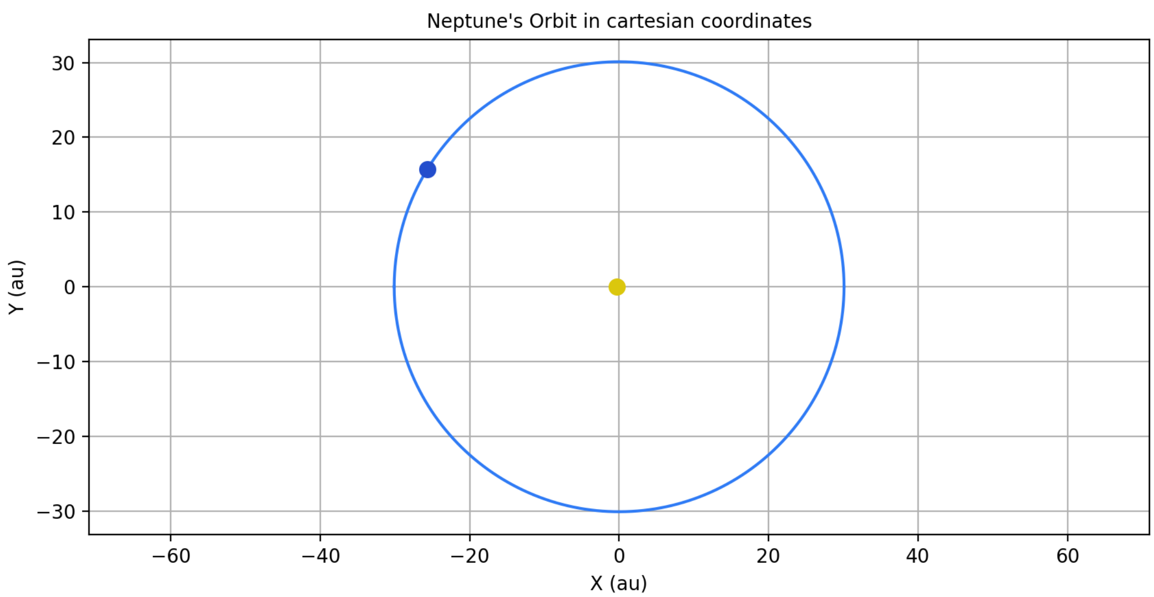
$$r_0 = \frac{b^2}{a} = \frac{(\sqrt{906.1068061})^2}{30.1035} \approx 30.0997161791$$

Replacing the values give us the following equation:

$$r = \frac{30.0997161791}{1 + 0.01121132094\cos(\theta)}$$

Figure 16

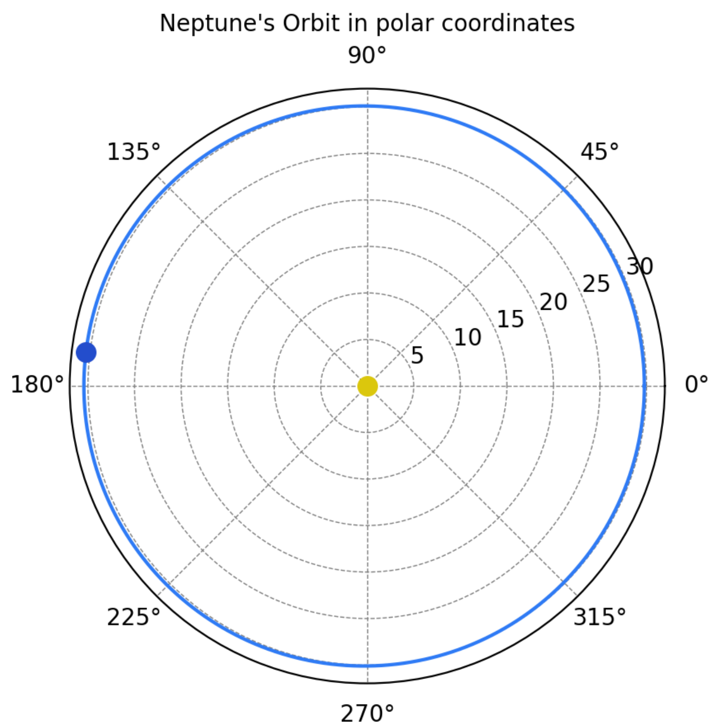
Neptune's Orbit Representation



Note. Figure created by the authors.

Figure 17

Neptune's Orbit in polar coordinates



Note. Figure created by the authors.

ANALYSIS

1. Enhancing Orbital Understanding Through Computational Simulation

This section provides a clear explanation of how the code was developed to simulate planetary orbits in Python. The main goal was to model the motion of a body in space by applying the laws of Newtonian mechanics. To achieve this, the Verlet numerical integration method was employed, enabling the precise calculation of the orbital trajectory over time. The process involved several key steps: calculating acceleration using Newton's Law of Universal Gravitation, establishing initial conditions and velocities, and iteratively integrating motion using the Verlet algorithm. Throughout the development, special attention was given to maintaining the physical consistency of the model, which involved carefully converting between different units—such as meters, Astronomical Units (AU), and days—to ensure the accuracy and reliability of the results.

1.1 Polar and Cartesian Coordinates

As mentioned earlier, the Verlet method is a numerical technique used to estimate an object's next position based on its two previous positions and the current acceleration, without needing to calculate velocity. In the code, the verlet function applies this method to update the planet's position at each step of the simulation. It takes the current and previous coordinates along with the acceleration caused by the Sun's gravity to figure out the next location of the planet.A key part of the project was the implementation of Cartesian and polar coordinate plots to represent the simulated planetary orbits. In the code, this visualization was made possible through the use of the NumPy and Matplotlib libraries, which allowed for efficient data handling and the generation of high-quality graphs.

For the Cartesian coordinates, the code generated a set of points along the x-axis representing the span of the semi-major axis. Based on these values, the corresponding y-values were calculated using the ellipse equation. This resulted in a complete two-dimensional representation of the planet's orbit. In each graph, the position of the Sun was marked at one of the foci of the ellipse, and the planet was also placed at an arbitrary point along its path to visualize its motion.

For the polar coordinates, a different approach was used. Multiple values of the angle were generated, covering a full revolution (from 0 to 2), and for each of these, the corresponding radial value r was calculated using the polar equation of the ellipse. This made it possible to plot the orbit with the Sun located at the center of the coordinate system, offering a complementary perspective to the Cartesian view. The plt.polar() function was used for this purpose, with visual parameters adjusted to ensure consistency with the other graphical representations in the project.

2. Verlet Integration Method

As mentioned earlier, the Verlet method is a numerical technique used to estimate an object's next position based on its two previous positions and the current acceleration, without needing to calculate velocity. In the code, the verlet function applies this method to update the planet's position at each step of the simulation. It takes the current and previous coordinates along with the acceleration caused by the Sun's gravity to figure out the next location of the planet.

2.1 Acceleration Calculation

The acceleration function calculates the gravitational acceleration that the orbiting body experiences due to the presence of the central body, using Newton's Law of Universal Gravitation. This function determines how strongly the orbiting body is pulled based on the mass of the central body and the distance between them. It computes the acceleration as a vector, breaking it into x and y components based on the relative positions of the two bodies.

2.2 Initialization

The simulation function begins by establishing the initial conditions for the orbiting body. A time step of one day is defined to represent the temporal resolution of the simulation. The perihelion and aphelion values of the orbiting body are converted from

Astronomical Units to meters to ensure consistency with the gravitational constant used in acceleration calculations. The initial position is set on the positive x-axis, aligning with the planet's semi-major axis, and the initial velocity is determined based on the principle of energy conservation, assuming all initial motion occurs in the y-direction. This setup reflects a physically accurate starting condition for an elliptical orbit. To enable the Verlet method, the position for the next time step is estimated using a simple Euler-like update, providing the two required positions to begin integration. Finally, the orbital period is estimated using Kepler's Third Law, converting the result into days.

2.3 Data Reduction

To manage the amount of data generated, the total number of position points is compared to a predefined threshold. If the total exceeds this threshold, the simulation applies a downsampling process. A step size is calculated to select every nth point from the original dataset, reducing the number of points while still preserving the shape and continuity of the orbit. The last point is always included to ensure that the full trajectory is represented. If the total number of points is already within the threshold, no reduction is applied, and the original dataset is retained in full.

3. Main Loop

The core of the simulation is a loop that runs for some days equivalent to the calculated orbital period. During each iteration, the simulation updates the orbiting body's position using the Verlet integration method. It begins by retrieving the current and previous positions from the trajectory lists. Then, it calculates the x and y components of the displacement vector between the central and orbiting bodies, taking into account that the central body is positioned at one focus of the ellipse rather than at the origin. The total distance is computed using these components, and the gravitational acceleration is recalculated for the current position. Using this acceleration, the new position is computed and stored.

4. Data Handling and Output

After completing the main simulation loop, the program generates a large set of x-position and y-position data points that represent the complete orbital trajectory. To improve visualization performance and reduce computational load during further processing, the simulation incorporates a data reduction step. This process selectively samples points from the full dataset to retain the overall shape and accuracy of the orbit while minimizing the amount of data used for plotting.

5. Plotting Setup Non-Animation Aspects

Although the focus of the simulation is not on animation, the code includes a well-structured setup for static plotting to visualize the computed orbital data. The simulation initializes an axes object to display both the orbit and the central body. The central body is plotted at one of the foci of the ellipse, consistent with its physical location in an elliptical orbit. The display window is dynamically adjusted to accommodate the minimum and maximum values of the calculated positions, with padding added to ensure clarity. The aspect ratio is fixed to “equal” to preserve the true proportions of the elliptical path and avoid distortion.

CONCLUSION

This project developed a computational model to simulate the planetary orbits of the solar system. Using only the aphelion and perihelion values of each planet, key orbital elements such as the semi-major axis, semi-minor axis, eccentricity, and focal distance were calculated. With this data, elliptical trajectories were generated using geometric equations, and the Verlet numerical integration method was applied to simulate dynamic motion over time. The entire process was implemented in Python, and the results were visualized in both Cartesian and polar coordinates.

The results demonstrated that it is possible to accurately represent the shape of an elliptical orbit using only the aphelion and perihelion values, without needing an extensive dataset. Through the use of geometric formulas, a coherent trajectory was constructed, and with the Verlet method, it was possible to simulate how the planet's position changes over time. This approach, although simplified, proved to be effective in capturing the principles of orbital motion, especially under the assumption of a two-body system.

When comparing the orbit generated through the exact ellipse formula with the trajectory obtained using the Verlet method, only minimal differences were observed. Visually, both curves showed a high degree of similarity, which validates the effectiveness of the numerical method as an approximation tool. While the geometric model is more immediate and exact in shape, the Verlet simulation adds a temporal dimension that allows the dynamics of motion to be observed. The small difference between both methods highlights the ability of the numerical approach to faithfully represent real physical phenomena.

Finally, it is important to note that the file to view the entire code and simulation is called 'main-simulation.py' and all the files are in github.

Link: https://github.com/omiher08/planetary-orbit-plotter

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