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Derivation steps for the most accurate Fouth-Order-Runge-Kutta method.

Question:

$$\frac{dy}{dx} = -2y + x + 4, u(0) = 1, h = 0.2$$

The following is the formula I did on my program and report.

$$k1 = f(x_n, y_n)$$
$$k2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} * k1\right)$$
$$k3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} * k2\right)$$
$$k4 = f(x_n + h, y_n + h * k3)$$
$$y_{n+1} = y_n + \frac{h}{6} * (k1 + 2 * k2 + 2 * k3 + k4)$$

Calculation steps:

K1 = f(0, y(0)) = f(0, 1) = -2*1+0+4 = 2

K2 = f(0.1, y(0)+2*0.2/2) = f(0.1, 1.2) = -2*1.2+0.1+4 = 1.7

K3 = f(0.1, y(0)+1.7*0.2/2) = f(0.1, 1.17) = -2*1.17+0.1+4 = 1.76

K4 = f(0.2, y(0)+1.76*0.2) = f(0.2, 1.352) = -2*1.352+0.2+4 = 1.496 ,

y(0.2) = y(0)+(1/6)*(2+2*1.7+2*1.76+1.496)*0.2 = 1.3472 .