Derivations of Activation Functions

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1 ReLU (Rectified Linear Unit)

f(x) = $\{0 \quad if \quad x < 0; x \quad if \quad x \geq 0$
$f'(x) = \{0 \quad if x < 0; \quad 1 \quad if x \geq 0$

2 Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$
$\sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$
$= \sigma(x)(1 - \sigma(x))$

3 tanh (Hyperbolic Tangent)

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$\tanh'(x) = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$
$= \frac{e^{2x}+2+e^{-2x} - (e^{2x}-2+e^{-2x})}{(e^x+e^{-x})^2}$
$= \frac{4}{(e^x+e^{-x})^2}$
$= 1 - \tanh^2(x)$
another derivation could be using hyperbolic functions $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
which will lead to the same results

4 $\tanh(x) = 2 \times \sigma(2x) - 1$

$2 \times \sigma(2x) = \frac{2}{1+e^{-2x}} =$
$\frac{2}{1+e^{-2x}} + \frac{-1}{1} =$
$\frac{-1-e^{-2x}+2}{1+e^{-2x}} =$
$\frac{1-e^{-2x}}{1+e^{-2x}}$