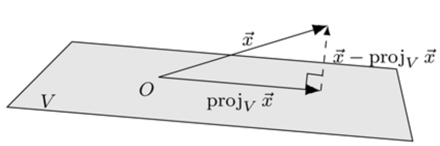
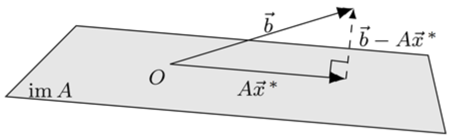
Linear Algebra Fact Sheet

1. There can be at most n linearly independent variables in $\mathbb{R}^n$.
2. Shears are invertible linear transformations.
3. Linear transformations are determined by what they do to a basis of their domain or image.
4. The columns of a matrix span its image, though eliminating redundancies is often times necessary.
5. If u is a linear combination of v and w, it does not necessarily have to be made of non-zero amounts of each vector.
6. A general 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a helpful way of finding out characteristics of the matrix's components.
7. The $\ker(A)$ always equals the $\ker(\mathit{rref}(A))$.
8. The $\operatorname{im}(A)$ doesn't necessarily equal the $\operatorname{im}(\mathit{rref}(A))$.
9. If $\vec{x}_1$ is a solution to $A\vec{x} = \vec{b}$, then $A\vec{x}_1 = \vec{b}$.
10. If A is invertible, then $\det(A^{-1}) = \frac{1}{\det(A)}$.
11. If A is not invertible, then $\det(A) = 0$
12. The kernel and image of a matrix A are not mutually exclusive.
13. A matrix A with dimensions $m \times n$ performs the following transformation: T:$\mathbb{R}^n \to \mathbb{R}^m$, and the sum of the dimensions of the kernel and image of A equals n.
14. For a given vector $\vec{x}$, there is more than one basis $\mathfrak{B}$, such that the $[\vec{x}]_{\mathfrak{B}} =$ some vector $\vec{b}$.
15. If A is an $n \times n$ matrix, then the trace of A, or $\operatorname{tr}(A) = \lambda_1 + \lambda_2 + ... + \lambda_n$
16. If A is an $n \times n$ matrix, then the determinant of A, or $\det(A) = \lambda_1 \cdot \lambda_2 \cdot ... \cdot \lambda_n$
17. The direction field of $\frac{dx}{dt} = -A\vec{x}$ has arrows that are exactly the negatives of those in $\frac{dx}{dt} = A\vec{x}$

18. For any $n \times n$ matrix A, $\det(A^T) = \det(A)$
19. For any $n \times n$ matrix A, $\ker(A^T) = (\mathrm{im} A)^\perp$
20. For any subspace V of R^n, $\dim V + \dim(V^\perp) = n$
21. For any symmetric $n \times n$ matrix A, $A = A^T$
22. If the matrices A and B are invertible $\rightarrow$ AB is invertible.
23. If the matrix AB is invertible $\nrightarrow$ A and B are invertible.
24. If the matrices A and B are invertible $\nrightarrow$ $A + B$ is invertible.
25. If the matrices A and B are symmetric $\nrightarrow$ AB is symmetric.
26. If the matrices A and B are symmetric $\rightarrow$ $A + B$ is symmetric.
27. If the matrices A and B are diagonal $\rightarrow$ AB is diagonal, and $A + B$ is diagonal.
28. If the matrices A and B are orthogonal $\rightarrow$ AB is orthogonal.
29. If the matrices A and B are orthogonal $\nrightarrow$ $A + B$ is orthogonal.
30. If the matrices A and B are diagonalizable with the same set of eigenvectors $\rightarrow$ AB is diagonalizable.
31. If the matrices A and B are diagonalizable with different sets of eigenvectors $\nrightarrow$ AB is diagonalizable.
32. If the matrices A and B are diagonalizable $\nrightarrow$ $A + B$ is diagonalizable.
33. A square matrix is invertible if and only if its rows are linearly independent. That means no row can be expressed as the weighted sum of other rows.
34. Although nothing such can be assumed about the sum of matrices, $\det(AB) = (\det A)(\det B)$
35. 

36. For any $n \times n$ matrix A, $\det(A)$ doesn't necessarily equal $\det(\text{rref}(A))$

37. 

38. The eigenspaces of a symmetric matrix are orthogonal to each other.

39. If λ is an eigenvalue of A, then λ^n is an eigenvalue of A^n. In both cases, eigenvectors remain the same.

40. If A is an orthogonal matrix, then $A^T A = I_n$

41. The zero matrix Z_n is always a good failsafe to test algebraic relationships.

42. The rules of inner products for functions $f, g, h \in V$ and scalar $c \in \mathbb{R}$ are as follows:

    - $\langle f, g \rangle = \langle g, f \rangle$
    - $\langle f + h, g \rangle = \langle g, f \rangle + \langle h, f \rangle$
    - $\langle cf, g \rangle = c\langle g, f \rangle$
    - $\langle f, f \rangle$ is positive for all nonzero f.

43. The inner product space allows us to define:

    - The *norm* of $f \in V$ is $||f|| = \sqrt{\langle f, f \rangle}$
    - The *distance* between $f, g \in V$ is $\sqrt{\langle f - g, f - g \rangle}$
    - Functions $f, g \in V$ are *orthogonal* when $\langle f, g \rangle = 0$

44. The inner product used for Fourier Analysis is $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x) dx$

45. Parseval's Theorem states that the sum of the squares of the coefficients of the Fourier Series of a function f equals $\langle f, f \rangle$.

46. Interval start and endpoints, as well as points of discontinuity are important to check to see if a Fourier or Fourier Sine Series equals a function f on a certain interval.