Table of Distributions

Distribution	Story	$\mathbf{E}(X)$	$\mathbf{Var}(X)$	PMF or PDF
Discrete Uniform $X \sim \text{DUnif}(a, b)$	Roll a dice with face values on the interval $[a, b]$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$, where $n = b - a + 1$	$P(X = k) = \frac{1}{n}$ $k \in \{a, a+1, \ldots, b-1, b\}$
Bernoulli $X \sim \text{Bern}(p)$	A trial is performed with probability p of "success", and X is the indicator of success: 1 means success, 0 means failure.	p	pq	$P(X = 1) = p$ $P(X = 0) = q = 1 - p$
Binomial $X \sim \text{Bin}(n, p)$	X is the number of "successes" that we will achieve in n independent trials, where each trial is either a success or a failure, each with the same probability p of success.	np	npq	$P(X = k) = \binom{n}{k} p^k q^{n-k}$ $k \in \{0, 1, 2, \ldots n\}$
Negative Binomial $X \sim \text{NBin}(r, p)$	X is the number of "failures" we will have before we achieve our rth success. Our successes have probability p.	$r \cdot \frac{q}{p}$	rq/p^2	$P(X = n) = \binom{r+n-1}{r-1} p^r q^n$ $n \in \{0,\ 1,\ 2,\ \ldots\}$
Geometric $X \sim \text{Geom}(p)$	X is the number of "failures" we will achieve before we achieve our first success. Our successes have probability p.	$\frac{q}{p}$	$\frac{q}{p^2}$	$P(X = k) = q^k p$ $k \in \{0,\ 1,\ 2,\ \ldots\}$
First Success $X \sim \text{FS}(p)$	X is the number of trials that we will run until we achieve our first success. Our successes have probability p.	$\frac{1}{p}$	$\frac{q}{p^2}$	$P(X = k) = pq^{k-1}$ $k \in \{1,\ 2,\ 3,\ \ldots\}$
Hyper Geometric $X \sim \text{HGeom}(w, b, n)$	In a population of w desired objects and b undesired objects, X is the number of "successes" we will have in a draw of n objects, without replacement.	$\mu = \frac{nw}{w+b}$	$\frac{w+b-n}{w+b-1} \cdot \mu(1 - \frac{\mu}{n})$	$P(X = k) = \binom{w}{k}\binom{b}{n-k} / \binom{w+b}{n}$ $k \in \{0, 1, 2, \ldots, n\}$
Negative Hyper Geometric $X \sim \text{NHGeom}(w, b, r)$	In an urn with w white balls and b black balls, sample balls without replacement until you get r white balls, then count the number of black balls in the sample.	$\frac{rb}{w+1}$	$\frac{rb(w+b+1)(w-r+1)}{(w+1)^2(w+2)}$	$\frac{\binom{r+k-1}{r-1}\binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$ $k \in \{0, 1, 2, \ldots, n\}$
Poisson $X \sim \text{Pois}(\lambda)$	There are rare events that occur many different ways at an average rate of λ occurrences per unit space or time. The occurrences in that unit of space or time is X.	λ	λ	$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$ $k \in \{0,\ 1,\ 2,\ \ldots\}$
Uniform $X \sim \text{Unif}(a, b)$	A world where every value on an open interval is equally as likely.	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$f(x) = \frac{1}{b-a}$ $x \in (a, b)$
Normal $X \sim \mathcal{N}(\mu, \sigma^2)$	For data influenced by various factors you'll see a central clustering of points on a graph around a mean. This creates a bell-shaped curve, known as the normal distribution.	μ	σ^2	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x\ -\ \mu)^2/(2\sigma^2)}$ $x \in (-\infty, \infty)$
Exponential $X \sim \text{Expo}(\lambda)$	You wait for a bus, and note the time it takes for the first bus to arrive. The exponential distribution describes the probability of waiting different durations for the next bus.	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$f(x) = \lambda e^{-\lambda x}$ $x \in (0, \infty)$
Gamma $X \sim \text{Gamma}(a, \lambda)$	You wait for n shooting stars, where the wait time for a star is $\sim \text{Expo}(\lambda)$. The total wait time for the nth star is $\text{Gamma}(n, \lambda)$.	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$f(x) = \frac{1}{\Gamma(a)}(\lambda x)^a e^{-\lambda x} \frac{1}{x}$ $x \in (0, \infty)$
Beta $X \sim \text{Beta}(a, b)$	The conjugate prior of the binomial distribution.	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{(a+b+1)}$	$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$ $x \in (0, 1)$
Log-Normal $X \sim \mathcal{LN}(\mu, \sigma^2)$	X's logarithm follows a Normal distribution with mean μ and variance σ^2.	$\theta = e^{\mu + \sigma^2/2}$	$\theta^2(e^{\sigma^2} - 1)$	$\frac{1}{x\sigma\sqrt{2\pi}} e^{-(\log x - \mu)^2/(2\sigma^2)}$ $x \in (0, \infty)$
Chi-Square $X \sim \chi_n^2$	A Chi-Square(n) is the sum of the squares of n independent standard Normal r.v.s.	n	$2n$	$\frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$ $x \in (0, \infty)$

Important Properties

Binomial

If $X \sim \text{Bin}(n, p), Y \sim \text{Bin}(m, p)$ & $X \perp\!\!\!\perp Y$:

- **Redefine Success:** $n - X \sim \text{Bin}(n, 1 - p)$
- **Conditional Distribution:** $X|(X + Y = r) \sim \text{HGeom}(n, m, r)$
- **Binomial-Poisson:** $\text{Bin}(n, p)$ is approximately $\text{Pois}(\lambda)$ if p is small.
- **Binomial-Normal:** $\text{Bin}(n, p)$ is approximately $\mathcal{N}(np, np(1 - p))$ if n is large and p is not near 0 or 1.

Poisson

Let $X \sim \text{Pois}(\lambda_1)$, $Y \sim \text{Pois}(\lambda_2)$, & $X \perp\!\!\!\perp Y$.

- **Conditional Distribution:** $X|(X + Y = n) \sim \text{Bin}\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$
- **Chicken-Egg:** For $Z \sim \text{Pois}(\lambda)$ items, if we randomly & independently "accept" each item with probability p, then the number of accepted items $Z_1 \sim \text{Pois}(\lambda p)$, and the number of rejected items $Z_2 \sim \text{Pois}(\lambda(1 - p))$, and $Z_1 \perp\!\!\!\perp Z_2$.

Uniform

- For $\text{Uniform}(0, 1)$, $F(x) = x$, for $x \in (0, 1)$.

Exponential

- **Rescaling:** $Y \sim \text{Expo}(\lambda) \to X = \lambda Y \sim \text{Expo}(1)$.
- **Memorylessness:** For $X \sim \text{Expo}(\lambda)$ and any positive numbers s and t, $P(X > s + t | X > s) = P(X > t)$.
- **CDF:** $F(x) = 1 - e^{-\lambda x}$, for $x \in (0, \infty)$.
- **Min. of Expos:** For $X_i \sim \text{Expo}(\lambda_i)$, $\min(X_1, \ldots, X_k) \sim \text{Expo}(\lambda_1 + \cdots + \lambda_k)$.

Normal

- **Scaling:** $Z = \frac{X-\mu}{\sigma} \sim \mathcal{N}(0, 1)$

Beta

If $X \sim \text{Gamma}(a, \lambda)$, $Y \sim \text{Gamma}(b, \lambda)$, & $X \perp\!\!\!\perp Y$:

- $\frac{X}{X+Y} \sim \text{Beta}(a, b)$.
- $X + Y \perp\!\!\!\perp \frac{X}{X+Y}$

Chi-Square

For $X \sim \chi_n^2$:

- **Definition:** X is distributed as $Z_1^2 + Z_2^2 + \cdots + Z_n^2$ for i.i.d. $Z_i \sim \mathcal{N}(0, 1)$.

Other Distributions

- **Student-t:** $t_n \sim \frac{Z}{\sqrt{V/n}}$, where Z is the standard Normal & $V \sim \chi_n^2$.
- **Cauchy:** A function with undefined mean and variance, equivalent to t_1; similar to the normal but with heavier tails.