Probability and Statistics (PHM111s)-Lecture 5

Part I: Introduction to Statistical Methods.

Part II: Methods of Descriptive Statistics.

1- Collecting Data.
2- Organizing Data.
3- Presenting Data.
4- Summarizing Data.

Part III: Introduction to Probability.

Part IV: Methods of Inferential Statistics.

Introduction to Probability

Random Experiment: An experiment whose results can't be predicted with certainty (or that can result in different outcomes, even though it is repeated in the same manner every time), is called a **random experiment**.

Sample Space: The set of all possible outcomes of a random experiment is called the **sample space** of the experiment. The sample space is denoted as S.

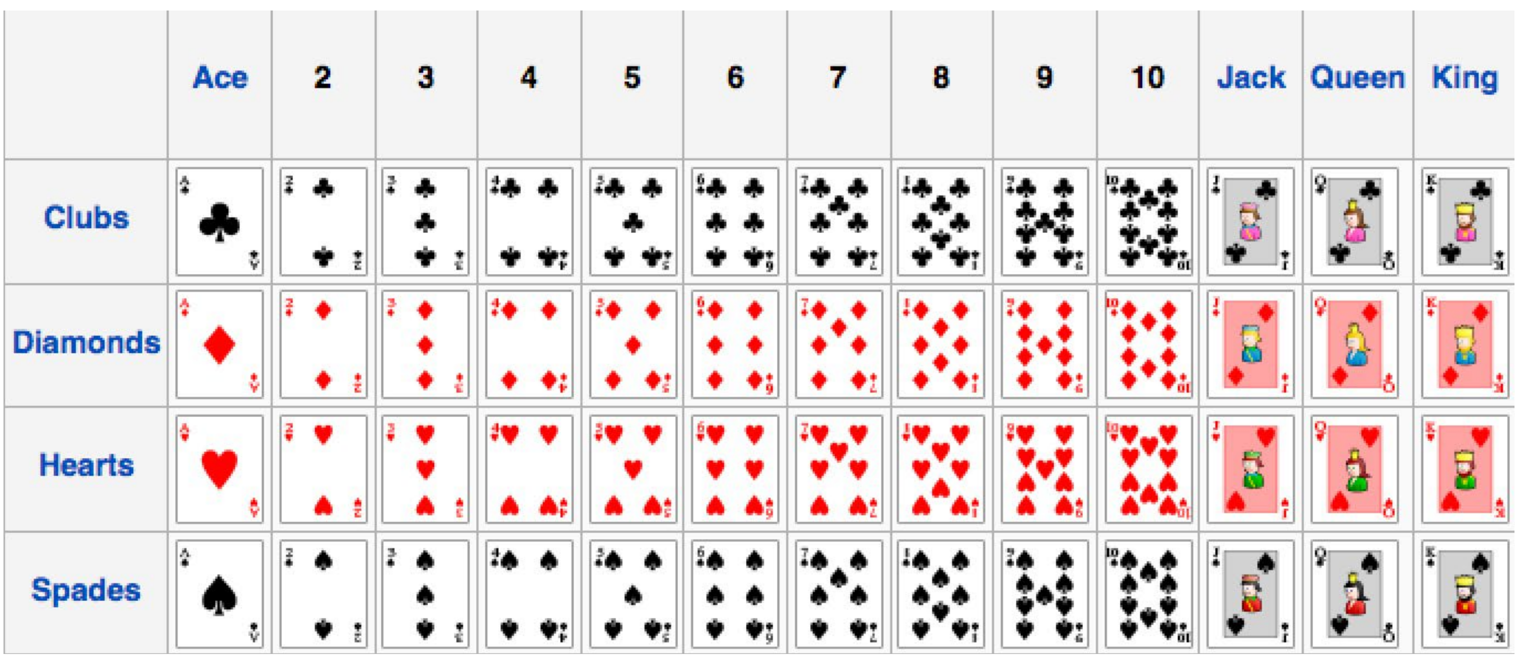
sample point: Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**. If the sample space has a finite number of elements, we may *list* the members separated by commas and enclosed in braces.

Event: An **event** is a subset of the sample space of a random experiment.

- Throwing (Rolling) a die, $S = \{1, 2, 3, 4, 5, 6\}$.
  Event A: even number appears. How many ways?

- Flipping a coin once, $S = \{H, T\}$, where H and T correspond to heads and tails, respectively.
  Event B: "H" appears. How many ways?

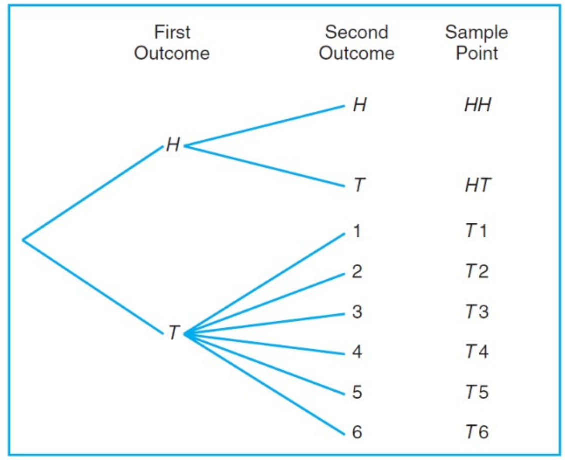
- Drawing (Picking) a card from a deck, $S = \{A\heartsuit, A\spadesuit, A\diamondsuit, A\clubsuit, 2\heartsuit, \ldots, K\clubsuit\}$.



Event C: "Heart" appears. How many ways?

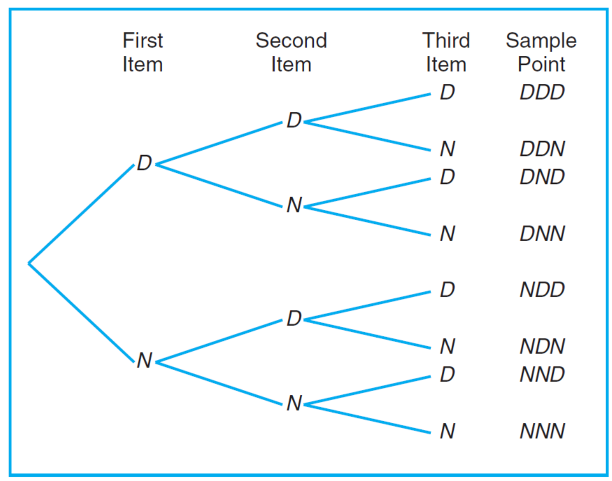
Sample spaces can also be described graphically with **tree diagrams**.

Example 5.2: An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. To list the elements of the sample space providing the most information, we construct the tree diagram of the opposite figure. The various paths along the branches of the tree give the distinct sample points. Starting with the top left branch and moving to the right along the first path, we get the sample point *HH*, indicating the possibility that heads occurs on two successive flips of the coin. Likewise, the sample point *T*3 indicates the possibility that the coin will show a tail followed by a 3 on the toss of the die. By proceeding along all paths, we see that the sample space is:



$$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}.$$

Example 5.3: Suppose that three items are selected at random from a manufacturing process. Each item is inspected and classified defective, *D*, or nondefective, *N*. To list the elements of the sample space providing the most information, we construct the tree diagram of the following figure. Now, the various paths along the branches of the tree give the distinct sample points. Starting with the first path, we get the sample point *DDD*, indicating the possibility that all three items inspected are defective. As we proceed along the other paths, we see that the sample space is:



$$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}.$$

Event A: one defective occurs (of course $\Rightarrow$also two nondefectives!!!). How many ways?

Consider the situation of sampling until *k* defectives are observed. Suppose the experiment is to sample items randomly until one defective item is observed. The sample space for this case is:

$$S = \{D, ND, NND, NNND, \ldots\}.$$

If the experiment stopped after the third test. Define the event A!!!

Example 5.4: Given the sample space $S = \{t \mid t \geq 0\}$, where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset $A = \{t \mid 0 \leq t < 5\}$.

Different Types of Probability:

Experimental (Observed) Probability

$$P(A) = \frac{\text{No. of times "A" occured}}{\text{No. of times Exp. was repeated}}.$$

Theoretical (Classical) Probability

$$P(A) = \frac{\text{No. of ways "A" } \underline{\underline{\text{should}}} \text{ occur}}{\text{No. of outcomes}}.$$

Subjective Probability

Subjective probability is based on a person's own personal reasoning and judgment (Educated guess)

Ex1. If the coin is flipped 50 times and it lands on heads 28 times:

Then the Experimental (Observed) probability is $\frac{28}{50}$.

Ex2. On a die, to get a "3":

Then the Theoretical (Classical) probability is $\frac{1}{6}$.

Ex3. You came to a Lecture late:

Then the Subjective probability that the lecturer will drive you out of the hall is

Some Important Events:

1- **Impossible event** and denoted by the symbol φ, which contains no elements at all.

$$P(\varphi) = 0.$$

2- **Sure (certain)**and denoted by the symbol S, which contains all elements.

$$P(S) = 1.$$

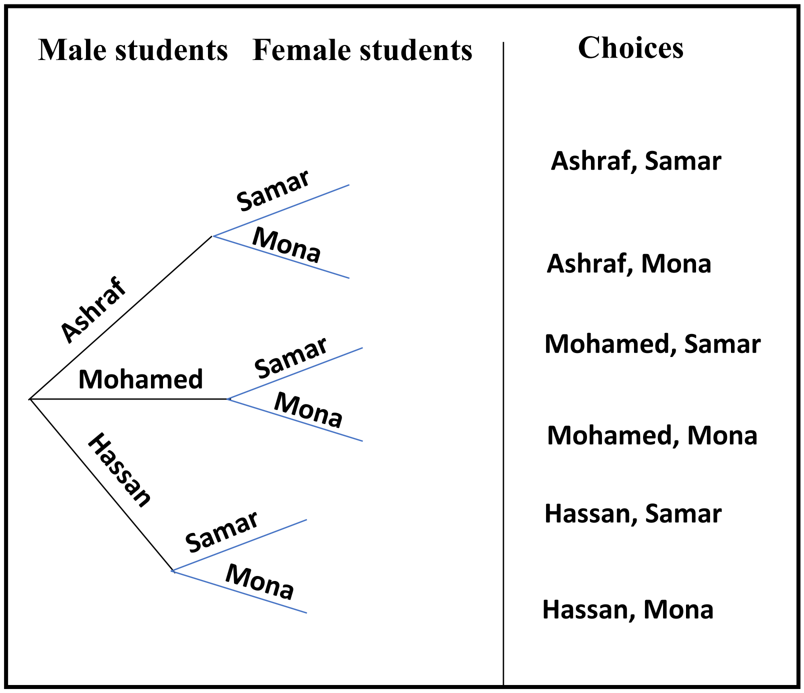
Counting Methods:

1- Multiplication rule of counting:

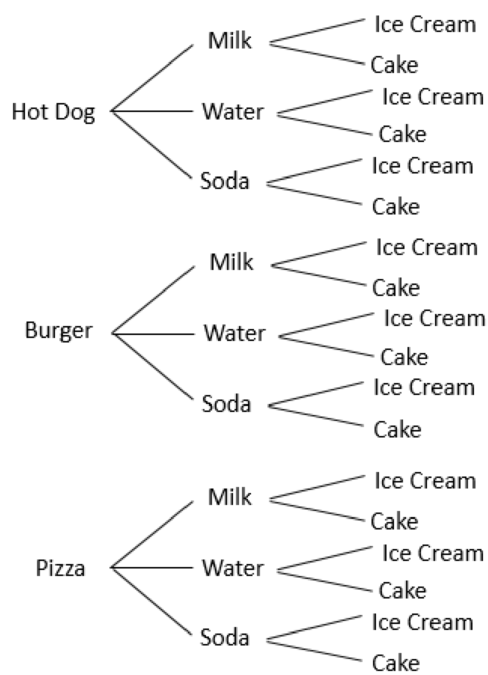
Rule 1: If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

Example 5.13: How many ways are there to choose a male student out of three students (Ashraf – Mohamed – Hassan) and a female student out of two students (Samar – Mona)?

Solution: Since the number of ways of choosing a male student out of three students (n_1) = 3 ways and the number of ways of choosing a female student out of two students (n_2) = 2 ways, then the number of ways of choosing is $n_1 n_2 = (3)(2) = 6$ ways.



Rule 2: If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 \cdots n_k$ ways.



Example 5.14: Sam is going to assemble a computer by himself. He has the choice of chips from two brands, a hard drive from four, memory from three, **and** an accessory bundle from five local stores. How many different ways can Sam order the parts?

Solution: Since $n_1 = 2$, $n_2 = 4$, $n_3 = 3$, and $n_4 = 5$, there are $n_l \times n_2 \times n_3 \times n_4 = 2 \times 4 \times 3 \times 5 = 120$ different ways to order the parts.

Example:

(a) How many four digit numbers can be formed using only the digits 1, 2, 3, 4, 5 and 6?
(b) How many four digit numbers from (a) have no repeated digits?
(c) How many four digit numbers from (b) are greater than 5000?
(d) How many even four digit numbers from (b)?

Solution: (a) 6 * 6 * 6 * 6 = 1296.
(b) 6 * 5 * 4 * 3 = 360.
(c) 2 * 5* 4 * 3 = 120.
(d) 3 * 5 * 4 * 3 =180.

2- Addition rule of counting:

If an operation can be performed in n_1 ways and a second operation can be performed in n_2 ways, and we cannot do both at the same time, then there are $n_1 + n_2$ways to choose one of the actions.

Example: Suppose that we are planning a trip and are deciding between bus **or** train transportation. If there are three bus routes and two train routes, then there are 3+2 = 5 different routes available for the trip.

3- Permutations:

Definition: A **permutation** is a different arrangement of a set of objects.

Consider the three letters *a*, *b*, and *c*. The possible permutations are *abc*, *acb*, *bac*, *bca*, *cab*, and *cba*. Thus, we see that there are 6 distinct arrangements.

$$n_1n_2n_3 = (3)(2)(1) = 6 \text{ permutations}$$

by Rule 2. In general, n distinct objects can be arranged in

$$n(n-1)(n-2)\cdots(3)(2)(1) = n! \text{ ways.}$$

The number of permutations of the four letters *a*, *b*, *c*, and *d* will be 4! = 24.

If "n" different items and "r" items from them to be arranged given the **order is important:**

Consider the number of permutations that are possible by taking two letters at a time from four. These would be *ab*, *ac*, *ad*, *ba*, *bc*, *bd*, *ca*, *cb*, *cd*, *da*, *db*, and *dc*. Using Rule 1 again, we have two positions to fill, with $n_1 = 4$ choices for the first and then $n_2 = 3$ choices for the second, for a total of

$$n_1n_2 = (4)(3) = 12$$

permutations. In general, n distinct objects taken r at a time can be arranged in

$$n(n-1)(n-2)\cdots(n-r+1) \quad \text{ways.}$$

Theorem: The number of permutations of n distinct objects taken r at a time is

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Example 5.16: How many ways can three letters be formed from the four letters A, B, C, and D?
***Solution*:** The total number of ways is

$$_{4}P_{3} = \frac{4!}{(4-3)!} = \frac{4!}{1!} = (4)(3)(2)(1) = 24.$$

A, B, C	C, A, B
A, B, D	C, A, D
A, C, D	C, B, A
A, C, B	C, B, D
A, D, B	C, D, A
A, D, C	C, D, B
B, A, C	D, A, B
B, A, D	D, A, C
B, C, A	D, B, A
B, C, D	D, B, C
B, D, A	D, C, A
B, D, C	D, C, B

4- Combinations:

Now consider the number of permutations that are possible by taking two letters at a time from four. These would be *ab*, *ac*, *ad*, *ba*, *bc*, *bd*, *ca*, *cb*, *cd*, *da*, *db*, and *dc*.
From the previous data, we notice that selecting *ab* is different from selecting *ba* and so on…
If we want to select from the previous **disregarding the order**, the all possible choices are: *ab*, *ac*, *ad*, *bc*, *bd* and *cd* and each choice of these choices is called "Combination"

Theorem: The number of combinations of n distinct objects taken r at a time is

$$^{n}C_{r} \equiv \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example 5.19: How many ways can three letters be formed from the four letters A, B, C, and D **disregarding the order**?
***Solution*:** The total number of ways is

$$^{4}C_{3} = \binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!1!} = (4).$$

A, B, C	C, A, B
A, B, D	C, A, D
A, C, D	C, B, A
A, C, B	C, B, D
A, D, B	C, D, A
A, D, C	C, D, B
B, A, C	D, A, B
B, A, D	D, A, C
B, C, A	D, B, A
B, C, D	D, B, C
B, D, A	D, C, A
B, D, C	D, C, B

Example 6.1: A coin is tossed twice. What is the probability that at least 1 head occurs?

***Solution*:** The sample space for this experiment is

$$S = \{HH, HT, TH, TT\}.$$

If the coin is balanced, each of these outcomes is equally likely to occur. Therefore, we assign a probability of ω to each sample point. Then $4\omega = 1$, or $\omega = 1/4$. If A represents the event of at least 1 head occurring, then

$$A = \{HH, HT, TH\} \text{ and } P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Example 6.2: A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, find $P(E)$.

***Solution*:** The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We assign a probability of w to each odd number and a probability of $2w$ to each even number. Since the sum of the probabilities must be 1, we have $9w = 1$ or $w = 1/9$. Hence, probabilities of 1/9 and 2/9 are assigned to each odd and even number, respectively. Therefore,

$$E = \{1, 2, 3\} \text{ and } P(E) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$$

Example 6.5: In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.

***Solution*:** The number of ways of being dealt 2 aces from 4 cards is

$$\binom{4}{2} = \frac{4!}{2!2!} = 6,$$

and the number of ways of being dealt 3 jacks from 4 cards is

$$\binom{4}{3} = \frac{4!}{3!1!} = 4.$$

By the multiplication rule, there are $n = (6)(4) = 24$ hands with 2 aces and 3 jacks. The total number of 5-card poker hands, all of which are equally likely, is

$$N = \binom{52}{5} = \frac{52!}{5!47!} = 2{,}598{,}960.$$

Therefore, the probability of getting 2 aces and 3 jacks in a 5-card poker hand is

$$P(C) = N = \frac{24}{2{,}598{,}960} = 0.9 \times 10^{-5}.$$