Fall 2022	Major Task (Part 2)	Total: 15 marks

PHM111: Probability and Statistics

1/15

Name:	**ID:**	**Deadline: Week 14**

Please, Solve each problem in its assigned place ONLY (the empty space below it)

Part I: Discrete Random Variables

1. Classify the following random variables as discrete or continuous:

a) X: the number of automobile accidents per year in Virginia. (...................)
b) Y: the length of time to play 18 holes of golf. (...................)
c) M: the amount of milk produced yearly by a particular cow. (...................)
d) N: the number of eggs laid each month by a hen. (...................)

2. Determine the value c so that the following function can serve as a probability distribution of the discrete random variable X: $f(x) = c\binom{2}{x}\binom{3}{3-x}, \quad \text{for} \quad x = 0,\ 1,\ 2.$

3. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X.

a) Express the results graphically as a probability histogram.
Histogram:

b) Find the cumulative distribution function of the random variable X representing the number of defectives.

c) Using F(x), find

i. $P(X = 1)$

ii. $P(0 < X \leq 2)$

d) Construct a graph of the cumulative distribution function.

4. Determine the probability mass function of X from the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \leq x < 0 \\ 0.7 & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

5. A coin is biased such that a head is three times as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice.

6. The distribution of the number of imperfections per 10 meters of synthetic fabric is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

a) Plot the probability function.

b) Find the expected number of imperfections, E(X) = μ.

c) Find $E\left(x^2\right)$

d) Find the variance and standard deviation of the number of imperfections.

7. If a random variable X is defined such that $E\left[\left(x-1\right)^2\right]=10$ and $E\left[\left(x-2\right)^2\right]=6$, find μ and σ^2.

Part II: Special Distributions of Discrete Random Variable

1. In a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district

a) exactly 2 resulted from the need for money to buy drugs.

b) at most 3 resulted from the need for money to buy drugs.

2. A national study that examined attitudes about antidepressants revealed that approximately 70% of respondents believe "antidepressants do not really cure anything; they just cover up the real trouble." According to this study:

a) what is the probability that at least 3 of the next 5 people selected at random will hold this opinion?

b) If X represents the number of people who believe that antidepressants do not cure but only cover up the real problem, find the mean and variance of X when 5 people are selected at random.

3. In a batch of 2000 calculators, there are, on average, 8 defective ones. If a random sample of 150 is selected, find the probability of 5 defective ones.

4. A mail-order company receives an average of 5 orders per 500 solicitations. If it sends out 100 advertisements, find the probability of receiving at least 2 orders.

5. A bookstore owner examines 5 books from each lot of 25 to check for missing pages. If he finds at least 2 books with missing pages, the entire lot is returned. If, indeed, there are 5 books with missing pages, find the probability that the lot will be returned.

Part III: Continuous Random Variables

1. The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \dfrac{20,000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of

a) at least 200 days.

b) anywhere from 80 to 120 days.

2. Consider the density function

$$f(x)=\begin{cases} k\sqrt{x}, & 0<x<1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Evaluate k.

b) Find F(x) and use it to evaluate P(0.3 < X < 0.6).

3. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function:

$$f(x)=\begin{cases} k(3-x^2), & -1\le x\le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Determine k that renders f(x) a valid density function.

b) Find the probability that a random error in measurement is less than 1/2.

c) For this particular measurement, it is undesirable if the magnitude of the error (i.e., |x|) exceeds 0.8. What is the probability that this occurs?

4. On a laboratory assignment, if the equipment is working, the density function of the observed outcome, X, is:

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Calculate P(X ≤ 1/3).

b) What is the probability that X will exceed 0.5?

c) Given that X ≥ 0.5, what is the probability that X will be less than 0.75?

5. The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$f(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-0.01x}, & 0 \le x. \end{cases}$$

Determine the probability density function of X. What proportion of reactions is complete within 200 milliseconds?

Part IV: Special Distributions of Continuous Random Variables

1. Given a standard normal distribution,

a) find the area under the curve that lies between $z = -0.48$ and $z = 1.74$.

b) Find the value of z if the area enclosed between $-z$ and z, is 0.9500.

c) find the value of k such that $P(-0.93 < Z < k) = 0.7235$.

2. Given a normal distribution with $\mu = 30$ and $\sigma = 6$, find the value of x that has 80% of the normal curve area to the left.

3. Given the normally distributed variable X with mean 18 and standard deviation 2.5, find the value of k such that $P(X > k) = 0.1814$.

4. The heights of 1000 students are normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights

a) less than 160.0 centimeters?

b) between 171.5 and 182.0 centimeters inclusive?

c) equal to 175.0 centimeters?

d) greater than or equal to 188.0 centimeters?

5. If a set of observations is normally distributed, what percent of these differ from the mean by

a) more than 1.3σ?

b) less than 0.52σ?

6. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

7. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

a) What is the probability that response time exceeds 5 seconds?

b) What is the probability that response time exceeds 10 seconds?

Part V: Joint Distributions

1. Determine the values of c so that the following function represents joint probability distribution of the random variables X and Y:
$f(x, y) = c|x - y|$, for $x = -2, 0, 2$; $y = -2, 3$.

2. If the joint probability distribution of X and Y is given by
$f(x,y) = \frac{x+y}{30}, \quad \text{for } x = 0,1,2,3;\ y = 0,1,2,$, Find

a) $P(X \leq 2, Y = 1)$

b) $P(X + Y = 4)$

c) the marginal distribution of X

d) the marginal distribution of Y

3. A fast-food restaurant operates both a drive through facility and a walk-in facility. On a randomly selected day, let X and Y , respectively, be the proportions of the time that the drive-through and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y)=\begin{cases} \frac{2}{3}(x+2y), & 0\le x\le 1, 0\le y\le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find the marginal density of X

b) Find the marginal density of Y

c) Find the probability that the drive-through facility is busy less than one-half of the time

4. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f(x,y)=\begin{cases} 24xy, & 0\le x\le 1, 0\le y\le 1, x+y=1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find the probability that in a given box the cordials account for more than 1/2 of the weight.

b) Find the marginal density for the weight of the creams.

5. Suppose that X and Y have the following joint probability distribution:

f(x, y)		x = 2	x = 4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

a) Find the marginal distribution of X

b) Find the marginal distribution of Y

c) Determine whether the two random variables X and Y are dependent or independent.

6. A coin is tossed twice. Let Z denote the number of heads on the first toss and W the total number of heads on the 2 tosses. If the coin is unbalanced and a head has a 40% chance of occurring, find

a) the joint probability distribution of W and Z

b) the marginal distribution of W

c) the marginal distribution of Z

d) the probability that at least 1 head occurs.

7. The joint probability density function of the random variables X, Y, and Z is

$$f(x,y,z)=\begin{cases}\dfrac{4xyz^2}{9}, & 0<x,y<1, 0<z<3,\\ 0, & \text{elsewhere.}\end{cases}$$, Find

a) the joint marginal density function of Y and Z;

b) the marginal density of Y

c) $P(\frac{1}{4}<X<\frac{1}{2}, Y>\frac{1}{3}, 1<Z<2)$;

8. Two tire-quality experts examine stacks of tires and assign a quality rating to each tire on a 3-point scale. Let X denote the rating given by expert A and Y denote the rating given by B. The following table gives the joint distribution for X and Y.

f(x, y)		y		
		1	2	3
x	1	0.10	0.05	0.02
	2	0.10	0.35	0.05
	3	0.03	0.10	0.20

Find μ_x and μ_y.

9. Random variables X and Y follow a joint distribution

$$f(x,y)=\begin{cases}2, & 0<x\le y<1,\\ 0, & \text{otherwise.}\end{cases}$$

Determine the correlation coefficient between X and Y.

10. If X and Y are independent random variables with variances σ_X^2 = 5 and σ_Y^2 = 3, find the variance of the random variable Z = −2X + 4Y − 3.

11. There are two service lines. The random variables X and Y are the proportions of time that line 1 and line 2 are in use, respectively. The joint probability density function for (X, Y) is given by

$$f(x,y)=\begin{cases}\frac{3}{2}(x^2+y^2), & 0\le x,y\le 1,\\ 0, & \text{otherwise.}\end{cases}$$

a) Determine whether or not X and Y are independent.

b) It is of interest to know something about the proportion of Z = X + Y, the sum of the two proportions. Find E(X + Y). Also find E(XY).

c) Find Var(X), Var(Y), and Cov(X,Y).

d) Find Var(X + Y).