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Center
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ENGINEERING MATHEMATICS

New Programs Students

Master

Probability and Statistics

Chapter (2)

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Probability

Random Experiment :

It has more than possible outcomes

Examples:

1. Tossing a coin has two outcomes : {Head, Tail}
2. Throwing (rolling) a die has six outcomes : {1,2,3,4,5,6}
3. Drawing سحب a playing card from a deck كوتشينة has 52 outcomes

heart "13" (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack ولد, Queen ملكة, King ملك)

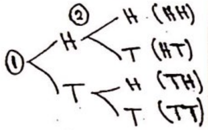
diamond "13" (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

spade "13" (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

club "13" (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King)

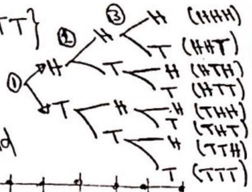
Sample space: It's the set of all possible outcomes in random variables

1. Tossing a coin ⇒ S = {H, T}
2. Tossing 2 coins ⇒ S = { HH, HT, TH, TT }



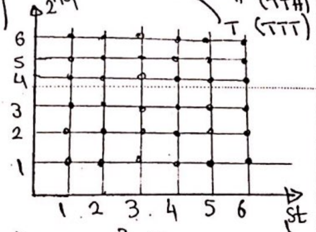
3. Tossing 3 coins ⇒

S = {HHH, HTH, THH, HHT, TTH, THT, HTT, TTT}



4. Throlling 2 dice ⇒

S = {(1,1), (1,2), ------, (2,1), ------, (6,6)}



Note

⇒ If a random experiment has (N) possible outcomes and is repeated (n) times then the total number of outcomes = N^n

Examples

1. Tossing a coin three times : number of outcomes = $2^3 = 8$
2. Rolling a die twice : number of outcomes = $6^2 = 36$

Probability of an event "A"

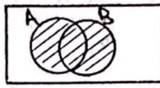
$$P(A) = \frac{\text{the number of ways that event "A" can occur}}{\text{the total number of possible outcomes}}$$

Examples

Event A	P(A)
• having a card of heart from a deck	$\frac{13}{52}$
• having a prime number from rolling a die	$\frac{3}{6}$
• having an odd <u>and</u> prime number when rolling a die	$\frac{2}{6}$
• having an odd <u>or</u> prime number when rolling a die	$\frac{4}{6}$
• having exactly two tails from throwing a coin three times	$\frac{3}{8}$
• Choosing a vowel from the alphabet	$\frac{5}{26}$
• having <u>at least</u> 2 tails from throwing 3 coins	$\frac{4}{8}$
• having <u>at most</u> 2 tails from throwing 3 coins	$\frac{7}{8}$
• A lot contains 10 good items, 4 minor defects and 2 major defects. An item is choosen 1) the probability it has no defects 2) it has a major defect 3) it is either good <u>or</u> has a major defect	 $\frac{10}{16}$ $\frac{2}{16}$ $\frac{10+2}{16} = \frac{12}{16}$
• having a sum of 7 when throwing 2 dice	$\frac{6}{36}$
• having a heart <u>or</u> Ace from a deck	$\frac{13+3}{52} = \frac{16}{52}$
• having a heart <u>and</u> Ace from a deck	$\frac{4}{52}$
• A student is chosen from a class of 16 girls and 14 boys. What is the probability that the student is <u>not</u> a girl?	$\frac{14}{30} = 1 - \frac{16}{30}$
• A jar contains 6 red, 5 green, 8 blue and 3 yellow marbles. If a single marble is chosen. 1) the probability it is red <u>or</u> green 2) the probability it is red <u>and</u> green	 $\frac{6+5}{22} = \frac{11}{22}$ 0 "mutually exclusive"

Axioms of Probability

① $0 \leq P(A) \leq 1$

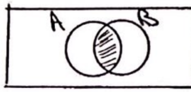


② $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

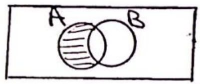
or ≡ any of them ≡ either of them ≡ at least one of them

③ $P(A \cap B) = \begin{cases} \text{Zero, if } A \text{ and } B \text{ are mutually exclusive (disjoint)} \\ P(A)\,P(B), \text{ if } A \text{ and } B \text{ are independent} \end{cases}$

and ≡ both of them ≡ occur together ≡ occur at the same time



④ $P(A \cap B') = P(A) - P(A \cap B)$



A but not B ≡ only A occurs

⑤ $P(A') = 1 - P(A)$ "not A"

⑥ $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$ "neither A nor B"

⑦ $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$ "at most one of them"

⑧ $P(A \cup B) - P(A \cap B)$ "only one of the two events"

⑨ $P(A \cup B') = 1 - [P(B) - P(A \cap B)] = 1 - P(A' \cap B)$

⑩ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

⇒ If A, B, C are mutually exclusive, then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

⑪ If $A \subset B$, then $P(A \cap B) = P(A)$ and $P(A \cup B) = P(B)$

⑫ Probability can be expressed as fractions, decimals or percentages

$$\frac{1}{2} = 0.5 = 50\%$$

⑬ $P(A) = P(A \cap B) + P(A \cap B') = P(A|B) \cdot P(B) + P(A|B') \cdot P(B')$

⑭ $P(A'|B) = 1 - P(A|B)$

⑮ $P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$

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Types of Events

1) Mutually exclusive events:

Two events are <u>mutually exclusive</u> if they cannot occur at the same time (they have no outcomes in common) : $\boxed{P(A\cap B)=0}$

<u>Examples</u>

- A = having a heart , B = having a diamond
- A = an odd number , B = an even number
- A = defective item , B = good item

⇒ If more than two events are mutually exclusive, then:

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)$$

② Independent events: (With replacement (سحب مع الإرجاع)

Two events A and B are <u>independent</u> if the fact that A occurs doesn't affect the probability of B occuring : $\boxed{P(A\cap B)=P(A)\cdot P(B)}$

<u>Examples</u>

- Choosing a "3" from a deck, <u>replacing</u> it, <u>AND</u> then choosing an "ace"
- Rolling a die and getting a "6", <u>AND</u> then rolling a die and getting a "3"

⇒ If more than two events are <u>independent</u>, then:

$$P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$$

③ Dependent events: (Without replacement (سحب مع عدم الإرجاع)

Two events are <u>dependent</u> if the outcome of the first affects the outcome of the second, so that the probability is changed.

$$P(A\cap B) = P(A)\cdot P(B|A) \neq P(A)\cdot P(B)$$

$P(B|A)$ → Probability of B occurs <u>given that</u> A has already occured. if

$P(A\cap B\cap C)=P(A)\cdot P(B|A)\cdot P(C|A\cap B)$

<u>Examples</u>

- Drawing a card from a deck, <u>not replacing</u> it, <u>AND</u> then drawing a second card.
- Selecting a ball from an urn (box), <u>not replacing</u> it, <u>AND</u> then selecting a second ball.

Conditional Probability:

① $P(A|B) = \dfrac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = P(B)\cdot P(A|B)$

↳ Probability that A occurs given that B has already occured.

② $P(B|A) = \dfrac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = P(A)\cdot P(B|A)$

Notes:

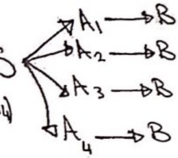
① If A and B are independent, then $\begin{cases} P(A|B) = P(A) \\ P(B|A) = P(B) \end{cases}$

② $P(A\cap B) = \begin{cases} \left.\begin{matrix} P(A|B)\cdot P(B) \\ P(B|A)\cdot P(A) \end{matrix}\right\}, & \text{if A and B are dependent} \\ P(A)\cdot P(B), & \text{if A and B are independent} \end{cases}$

Total Probability:

It expresses the total probability of an outcome which can be realized via several distinct events.

$$P(B) = P(A_1\cap B) + P(A_2\cap B) + P(A_3\cap B) + P(A_4\cap B)$$
$$= P(A_1)\cdot P(B|A_1) + P(A_2)\cdot P(B|A_2) + P(A_3)\cdot P(B|A_3) + P(A_4)\cdot P(B|A_4)$$



• If an item is selected from B and we want to know the Probability that it comes from a certain event:

$$P(A_i|B) = \frac{P(A_i\cap B)}{P(B)} = \frac{P(A_i)\cdot P(B|A_i)}{P(B)}$$

↳ Probability that an item is from $\underline{\underline{A}}$, given that (known) (found to be) $\underline{\underline{B}}$

Tree Diagram:

• It's mainly used when there's a sequence of events.
• It's useful in multi-stage random experiments and in finding Conditional Probability.

10 If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9 ,

a) What is the probability that the last digit is 0 ?

b) What is the probability that the last digit is greater than or equal 5 ?

Sol

a) $P(A) = \frac{1}{10} = \boxed{0.1}$ b) $P(B) = \frac{5}{10} = \boxed{0.5}$

2-47 Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

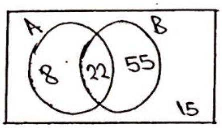
		Conforms Yes	Conforms No
Supplier	1	22	8
	2	25	5
	3	30	10

let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. If a sample is selected at random, determine :

(a) $P(A)$ (b) $P(B)$ (c) $P(A')$ (d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A' \cup B)$ (g) $P(B|A)$

(h) Are events A and B independent?

Sol



(a) $P(A) = \frac{22+8}{100} = \boxed{0.3}$

(b) $P(B) = \frac{22+25+30}{100} = \boxed{0.77}$

(c) $P(A') = 1 - P(A) = 1 - 0.3 = \boxed{0.7}$

(d) $P(A \cap B) = \frac{22}{100} = \boxed{0.22}$

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.77 - 0.22 = \boxed{0.85}$

(f) $P(A' \cup B) = 1 - [P(A) - P(A \cap B)] = 1 - [0.3 - 0.22] = \boxed{0.92}$

(g) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.22}{0.3} = \boxed{0.733}$

(h) $\because P(A) \cdot P(B) = (0.3)(0.77) = 0.231 \neq P(A \cap B)$

$\therefore$ A and B are not independent

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55 A manufacturer of front lights for automobiles tests under a high humidity, high temperature environment using intensity and useful life as the response of interest. The following table shows the performance of 130 lamps:

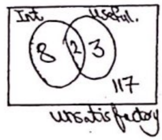
		Useful life: Satisfactory	Useful life: unsatisfactory
Intensity	Satisfactory	117	3
	unsatisfactory	8	2

(a) Find the probability that a randomly selected lamp will yield unsatisfactory results under <u>any</u> criteria.

(b) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

Sol

(a) $P(\text{unsatisfactory}) = \dfrac{8+2+3}{130} = \boxed{\dfrac{13}{130}}$



OR $\dfrac{10}{130} + \dfrac{5}{130} - \dfrac{2}{130} = \boxed{\dfrac{13}{130}}$

(b) $P(\underline{\text{both}}\ \text{criteria satisfactory}) = \dfrac{117}{130} = \boxed{0.90} < 0.95$ (No)

Then, the lamp manufacturer <u>can't</u> meet the required demand.

2-58 Samples of a cast aluminium part are classified on the basis of surface finish (in microinches) and length measurements. The results of 100 parts are summarized as follows:

		Length: excellent	Length: good
Surface finish	excellent	80	2
	good	10	8

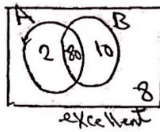
let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent length. Determine:

(a) $P(A)$ (b) $P(B)$ (c) $P(A|B)$ (d) $P(B|A)$

(e) If the selected part has excellent surface finish, what is the probability that the length is excellent?

(f) If the selected part has good length, what is the probability that the surface finish is excellent?

$P(A) = \frac{80+2}{100} = \boxed{0.82}$



(b) $P(B) = \frac{80+10}{100} = \boxed{0.9}$

(c) $\because P(A \cap B) = \frac{80}{100} = 0.8$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.8}{0.9} = \boxed{\frac{8}{9}}$

(d) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.8}{0.82} = \boxed{0.9756}$

(e) $P(B|A) = \frac{0.8}{0.82} = \boxed{0.9756}$

(f) $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{2}{100}}{0.1} = \boxed{0.2}$

$\boxed{2\text{-}109}$ The probability of getting through by telephone to buy concert tickets is 0.92. For the same event, the probability of accessing the vendor's Website is 0.95. Assume that these two ways to buy tickets are independent. What is the probability that someone who tries to buy tickets through the Internet and by phone will obtain tickets?

Sol

let A: denote the ticket through telephone & $P(A \cap B) = P(A) \cdot P(B)$

B denote the ticket through Web

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (0.92) + (0.95) - (0.92)(0.95) = \boxed{0.966}$

$\boxed{2020}$ True or False

a) If $P(A|B) = 0.4$, $P(B) = 0.8$ and $P(A) = 0.5$, then A and B are independent.

b) If $P(A) = 0.2$, $P(B) = 0.2$ and A, B are mutually exclusive, then A and B are independent.

c) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) + P(A \cap B \cap C)$.

Sol

a) False. $\because P(A|B) \neq P(A)$, then A and B are not independent

b) False. $\because P(A \cap B) = 0 \neq P(A) \cdot P(B)$, then A and B are not Independent

c) False. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

45 Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The result from 100 disks are summarized as follows:

		Shock resistance	
		High	Low
Scratch resistance	High	70	9
	Low	16	5

Let A denote the event that a disk has a high shock resistance, and let B denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following:

(a) $P(A)$ (b) $P(B)$ (c) $P(A')$ (d) $P(A \cap B)$

(e) $P(A \cup B)$ (f) $P(A' \cup B)$ (g) $P(A|B)$ (h) Are A, B independent?

Sol

(a) $P(A) = \frac{70+16}{100} = 0.86$

(b) $P(B) = \frac{70+9}{100} = 0.79$

(c) $P(A') = 1 - P(A) = 1 - 0.86 = 0.14$

(d) $P(A \cap B) = \frac{70}{100} = 0.7$

(e) $P(A \cup B) = \frac{70+16+9}{100} = 0.95$ $\boxed{OR}$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.95$

(f) $P(A' \cup B) = 1 - [P(A) - P(A \cap B)] = 1 - [0.86 - 0.7] = 0.84$

$\boxed{OR}$ $P(A' \cap B) = P(A') + P(B) - P(A' \cap B) = 0.14 + 0.49 - 0.9 = 0.84$

(g) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.7}{0.79} = \frac{70}{79}$

(h) $\because P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ A and B are dependent

2-52 In 2-45

a) If a disk is selected at random, what is the probability that its scratch resistance is high <u>and</u> its shock resistance is high?

b) If a disk is selected at random, what is the probability that its scratch resistance is high <u>or</u> its shock resistance is high?

c) Consider the event that a disk has high scratch resistance and the event that a disk has high shock resistance. Are these two events mutually exclusive?

a) $P(A \cap B) = \frac{70}{100} = 0.7$ (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{79}{100} + \frac{86}{100} - \frac{70}{100} = 0.95$

b) Since $P(A \cap B) \neq 0$, the events are not mutually exclusive.

2-67 A maintenance firm has gathered the following information regarding the failure mechanisms for air-conditioning systems:

		evidence of gas leaks	
		yes	no
evidence of electrical failure	yes	55	17
	no	32	3

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability

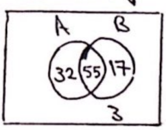
(a) That failure involves a gas leak.

(b) That there is evidence of electrical failure given that there was a gas leak.

(c) That there is evidence of gas leak given that there is evidence of electrical failure

Let A denote the event that failure involves a gas leak
& B denote the event that failure involves an electrical failure

(a) $P(A) = \frac{55+32}{55+32+17+3} = \boxed{\frac{87}{107}}$



(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{55}{107}}{87/107} = \boxed{\frac{55}{87}}$

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{55}{107}}{\frac{72}{107}} = \boxed{\frac{55}{72}}$

103 Computers in a shipment of 100 units contain a portable hard drive, CD RW drive, or both according to the following table:

CDRW	Portable hard drive: Yes	Portable hard drive: no
Yes	15	80
no	4	1

let A denote the events that a computer has a portable hard drive and B denote the event that a computer has a CD RW drive. If one computer is selected at random, compute:

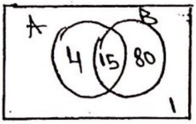
(a) $P(A)$

(b) $P(A\cap B)$

(c) $P(A\cup B)$

(d) $P(A'\cap B)$

(e) $P(A|B)$



(a) $P(A) = \dfrac{15+4}{15+80+4+1} = \dfrac{19}{100} = \boxed{0.19}$

(b) $P(A\cap B) = \dfrac{15}{100} = \boxed{0.15}$

(c) $P(A\cup B) = P(A)+P(B)-P(A\cap B) = \dfrac{19}{100} + \dfrac{95}{100} - \dfrac{15}{100} = \boxed{0.99}$

OR $P(A\cup B) = \dfrac{4+15+80}{100} = 0.99$

(d) $P(A'\cap B) = P(B) - P(A\cap B) = \dfrac{95}{100} - \dfrac{15}{100} = \boxed{0.8}$

OR $P(A'\cap B) = P(B) - P(A\cap B) = 0.8$

(e) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.15}{0.95} = \boxed{0.158}$

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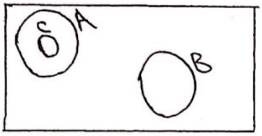
[69] Suppose A and B are mutually exclusive events. Construct a Venn diagram that contains three events A, B and C such that $P(A|C)=1$ and $P(B|C)=0$

Sol

$P(A|C) = \frac{P(A\cap C)}{P(C)} = 1$ means "$C \subset A$"

$P(B|C) = 0$ means $P(B\cap C)=0$ ($\therefore$ B and C mutually exclusive)



[2-70] (2019, 2020) Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine:

(a) $P(A\cap B)$ (b) $P(A'\cap B)$

Sol

(a) $\because P(A|B) = \frac{P(A\cap B)}{P(B)} \Rightarrow 0.4 = \frac{P(A\cap B)}{0.5} \Rightarrow \boxed{P(A\cap B) = 0.2}$

(b) $P(A'\cap B) = P(B) - P(A\cap B) = 0.5 - 0.2 = \boxed{0.3}$

[2-71] (2019) Suppose that $P(A|B) = 0.2$, $P(A|B') = 0.3$ and $P(B) = 0.8$. What is $P(A)$?

Sol

Using the Total Probability Rule

$P(A) = P(A\cap B) + P(A\cap B')$

$= P(A|B)\cdot P(B) + P(A|B')\,P(B')$

$= (0.2)(0.8) + (0.3)(0.2) = \boxed{0.22}$

[2-100] [2020] A lot of 100 semiconductor chips contains 20 that are defective.

(a) Two are selected at random, without replacement, determine the probability that the second chip is defective.

(b) Three are selected at random, without replacement, find the probability that all are defective.

Sol

Let D_n denote the event that the n^{th} chip is defective

D	D'
20	80

$n = 100$

(a) $P(D_2) = P(D_1 \cap D_2) + P(D'_1 \cap D_2)$

$$= \left(\frac{20}{100}\right)\left(\frac{19}{99}\right) + \left(\frac{80}{100}\right)\left(\frac{20}{99}\right) = \boxed{0.2}$$

(b) $P(D_1 D_2 D_3) = \left(\frac{20}{100}\right)\left(\frac{19}{99}\right)\left(\frac{18}{98}\right) = \boxed{0.00705}$

[2-86] If $P(A) = 0.2$, $P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

$\because$ A and B are mutually exclusive, then $P(A \cap B) = 0$

but $P(A) \cdot P(B) = 0.04 \neq P(A \cap B)$

Therefore, A and B are not independent.

[2-51] [2020] If A, B and C are mutually exclusive events, is it possible for $P(A) = 0.3$, $P(B) = 0.4$ and $P(C) = 0.5$? Why or why not?

Sol

If A, B and C are mutually exclusive, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2 > 1$

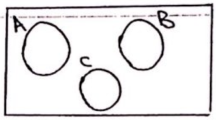
, then $P(A)$, $P(B)$ and $P(C)$ can't equal the given values.

0] If A, B and C are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.4$, determine the following:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$

(d) $P[(A \cup B) \cap C]$ (e) $P(A' \cap B' \cap C')$

Sol



a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.2 + 0.3 + 0.4 = 0.9$

(b) $P(A \cap B \cap C) = 0$

(c) $P(A \cap B) = 0$

(d) $P([A \cup B] \cap C) = 0 = P(A \cap B) \cup P(B \cap C)$

(e) $P(A' \cap B' \cap C') = P(A \cup B \cup C)' = 1 - P(A \cup B \cup C) = 1 - 0.9 = 0.1$

$\boxed{2\text{-}49}$ If $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, determine:

a) $P(A')$ (b) $P(A \cup B)$ (c) $P(A' \cap B)$ (d) $P(A \cap B')$

e) $P(A \cup B)'$ (f) $P(A' \cup B)$

Sol

(a) $P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = 0.4$

(c) $P(A' \cap B) = P(B) - P(A \cap B) = 0.2 - 0.1 = 0.1$

(d) $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

(e) $P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0.4 = 0.6$

(f) $P(A' \cup B) = 1 - P(A \cap B') = 1 - 0.2 = 0.8$

$\boxed{\text{OR}} = 1 - [P(A) - P(A \cap B)] = 0.8$

ample If A and B are independent events where $P(A) = 0.6$, $P(A \cap B) = 0.3$, find:

(a) $P(B)$ (b) $P(A|B)$ (c) $P(B'|A)$

Sol

(a) $\because$ A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$

$\therefore \quad 0.3 = 0.6\, P(B) \Rightarrow P(B) = \frac{1}{2}$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = P(A) = 0.6$

(c) $P(B'|A) = \dfrac{P(B' \cap A)}{P(A)} = P(B') = 1 - P(B) = \frac{1}{2}$

$\Rightarrow$ $\boxed{\text{Note}}$

- If A and B are independent, then:
  - A, B' are independent
  - A', B are independent
  - A', B' are independent
- A and B are independent if <u>one</u> of the following occurs:

(a) $P(A|B) = P(A)$ (b) $P(B|A) = P(B)$ (c) $P(A \cap B) = P(A) \cdot P(B)$

$\boxed{2\text{-}81}$ If $P(A|B) = 0.4$, $P(B) = 0.8$, and $P(A) = 0.5$, are the events A and B independent?

Sol

$\because P(A|B) \neq P(A)$, then events are <u>not</u> independent (dependent)

$\boxed{2\text{-}82}$ If $P(A|B) = 0.3$, $P(B) = 0.8$, and $P(A) = 0.3$, are the events B and the complement of A independent?

Sol

We need to show that $P(A'|B) = P(A')$ $\boxed{\text{OR}}$ $P(A' \cap B) = P(A') \cdot P(B)$

Now, $P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

$\because P(A|B) = \dfrac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = (0.3)(0.8) = 0.24$

$\therefore P(A' \cap B) = P(B) - P(A \cap B) = 0.8 - 0.24 = 0.56 \rightarrow (1)$

$P(A') \cdot P(B) = (0.7)(0.8) = 0.56 \rightarrow (2)$

Therefore, $P(A' \cap B) = P(A') \cdot P(B) \Rightarrow A'$ and B are independent

<u>OR</u> $P(A'|B) = 1 - P(A|B) = 0.7 = P(A') \Rightarrow A'$ and B are independent

Example Two balls are drawn without replacement from a box containing 10 red, 20 white and 15 blue. What is the probability that:

(a) both balls are red ?.

(b) the first is red and the second is blue ?.

(c) one is red and the other is blue ?.

(d) at least one is red?.

Sol

R	W	B
10	20	15

$N = 45$

(a) $P(R_1 \cap R_1) = P(R_1) \cdot P(R_2 | R_1) = \frac{10}{45} \cdot \frac{9}{44} = \frac{1}{22}$

(b) $P(R_1 \cap B_2) = P(R_1) \cdot P(B_2 | R_1) = \frac{10}{45} \cdot \frac{15}{44} = \frac{5}{66}$

(c) $P(R_1 \cap B_2) + P(B_1 \cap R_2) = \frac{10}{45} \cdot \frac{15}{44} + \frac{15}{45} \cdot \frac{10}{44} = \frac{5}{33}$

(d) $P(R_1 \cap R_2) + P(R_1 \cap \text{not } R_2) + P(\text{not } R_1 \cap R_2)$

$= \frac{10}{45} \cdot \frac{9}{44} + \frac{10}{45} \cdot \frac{35}{44} + \frac{35}{45} \cdot \frac{10}{44} = \frac{79}{198}$

Fall 2016 Let A and B be two events such that $P(A) = 0.5$, $P(B) = 0.3$, $P(A \cap B) = 0.1$, Find:

(a) $P(A|B)$ (b) $P(B|A)$ (c) $P[A|(A \cup B)]$ (d) $P[A|(A \cap B)]$

(e) $P[(A \cap B)|(A \cup B)]$

Sol

(a) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$

(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.5} = \frac{1}{5}$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.1 = 0.7$

, $P[A \cap (A \cup B)] = P(A) = 0.5$

$\therefore P[A|(A \cup B)] = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{0.5}{0.7} = \frac{5}{7}$

(d) $P[A|(A \cap B)] = \frac{P[A \cap (A \cap B)]}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1$

(e) $P[(A \cap B)|(A \cup B)] = \frac{P[(A \cap B) \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{1}{7}$

Fall 2016

A day's production of 850 manufactured parts contain 50 parts that don't meet customer's requirements.

(a) If two parts are selected without replacement, what is the probability that the second is defective given that the first is defective?

(b) If three parts are selected, what is the probability that the first two parts are defective and the third is not defective? i) Without replacement ii) With replacement

Sol

(a) $P(D_2|D_1) = \frac{49}{849}$

850 → 50 D
850 → 800 N

(b) i) Without replacement

$P(D_1 \cap D_2 \cap N_3) = \frac{50}{850} \cdot \frac{49}{849} \cdot \frac{800}{848} = 0.0032$

ii) With replacement

$P(D_1 \cap D_2 \cap N_3) = \frac{50}{850} \cdot \frac{50}{850} \cdot \frac{800}{850} = \frac{16}{4913}$

c) What is the probability that the second part is defective?

$P(D_2) = P(D_1 \cap D_2) + P(N_1 \cap D_2) = \frac{50}{850} \cdot \frac{49}{849} + \frac{800}{850} \cdot \frac{50}{849} = \frac{50}{850}$

(Without replacement)

Fall 2016 Suppose your vehicle is licensed in a state that issues license plates that consist of three digits (between 0 and 9) followed by three letters (between A and Z). If a license number is selected randomly, what is the probability that yours is the one selected?

Sol

$P(\text{three numbers}) \cdot P(\text{three letters})$

$= \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{26} \cdot \frac{1}{26} \cdot \frac{1}{26}\right) = 5.7 * 10^{-8}$

10 numbers → $\frac{1}{10}$ 1st, $\frac{1}{10}$ 2nd, $\frac{1}{10}$ 3rd

26 letters → $\frac{1}{26}$ 1st, $\frac{1}{26}$ 2nd, $\frac{1}{26}$ 3rd

-107 A steel plate contains 20 bolts. Assume that 5 bolts are not torqued to the proper limit. Four bolts are selected at random, without replacement, to be checked for torque.

(a) What is the probability that all four of the selected bolts are torqued to the proper limit?

(b) What is the probability that at least one of the selected bolts is not torqued to the proper limit?

Sol

A	A'
15	5

let A_n denote the event that the n^{th} bolt is not torqued.

(a) $P(A_1 A_2 A_3 A_4) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17} = \boxed{0.282}$

(b) $P(\text{at least one is not torqued}) = 1 - P(\text{all are torqued})$

$= 1 - 0.282 = \boxed{0.718}$

2-114 A robotic insertion tool contains 10 primary components. The probability that any component fails during the warranty period is 0.01. Assume the components fail independently and that the tool fails if any component fails. What is the probability that the tool fails during the warranty period?

Sol

let F denote the event that the tool fails, then $P(F') = (0.99)^{10}$ by independence.

and $P(F) = 1 - (0.99)^{10} = \boxed{0.0956}$

93] (2019) A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected at random, without replacement, from the batch. Let A and B denote the events that the first and second containers selected is defective, respectively.

(a) Are A and B independent events?

(b) If the sampling were done with replacement, would A and B be independent?

D	D'
5	495

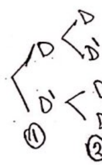
$n = 500$

(a)

$\because P(B|A) = \boxed{\frac{4}{499}}$

& $P(B) = P(B|A)P(A) + P(B|A')P(A')$

$= \left(\frac{4}{499}\right)\left(\frac{5}{500}\right) + \left(\frac{5}{499}\right)\left(\frac{495}{500}\right) = \boxed{\frac{5}{500}}$



$\therefore P(B|A) \neq P(B)$

Therefore, A and B are not independent

(b) $P(B|A) = \boxed{\frac{5}{500}}$

& $P(B) = \left(\frac{5}{500}\right)\left(\frac{5}{500}\right) + \left(\frac{495}{500}\right)\left(\frac{5}{500}\right) = \boxed{\frac{5}{500}}$

$\therefore P(B|A) = P(B)$

Therefore, A and B are independent

(c) What is the probability that both are defective? "without replace."

$P(D_1 D_2) = \frac{5}{500} \cdot \frac{4}{499} = \boxed{0.000080}$

2-63, 64

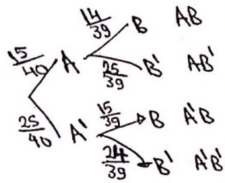
A lot contains 15 castings from a local supplier and 25 castings from a supplier in the next state. Two castings are selected randomly, without replacement, from the lot of 40.
Let A denote the event the first casting selected is from the local supplier, and let B denote the event the second casting is selected from the local supplier. Determine:

(a) $P(A)$ (b) $P(B|A)$ (c) $P(A\cap B)$ (d) $P(A\cup B)$

(e) Suppose three castings are selected at random, without replacement, from the lot. In addition to the definitions of events A and B, let C denote the event that the third casting selected is from the local supplier. Determine:

i) $P(A\cap B\cap C)$ ii) $P(A\cap B\cap C')$

Sol



(a) $P(A) = \boxed{\frac{15}{40}}$ OR $P(AB)+P(AB') = \frac{15}{40}\cdot\frac{14}{39}+\frac{15}{40}\cdot\frac{25}{39}$

(b) $P(B|A) = \boxed{\frac{14}{39}}$

(c) $P(A\cap B) = P(AB) = \left(\frac{15}{40}\right)\left(\frac{14}{39}\right) = \boxed{0.135}$

(d) $P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{15}{40}+\frac{14}{39}-\left(\frac{15}{40}\right)\left(\frac{14}{39}\right) = \boxed{0.599}$

(e) $P(A\cap B\cap C) = \left(\frac{15}{40}\right)\left(\frac{14}{39}\right)\left(\frac{13}{38}\right) = \boxed{0.046}$

6 $P(A\cap B\cap C') = \left(\frac{15}{40}\right)\left(\frac{14}{39}\right)\left(\frac{25}{38}\right) = \boxed{0.089}$

2-76 Samples of laboratory glass are in small, light packing or heavy, large packing. Suppose that 2 and 1 % of the sample shipped in small and large packages, respectively, break during transit. If 60 % of the samples are shipped in large packages and 40 % are shipped in small packages, what proportion of samples break during shipment?.

Sol

Let B denote the event that a glass breaks
L denote the event that large packing is used

$$P(B) = P(B|L)\,P(L) + P(B|L')\,P(L')$$
$$= (0.01)(0.60) + (0.02)(0.40) = \boxed{0.014}$$

2-79 A batch of 25 injection-modeled parts contains 5 that have suffered excessive shrinkage.

(a) If two parts are selected at random, and without replacement, what is the probability that the second part selected is one with excessive shrinkage?.

(b) If three parts are selected at random, and without replacement, what is the probability that the third part selected is one with excessive shrinkage?.

$\underset{E}{\boxed{5}}$ $\underset{E'}{\boxed{20}}$

Sol

Let A denote the event that the first part selected has excessive shrinkage
B " " " " second " " " "

(a) $P(B) = P(B|A)\,P(A) + P(B|A')\,P(A') = \left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \left(\frac{5}{24}\right)\left(\frac{20}{25}\right) = \boxed{0.2}$

(b) Let C denote the event that the third has excessive shrinkage.

$$P(C) = P(C|A\cap B)\,P(A\cap B) + P(C|A\cap B')\,P(A\cap B') + P(C|A'\cap B)\,P(A'\cap B) + P(C|A'\cap B')\,P(A'\cap B')$$
$$= \left(\frac{3}{23}\right)\left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \left(\frac{4}{23}\right)\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \left(\frac{4}{23}\right)\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \left(\frac{5}{23}\right)\left(\frac{19}{24}\right)\left(\frac{20}{25}\right) = \boxed{0.2}$$

[2-112] An encryption-decryption system consists of three elements; encode, transmit and decode. A faulty encode occurs in 0.5% of the messages processed, transmission errors occur in 1% of the messages, and a decode error occurs in 0.1% of the messages. Assume the errors are indefendent.

(a) What is the probability of a completely defect-free messages?

(b) What is the probability of a message that has either an encode or a decode error?

Sol

$\because P(E) = 0.005 \quad , P(T) = 0.01 \quad , P(D) = 0.001$

(a) $P(E' \cap T' \cap D') = (0.995)(0.99)(0.999) = \boxed{0.984}$

(b) $P(E \cup D) = P(E) + P(D) - P(E \cap D)$

$= (0.005) + (0.001) - (0.005)(0.0001) = \boxed{0.005995}$

2-104 The probability that a customer's order is not shipped on time is 0.05. A particular customer places three orders, and the orders are placed far enough apart in time that they can be considered to be independent events.

(a) What is the probability that all are shipped on time?.

(b) What is the probability that exactly one is not shipped on time?.

(c) What is the probability that two or more orders are not shipped on time?

Sol

Let A_n denote the event that the n^{th} order is shipped on time

(a) $P(A_1 A_2 A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = (0.95)^3 = \boxed{0.857}$

(b) $P(A_1' A_2 A_3) + P(A_1 A_2' A_3) + P(A_1 A_2 A_3') = 3\left((0.05)(0.95)^2\right)$
$= \boxed{0.135}$

(c) $P(A_1' A_2' A_3) + P(A_1' A_2 A_3') + P(A_1 A_2' A_3') + P(A_1' A_2' A_3')$
$= 3\left((0.05)^2(0.95)\right) + (0.05)^3 = \boxed{0.00725}$

-87 2019 The probability that a lab specimen contains high-levels of contamination is 0.10. Five samples are checked, and the samples are independent.

(a) What is the probability that none contains high levels of contamination?

(b) What is the probability that exactly one contains high levels of contamination?

(c) What is the probability that at least one contains high levels of contamination?

Sol

let H_n denote the events that $\underline{n^{th}}$ sample contains high levels of contamin.

(a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = (0.9)(0.9)(0.9)(0.9)(0.9) = (0.9)^5 = \boxed{0.59}$

(b) $P(H_1 H_2' H_3' H_4' H_5') + P(H_1' H_2 H_3' H_4' H_5') + P(H_1' H_2' H_3 H_4' H_5')$

$+ P(H_1' H_2' H_3' H_4 H_5') + P(H_1' H_2' H_3' H_4' H_5) = 5\left((0.9)(0.9)(0.9)(0.9)(0.1)\right)$

$= \boxed{0.328}$

(c) $P(\text{at least one}) = 1 - P(\text{none}) = 1 - 0.59 = \boxed{0.41}$

09 / 2020 Eight cavities in an injection-molding tool produce plastic connectors that fall into a common stream. A sample is chosen every several minutes. Assume that the samples are independent.

(a) What is the probability that five successive samples were all produced in cavity one of the mold?

(b) What is the probability that five successive samples were all produced in the same cavity of the mold?

(c) What is the probability that four out of five successive samples were produced in cavity one of the mold?

Sol

let A denote the event that a sample is produced in cavity one of the mold.

$\therefore P(\text{cavity one}) = \frac{1}{8}$

(a) $P(\text{cavity one in all five samples}) = P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot P(A_4) \cdot P(A_5) = \left(\frac{1}{8}\right)^5 = \boxed{0.00003}$

(b) Let B_i be the event that all five samples are produced in cavity i.

Because the B's are mutually exclusive, then:

$P(B_1 \cup B_2 \cup \dots \cup B_8) = P(B_1) + P(B_2) + \dots + P(B_8)$

From part a), $P(B_i) = \left(\frac{1}{8}\right)^5$

$\therefore P(B_1 \cup B_2 \cup \dots \cup B_8) = 8\left(\frac{1}{8}\right)^5 = \boxed{0.00024}$

(c) $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') + P(A_1 \cap A_2 \cap A_3 \cap A_4' \cap A_5) + P(A_1 \cap A_2 \cap A_3' \cap A_4 \cap A_5) + P(A_1 \cap A_2' \cap A_3 \cap A_4 \cap A_5) + P(A_1' \cap A_2 \cap A_3 \cap A_4 \cap A_5) = 5\left(\frac{1}{8}\right)^4\left(\frac{7}{8}\right) = \boxed{0.00107}$

Example Two cards are drawn at random without replacement from a deck of 52 cards. What is the probability that the first card is diamond and the second card is red?

Sol

$$P(\text{diamond} \cap \text{red}) = P(\text{diamond}) \cdot P(\text{red} \mid \text{diamond})$$

$$= \frac{13}{52} \cdot \frac{25}{51} = \frac{25}{204}$$

⇒ **Note**

52 → 26 black → 13 spade ♤ (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)

→ 13 club ♧ ( )

52 → 26 red → 13 diamond ◇ ( )

→ 13 heart ♡ ( )

Example A lot contains 20 steel rods of which 5 are defective. Two rods are drawn at random without replacement.

(a) What is the probability that the second is defective given that the first is defective?

(b) What is the probability that the first is defective?

(c) What is the probability that the first is not defective and the second is defective?

Sol

20 → 5 (D), 15 (ND)

$P(D) = \frac{5}{20}$

$P(ND) = \frac{15}{20}$

(a) $P(D_2 \mid D_1) = \frac{4}{19}$

(b) $P(D_1) = P(D_1 \cap D_2) + P(D_1 \cap N_2)$

$= P(D_1) \cdot P(D_2 \mid D_1) + P(D_1) \cdot P(N_2 \mid D_1)$

$= \frac{5}{20} \cdot \frac{4}{19} + \frac{5}{20} \cdot \frac{15}{19} = \frac{5}{20}$

(c) $P(N_1 \cap D_2) = P(N_1) \cdot P(D_2 \mid N_1) = \frac{15}{20} \cdot \frac{5}{19} = \frac{15}{76}$

(d) What is the probability the first is satisfactory given that the second is defective?

$$P(N_1 \mid D_2) = \frac{P(N_1 \cap D_2)}{P(D_2)} = \frac{\frac{15}{20} \cdot \frac{5}{19}}{P(D_1 \cap D_2) + P(N_1 \cap D_2)} = \frac{\frac{15}{20} \cdot \frac{5}{19}}{\frac{5}{20} \cdot \frac{4}{19} + \frac{15}{20} \cdot \frac{5}{19}} = \frac{15}{19}$$

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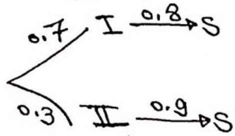
Fall 2015

A Company has two plants to manufacture motorcycles. 70% motorcycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% motorcycles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motorcycle, randomly picked up, is found to be of standard quality. Find the probability that it has come out from the second plant.

Sol

E_1 = 1^{st} plant

E_2 = 2^{nd} plant ; S: standard



$$\because P(S) = P(I)\cdot P(S|I) + P(II)\cdot P(S|II)$$
$$= (0.7)(0.8) + (0.3)(0.9) = 0.83$$
$$\Rightarrow P(II|S) = \frac{P(II \cap S)}{P(S)} = \frac{(0.3)(0.9)}{0.83} = \frac{27}{83}$$

2019 A sample space contains 20 equally likely outcomes. If the probability of event A is 0.3, how many outcomes are in event A?

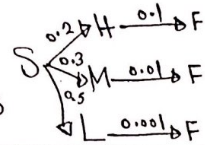
$$\because P(A) = \frac{X}{20} \Rightarrow 0.3 = \frac{X}{20} \Rightarrow \boxed{X=6}$$

Example Assume the following Probabilities for product failure subject to the level of Contamination in manufacturing

Probability of failure	level of Contamination
0.1	High
0.01	Medium
0.001	Low

In a Particular Production, 20% of chips are subjected to high level of Contamination, 30% medium and 50% low levels. What is the Probability that a Product using one of these chips fails?

Sol



$$P(F) = P(H)\cdot P(F|H) + P(M)\cdot P(F|M) + P(L)\cdot P(F|L)$$

$$= (0.2)(0.1) + (0.3)(0.01) + (0.5)(0.001) = 0.0235$$

Example Suppose 2% of Cotton fabric rolls and 3% of nylon fabric contain flaws. Of the rolls used by the manufacturer, 70% are Cotton and 30% are nylon. What is the Probability that a randomly selected roll used by the manufacturer contains flaws?

Sol

$$P(F) = P(C)\cdot P(F|C) + P(N)\cdot P(F|N)$$

$$= (0.7)(0.02) + (0.3)(0.03) = 0.023$$

__all 2015__

A firm offers routine physical examinations as part of a health service program for its employees. The exams showed that 8% of the employees needed corrective shoes, 15% needed dental work, and 3% needed both. What is the probability that an employee selected at random will need either corrective shoes or dental work?

Sol

Let A = event that an employee needs corrective shoes

B = event that an employee needs dental work

$\because P(A) = 0.08 \quad , \quad P(B) = 0.15 \quad , \quad P(A \cap B) = 0.03$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.08 + 0.15 - 0.03 = 0.2$

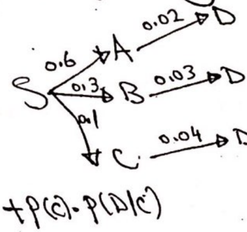
__Fall 2015, 2016__ Three machines A, B and C produce 6000, 3000 and 1000 items respectively. The percentages of defective items of these machines are 2%, 3% and 4% respectively. If an item is selected at random and is found to be defective, find the probability it was produced by machine C.

Sol

$P(A) = \frac{6000}{10000} = 0.6$

$P(B) = \frac{3000}{10000} = 0.3$

$P(C) = \frac{1000}{10000} = 0.1$



$\because P(D) = P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)$

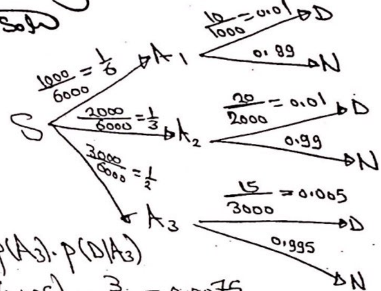
$= (0.6)(0.02) + (0.3)(0.03) + (0.1)(0.04) = 0.025$

$\therefore P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{(0.1)(0.04)}{0.025} = 0.16$

Example Three machines in a certain factory A_1, A_2 and A_3 produce respectively 1000, 2000 and 3000 items/day. The number of defective output items of these machines are 10, 20 and 15 items/day, respectively. If an item is selected at random from the daily production. Find the probability that the selected item is defective and the probability it was from machine A_3.

Sol

$S = 1000 + 2000 + 3000 = 6000$



$$P(D) = P(A_1)\cdot P(D|A_1) + P(A_2)\cdot P(D|A_2) + P(A_3)\cdot P(D|A_3)$$
$$= \left(\tfrac{1}{6}\right)(0.01) + \left(\tfrac{1}{3}\right)(0.01) + \left(\tfrac{1}{2}\right)(0.005) = \frac{3}{400} = 0.0075$$

$$, \; P(A_3|D) = \frac{P(A_3 \cap D)}{P(D)} = \frac{\left(\tfrac{1}{2}\right)(0.005)}{0.0075} = \frac{1}{3}$$

Another method

$$P(D) = \frac{45}{6000} = 0.0075$$
$$, \; P(A_3|D) = \frac{15}{45} = \frac{1}{3}$$

2019 If A and B are events in a sample space for which $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$. Find:

(a) $P(A \cap B)$ (b) $P(A \backslash B)$ (c) $P(A' \backslash B')$

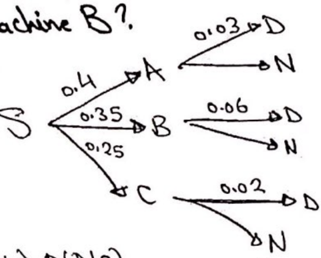
(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.4 - 0.8 = \boxed{0.1}$

(b) $P(A \backslash B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.4} = \boxed{\tfrac{1}{4}}$

(c) $P(A' \backslash B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1 - 0.8}{0.6} = \boxed{\tfrac{1}{3}}$

Example H.T Three machines A, B and C produce respectively 40%, 35% and 25% of the total number of items. The percentages of defective items are 3%, 6% and 2% respectively. If an item is selected at random, what is the probability that the selected item is defective? Given that the selected item is defective, what is the probability it was produced by machine B?

Sol



$$P(D) = P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)$$
$$= (0.4)(0.03) + (0.35)(0.06) + (0.25)(0.02) = \boxed{0.038}$$

$$\text{, } P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{(0.35)(0.06)}{0.038} = \boxed{0.5526}$$

Example Let A and B be two events such that $P(A) = 0.8$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$. Find $P(A^c|B^c)$ and $P(B^c|A^c)$

Sol

$$P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] = 1 - (0.8 + 0.4 - 0.3)$$
$$= 0.1$$

$$\therefore P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{0.1}{1 - 0.4} = \boxed{\frac{1}{6}}$$

$$\text{, } P(B^c|A^c) = \frac{P(A^c \cap B^c)}{P(A^c)} = \frac{0.1}{1 - 0.8} = \boxed{\frac{1}{2}}$$

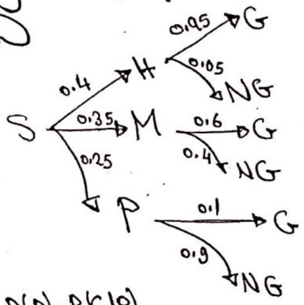
2017 Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

(a) What is the probability that a product attains a good review?

(b) If a new design attains a good review, what is the probability that it will be a highly successful product?

(c) If a product does not attain a good review, what is the probability that it will be a highly successful product?

Sol



a) $P(G) = P(H)\cdot P(G|H) + P(M)\cdot P(G|M) + P(P)\cdot P(G|P)$

$= (0.4)(0.95) + (0.35)(0.6) + (0.25)(0.1) = 0.615$

b) $P(H|G) = \dfrac{P(H \cap G)}{P(G)} = \dfrac{(0.4)(0.95)}{0.615} = 0.618$

c) $P(H|NG) = \dfrac{P(H \cap NG)}{P(NG)} = \dfrac{(0.4)(0.05)}{1 - 0.615} = 0.052$

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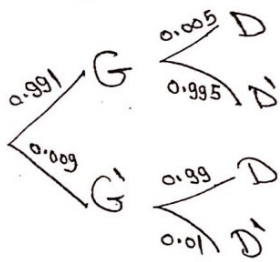
[8] (2019) An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that its line produced 0.9% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

(b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?

G denotes good (conforming)

G' denotes non-conforming



(a) $P(D) = P(G) \cdot P(D|G) + P(G')\,P(D|G')$

$= (0.991)(0.005) + (0.009)(0.99) = \boxed{0.013865}$

(b) $P(G|D') = \dfrac{P(G \cap D')}{P(D')} = \dfrac{P(G) \cdot P(D'|G)}{1 - P(D)}$

$= \dfrac{(0.991)(0.995)}{1 - 0.013865} = \boxed{0.9999}$

2-94 Suppose that $P(A|B)=0.7$, $P(A)=0.5$ and $P(B)=0.2$. Determine $P(B|A)$

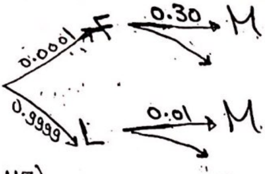
$\because P(A|B)=\dfrac{P(A\cap B)}{P(B)} \Rightarrow \therefore P(A\cap B)=(0.7)(0.2)=0.14$

$\therefore P(B|A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{(0.7)(0.2)}{0.5}=\boxed{0.28}$

2-95 Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It's found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent?

Sol

Let F denote a fraudulent user, L denote a legitimate user
M denote a user that originates calls from two or more metropolitan areas in a day.



$P(M)=P(F)\cdot P(M|F)+P(L)\cdot P(M|L)$

$=(0.0001)(0.30)+(0.999)(0.01)=\boxed{0.01002}$

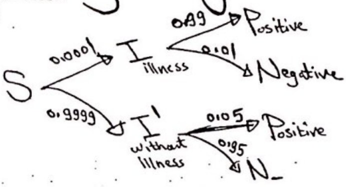
$\therefore P(F|M)=\dfrac{P(F\cap M)}{P(M)}=\dfrac{P(F)\,P(M|F)}{P(M)}=\dfrac{(0.30)(0.0001)}{0.01002}$

$\therefore \boxed{P(F|M)=0.003}$

Example 2-30

Because a new medical procedure has been shown to be effective in the early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies someone without the illness as negative is 0.95, and the probability that the test correctly identifies someone with the illness as positive is 0.99. The incidence of the illness in the general population is 0.0001. You take the test, and the result is positive. What is the probability that you have the illness?

Sol



Let $P(I) = 0.0001$

$\therefore P(I') = 0.9999$

$$P(P_{+ve}) = P(I)\cdot P(P_+|I) + P(I')\cdot P(P_+|I')$$

$$= (0.0001)(0.99) + (0.9999)(0.05) = \boxed{0.050094}$$

$$P(I|P_+) = \frac{P(I)\cdot P(P_+|I)}{P(P_+)} = \frac{(0.0001)(0.99)}{0.050094} = \frac{1}{506} \simeq \boxed{0.002}$$

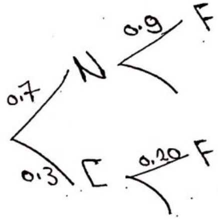
Example 2-26 Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent. If 15 wafers are analyzed, what is the probability that no large particles are found?

Let E_i denote the event that the ith wafer contains no large particle, then $P(E_i) = 1 - 0.01 = 0.99$

$$\therefore P(E_1 \cap E_2 \cap \cdots \cap E_{15}) = P(E_1)\cdot P(E_2)\cdots P(E_{15}) = (0.99)^{15} = \boxed{0.86}$$

106 Transactions to a computer database are either new items or changes to previous items. The addition of an item can be completed less than 100 milliseconds 90% of the time, but only 20% of changes to a previous item can be completed in less than this time. If 30% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds?

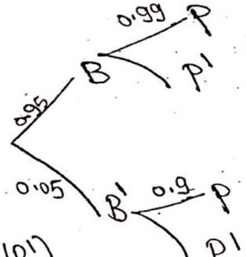
let N denote new
C denote change
F denote complete (full)



$$\therefore P(F) = P(N)\cdot P(F|N) + P(C)\cdot P(F|C)$$

$$= (0.7)(0.9) + (0.3)(0.20) = \boxed{0.69}$$

2-110 The British government has stepped up its information campaign regarding foot and mouth disease by mailing brochures to farmers around the country. It is estimated that 99% of Scottish farmers who receive the brochures possess enough information to deal with an outbreak of the disease, but only 90% of those without the brochure can deal with an outbreak. After the first three month of mailing, 95% of the farmers in Scotland received the information. Compute the probability that a randomly selected farmer will have enough information to deal effectively with an outbreak of the disease.



let B the event of receiving brochures.
, P the event of possessing information.

$$\therefore P(P) = P(B) \cdot P(P|B) + P(B') \cdot P(P|B')$$

$$= (0.95)(0.99) + (0.05)(0.90) = \boxed{0.9855}$$

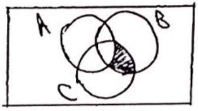
ll 2017 The events A, B and C satisfy : $P(A|B\cap C) = \frac{1}{4}$, $P(B|C) = \frac{1}{3}$, and $P(C) = \frac{1}{2}$. Calculate $P(A^c \cap B \cap C)$

Sol

$\because P(B|C) = \dfrac{P(B\cap C)}{P(C)} \Rightarrow P(B\cap C) = (\frac{1}{3})(\frac{1}{2}) = \frac{1}{6}$

& $P(A|B\cap C) = \dfrac{P(A\cap B\cap C)}{P(B\cap C)} \Rightarrow P(A\cap B\cap C) = (\frac{1}{4})(\frac{1}{6}) = \frac{1}{24}$

Now, $P(A^c\cap B\cap C) = P(B\cap C) - P(A\cap B\cap C) = \frac{1}{6} - \frac{1}{24} = \frac{1}{8}$



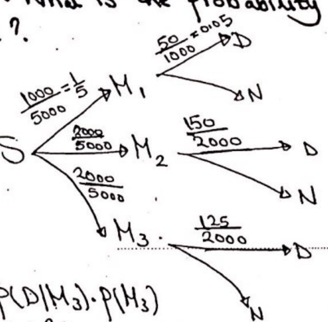
Lecture A factory of electric bulbs contains 3 machines M_1, M_2 and M_3. Machine M_1 produced daily 1000 bulbs out of which 50 are defective. Machine M_2 produced daily 2000 bulbs out of which 150 are defective. Machine M_3 produced daily 2000 bulbs out of which 125 are defective. A bulb is choosen at random from the total daily production and it was found to be defective. What is the probability it was produced by machine M_2?

Sol

$S = 1000 + 2000 + 2000 = 5000$

$P(M_1) = \frac{1000}{5000} = \frac{1}{5}$

$P(M_2) = P(M_3) = \frac{2000}{5000} = \frac{2}{5}$



$P(D) = P(D|M_1)\cdot P(M_1) + P(D|M_2)\cdot P(M_2) + P(D|M_3)\cdot P(M_3)$

$= (0.05)(\frac{1}{5}) + (0.075)(\frac{2}{5}) + (0.0625)(\frac{2}{5}) = 0.065$

$\therefore P(M_2|D) = \dfrac{P(M_2\cap D)}{P(D)} = \dfrac{P(M_2)\cdot P(D|M_2)}{P(D)} = \dfrac{(\frac{2}{5})(0.075)}{0.065} = \dfrac{6}{13}$

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Quiz (1): (A)

1. A lot of 100 semiconductor chips contains 20 that are defective. Two are selected at random, without replacement.

a) What is the probability that the first one is defective?

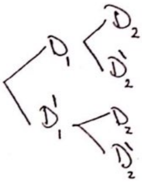
b) What is the probability that the second one is defective given that the first one is defective?

c) What is the probability that both are defective?

d) How does the answer to part (b) change if chips selected were replaced prior to the next selection?

Sol

a) $P(D_1 D_2) + P(D_1 D_2') = \frac{20}{100} \cdot \frac{19}{99} + \frac{20}{100} \cdot \frac{80}{99} = \boxed{0.2}$



b) $P(D_2 | D_1) = \boxed{\frac{19}{99}}$

c) $P(D_1 D_2) = \frac{20}{100} \cdot \frac{19}{99} = \boxed{0.038}$

d) $P(D_2 | D_1) = \frac{20}{100} = \boxed{0.2}$

2. If $P(A) = 0.2$, $P(B) = 0.2$ and A, B are mutually exclusive, are they independent?

$\because P(A \cap B) = 0 \neq P(A) \cdot P(B) = 0.04$

Therefore, A and B are <u>not</u> independent

3. Suppose that $P(A|B) = 0.7$, $P(A) = 0.5$ and $P(B) = 0.2$. Determine $P(B|A)$.

Sol

$\because P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.7 = \frac{P(A \cap B)}{0.2} \Rightarrow P(A \cap B) = 0.14$

$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.14}{0.5} = \boxed{0.28}$

4. If $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$. Determine the following:

a) $P(A') = 1 - P(A) = \boxed{0.7}$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = \boxed{0.4}$

c) $P(A' \cap B) = P(B) - P(A \cap B) = 0.2 - 0.1 = \boxed{0.1}$

d) $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.1 = \boxed{0.2}$

e) $P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.4 = \boxed{0.6}$

f) $P(A' \cup B) = 1 - P(A \cap B') = \boxed{0.8}$

Quiz (1): (B)

1. A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected at random, without replacement. Let A and B denote the events that the first and the second is defective, respectively.

(a) What is the probability that the second one selected is defective given that the first one is defective?

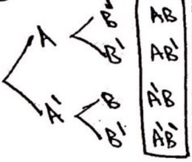
(b) What is the probability that both are defective?

(c) What is the probability that both are acceptable?

(d) Are A and B independent events?

(e) If the sampling was with replacement, would A and B be independent?

Sol



(a) $P(B|A) = \boxed{\frac{4}{499}}$

(b) $P(AB) = \left(\frac{5}{500}\right)\left(\frac{4}{499}\right) = \boxed{0.000080}$

(c) $P(A'B') = \left(\frac{495}{500}\right)\left(\frac{494}{499}\right) = \boxed{0.98}$

(d) $P(B) = P(AB) + P(A'B) = \left(\frac{4}{499}\right)\left(\frac{5}{500}\right) + \left(\frac{5}{499}\right)\left(\frac{495}{500}\right) = \frac{5}{500}$

$\because P(B|A) \neq P(B)$, therefore, A and B are not independent

(e) $\because P(B|A) = \frac{5}{500}$ & $P(B) = \left(\frac{5}{500}\right)\left(\frac{5}{500}\right) + \left(\frac{5}{500}\right)\left(\frac{495}{500}\right) = \frac{5}{500}$

$\therefore P(B|A) = P(B)$, therefore A and B are independent.

---

2. If $P(A|B) = 0.3$, $P(B) = 0.8$ and $P(A) = 0.3$. Are the events B and the complement of A independent?

$\because P(A'|B) = 1 - P(A|B) = 0.7$ and $P(A') = 0.7$

$\therefore P(A'|B) = P(A')$ Therefore A' and B are independent.

---

3. If A and B are independent, show that A' and B' are independent.

$P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A) \cdot P(B)] = 1 - P(A) - P(B) + P(A) \cdot P(B)$

$\therefore P(A' \cap B') = [1 - P(A)][1 - P(B)] = P(A')\, P(B')$

---

4. Each of the possible five outcomes is equally likely. The sample space is $\{a, b, c, d, e\}$. Let A denote $\{a, b\}$, B denote $\{c, d, e\}$. Find:

(a) $P(A) = \boxed{\frac{2}{5}}$ (b) $P(B) = \boxed{\frac{3}{5}}$ (c) $P(A') = \boxed{\frac{3}{5}}$

(d) $P(A \cup B) = \boxed{1}$ (e) $P(A \cap B) = \boxed{0}$

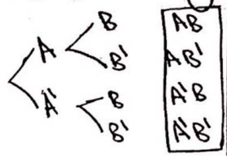
Quiz (1): (C)

[1] A batch of 25 injection-modeled parts contains 5 that have suffered excessive shrinkage.

(a) If two are selected at random, without replacement, what is the probability that the second part is one with excessive shrinkage?

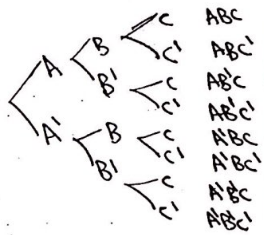
(b) If three are selected at random, without replacement, what is the probability that the third part selected is one with excessive shrinkage?

(Sol)



(a) $P(B) = P(AB) + P(A'B)$

$= \left(\frac{5}{25}\right)\left(\frac{4}{24}\right) + \left(\frac{20}{25}\right)\left(\frac{5}{24}\right) = \boxed{0.2}$



(b) $P(C) = P(ABC) + P(AB'C) + P(A'BC) + P(A'B'C)$

$= \left(\frac{5}{25}\right)\left(\frac{4}{24}\right)\left(\frac{3}{23}\right) + \left(\frac{5}{25}\right)\left(\frac{20}{24}\right)\left(\frac{4}{23}\right) + \left(\frac{20}{25}\right)\left(\frac{5}{24}\right)\left(\frac{4}{23}\right)$

$+ \left(\frac{20}{25}\right)\left(\frac{19}{24}\right)\left(\frac{5}{23}\right) = \boxed{0.2}$

(c) P(all three are with excessive shrinkage)

$= P(ABC) = \left(\frac{5}{25}\right)\left(\frac{4}{24}\right)\left(\frac{3}{23}\right) = \boxed{\frac{1}{230}}$

[2] If A, B and C are mutually exclusive events, is it possible for $P(A) = 0.3$, $P(B) = 0.4$ and $P(C) = 0.5$? Why or why not?

$\because P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2 > 1$ (impossible)

then $P(A)$, $P(B)$ and $P(C)$ can't equal the given values.

[3] If $P(A|B) = 0.4$, $P(B) = 0.8$ and $P(A) = 0.5$, are A and B independent?

$\because P(A|B) \neq P(A)$, then A and B are not independent.

[4] The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4 and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and B denote $\{c, d, e\}$. Determine the following:

(a) $P(A) = 0.1 + 0.1 + 0.2 = \boxed{0.4}$

(b) $P(B) = 0.2 + 0.4 + 0.2 = \boxed{0.8}$

(c) $P(A') = 1 - P(A) = \boxed{0.6}$

(d) $P(A \cup B) = \boxed{1}$

(e) $P(A \cap B) = P(c) = \boxed{0.2}$

Quiz (1): (D)

[1] A day's production of 850 manufactured parts contain 50 parts that don't meet customer requirements.

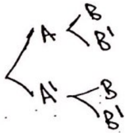
(a) If two parts are selected without replacement, what is the probability that the second is defective, given that the first is defective?

(b) What is the probability that the second part is defective?

(c) What is the probability that both parts are defective?

(d) If three parts are selected, what is the probability that the first two parts are defective and the third is not defective?

i) without replacement ii) With replacement



(a) $P(B|A) = \boxed{\frac{49}{849}}$

(b) $P(B) = P(AB) + P(A'B) = \frac{50}{850}\cdot\frac{49}{849} + \frac{800}{850}\cdot\frac{50}{849} = \boxed{\frac{50}{850}}$

(c) $P(AB) = \frac{50}{850}\cdot\frac{49}{849} = \boxed{0.003395}$

(d) i) $P(ABC') = \frac{50}{850}\cdot\frac{49}{849}\cdot\frac{800}{848} = \boxed{0.0032}$

ii) $P(ABC') = \frac{50}{850}\cdot\frac{50}{850}\cdot\frac{800}{850} = \boxed{0.003257}$

[2] Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine: a) $P(A\cap B)$ b) $P(A'\cap B)$

(a) $\because P(A|B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = (0.4)(0.5) = \boxed{0.2}$

(b) $P(A'\cap B) = P(B) - P(A\cap B) = 0.5 - 0.2 = \boxed{0.3}$

[3] Suppose $P(A|B) = 0.2$, $P(A|B') = 0.3$ and $P(B) = 0.8$. What is $P(A)$?

$P(A) = P(A\cap B) + P(A\cap B')$

$= P(A|B)\cdot P(B) + P(A|B')\cdot P(B') = (0.2)(0.8) + (0.3)(0.2) = \boxed{0.22}$

[4] A part selected for testing is equally likely to have been produced on any one of six cutting tools.

(a) What is the sample space? $S = \{1,2,3,4,5,6\}$

(b) What is the probability that the part is from tool 1? $P(1) = \boxed{\frac{1}{6}}$

(c) What is the probability that the part is from tool 1 **or** tool 3? $P(1) + P(3) = \boxed{\frac{2}{6}}$

(d) What is the probability that the part is **not** from tool 4? $P(4') = 1 - \frac{1}{6} = \boxed{\frac{5}{6}}$

Scanned with CamScanner

Quiz (1): E

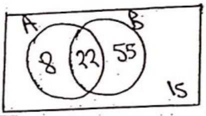
Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized as follows:

		Conforms Yes	Conforms No
Supplier	1	22	8
	2	25	5
	3	30	10

Let A denote the event that a sample is from supplier 1, and let B denote the event that a sample conforms to specifications. If a sample is selected at random, determine:

(a) $P(A)$ (b) $P(B)$ (c) $P(A')$ (d) $P(A\cap B)$ (e) $P(A\cup B)$

(f) $P(A'\cup B)$ (g) $P(B|A)$ (h) Are A and B independent?



(a) $P(A) = \frac{22+8}{100} = \boxed{0.3}$ (b) $P(B) = \frac{22+25+30}{100} = \boxed{0.77}$

(c) $P(A') = \boxed{0.7}$ (d) $P(A\cap B) = \frac{22}{100} = \boxed{0.22}$

(e) $P(A\cup B) = P(A)+P(B)-P(A\cap B) = 0.3+0.77-0.22 = \boxed{0.85}$

OR $P(A\cup B) = \frac{8+22+55}{100} = \boxed{0.85}$

(f) $P(A'\cup B) = 1-[P(A)-P(A\cap B)] = 1-[0.3-0.22] = \boxed{0.92}$

(g) $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0.22}{0.3} = \boxed{0.733}$

(h) $\because P(A)\cdot P(B) = (0.3)(0.77) = 0.231 \neq P(A\cap B)$ $\therefore$ A and B are not indepen.

OR $P(B|A) \neq P(B)$

[2] If A, B and C are mutually exclusive with $P(A)=0.2$, $P(B)=0.3$ and $P(C)=0.4$. Determine the following:

(a) $P(A\cup B\cup C) = P(A)+P(B)+P(C) = \boxed{0.9}$

(b) $P(A\cap B\cap C) = \boxed{0}$

(c) $P(A\cap B) = \boxed{0}$

(d) $P(A\cup B\cap C) = \boxed{0}$

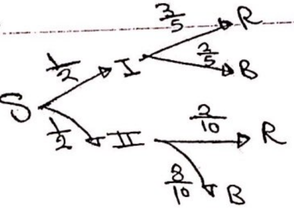
(e) $P(A'\cap B'\cap C') = 1-P(A\cup B\cup C) = 1-0.9 = \boxed{0.1}$

[3] If the last digit of a weight is equally likely to be any digit from 0 through 9. a) What is the probability that the last digit is 0? $P(A) = \boxed{\frac{1}{10}}$

b) What is the probability the last digit is greater than or equal 5? $P(B) = \boxed{\frac{5}{10}}$

Example Box I contains 3 red and 2 blue balls, while box II contains 2 red and 8 blue ones. A fair coin is tossed. If the coin shows a head, a ball is chosen from box I. If it shows a tail, a ball is chosen from box II. Find the probability that a red ball is chosen, and the probability that it comes from box I.

(Sol.)



$$P(R) = P(R|I)\cdot P(I) + P(R|II)\cdot P(II)$$
$$= \left(\tfrac{1}{2}\right)\left(\tfrac{3}{5}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{2}{10}\right) = \frac{2}{5}$$

$$P(I|R) = \frac{P(R|I)\cdot P(I)}{P(R)} = \frac{(3/5)(1/2)}{2/5} = \frac{3}{4}$$

Example 3 marbles are drawn, with replacement, from a lot of 18 marbles (10 black, 5 red, 3 white). Find the probability of having:

i) exactly one black only.
ii) Red, black, then white, respectively.
iii) Three of the same colour.
iv) At most 1 white
v) At least 1 white

(Sol.)

i) $P(\text{one black}) = P(B_1 \cap B_2^c \cap B_3^c) + P(B_1^c \cap B_2 \cap B_3^c) + P(B_1^c \cap B_2^c \cap B_3)$
$$= 3\left(\frac{10}{18}\cdot\frac{8}{18}\cdot\frac{8}{18}\right) = \frac{80}{243}$$

ii) $P(R_1 \cap B_2 \cap W_3) = \frac{5}{18}\cdot\frac{10}{18}\cdot\frac{3}{18} = \frac{25}{972}$

iii) $P(R\cap R\cap R) + P(B\cap B\cap B) + P(W\cap W\cap W) = \left(\frac{5}{18}\right)^3 + \left(\frac{10}{18}\right)^3 + \left(\frac{3}{18}\right)^3$

iv) $P(\text{at most one white}) = P(\text{no white}) + P(\text{only one white})$
$$= \left(\frac{15}{18}\right)^3 + 3\left(\frac{3}{18}\right)\left(\frac{15}{18}\right)^2$$

v) $P(\text{at least one white}) = P(\text{one white}) + P(\text{2 white}) + P(\text{3 white})$

Example
I.T

A box contains 6 tubes of which 2 are defective. The tubes are tested until the 2 defective tubes are discovered. i) What is the probability that the process stopped on the third test?

ii) If the process stopped on the third test, what is the probability that the first tube is non-defective?

Sol

2D
4N
6

i) $P(3^{rd}\ test) = P(N_1 \cap D_2 \cap D_3) + P(D_1 \cap N_2 \cap D_3)$
$= \left(\frac{4}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) = \boxed{\frac{2}{15}}$

ii) $P(N_1 | 3^{rd}\ test) = \frac{P(N_1 \cap D_2 \cap D_3)}{P(3^{rd}\ test)} = \boxed{\frac{1}{2}}$

Example A box contains 5 tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. What is the probability that the process stopped on the:

i) Second test

ii) third test

iii) If the process stopped on the third test, what is the probability that the first tube is non-defective?

Sol

i) $P(2^{nd}\ test) = P(D_1 \cap D_2) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \boxed{\frac{1}{10}}$

ii) $P(3^{rd}\ test) = P(D_1 \cap N_2 \cap D_3) + P(N_1 \cap D_2 \cap D_3) + P(N_1 \cap N_2 \cap N_3)$
$= \left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) = \boxed{\frac{3}{10}}$

iii) $P(N_1 | 3^{rd}\ test) = \frac{P(N_1 \cap D_2 \cap D_3) + P(N_1 \cap N_2 \cap N_3)}{P(3^{rd}\ test)} = \boxed{\frac{2}{3}}$

Example A and B are two events such that $P(A) = 0.5$, $P(B) = 0.3$, $P(A\cup B) = 0.7$

1) Are A and B independent or not?
2) Are A and B mutually exclusive or not?
3) Find $P(A|B)$, $P(B|A)$, $P(A|A\cup B)$, $P(A|A\cap B)$, $P(A\cap B|A\cup B)$, $P((A^c\cap B^c)\cup B)$, $P((A^c\cap B)|(A\cup B))$

Sol

$\because P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\therefore 0.7 = 0.5 + 0.3 - P(A\cap B) \Rightarrow P(A\cap B) = 0.1$

1) $P(A\cap B) \neq P(A)\cdot P(B) \Rightarrow$ A and B are dependent

2) $P(A\cap B) \neq 0 \Rightarrow$ A and B are not mutually exclusive

3) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$, $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0.1}{0.5} = \frac{1}{5}$

$P(A|A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)} = \frac{P(A)}{P(A\cup B)} = \frac{0.5}{0.7} = \frac{5}{7}$

$P(A|A\cap B) = \frac{P(A\cap(A\cap B))}{P(A\cap B)} = \frac{P(A\cap B)}{P(A\cap B)} = 1$

$P(A\cap B|A\cup B) = \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)} = \frac{P(A\cap B)}{P(A\cup B)} = \frac{1}{7}$

$\Rightarrow P[(A^c\cap B^c)\cup B] = P(A^c\cap B^c) + P(B) - P(A^c\cap B^c\cap B)$

$= P(A\cup B)^c + P(B) - 0 = (1-0.7) + 0.3 - 0 = 0.6$

or $= 1 - P(A\cap B^c) = 1 - [P(A) - P(A\cap B)] = 1 - [0.5 - 0.1] = 0.6$

$\Rightarrow P[(A^c\cap B)|(A\cup B)] = \frac{P[(A^c\cap B)\cap(A\cup B)]}{P(A\cup B)} = \frac{P(B) - P(A\cap B)}{P(A\cup B)} = \frac{2}{7}$

Example Let A and B be two events such that $P(A^c \cap B^c) = 0.2$, $P(A \cap B^c) = 0.2$ and $P(A^c \cap B) = 0.5$

Find $P(A|B)$ and $P(B|A)$.

Sol

$P(A^c \cap B^c) = P(A \cup B)^c = 1 - [P(A \cup B)] = 0.2 \Rightarrow P(A \cup B) = 0.8$

$P(A \cap B^c) = P(A) - P(A \cap B) = 0.2 \rightarrow (1)$

$P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 \rightarrow (2)$

(1)+(2): $\underbrace{P(A) + P(B) - P(A \cap B)}_{} - P(A \cap B) = 0.7$

$0.8 - P(A \cap B) = 0.7 \Rightarrow P(A \cap B) = 0.1$

in (1): $P(A) = 0.3$

in (2): $P(B) = 0.6$

$\therefore P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.6} = \dfrac{1}{6}$

$P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$

Example Two fire engines are operating independently. The probability that a specific engine is available when needed is 0.99. Find the probability that:

1) a fire engine is available when needed.

2) neither is available when needed.

Sol

Let E_i be the event that the engine i is available when needed

$P(E_i) = 0.99 \Rightarrow P(E_i^c) = 0.01$

1) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = (0.99) + (0.99) - (0.99)(0.99)$

$= \boxed{0.9999}$ independent

2) $P(E_1^c \cap E_2^c) = P(E_1^c)\, P(E_2^c) = (0.01)(0.01) = \boxed{0.0001}$

OR $= 1 - P(E_1 \cup E_2) = 1 - 0.9999 = 0.0001$