Omar Aguilar

1)

Time(s)	0	2	4	6	8	10
Distance(m)	0	40	80	130	180	250

@ t=0

$$3\text{ pt. FD} = \frac{-3f(x_i)+4f(x_{i+1})-f(x_{i+2})}{2h}$$

$$= \frac{-3(0)+4(40)-(80)}{2(2)}$$

$$= \frac{80}{4} = 20 \text{ m/s}$$

$$\frac{20\text{ m}}{\text{s}} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{3600\text{ s}}{1\text{ hr}} = 72 \text{ km/hr}$$

@ t=4

$$2\text{ pt. CD} = \frac{f(x_{i+1})-f(x_{i-1})}{2h}$$

$$= \frac{130-40}{2(2)}$$

$$= \frac{90}{4} = 22.5 \text{ m/s}$$

$$\frac{22.5\text{ m}}{\text{s}} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{3600\text{ s}}{1\text{ hr}} = 81 \text{ km/hr}$$

@ t=10

$$3\text{ pt. BD} = \frac{3f(x_i)-4f(x_{i-1})+f(x_{i-2})}{2h}$$

$$= \frac{3(250)-4(180)+(130)}{2(2)}$$

$$= 40 \text{ m/s}$$

$$\frac{40\text{ m}}{\text{s}} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{3600\text{ s}}{1\text{ hr}} = 144 \text{ km/hr}$$

2) b) For FD, theory states that as the h is cut in half, the error will also be cut in half. This is signified by the error for FD being $O(h)$. Due to the mathematical characteristics of $\sin(x)$ in combination with the structure of the Taylor Series expansion seen below, when using $x=0$, the higher order terms matter more since the even derived terms cancel out. This causes a heavier lean on the higher order terms, causing the convergence rate to be higher.

c) $x=0, n=6$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f^3(x_0)}{3!}(x-x_0)^3 + \frac{f^4(x_0)}{4!}(x-x_0)^4 + \dots + \frac{f^7(c)}{7!}(x-x_0)^7$$

$f(x)=\sin(x)$ $f'(x)=\cos(x)$ $f''(x)=-\sin(x)$ $f^3(x)=-\cos x$

$$f(0) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x\cos(c)}{7!}$$

When using $x=0$, a lot of the terms get cancelled out, therefore the contributions of higher order terms becomes significant as compared to using $x=1$, where no terms would cancel out and input is seen from every derivative.