Omar Aguilar

3) $\begin{pmatrix} 1 & 1 & 2 \\ 4 & 3 & 0 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix}$

a) $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 4 & 3 & 0 & 1 \\ 0 & 2 & 2 & 4 \end{array}\right) \xrightarrow{R_2 = R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & -1 & -8 & -23 \\ 0 & 2 & 2 & 4 \end{array}\right) \xrightarrow{R_3 = R_3 + 2R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & -1 & -8 & -23 \\ 0 & 0 & -14 & -42 \end{array}\right)$

$-14x_3 = -42$ $\quad -x_2 - 8x_3 = -23$ $\quad x_1 + x_2 + 2x_3 = 6$

$x_3 = 3$ $\quad -x_2 - 24 = -23$ $\quad x_1 - 1 + 6 = 6$

$x_2 = -1$ $\quad x_1 = 1$

$x = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

b) $U = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -8 \\ 0 & 0 & -14 \end{pmatrix}$ $\quad L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ $\quad b = \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix}$

$Ly = b$

$y_1 = 6$ $\quad 4y_1 + y_2 = 1$ $\quad -2y_2 + y_3 = 4$

$24 + y_2 = 1$ $\quad 46 + y_3 = 4$

$y_2 = -23$ $\quad y_3 = -42$

$y = \begin{pmatrix} 6 \\ -23 \\ -42 \end{pmatrix}$

$Ux = y$

$-14x_3 = -42$ $\quad -x_2 - 8x_3 = -23$ $\quad x_1 + x_2 + 2x_3 = 6$

$x_3 = 3$ $\quad -x_2 - 24 = -23$ $\quad x_1 - 1 + 6 = 6$

$x_2 = -1$ $\quad x_1 = 1$

$x = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

4) $f(x) = 1 + e^x$, $x = 0.5$, $x_0 = 1$

$f'(x) = e^x$, $f''(x) = e^x$, ...

at $n = 1$:

$f(0.5) = f(1) + f'(1)(0.5 - 1)$

$f(0.5) \approx 1 + e - 0.5e = 1 + 0.5e = 2.359$

error term: $\frac{e^c}{2}(x-1)^2 \Rightarrow 0.5 \le c \le 1 \rightarrow$ upper bound for c

upper bound $= \frac{e}{2}(-0.5)^2$

$= 0.125e$

$= 3.398 \times 10^{-1}$

at $n = 2$

$f(0.5) = f(1) + f'(1)(0.5-1) + \frac{f''(1)}{2}(0.5-1)^2$

$f(0.5) \approx 1 + e - 0.5e + 0.125e = 2.699$

error term: $\frac{e^c}{6}(x-1)^3$

upper bound $= \frac{e}{6}(-0.5)^3$

$= -5.663 \times 10^{-2}$

at $n = 4$

$f(0.5) = f(1) + f'(1)(0.5-1) + \frac{f''(1)}{2}(0.5-1)^2 + \frac{f'''(1)}{6}(0.5-1)^3 + \frac{f^{(4)}(1)}{24}(0.5-1)^4$

$f(0.5) \approx 1 + e - 0.5e + 0.125e - \frac{e}{48} + \frac{e}{384} = 2.649$

error term: $\frac{e^c}{120}(x-1)^5$

upper bound $= \frac{e}{120}(-0.5)^5$

$= -7.879 \times 10^{-4}$