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1) $A = \begin{pmatrix} 0 & 4 & 1 \\ -5 & 4 & 7 \\ -1 & 3 & 2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 0 & 4 & 1 & 1 & 0 & 0 \\ -5 & 4 & 7 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} -5 & 4 & 7 & 0 & 1 & 0 \\ 0 & 4 & 1 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 = \frac{R_1}{-5}} \left(\begin{array}{ccc|ccc} 1 & -\frac{4}{5} & -\frac{7}{5} & 0 & -\frac{1}{5} & 0 \\ 0 & 4 & 1 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 = R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & -\frac{4}{5} & -\frac{7}{5} & 0 & -\frac{1}{5} & 0 \\ 0 & 4 & 1 & 1 & 0 & 0 \\ 0 & \frac{11}{5} & \frac{3}{5} & 0 & -\frac{1}{5} & 1 \end{array}\right) \xrightarrow{R_2 = \frac{R_2}{4}} \left(\begin{array}{ccc|ccc} 1 & -\frac{4}{5} & -\frac{7}{5} & 0 & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{11}{5} & \frac{3}{5} & 0 & -\frac{1}{5} & 1 \end{array}\right)$$

$$\xrightarrow{R_1 = R_1 + \frac{4}{5}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{6}{5} & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{11}{5} & \frac{3}{5} & 0 & -\frac{1}{5} & 1 \end{array}\right) \xrightarrow{R_3 = R_3 - \frac{11}{5}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{6}{5} & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{20} & -\frac{11}{20} & -\frac{1}{5} & 1 \end{array}\right)$$

$$\xrightarrow{R_3 = 20R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{6}{5} & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & -11 & -4 & 20 \end{array}\right) \xrightarrow{R_1 = R_1 + \frac{6}{5}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & -5 & 24 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & -11 & -4 & 20 \end{array}\right)$$

$$\xrightarrow{R_2 = R_2 - \frac{1}{4}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & -5 & 24 \\ 0 & 1 & 0 & 3 & 1 & -5 \\ 0 & 0 & 1 & -11 & -4 & 20 \end{array}\right) \qquad \boxed{A^{-1} = \begin{pmatrix} -13 & -5 & 24 \\ 3 & 1 & -5 \\ -11 & -4 & 20 \end{pmatrix}}$$

From MATLAB: $A^{-1} = \begin{pmatrix} -13-5.15e{-14} & -5-1.87e{-14} & 24+8.88e{-14} \\ 3+1.02e{-14} & 1+3.55e{-15} & -5-1.78e{-14} \\ -11-4.09e{-14} & -4-1.42e{-14} & 20+7.11e{-14} \end{pmatrix}$

The $\boxed{\text{hand computed}}$ solution is correct:

Hand Comp: $AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

MATLAB: $AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1.42e{-14} & 1+7.11e{-15} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2)

$$x=\begin{pmatrix}0\\4\\0\\-3\end{pmatrix} \quad A=\begin{pmatrix}0&4&1\\-5&4&7\\-1&3&2\end{pmatrix} \quad B=\begin{pmatrix}0&2\\3&0\end{pmatrix}$$

a) $\|x\|_2=\sqrt{0^2+4^2+0^2+(-3)^2}$ $\quad \|x\|_\infty=\max|x|$

$=\sqrt{25}\ \boxed{=5}$ $\quad \boxed{=4}$

$\|A\|_\infty=\max(5,16,6)$ $\quad \|B\|_\infty=\max(3,2)$

$\boxed{=16}$ $\quad \boxed{=3}$

b) $\text{cond}_\infty(A)=\|A\|_\infty\|A^{-1}\|_\infty \rightsquigarrow \max(42,9,35)=42$

$=(16)(42)$

$\boxed{=672}$

$$\left(\begin{array}{cc|cc}0&2&1&0\\3&0&0&1\end{array}\right) \xrightarrow{R_2\leftrightarrow R_1} \left(\begin{array}{cc|cc}3&0&0&1\\0&2&1&0\end{array}\right) \xrightarrow{R_1=\frac{R_1}{3}} \left(\begin{array}{cc|cc}1&0&0&\frac{1}{3}\\0&2&1&0\end{array}\right) \xrightarrow{R_2=\frac{R_2}{2}} \left(\begin{array}{cc|cc}1&0&0&\frac{1}{3}\\0&1&\frac{1}{2}&0\end{array}\right)$$

$\text{cond}_\infty(B)=\|B\|_\infty\|B^{-1}\|_\infty \longrightarrow \max(\frac{1}{2},\frac{1}{3})=\frac{1}{2}$

$=(3)(\frac{1}{2})$

$\boxed{=1.5}$

c) $\|B\|_2=\max_{x\in\mathbb{R}^n}\dfrac{\|Ax\|_2}{\|x\|_2}$

$$Ax=\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}2x_2\\3x_1\end{pmatrix}$$

$\|Ax\|_2=\sqrt{(2x_2)^2+(3x_1)^2}$ $\quad \|x\|_2=\sqrt{x_1^2+x_2^2}$

$=\sqrt{4x_2^2+9x_1^2}$

When $x=\begin{pmatrix}1\\0\end{pmatrix}$: $\|Ax\|_2=3$ and $\|x\|_2=1$, so $\dfrac{\|Ax\|_2}{\|x\|_2}=3$

When $x=\begin{pmatrix}0\\1\end{pmatrix}$: $\|Ax\|_2=2$ and $\|x\|_2=1$, so $\dfrac{\|Ax\|_2}{\|x\|_2}=2$

$\|B\|_2=\max(3,2)$

$\boxed{=3}$