Omar Aguilar

4) a) $x = \begin{pmatrix} 0 \\ \pi/2 \\ \pi \end{pmatrix}$ $y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & \pi/2 & \pi^2/4 \\ 1 & \pi & \pi^2 \end{pmatrix}$ $c = A \backslash y$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & \pi/2 & \pi^2/4 & 1 \\ 1 & \pi & \pi^2 & 0 \end{array}\right) \xrightarrow{R2 = R2 - \frac{R_3}{4}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 3/4 & \pi/4 & 0 & 1 \\ 1 & \pi & \pi^2 & 0 \end{array}\right)$

$c_1 = 1$ $\frac{3}{4}c_1 + \frac{\pi}{4}c_2 = 1$ $c_1 + \pi c_2 + \pi^2 c_3 = 0$

$\frac{\pi}{4}c_2 = \frac{1}{4}$ $\pi^2 c_3 = -2$

$c_2 = \frac{1}{\pi}$ $c_3 = -\frac{2}{\pi^2}$

$c = \begin{pmatrix} 1 \\ 1/\pi \\ -2/\pi^2 \end{pmatrix}$ $p(x) = 1 + \frac{x}{\pi} - \frac{2x^2}{\pi^2}$

This polynomial is unique, as it is the only polynomial that can fit these points.

b) $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ $y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $a = A \backslash y$

$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1 = R_1 + R_2} \left(\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{array}\right)$

$2a_1 = 1$ $a_1 - a_2 = 0$ $a_1 + a_3 = 1$

$a_1 = \frac{1}{2}$ $a_2 = \frac{1}{2}$ $a_3 = 1 - \frac{1}{2}$

$= \frac{1}{2}$