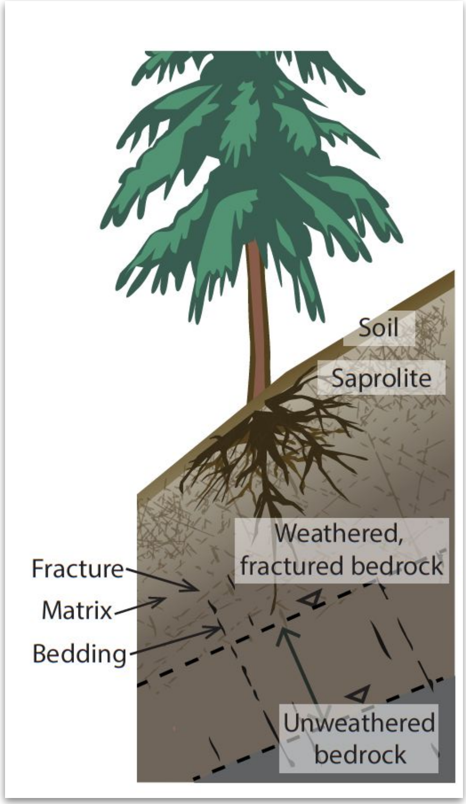
Fracture Measurement in CT

Omar Alamoudi and Logan Schmidt

Motivation: Water storage in unsaturated fractured bedrock is important



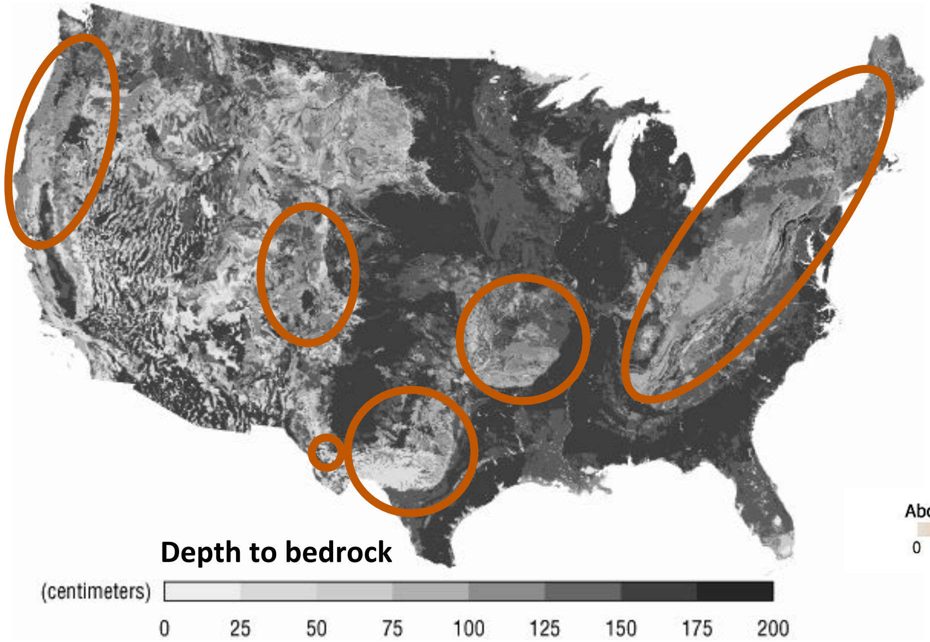
Eel River, CA Sandstone

McDonald Observatory, TX Volcanics

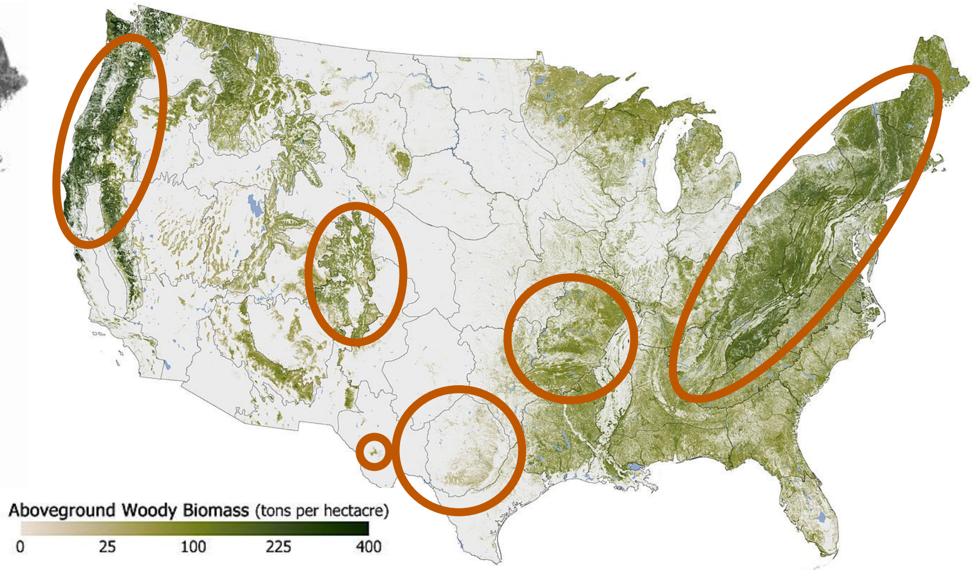
White Property, TX Karst

Gothic, CO Mudstone

Motivation: Water storage in unsaturated weathered bedrock is important

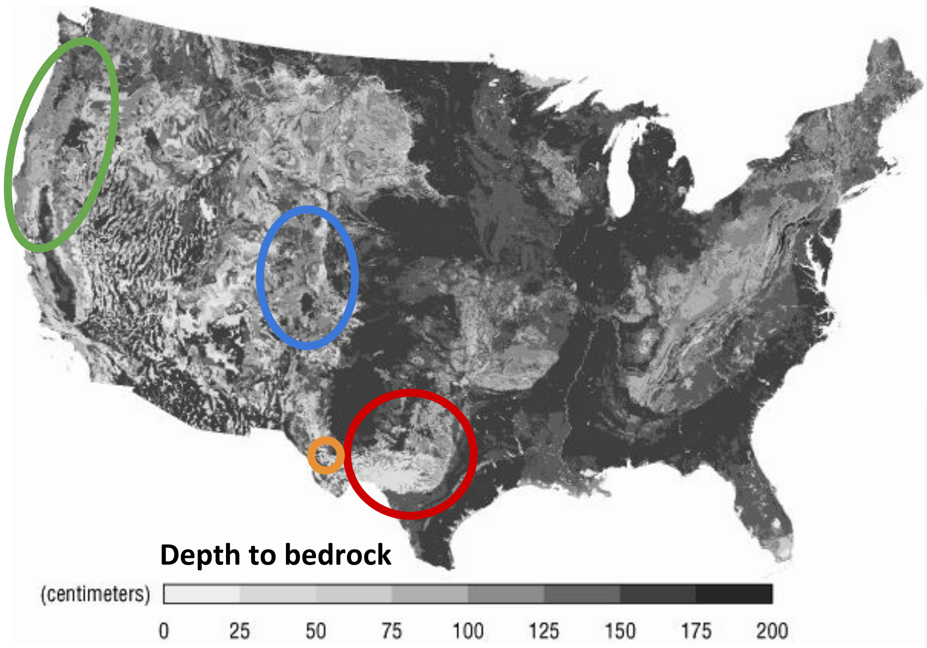


(Miller and White, 1998)



(NASA Earth Observatory map by Robert Simmon)

Proposed Research Objectives



(Miller and White, 1998)

Eel River, CA Sandstone

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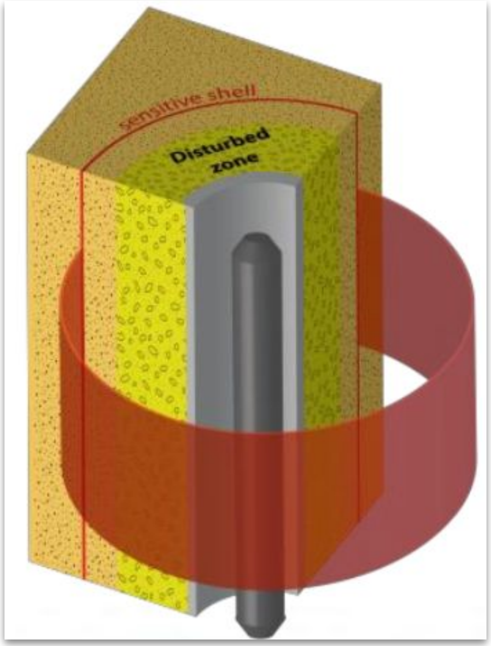
Motivation: NMR is uniquely suited for hydrological investigations in the weathered bedrock vadose zone



(Vista Clara, Inc.)

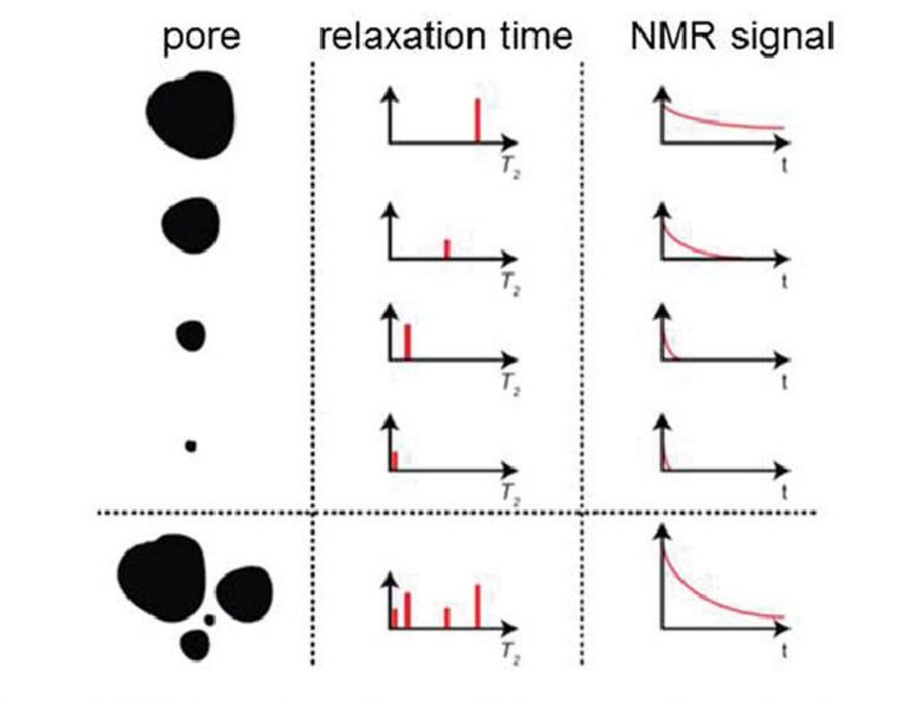
- Directly sensitive to water
- **No site-specific calibration**
- **Matrix-independent water content measurement**
- **Fixed, well-defined volume of investigation**
- High vertical resolution
- Decades of use in petroleum exploration
- Sensitive to pore environment

Motivation: NMR is uniquely suited for hydrological investigations in the weathered bedrock vadose zone

Initial amplitude directly proportional to water content

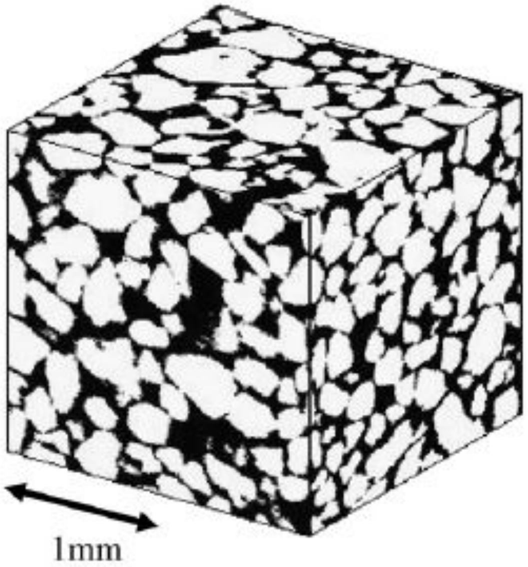
Signal decay (relaxation) influenced by pore environment

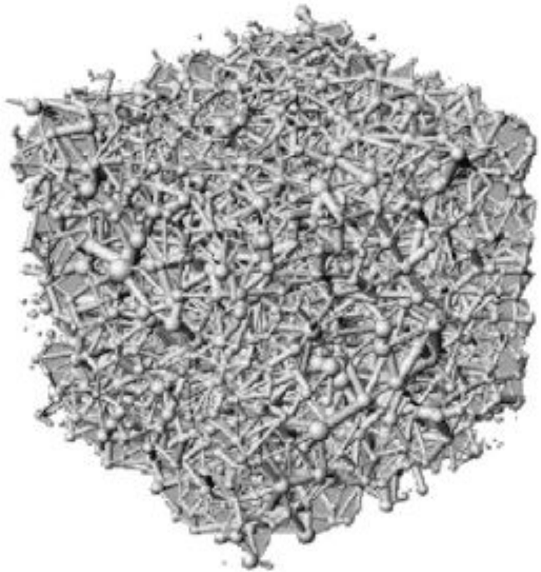
$$M(t) = M_0 \sum_{i=1}^{n} a_i e^{-t/T_{2i}}$$



(Coates et al., 1999)

Part 3 *Question*: What is the water potential of rock moisture

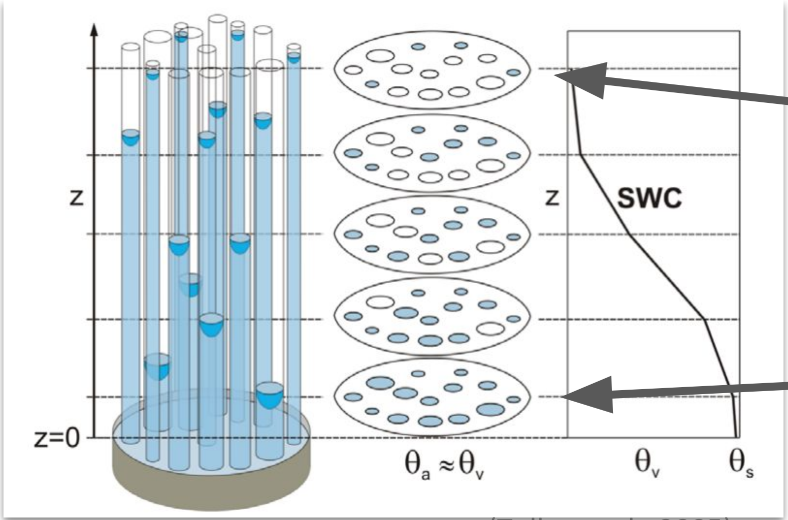




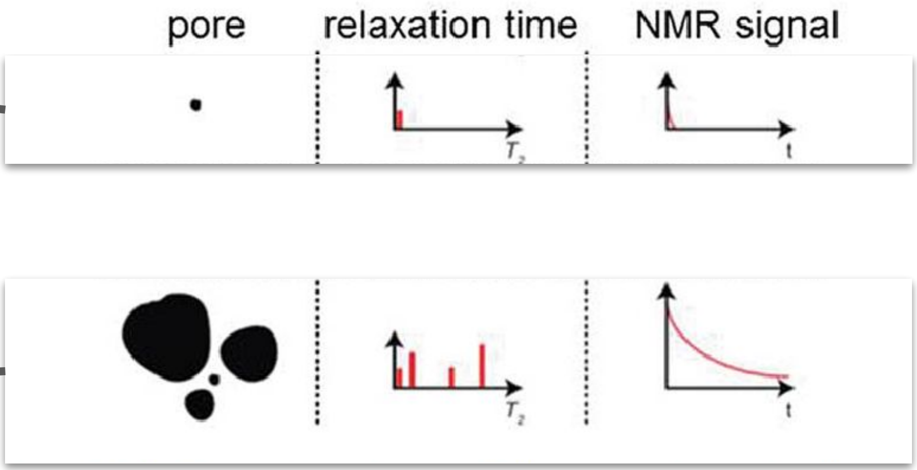
(Talabi et al., 2009)

Part 3 *Methods*: NMR relaxation is controlled by pore radius and can be related to water potential

$$\Delta P \propto \frac{\sigma}{r} \qquad \frac{1}{T_2} = \rho_2\left(\frac{\alpha}{r}\right) \qquad M(t) = \mathbf{M}_0 \sum_i M_i e^{(-t/T_{2,i})}$$

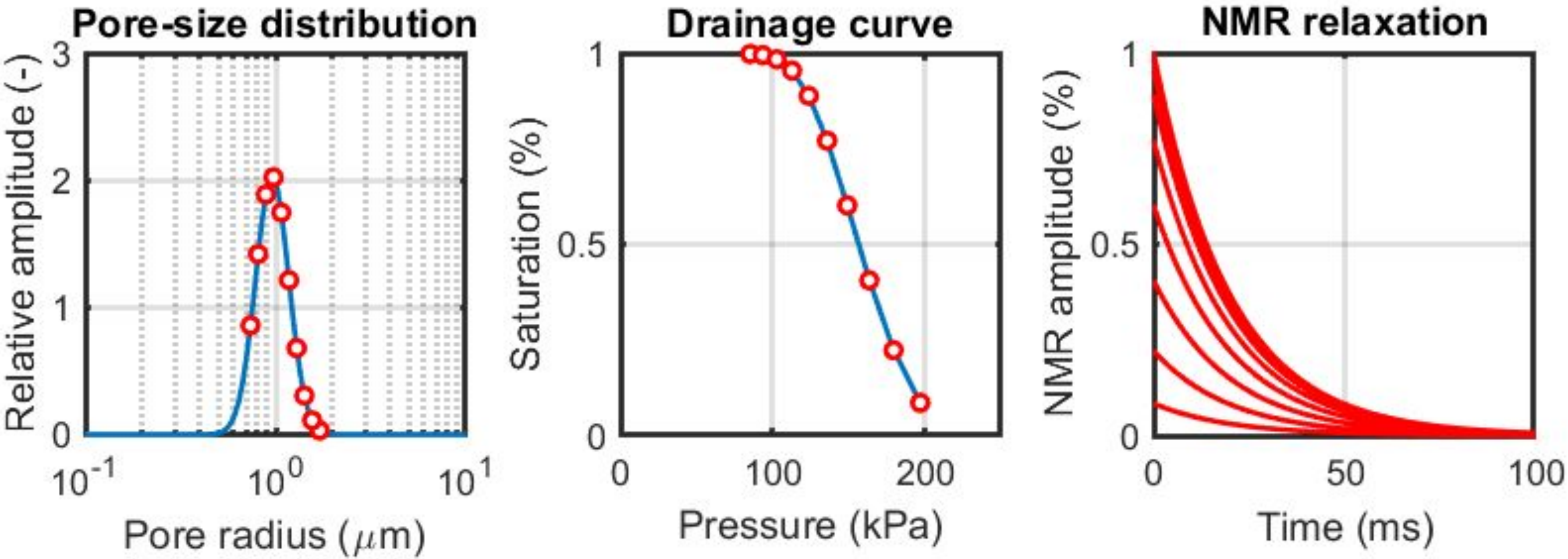


(Tuller et al., 2005)

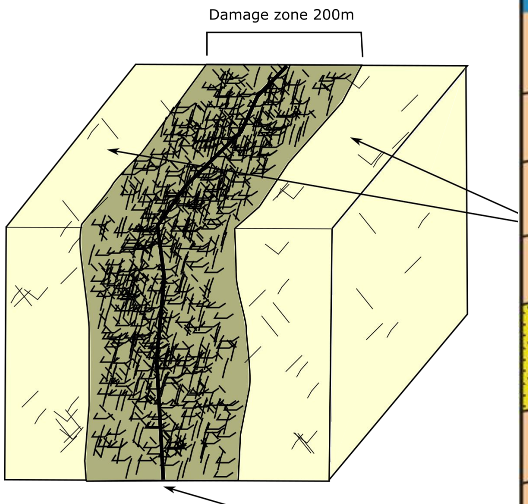


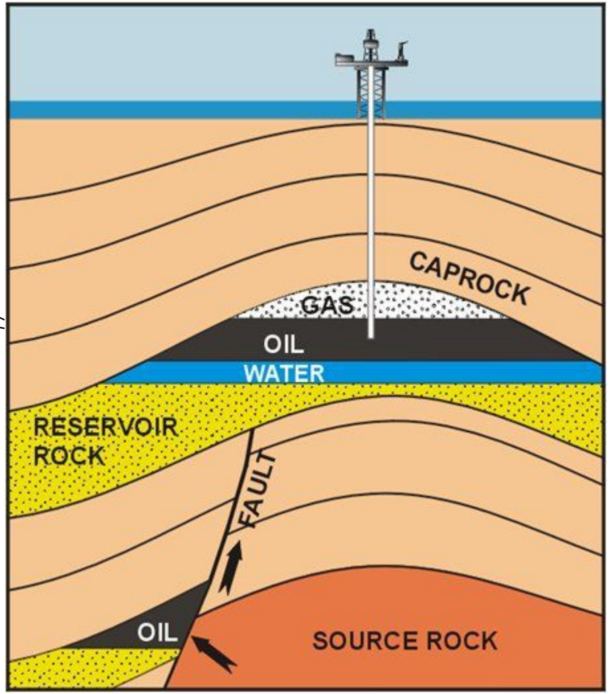
(Coates et al., 1999)

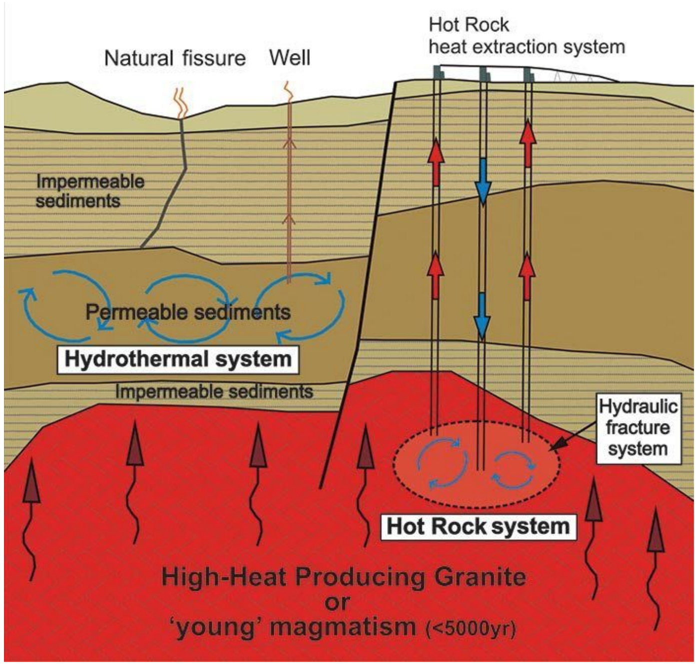
Part 3 *Methods*: NMR relaxation can be related to water potential



Motivation





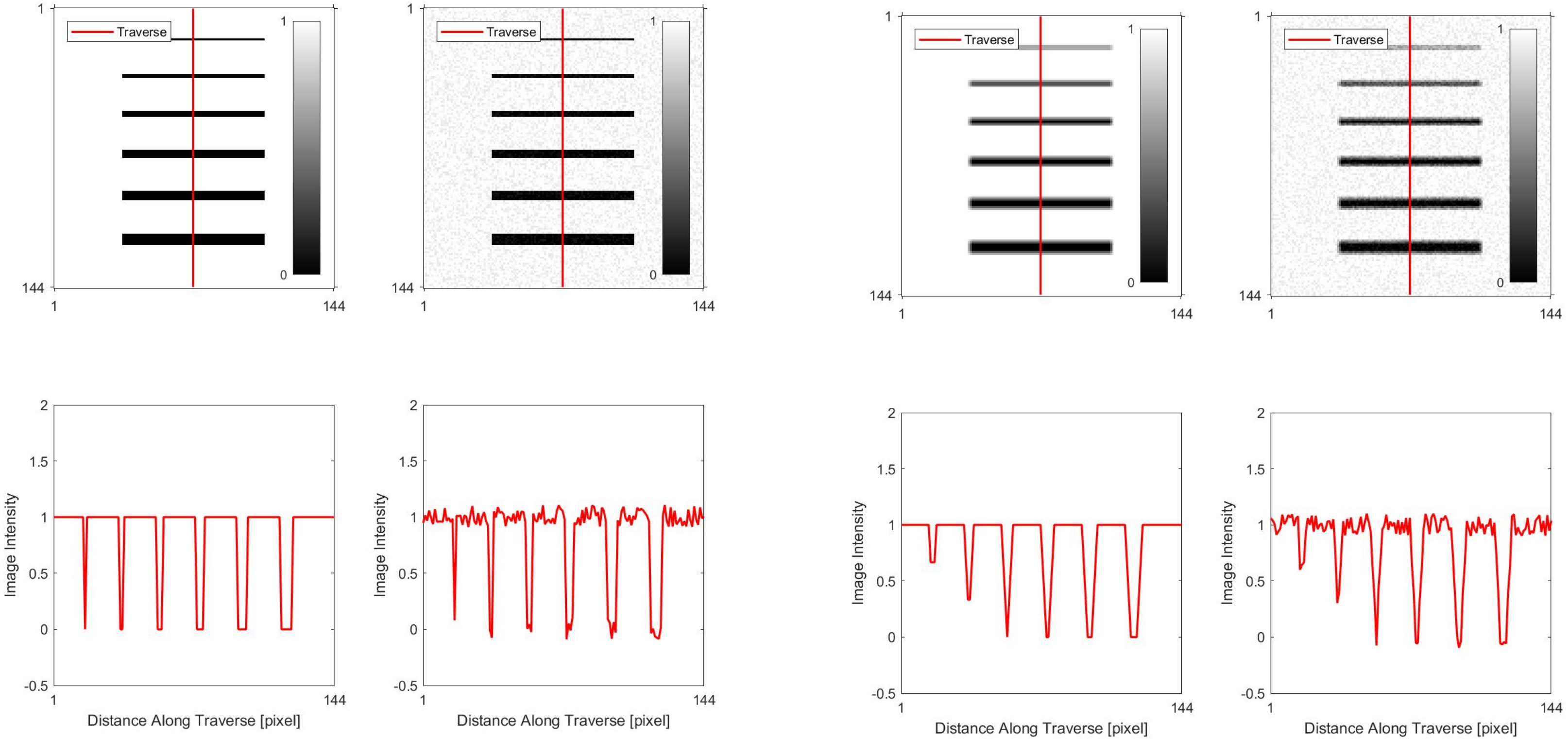


https://structuredatabase.wordpress.com/fault-zone-structure-and-permeability/

https://www.science.org.au/curious/technology-future/feeling-heat-geothermal-energy

Motivation

Methods: Generating Synthetic Fractures



Methods: Fracture Detection (The Hessian Method)

$$g(x) = ae^{-\frac{1}{2}\left(\frac{x-b}{c}\right)^2}$$

$$g'(x) = (-\frac{a}{c^2})(x-b)e^{-\frac{1}{2}\left(\frac{x-b}{c}\right)^2}$$

$$g''(x) = \frac{a\left(x^2 - 2bx - c^2 + b^2\right)\mathrm{e}^{-\frac{(x-b)^2}{2c^2}}}{c^4}$$

$$g(x,y) = A\exp\left[-\frac{1}{2}\left(\left(\frac{x-x_0}{c_x}\right)^2 + \left(\frac{y-y_0}{c_y}\right)^2\right)\right]$$

Methods: Computing The Hessian

$$H = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

$$I_{ij} = I(x, y, z) * (\alpha G_{ij}(x, y, z, s))$$

$$\text{where} \qquad i, j = x, y, z$$

Voorn et al. 2013

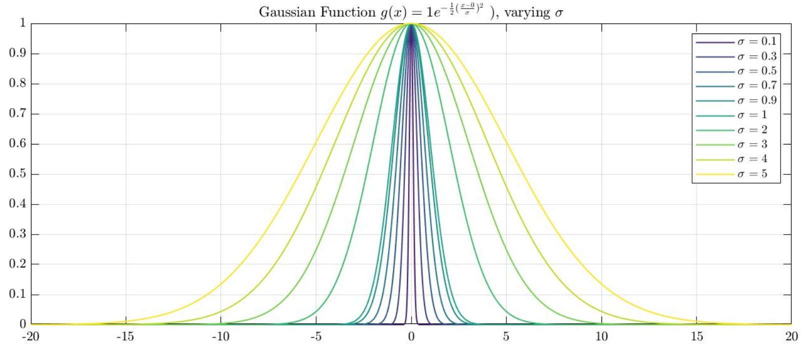
Methods: Computing The Hessian

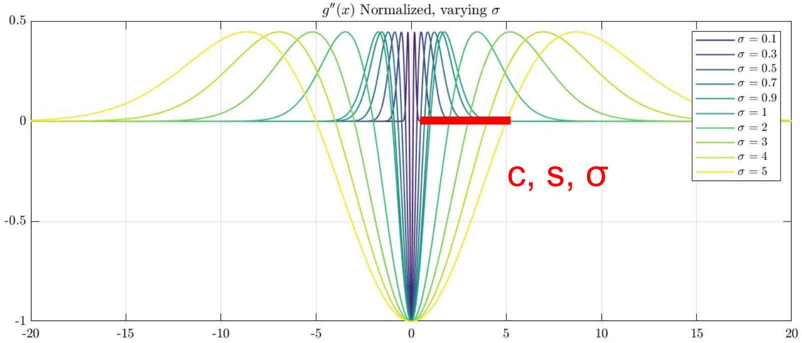
Let's walk through applying the following 3x3 **sharpen** kernel to the image of a face from above.

sharpen

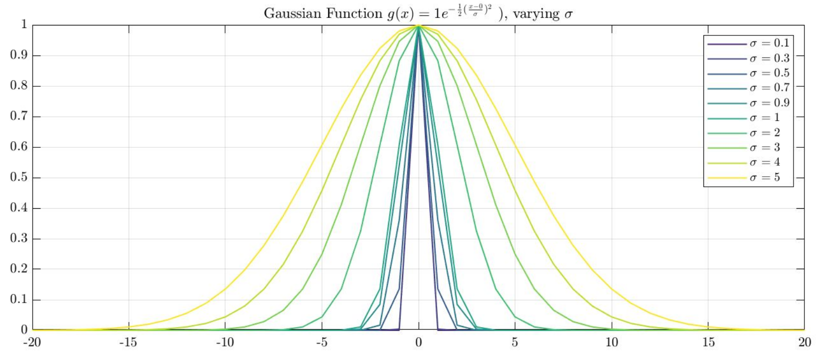
$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

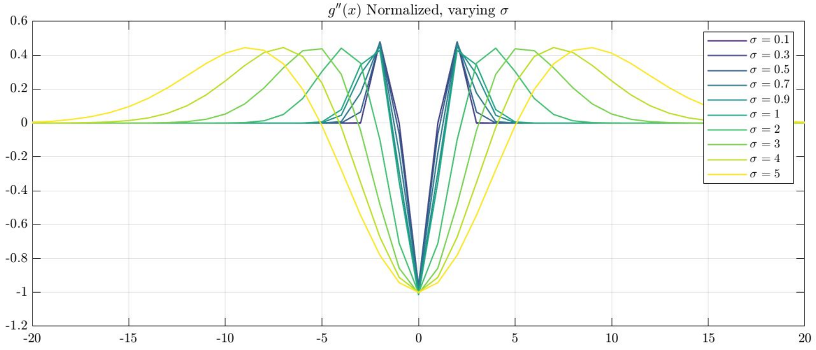
Methods: Continuous G_xx



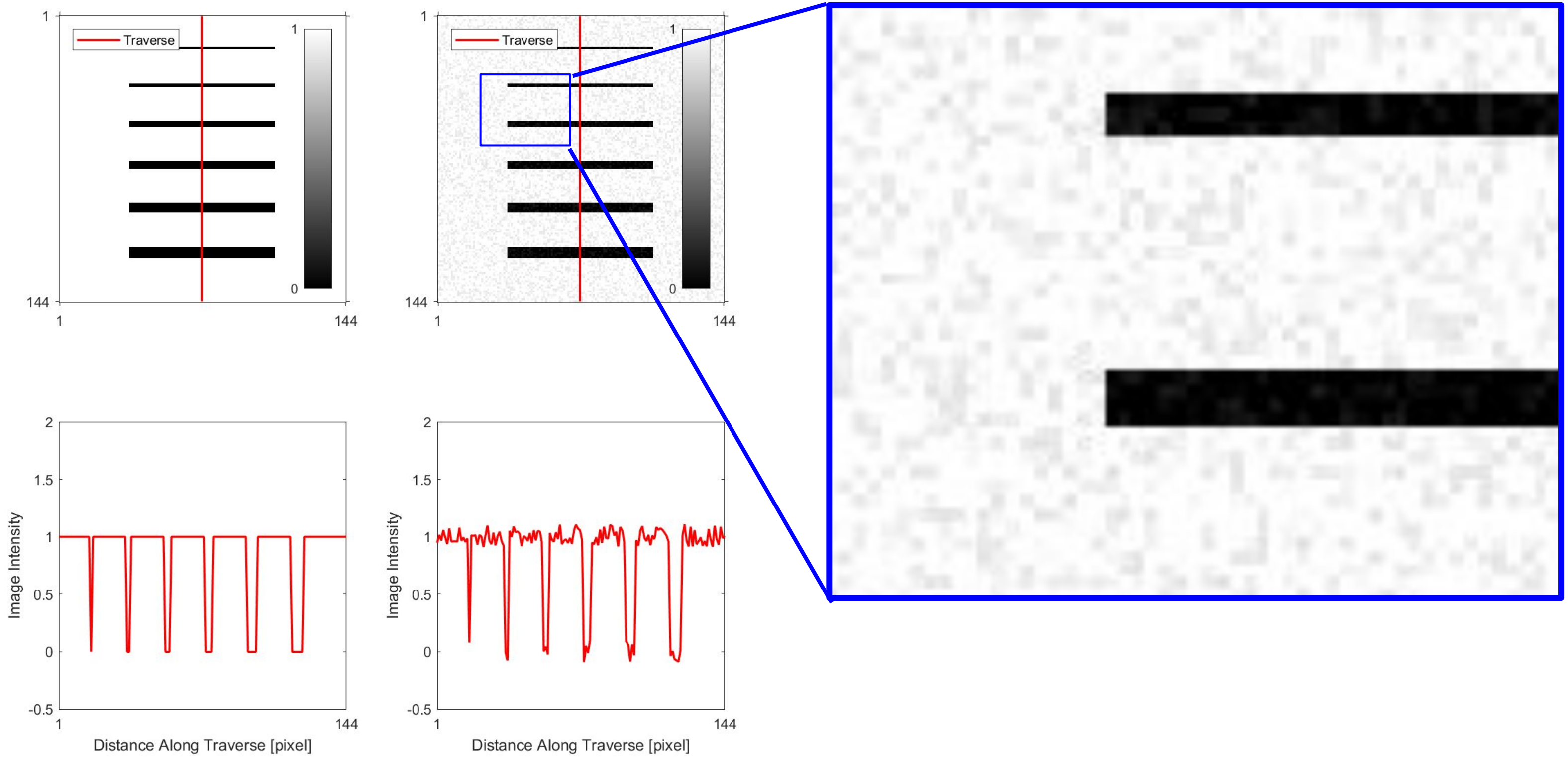


Methods: Discrete G_xx

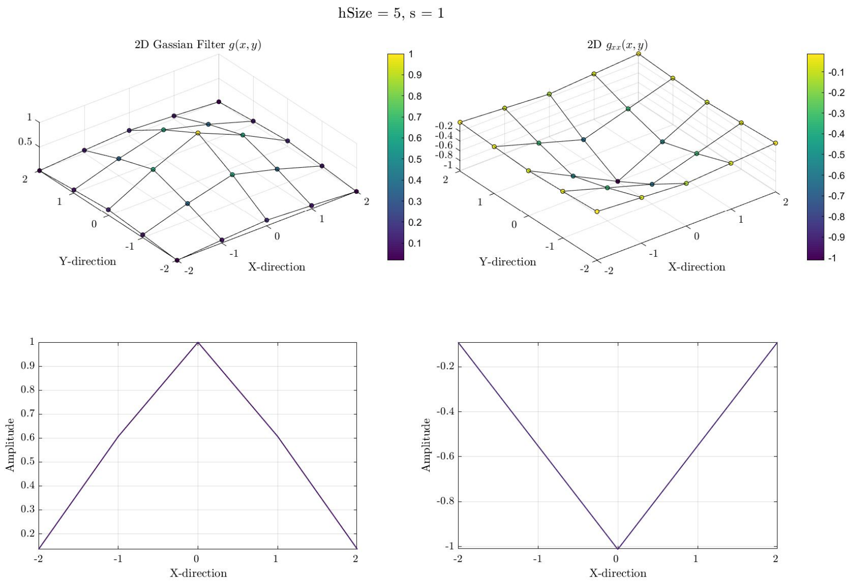


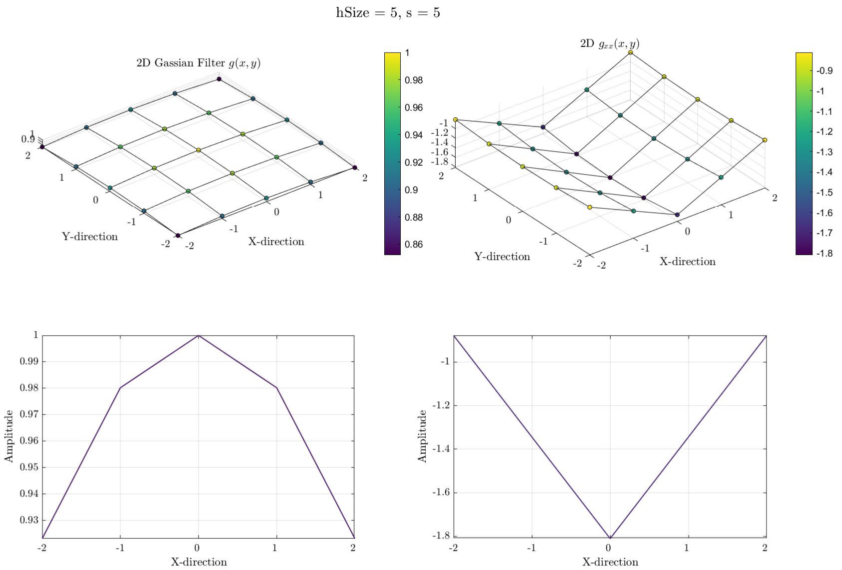


Method: Images are discrete (and 3D)

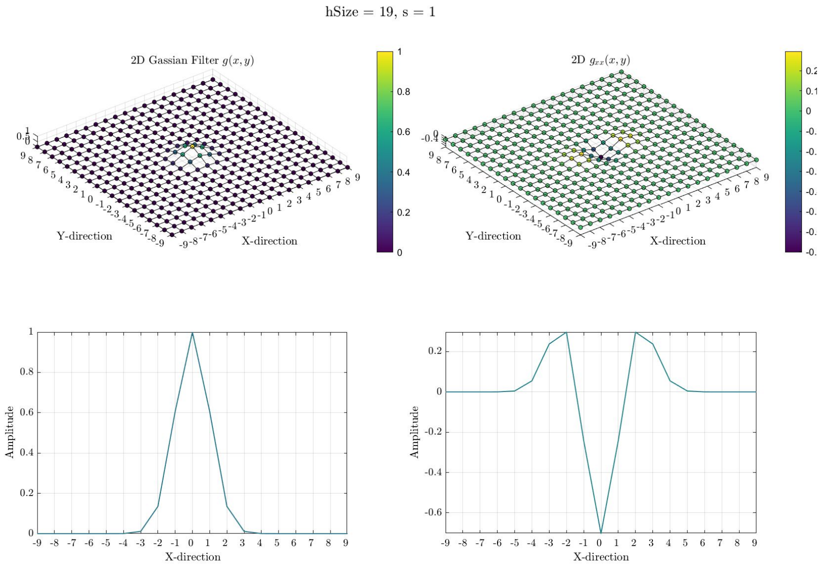


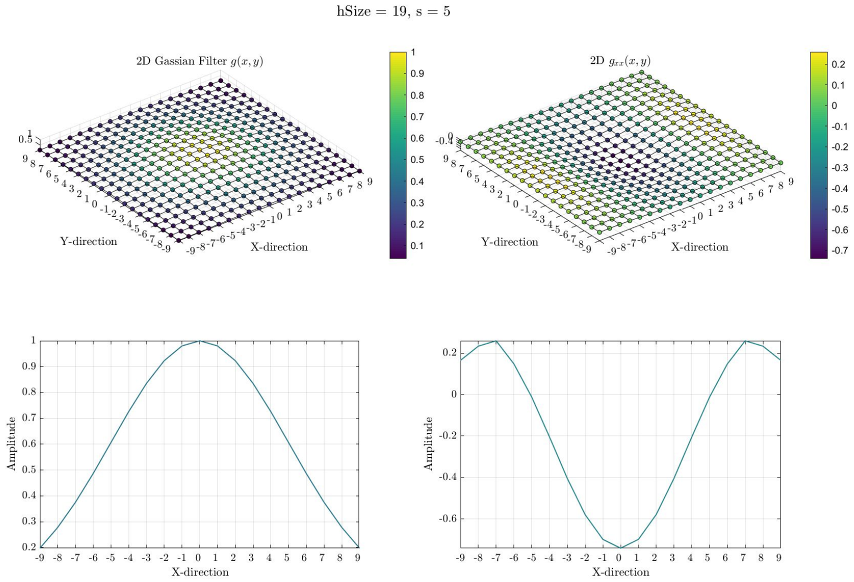
Methods: Discrete G_xx in 2D, shape is not appropriate





Methods: Discrete G_xx in 2D, shape looks good





Methods: Computing the Hessian

$$H = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

$$I_{ij} = I(x, y, z) * (\alpha G_{ij}(x, y, z, s))$$

$$\text{where} \qquad i, j = x, y, z$$

Methods: How is this useful?

2D		3D			Orientation pattern
λ_1	λ_2	λ_1	λ_2	λ_3	
N	N	N	N	N	Noisy, no orientation pattern
		L	L	H+	Plate-like structure (dark)*
		L	L	H–	Plate-like structure (bright)
L	H+	L	H+	H+	Tubular structure (dark)
L	H–	L	H–	H–	Tubular structure (bright)
H+	H+	H+	H+	H+	Blob-like structure (dark)
H–	H–	H–	H–	H–	Blob-like structure (bright)

$|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$. H = high value; L = low value; N = noise; +/– = positive or negative eigenvalue. Dark = dark feature on a bright background. Bright = bright feature on a dark background. The fracture representation we are interested in is indicated with a *. After Frangi et al. (1998).

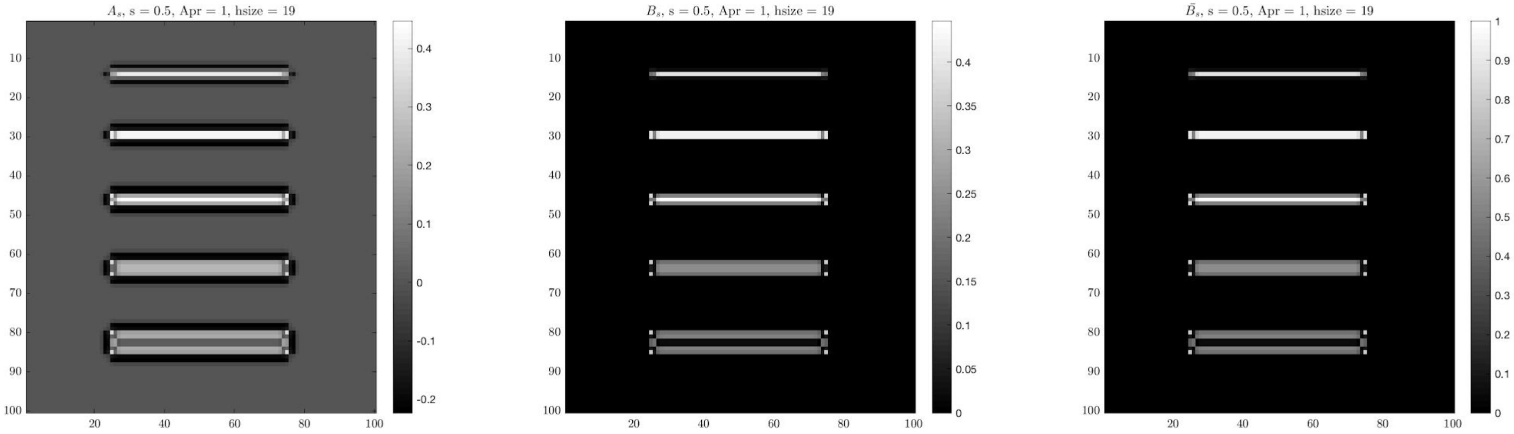
Voorn et al. 2013

Methods: Computing the "Hessian"

Algorithm

- For each s (scaling factor ~ ½ aperture of interest) in our case [1,2,3,4,6]
- Compute all Hessian elements by convolution for each s
- Loop over all voxels and compute the eigenvalues and eigenvectors
- Sort the eigenvalues (from smallest to largest)
- *For each voxel compute $A_s = \lambda_3 - |\lambda_2| - |\lambda_1|$*
- *Compute $B_s = A_s$ if $A_s > 0$, or 0 otherwise*
- *Normalize B_s such that $0 < B_s < 1$*
- Determine C_s such that $C_s = 1$ if $A_x > 1 - tol$, or 0 otherwise
- The sum up all of C_s for each s value

Method: the values for A_s, B_s, and Normalized B_s



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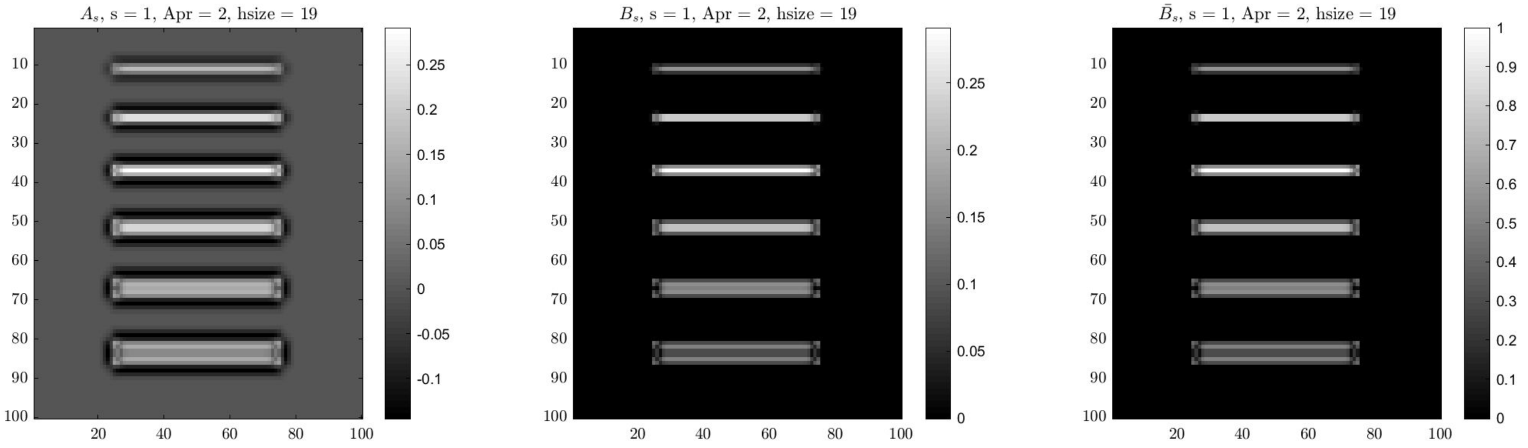
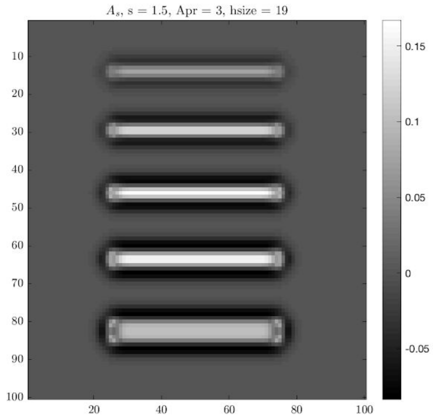
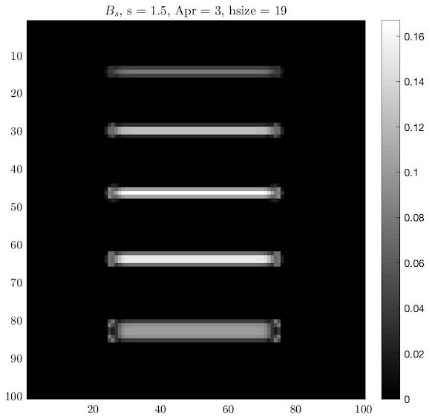
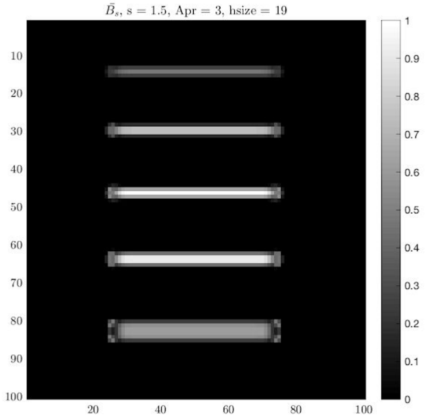


Method: the values for A_s, B_s, and Normalized B_s







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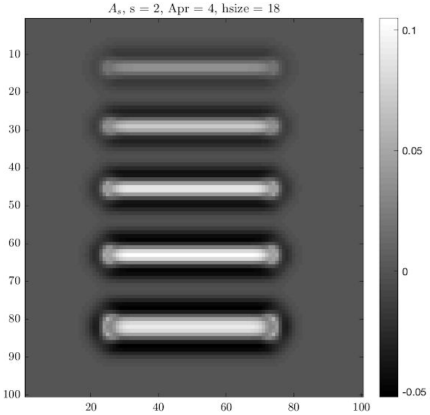

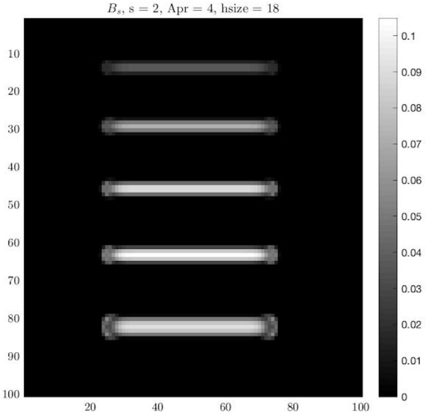

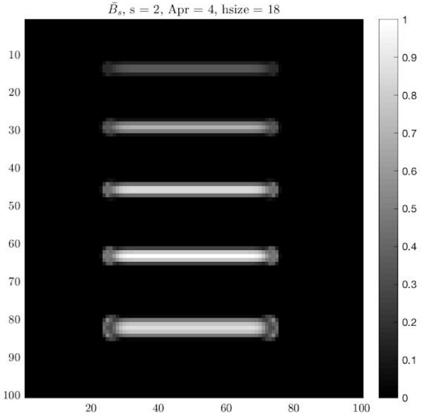

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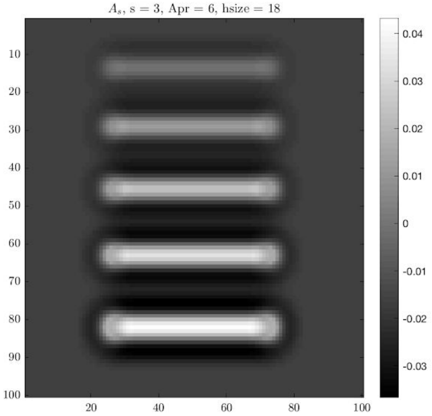
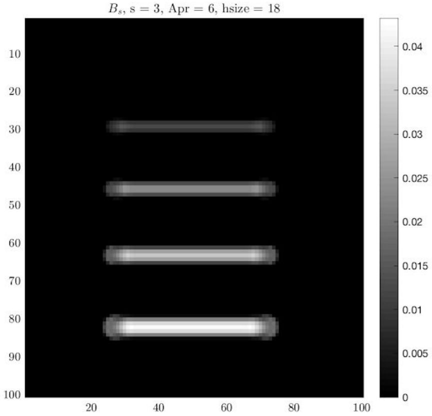
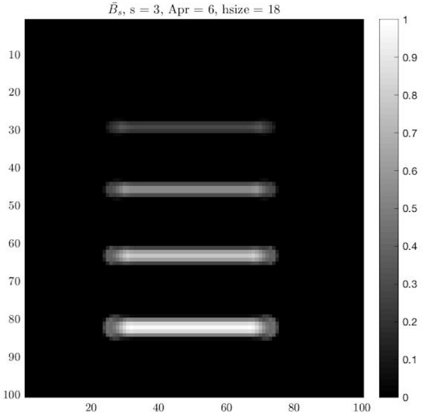


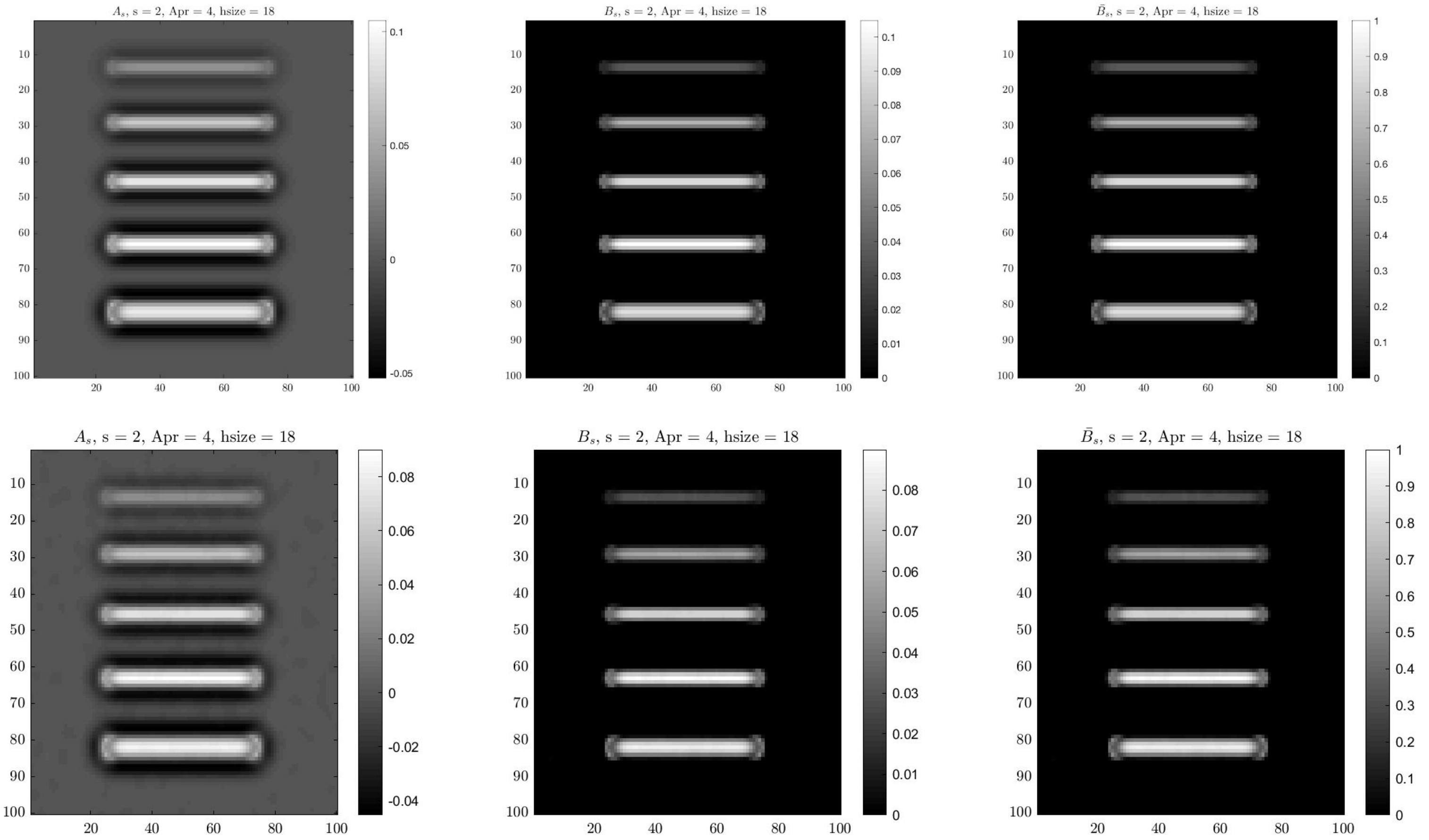




Method: the values for A_s, B_s, and Normalized B_s

Crisp image

Blurry and noisy image



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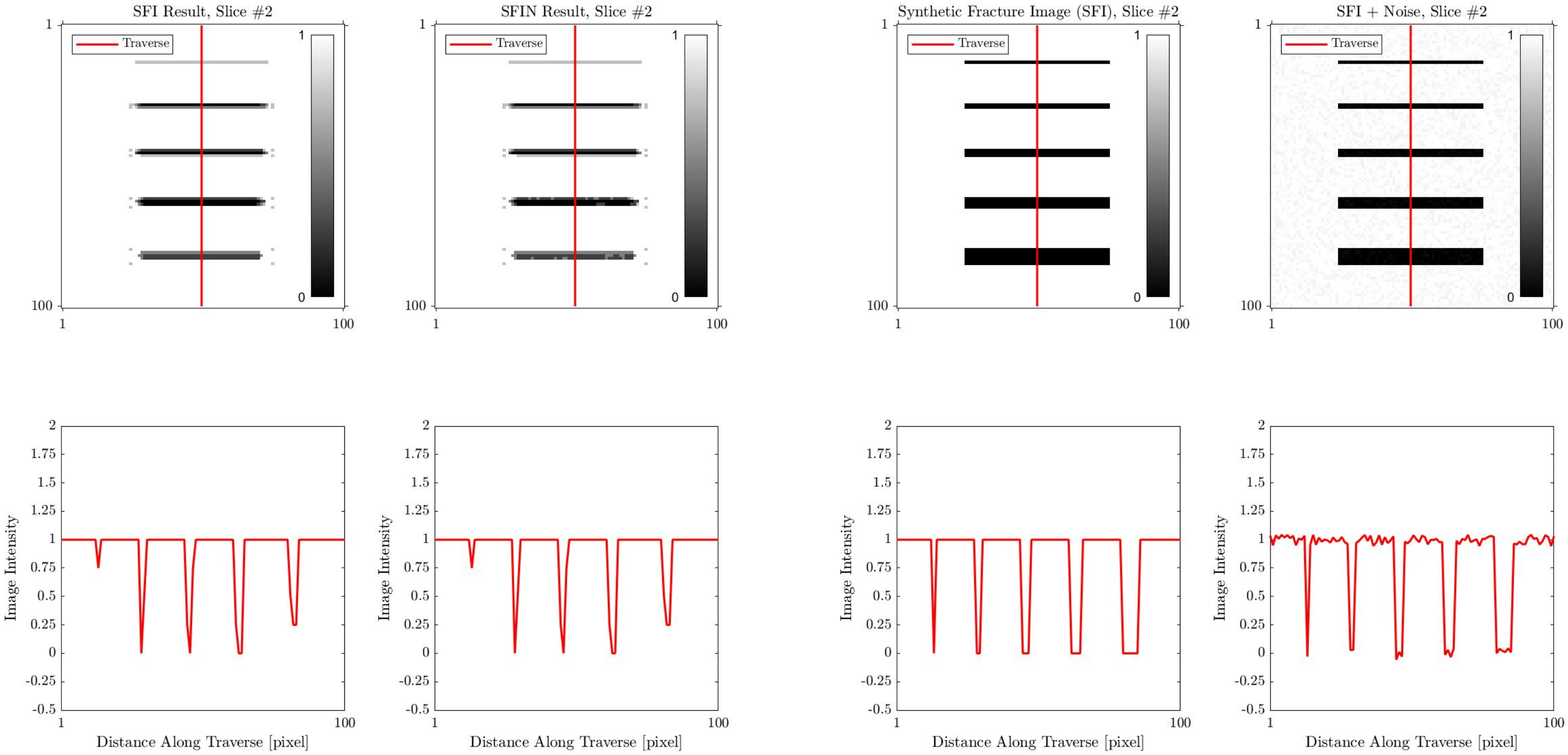
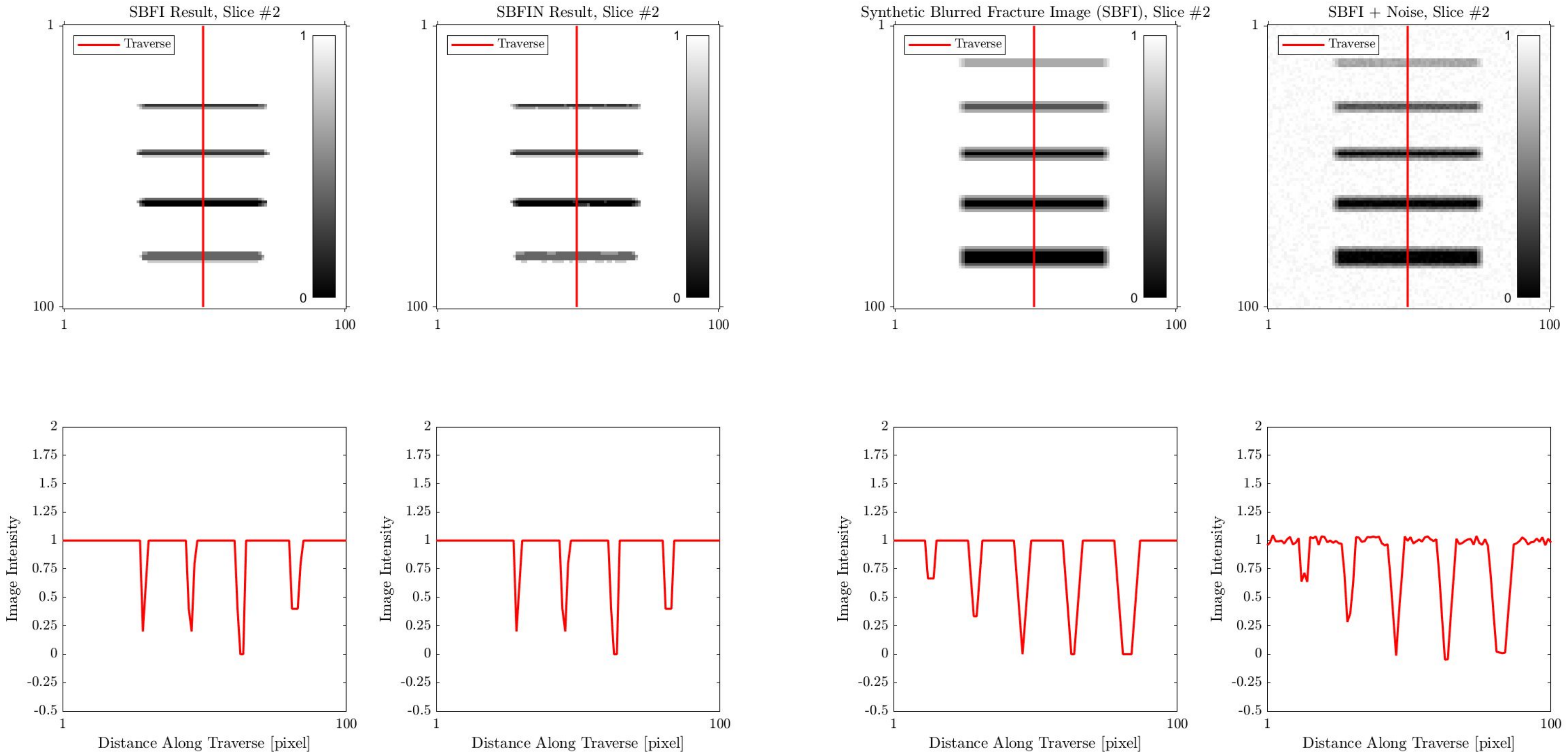
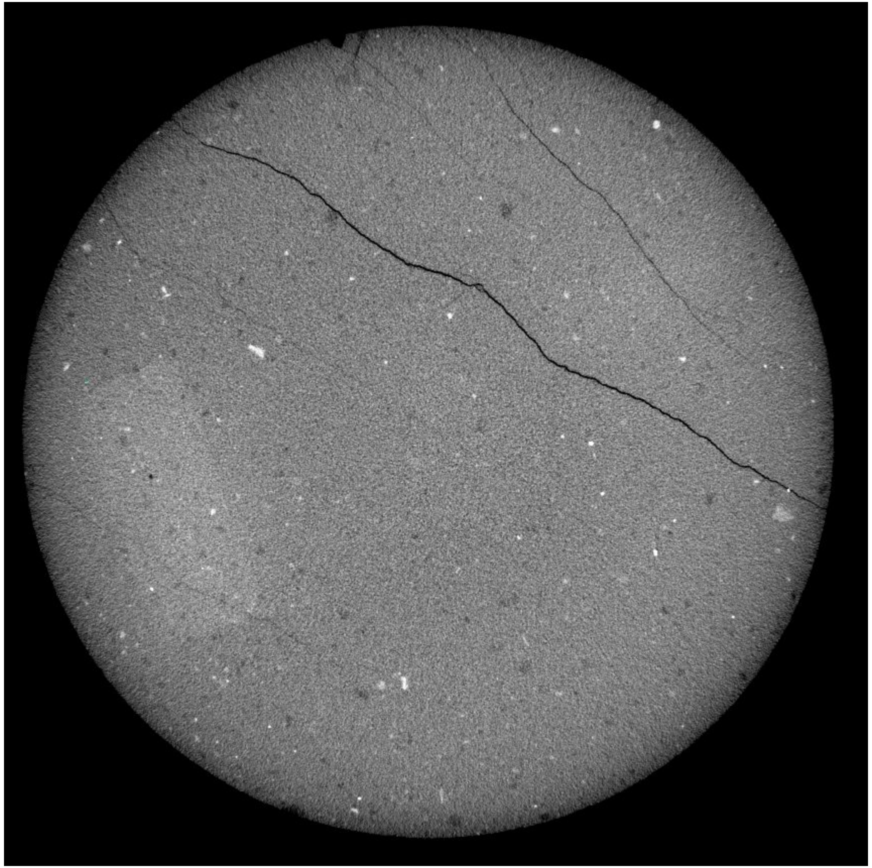
Methods: Hessian Result (crisp images)



Methods: Hessian Result (blurry images)



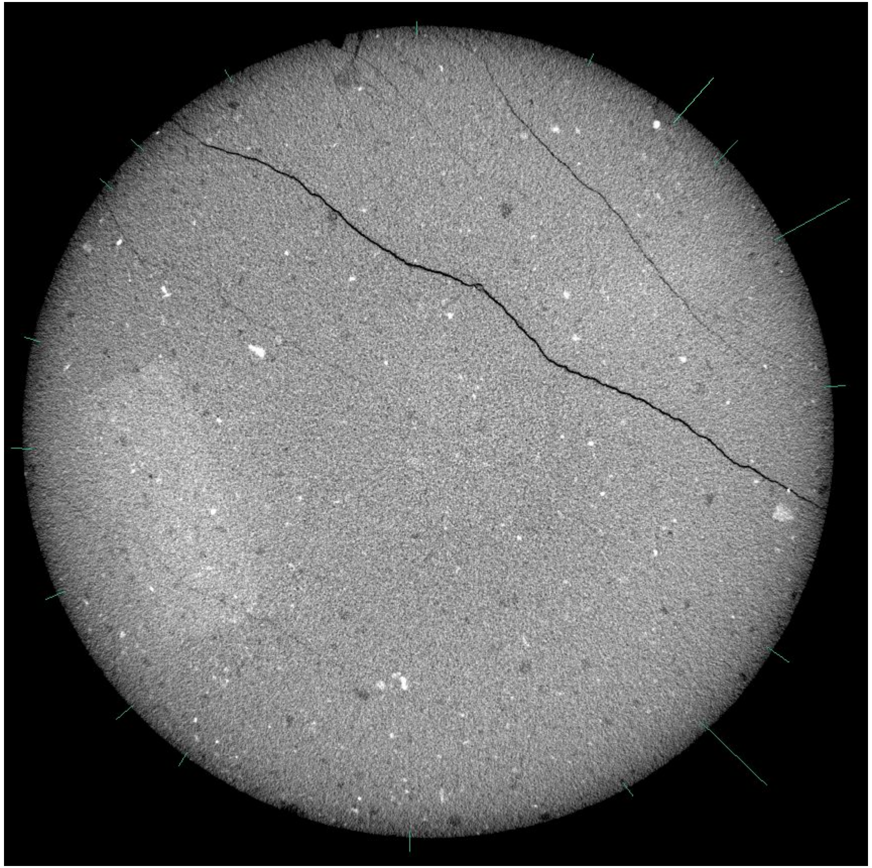
Materials

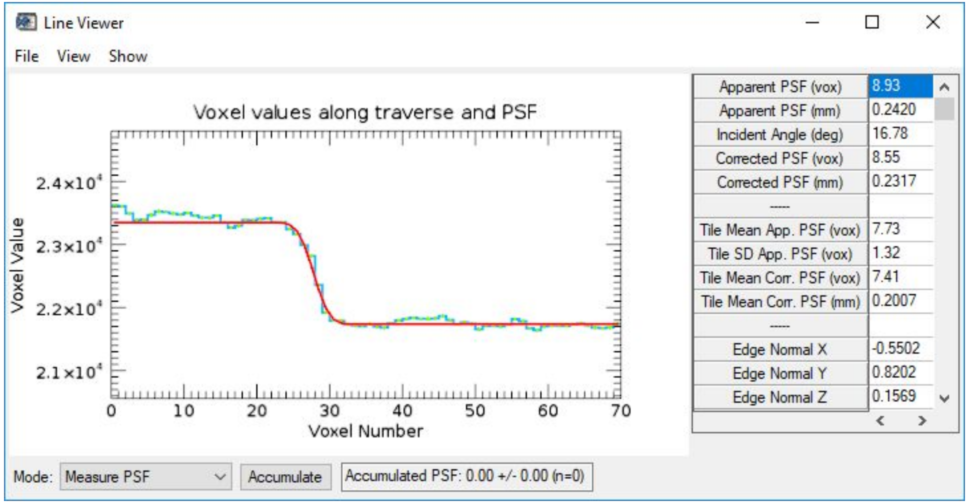


Argillite mudrock

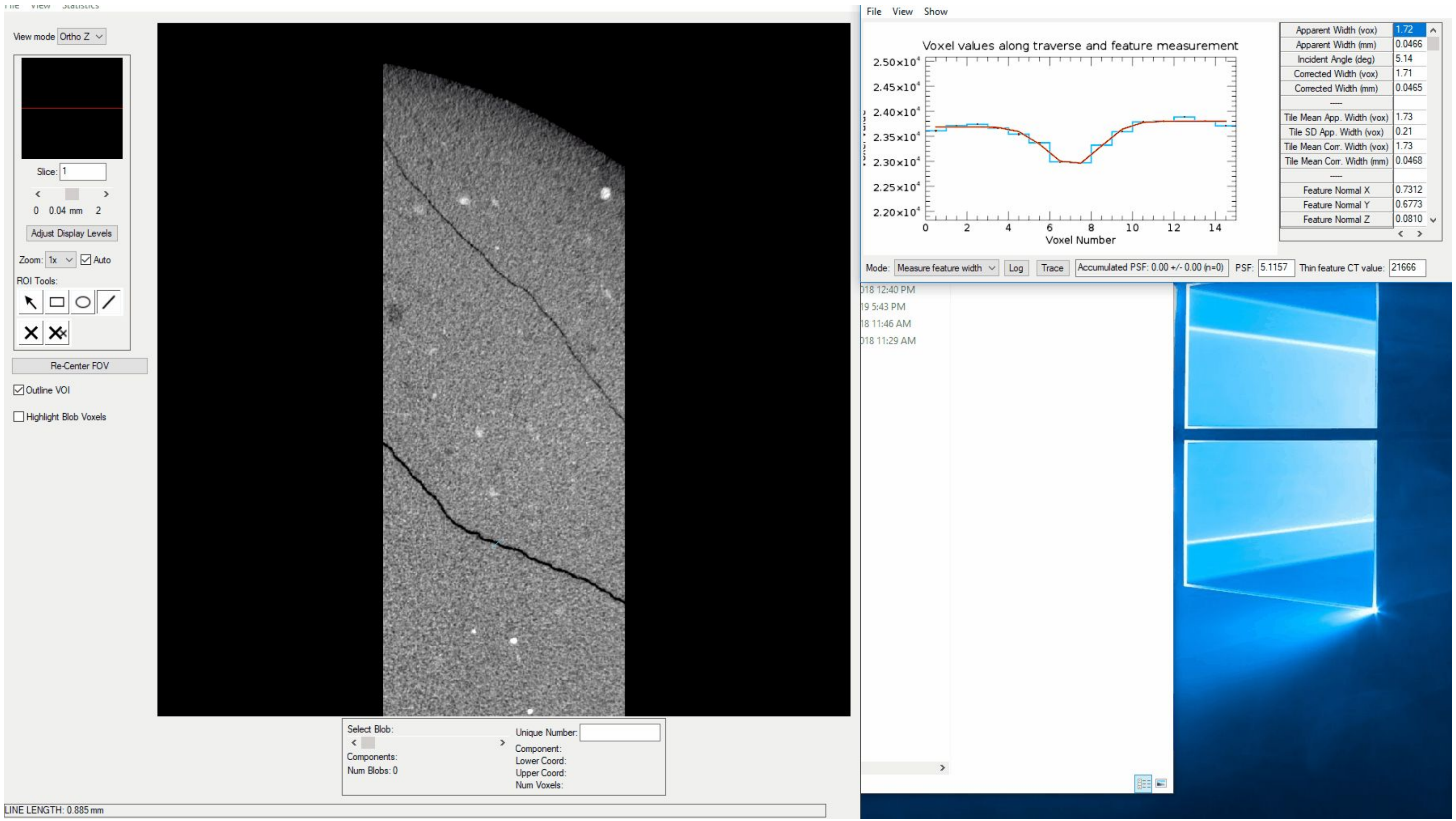
Core hand-drilled from creekbed

Point Spread Function in Blob3D





No segmentation: Blob3D



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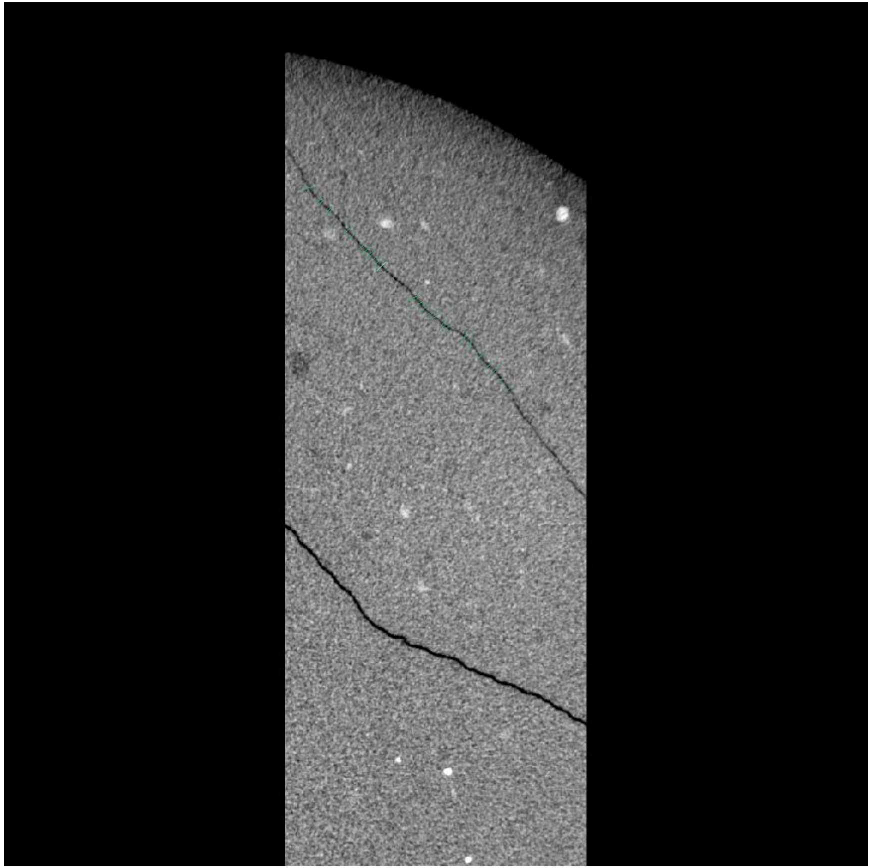

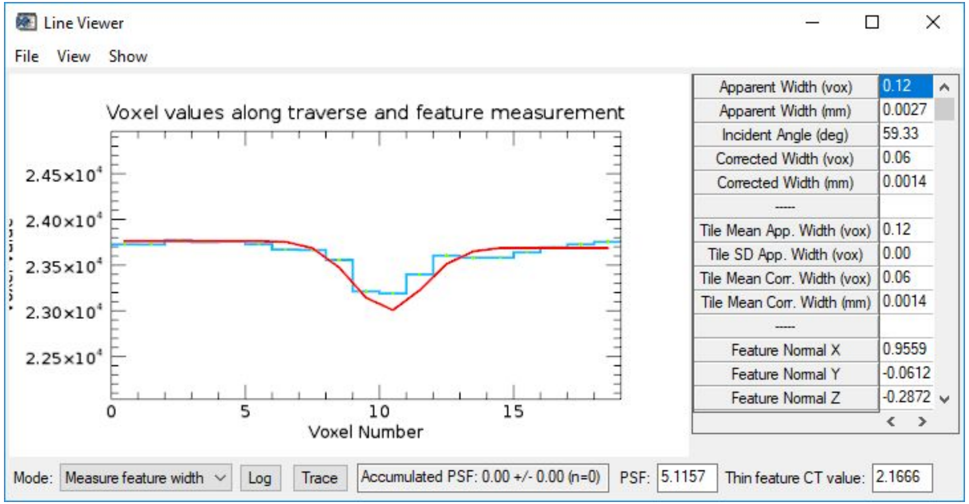

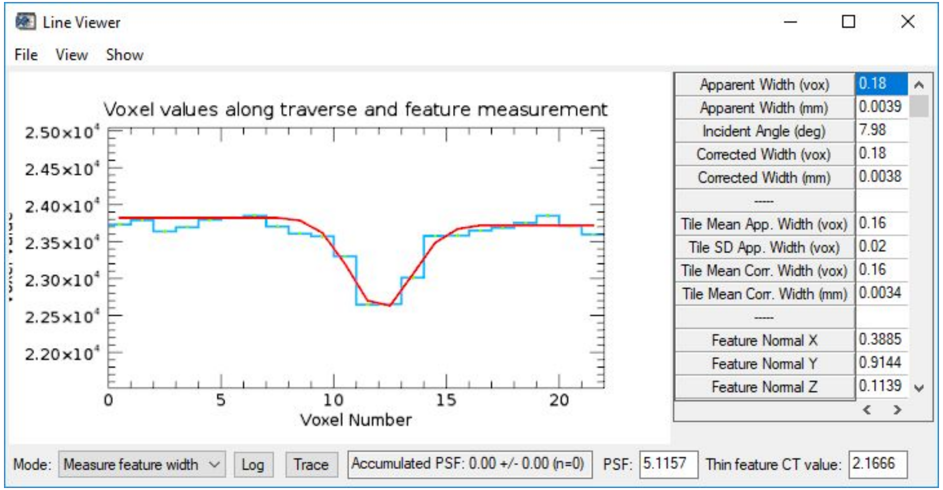

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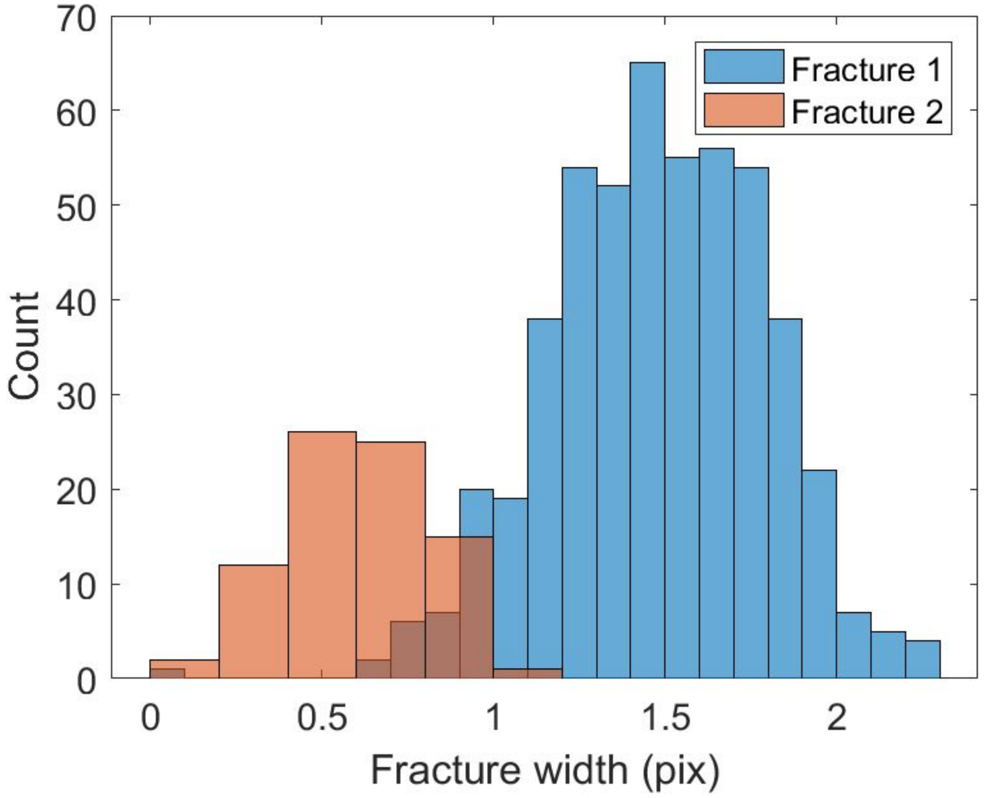
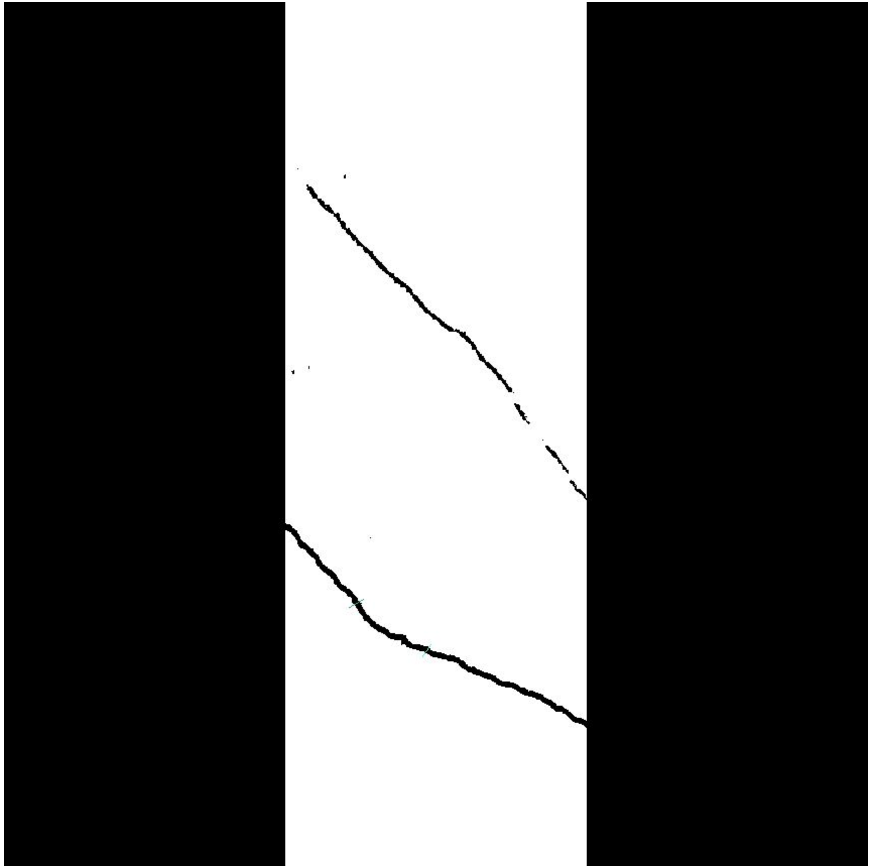
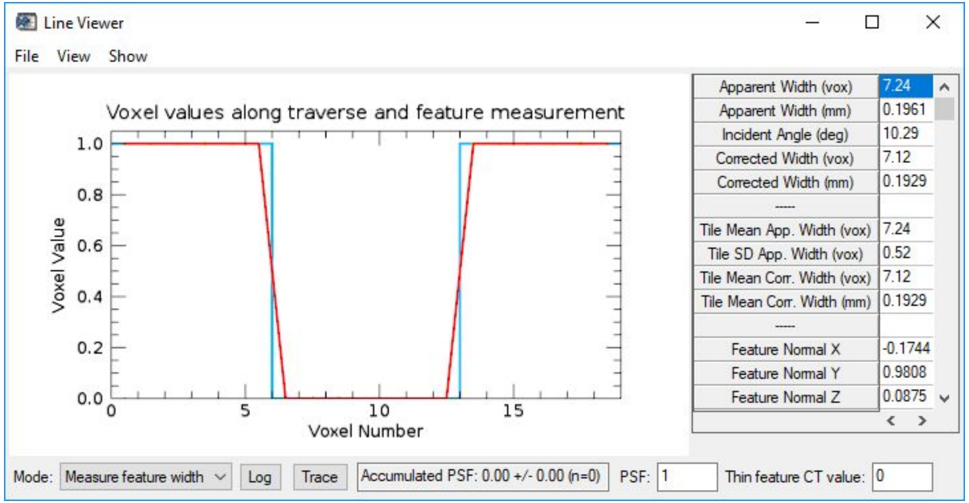


Fracture 1 mean: 1.5+-0.3

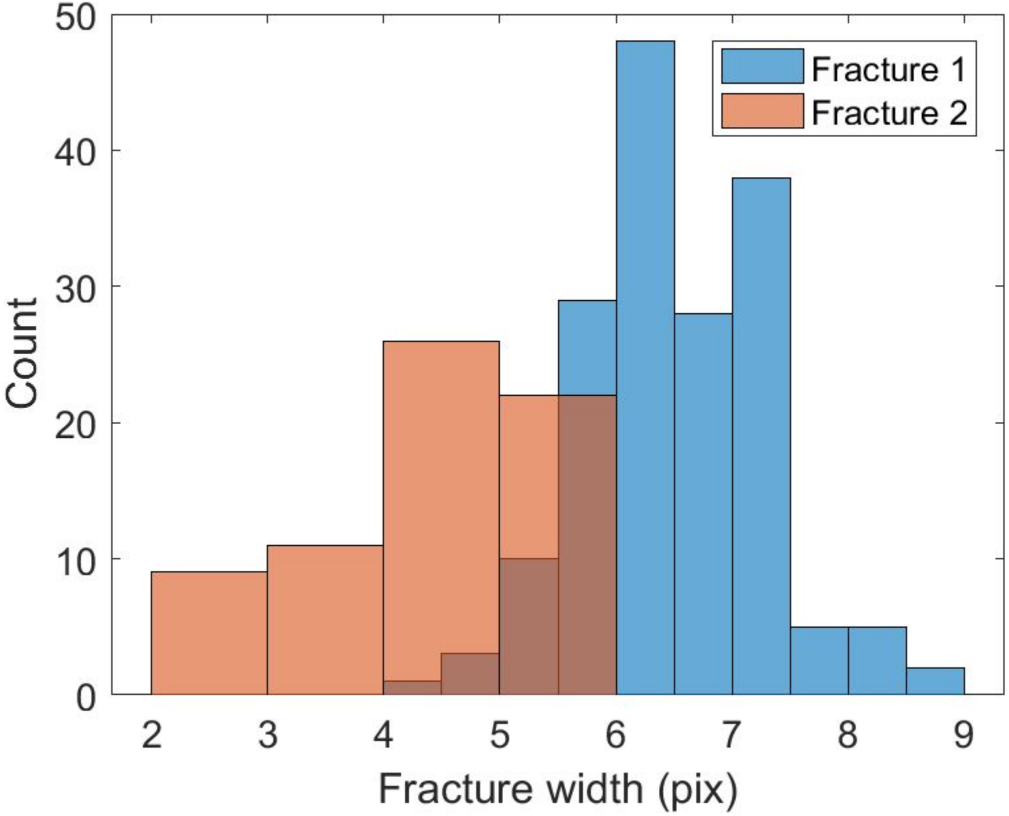
Fracture 2 mean: 0.6+-0.2

Trainable segmentation: Blob3D





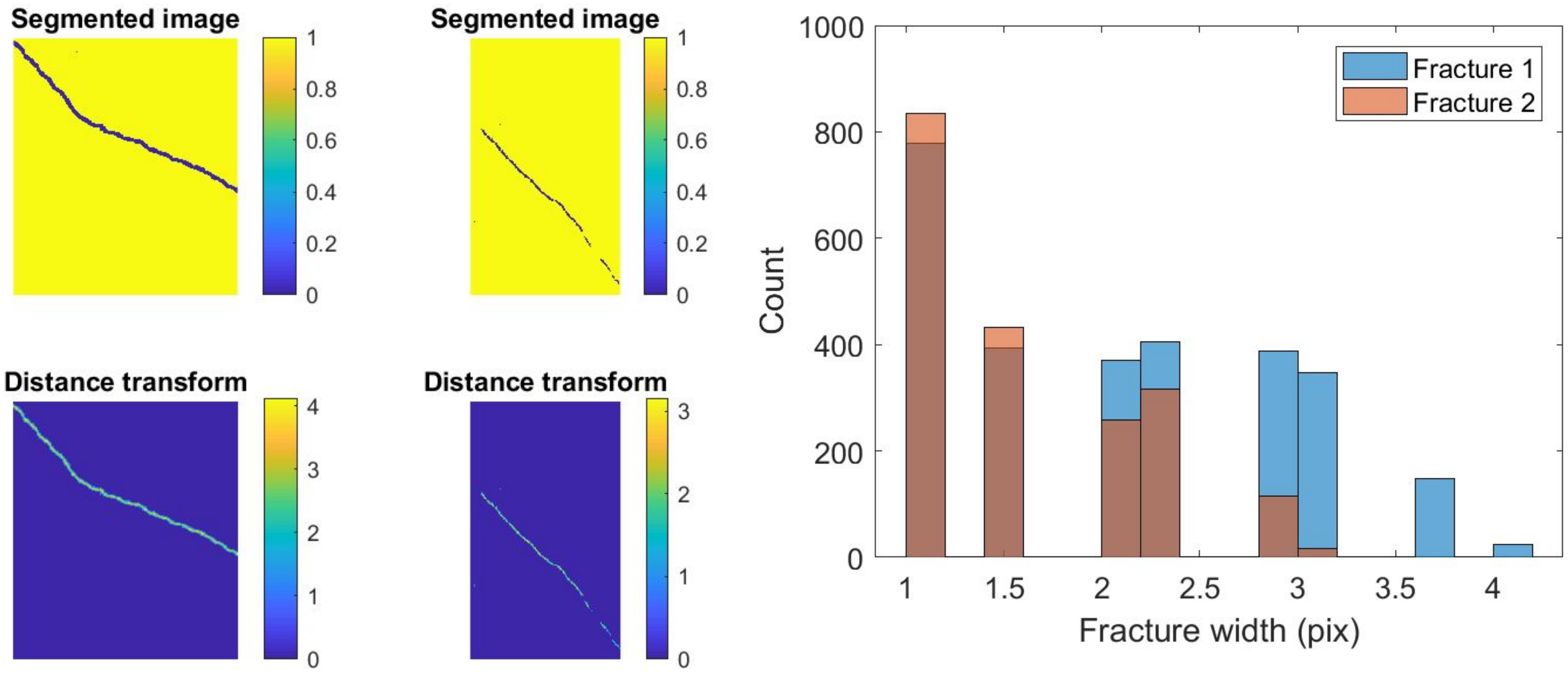
Trainable segmentation: Blob3D



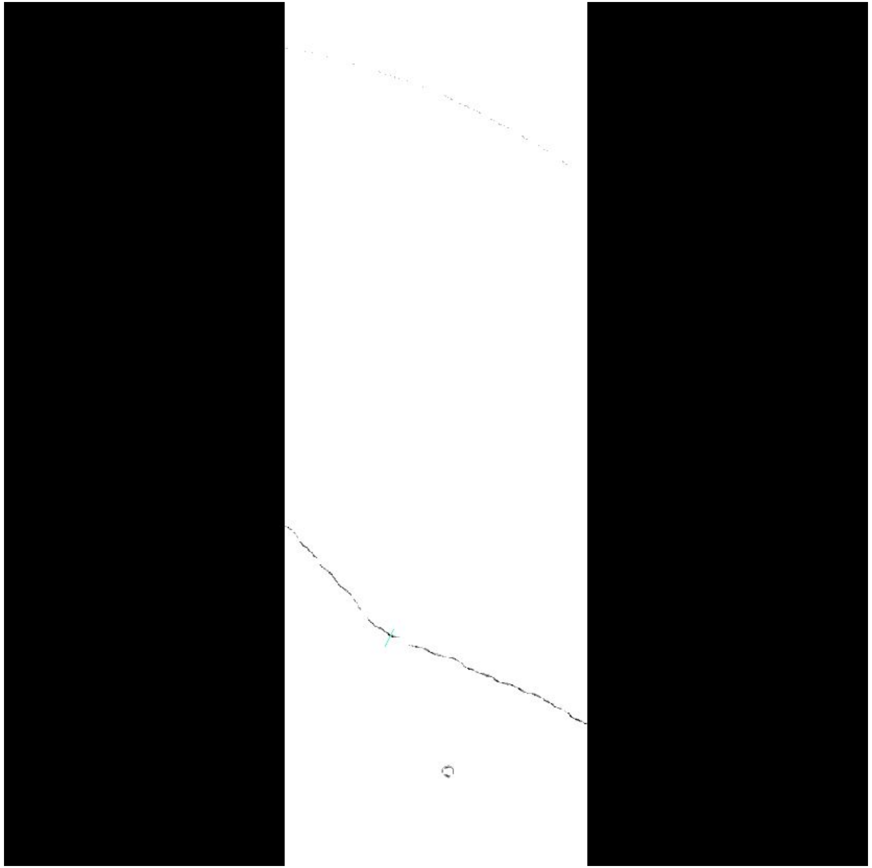
Fracture 1 mean: 6.5+-0.8

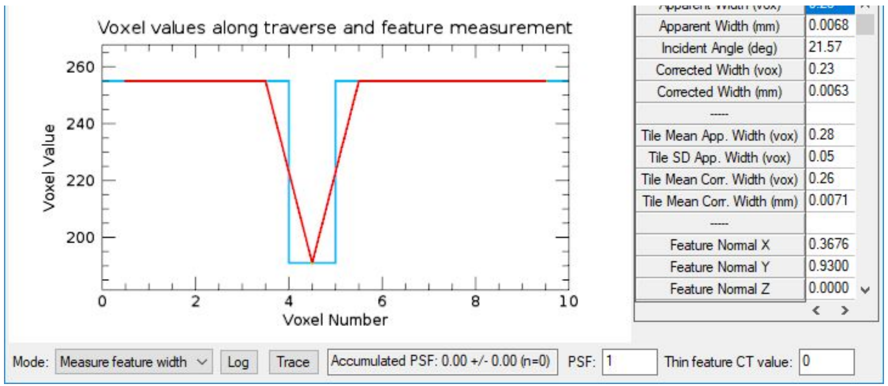
Fracture 2 mean: 4.5+-1.0

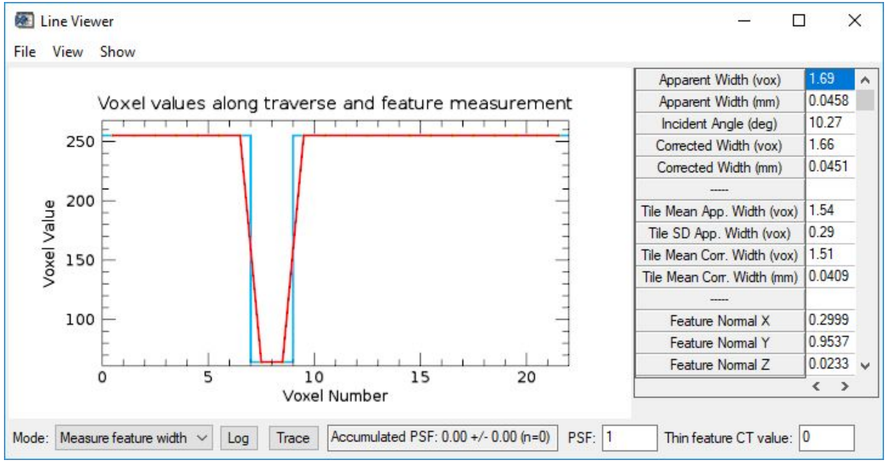
Trainable segmentation: Distance function



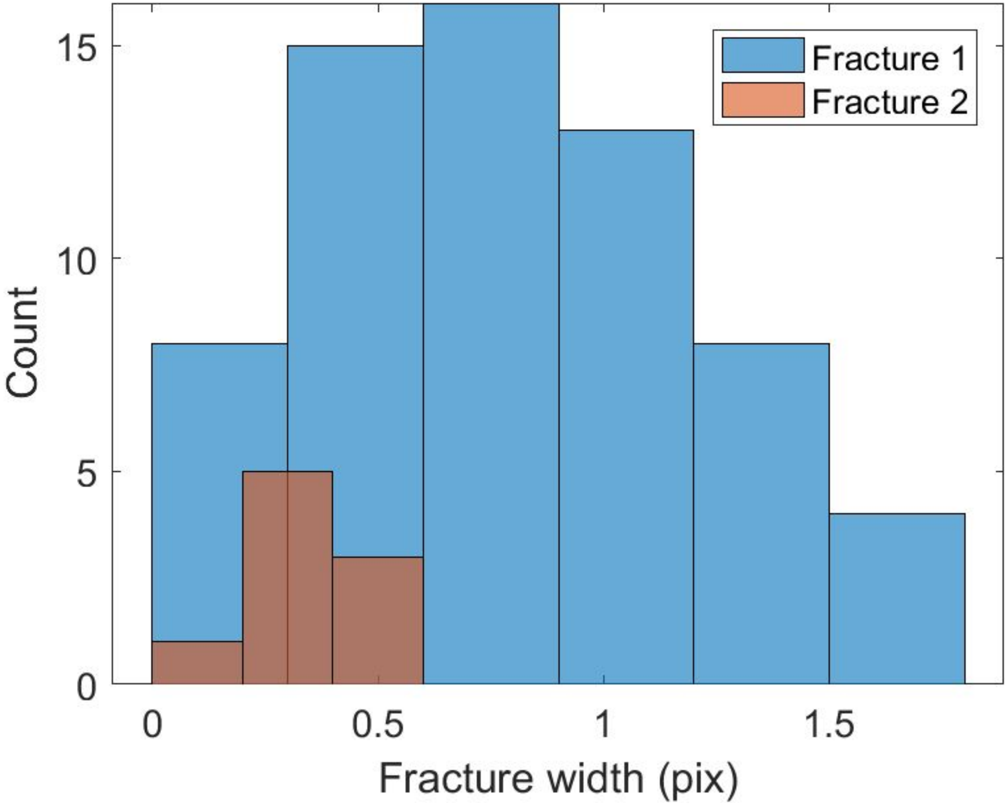
Hessian segmentation: distance function







Hessian segmentation: distance function



Fracture 1 mean: 0.8+-0.4

Fracture 2 mean: 0.3+-0.1

Hessian segmentation: distance function