Assignment 3

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COMP 5107

In this assignment we have two classes X_1 and X_2 with means same as the previous assignment M_1 and M_2 where:

$$M_1 = \begin{bmatrix} 3 & 1 & 4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} -3 & 1 & -4 \end{bmatrix}$$

and covariance matrices as follows:

$$\sum_{X_1} = \begin{bmatrix} a^2 & \beta ab & \alpha ac \\ \beta ab & b^2 & \beta bc \\ \alpha ac & \beta bc & c^2 \end{bmatrix}, \quad \sum_{X_2} = \begin{bmatrix} c^2 & \alpha bc & \beta ac \\ \alpha bc & b^2 & \alpha ab \\ \beta ac & \alpha ab & a^2 \end{bmatrix}$$

The parameters used in this assignment is as follows:

$$a = 2, \quad b = 3, \quad c = 4, \quad \alpha = 0.1, \quad \beta = 0.2, \quad \#points = 2000$$

This resulted the covariance matrices to have the following values:

$$\sum_{X_1} = \begin{bmatrix} 4 & 1.2 & 0.8 \\ 1.2 & 9 & 2.4 \\ 0.8 & 2.4 & 16 \end{bmatrix}, \quad \sum_{X_2} = \begin{bmatrix} 16 & 1.2 & 1.6 \\ 1.2 & 9 & 0.6 \\ 1.6 & 0.6 & 4 \end{bmatrix}$$

a. Create points for each distribution:

Here we used the same exact methods for creating the points from the previous assignment. And the plots of the points are available below.

Listing 1: Gaussian vector generation

```
# create point matrices for the two classes X1 and X2
z1_matrix, x1_matrix = h.generate_point_matrix(v_x1, lambda_x1, m1, number_of_points)
z2_matrix, x2_matrix = h.generate_point_matrix(v_x2, lambda_x2, m2, number_of_points)

# PLOTTING #
```

```
# plot the first class as blue for (d1 - d2) domain and second class as red
h.plot_2d_graph(x1_matrix, x2_matrix, 1, 2, 'x1', 'x2', 'x1-x2')

# plot the first class as blue for (d1 - d3) domain and second class as red
h.plot_2d_graph(x1_matrix, x2_matrix, 1, 3, 'x1', 'x3', 'x1-x3')
```

b. Compute the optimal Bayes discriminant function:

For calculating the Bayes discriminant function we use the quadratic function form of the matrices using the following equation:

$$X^T AX + B^T X + C \lesseqgtr 0$$

Then setting one variable to 0 and solving for the other two which will result in a quadratic equation. For $(x_1 - x_2)$:

$$(a_{22})x_2^2 + (a_{12}x_1 + a_{21}x_1 + b_{12})x_2 + (a_{11}x_1^2 + b_{11}x_1 + c) = 0$$

For $(x_1 - x_3)$:

$$(a_{33})x_3^2 + (a_{13}x_1 + a_{31}x_1 + b_{13})x_3 + (a_{11}x_1^2 + b_{11}x_1 + c) = 0$$

Listing 2: Simultaneous Diagonalization

```
a = ((np.linalg.inv(sigma_x2) - np.linalg.inv(sigma_x1)) / 2)
b = np.array(m1.transpose() @ np.linalg.inv(sigma_x1) - m2.transpose() @ np.linalg.inv(sigma_x2))
c = np.math.log(p1 / p2) + np.log(np.linalg.det(sigma_x2) / np.linalg.det(sigma_x1))

equation_points = []
roots_1 = []
roots_2 = []

min_w = min(min(min(x1_matrix[0, :]), min(x2_matrix[0, :])),
            min(min(x1_matrix[1, :]), min(x2_matrix[1, :])))
max_w = max(max(max(x1_matrix[0, :]), max(x2_matrix[0, :])),
            max(max(x1_matrix[1, :]), max(x2_matrix[1, :])))

# get the roots for the discriminant function for (x1-x2)
for x1 in range(-15, 10, 1):
    equation_points.append(x1)
```

```
    x2_square_coefficient = a[1      ][1]
    x2_coefficient = (a[0][1] * x1) + (a[1][0] * x1) + b[0][1]
    constant = a[0][0] * np.math.pow(x1, 2) + b[0][0] * x1 + c

    poly_coefficients = [x2_square_coefficient, x2_coefficient, constant]
    roots = np.roots(poly_coefficients)
    roots_1.append(roots[0])
    roots_2.append(roots[1])

# get the roots for the discriminant function for (x1-x3)
for x1 in range(-15, 15, 1):
    equation_points.append(x1)
    x2_square_coefficient = a[2][2]
    x2_coefficient = (a[0][2] * x1) + (a[2][0] * x1) + b[0][2]
    constant = a[0][0] * np.math.pow(x1, 2) + b[0][0] * x1 + c

    poly_coefficients = [x2_square_coefficient, x2_coefficient, constant]
    roots = np.roots(poly_coefficients)
    roots_1.append(roots[0])
    roots_2.append(roots[1])
```

c. Generating 200 testing points and report the accuracy:

Here we create 200 points in the X-world and try to classify them using the discriminant function that we calculated before.

If the $discriminantvalue > 0$, then class 1 and if $discriminantvalue < 0$, then class 2

Listing 3: Generating 5000 points

```
+---------+---------+---------+----------+
|  Prd\Tr | class 1 | class 2 | Accuracy |
+---------+---------+---------+----------+
| class 1 |  180.0  |   20.0  |   90.0   |
| class 2 |   3.0   |  197.0  |   98.5   |
+---------+---------+---------+----------+
```

d. Diagonalize the points:

Listing 4: Generating 5000 points

```python
# diagonalizing the original points and the testing points
v1_matrix, v2_matrix, sigma_v1, sigma_v2, v1_mean, v2_mean = h.diagonalize_simultaneously(x1_matrix, x2_matrix, sigma_x1, sigma_x2, m1, m2)
v1_test_points, v2_test_points, _, _, _, _ = h.diagonalize_simultaneously(x1_test_points, x2_test_points, sigma_x1, sigma_x2, m1, m2)
```

e. Computing the discriminant function for V-world:

For this part i used the same method as part b, but instead I used the covariance and means of the diagonalized points.

f. Classify the same testing points in the V-world:

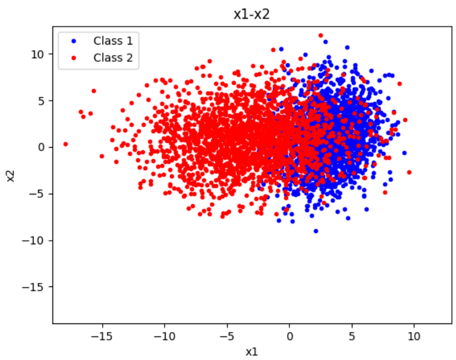
We used the same exact 200 points here to test the new discriminant function that we created for the V-world. The values returned by the function were almost identical, they just differed in floating points which did not affect the classification accuracy. hence we got the results as follows:

Listing 5: Generating 5000 points

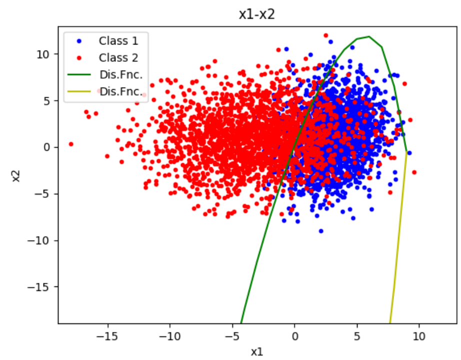
```
After diagonalizing:
+---------+---------+---------+----------+
|  Prd\Tr | class 1 | class 2 | Accuracy |
+---------+---------+---------+----------+
| class 1 |  180.0  |   20.0  |   90.0   |
| class 2 |   3.0   |  197.0  |   98.5   |
+---------+---------+---------+----------+
```

g. The graphs for the points and the discriminant function:

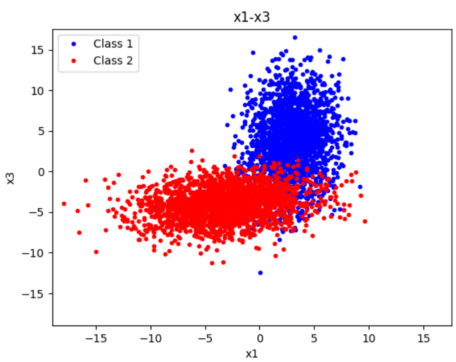
The following graphs show the points generated in the X world before diagonalization and also the calculated *discriminant function*.



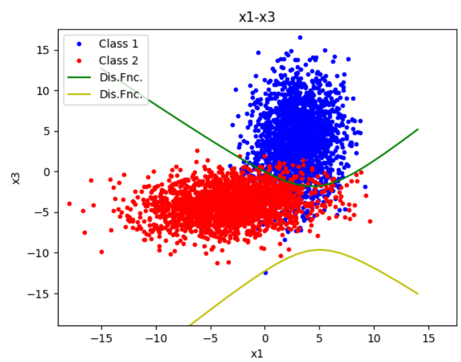
(a) x1-x2



(b) x1-x2

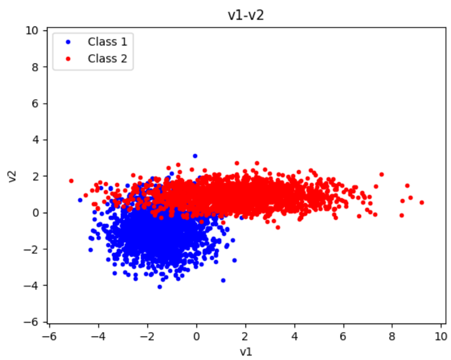


(c) x1-x3

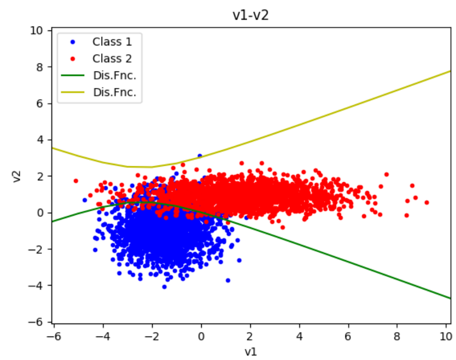


(d) x1-x3

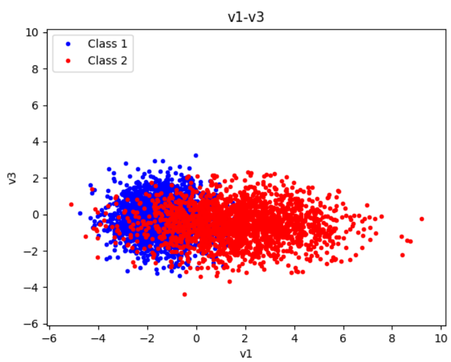
Figure 1: The generated points in X-world and the *discriminant function*



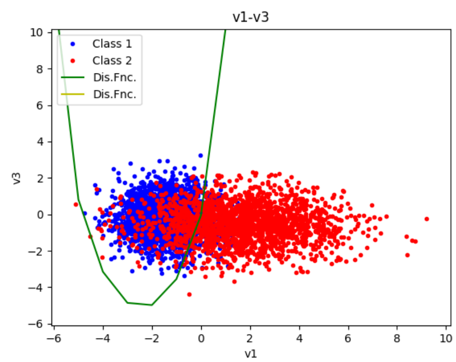
(a) v1-v2



(b) v1-v2



(c) v1-v3



(d) v1-v3

Figure 2: The generated points in V-world and the *discriminant function*