German University in Cairo
Media Engineering and Technology
Prof. Dr. Slim Abdennadher

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Practice Assignment10

Exercise 10-1

a) What is the point of description logics (DL) and other ontology languages? Why don't knowledge representation professionals use first-order logic for everything?

Solution:

The main reason for using restricted ontology languages is that restriction in expressive power makes reasoning in them much more efficient than in first order logic. Another reason is that hierarchies of concepts, and entity-relationship diagrams are familiar to many users and are considered more intuitive.

b) Express the following concepts and sentences in DL (SHIQ) by only using the roles *Module* and *Supervision* and atomic concepts *Academic*, *Lecturer*and *Compulsory*:

C1: concept of an academic who supervises some students

C2: concept of an academic who teaches at least two modules

C3: concept of an academic who teaches only compulsory modules

C4: a lecturer is an academic who supervises at least 8 students and teaches at least 2 modules

Solution:

C1: $Academic \sqcap \exists_{\geq 1} Supervision$

C2: $Academic \sqcap \exists_{\geq 2} Module$

C3: $Academic \sqcap \forall Module.Compulsory$

C4: $Lecturer \doteq Academic \sqcap \exists_{\geq 8} Supervision \sqcap \exists_{\geq 2} Module$

Exercise 10-2

Given the following Description Logic knowledge base:

$$\begin{aligned} Cow &\doteq Animal \sqcap Vegetarian \\ Sheep &\sqsubseteq Animal \\ Vegetarian &\doteq \forall eats.\neg Animal \\ MadCow &\doteq Cow \sqcap \exists eats.Sheep \end{aligned}$$

Which of the following can be concluded from this knowledge base. Justify your answer in natural language!

a)
- $MadCow$ is inconsistent
- Vegetarians eat vegetables

b)
- Vegetarian is inconsistent
- Vegetarians eat vegetables

c)
- $MadCow$ is inconsistent
- Every vegetarian animal is a cow

d)
- Vegetarian is inconsistent
- Every vegetarian animal is a cow

Solution:

- Vegetables are not a concept in the knowledge base and thus the statement Vegetarians eat vegetables is meaningless.
- $MadCow$ is inconsistent, because a MadCow is a Cow and a Cow is a Vegetarian.
- Now, everything a vegetarian (and thus a $MadCow$) eats is not an animal.
- However, a $MadCow$ eats (at least one) Sheep (indicated with the existential quantification) and a sheep is an Animal.
- Thus, because a $MadCow$ is a vegetarian, it does not eat animals, but it does eat sheep. This is inconsistent.

Exercise 10-3

a) Construct a Tableau to either prove or refute the following axiom:

$$(\neg A \sqcup \forall R.C) \sqcap \neg C \sqsubseteq \exists R.C \sqcap \neg A$$

Does the subsumption hold?

Solution:
To check whether the subsumption:

$$(\neg A \sqcup \forall R.C) \sqcap \neg C \sqsubseteq \exists R.C \sqcap \neg A$$

holds we should check whether

$$((\neg A \sqcup \forall R.C) \sqcap \neg C) \sqcap \neg(\exists R.C \sqcap \neg A)$$

is inconsistent.
To use the tableau rules, first we have to convert the formula into Negation Normal Form as follows:

$$((\neg A \sqcup \forall R.C) \sqcap \neg C) \sqcap \neg(\exists R.C \sqcap \neg A)$$
$$((\neg A \sqcup \forall R.C) \sqcap \neg C) \sqcap (\forall R.\neg C \sqcup A)$$

- $A_0 = \{x : ((\neg A \sqcup \forall R.C) \sqcap \neg C) \sqcap (\forall R.\neg C \sqcup A)\}$
- $A_1 = A_0 \cup \{x : (\neg A \sqcup \forall R.C), x : \neg C, x : (\forall R.\neg C \sqcup A)\}$
- $A_2 = A_1 \cup \{x : \neg A\}$
- $A_3 = A_2 \cup \{x : (\forall R.\neg C)\}$ No more rules appicable
- $A_3' = A_2 \cup \{x : A\}$ Clash
- $A_2' = A_1 \cup \{x : \forall R.C\}$

- $A_3'' = A_2' \cup \{x : (\forall R.\neg C)\}$ No more rules applicable
- $A_3''' = A_2' \cup \{x : A\}$ Clash

The formula is consistent since there is a branch in the tableau where no more rules are applicable and no clash has occurred. Thus, the subsumption does not hold.

b) Construct a Tableau to either prove or refute the following Description Logic axiom:

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqsubseteq \neg(A \sqcap \neg B) \sqcap \forall R.A$$

Does the subsumption hold?

Solution:

To check whether the subsumption:

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqsubseteq \neg(A \sqcap \neg B) \sqcap \forall R.A$$

holds we should check whether

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqcap \neg(\neg(A \sqcap \neg B) \sqcap \forall R.A)$$

is inconsistent.

To use the tableau rules, first we have to convert the formula into Negation Normal Form as follows:

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqcap \neg(\neg(A \sqcap \neg B) \sqcap \forall R.A)$$
$$(\forall R.A \sqcap B \sqcap C) \sqcap ((A \sqcap \neg B) \sqcup \exists R.\neg A)$$

- $A_0 = \{x : (\forall R.A \sqcap B \sqcap C) \sqcap ((A \sqcap \neg B) \sqcup \exists R.\neg A)\}$
- $A_1 = A_0 \cup \{x : (\forall R.A \sqcap B \sqcap C), x : ((A \sqcap \neg B) \sqcup \exists R.\neg A)\}$
- $A_2 = A_1 \cup \{x : \forall R.A, x : B, x : c\}$
- $A_3 = A_2 \cup \{x : (A \sqcap \neg B)\}$
- $A_4 = A_3 \cup \{x : A, x : \neg B\}$ Clash
- $A_3' = A_2 \cup \{x : \exists R.\neg A\}$
- $A_4' = A_3' \cup \{(x, y) : R, y : \neg A\}$
- $A_5' = A_4' \cup \{y : A\}$ Due to $x : \forall R.A$ in A_2. This leads to a clash

Every branch in the tableau contains a clash thus the formula

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqcap \neg(\neg(A \sqcap \neg B) \sqcap \forall R.A)$$

is inconsistent.

Thus the subsumption

$$\neg\exists R.\neg A \sqcap B \sqcap C \sqsubseteq \neg(A \sqcap \neg B) \sqcap \forall R.A$$

holds.