German University in Cairo
Media Engineering and Technology
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Practice Assignment8

Exercise 8-1

Let `p, q` and `r` be the following propositions:

- p: You get an A in the final exam.
- q: You do every excercise in the book.
- r: You get an A in this class.

Write the following formulas using `p, q` and `r`.

a) to get an A in this class, but you do not do every exercise in the book.

Solution:

$p \wedge \neg q$

b) To get an A in this class, it is necessary for you to get an A on the final.

Solution:

$r \rightarrow p$

c) Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class.

Solution:

$(p \wedge q) \rightarrow r$

Exercise 8-2

Solve the following questions

a) Express the following statements in propositional logic:

- Statement 1: Slim lives in Cairo.

  Solution:

  p
- Statement 2: Slim lives in Egypt.

  Solution:

  r
- Statement 3: If Slim lives in Cairo then he lives in Egypt.

  Solution:

  $p \rightarrow r$

b) Prove formally the following: Statement 3 and Statement 2 do not entail Statement 1.

Solution:

Through using resolution and trying to prove that Statement3, Statement 2 entail Statement1):

1. $\neg p \vee r$
2. r
3. $\top$ (from 1 and 2)

This means that we cannot reach the empty set whic means that we cannot prove the statment and that it does not hold.

Exercise 8-3

Prove or disprove (by giving a counter example):

a) $\{\varphi 1, \varphi 2\} \models \psi$ if and only if $\varphi 1 \models \psi$ and $\varphi 2 \models \psi$.

Solution:

This statement is incorrect since it is not the case that if $\varphi 1 \models \psi$ and $\varphi 2 \models \psi$ then $\{\varphi 1, \varphi 2\}$.
The counter example is whenever $\varphi 1 = p \rightarrow q$ and $\varphi 2 = p$ and $\psi = r$

b) Given sentences φ, ψ, then $\varphi \models \psi$ if and only if $\varphi \not\models \neg \psi$.

Solution:

The statement is incorrect since if $\varphi \not\models \neg \psi$ then this means that we can not derive $\bot$ from $\{\varphi, \psi\}$. However, this does not make $\varphi \models \psi$ correct.
An example is the case where $\varphi = p$, $\psi = r$. In this case $\varphi \not\models \neg \psi$ holds since with the set $\{p, r\}$, we cannot reach a contradiction. However, also for the set $\{p, \neg r\}$, we cannot reach a constradiction. This means that $\varphi \models \psi$ does not hold which means that although $\varphi \not\models \neg \psi$ holds, $\varphi \models \psi$ does not hold which makes the initial statement incorrect.

c) If the sentence φ has no model, then $\varphi \models \psi$ for any sentence φ. Give an example for such a sentence φ.

Solution:

Such sentence can be $p \wedge \neg p$. Since such sentence produces $\bot$ rightaway. Therefore to test whether $\varphi \models \psi$, we try to prove that $\varphi \wedge \neg\psi$ produces $\bot$ which is true for any ψ.
As for a model: $p = \top, r = \bot, q = \top$

d) Given sets of sentences M, M', with $M \subseteq M'$ and a sentence M, then $M \models \varphi$ if and only if $M' \models \varphi$.

Solution:

This statement is incorrect. A counter example is when $M = \{p\}$, $M' = \{p \rightarrow q, p\}$ and $\varphi = r$.
As for a model: $p = \top, r = \bot, q = \top$

Exercise 8-4

Prove by means of the tableau method and resolution method that the following entailements are valid. If not give a counter example.

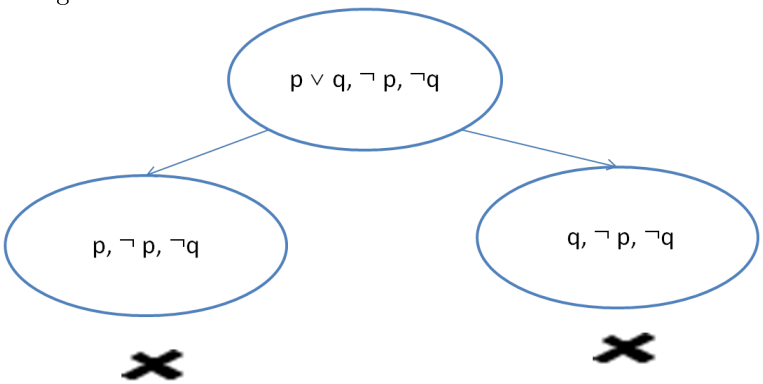
a) $\{p \vee q, \neg p\} \models q$

Solution:

Using Resolution: Trying to prove that $\{p \vee q, \neg p, \neg q \models \bot\}$

1. $p \vee q$
2. $\neg p$
3. $\neg q$
4. q (From 1,2)
5. $\bot$ (From 3,4)

Using Semantic Tableau:



b) $\{p \to q, q\} \models p$

Solution:

A counter example: $p = \bot, q = \top$

c) $\{p \to q\} \models \neg(q \to p)$

Solution:

A counter example: $p = \bot, q = \bot$