German University in Cairo
Media Engineering and Technology Faculty
Prof. Dr. Slim Abdennadher

December 12, 2016

Semantic Web and Internet of Things
Winter Semester 2016-2017
Final Exam

Name:

ID Number:

Instructions: Read carefully before proceeding.

1) Duration of the exam: 1 hour.
2) The exam consists of 5 exercises. One of the questions will be taken as a bonus exercise. Try to solve all of them.
3) This exam booklet contains 10 pages, including this one. Three extra sheets of scratch paper are attached and have to be kept attached. **Note that if one or more pages are missing, you will lose their points. Thus, you must check that your exam booklet is complete**.
4) Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem or on the four extra sheets and make an arrow indicating that. **Scratch sheets will not be graded unless an arrow on the problem page indicates that the solution extends to the scratch sheets**.
5) When you are told that time is up, stop working on the test.

Good Luck!

Don't write anything below `;-)`

Exercise	1	2	3	4	5	$\sum$
Possible Marks	10	10	10	10	10	50
Final Marks						

Exercise 1

Consider the following hypotheses:

'if I take the highway and there is no accident I will arrive on time but if there is an accident I will arrive late unless I take surface streets. I will arrive on time.'.

Which conclusions can be inferred from the hypothesis? To solve the problem, first you have to model it and then provide formal proofs using resolution for the first hypothesis and using tableaux for the second hypothesis.

a) I will take the highway

Solution:

- Formalize the given knowledge base:
  1. F1: $(h \wedge \neg a) \rightarrow \neg l$
  2. F2: $\neg s \rightarrow (a \rightarrow l)$
  3. F3: $\neg l$
- Check whether the statement (h) is a logical consequence of the knowledge base or not. In other words, check if $F1 \wedge F2 \wedge F3 \models h$. We can do so using resolution
- $F1 \wedge F2 \wedge F3 \wedge \neg h$ should be unsatisfiable. $[(h \wedge \neg a) \rightarrow \neg l] \wedge [\neg s \rightarrow (a \rightarrow l)] \wedge [\neg l] \wedge [\neg h]$
- Put the formula in CNF:
  - $[\neg(h \wedge \neg a) \vee \neg l] \wedge [s \vee (\neg a \vee l)] \wedge [\neg l] \wedge [\neg h]$
  - $[\neg h \vee a \vee \neg l] \wedge [s \vee \neg a \vee l] \wedge [\neg l] \wedge [\neg h]$
- Form the clauses:
  - c1: $\{\neg h, a, \neg l\}$
  - c2: $\{s, \neg a, l\}$
  - c3: $\{\neg l\}$
  - c4: $\{\neg h\}$
- Since we can not reach the empty clause from these clauses then $F1 \wedge F2 \wedge F3 \not\models h$

b) If there is an accident, then I took surface streets.

Solution:

- Formalize the given knowledge base:
  1. F1: $(h \wedge \neg a) \rightarrow \neg l$
  2. F2: $\neg s \rightarrow (a \rightarrow l)$
  3. F3: $\neg l$
- Check whether the statement $(a \rightarrow s)$ is a logical consequence of the knowledge base or not. In other words, check if $F1 \wedge F2 \wedge F3 \models a \rightarrow s$.
-
- $F1 \wedge F2 \wedge F3 \wedge \neg(a \rightarrow s)$ should be unsatisfiable.
- $A_0 = [(h \wedge \neg a) \rightarrow \neg l] \wedge [\neg s \rightarrow (a \rightarrow l)] \wedge [\neg l] \wedge [\neg(a \rightarrow s)]$
- Tableau method:
  - $A_1 = A_0 \cup \{a, \neg s\}$
  - $A_2 = A_1 \cup \{s\}$ Clash
  - $A_2' = A_1 \cup \{a \rightarrow l\}$
  - $A_3' = A_2' \cup \{\neg a\}$ Clash
  - $A_3'' = A_2' \cup \{l\}$ Clash
- Tableau is closed. Thus the subsumption holds.

Exercise 2

Let $V(x, y)$ be the predicate *x voted for y*, let $M(x, y)$ be the predicate *x received more votes than y*, and let the universe for all variables be the set of all people. Express each of the following statements as predicate logic formulas using V and M:

a) Everybody received at least one vote.

Solution:

$\forall y \exists x[V(x, y)]$

b) Jane and John voted for the same person.

Solution:

$\exists x[V(jane, x) \wedge V(john, x)]$

c) Trump won the election. (The winner is the person who received the most votes.)

Solution:

$\forall x[M(Trump, x)]$

d) Nobody who votes for him/herself can win the election.

Solution:

$\forall x[V(x, x) \rightarrow \exists y M(y, x)]$

e) Everybody can vote for atmost one person.

Solution:

$\forall x[\neg \exists y \exists z[V(x, y) \wedge V(x, z) \wedge \neg(y = z)]]$

Exercise 3

Prove the validity of the following formula using resolution

$$(\exists x \forall y Q(x,y) \wedge \forall x(Q(x,x) \rightarrow \exists y R(y,x))) \rightarrow \exists y \exists x R(x,x)$$

Solution:

Note that if F is a tautology, then the $\neg F$ is unsatisfiable.

- Negate the given formula:

$$\neg[(\exists x \forall y Q(x,y) \wedge \forall x(Q(x,x) \rightarrow \exists y R(y,x))) \rightarrow \exists y \exists x R(x,x)]$$

- Resolve all implications before pushing any negation

$$\neg[\neg(\exists x \forall y Q(x,y) \wedge \forall x(Q(x,x) \rightarrow \exists y R(y,x)))] \vee \exists y \exists x R(x,x)]$$

$$\neg[\neg(\exists x \forall y Q(x,y) \wedge \forall x(\neg Q(x,x) \vee \exists y R(y,x)))] \vee \exists y \exists x R(x,x)]$$

- Push the inner negation

$$\neg[(\forall x \exists y \neg Q(x,y) \vee \exists x(Q(x,x) \wedge \forall y \neg R(y,x)))] \vee \exists y \exists x R(x,x)]$$

- Push the outer negation

$$(\exists x \forall y Q(x,y) \wedge \forall x(\neg Q(x,x) \vee \exists y R(y,x))) \wedge \forall y \forall x \neg R(x,x)$$

- Skolemize

$$(\forall y Q(a,y) \wedge \forall x(\neg Q(x,x) \vee R(b,x))) \wedge \forall y \forall x \neg R(x,x)$$

- Drop universal quantifier

$$(Q(a,y) \wedge (\neg Q(x,x) \vee R(b,x))) \wedge \neg R(x,x)$$

- Create clauses
  c1: $\{Q(a,y)\}$
  c2: $\{\neg Q(x,x), R(b,x)\}$
  c3: $\{\neg R(x,x)\}$

- Resolve clauses
  c4 (c1 R c2): $\{R(b,x)\}$
  c5 (c4 R c3): $\{\ \}$

- We have reached the empty clause, therefore $\neg F$ is unsatisfiable, so we can say that F is valid.

Exercise 4 (10 Marks)

a) Express the following concepts/sentences in description logic using atomic concepts `School`, `Female`, and `GirlsSchool` and roles `Pupil` and `Employee`.

1. A girls school is defined as school where all pupils are girls.

   Solution:

$$GirlsSchool = School \sqcap \forall Pupil.Female$$

2. In girls schools all employees are female.

   Solution:

$$GirlsSchool \sqsubseteq \forall Employee.Female$$

3. A school which has at least 30 pupils and 5 employees.

   Solution:

$$GirlsSchool = School \sqcap \exists_{\geq 30} Pupil.\top \sqcap \exists_{\geq} Employee.\top$$

b) Express the following concepts in description logic using the atomic concepts `Cat`, `Fish` and `Animal`, the roles `Eat`, and the constant `Minou`.

1. An animal that eats only things that themselves eat only fish.

   Solution:

$$Animal \sqcap \forall Eat.(\forall Eat.Fish)$$

2. Minou is a cat who eats only fish.

   Solution:

$$Minou : Cat \sqcap \forall Eat.Fish$$

Exercise 5 (5+5=10 Marks)

Using the Tableaux Method rules stated in the next page, check if the following subsumptions are valid:

a)

$$\exists R.(A \sqcap B) \sqsubseteq \exists R.A \sqcap \exists R.B$$

Solution:

- $A_0 = \{a : (\exists R.(A \sqcap B) \sqcap \forall R.\neg A \sqcup \forall R.\neg B)\}$
- $A_1 = A_0 \cup \{a : \exists R.(A \sqcap B), a : \forall R.\neg A \sqcup \forall R.\neg B)\}$
- $A_2 = A1 \cup \{(a, b) : R, b : (A \sqcap B)\}$
- $A_3 = A_2 \cup \{b : A, b : B\}$
- $A_4 = A_3 \cup \{a : \forall R.\neg A\}$
- $A_5 = A_4 \cup \{b : \neg A\}$ Clash
- $A_4' = A_3 \cup \{a : \forall R.\neg B\}$
- $A_5' = A_4' \cup \{b : \neg B\}$ Clash

Subsumption holds.

b)

$$\exists R.A \sqcap \exists R.B \sqsubseteq \exists R.(A \sqcap B)$$

Solution:

- $A_0 = \{a : \exists R.A \sqcap \exists R.B \sqcap \forall R.(\neg A \sqcup \neg B)\}$
- $A_1 = A_0 \cup \{a : \exists R.A, a : \exists R.B, a : \forall R.(\neg A \sqcup \neg B)\}$
- $A_2 = A1 \cup \{(a, b) : R, b : A\}$
- $A_3 = A_2 \cup \{(a, c) : R, c : B\}$
- $A_4 = A_3 \cup \{b : \neg A \sqcup \neg B\}$
- $A_5 = A_4 \cup \{c : \neg A \sqcup \neg B\}$
- $A_6 = A_5 \cup \{b : \neg A\}$ Clash
- $A_6' = A_5 \cup \{b : \neg B\}$
- $A_7' = A_6' \cup \{c : \neg A\}$ Open branch

Subsumption does not hold.

Tableaux Method Rules for Description Logic

- Conjunction Rule

  If $a : C \sqcap D \in A\ and\ \{a : C, a : D\} \not\subseteq A$ then $A' = A \cup \{a : C, a : D\}$

- Disjunction

  If $a : C \sqcup D \in A\ and\ \{a : C, a : D\} \cap A = \emptyset$ then $A' = A \cup \{a : C\}$ or $A' = A \cup \{a : D\}$

- Existential Quantification

  If $a : \exists R.C \in A$ and there is no z such that $\{(a, z) : R, z : C\} \subseteq A$ then $A' = A \cup \{(a, b) : R, b : C\}$ with b fresh in A

- Universal Quantification

  If $a : \forall R.C \in A$ and $(a, b) : R \in A$ and $b : C \notin A$ then $A' = A \cup \{b : C\}$

Scratch paper

Scratch paper

Scratch paper