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Practice Assignment9

Exercise 9-1

a) Express the following sentences in first-order logic, by only using the following unary predicates $fragile, break, fall, tennisBall$.

S1: Fragile things break if they fall

S2: Tennis balls are not fragile

S3: Tennis balls do not break if they fall

Solution:

$$\begin{array}{ll} S1 & \forall x(Fragile(x) \wedge Fall(x) \rightarrow Break(x)) \\ S2 & \forall x(TennisBall(x) \rightarrow \neg Fragile(x)) \\ S3 & \forall x(TennisBall(x) \wedge Fall(x) \rightarrow \neg Break(x)) \end{array}$$

b) Give a definition of logical entailment for first-order logic, that is, what it means for a set of sentences S to logically entail a sentence A.

Solution:

S logically entails A if for every interpretation I, if all sentences in S hold for I, then A holds for I too.

c) Do the above sentences sentences $S1$ and $S2$ logically entail $S3$? Justify your answer; if you answered yes, show why the entailment holds, if you answered no, give a counterexample.

Solution:

No, $S1$ and $S2$ do not logically entail $S3$. An interpretation which satisfies $S1$ and $S2$ but not $S3$ would be for example where all things break if they fall: the domain is $D = \{d1, d2\}$, $I(Fragile) = \{d1\}$, $I(TennisBall) = \{d2\}$, $I(Fall) = \{d1, d2\}$, and $I(Break) = \{d1, d2\}$.

d) Is there a fully automatic terminating procedure to establish, for any finite set of formulas S and a formula A, whether S logically entails A in either propositional or first-order logic? If the answer is yes to either of these, then describe that procedure.

Solution:

There is no such procedure for first-order logic. For propositional logic, one could for example construct a truth table for S and A and check whether all assignments satisfying S also satisfy A.In addition, resolution method is decidable procedure for propositional logic.

Exercise 9-2

a) Reduce the following statements to clausal form:

S1: $\forall x \exists y \forall z \exists u P(x, y, z, u)$

S2: $\forall x \forall y (R(x, y) \vee \exists z R(y, z))$

S3: $\forall x \forall y ((R(x, y) \vee Q(x, y)) \rightarrow (R(y, x) \vee Q(y, x)))$

Solution:

S1: $\{P(x, f(x), z, g(x, z))\}$

S2: $\{R(x, y) \vee R(y, f(x, y))\}$

S3: $\{\neg R(x, y) \vee R(y, x) \vee Q(y, x), \neg Q(x, y) \vee R(y, x) \vee Q(y, x)\}$

b) Show by resolution that clauses C1, C2, an C3 below entail $P(a, f(f(a)))$.

C1: $\{\neg P(x, y), \neg P(y, z), P(x, z)\}$

C2: $\{\neg P(x, f(x)), P(f(x), f(f(x)))\}$

C3: $\{P(a, f(a))\}$

Solution:

C4: $\neg P(a, f(f(a)))$

C5: $\{\neg P(a, y), \neg P(y, f(f(a)))$ resolution C1 and C4 with $x = a; z = f(f(a))$

C6: $\{\neg P(f(a)), f(f(a)))\}$ resoultion C5 and C3 with $y = f(a)$

C7: $\{P(f(a)); f(f(a)))\}$ resolution C2 with C6 with $x = a$

C8: $\perp$ resolution C6 and C7