# Kinematic Model for a Three-Wheeled Robot with 120° Wheel Separation

Group3 MIA Phase 2

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## Forward Kinematics

The forward kinematics describe how the robot's wheel velocities $(v_1, v_2, v_3)$, linear velocities $(V_x, V_y)$, and total angular velocity $(\omega)$ are related.

### Wheel Velocities $(v_1, v_2, v_3)$

The linear velocity of each wheel $(v_i)$ is related to its angular velocity $(\omega_i)$ by $v_i = R \cdot \omega_i$, where $R$ is the wheel radius.

### Robot Velocities $(V_x, V_y)$

The robot's linear velocities in the $x$ and $y$ directions are determined by combining the individual wheel velocities. Since the wheels are positioned at 120° angles to each other, we can calculate $V_x$ and $V_y$ as follows:

## Wheel(Linear) Velocities $(v_1, v_2, v_3)$

For wheel 1 (W1):

$$\begin{aligned} V_{W1X} &= (-)V_{W1}\cos(\alpha) \\ V_{W1Y} &= (+)V_{W1}\sin(\alpha) \end{aligned}$$

For wheel 2 (W2):

$$\begin{aligned} V_{W2X} &= (-)V_{W2}\cos(\alpha) \\ V_{W2Y} &= (+)V_{W2}\sin(\alpha) \end{aligned}$$

For wheel 3 (W3):

$$\begin{aligned} V_X &= V_{W3} - V_{W1}\cos(\alpha) - V_{W2}\cos(\alpha) \\ V_Y &= V_{W1}\sin(\alpha) - V_{W2}\sin(\alpha) \end{aligned}$$

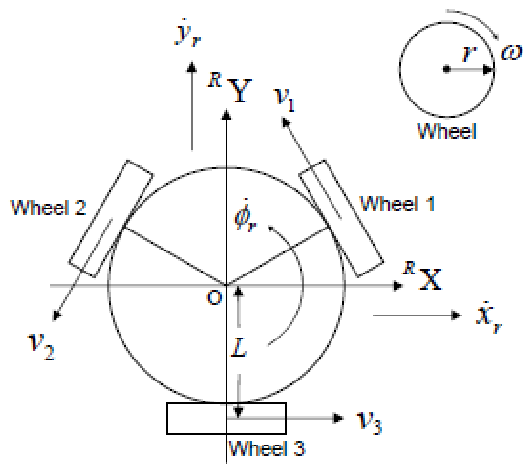


Figure 1: Kinematics of robot with three wheels

## Robot Velocities $(V_x, V_y)$

The robot velocities in the Cartesian coordinates $(V_x, V_y)$ are calculated based on the following equations:

$\mathrm{V}_{Wi}(1,2,3) = \omega \cdot r$

$\omega$ = Angular velocity of the omni-directional wheel (rad/sec)

$r$ = Omni-directional wheel radius (cm)

$V_\theta$ = Robot movement velocity

Given the wheels are symmetrically arranged 120 degrees apart, we have $\alpha = 60°$

$V_X = V_{W3} - \frac{V_{W1}}{2} - \frac{V_{W2}}{2}$

$V_Y = V_{W1}\left(\frac{\sqrt{3}}{2}\right) - V_{W2}\left(\frac{\sqrt{3}}{2}\right)$

## Total Angular Velocity $(\omega)$

$$\omega = \frac{V_{W1}}{L} + \frac{V_{W2}}{L} + \frac{V_{W3}}{L}$$

# Inverse Kinematics

The inverse kinematics allow us to determine the individual wheel angular velocities $(\omega_1, \omega_2, \omega_3)$ based on desired robot velocities $(V_x, V_y)$ and total angular velocity $(\omega)$.

### Wheel Angular Velocities $(\omega_1, \omega_2, \omega_3)$

Given the desired robot velocities $(V_x, V_y)$ and total angular velocity $(\omega)$, the inverse kinematic equations express the individual wheel angular velocities in terms of these desired values:

$$\begin{aligned}
\omega_1 &= -\frac{1}{2} \cdot V_y + \frac{\sqrt{3}}{2} \cdot L \cdot \omega \\
\omega_2 &= -\frac{1}{2} \cdot V_y - \frac{\sqrt{3}}{2} \cdot L \cdot \omega \\
\omega_3 &= V_x
\end{aligned}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ L \cdot \omega \end{bmatrix}$$

These equations account for the 120° wheel separation and relate the desired robot motion to the required wheel angular velocities or vice versa.