Module-2 REGULAR EXPRESSIONS

Regular Expressions (RE) is a string that can be framed according to the following rules.

1. $\emptyset$ is a regular expression
2. ϵ " "
3. Every element in Σ is a RE
4. Given 2 REs α and β, $\alpha\beta$ is RE
5. " " " $\alpha \cup \beta$ is RE
6. Given a RE α, α^* is a RE
7. " " " α^+ is a RE
8. " " " (α) is a RE

Every regular expression contains 2 kinds of symbols

1. an alphabet Σ
2. Special symbol $\emptyset$, $\cup$, ϵ, $()$, $*$, $+$

Example:

Analyze simple RE

$$L((a \cup b)^* b) = L((a \cup b)^*) L(b)$$
$$= (L((a \cup b)))^* L(b)$$
$$= (L(a) \cup L(b))^* L(b)$$
$$= (\{a\} \cup \{b\})^* \{b\}$$
$$= \{a, b\}^* \{b\}$$

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The meaning of RE $(a \cup b)^* b$ is the set of all strings over the alphabet $\{a, b\}$ that end in b.

Given a language, find a regular expression.

Ex1. $L = \{w \in \{a,b\}^*, |w| \text{ is even}\}$

$((a \cup b)(a \cup b))^*$ or $((a+b)(a+b))^*$

Ex2. $L = \{w \in \{a,b\}^* : |w| \leq 2\}$

ie $|w|$ can be 0, 1 or 2

ϵ for 0

a, b for 1 ie $(\epsilon + a + b)$

for $|w| = 2$ $(\epsilon + a + b)(\epsilon + a + b)$

$= (\epsilon + a + b)^2$ or $(\epsilon \cup a \cup b)^2$

Ex.3. $L = \{w \in \{a,b\}^* : |w| \text{ is odd}\}$

$(a \cup b)((a \cup b)(a \cup b))^*$

OR

$(a+b)((a+b)(a+b))^*$

Ex.4. $L = \{w \in \{a,b\}^* : w \text{ contains alternate a's and b's}\}$

$(ab)^*$ - but this always begins with a and ends with b

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Hence add $(\epsilon+b)$ at begining &
$(\epsilon+a)$ at end.

i.e. $(\epsilon+b)(ab)^*(\epsilon+a)$.

Ex 5. $L = \{w \in \{0,1\}^* :$ w contains at most one pair of consecutive 0's $\}$.

- No zero's 1^*
- with one zero 01 or 10
- with a single pair of 00.

$(1+01)^* 0 (1+01)^*$ with 01

$(1+10)^* 0 (1+10)^*$ with 10.

Hence $RE = 1^* + (1+01)^* 0 (1+01)^* + (1+10)^* 0 (1+10)^*$

Ex. 6. $L = \{w \in \{a,b,c\}^* :$ w contains at least one a and at least one b $\}$

minimum w = ab or ba

It can begin with anything, end with anything and in between also can be anything.

$(a+b)^* a (a+b)^* b (a+b)^*$ for ab

$(a+b)^* b (a+b)^* a (a+b)^*$ for ba

hence $RE = [(a+b)^* a (a+b)^* b (a+b)^*] + [(a+b)^* b (a+b)^* a (a+b)^*]$

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Ex. 7.

$L = \{ w \in \{a, b\}^* :$ w ends with b and has no substring aa$\}$

end with b but no aa.

is either b or ab.

$(b+ab)$ b or ab

but b or ab can come any No. of times

$(b+ab)(b+ab)^* = (b+ab)^+$

Ex. 8. $L = \{ w \in \{a, b\}^* :$ w begins with a and ends with b$\}$

begins with a and ends with b = ab

in between it could be anything.

RE = $a(a+b)^*b$.

Ex. 9. $L = \{ w \in \{a, b\}^* :$ second symbol from right-end is a$\}$

Initially anything. $(a+b)^*$

Second sym from right is a $(a+b)^*a(a+b)$

last sym can be either a or b.

RE = $(a+b)^*a(a+b)$

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Ex. 10.

$L = \{ w \in \{a,b\}^* : w$ begins & ends with same symbol$\}$

begins ~~with~~ & ends with a $= a(a+b)^*a$

" " " b $= b(a+b)^*b$

$\therefore RE = a(a+b)^*a + b(a+b)^*b.$

Ex. 11.

$L = \{ w \in \{a,b\}^* : |w|$ is even or multiple of 3$\}$

length is even $[(a+b)(a+b)]^*$

length multiple of three $((a+b)(a+b)(a+b))^*$.

$RE = ((a+b)(a+b))^* + ((a+b)(a+b)(a+b))^*.$

Ex. 12.

$L = \{ w \in \{a,b\}^* : w = a^m b^n$ m is even & n is odd$\}$

ie even No. of a's followed by odd No. of b's

even No. of a's. $(aa)^*$

odd " " b's $b(bb)^*$

$RE = (aa)^* b(bb)^*.$

Ex. 13. $L = \{ w \in \{a,b\}^* : w = a^m b^n$ m+n is even$\}$

we know that even + even = even

odd + odd = even

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even No of a's & even No of b's $= (aa)^*(bb)^*$

odd " " & odd " " $= a(aa)^* b(bb)^*$

$RE = (aa)^*(bb)^* + a(aa)^* b(bb)^*$

Ex. 14. $L = \{a^n b^m \mid m \geq 1, n \geq 1 \quad mn \geq 3\}$

we know that

$mn \geq 3$ when

1) $n = 1$ & $m \geq 3$ or
2) $n \geq 3$ & $m = 1$ or
3) $n \geq 2$ & $m \geq 2$

1) $abbbb^*$ $\quad |a| = 1 \quad |b| \geq 3$
2) $aaaa^*b$ $\quad |a| \geq 3 \quad |b| = 1$
3) aaa^*bbb^* $\quad |a| \geq 2 \quad |b| \geq 2$

$RE = abbbb^* + aaaa^*b + aaa^*bbb^*.$

Ex. 15. $L = \{w \in \{a,b\}^* : |w| \bmod 3 = 0\}$

$((a+b)(a+b)(a+b))^*$.

Ex. 16. $L = \{w \in \{a,b\}^* : \#_a(w) \bmod 3 = 0\}$

ϵ, aaa, abaa, aaabababba,

$b^* + a \quad (b^*ab^*ab^*ab^*)^*$

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Building an FSM from a regular expression (RE)

theorem :- ~~for any~~ every RE there is an equivalent FSM.

proof :- The proof is by construction of FSM.
we will show that, given a RE α, we can construct an FSM M. such that $L(\alpha) = L(M)$.

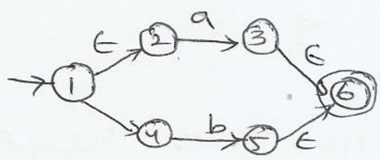
There exists an FSM that corresponds to each premitive RE.

If α is any $c \in \Sigma$

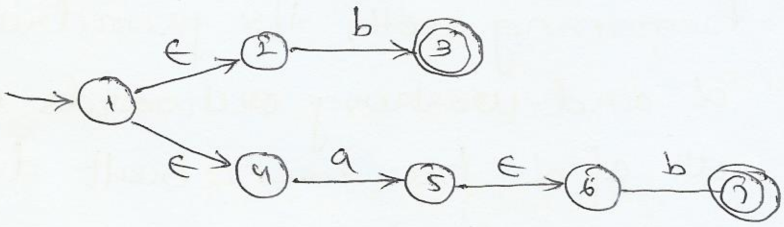
If α is $\emptyset$

If α is ϵ

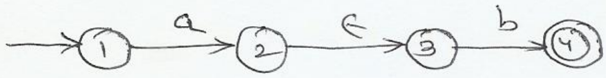
If α is RE $= \beta \cup \gamma$ $\alpha = a \cup b$



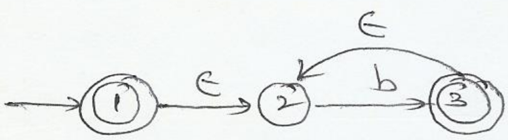
$\alpha = \beta \cup \gamma$
$\beta = b$ $\gamma = ab$
ie $\alpha = b \cup ab$



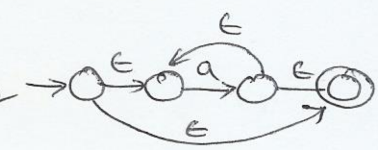
If $\alpha = ab$



If $\alpha = b^*$



or



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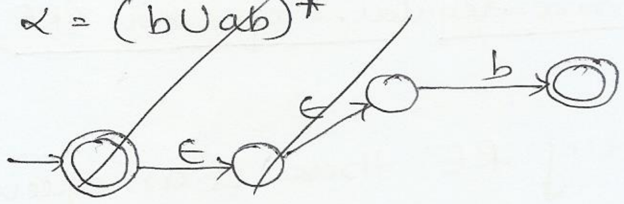
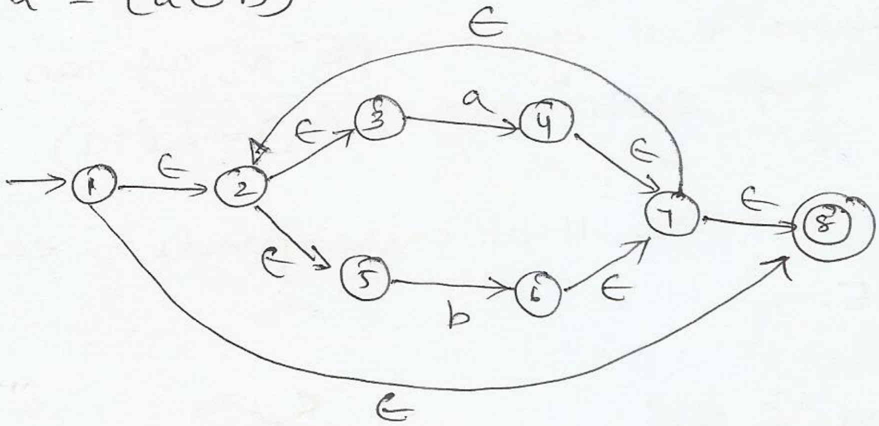
If $\alpha = (b \cup ab)^*$



If $\alpha = (a \cup b)^*$



Based on the construction, we define algorithm to construct, given a RE α, a corresponding FSM.

Algorithm regextofsm (α : RE) =

Beginning with the primitive subexpression of α and working outwards until an FSM for all of α has been built do:

Construct an FSM as described above.

~~Butt~~:

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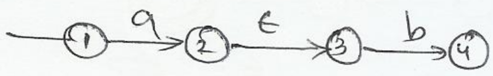
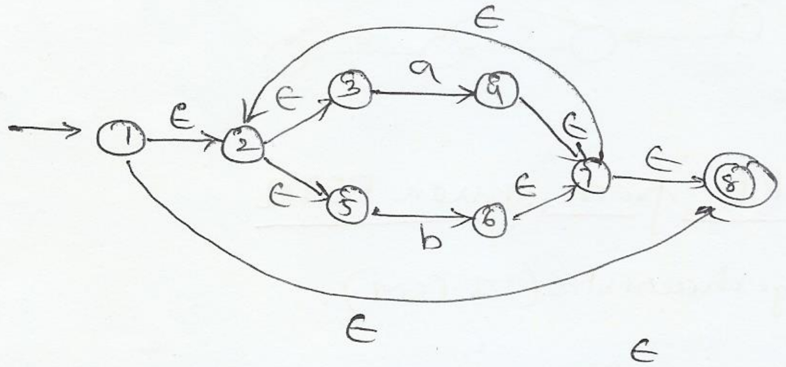
Example 1 : Build FSM from a given RE

RE = ab(a+b)* or ab(a∪b)*.

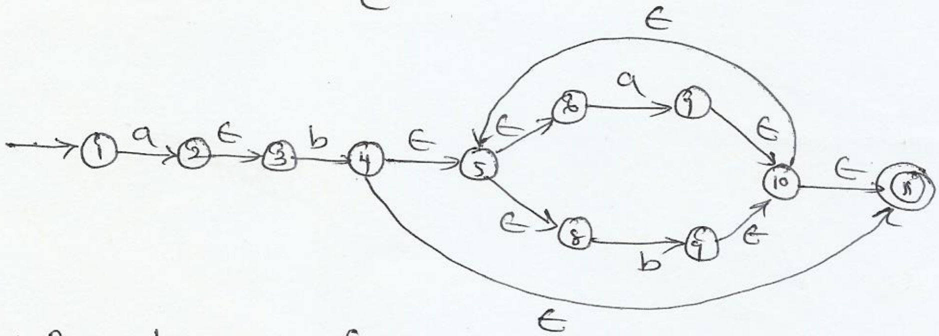
ab =



(a+b)* =



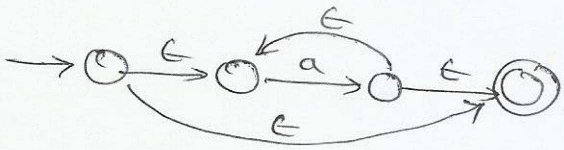
final
∴ FSM =



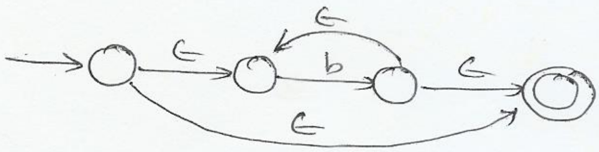
Example-2. Build FSM from a given RE

RE = a* + b* + c*

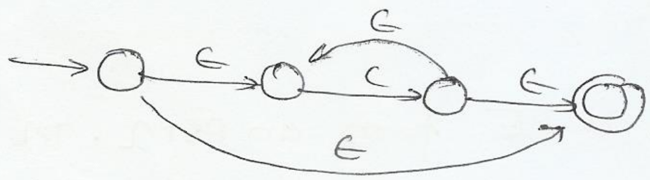
a* =



b* =



c* =



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