

Modeling Hedge Fund Alpha: Factor Selection, AUM Optimization, and Crisis Stress Testing

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1 Introduction

This project develops a comprehensive pipeline for modeling hedge fund performance using factor-based regressions. The analysis applies traditional asset pricing models (CAPM, Fama-French, Carhart), elastic net for variable selection, and an OLS refit to estimate fund alpha. It further explores the relationship between alpha and assets under management (AUM), following the framework of Berk and Green (2004), and simulates how the fund might have performed during the 2008–2009 financial crisis using historical factor data.

Note: The original return and AUM data were obtained from WRDS and are proprietary, hence not publicly available in this repository.

2 Strategy Overview

The fund employs a global macroeconomic strategy aimed at capturing returns driven by inefficiencies in how markets incorporate new information. This strategy leverages both qualitative and quantitative analyses of macroeconomic and geo-political trends, focusing on key variables such as inflation, monetary policy, international trade, growth, and investor sentiment. The fund aims to exploit the market’s underreaction to changes in these factors, identifying it as a recurring inefficiency. It maintains a market-neutral stance, seeking minimal correlation with broad market indices.

3 Analysis of Abnormal Returns

3.1 CAPM Regression

$$R_i - R_f = \alpha + \beta(R_m - R_f) + \varepsilon$$

The model yields a low R^2 of 0.0042, indicating minimal explanatory power of the S&P 500 on the fund’s returns. The gross alpha is 0.0017, suggesting a small positive return uncorrelated with the market, while beta is effectively zero (-0.0309), consistent with a market-neutral strategy.

3.2 Fama-French Regression

$$R_i - R_f = \alpha + \beta_{MKT}(R_m - R_f) + \beta_{SMB} SMB + \beta_{HML} HML + \varepsilon$$

The Fama-French model yields an R^2 of 0.0379 and gross alpha of 0.0021. Factor loadings are:

$$\beta_{MKT} = -0.0487, \quad \beta_{SMB} = 0.0805, \quad \beta_{HML} = 0.0672,$$

supporting the fund's market-neutral stance with minimal size or value tilts.

3.3 Carhart 4-Factor Model

$$R_i - R_f = \alpha + \beta_{MKT}(R_m - R_f) + \beta_{SMB} \text{SMB} + \beta_{HML} \text{HML} + \beta_{MOM} \text{MOM} + \varepsilon$$

The Carhart model yields the highest R^2 among the classical models at 0.0513, but still explains little of the fund's return variation. Estimated coefficients are:

$$\alpha = 0.0018, \quad \beta_{MKT} = -0.0275, \quad \beta_{SMB} = 0.1044, \quad \beta_{HML} = 0.0845, \quad \beta_{MOM} = 0.0758.$$

This reinforces the fund's detachment from U.S. equity factors and supports using models incorporating global macroeconomic and asset-specific variables.

3.4 Benchmark Analysis – Vanguard Funds

The three classical models proved largely ineffective in explaining the behavior of the fund's returns. To address this, we implemented a model using the returns of 11 Vanguard mutual funds as predictors. These funds were selected for their diverse asset-class exposures, serving as a benchmark for the fund's global and multi-asset investment strategy:

$$R_i - R_f = \alpha + \sum_{j=1}^{11} \beta_j v_j + \varepsilon.$$

The full model ($R^2 = 0.3884$) showed high multicollinearity. To address this, we applied an Elastic Net 10-fold cross-validation to identify key drivers. We chose to set $\alpha = 0.4$ as it produced a parsimonious set of predictors with VIFs below 5 and yielded the highest adjusted R^2 in the OLS refit. The final 8-factor model ($R^2 = 0.3769$) included seven predictors with p -values below 0.10, two of which were statistically significant:

$$\beta_{VBINX} = -2.1332, \quad \beta_{VISGX} = 0.5519.$$

VBINX (a 60/40 U.S. stock-bond fund) has a strong negative loading, suggesting the fund moves away from traditional U.S. assets. VISGX (small-cap growth stocks) has a positive loading, indicating some indirect exposure, likely through shared factors such as momentum or value.

3.5 Extended Factor Model

To model the fund's excess returns, we began with 40 market premia factors spanning asset classes, investment styles, and macroeconomic themes. Similar to above, we apply an Elastic Net with $\alpha = 0.7$ to prioritize sparsity and isolate a minimal set of predictors. We then confirmed that all selected variables had VIFs below 5 before proceeding with the OLS refit.

$$\begin{aligned} R_i - R_f = & \alpha + \beta_{SMB} \text{SMB} + \beta_{US.Def} \text{U.S.Stock.Selection.Defensive} + \beta_{FI.Carry} \text{Fixed.Income.Carry} \\ & + \beta_{FI.Def} \text{Fixed.Income.Defensive} + \beta_{FI.Multi} \text{Fixed.Income.Multi-Style} \\ & + \beta_{Com.Val} \text{Commodities.Value} + \beta_{Com.Mom} \text{Commodities.Momentum} \\ & + \beta_{Macro.Carry} \text{All.Macro.Carry} + \beta_{FI.Market} \text{Fixed.Income.Market} + \varepsilon. \end{aligned} \tag{1}$$

The final model included 9 factors and achieved an adjusted R^2 of 0.3152, with a residual standard error of 0.01778 and an F -statistic of 7.34 ($p < 0.001$), confirming joint significance. The estimated alpha was 0.001552 ($p = 0.3523$), indicating no statistically significant unexplained return. Notable exposures included negative loadings to fixed income carry (-0.5892) and defensive (-0.5433) strategies, and positive exposures to fixed income multi-style (1.1223), commodity momentum (0.1013), and macro carry (0.5621), all statistically significant. These results suggest the fund actively avoids traditional fixed income exposures while tilting toward momentum and multi-style strategies across asset classes. Compared to the Vanguard fund model, which achieved a similar R^2 , the market premia model showed superior predictive behavior based on diagnostic checks (e.g., RMSE, residual patterns), supporting its use as the basis for further analysis.

4 AUM and Value Added

4.1 Asset Allocation

We aggregate the capital allocated to all share classes and conduct our analysis on the aggregate AUM of the entire fund.

4.2 AUM Regression

$$\alpha_i = a + b \text{AUM}_i + \varepsilon_i.$$

The model yields an R^2 of 0.0247, with an intercept (a) of 0.003468 and slope (b) of 6.05×10^{-5} . The low R^2 suggests a weak relationship between fund performance and size. Despite the low R^2 , the model still estimates the AUM level where marginal Alpha reaches zero.

4.3 Optimal AUM and Value Added

Using the Berk and Green (2004) framework, we calculate performance at the fund's optimal size:

Gross Alpha at Optimum: 0.001734, Net Alpha at Optimum: 0.000901, Optimal AUM: \$28.66M.

The optimal AUM is over \$170 million below the fund's current size. The value added at the optimum is 0.04969845M, indicating potential value generation at smaller scale.

4.4 Current Performance and Investor Outcomes

Using alpha estimates from the most recent month (based on the market premia model):

Current Gross Alpha: -0.0371 , Current Net Alpha: -0.0380 .

These negative values indicate underperformance even before fees. Current value added from both manager and investor perspectives is around -\$6 million, and aggregate value added is also negative, suggesting that despite the fund's potential at smaller scale, it is currently destroying value. However, investors appear confident in the fund's downside protection claims, as capital outflows remain limited.

5 Stress-Test: 2008 Financial Crisis

Return analysis alone may not fully capture the fund's hedging objective. We simulate its hypothetical performance during the 2008 financial crisis using the market premia model. Defining the crisis period as December 2007 to June 2009, we observe that the fund would have outperformed the S&P 500 but consistently underperformed 3-month Treasury bills. While this suggests some equity-risk hedging, it fails to surpass a risk-free benchmark, undermining its value proposition.

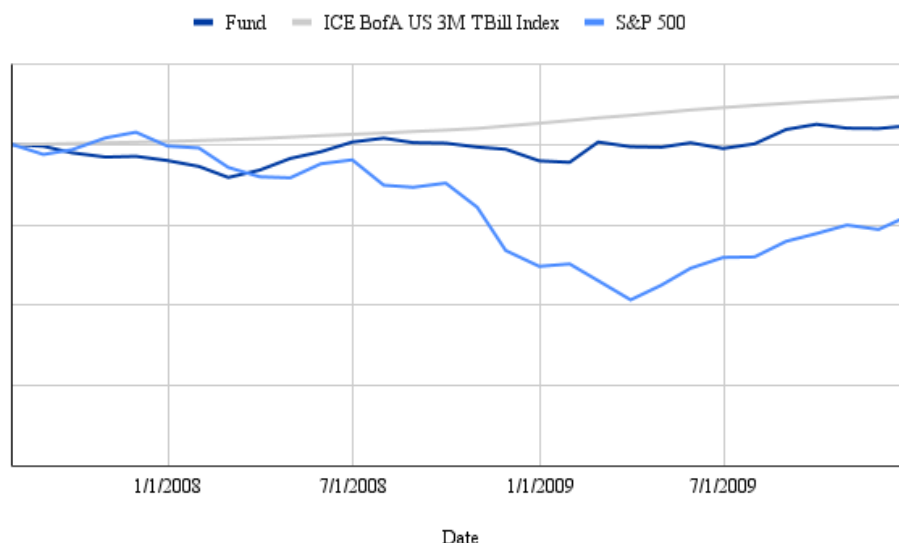


Figure 1: Value of \$1 invested in Fund, S&P 500, and ICE BofA US 3M T-Bill (2007–2009)

6 Conclusion

Our analysis, based on the market premia model, suggests the fund is overfunded, delivers no consistent alpha, and underperforms its benchmark, even in times of crisis. With an optimal AUM of \$28.66M and weak investor outcomes at current scale, the fund fails to justify its fees or capital base.

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