

Wednesday 17 December 2025 – Morning

A Level Further Mathematics A

Y542/Y543 Statistics/Mechanics

Time allowed: 1 hour 30 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

ADVICE

- Read each question carefully before you start your answer.

Section A – Statistics

- 1 A six-sided dice may or may not be equally likely to land on any one of its faces. It has two faces numbered 5 and the other four faces are numbered 1, 2, 3 and 4. The dice is thrown 60 times, and the outcome on each throw is recorded. The results are shown in the table.

Outcome	1	2	3	4	5
Frequency	7	13	11	15	14

It is required to carry out a test at the 5% significance level of whether the probabilities of the outcomes 1, 2, 3, 4 and 5 are in the ratio 1 : 1 : 1 : 1 : 2.

- (a) State suitable hypotheses for the test. [1]
- (b) Carry out the test. [6]

- 2 On any day, flights of helicopters and aeroplanes pass within sight of a certain building only during a daytime period of 12 hours, from 7.00 am to 7.00 pm.

The random variable H is the number of helicopters that pass within sight of the building during a randomly chosen daytime period of 12 hours.

- (a) State **two** assumptions needed for H to be well modelled by a Poisson distribution. [2]

Assume now that H can be well modelled by the distribution $Po(1.8)$.

- (b) Find the probability that, during a randomly chosen daytime period of 12 hours, the number of helicopters that pass within sight of the building is 2, 3 or 4. [2]
- (c) During a period of t hours (within the daytime period of 12 hours), the probability that no helicopters pass within sight of the building is 0.95.

Use an algebraic method to determine the value of t . [3]

- (d) During a randomly chosen daytime period of t hours (where t is the value found in part (c)) two helicopters pass within sight of the building.

Explain whether this casts doubt on the validity of the model. You do not need to carry out any calculations. [1]

Throughout the daytime period of 12 hours on any day, aeroplanes pass within sight of the building at a fixed constant rate of 1 aeroplane every 4 minutes.

- (e) Explain whether the number of aeroplanes that pass within sight of the building on any day is likely to be well modelled by a Poisson distribution. [1]
- (f) Find the expected value of the total number of helicopters and aeroplanes that pass within sight of the building on a randomly chosen day. [1]

- 3** In a science experiment, the values of a variable W are selected by the scientist. The experiment involves measuring the corresponding values of the variable V . The results are summarised as follows.

$$n = 10 \quad \Sigma v = 176.2 \quad \Sigma w = 87.5 \quad \Sigma v^2 = 4077.64 \quad \Sigma w^2 = 951.25 \quad \Sigma vw = 1963.4$$

- (a) Calculate the value of the product-moment correlation coefficient. [2]
- (b) By identifying the controlled variable in the experiment, calculate the equation of the appropriate least squares regression line. [3]

It is required to obtain an estimate of the value of the response variable when the value of the controlled variable is 10.0.

- (c) (i) Find the value of the required estimate. [1]
- (ii) Use the appropriate mean and standard deviation to determine whether 10.0 is likely to be within the range of the data for the controlled variable. [2]
- (iii) Explain whether the estimate found in part (c)(i) is likely to be reliable. [2]
- (d) Explain what is meant by the words “least squares” in the phrase “least squares regression line”. [1]

- 4** The probability distribution of a discrete random variable U is given in the table.

u	0	1	2	3
$P(U = u)$	r	0.3	p	$0.5 - p$

- (a) Find the value of the constant r . [1]
- (b) Determine an expression for $\text{Var}(U)$ in terms of p . Give your answer in the form $a - (p + b)^2$, where a and b are constants. [4]
- (c) Use your expression in part (b) to determine the smallest possible value of $\text{Var}(U)$. [3]
- (d) Find the value of $E(U)$ that corresponds to the smallest possible value of $\text{Var}(U)$. [1]

Section B – Mechanics

- 5 A cricket ball of mass 0.16 kg is at rest on rough horizontal ground when it is struck by a bat. Immediately after the ball is struck its speed is 15 m s^{-1} in a horizontal direction.

(a) Find the magnitude of the impulse exerted by the bat on the ball. [1]

The ball remains in contact with the ground and the resistance to the motion of the ball is modelled as constant.

The ball comes to rest again after it has travelled 30 m along the ground.

(b) Use the work-energy principle to find the magnitude of the resistance to the motion of the ball. [3]

- 6 A car of mass 900 kg has an engine that is capable of working at a maximum power of 12 kW .

When the car is travelling along a straight horizontal road, the maximum constant speed that it can maintain is 40 m s^{-1} .

In an initial model of the motion of the car it is assumed that the total resistance to motion is constant.

(a) (i) Show that when the car is travelling at 15 m s^{-1} along the horizontal road with the engine working at a constant rate of 10 kW , the acceleration of the car is 0.407 m s^{-2} correct to 3 significant figures. [3]

(ii) Find the speed of the car when it is accelerating at 0.2 m s^{-2} along the horizontal road with the engine working at a constant rate of 9.6 kW . [2]

(iii) The car now starts to **descend** a straight road which is inclined at 1° to the horizontal.

Find the power produced by the engine of the car when it is travelling along this road at a constant speed of 40 m s^{-1} . [2]

In a refined model of the motion of the car it is assumed that the total resistance to motion is proportional to the speed of the car.

(b) (i) Show that the total resistance to the motion of the car is now given by $7.5v\text{ N}$ where $v\text{ m s}^{-1}$ is the speed of the car. [1]

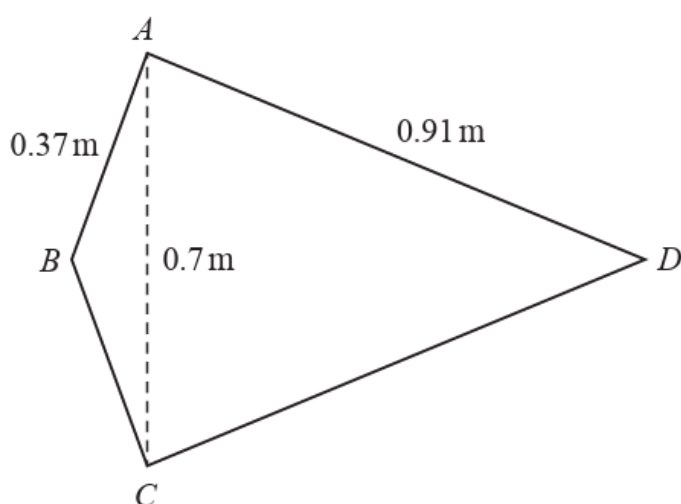
(ii) State **one** way in which the refined model is an improvement over the initial model. [1]

- 7 Two particles, A of mass 4 kg and B of mass 2 kg , are free to move along the same straight line on a smooth horizontal surface. They are projected directly **towards** each other with speeds of 6 ms^{-1} and 5 ms^{-1} respectively.

After A and B collide the speed of A is 1 ms^{-1} .

- (a) Determine the possible values of the coefficient of restitution between A and B . [6]
- (b) State whether the collision between A and B is perfectly elastic. Justify your answer. [1]

- 8 $ABCD$ is a uniform lamina in the shape of a kite with $BA = BC = 0.37\text{ m}$, $DA = DC = 0.91\text{ m}$ and $AC = 0.7\text{ m}$ (see diagram). The centre of mass of $ABCD$ is G .



- (a) Explain why G lies on BD . [1]
- (b) Show that the distance of G from B is 0.36 m . [4]

The lamina $ABCD$ is freely suspended from the point A .

- (c) Determine the acute angle that CD makes with the horizontal, stating which of C or D is higher. [4]

- 9** A body B of mass 1.5 kg is moving along the x -axis. At the instant that it is at the origin, O , its velocity is $u \text{ ms}^{-1}$ in the positive x -direction.

At any instant, the resistance to the motion of B is modelled as being directly proportional to v^2 where $v \text{ ms}^{-1}$ is the velocity of B at that instant. The resistance to motion is the only horizontal force acting on B .

At an instant when B 's velocity is 2 ms^{-1} , the resistance to its motion is 24 N .

- (a)** Show that B 's motion can be modelled by the differential equation $\frac{1}{v} \frac{dv}{dx} = -4$. **[3]**
- (b) (i)** Solve the differential equation in part **(a)** to find the particular solution for v in terms of x and u . **[4]**
- (ii)** By considering the behaviour of v as $x \rightarrow \infty$ describe **one** feature of the model that is not realistic. **[1]**

END OF QUESTION PAPER