A Kernel-Based Approach to Non-Stationary Reinforcement Learning in Metric Spaces

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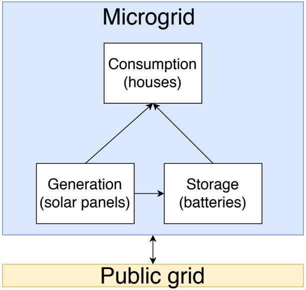
Motivation

Real word applications:

- Continuous states and actions
- Environment may change over time

Microgrid management:

- Electrical network with energy generation, storage and consumption
- Can buy energy from an external source (public grid)
- Goal: minimize cost

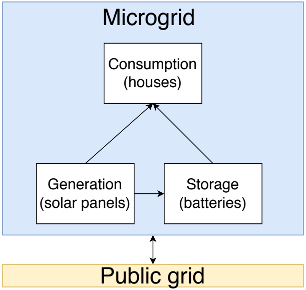


Motivation

State: [production, consumption, batteries] $\in \mathbb{R}^d$
Actions: [charge, discharge] the batteries
Reward: $-1 \times$(cost of generating/buying energy)

Non-stationary behavior:

- Fluctuations of the energy price
- New consumer in the network
- Degradation of batteries



Model

Episodic RL: K episodes of horizon H

Non-stationary MDP:

At time (k, h):

- State $x_h^k \in \mathcal{X}$
- Action $a_h^k \in \mathcal{A}$
- Reward $\widetilde{r}_h^k \in [0,1]$ with mean $r_h^k(x_h^k, a_h^k)$
- Next state $x_{h+1}^k \sim \mathrm{P}_h^k(\cdot | x_h^k, a_h^k)$

Optimal value functions in episode k:

$$\mathrm{Q}_{k,h}^*(x,a) = r_h^k(x,a) + \mathrm{P}_h^k \mathrm{V}_{k,h+1}^*(x,a)$$
$$\mathrm{V}_{k,h}^*(x) = \max_{a \in \mathcal{A}} \mathrm{Q}_{k,h}^*(x,a)$$

Goal & Regularity Assumptions

Goal: minimize the dynamic regret

$$\underbrace{\mathcal{R}(K)}_{\text{regret}} = \sum_{k=1}^{K} \left(\underbrace{V^*_{1,k}(x_1)}_{\text{value of an optimal agent}} - \underbrace{V^{\pi_k}_{1,k}(x_1)}_{\text{value of the algorithm}} \right)$$

Assumption: Lipschitz continuity w.r.t. a known metric ρ:

$$\left| r_h^k(x,a) - r_h^k(x',a') \right| \leq L_{\mathrm{r}} \rho \left[(x,a),(x',a') \right]$$

$$\underbrace{\mathbb{W}_1 \left(\mathrm{P}_h^k(\cdot|x,a), \mathrm{P}_h^k(\cdot|x',a') \right)}_{\text{1-Wasserstein distance}} \leq L_{\mathrm{p}} \rho \left[(x,a),(x',a') \right]$$

$\Longrightarrow$ optimal Q-functions are L-Lipschitz

Kernel-Based RL (KBRL)

Non-parametric model of the MDP:

$$\widehat{r}_h^k(x,a) = \frac{\sum_{s=1}^{k-1} w_h^{k,s}(x,a)\, \widetilde{r}_h^s}{\beta + \sum_{s=1}^{k-1} w_h^{k,s}(x,a)}, \quad \widehat{P}_h^k(y|x,a) = \frac{\sum_{s=1}^{k-1} w_h^{k,s}(x,a)\, \delta_{x_{h+1}^s}(y)}{\beta + \sum_{s=1}^{k-1} w_h^{k,s}(x,a)}$$

$w_h^{k,s}(x,a)$: similarity between (x,a) and (x_h^s, a_h^s) in episode k

- Depends on the distance $\rho\left[(x,a),(x_h^s,a_h^s)\right]$
- Depends on the time interval $(k-s)$

KBRL for stationary MDPs:

$\rightarrow$ First studied by [Ormoneit and Sen, 2002] (offline)

$\rightarrow$ Regret bounds provided by [Darwiche Domingues et al., 2020] (online)

Examples of similarity measures

$\{$sliding window, exponential discount$\} \times$ Gaussian kernel

sliding window [Garivier and Moulines, 2011, Gajane et al., 2018, Cheung et al., 2019]

$$w_h^{k,s}(x,a) = \mathbb{I}\{s \geq k - W\} \exp\left(-\frac{\rho[(x,a),(x_h^s,a_h^s)]^2}{2\sigma^2}\right)$$

discount [Kocsis and Szepesvári, 2006, Garivier and Moulines, 2011, Russac et al., 2019]

$$w_h^{k,s}(x,a) = \eta^{k-s-1} \exp\left(-\frac{\rho[(x,a),(x_h^s,a_h^s)]^2}{2\sigma^2}\right)$$

Space- and Time-Dependent Kernels

Kernel function $\overline{\Gamma}_{\eta,W} : \mathbb{N} \times \mathbb{R} \to [0,1]$ such that:

$$w_h^{k,s}(x,a) = \overline{\Gamma}_{\eta,W}\left(\underbrace{k-s-1}_{\text{time interval}}, \underbrace{\rho\left[(x,a),(x_h^s,a_h^s)\right]/\sigma}_{\text{distance}}\right)$$

σ: kernel bandwidth, η: discount parameter, W: window parameter

Assumptions:

- $z \mapsto \overline{\Gamma}_{\eta,W}(t,z)$ is Lipschitz continuous
- $\overline{\Gamma}_{\eta,W}(t,z) \leq c_1 \exp(-z^2/2)$
- $\overline{\Gamma}_{\eta,W}(t,z) \leq c_2 \eta^t$ if $t \geq W$: $\rightarrow$ forget old data (bias)
- $\overline{\Gamma}_{\eta,W}(t,z) \geq G(z)\eta^t$ if $t < W$: $\rightarrow$ remember recent data (variance)

Algorithm

KeRNS

For all state-action pairs (x_h^s, a_h^s) that were seen before, compute:

$$\textbf{(``training'')} \quad \widetilde{Q}_h^k(x_h^s, a_h^s) = \widehat{r}_h^k(x_h^s, a_h^s) + \widehat{P}_h^k V_{h+1}^k(x_h^s, a_h^s) + \mathtt{B}_h^k(x_h^s, a_h^s)$$

For any new state-action pair (x, a), define:

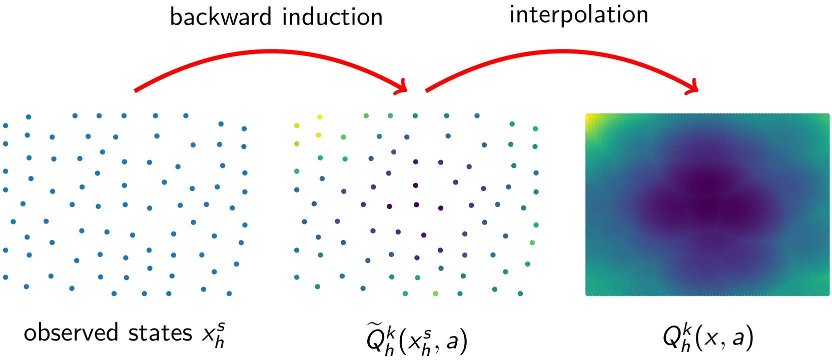
$$\textbf{(generalizing)} \quad Q_h^k(x, a) \stackrel{\text{def}}{=} \min_{s \in [k-1]} \left[\widetilde{Q}_h^k(x_h^s, a_h^s) + L\rho\left[(x, a), (x_h^s, a_h^s)\right] \right]$$

Play the policy: **(acting)** $\pi^k(h, x) \in \operatorname{argmax}_a Q_h^k(x, a)$

Exploration Bonus: $\mathtt{B}_h^k(x, a) \approx H/\sqrt{\mathbf{C}_h^k(x, a)} + \beta H/\mathbf{C}_h^k(x, a) + L\sigma$

$$\mathbf{C}_h^k(x, a) \stackrel{\text{def}}{=} \beta + \sum_{s=1}^{k-1} w_h^{k,s}(x, a) \approx \text{ number of recent visits to } (x, a)$$

Algorithm


backward induction
interpolation
observed states x_h^s
$\widetilde{Q}_h^k(x_h^s, a)$
$Q_h^k(x, a)$

Regret Bound

Variation of the MDP: $\Delta = \Delta^{\mathrm{r}} + L\Delta^{\mathrm{p}}$, where

$$\Delta^{\mathrm{r}} = \sum_{i,h} \sup_{x,a} \left| r_h^i(x,a) - r_h^{i+1}(x,a) \right|, \quad \Delta^{\mathrm{p}} = \sum_{i,h} \sup_{x,a} \mathbb{W}_1 \left(\mathrm{P}_h^i(\cdot|x,a), \mathrm{P}_h^{i+1}(\cdot|x,a) \right)$$

With probability at least $1-\delta$, the regret of KeRNS is bounded by $\approx$

$$\underbrace{H^2 K \sqrt{\log\left(\frac{1}{\eta}\right)} \sqrt{|\mathcal{C}'_\sigma| \, |\mathcal{C}_\sigma|}}_{\text{sum of exploration bonuses}} + \underbrace{HW\Delta + \frac{\eta^W}{1-\eta} KH^3}_{\text{bias due to changes}} + \underbrace{LKH\sigma}_{\text{bias due to smoothing}}$$

$\rightarrow |\mathcal{C}_\sigma|, |\mathcal{C}'_\sigma|$: σ-covering numbers of $\mathcal{X} \times \mathcal{A}$ and $\mathcal{X}$

Regret Bound

Tuning the kernel parameters to minimize the regret:

$$\log(1/\eta) = \Delta^{\frac{2}{3}} K^{-\frac{2d+2}{2d+3}}, \quad W = \log_\eta((1-\eta)/K), \quad \sigma = K^{-\frac{1}{2d+3}}$$

gives a regret bound

$$\mathcal{R}(K) \lesssim H^2 \Delta^{\frac{1}{3}} K^{\frac{2d+2}{2d+3}}$$

where $d =$ covering dimension of $\mathcal{X} \times \mathcal{A}$.

Requirement for sublinear regret: $\Delta < K^{3/(2d+3)}$

$\rightarrow$ Another analysis gives a bound $\mathcal{R}(K) \lesssim H^2 \Delta^{\frac{1}{2}} K^{\frac{2d+1}{2d+2}}$ (for $d > 0$)

$\rightarrow$ Optimal choice of parameters requires an upper bound on Δ

$\rightarrow$ For $d = 0$, matches the lower bound for bandits in terms of Δ and K [Besbes et al., 2014]

Conclusion & Perspectives

+ dynamic regret bounds for continuous non-stationary MDPs
+ flexible and easy to implement
+ weak assumptions on the MDP

– backward induction is slow
– curse of dimensionality

◦ lower bounds for $d > 0$?
◦ what if we don't know (an upper bound on) Δ?
◦ can we use these kernel-based bonuses in deep RL?

Thank you!

Questions?

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Runtime

KeRNS requires a backward induction over (x_h^s, a_h^s) for $s = 1, \ldots, k-1$
$\implies \mathcal{O}\left(k^2\right)$ time per episode k

Idea: use m representative state-action pairs (x_i, a_i) instead!
[Kveton and Theocharous, 2012, Barreto et al., 2016]
$\implies$ constant time per episode: $\mathcal{O}\left(m^2\right)$

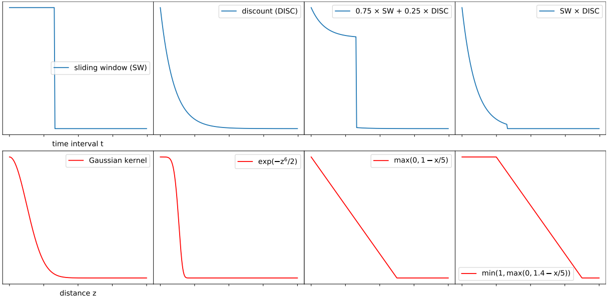
Impact on the regret: additive term $(\varepsilon/\sigma)H^3 K$, where

$$\varepsilon = \max_{s,h} \min_i \rho\left[(x_h^s, a_h^s), (x_i, a_i)\right]$$

and assuming a Gaussian kernel.

Examples of kernels

Simplest examples: time-dependent kernel $\times$ space-dependent kernel

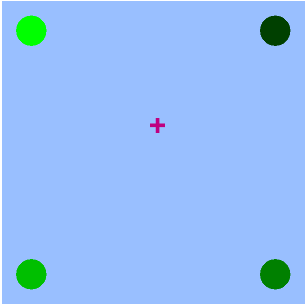


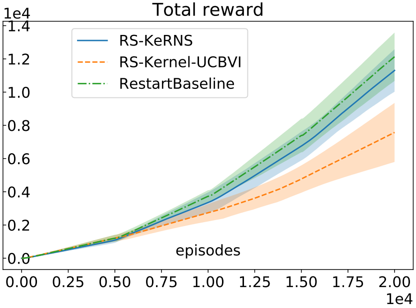
More generally, the kernel design can be adapted to the problem, e.g. :

- If the MDP changes only in certain regions, we can choose a kernel that forgets data only in those regions.
- If we know that a task is appearing periodically, we might use a periodic time-dependent kernel.

Experiments

`RS-KeRNS` = `KeRNS` on Representative States





Environment: continuous GridWorld, rewards change in the corners.