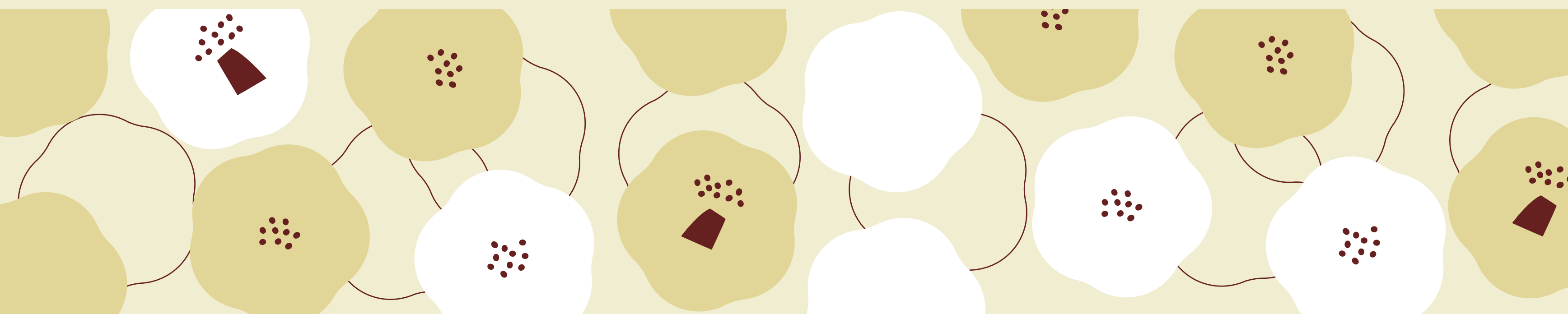

0. Utilizando el algoritmo de Euclides, encuentre x, y que satisfacen $(27, 15) = 27x + 15y$

Dado $(27, 15)$

$$27 = 15 \cdot 1 + 12$$
$$15 = 12 \cdot 1 + 3$$
$$12 = 3 \cdot 4 + 0$$

$$3 = 15 - 12$$
$$3 = 15 - (27 - 15)$$
$$3 = 2 \cdot 15 - 27$$
$$3 = 2 \cdot 15 - 1 \cdot 27$$

Tenemos $x = -1$, $y = 2$

1. En $(\mathbb{Z}_{27}, \oplus)$, encuentre<18>.

$\langle 18 \rangle$

$$18^0 = e = 0$$
$$18^1 = 18 \bmod 27 = 18$$
$$18^2 = 36 \bmod 27 = 9$$
$$18^3 = 54 \bmod 27 = 0$$

Por lo tanto $\langle 18 \rangle = \{0, 18, 9\}$

$$o(18) = 3$$

$$o(x) = x^n$$

2. En $(\mathbb{Z}_{27}, \oplus)$, encuentre el orden de los elementos 3, 9 y 21.

$$o(3) = 3^9 = 9 \cdot 3 \bmod 27$$
$$= 27 \bmod 27 = 0$$

$$o(9) = 9^3 = 3 \cdot 9 \bmod 27$$
$$= 27 \bmod 27 = 0$$

$$o(12) = 12^9 = 9 \cdot 12 \bmod 27$$
$$= 108 \bmod 27 = 0$$

3. Liste los elementos de $(\mathbb{Z}_{27}, \oplus)$, que tienen orden 9.

Teorema

Dado $g \in G$, si $o(g) = n$ y $(m, n) = d$, entonces $o(g^m) = n/d$

$$\{x \in \mathbb{Z}_{27} : o(x) = 9\}$$

$$9 = \frac{27}{(m, 27)} \Rightarrow (m, 27) = d \qquad \boxed{d = 3}$$

Dado que $d = 3$, entonces m es múltiplo de 3

$$o(3) = 3^9 = 3 \cdot 9 \bmod 27 = 0 \checkmark$$
$$o(6) = 6^9 = 6 \cdot 9 \bmod 27 = 0 \checkmark$$

$o(9) = 9^3 = 3 \cdot 9 \mod 27 = 0$ ✗

$o(12) = 12^9 = 12 \cdot 9 \mod 27 = 0$ ✓

$o(15) = 15^9 = 15 \cdot 9 \mod 27 = 0$ ✓

$o(18) = 18^3 = 18 \cdot 3 \mod 27 = 0$ ✗

$o(21) = 21^9 = 21 \cdot 9 \mod 27 = 0$ ✓

$o(24) = 24^9 = 24 \cdot 9 \mod 27 = 0$ ✓

Los elementos que cumple $(m, 27) = 3$ son

$6, 12, 15, 21, 24, 3$