QFGB8960 Advanced C++ for Finance
Homework 2

Spring 2025

Problem 1 (50 points) Quanto Forward Price

A quanto forward is a forward contract on a foreign asset with a contractually fixed exchange rate. Suppose that the underlying asset is an equity traded in London in GBP (British pounds). A US-based investor that is long a quanto forward expiring at T will receive/pay in USD

$$\bar{Q}_{\text{FIX}} \cdot (S(T) - K),$$

where $S(T)$ is the price of the equity in GBP, K is the strike in GBP, and $\bar{Q}_{\text{FIX}}$ is the fixed price of 1 GBP in USD. The exchange rate is fixed at the inception of the contact. The intention of fixing the FX rate is to make the contract value at expiration not depend on the prevailing FX rate $\bar{Q}(T)$, or its inverse $Q(T) = 1/\bar{Q}(T)$, which is the price of one USD in GBP. It turns out that by fixing the FX rate to $\bar{Q}_{FIX}$, the quanto forward price F^Q becomes both volatility and correlation dependent. The formula is

$$F^Q(t,T) = e^{\left(r_f - q + \rho\sigma_S\sigma_Q\right)(T-t)}\bar{Q}_{\text{FIX}}S(t) = e^{\rho\sigma_S\sigma_Q(T-t)}\bar{Q}_{\text{FIX}}F(t,T), \tag{1}$$

where r_f is the foreign risk free rate, σ_S is the volatility of the equity S, σ_Q is the volatility of the FX rate Q, and ρ is the correlation of returns of S and Q. $F(t,T)$ is the regular forward price of the foreign asset in its own (foreign) currency.

(a) Implement a C++ function `qf::quantoFwdPrice` and expose it to Python as `qf.qFwdPrice`. The function computes the quanto forward price as in eq. (1) using the following inputs

1. spot
2. time to expiration
3. risk free rate (foreign)
4. dividend yield (of asset)

5. asset vol
6. FX vol
7. asset-FX correlation

The fixed FX rate $\bar{Q}_{\text{FIX}}$ is assumed to be 1. Time to expiration is measured in years and all rates and volatilities are annualized (per year).

The function should be declared in `qflib/pricers/simplepricers.hpp` and implemented in `qflib/pricers/simplepricers.cpp`. Make sure that you validate the input arguments. Parts of the calculations for the quanto forward are already implemented in the function `fwdPrice`. Call this function instead of copy-pasting the code.

(b) Create a Python notebook where you price a forward contract on XYZ (Japanese equity), which is traded in JPY (Japanese yen), quanto-ed into USD (US dollars). This would be relevant for a US-based investor that wants exposure to the Japanese equity, but does not want exposure to the USDJPY FX rate.

The quanto forward parameters are:

1. XYZ Spot: 100 USD (the spot can be set in USD equivalent terms)
2. Time to expiration: 1 year
3. Japanese risk free rate: 4% p.a.
4. XYZ dividend yield: 1% p.a.
5. XYZ volatility: 40% p.a.
6. USDJPY volatility: 10% p.a.
7. XYZ-USDJPY correlation: -1.0, -0.9, -0.8, ..., 0.8, 0.9, 1.0

Plot the value of the quanto forward as a function of the equity-FX correlation. Is it increasing or decreasing? Explain in economic terms why. Write your explanation in the notebook.

Problem 2 (50 points) Forward Price from Simulation

Use the numpy function `random.random_sample` to generate 10,000 random numbers in the interval $(0, 1)$ and the Python function `qf.normalInvCdf` to transform them to standard normal random numbers. Then use eq. (23) in the lecture notes to generate 10,000 samples of the spot price $S(T)$ given the following:

1. Initial spot $S(t = 0) = 100$

2. Time $T = 1$ yr
3. Risk-free rate $r = 4\%$ p.a.
4. Dividend yield $q = 1\%$ p.a.
5. Volatility $\sigma = 40\%$ p.a.

Compute numerically the mean of the sample and compare it with the exact answer given by eq. (25) in the lecture notes. Report the difference as a multiple of the standard error of the sample mean (standard deviation / $\sqrt{\mathrm{nsamples}}$).

Provide your answer on the same notebook as in problem 1.