# QFGB8960 Advanced C++ for Finance
# Homework 3

Spring 2025

## Problem 1 (30 points) Knockout Forward Contract

Repeat the derivation of the time-$t$ value of the single point knockout forward contract in the lecture (eq. 16-17), but using non-zero dividend yield and interest rate. The forward has strike $K$ and expires at time $T$. It may knock out if the spot at time $T_1$ falls below level $H$. Express the result in terms of a European call and a digital call, both with strike $H$ and expiration $T_1$. Use the relation

$$\mathbb{E}_{T_1}\left[S\left(T\right)\right] = e^{(r-q)(T-T_1)} S\left(T_1\right). \tag{1}$$

Since the forward contract is settled at expiration $T$, do not forget to apply discount factor $e^{-r(T-t)}$.

## Problem 2 (40 points) Knockout Forward Implementation

Implement a C++ function `orf::knockoutFwd` and expose it to Python as `qf.koFwd`. The function computes the present value of a single point knockout forward contract using the Black-Scholes formula.The inputs should be

1. Spot $(S)$
2. Strike $(K)$
3. KOLevel $(H)$
4. TimeToExp (in years) $(T)$
5. TimeToKO (in years) $(T_1)$
6. IntRate (cont cmpd) $(r)$
7. DivYield (cont cmpd) $(q)$

8. Vol (annualized) ($\sigma$)

Create a Python notebook that uses the `qf.koFwd` function to price a single point knockout forward contract. Fix the spot and strike at $S = K = 100$, and vary the knockout level $H$ in the range $[0, 150]$ stepping by 10. Set all other parameters as follows: expiration $T = 1$ yr, time to knock out $T_1 = 1/2$ yr, dividend yield $q = 2\%$, interest rate $r = 4\%$ and volatility $\sigma = 40\%$. Plot the PV of the knockout forward as a function of the level $H$. Compute the present value of the standard forward for the same inputs and comment on your plot.

## Problem 3 (30 points) Capped Forward Contract

A standard forward contract pays at maturity $T$ the difference between the time-$T$ spot price and the strike $K_1$.

A forward contract capped at $K_2$, with $K_2 > K_1$ pays at maturity $T$ the minimum of $K_2$ and the difference between the time-$T$ spot price and the strike $K_1$.

- Standard forward payoff: $S_T - K_1$
- Capped forward payoff: $\min(K_2, S_T - K_1)$

(a) Using the payoff decomposition formulae in the lecture notes, express the capped forward contract in terms of fixed cash payments and European options.

(b) On a Python notebook price the standard forward contact with strike $K_1$ using the function `qf.fwdPrice`, and the capped forward contract using the function `qf.euroBS`.

The standard and capped forward contract parameters are

1. Spot: from 50 to 250 step by 10
2. Time to expiration: 2 years
3. Strike $K_1$: 100
4. Cap $K_2$: 110
5. Risk-free rate: 4% p.a. (continuous compounding)
6. Dividend yield: 2% p.a. (continuous compounding)
7. Volatility: 25% p.a

Recall that the function `qf.fwdPrice` returns the undiscounted price, but the option pricing functions return discounted prices.

Plot the PV (value as of time 0) of the standard and capped forward contracts as a function of the spot for the range of spot values above.

Reprice the capped forward contract keeping all parameters the same as above except change the volatility to 50%. Add another curve to your plot with the capped forward PV at 50% volatility. What is the effect of volatility on the value of the capped forward? Give an economic argument why.

Provide your answer on the same notebook as in problem 1.