QFGB8960 Advanced C++ for Finance
Homework 4

Spring 2025

Problem 1 (30 points) Vega of a Digital

Compute analytically the Vega of a digital call and put option in the Black-Scholes model.

Now consider a short digital call position with strike K and expiration T. The forward price of the underlying is F. Under what condition is the Vega of the short call position positive? Answer the same question for a short digital put position.

Problem 2 (50 points) Calculating Black-Scholes Greeks

Version 0.3.0 of qflib contains the C++ functions `qf::digitalOptionBS` and `qf::europeanOptionBS` and exposes them to Python as `qf.digiBS` and `qf.euroBS` respectively. Modify these C++ functions by making them return an `qf::Vector` instead of a `double`. The vector should contain the following in order: Price, Delta, Gamma, Theta and Vega. The calculations of the Greeks are defined in the lecture notes, in equations (8), (10), (12), (13), (14), (18) and in the previous exercise. Derive the Theta of the digital call and put and add it to the implementation.

Modify the Python functions to return a 1-D numpy array with the above Greeks. This can be done using the `asNumpy` utility function defined in the file pyqflib/pyutils.hpp.

Problem 3 (20 points) Checking the Black-Scholes Equation

As we saw in equation (20) in the lecture notes, the Black-Scholes equation is a statement about the balance of Greeks.

Use the function `qf.euroBS` from the previous exercise to verify this as follows.

In a Python notebook create a Pandas data frame with the column names below.

Payoff	Spot	Strike	TimeToExp	IntRate	DivYield	Volatility	Price	Delta	Gamma	Theta	Diff
1	110.0	100.0	0.75	0.04	0.00	0.30					
-1	95.0	100.0	0.50	0.04	0.03	0.40					
...	...	...	...	...	...						

Choose ten different random combinations of Payoff, Spot, Strike, ..., Volatility. Use the Python function `qf.euroBS` to calculate Price, Delta, Gamma and Theta. Use the returned values of this function to fill in the coresponding columns of the above data frame. In the last column "Diff" put the result of the expression below:

$$\Theta + (r - q)S\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma - rV \tag{1}$$

Show that for all your combinations, the "Diff" column is always zero to high precision.

Repeat the above process using the function `qf.digiBS`, use the same ten random combinations of Payoff, Spot, Strike, ..., Volatility and check the Black Scholes equation with digital options.

- Put your Python notebook in the folder qflib-0.4.0/examples/Python.