Pricing Quanto Options

QFGB8960 Advanced C++ for Finance

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April 30, 2025

Quanto Options - Overview

- **Definition**: Cross-currency derivatives that provide exposure to foreign asset performance with fixed domestic currency payoff
- **Key feature**: Eliminates currency risk while maintaining foreign market exposure
- **Payoff**: $(\phi(S(T) - K))^+$ units of domestic currency
  - $\phi = 1$ for call and $\phi = -1$ for put
- **Challenge**: Requires adjustment to standard Black-Scholes to account for asset-FX correlation

Theoretical Framework and Implementation

- **Quanto-adjusted drift**: $r_f - q + \rho\sigma_S\sigma_Q$
- **Quanto forward**: $F^Q(t,T) = e^{(r_f - q + \rho\sigma_S\sigma_Q)(T-t)}S(t)$
- **Modified dividend yield**: $q' = r_d - r_f + q - \rho\sigma_S\sigma_Q$
- **Intuition**: Correlation between asset and exchange rate affects effective growth rate
  - $\rho > 0$: increases drift
  - $\rho < 0$: reduces drift
- **Analytical Solution**:
  - Modified Black-Scholes with quanto drift
  - `quantoEuropeanOptionBS` $\rightarrow$ `qf.qEuroBS`
- **Monte Carlo**:
  - `BsMcQuantoPricer` extends `BsMcPricer`
  - Python: `qf.qEuroBSMC`
- **PDE Method**:
  - `Pde1DSolver` with quanto drift
  - Python: `qf.qEuroBSPDE`

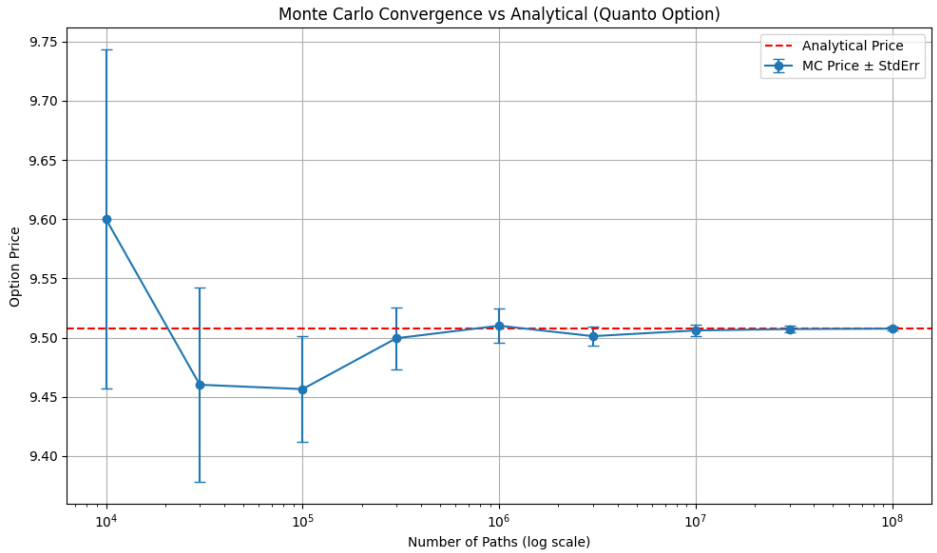
Verification of Methods

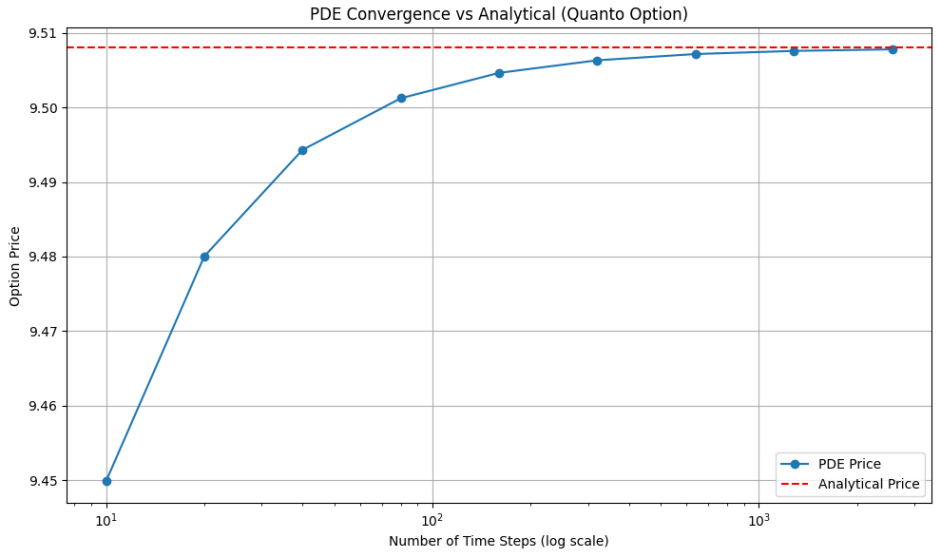
Test Parameters:

- Spot: 100, Strike: 100
- Time to exp: 1 year
- Asset vol: 0.2, FX vol: 0.15
- Correlation: 0.5
- Domestic rate at exp: 0.02
- Foreign rate at exp: 0.025

Results:

- Analytical: 9.50800966
- Monte Carlo (30M paths): 9.50758241 (Rel. Error: 4.49e-05)
- PDE (25600 nodes): 9.50780061 (Rel. Error: 2.20e-05)





Quanto Call-Put Parity

- **Theoretical relationship**:

$$\text{Call}_{\text{quanto}}(K,T) - \text{Put}_{\text{quanto}}(K,T) = e^{-r_d T} \cdot \left(S_0 \cdot e^{(r_f - q + \rho\sigma_S\sigma_Q)T} - K\right)$$

- **Verification results**:
  - Tested across multiple strikes (80–120) and maturities (0.5–2.0 years) using analytical approach (qf.qEuroBS)
  - Maximum relative error: $\sim 10^{-15}$
  - Confirms theoretical consistency of implementation
- Compared with standard call-put parity: $\text{Call} - \text{Put} = e^{-rT} \cdot (S_0 e^{(r-q)T} - K)$

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=== Quanto Put-Call Parity Verification  ===

=== Maturity (T) - 0.5 year(s) ===

   Strike                LHS                RHS          Abs Error          Rel Error
--------------------------------------------------------------------------------------
    80.00        21.29726539        21.29726539           1.07e-14           5.00e-16
    90.00        11.39676705        11.39676705           0.00e+00           0.00e+00
   100.00         1.49626871         1.49626871           6.88e-15           4.60e-15
   110.00        -8.40422963        -8.40422963           5.33e-15           6.34e-16
   120.00       -18.30472796       -18.30472796           0.00e+00           0.00e+00


=== Maturity (T) - 1.0 year(s) ===

   Strike                LHS                RHS          Abs Error          Rel Error
--------------------------------------------------------------------------------------
    80.00        22.58912284        22.58912284           0.00e+00           0.00e+00
    90.00        12.78713611        12.78713611           7.11e-15           5.56e-16
   100.00         2.98514938         2.98514938           0.00e+00           0.00e+00
   110.00        -6.81683736        -6.81683736           0.00e+00           0.00e+00
   120.00       -16.61882409       -16.61882409           0.00e+00           0.00e+00


=== Maturity (T) - 2.0 year(s) ===

   Strike                LHS                RHS          Abs Error          Rel Error
--------------------------------------------------------------------------------------
    80.00        25.15697887        25.15697887           3.55e-15           1.41e-16
    90.00        15.54908448        15.54908448           1.07e-14           6.85e-16
   100.00         5.94119009         5.94119009           7.11e-15           1.20e-15
   110.00        -3.66670430        -3.66670430           8.44e-15           2.30e-15
   120.00       -13.27459870       -13.27459870           2.31e-14           1.74e-15
```

Fig. Put-Call Parity Verification

Impact of Correlation

Key findings:

- Correlation significantly affects quanto option prices
- Quanto call prices increase with correlation
- Quanto put prices decrease with correlation

Economic interpretation:

- $\rho > 0$: Strengthening FX with an increase in asset price
  - Benefits call options, reduces put value
- $\rho < 0$: Weakening FX with increasing asset price
  - Reduces call value, benefits put options

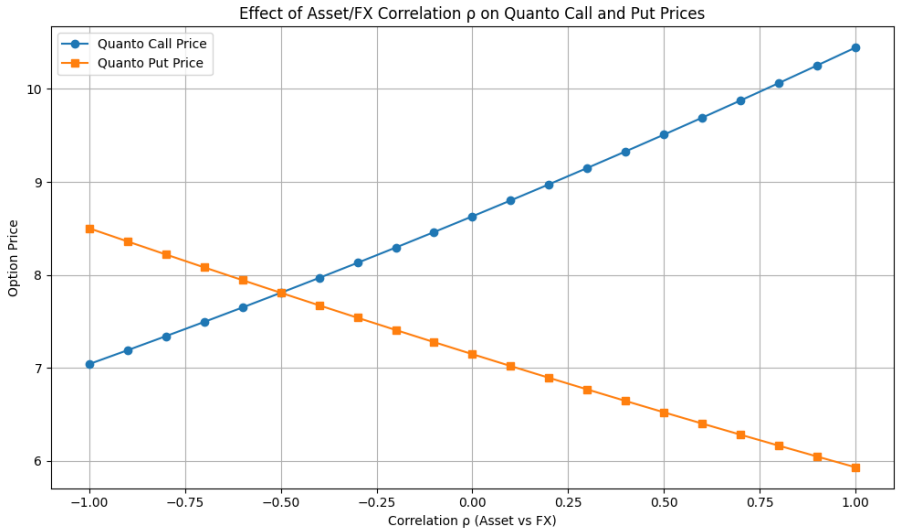

Fig. Effect of correlation

Mathematical Impact of Correlation

- **Drift Adjustment**: Correlation affects option pricing through the drift adjustment term $\rho\sigma_S\sigma_Q$
- **Effect on Forward Price**:

$$F^Q(t,T) = S_0 \cdot e^{(r_f - q + \rho\sigma_S\sigma_Q)T} \tag{1}$$

$$= F^{std}(t,T) \cdot e^{\rho\sigma_S\sigma_Q \cdot T} \tag{2}$$

  where F^{std} is the standard forward price

- **Shift in Probability Measure**:
  - Positive ρ shifts the forward price upward (favoring calls)
  - Negative ρ shifts the forward price downward (favoring puts)

Summary & References

Summary:

- Successfully implemented three pricing methods for quanto options
- Verified consistency across methods with high numerical accuracy
- Confirmed theoretical properties (call-put parity)
- Analyzed correlation effects on pricing

References:

1. Hull, J. C. (2018). *Options, Futures, and Other Derivatives*. Pearson.
2. Wilmott, P. (2006). *Paul Wilmott on Quantitative Finance*. Wiley.
3. Shreve, S. E. (2004). *Stochastic Calculus for Finance II: Continuous-Time Models*. Springer.