

# MACHINE LEARNING APPROACH TO CHIRP RATE ESTIMATION OF LINEAR FREQUENCY MODULATED RADARS

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## Abstract

The detection and parametric estimation of low-SNR radar signals, particularly linear frequency modulated (LFM) radar signals, is a problem of considerable interest. In prior work, this problem has been investigated using various signal processing techniques, such as maximum likelihood estimation, fractional Fourier transform and Wigner-Ville-based methods, to analyze the signal parameters of a complex linear frequency modulated signal. Other work has focused on applying deep learning to automatically recognize various radar waveform types and their features, such as linear frequency modulation (LFM), Barker code and rectangular waveforms. In this paper, we investigate this problem from a machine learning perspective for multiple LFM radar signals given a priori information. We explore the use of naive Bayes, support vector machine and neural network classifiers to identify the LFM chirp rate, out of a set of known chirp rates, from a specific radar emitter under varying SNR conditions. Simulation results demonstrate the viability of this technique to identify the radar LFM mode in very low signal-to-noise ratio conditions down to -20 dB where using existing approaches (e.g., Wigner-Ville) fail.

Index Terms - Linear frequency modulation, chirp, radar

## I. Introduction

Radar receivers are designed using a matched filter which maximizes the output peak signal to mean noise power ratio [1-3]. A matched filter is the optimum linear filter for detecting a known signal in the presence of noise. The detection and parametric estimation of linear frequency modulated (LFM) signals in particular has been an issue in the radar domain, but approaches based on

matched filtering have not been well explored. Many current approaches are based on the sampling theorem and fractional broadening [4]. Synthetic aperture radars, commonly found on airborne and spaceborne platforms, typically use LFM radar signals. The detection and parametric estimation of the LFM signal has been a long standing problem in the radar domain. The main methods used for parametric estimation include maximum likelihood estimation, dechirp parameter estimation, Wigner-Ville time-frequency analysis, and the fractional Fourier transform (FRFT). Wigner-Ville provides good estimates if the signal-to-noise ratio (SNR) is high; however, it is ineffective when receiving low-SNR LFM signals. FRFT can be used to estimate LFM signal parameters but the computational efficiency is dependent upon the search range and evaluation of the optimum FRFT order.

Several techniques have been proposed to conduct parametric estimation of LFM signals using machine learning algorithms. Convolutional neural networks (CNNs) have been used to classify waveform types [5] and signal parameters [6]. These machine learning algorithms have been used with various types of preprocessing. For example, [7] used a short-time Fourier transform (STFT) to recognize micro-Doppler features in radar signals, then applied Markov and support vector machine (SVM) classifiers. The Wigner-Ville transform has also been used [8].

In this paper, we investigate the case of parametric characterization of pulsed radar waveforms, specifically LFM signals, in the case of low SNR for calibration in noisy environments. We ask the question: can machine learning algorithms classify a low-SNR radar signal when it is known that the parameters for the incoming signal are drawn from a predetermined set of parameters? To answer this question, we

investigate the combination of matched filtering with machine learning classifiers to estimate the LFM chirp rate from a specific radar emitter in a congested radar spectrum. We report that, in this preliminary work, we can successfully classify the LFM chirp rate in low SNR conditions.

## II. Distinguishing Chirp Rates

In general, LFM signals can be written in the form

$$x_{\text{LFM}}(t) = \exp \left[ j\phi_0 + 2\pi j \left( \alpha t + \frac{\beta}{2} t^2 \right) \right], \quad (1)$$

where  $\phi_0$  is the initial phase,  $\alpha$  is the starting frequency, and  $\beta$  is the chirp rate. Our goal is to determine the chirp rate for an arbitrary LFM signal where the chirp rate is unknown, but is drawn from a list of known chirp rates. To simplify the problem, we will only consider the special case where  $\phi_0 = 0$  and  $\alpha = 0$  in this paper, leaving the more general case for future work.

We assume that all the signals involved have been sampled to yield discrete-time signals. Thus, the ideal LFM signal  $x_{\text{LFM}}(t)$  would be sampled at a constant sampling frequency  $f_s$ , yielding the discrete-time signal

$$x_\beta[n] = \exp \left[ 2\pi j \cdot \frac{\beta}{2} \left( \frac{n}{f_s} \right)^2 \right], \quad n = 0, \dots, N-1 \quad (2)$$

where  $N$  is the length of the sampled signal. In general, a received LFM signal may be corrupted by noise, yielding

$$x_\beta[n] + w[n]. \quad (3)$$

For the purposes of this paper, we assume that the noise,  $w[n]$ , is complex-valued Gaussian white noise with half the power in the real part and half the power in the imaginary part. We, furthermore, assume that any incoming signal contains at most one LFM signal; the case of superpositions of multiple LFM signals will be left for future work.

The problem we will consider may be stated as follows: given a signal of the form  $x_\beta[n] + w[n]$ , estimate  $\beta$  if it is known that  $\beta$  is drawn from a predetermined set of  $m$  chirp rates  $\{\beta_1, \beta_2, \dots, \beta_m\}$ .

### A. Matched Filtering and Machine Learning

The approach we will explore for determining  $\beta$  is to perform matched filtering of an incoming signal with an unknown chirp rate, which we may denote simply as  $x[n]$ , against all the ideal signals  $x_{\beta_1}[n], \dots, x_{\beta_m}[n]$ . The outputs of these matched filters are then fed into a machine learning classifier which outputs a prediction of the chirp rate of the signal.

In the case of additive white noise as assumed above, the matched filter  $h_\beta[n]$  corresponding to a LFM signal  $x_\beta[n]$  is

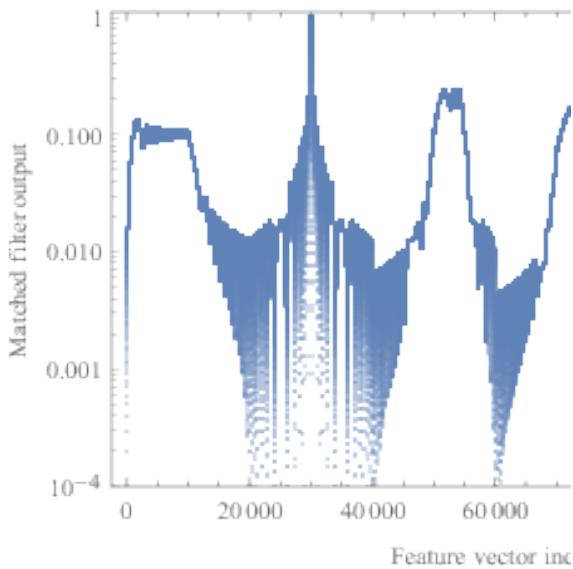
$$h_\beta[n] = \frac{x_\beta^*[-n]}{\sum_{k=0}^{N-1} x_\beta[k] x_\beta^*[k]} \quad (4)$$

where the superscript  $*$  indicates complex conjugation and the denominator is simply a normalization factor. The output of the matched filter is simply the convolution  $x[n] * h_\beta[n]$ . This amounts, in fact, to calculating the cross-correlation between the signals  $x[n]$  and  $x_\beta[n]$ , so we expect a large output when the incoming signal is similar to the ideal LFM signal  $x_\beta[n]$  and a small output when the two signals are very different. In the following, we will assume that the input  $x[n]$  is normalized just like the filter  $h_\beta[n]$  is. In effect, we calculate the normalized cross-correlation instead of the cross-covariance.

Because we know the set that  $\beta$  is drawn from, we can perform matched filtering of the incoming signal against all of the signals  $x_{\beta_1}[n], \dots, x_{\beta_m}[n]$ . Heuristically speaking, we would expect that the matched filter output with the highest peak will tell us which of the chirp rates best describes the incoming signal. In order to automate the process of examining all the matched filter outputs, we turn to machine learning.

The machine learning portion of our procedure involves concatenating all of the matched filter outputs  $x[n] * h_{\beta_1}[n], \dots, x[n] * h_{\beta_m}[n]$  into one long vector. This vector is then considered as a feature vector to be fed into a classifier. An example of such a feature vector, assuming no noise, is shown in Figure 1. By training a classifier on a large number of matched filter outputs for various chirp rates and

signal-to-noise ratios (SNRs), we expect to obtain a classifier which will accurately predict the chirp rate of the incoming signal.



**Figure 1. Example of a Feature Vector Obtained by Concatenating All the Matched Filter Outputs of an Ideal Chirp**

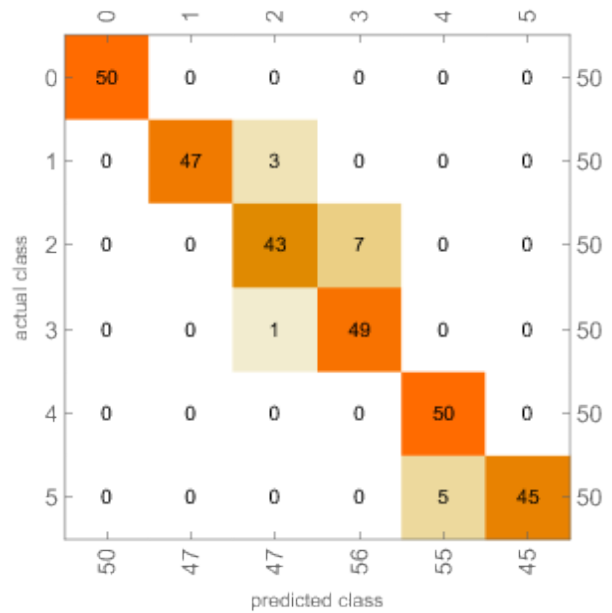
### III. Results

For our proof-of-concept classifier, we simulated LFM signals with chirp rates of  $\beta = 0, 1, 2, 3, 4,$  and  $5$  Hz/s. We used a sampling frequency of  $f_s = 2000$  Hz, and the number of samples in each sampled signal was  $N = 10\,000$ . We considered signals with SNRs of 20, 10, 0,  $-10$ ,  $-20$ ,  $-30$ , and  $-40$  dB.

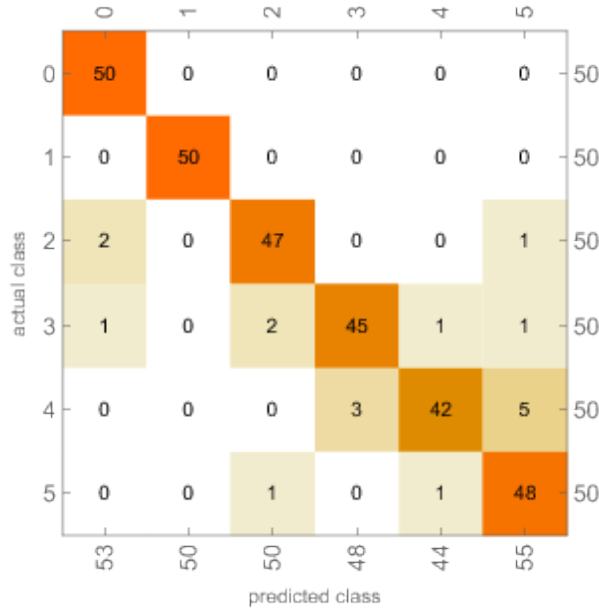
We trained a naive Bayes classifier, a SVM classifier, and a neural network (NN) classifier using Wolfram Mathematica's Classify command. Each feature vector consists of the matched filter outputs that result from filtering a given signal against all six chirp rates, resulting in a vector of length 119 994 (each matched filter output being 19 999 samples long). Our training set consisted of 200 noisy LFM signals for each  $\beta$  and each SNR listed above, for a total of 8400 signals. Our

training set therefore has size  $8400 \times 119\,994$ . We then trained the classifiers on these feature vectors. The trained classifiers were tested on 50 LFM signals for each  $\beta$  and SNR, and confusion matrices were generated separately for each SNR.

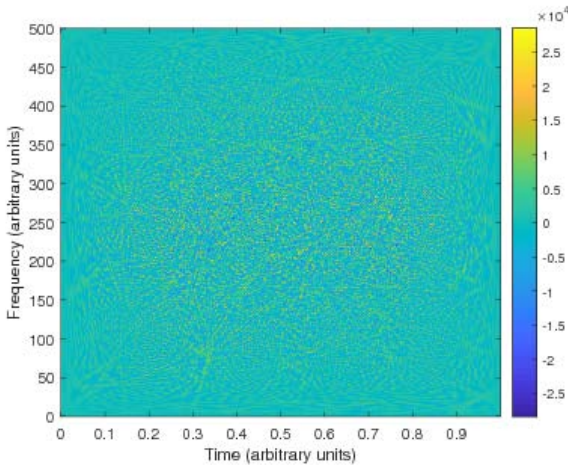
We found that the naive Bayes and SVM classifiers classified the test set perfectly for all signals with an SNR of  $-10$  dB or above. Performance begins to degrade at  $-20$  dB, though at this level of noise the performance was still quite good, as can be seen in Figures 2 and 3. However, performance at  $-30$  dB and below was poor. We find, in summary, that good classification performance was achievable down to an SNR of  $-20$  dB even with the simple, unsophisticated approach taken here. This is significantly better than what is possible with the Wigner-Ville transform. As seen in Figure 4, which shows the Wigner-Ville transform of a LFM signal at a SNR of  $-20$  dB, the straight line which is the signature of the Wigner-Ville transform of a LFM signal is entirely lost in noise.



**Figure 2. Confusion Matrix for a Naive Bayes Classifier at a SNR of  $-20$  dB**



**Figure 3. Confusion Matrix for a SVM Classifier at a SNR of  $-20$  dB**



**Figure 4. Wigner-Ville Transform of a Noisy LFM Signal at a SNR of  $-20$  dB**

#### IV. Conclusions

The work presented in this paper represents only a preliminary application of the combination of matched filtering and machine learning. Note that we did not conduct any feature selection or extraction: the raw matched filter outputs were presented to the classifiers considered in this paper. Even so, we were able to show that our method shows promise for radar parametric estimation for LFM signals when we have a priori information.

Our method has the drawback that it requires the set of possible LFM parameters to be known in advance. However, this is not an unreasonable assumption in the context of calibration. We can use this method to determine the presence or absence of known radar emitters, or to show the operational or non-operational state of these emitters in a low-SNR scenario.

This paper represents the first phase in the analysis of multiple radar emitters in a low-SNR environment and the differentiation of these radar emitters. Further studies will investigate the issue of feature extraction, the use of Markov-based classifier algorithms, and the application of this approach to more complicated modulations.

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