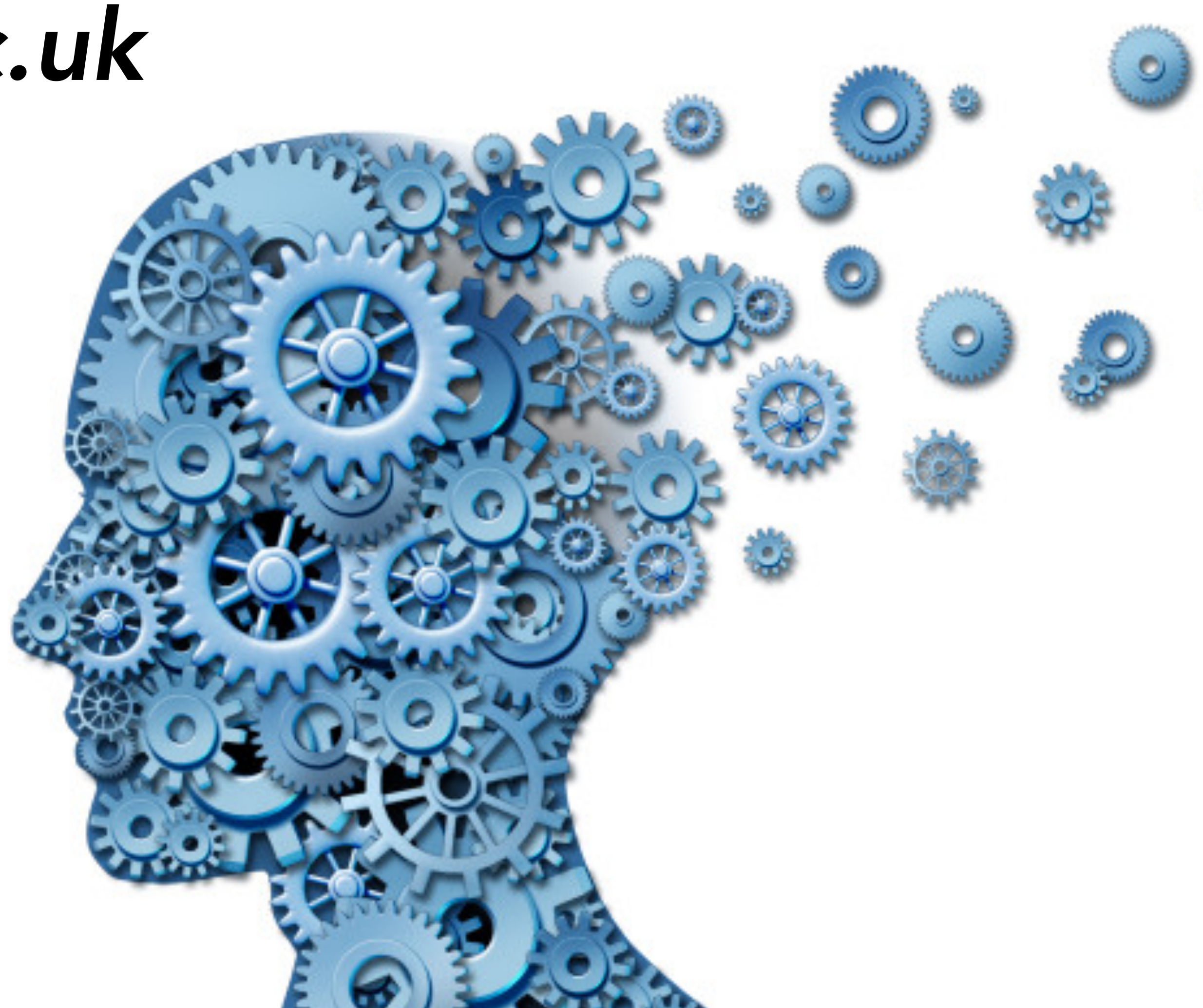
COMS30017
Computational Neuroscience

Week 6 / Video 5 / Topographic maps and sparse coding

Dr. Laurence Aitchison

laurence.aitchison@bristol.ac.uk



slides adapted from Dr. Cian O'Donnell

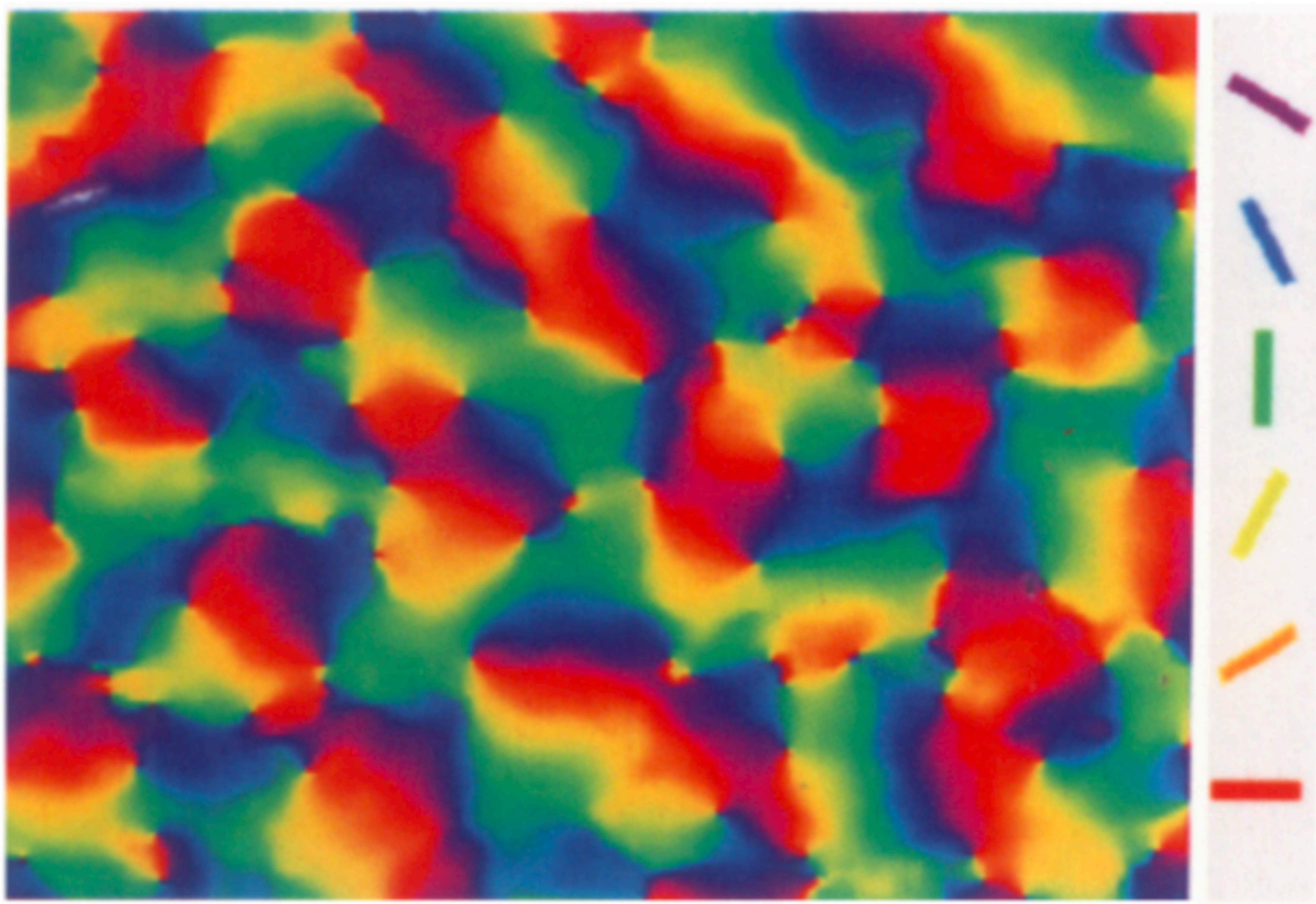
Intended Learning Outcomes

- Topographic maps.
- Sparse coding.

Topographic maps

- Topographic maps are when anatomically nearby neurons in cortex demonstrate similar functional properties.
- First described by Vernon Mountcastle (*J Neurophysiol*, 1957).
- The retinotopic map (nearby locations in the visual field map to anatomically nearby locations in cortex) is one example.
- The visual cortex in most mammals has multiple topographic maps superimposed, according to: orientation preference, ocular dominance, motion direction preference, spatial frequency sensitivity, etc.
- Also exist in other parts of the brain: auditory cortex has a tonotopic map, somatosensory cortex has a somatotopic map, etc.

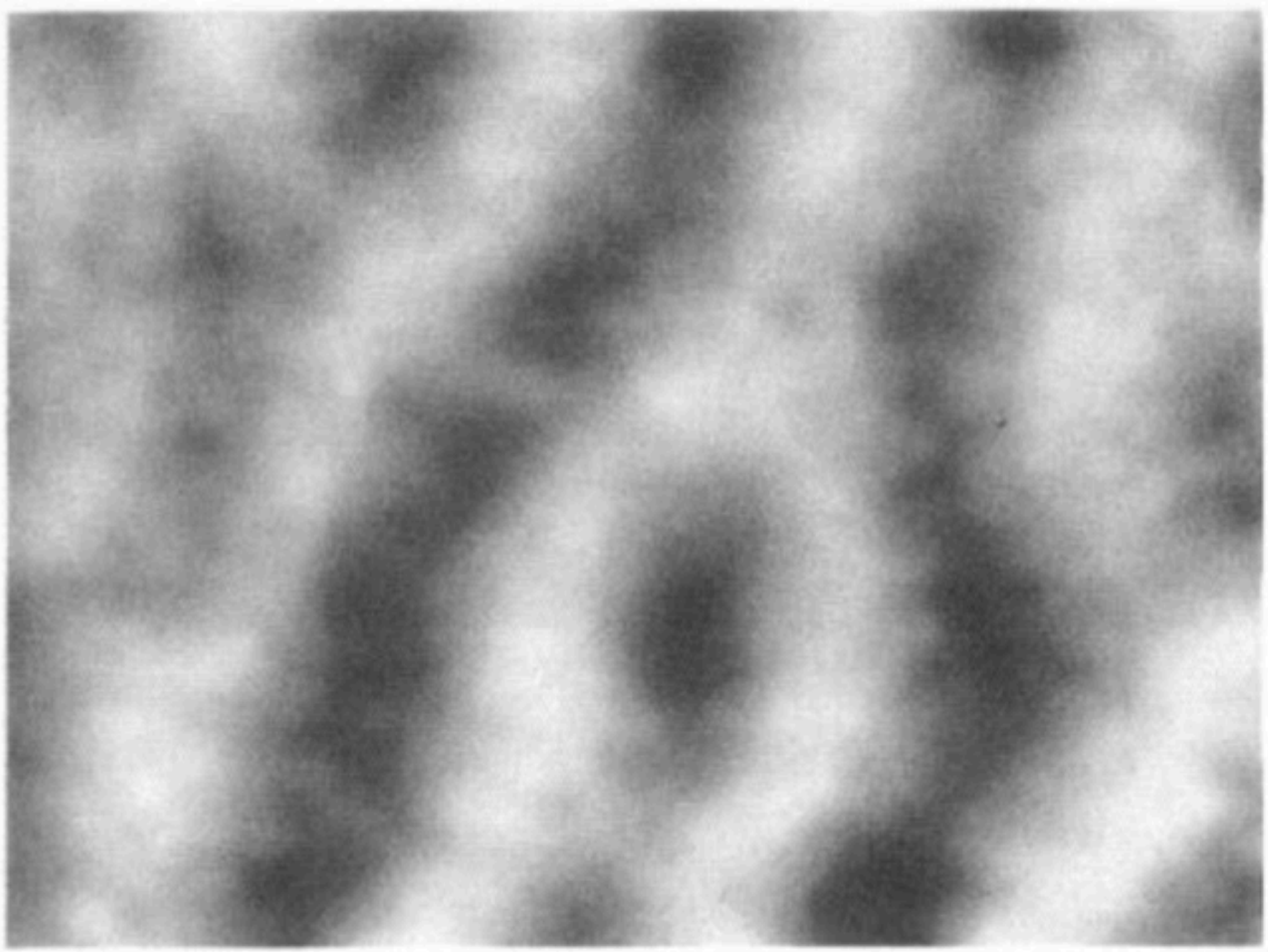
Orientation preference maps



Intrinsic imaging of orientation preference patches at the surface of macaque V1.

Blasdel, *J Neurosci* (1992)

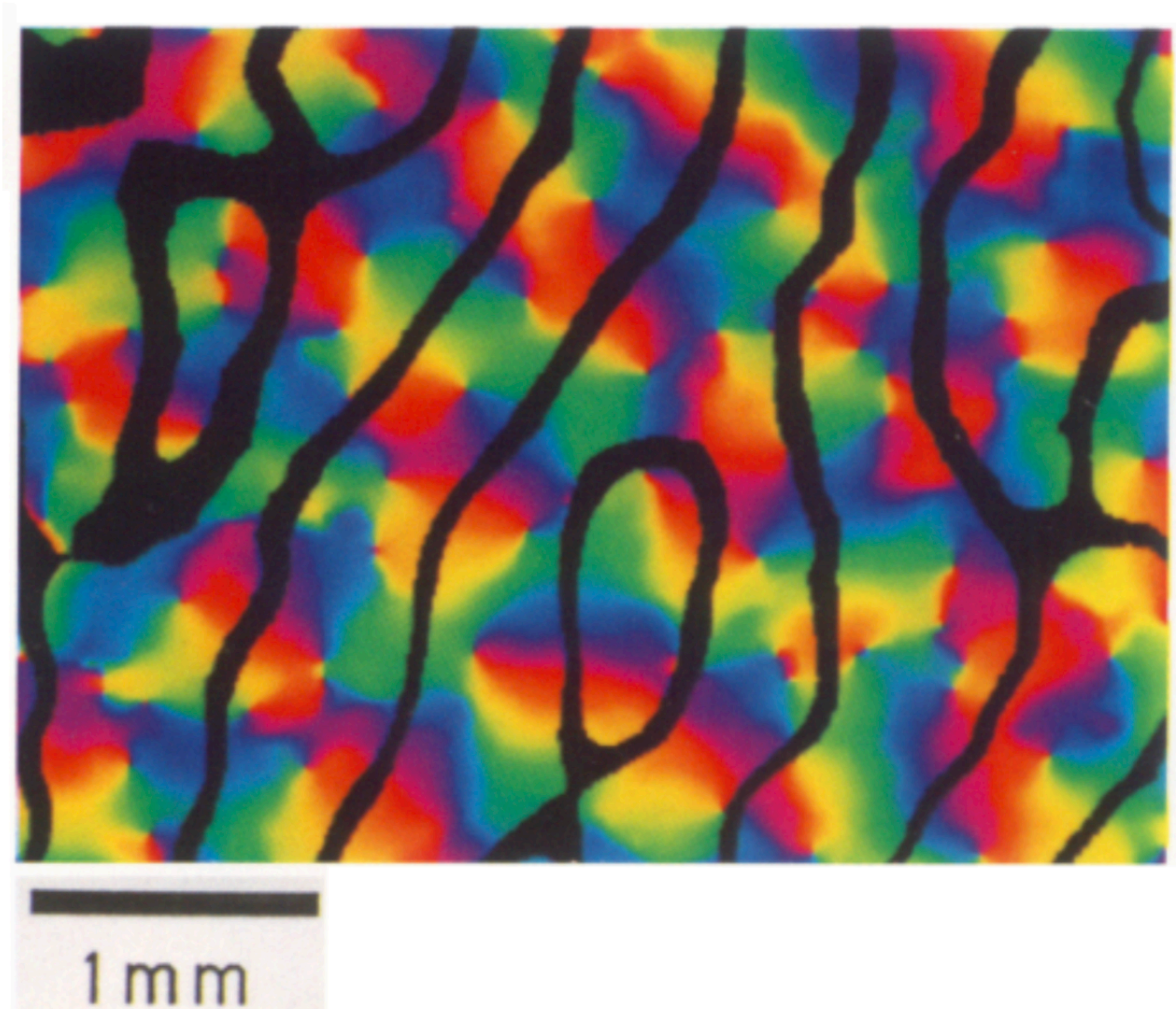
Ocular dominance maps



Intrinsic imaging of ocular dominance patches at the surface of macaque V1.

Blasdel, *J Neurosci* (1992)

Interaction between OR and OD maps



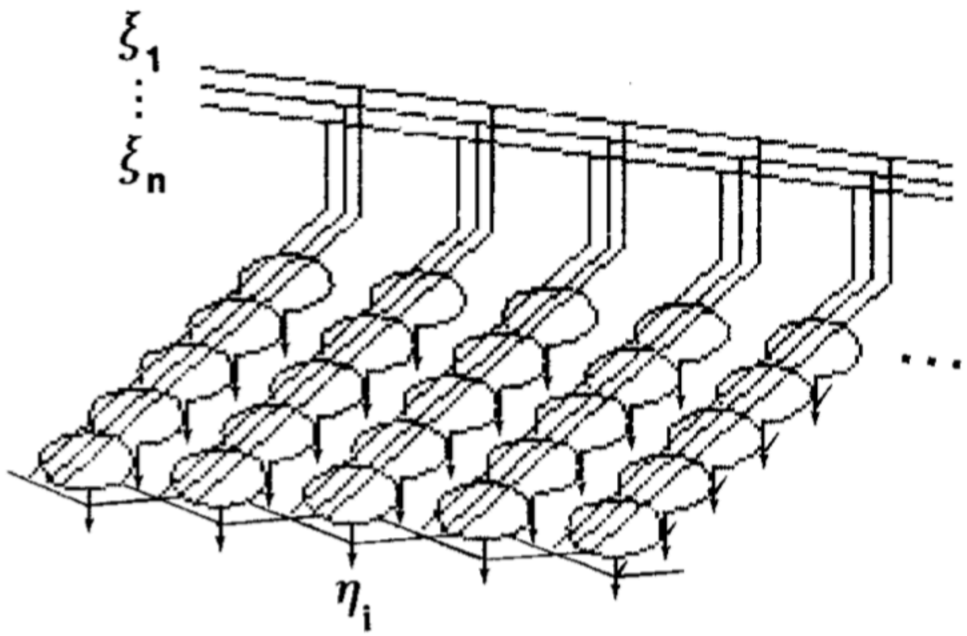
Blasdel, *J Neurosci* (1992)

Models of V1 development

- Various computational models have been proposed to explain aspects of visual cortex development.
- Ocular dominance columns classic example (Kenneth Miller). Requires symmetry-breaking.
- Kohonen map (a.k.a. self-organising map) can generically explain topographic map formation.

Kohonen map

Algorithm for learning self-organised maps



- Neuron i's output y_i is a weighted sum of inputs x_js
$$y_i = \sum_j w_{ij} x_j$$
- Present an input pattern, $\boldsymbol{x}$, find the most active neuron in the array, $\max(\boldsymbol{y})$
- Only update the weights of the most active neuron plus its nearest neighbours in the grid, according to the rule:
$$w_i(t+1) = \frac{w_i(t) + \alpha x_i(t)}{C}$$
where C is a normalising constant to keep the weight vectors of unit length.
- Repeat from step 2. This results in the weights of neighbouring units becoming more similar with training.

Kohonen, *Biol Cybern* (1982)

Kohonen map

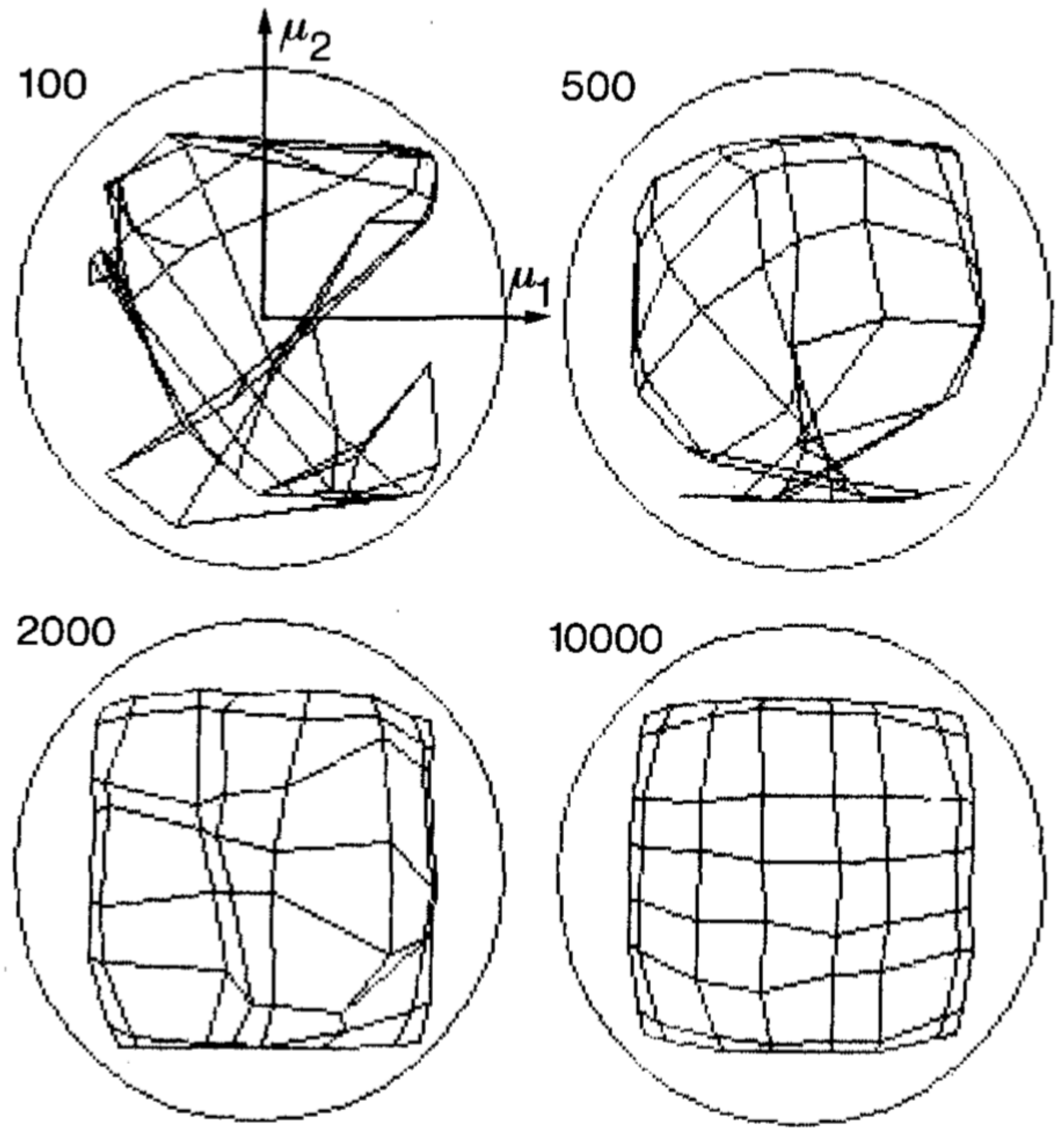


Fig. 4. Distribution of the weight vectors $m_i(t)$ at different times. The number of training steps is shown above the distribution. Interaction between nearest neighbours only

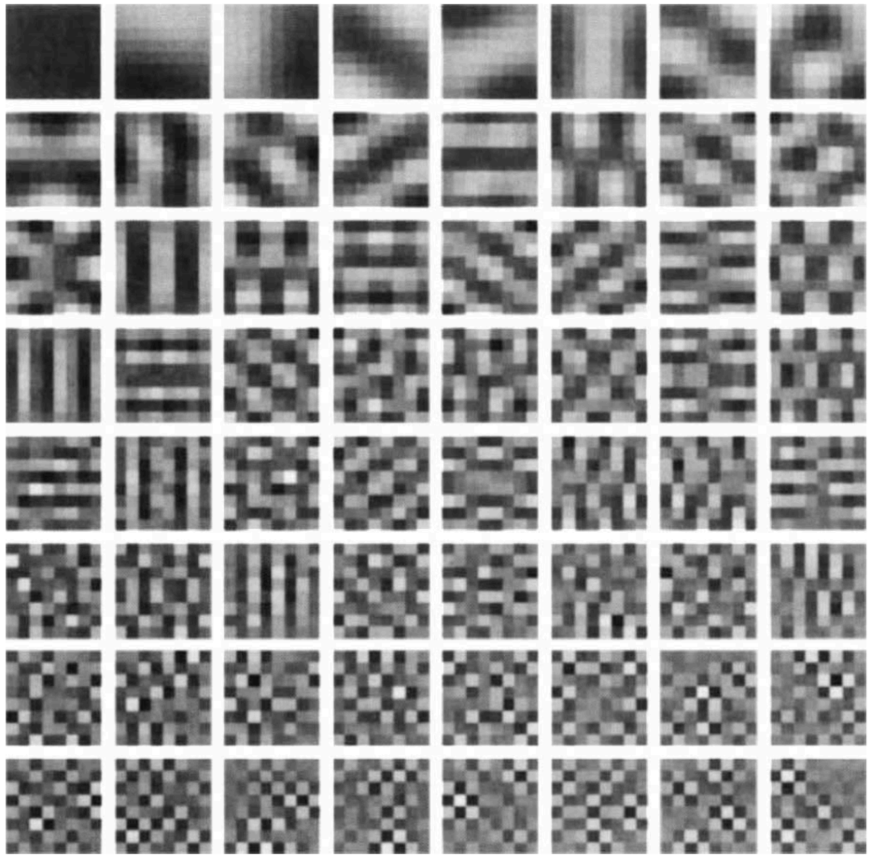
Kohonen, *Biol Cybern* (1982)

Sparse coding

- The activity in many brain regions is sparse: many neurons are often silent.
- Sparse codes sit somewhere between dense codes (all neurons participate in all patterns) and fully local codes (only one neuron active for each type of sensory signal).
- Sparsity may resolve a tradeoff between energy efficiency and representational capacity.
- Theoretical neuroscientists have found that sparse coding can also provide computational benefits:
  - Sparsity can enhance discriminability; less overlap between patterns.
  - Sparsity can act as a form of regularisation; many signals in the outside world are inherently sparse, so sparse brain coding can speed up learning of sensory signal statistics.
- Further reading: http://www.scholarpedia.org/article/Sparse_coding and Olshausen and Fields, *Curr Opin Neurbiol*, 2004.

Sparse coding encourages V1-like receptive fields

- A computer science approach: find a set of basis functions that we can linearly sum to reconstruct an image: $I(x,y) = \sum_i a_i \phi_i(x,y)$
- Principal components analysis (PCA) is a classic method, but when applied to a set of natural images gives basis functions that look nothing like V1 receptive fields.



PCA components

Olshausen and Field, *Nature* (1996)

Sparse coding encourages V1-like receptive fields

- Olshausen and Field searched for basis functions that can be combined to reconstruct images, but also have a sparseness penalty that encourages the a_i terms to be small or zero.
- Want to minimise: $E = -[\text{preserve information}] - \lambda[\text{sparseness of } a_i]$

$$[\text{preserve information}] = -\sum_{x,y}\left[I(x,y) - \sum_i a_i \phi_i(x,y)\right]^2$$

Encourages good reconstructions

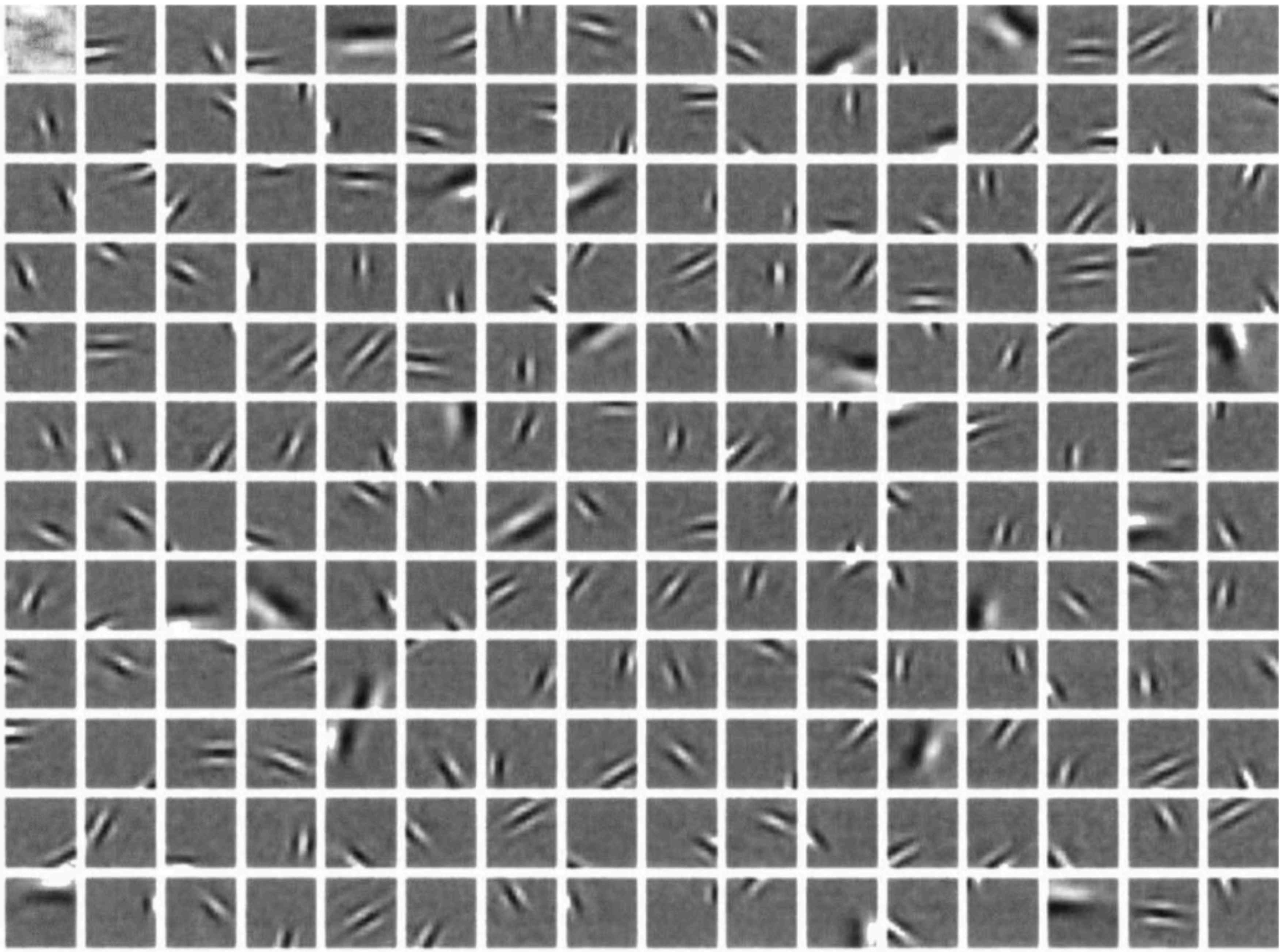
$$[\text{sparseness of } a_i] = -\sum_i S\left(\frac{a_i}{\sigma}\right)$$

Encourages a_is to be small

- This results in finding a sparse, "overcomplete" basis set where components are typically localised, oriented, and bandpass.

Olshausen and Field, *Nature* (1996)

Sparse coding encourages V1-like receptive fields



Gabor-like basis functions learned from a set of natural images from the Amercian Northwest, by including a penalty term that encourages sparseness.

Olshausen and Field, *Nature* (1996)

End