Practical Assignment:
-Estimating VAR of a Portfolio-

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The Problem:

Consider a Russian investor holding a US and German stock portfolio on 09/03/2022. Suppose that the investment is made in rubles and that:

- 60% of the portfolio is held in US stocks.
- 40% of the portfolio is held in German stocks.
- The US portfolio has a β of 1.6 relative to the NASDAQ index and the German portfolio has a β of 1.3 relative to the DAX index.

In order to evaluate the portfolio risk, you are given the data from NASDAQ 100, DAX and EUR/RUB and USD/RUB exchange rates from 09/03/2017 to 09/03/2022.

Answer all the questions and explain your answers and choices on a report.

Some Notes on the Formulation:

1. Note that, for instance, the price of a given US stock, in rubles, is the dollar price multiplied by the USD/RUB exchange rate, ie,

$$P_t^{RUB} = P_t^{USD} X_t^{USD/RUB}$$

Hence, the log equity return and the log forex return are additive, ie,

$$ln\left(\frac{P_{t+1}^{RUB}}{P_t^{RUB}}\right) = ln\left(\frac{P_{t+1}^{USD}}{P_t^{USD}}\right) + ln\left(\frac{X_{t+1}^{USD/RUB}}{X_t^{USD/RUB}}\right) \iff$$
$$r_t^{RUB} = r_t^{USD} + r_t^{USD/RUB}$$

This means that the risk of a US stock portfolio to a Russian investor has an equity component, based on the risk of the dollar returns on the portfolio and a forex component, which is based on the USD/RUB foreign exchange rate.

2. Considering:

ω_1 the proportion of total capital invested in German market.
ω_2 - the proportion of total capital invested in US market.
y_{1t} - daily log returns on German index .
y_{2t} - daily log returns on US index.
r_t^{RUB} - daily log returns of the total portfolio, in rubles.
r_{1t}^{RUB} - daily log returns on the German portfolio.
r_{2t}^{USD} - daily log returns on the US portfolio.
$r_t^{EUR/RUB}$ - daily log returns of the forex rate EUR/RUB.
$r_t^{USD/RUB}$- daily log returns of the forex rate USD/RUB.
β_1 - sensitivity of German stocks to DAX index.

β_2 - sensitivity of USD stocks to NASDAQ100.
We may notice that:

$$\begin{aligned} r_t^{RUB} &\simeq \omega_1 r_{1t}^{RUB} + \omega r_{2t}^{RUB} \\ &= \omega_1 \left(r_{1t}^{EUR} + r_t^{EUR/RUB} \right) + \omega_2 \left(r_{2t}^{USD} + r_t^{USD/RUB} \right) \\ &= \left(\omega_1 \beta_1 y_{1t}^{EUR} + \omega_2 \beta_2 y_{2t}^{USD} \right) + \left(\omega_1 r_t^{EUR/RUB} + \omega_2 r_t^{USD/RUB} \right) \end{aligned}$$

Where:

- $\omega_1 \beta_1 y_{1t}^{EUR} + \omega_2 \beta_2 y_{2t}^{USD}$ corresponds to the net equity return of the portfolio.
- $\omega_1 r_t^{EUR/RUB} + \omega_2 r_t^{USD/RUB}$ corresponds to the net forex return of the portfolio.

Part I

(4 Questions):

1. Plot the evolution of both indexes and of the foreign exchange rates over the sample period. Comment.

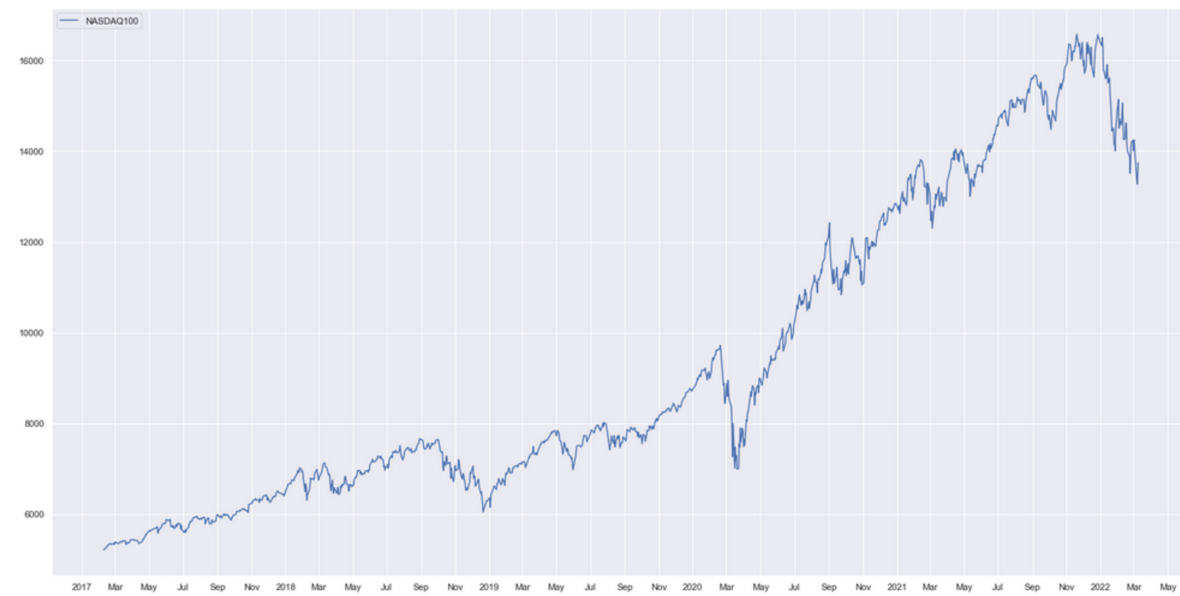

Figure 0.1: NASDAQ100 evolution.

We can see a decrease in the value of the NASDAQ100 index in February 2022 which corresponds to the beginning on Russia war on Ukraine, and on February 2020 which corresponds to the beginning of the covid pandemic.

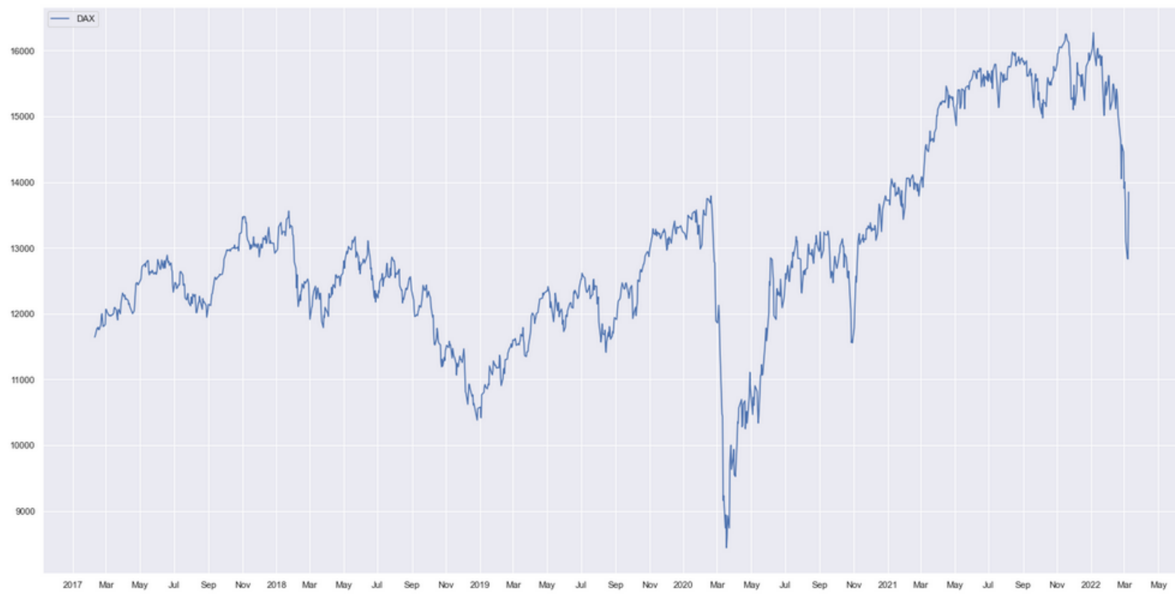

Figure 0.2: DAX evolution.

We can see a decrease in the value of the DAX index in February 2022 which corresponds to the beginning on Russia war on Ukraine, and on February 2020 which corresponds to the beginning of the covid pandemic.

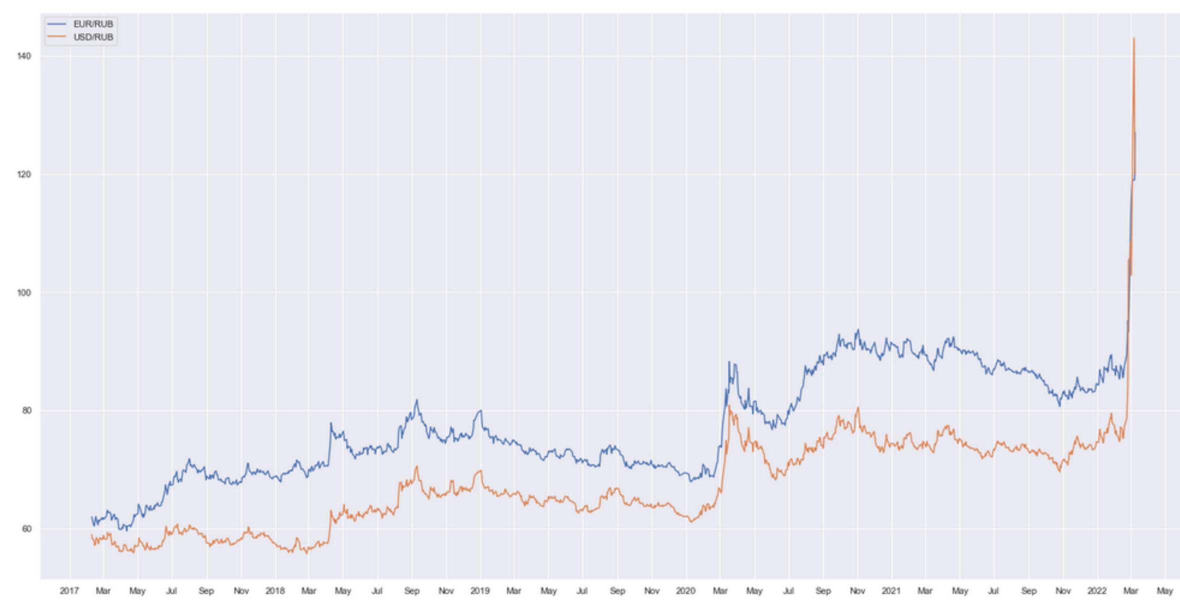

Figure 0.3: Indexes evolution.

From the plot above we can see that the exchange values of EUR/RUB and USD/RUB increase, meaning the rubble loses its value starting February 2022, which corresponds to the beginning of Russia war on Ukraine. We can also see a correlation in both exchange rates and a similar direction of fluctuations

2. Express the DAX and NASDAQ100 values in rubles and plot them again. Comment.

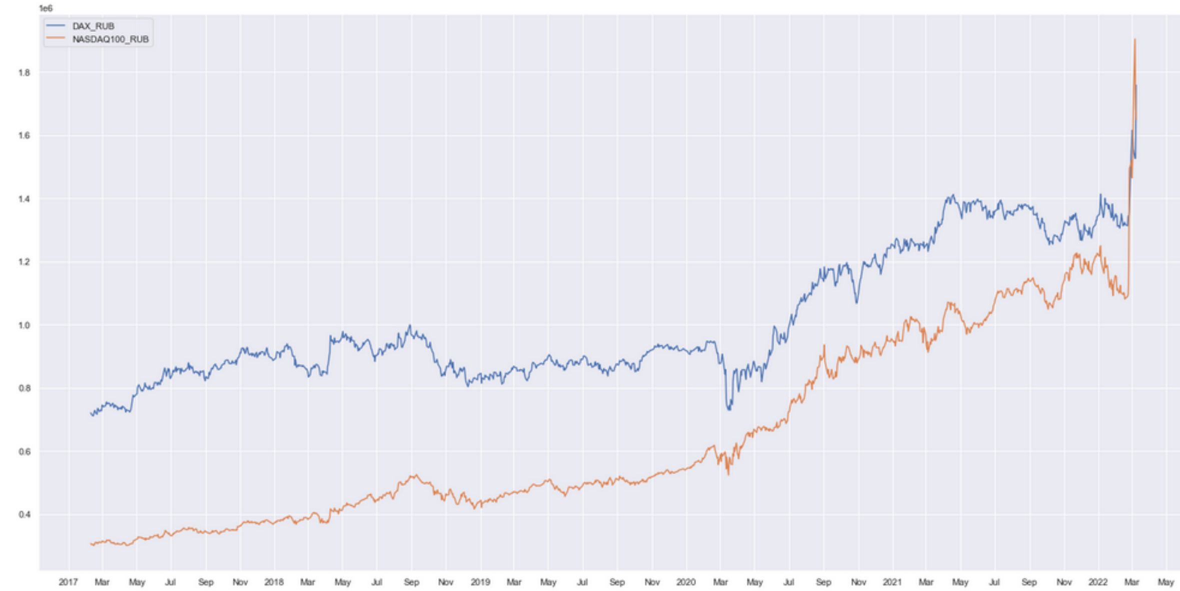

Figure 0.4: DAX and NASDAQ in rubles.

From the plot above we can see that the value of indexes in RUB increases a lot in February 2022, which corresponds to the beginning of Russia war on Ukraine.

3. Obtain the daily net equity returns and the daily net forex returns of the portfolio. Plot them and comment.

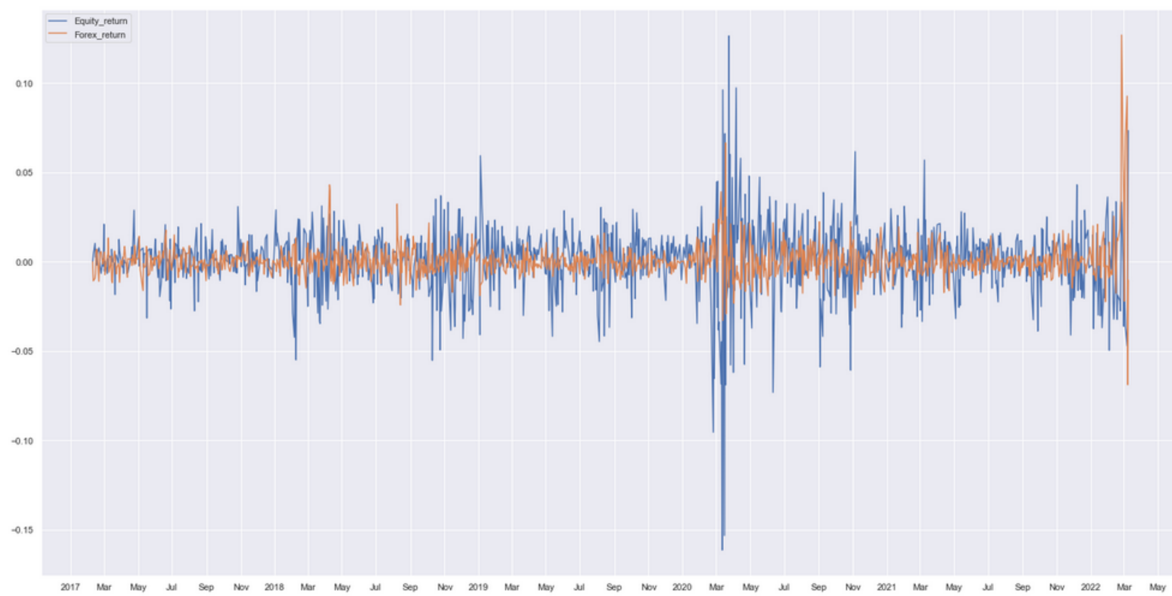

Figure 0.5: Equity and Forex daily returns.

We can see a big variability of returns on march 2020 which corresponds to the beginning of the covid19 pandemic, and also on February 2022 which correspond to the beginning of Russia war on Ukraine. Volatility clustering is observed in both equity returns and forex returns. In addition periods of extreme volatility are shared by both Equity Returns and forex returns.

4. Obtain the daily portfolio returns and plot them on an histogram. Comment on the shape of the distribution.

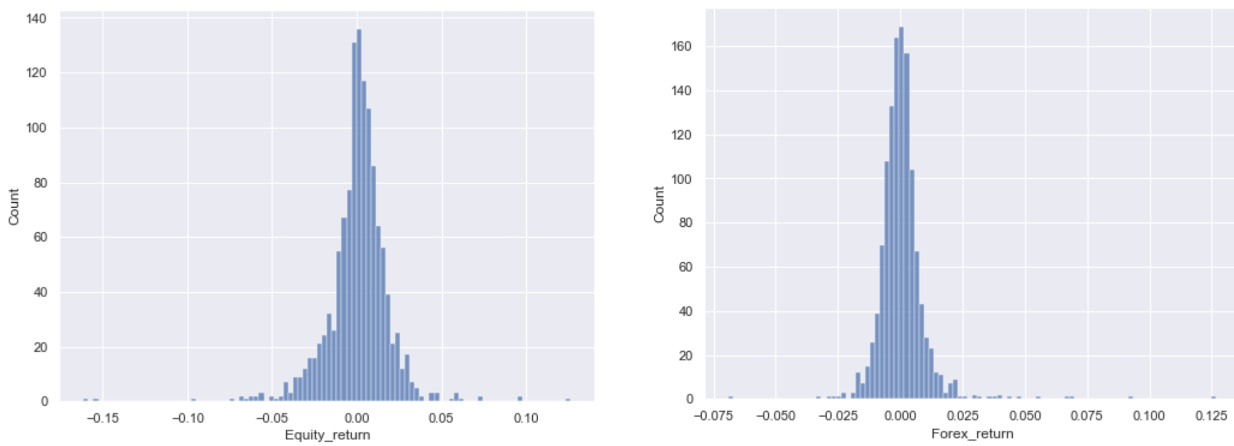

Figure 0.6: Equity and Forex returns histograms.

As expected, Equity returns tend to be negatively skewed.

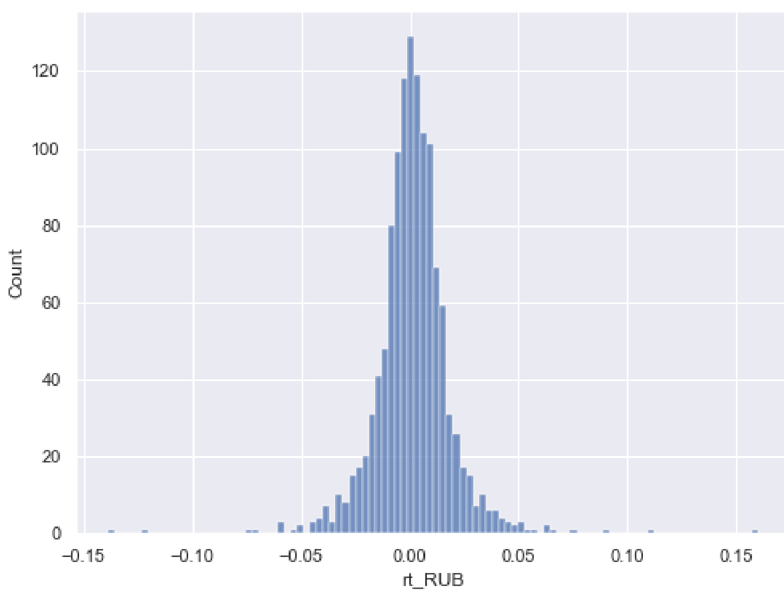

Figure 0.7: Portfolio returns histogram.

The distribution looks symmetrical with a bell shape, which makes us guess of a normally distributed returns.

Part II

(3 Questions):

Considering a smoothing parameter of $\lambda = 0.94$, obtain:

1. The EWMA volatilities and correlations over the sample period for the net equity returns and forex returns.

- Variance Estimation:

The EWMA for the variance estimate, at time t, for a series of returns $\{r_t\}_{t=1}^{T}$ can be obtained recursively from:

$$\hat{\sigma}_t^2 = (1-\lambda)\, r_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2, \quad t = 2, ..., T$$

and

$$\hat{\sigma}_1^2 = \frac{1}{T}\sum_{i=1}^{T} r_i^2$$

The values of volatilities are then obtained by taking the square root of the variances.

- Covariance Estimation:

The EWMA covariance estimate, at time t, for two contemporaneous time series of returns $\{r_{1,t}\}_{t=1}^{T}$ and $\{r_{2,t}\}_{t=1}^{T}$ can be obtained recursively from:

$$cov\left(r_{1,t}, r_{2,t}\right) = (1-\lambda)\, r_{1,t-1} r_{2,t-1} + \lambda cov\left(r_{1,t-1}, r_{2,t-1}\right), \quad t = 2, ..., T$$

and

$$cov\left(r_{1,1}, r_{2,1}\right) = \frac{1}{T}\sum_{i=1}^{T} r_{1,i} r_{2,i}$$

The values of correlations can be derived from the volatilities and the co-variances values.

	Date	Equity_Adjusted_Volatility	Forex_Adjusted_Volatility	Correlation
0	2017-02-09	0.018434	0.009555	-0.341411
1	2017-02-10	0.017873	0.009264	-0.341411
2	2017-02-13	0.017353	0.009359	-0.342061
3	2017-02-14	0.017012	0.009358	-0.364239
4	2017-02-15	0.016505	0.009226	-0.364632
...	...	...	...	...
1229	2022-03-03	0.023034	0.035537	-0.022177
1230	2022-03-04	0.023188	0.036351	-0.106063
1231	2022-03-07	0.024243	0.039197	-0.252235
1232	2022-03-08	0.026182	0.044264	-0.420273
1233	2022-03-09	0.025402	0.046110	-0.377260

Figure 0.8: The EWMA volatilities and correlations.

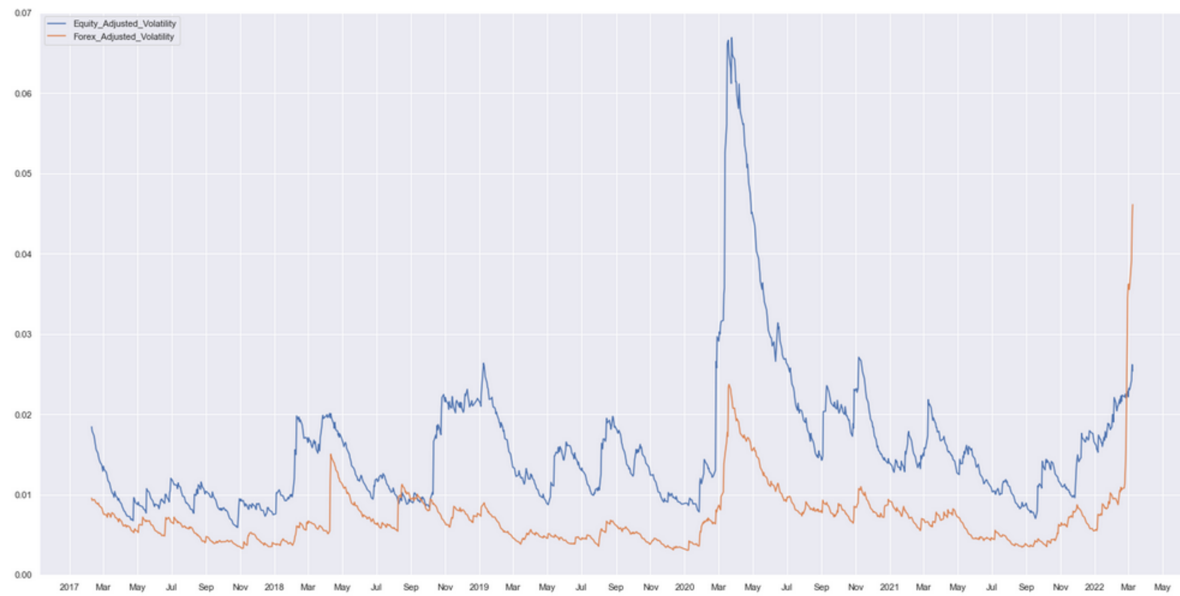

Figure 0.9: Equity and Forex adjusted volatilies.

We can see a spike on the Equity adjusted volatility on march 2020 maybe due to covid19 pandemic, same for forex adjusted volatility but with a less magnitude, on February 2022 the forex adjusted volatility increases exponentially probably due to Russia war on Ukraine.

2. Adjust the portfolio returns, using $\tilde{r}_{t,T} = \frac{\hat{\sigma}_T}{\hat{\sigma}_t} r_t$***,*** $t = 1, ..., T$***. Compare them with the original ones and comment.***

Figure 0.10: Equity and Adjusted Equity returns.

From the plot above we can see that the adjusted Equity returns are a lot more volatile that the Equity returns, which is normal as this methodology is designed to weight returns in such a way that we adjust their volatility to the current volatility.

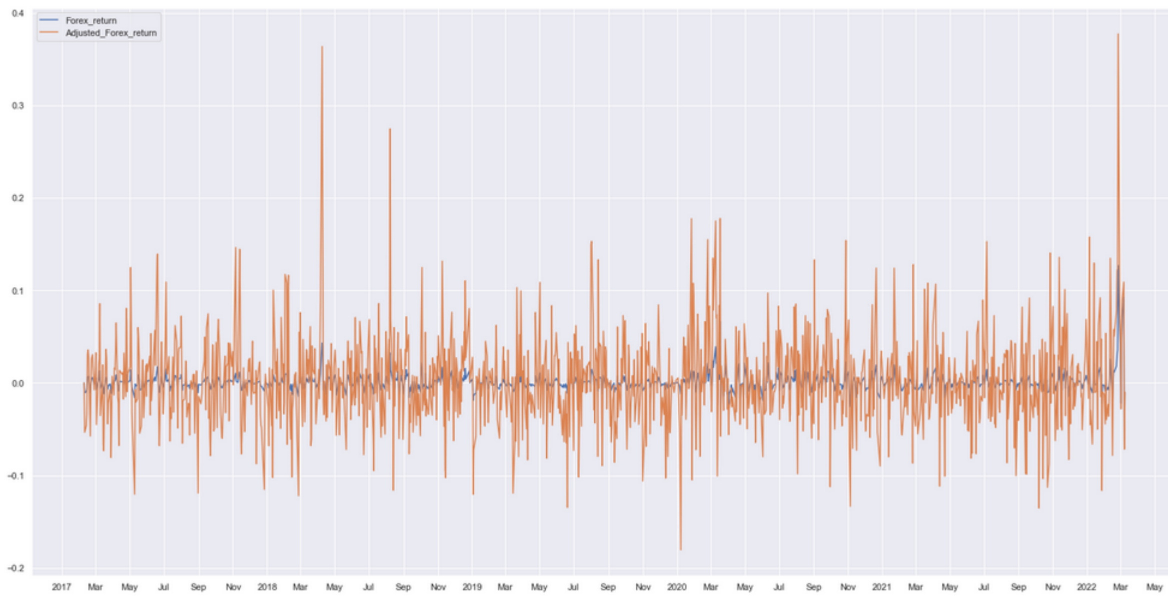

Figure 0.11: Forex and Adjusted Forex returns.

Same observations as before.

3. Estimate the EWMA annual co-variance matrix of the portfolio on 09/03/2022. Compare to the obtained from data at 23/02/2022. Comment.

The annual co-variance matrix of the portfolio formula is given by:

$$\Omega_h = h \begin{pmatrix} \hat{\sigma}^2_{Equity,T} & \hat{\sigma}_{Equity,T} \times \hat{\sigma}_{Forex,T} \times \rho_{E,F,T} \\ \hat{\sigma}_{Equity,T} \times \hat{\sigma}_{Forex,T} \times \rho_{E,F,T} & \hat{\sigma}^2_{Forex,T} \end{pmatrix}$$

Taking a value of $h = 250$, we have:

On the 23/02/2022:

	Equity	Forex
Equity	0.122700	-0.038781
Forex	-0.038781	0.032370

Figure 0.12: Annual co-variance matrix on the 23/02/2022.

On the 09/03/2022:

	Equity	Forex
Equity	0.161312	-0.110467
Forex	-0.110467	0.531522

Figure 0.13: Annual co-variance matrix on the 09/03/2022.

The variances went up after the war started, especially the forex variance that went from a value equal to 0.032 to a value of 0.53, the Equity and Forex are also more negatively correlated, meaning that when the Equity go down the forex goes up and vice-versa.

Part III
(4 Questions):

Using the adjusted returns obtained in Question II.2, considering both dates 09/03/2022 and 23/02/2022 and setting a 1% significance level:

1. Obtain the 1-day and 10-day Normal Parametric Systematic Var for the portfolio. Comment.

We assume the values of $\theta_1 = \theta_2 = 1$, we have the formula for the systematic Var:

$$Systematic \qquad Var_{h,\alpha} = \phi^{-1}(1-\alpha)\sqrt{\theta'\Omega_h\theta}$$

After calculation we obtain:

	23/02/2022	09/03/2022
Var_1	0.040962	0.101072
Var_10	0.129533	0.319616

Figure 0.14: The values of 1 and 10 days Vars.

We can see that the value of the VaR of the portfolio increased with time, the portfolio 10 days VaR on the 23/02/2022 is equal to 0.13 while on 09/03/2022 the portfolio days Var is equal to 0.32. Same for the value of 1 day VaR that increased from 0.04 to 0.10.

2. Estimate the 10-day Normal Stand-Alone VaR for the equity and forex risk factors on 09/03/2022. Comment.

We know that the Equity and Forex VaRs are given by the following formulas:

$$Equity \qquad Var_{h,\alpha} = \phi^{-1}(1-\alpha)\sqrt{\theta_E'\Omega_{Eh}\theta_E}$$

$$Forex \qquad Var_{h,\alpha} = \phi^{-1}(1-\alpha)\sqrt{\theta_F'\Omega_{Fh}\theta_F}$$

After calculations we obtain:

	23/02/2022	09/03/2022
Equity	0.162977	0.186869
Forex	0.083710	0.339207

Figure 0.15: Stand-Alone 10 days VaRs.

We can see a high increase on the Forex Value that moved from 0.08 to 0.34, the Equity VaR increased as well but not with the same range.

Due to the diversification effect between risk factor types (and variance properties):

- The sum of the stand-alone VaR's is greater than or equal to the Systematic VaR.
- Equality only is observed in the trivial case where all risk factors are perfectly correlated.

3. Obtain the 1% 1-day Historical VaR. Comment.

The historical simulation (HS) approach is a non-parametric method that makes no specific assumption about the distribution of risk factors. It consists of going back in time and replaying the tape of history on the current positions.

Advantages of Historical VaR:

- Historical VaR does not need any parametric assumptions.
- The dynamic evolution and dependencies of the risk factors are inferred directly from historical observations.
- This allow the model to access the risk of complex path-dependence products.
- Historical VaR includes the dynamic behaviour of risk factors in a natural and realistic manner.
- Historical VaR is not limited to linear portfolios.

Method 1: Equally Weighted Returns Distribution:

With equal weighting, the ordering or the observations is irrelevant, the historical VaR correspond to minus the α quantile value.

After calculations we obtain:

	23/02/2022	09/03/2022
Hist_Var_EWRD	0.041651	0.043756

Figure 0.16: 1% 1-day Historical VaR.

We can see that the value of the Var slightly increased with time.

Method 2: EWMA Returns Distribution:

This method works as follow:

1. Fix a smoothing constant $0 < \lambda < 1$.
2. Consider a data set $\{r_1, ..., r_T\}$ of T most recent portfolio returns.
3. Assign a probability weight to each of the returns as follows:

$$\omega_T = 1 - \lambda \qquad and \qquad \omega_i = \lambda \omega_{i-1}, \quad i = T-1, ..., 1$$

4. Use the probability weights to find the cumulative probability associated with the returns, when they are put in increasing order of magnitude.
   (a) Order increasingly the returns and record it's associated probability weight.
   (b) Add the weight associated with the next smallest return until reach the significance level.
5. The $100 \times \alpha\%$ historical VaR, as a percentage of the portfolio's value, is equal to minus the last return taken into the sum.
6. The risk horizon for the VaR is the holding period of returns (usually 1 day).

After calculations we obtain:

	23/02/2022	09/03/2022
Hist_Var_EWMA	0.044871	0.044871

Figure 0.17: 1% 1-day Historical VaR.

The values of the VaR does not seem to change, meaning that this method is not suitable for estimating the historical VaRs.

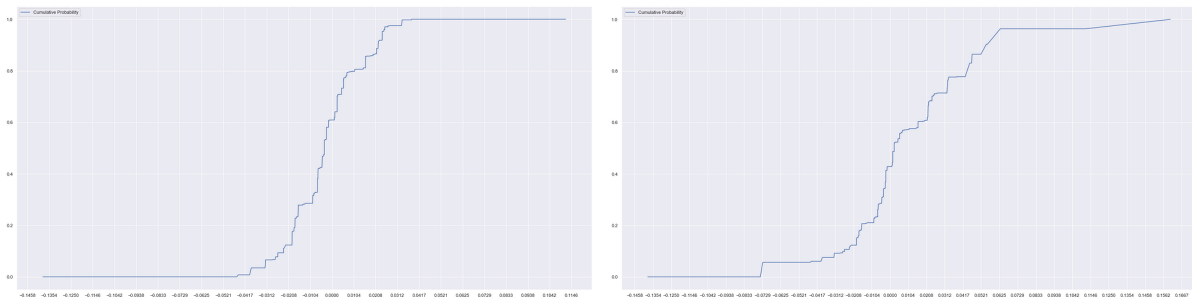

Figure 0.18: Cumulative distribution on both 23/02/2022 and 09/03/2022.

Method 3: Historical VaR with Volatility Weighting Methodology:

We need to calculate the adjusted return of the portfolio given by: $\tilde{r}_{t,T} = \frac{\hat{\sigma}_T}{\hat{\sigma}_t} r_t,\ t = 1, ..., T$, but before we need to get the values of the adjusted volatility of returns.

After calculations we obtain:

	23/02/2022	09/03/2022
Hist_Var_VolWeight	0.052063	0.128306

Figure 0.19: 1% 1-day Historical VaR.

We can see that the value of the portfolio historical VaR increased with time, these values are different from the ones obtained previously this is due the this last method taking into consideration the adjusted volatility effect.

The historical VaR values are lower than the parametric systematic VaR values of the same portfolio, this might be due to the fact that historical VaR fails to capture volatilities increase.