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MILANO 1863

Scaling Relations for the Geometry of Wire-to-Airfoil Atmospheric Ionic Thrusters

Omar Kahol

04/05/2023

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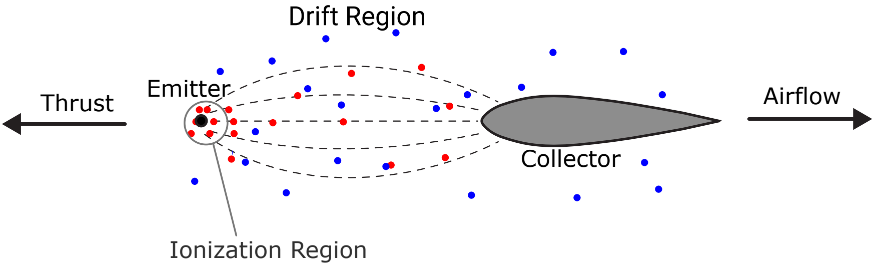
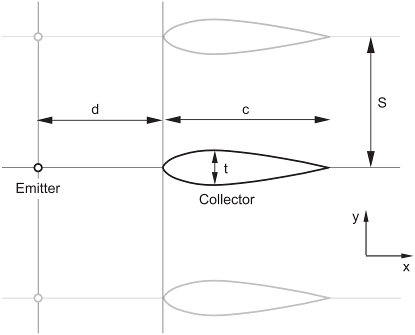
EHD phenomenology



- Positive or negative voltage applied between the electrodes
- Collisions generate airflow and thrust

Geometrical Parameters

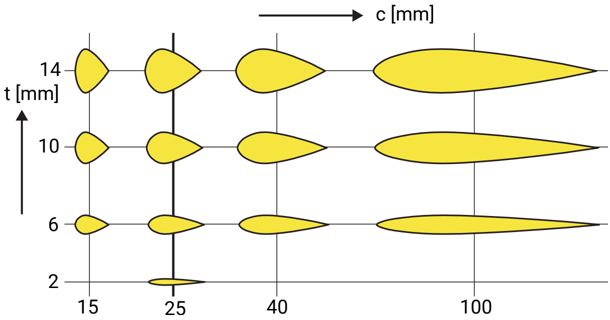


- Gap, d
- Collector chord, c
- Collector thickness, t
- Inter-collector spacing, S
- Airfoil span, b

Goal

Create a model for the performance of the thruster and optimize the performance

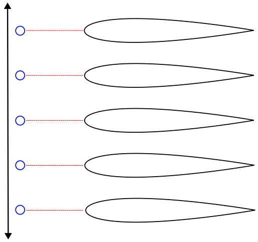
Research Parameters, Airfoil Shape



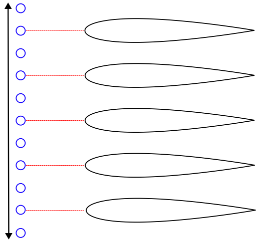
Airfoil Nomenclature

Each airfoil is called using CxxTyy where xx indicates the chord in mm and yy the thickness in mm

Research Parameters, Supplementary Tests



Single Emitter Configuration



Double Emitter Configuration

Supplementary Tests

The C25 airfoils were also tested at variable spacings and the C25T6 airfoil at different gaps.

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Governing Equations

$$
\begin{cases}
\nabla \cdot \mathbf{E} = \rho_q / \epsilon_0 \\
\nabla \cdot \rho_q \left(\mu_q \mathbf{E} + \mathbf{u} \right) = 0 \\
\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u} + \rho_q \mathbf{E} \\
\mathbf{E}(r_E) = E_i \ \mathbf{n}
\end{cases}
$$

Equations

1. Maxwell's law
2. Charge Conservation
3. Momentum conservation

\+ Peek's boundary condition

Scaling Variables

- Reference Length, d
- Reference Potential, V_a

Performance Indicators

Thrust and Power coefficients

$$
\begin{cases}
\dfrac{T_e}{b} = \displaystyle\int_\Omega \rho_q E_x \, \mathrm{d}x - \frac{1}{2}\rho u_0^2 c \; C_D \;\rightarrow\; \epsilon_0 \frac{V_a^2}{d} \; C_{Te} \\
\dfrac{P}{b} = \displaystyle\int_\Omega \mathbf{j} \cdot \mathbf{E} \, \mathrm{d}\Omega \;\rightarrow\; \mu_q \epsilon_0 \frac{V_a^3}{d^2} \; C_P
\end{cases}
$$

Derived indicators

Name	Symbol	Reference Value	Coefficient
Thrust to Power Ratio	$\dfrac{T_e}{P}$	$\dfrac{\mu_q d}{V_a}$	$\dfrac{C_{Te}}{C_P}$
Surface Thrust Density	$\dfrac{T_e}{A}$	$\epsilon_0 \dfrac{V_a^2}{d^2}$	$\dfrac{d}{S} C_{Te}$

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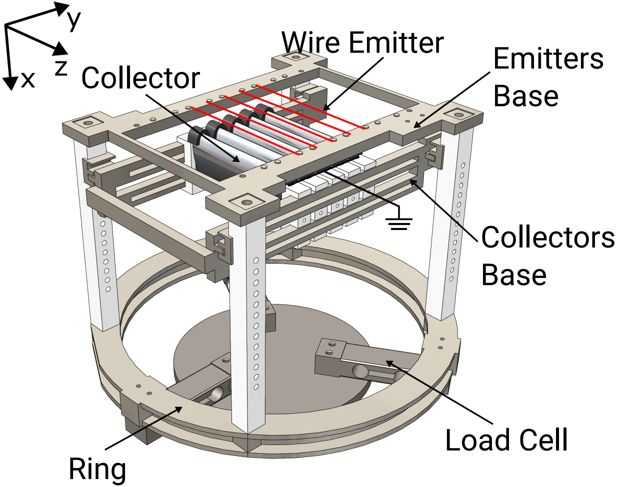
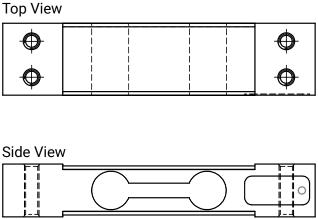
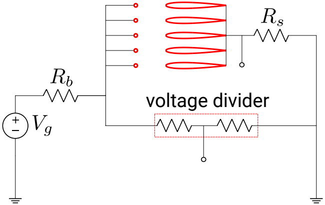
Load Measurement Setup



Measurement Chain





HBM PW6 Specifications

- 0.75 kg full scale
- 0.1 gram precision
- Output 0-10 V for 0-1 kg

Circuit Specifications

- $Rb = 0.996\text{M}\Omega$
- $Rs = 1.16\text{k}\Omega$
- voltage divider gain of 1/1000

Multiple 50 kHz 2 s acquisitions at 11 bit per point

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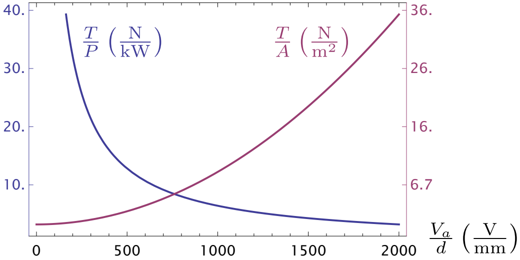
Scaling Variables, Theory

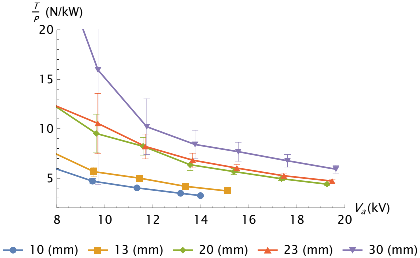
$$\begin{cases} \dfrac{T_e}{P} = \dfrac{\mu_q d}{V_a} \; C_{TPe} \\ \dfrac{T_e}{A} = \epsilon_0 \dfrac{V_a^2}{d^2} \; C_{TAe} \end{cases}$$

Focusing on the reference values only

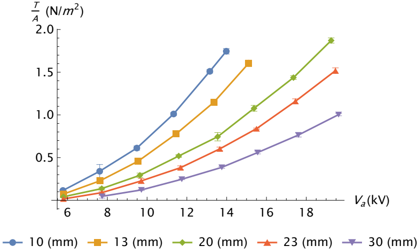


Scaling Variables, Experiments

Tests at $S = 35$ mm for the C25T6 airfoil



Effective Thrust to Power



Effective Thrust Density

Hypothesis

Negligible convection, 1D geometry

1D Equations and boundary conditions

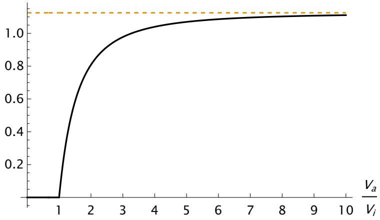
$$\begin{cases} \dfrac{\partial \hat{E}}{\partial \hat{x}} = \hat{\rho}_q \\ \dfrac{\partial}{\partial \hat{x}} \; \hat{\rho}_q \hat{E} = 0 \end{cases} + \begin{cases} \hat{E}(0) = \hat{V} \\ \int_0^1 \hat{E} \; \mathrm{d}\hat{x} = 1 \end{cases}$$

Series solution for $\hat{V} = V_a / V_i$

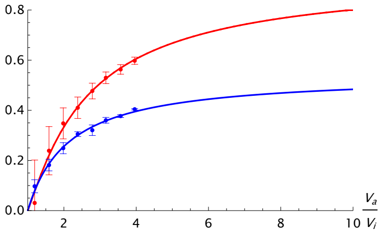
$$C_T = C_P = \begin{cases} 2 \; (\hat{V} - 1) + o[\; (\hat{V} - 1)^2] \;, \; \hat{V} \to 1 \\ \dfrac{9}{8} - \dfrac{3}{2} \hat{V}^{-2} + o(\hat{V}^{-3}) \;, \; \hat{V} \to \infty \end{cases}$$

1D Theory

$$\text{Model} \rightarrow f(\hat{V}) = (c_1 + c_2) - \frac{c_1}{\hat{V}} - \frac{c_2}{\hat{V}^2}$$



Theory



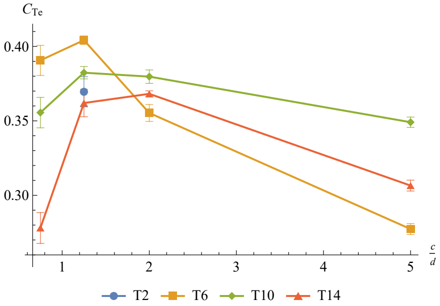
Experiment

Data for the C25T6 airfoil at $S = 35$ mm and $d = 20$ mm. C_{Te} in blue and C_P in red

Effective Thrust Coefficient

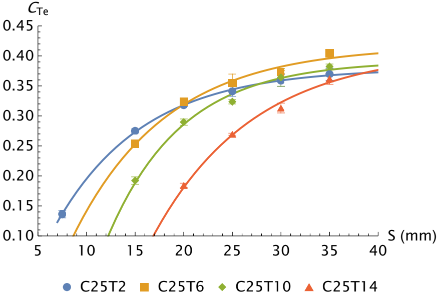
$$C_{Te} = C_T - \frac{1}{2}\frac{c}{d}C_D$$

Tests at $S = 35$ mm, $d = 20$ mm, $V = 20$ kV.



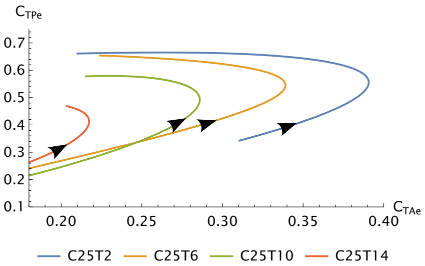
$$\text{Model} \rightarrow f\left(\frac{S}{d}\right) = k_1 \left[1 - \exp\left(-k_2\left(\frac{S}{d} - k_3\right)\right)\right]$$

Data

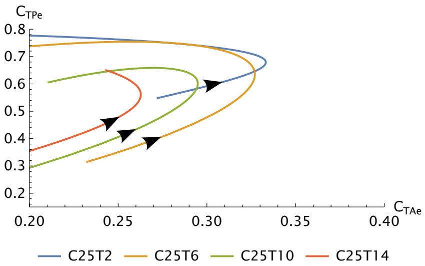


Tests at $d = 20$ mm, $V = 20$ kV for the C25 airfoils.

SE and DE configurations



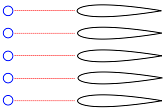
Single Emitter



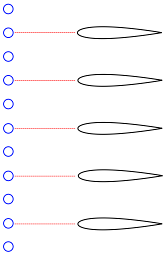
Double Emitter

Arrows indicate the direction of increasing spacing between units

Optimized Configurations



S10SE, small spacing



S40DE, large spacing

Configuration	C_{TAe}	T/A	C_{TP}	T/P	P/A
S10SE	0.39	4.42 N/m^2	0.55	3.5 N/kW	1.25 kW/m^2
S40DE	0.22	1.94 N/m^2	0.79	5.06 N/kW	0.38 kW/m^2

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Conclusions

Achievements

1. A scaling model
2. Detailed study of the effects of the dimensionless coefficients
3. Two optimized configurations

Further Developments

Ideas

1. Increase the temperature of the emitter
2. Effect of a mean flow
3. Design of an airship powered by EHD thrusters

Thank you for your attention

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Dimensionless Equations

$$\begin{cases} \hat{\nabla} \cdot \hat{\mathbf{E}} = \hat{\rho_q} \\ \hat{\nabla} \cdot \hat{\rho_q} \left(\hat{\mathbf{E}} + R_v \hat{\mathbf{u}} \right) = 0 \\ \hat{\mathbf{u}} \cdot \hat{\nabla} \hat{\mathbf{u}} = -\hat{\nabla} \hat{P} + \hat{\rho}_q \hat{\mathbf{E}} + \dfrac{1}{Re_d} \hat{\nabla}^2 \hat{\mathbf{u}} \\ \hat{\mathbf{E}}(\hat{r}_E) = \hat{V} \mathbf{n} \end{cases}$$

Reference Varibles

1. Charge density: $\rho_0 = \epsilon_0 \frac{V_a}{d^2}$
2. Velocity: $u_0 = \frac{V_a}{d} \sqrt{\frac{\epsilon_0}{\rho}}$
3. Pressure: $P_0 = \epsilon_0 \frac{V_a^2}{d^2}$

Dimensionless Numbers

$$Re_d = \frac{V_a}{\nu} \sqrt{\frac{\epsilon_0}{\rho}}, \text{ Gap Reynolds Number}$$

$$R_v = \frac{1}{\mu_q} \sqrt{\frac{\epsilon_0}{\rho}}, \text{ Ratio between mean flow and ion velocity}$$

$$\hat{V} = \frac{V_a}{V_i}, \text{ Dimensionless voltage}$$