Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1341x – The Midterm Test II (v.1) **Instructor: C. Rada**

Last name: ______________________________

First name: ______________________________

Student number:

Please, read the following instructions carefully:

- You have 80 minutes to complete this test. 1 file, 1 submission ONLY. TRY PDF! ONLY in Assignments, only in Brightspace. WE do NOT mark papers submitted elsewhere! WE do not reply to emails containing submissions! Faculty of science approved Calculators: basic, no graphing capabilities, no integral, derivative button. After 80 min stop writing, and you have 10 minutes to scan, create one file, and upload your file back into Assignments. SHOW ALL WORK for questions: 6,7. Marks are indicated for each exercise! Read that!
- This is a closed book exam, and no notes of any kind are allowed. The use of programmable calculators, cell phones, laptops, pagers or any text storage or communication device is not permitted. MUST be in zoom, muted, camera ON. NOT in zoom, no marks.

THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	5	6	7	Total
Mark								
Out of	2	1	1	1	2	7	6	20

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1. Let $U = \{(x, y, z) \in \mathbf{R}^3 \mid xy = 0\}$. Then (2)

cross (X) the correct answer

- $\boxed{\text{A}}$ $(0, 0, 0) \in U$ but U is closed under addition
- $\boxed{\text{B}}$ $(0, 0, 0) \in U$ but U is not closed under multiplication by scalars
- $\boxed{\text{C}}$ U is closed under addition and U is closed under multiplication by scalars.
- $\boxed{\text{D}}$ U is closed under addition but U is not closed under multiplication by scalars
- $\boxed{\text{E}}$ U is not closed under addition but U is closed under multiplication by scalars
- $\boxed{\text{F}}$ None of the above is true

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2. Which of the following are subspaces of $\mathbb{R}^3$? (1)

$$U = \{(x, y, z) \in \mathbb{R}^3 \mid 2x - 3y + z = 0\}$$

$$V = \{(x, y, z) \in \mathbb{R}^3 \mid (x - z)(x - y) = 0\}$$

$$W = \{(x + y, 2y, x - y) \in \mathbb{R}^3 \mid x, y \in \mathbb{R}\}$$

$$X = \{(x, xy, y) \in \mathbb{R}^3 \mid x, y \in \mathbb{R}\}$$

cross (X) the correct answer

- $\boxed{\text{A}}$ Only U and V
- $\boxed{\text{B}}$ Only U and X
- $\boxed{\text{C}}$ Only U and W
- $\boxed{\text{D}}$ Only V and W
- $\boxed{\text{E}}$ Only V and X
- $\boxed{\text{F}}$ Only W and X

3. Which of the following are subspaces of the vector space $M_{2\times 2}(\mathbf{R})$? (1)

cross (X) the correct answer

A $\left\{ \begin{pmatrix} a & 2 \\ b & b \end{pmatrix} \mid a, b \in \mathbf{R} \right\}$

B $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ac = 2,\ a, b, c, d \in \mathbf{R} \right\}$

C $\left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \text{ are integers} \right\}$

D $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ab = 0,\ a, b, c, d \in \mathbf{R} \right\}$

E $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid c + d = 0,\ a, b, c, d \in \mathbf{R} \right\}$

F None of the above

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4. Let U be a subspace of $\mathbf{R}^{10}$. Suppose U has 7 linearly independent vectors and at the same time it is spanned by 9 vectors. Then for **any** such U it is true that (1)

cross (X) the correct answer

A $\dim U < 7$

B $\dim U > 7$

C $\dim U < 9$

D $\dim U > 9$

E $\dim U \leq 9$

F $\dim U \leq 7$

5. Let u, v, w be non-zero vectors in a vector space V. Suppose that u, v, w are linearly dependent (coplanar) but at the same time u and v are linearly independent (non-colinear). Then for **any** such u, v and w the following is true (2)

cross (X) the correct answer

- A u and w are colinear
- B v and w are colinear
- C $v \in Span\{u, w\}$
- D $w \in Span\{u, v\}$
- E $u \in Span\{v, w\}$
- F None of the above

6. Consider the vector space $\mathbf{P}_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbf{R}\}$ of polynomial functions of degree at most 2. Consider the subset U of $\mathbf{P}_2$ defined by

$$U = \{f \in \mathbf{P}_2 \mid f(-1) = 0\}.$$

(the subset U consists of all polynomials of degree at most 2 which have root -1)

a) Show that U is a subspace using the **Subspace Test** (2)

b) Show that $U = Span\{1 + x, x + x^2\}$ (2)

c) Find a basis for U and explain why this is a basis

ANSWER (the basis is): $\boxed{}$ (1)

Justification: (2)

7. State whether each of the following statements is always true **(T)**, or is possibly false **(F)**, in the box after the statement.

- If you say the statement may be false, you must either give an explicit example where it fails or explain clearly why it fails.
- If you say the statement is always true, you must give a clear explanation.

a) If V is a vector space and $\{v_1, v_2, v_3, v_4\} \subset V$ is linearly dependent, then $\{v_1, v_2, v_3\} \subset V$ is also linearly dependent.

Answer (T/F): $\boxed{}$ (1/2)

Justification: (1)

b) Let v_1, v_2, v_3, v_4 be non-zero vectors in a vector space V and let $U = Span\{v_1, v_2, v_3, v_4\}$. Then $\dim U = 4$.

Answer (T/F): $\boxed{}$ (1/2)

Justification: (1)

c) If U and W are subspaces of $\mathbf{R}^3$, then their intersection

$$U \cap W = \{v \in \mathbf{R}^3 \mid v \in U \text{ and } v \in W\}$$

is closed under scalar multiplication

Answer (T/F): $\boxed{}$ (1/2)

Justification: (1)

d) Let $U = Span\{x^2 \sin^2(x), x^2 \cos^2(x), x^2\}$ be a subspace of the vector space of all functions $F(\mathbb{R})$. Then $\dim U = 3$.

Answer (T/F): $\boxed{}$ (1/2)

Justification: (1)

The last page (use it for computations)